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A Heuristic Parameter-Dependent Open-Loop Model Predictive Control

XIN ZAN[1], LU JIN[2], AND JUN WANG[2], (Member, IEEE)
[1]Faculty of Electrical and Information Engineering, School of Automation Science and Engineering, Xi'an Jiaotong University, Xi'an 710049, China
[2]College of Automation, Chongqing University of Posts and Telecommunications, Chongqing 400065, China

Corresponding author: Jun Wang (hellowangjun@yeah.net)

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ABSTRACT For the uncertain linear systems described by the linear parameter varying (LPV) model, a parameter-dependent open-loop model predictive control (MPC) is proposed. The controller applies a tree trajectory to generate the vertices of uncertainty state predictions. Based on the state prediction tree, the future free control moves are parameter-dependent, whose vertices correspond to those of state predictions. The cost function penalizes the deviations of all the vertices of state/input from their steady-state target values. It is shown that the offset-free property is achieved by this method. A simulation example is given to demonstrate effectiveness of the approach.

INDEX TERMS Model predictive control, linear parameter varying model, heuristic algorithm.

I. INTRODUCTION

Model predictive control (MPC) has been widely applied as a representative advanced process control (APC) algorithm in industrial circle since 1978 [1]. Nowadays, MPC expands to both theoretical and industrial fields to obtain insightful results (e.g., referring to [2]–[10] for several good algorithms). The main feature of MPC is its ability to handle physical constraints and multiple variables in a systematic manner (i.e., pose into an receding-horizon optimization problem). At each control interval, MPC optimizes a cost function associated with the future state/output/input predictions based on an explicit model of the system satisfying some physical constraints, then yields a sequence of control moves. However, only the first control move among this sequence is implemented, and the refreshed optimization is performed at the next control interval. Since the future predictions for state/output/input are needed from the system model, the accuracy of this model is crucial for the future prediction, which as a result, can influence the control performance.

An efficient system model should be representative for a wide class of systems. In MPC studies, there is a widely accepted system model called linear parameter varying (LPV) model. LPV model can include the dynamics of both nonlinear and uncertain systems by the utilizing the dynamic vector for a family of linear models [11]. Usually, thanks to the convexity property of the LPV model, applying LPV model in MPC can yield the convex optimization which can be solved very efficiently using the current optimization method such as interior point method. Another advantage for LPV model is the local linearity for each sub-linear model, which allows for the application of powerful linear design tools. MPC for LPV models, with guaranteed recursive feasibility (of optimization problem) and stability, has been referred to as the synthesis approach. Usually, a synthesis approach is designed based the state-or output-feedback laws (see e.g., [12]–[16]). Some free perturbation items have been added to the state-feedback law in order to enlarge regions of attraction [17]. The class of linearly parameter-dependent Lyapunov functions are proposed for MPC of LPV model in [18], which gives rise to less conservative stability conditions than those arising from classical quadratic Lyapunov functions in e.g. [19]. In [20], the authors presented a class of nonlinearly parameterized Lyapunov functions instrumental to the achievement of more efficient relaxed stability conditions. In [21], an efficient algorithm is given which constructs the maximal admissible set for LPVs. Considering the high-speed control for constrained LPV models, some explicit MPCs have been developed [22], [23]. MPC for LPV models with bounded parameter variations have been investigated in [24]–[26]. In [27], the output feedback MPC is proposed for LPV models based on the quasi-min-max algorithm. In [28], the authors considered the robust MPC for LPV models, where the scheduling parameter of LPV model is known online (advantageous for feedback).

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Although there are excellent theoretical results in pursuing the synthesis approaches of MPC, they have not been reported for wide applications in real industrial systems. Instead, the widely applied MPC is the heuristic approach without guaranteeing the stability and recursive feasibility. For practical applications, one of the major drawbacks with synthesis MPC is its high computational burden, as compared with the heuristic one. Another disadvantage of MPC synthesis approach is its conservativeness since a min-max optimization is usually formulated which take into account all possible realization of the system model. Hence, the linear model is typically addressed for heuristic MPC. However, this can be risky for an intrinsic uncertain systems. This paper aims at LPV model, with consideration of state and input constraints, and adopts a parameter-dependent open-loop MPC scheme based on the heuristic MPC. The proposed controller uses a tree trajectory to forecast the vertices of future state predictions, which is inspired by [29]. The optimization problem is properly formulated as a classic quadratic programming (QP) with the cost function involving all vertices of state predictions, input predictions and steady-state targets. By solving this QP, it yields the vertex control moves. By applying the scheme, the computational burden is less than the synthesis MPC and the offset-free control is achieved.

Notation: $\mathbb{R}^n$ is the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ the $m\times n$-dimensional real matrix space. For any matrix A, A^T denotes its transpose. For the variable x, $x(i|k)$ denotes the value at the future time $k+i$, predicted at k. The symbol $\star$ implies that the element can be deduced from the symmetry of the matrix. A variable with $*$ as superscript indicates that it is the optimal solution of the optimization problem. For the column vectors x and y, $[x; y] = [x^T, y^T]^T$. The time-dependence of MPC decision variables is often omitted for simplicity.

II. PROBLEM STATEMENT

Consider the discrete-time LPV model, i.e.,

$$\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k), \\ y(k) &= Cx(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ are measurable state and input, respectively. We assume that $[A(k)|B(k)] \in \Omega$, for all $k \geq 0$, where

$$\Omega = \text{Co}\{[A_1\,|B_1\,],\ [A_2\,|B_2\,],\ \ldots,\ [A_L\,|B_L\,]\}$$

i.e., there exist l time-varying nonnegative combining parameter $\omega_l(k)$, $l \in \{1, \ldots, L\}$ such that

$$\sum_{l=1}^{L}\omega_l(k) = 1,\ \ [A(k)\,|B(k)\,] = \sum_{l=1}^{L}\omega_l(k)\,[A_l\,|B_l\,], \tag{2}$$

where $[A_l\,|B_l\,]$ are known vertices of the polytope.

The input and state constraints are

$$\begin{aligned} u(k) &\in \mathbb{U} = \{u \in \mathbb{R}^m \big| -\underline{u} \leq u \leq \bar{u}\}, \\ x(k+1) &\in \mathbb{X} = \{x \in \mathbb{R}^n \big| -\underline{\psi} \leq \Psi x \leq \bar{\psi}\}, \end{aligned} \tag{3}$$

where $\underline{u} := [\underline{u}_1, \underline{u}_2, \ldots, \underline{u}_m]$, $\bar{u} := [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m]$; $\underline{\psi} := [\underline{\psi}_1, \underline{\psi}_2, \ldots, \underline{\psi}_q]$; $\bar{\psi} := [\bar{\psi}_1, \bar{\psi}_2, \ldots, \bar{\psi}_q]$; $\underline{u}_i > 0$, $\bar{u}_i > 0$, $i = 1\ldots m$; $\underline{\psi}_j > 0$, $\bar{\psi}_j > 0$, $j = 1\ldots q$; $\Psi \in \mathbb{R}^{q\times n}$.

In practice, (1) is short for

$$\nabla x(k+1) = A(k)\nabla x(k) + B(k)\nabla u(k). \tag{4}$$

Namely, (1) neglects the symbol ∇ in (4), where $\nabla x = x - x_{eq}$ and $\nabla u = u - u_{eq}$. x_{eq} and u_{eq} denote the steady-state operating points (equilibrium) of the system.

The system is said to be at steady state at time T if

$$\begin{aligned} x_{ss} &= x(k) = x(k+1), \quad \forall\, k \geq T \\ u_{ss} &= u(k) = u(k+1), \quad \forall\, k \geq T \\ y_{ss} &= y(k) = y(k+1), \quad \forall\, k \geq T \end{aligned} \tag{5}$$

where y_{ss}, u_{ss} and x_{ss} are the steady-state targets (setpoints) of y, u and x, respectively. Since there is no uncertainty in matrix C, we have, at the steady state,

$$y_{ss} = Cx_{ss} \tag{6}$$

III. THE OPEN-LOOP CONTROL PROBLEM

In this section, in order to effectively counteract the time-varying uncertainty, an effective method is designed which calculates the vertex control moves for all corners of the uncertainty evolution.

Define the vertex control moves

$$\pi_u = \left\{u(0|k), u^{l_0}(1|k), \ldots, u^{l_{N-2}\ldots l_0}(N-1|k)\right\}, \quad l_j = 1\ldots L,\ j = 0\ldots N-2,$$

where N is the control horizon. Note that the future control moves are based on the vertices of the uncertain polytope. With the increase of N, the number of vertices are increased dramatically. Then, the corresponding vertex state predictions are

$$\begin{aligned} x^{l_0}(1|k) &= A_{l_0}x(k) + B_{l_0}u(0|k), \\ x^{l_i\ldots l_0}(i+1|k) &= A_{l_i}x^{l_{i-1}\ldots l_0}(i|k) + B_{l_i}u^{l_{i-1}\ldots l_0}(i|k), \\ & i=1\ldots N-1,\quad l_j=1\ldots L,\ j=0\ldots N-1, \end{aligned}$$

where $\pi_x = \left\{x^{l_0}(1|k),\ x^{l_1l_0}(2|k), \ldots, x^{l_{N-1}\ldots l_1l_0}(N|k)\right\}$ are vertex state predictions.

The true control move $u(i|k)$ for $i > 0$ is defined by

$$\begin{aligned} u(i|k) &= \sum_{l_0\ldots l_{i-1}=1}^{L}\left(\left(\prod_{h=0}^{i-1}\omega_{l_h}(k+h)\right)u^{l_{i-1}\ldots l_0}(i|k)\right), \\ &\sum_{l_0\ldots l_{i-1}=1}^{L}\left(\prod_{h=0}^{i-1}\omega_{l_h}(k+h)\right) = 1, \\ &i = 1\ldots N-1 \end{aligned} \tag{7}$$

where $u(i|k)$ is parameter-dependent, i.e., each $u(i|k)$ is a convex combination through the parameters $\omega_{l_h}(k+h)$. According to (1) and (7), the future state predictions are found as

$$x(1|k) = A(k)x(0|k) + B(k)u(0|k)$$

$$
\begin{aligned}
&= \sum_{l_0=1}^{L} \omega_{l_0}(k)\left[A_{l_0}x(k) + B_{l_0}u(0|k)\right] \\
&= \sum_{l_0=1}^{L} \omega_{l_0}(k)x^{l_0}(1|k) \\
x(2|k) &= A(k+1)x(1|k) + B(k+1)u(1|k) \\
&= \sum_{l_1=1}^{L} \omega_{l_1}(k+1)\Big[A_{l_1}x(1|k) + B_{l_1}u(1|k)\Big] \\
&= \sum_{l_1=1}^{L} \omega_{l_1}(k+1)\Big[A_{l_1}\sum_{l_0=1}^{L} \omega_{l_0}(k)x^{l_0}(1|k) \\
&\quad + B_{l_1}\sum_{l_0=1}^{L} \omega_{l_0}(k)u^{l_0}(1|k)\Big] \\
&= \sum_{l_1=1}^{L}\sum_{l_0=1}^{L} \omega_{l_1}(k+1)\omega_{l_0}(k)x^{l_1 l_0}(2|k) \\
&\;\;\vdots
\end{aligned}
$$

Apparently, $x(i|k)$ belongs to polytope, i.e., (8)–(10), as shown at the bottom of the page.

Note that since the parameters $w(k)$ is completely unknown, so the accurate value of state is unknown. Hence, we utilize the vertex state predictions for MPC controller design, where the accurate value of state is allowed to vary in the polytope.

We define the following positive definite quadratic function with respect to vertices π_x and π_u as (10). In (10), $\mathscr{Q}_{1,l_0}, \mathscr{R}_0, \mathscr{Q}_{2,l_1,l_0}, \mathscr{R}_{1,l_0} \ldots \mathscr{Q}_{N,l_{N-1}\ldots l_1 l_0}, \mathscr{R}_{N-1,l_{N-2}\ldots l_1 l_0}$ are nonnegative weighing matrices. It is unnecessary that the steady-state targets are always equal to the equilibrium.

The objective of the control problem is to find the control actions that, once implemented, drive all branches (vertices) in the tree trajectory to converge to x_{ss} and u_{ss}. Accordingly, let vertices π_x and π_u be the decision variables. The optimization problem at each control interval k is formulated as a QP problem

$$
\begin{aligned}
&\min_{\pi_u,\pi_x} \hat{J}_0^N(k), \\
&\text{s.t. (9), (11b), (11c)} \qquad (11a)\\
&\quad -\underline{u} \le u(0|k) \le \bar{u}, \quad -\underline{u} \le u^{l_{i-1}\ldots l_1 l_0}(i|k) \le \bar{u}, \\
&\quad i = 1\ldots N-1, \; l_{i-1} = 1\ldots L \qquad (11b)\\
&\quad -\begin{bmatrix} \underline{\psi} \\ \underline{\psi} \\ \vdots \\ \underline{\psi} \end{bmatrix} \le \tilde{\Psi} \begin{bmatrix} x^{l_0}(1|k) \\ x^{l_1 l_0}(2|k) \\ \vdots \\ x^{l_{N-1}\cdots l_1 l_0}(N|k) \end{bmatrix} \le \begin{bmatrix} \bar{\psi} \\ \bar{\psi} \\ \vdots \\ \bar{\psi} \end{bmatrix}. \\
&\quad l_j = 1\ldots L, \quad j = 0\ldots N-1 \qquad (11c)
\end{aligned}
$$

After the optimization problem is solved, only $u(0|k)$ is implemented on the plant. Each diagonal block of $\tilde{\Psi}$ is Ψ. The approach based on the optimization problem (11) is called the open-loop model predictive heuristic control (MPHC).

$$
\begin{bmatrix} x(1|k) \\ x(2|k) \\ \vdots \\ x(N|k) \end{bmatrix} = \sum_{l_0\ldots l_{N-1}=1}^{L} \left(\prod_{h=0}^{N-1} \omega_{l_h}(k+h) \begin{bmatrix} x^{l_0}(1|k) \\ x^{l_1 l_0}(2|k) \\ \vdots \\ x^{l_{N-1}\cdots l_1 l_0}(N|k) \end{bmatrix} \right), \quad \sum_{l_0\ldots l_{i-1}=1}^{L} \left(\prod_{h=0}^{i-1} \omega_{l_h}(k+h) \right) = 1, \; i = 1\ldots N, \tag{8}
$$

$$
\begin{aligned}
\begin{bmatrix} x^{l_0}(1|k) \\ x^{l_1 l_0}(2|k) \\ \vdots \\ x^{l_{N-1}\cdots l_1 l_0}(N|k) \end{bmatrix} &= \begin{bmatrix} A_{l_0} \\ A_{l_1}A_{l_0} \\ \vdots \\ \prod_{i=0}^{N-1} A_{l_{N-1-i}} \end{bmatrix} x(k) \\
&\quad + \begin{bmatrix} B_{l_0} & 0 & \cdots & 0 \\ A_{l_1}B_{l_0} & B_{l_1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \prod_{i=0}^{N-2} A_{l_{N-1-i}}B_{l_0} & \prod_{i=0}^{N-3} A_{l_{N-1-i}}B_{l_1} & \cdots & B_{l_{N-1}} \end{bmatrix} \begin{bmatrix} u(0|k) \\ u^{l_0}(1|k) \\ \vdots \\ u^{l_{N-2}\cdots l_1 l_0}(N-1|k) \end{bmatrix},
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
\hat{J}_0^N(k) &= \sum_{l_0=1}^{L} \|Cx^{l_0}(1|k) - y_{ss}\|^2_{\mathscr{Q}_{1,l_0}} + \|u(0|k) - u_{ss}\|^2_{\mathscr{R}_0} + \sum_{l_1=1}^{L}\sum_{l_0=1}^{L} \|Cx^{l_1 l_0}(2|k) - y_{ss}\|^2_{\mathscr{Q}_{2,l_1,l_0}} \\
&\quad + \sum_{l_0=1}^{L} \|u^{l_0}(1|k) - u_{ss}\|^2_{\mathscr{R}_{1,l_0}} + \cdots + \sum_{l_{N-1}=1}^{L} \ldots \sum_{l_1=1}^{L}\sum_{l_0=1}^{L} \|Cx^{l_{N-1}\ldots l_1 l_0}(N|k) - y_{ss}\|^2_{\mathscr{Q}_{N,l_{N-1}\ldots l_1 l_0}} \\
&\quad + \sum_{l_{N-2}=1}^{L} \ldots \sum_{l_1=1}^{L}\sum_{l_0=1}^{L} \|u^{l_{N-2}\ldots l_1 l_0}(N-1|k) - u_{ss}\|^2_{\mathscr{R}_{N-1,l_{N-2}\ldots l_1 l_0}}
\end{aligned} \tag{10}
$$

Remark 1: Vertex control moves, vertex state predictions and the cost function as $\hat{J}_0^N(k)$ are found in [29]. This open-loop MPHC has some features, i.e., i) the computation load is less than the synthesis MPC; ii) stability of the system cannot be proved theoretically; iii) suppose that weighing matrices $\mathscr{Q}_\bullet$ and $\mathscr{R}_\bullet$ are positive definite, when $y_{ss} \neq 0$ and $u_{ss} \neq 0$, it is not easy to achieve $\hat{J}_0^N(\infty) = 0$, i.e., there may exist the offset. This is because usually, $Cx^{l_{i-1}\ldots l_1 l_0}(N-1|k) = y_{ss} \neq 0$ cannot hold for all $i = 1, 2, \ldots, N$ and $l_{i-1}\ldots l_1 l_0$ even if the closed-loop system is stable.

IV. AN IMPROVED OPEN-LOOP MPHC

Since we cannot give deterministic $[A(k)|B(k)]$, usually, y_{ss} and u_{ss} do not always satisfy the following equations:

$$\begin{cases} x_{ss} = A_{ss}x_{ss} + B_{ss}u_{ss} \\ y_{ss} = Cx_{ss} \end{cases} \tag{12}$$

However, there is an exception, i.e., when $k \to \infty$, $[A(k)|B(k)]$ can converge to the fixed $[A_{ss}|B_{ss}]$. In this case, y_{ss} and u_{ss} in the optimization problem (11a) must satisfy (12).

In general, in order to achieve offset-free control, we can assume that x_{ss} and u_{ss} satisfy the steady-state nonlinear equation

$$g(x_{ss} + x_{eq}, u_{ss} + u_{eq}) = 0, \tag{13}$$

where $g(\cdot)$ is assumed to be Lipschitz continuous and differentiable with respect to x and u in $\mathbb{X} \times \mathbb{U}$, with $g(0, 0) = 0$. When we obtain x_{ss} and u_{ss} from (13), we calculate (14), as shown at the bottom of the page.

Then, the cost function $\hat{J}_0^N(k)$ is replaced by $\tilde{J}_0^N(k)$, as shown at the bottom of the page.

The optimization problem (11) is updated to

$$\min_{\pi_u, \pi_x} \tilde{J}_0^N(k), \tag{15a}$$
$$\text{s.t. } (9),\ (11b),\ (11c) \tag{15b}$$

Remark 2: In [29], if $\hat{J}_0^N(k)$ is utilized, the offset-free control is achieved because x_{ss} and u_{ss} are obtained by a special procedure. Otherwise the proof of Theorem 5.2 in [29] must take advantage of the cost function $\tilde{J}_0^N(k)$ in this paper.

The open-loop MPHC algorithm is summarized as follows.

Algorithm 1.

Off-line Stage:

i) Find appropriate weighing matrices $\mathscr{Q}_{1,l_0}, \mathscr{Q}_{2,l_1,l_0}, \ldots, \mathscr{Q}_{N,l_{N-1}\ldots l_1 l_0}, \mathscr{R}_0, \mathscr{R}_{1,l_0}, \ldots, \mathscr{R}_{N-1,l_{N-2}\ldots l_1 l_0}$.
ii) Find the steady-state equations $g(\cdot)$ according to a given nonlinear system. Calculate the steady-state x_{ss} and u_{ss} in (13).
iii) Calculate $x_{ss}^{l_0}, x_{ss}^{l_1 l_0}, \ldots, x_{ss}^{l_{N-1}\ldots l_1 l_0}, u_{ss}^{l_0}, u_{ss}^{l_1 l_0}, \ldots, u_{ss}^{l_{N-1}\ldots l_1 l_0}$ according to (14).

On-line Stage:

i) Solve optimization problem (15) and obtain optimal control sequence π_u^* and predictive state sequence π_x^*.
ii) Implement $u^*(0|k)$ into the plant.
iii) Set $k = k + 1$, go to i).

V. NUMERICAL EXAMPLE

A nonlinear model of a continuous stirred tank reactor (CSTR) is adopted in this simulation (see Figure 1). With constant volume, the CSTR for an exothermic, irreversible reaction A → B is described by

$$\begin{aligned} \dot{C}_{\mathrm{A}}(t) &= \frac{\mathrm{q}}{\mathrm{V}}(C_{\mathrm{Af}} - C_{\mathrm{A}}(t)) - k_0 \exp\left(-\frac{E/R}{T(t)}\right) C_{\mathrm{A}}(t), \\ \dot{T}(t) &= \frac{\mathrm{q}}{\mathrm{V}}(T_{\mathrm{f}} - T(t)) + \frac{(-\Delta H)}{\rho C_{\mathrm{p}}} k_0 \exp\left(-\frac{E/R}{T(t)}\right) C_{\mathrm{A}}(t) \\ &\quad + \frac{\mathrm{UA}}{\mathrm{V}\rho C_{\mathrm{p}}}(T_{\mathrm{c}}(t) - T(t)), \end{aligned} \tag{16}$$

where C_{A} is the concentration of material A in the reactor, T the reactor temperature, T_{c} the coolant stream temperature. V and UA denote the volume of the reactor and the rate of

$$\begin{bmatrix} x_{ss}^{l_0} \\ x_{ss}^{l_1 l_0} \\ \vdots \\ x_{ss}^{l_{N-1}\cdots l_1 l_0} \end{bmatrix} = \begin{bmatrix} A_{l_0} \\ A_{l_1}A_{l_0} \\ \vdots \\ \prod_{i=0}^{N-1} A_{l_{N-1-i}} \end{bmatrix} x_{ss} + \begin{bmatrix} B_{l_0} & 0 & \cdots & 0 \\ A_{l_1}B_{l_0} & B_{l_1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \prod_{i=0}^{N-2} A_{l_{N-1-i}}B_{l_0} & \prod_{i=0}^{N-3} A_{l_{N-1-i}}B_{l_1} & \cdots & B_{l_{N-1}} \end{bmatrix} \begin{bmatrix} u_{ss} \\ u_{ss} \\ \vdots \\ u_{ss} \end{bmatrix}. \tag{14}$$

$$\begin{aligned} \tilde{J}_0^N(k) = &\sum_{l_0=1}^{L} \|x^{l_0}(1|k) - x_{ss}^{l_0}\|^2_{\mathscr{Q}_{1,l_0}} + \|u(0|k) - u_{ss}\|^2_{\mathscr{R}_0} + \sum_{l_1=1}^{L}\sum_{l_0=1}^{L} \|x^{l_1 l_0}(2|k) - x_{ss}^{l_1 l_0}\|^2_{\mathscr{Q}_{2,l_1,l_0}} \\ &+ \sum_{l_0=1}^{L} \|u^{l_0}(1|k) - u_{ss}\|^2_{\mathscr{R}_{1,l_0}} + \cdots + \sum_{l_{N-1}=1}^{L} \cdots \sum_{l_1=1}^{L}\sum_{l_0=1}^{L} \|x^{l_{N-1}\ldots l_1 l_0}(N|k) - x_{ss}^{l_{N-1}\ldots l_1 l_0}\|^2_{\mathscr{Q}_{N,l_{N-1}\ldots l_1 l_0}} \\ &+ \sum_{l_{N-2}=1}^{L} \cdots \sum_{l_1=1}^{L}\sum_{l_0=1}^{L} \|u^{l_{N-2}\ldots l_1 l_0}(N-1|k) - u_{ss}\|^2_{\mathscr{R}_{N-1,l_{N-2}\ldots l_1 l_0}} \end{aligned}$$

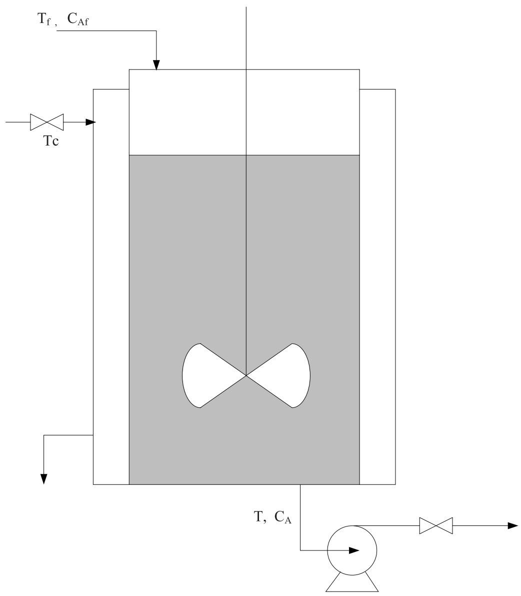

FIGURE 1. Continuous stirred-tank reactorsR.

heat input, respectively. k_0, E, and ΔH denote the preexponential constant, the activation energy, and the enthalpy of the reaction, respectively. C_p and ρ stand for the heat capacity and density of the fluid in the reactor, respectively. The objective is to regulate T by manipulating T_c satisfying $328\ \mathrm{K} \leq T_\mathrm{c} \leq 348\ \mathrm{K}$.

Denote the non-zero equilibrium as $\{C_\mathrm{A}^\mathrm{eq}, T^\mathrm{eq}, T_\mathrm{c}^\mathrm{eq}\}$. Choose $C_\mathrm{A}^\mathrm{eq} = 0.5$ mol/l, $T^\mathrm{eq} = 350$ K, $T_\mathrm{c}^\mathrm{eq} = 338$ K, $340\ \mathrm{K} \leq T \leq 360\ \mathrm{K}$, $0 \leq C_A \leq 1$ mol/l, q $= 100$ l/min, $C_\mathrm{Af} = 0.9$ mol/l, $T_\mathrm{f} = 350$ K, V $= 100$ l, $\rho = 1000$ g/l, $C_\mathrm{p} = 0.239$ J/(g K), $\Delta H = -2.5 \times 10^4$ J/mol, $E/R = 8750$ K, $k_0 = 3.456 \times 10^{10}\ \mathrm{min}^{-1}$, UA $= 5 \times 10^4$ J/(min K).

Define $x = \left[C_\mathrm{A} - C_\mathrm{A}^\mathrm{eq},\ T - T^\mathrm{eq}\right]^T$, $u = T_\mathrm{c} - T_\mathrm{c}^\mathrm{eq}$. Denote the bounds on u and x as $\underline{u} \leq u \leq \bar{u}$ $(-10 \leq u \leq 10)$, $\underline{x}_1 \leq x_1 \leq \bar{x}_1$ $(-0.5 \leq x_1 \leq 0.5)$, and $\underline{x}_2 \leq x_2 \leq \bar{x}_2$ $(-10 \leq x_2 \leq 10)$.

Define

$$\varphi_1(x_2) = k_0 \exp\left(-\frac{E/R}{x_2 + T^\mathrm{eq}}\right),$$

$$\varphi_2(x_2) = k_0 \left[\exp\left(-\frac{E/R}{x_2 + T^\mathrm{eq}}\right) - \exp\left(-\frac{E/R}{T^\mathrm{eq}}\right)\right] C_\mathrm{A}^\mathrm{eq} \frac{1}{x_2},$$

$$\varphi_1^0 = [\varphi_1(\underline{x}_2) + \varphi_1(\bar{x}_2)]/2,$$

$$\varphi_2^0 = [\varphi_2(\underline{x}_2) + \varphi_2(\bar{x}_2)]/2, \quad g_1(x_2) = \varphi_1(x_2) - \varphi_1^0,$$

$$g_2(x_2) = \varphi_2(x_2) - \varphi_2^0,$$

$$h_1 = \frac{1}{2}\frac{g_1(x_2) - g_1(\underline{x}_2)}{g_1(\bar{x}_2) - g_1(\underline{x}_2)}, \quad h_2 = \frac{1}{2}\frac{g_1(\bar{x}_2) - g_1(x_2)}{g_1(\bar{x}_2) - g_1(\underline{x}_2)},$$

$$h_3 = \frac{1}{2}\frac{g_2(x_2) - g_2(\underline{x}_2)}{g_2(\bar{x}_2) - g_2(\underline{x}_2)}, \quad h_4 = \frac{1}{2}\frac{g_2(\bar{x}_2) - g_2(x_2)}{g_2(\bar{x}_2) - g_2(\underline{x}_2)}.$$

Then (16) can be exactly represented by

$$x(t) = \tilde{A}(t)x(t) + \tilde{B}(t)u(t),$$
$$\tilde{A}(t) = \sum_{l=1}^{4} \omega_l(t)\tilde{A}_l, \quad \tilde{B}(t) = \sum_{l=1}^{4} \omega_l(t)\tilde{B}_l, \tag{17}$$

where $\tilde{A}_1$–$\tilde{A}_4$ and $\tilde{B}_1$, as shown at the bottom of the page.

By discretizing the continuous system (17) with sampling period $T_s = 0.05$min, we obtain the discrete-time LPV model, i.e.,

$$A_1 = \begin{bmatrix} 0.8837 & -0.0011 \\ 6.9390 & 0.9645 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.9568 & -0.0011 \\ -0.70853 & 0.96459 \end{bmatrix},$$

$$\tilde{A}_1 = \begin{bmatrix} -\frac{\mathrm{q}}{\mathrm{V}} - \varphi_1^0 - 2g_1(\bar{x}_2) & -\varphi_2^0 \\ \frac{(-\Delta H)}{\rho C_\mathrm{p}}\varphi_1^0 + 2\frac{(-\Delta H)}{\rho C_\mathrm{p}} g_1(\bar{x}_2) & -\frac{\mathrm{q}}{\mathrm{V}} - \frac{\mathrm{UA}}{\mathrm{V}\rho C_\mathrm{p}} + \frac{(-\Delta H)}{\rho C_\mathrm{p}}\varphi_2^0 \end{bmatrix},$$

$$\tilde{A}_2 = \begin{bmatrix} -\frac{\mathrm{q}}{\mathrm{V}} - \varphi_1^0 - 2g_1(\underline{x}_2) & -\varphi_2^0 \\ \frac{(-\Delta H)}{\rho C_\mathrm{p}}\varphi_1^0 + 2\frac{(-\Delta H)}{\rho C_\mathrm{p}} g_1(\underline{x}_2) & -\frac{\mathrm{q}}{\mathrm{V}} - \frac{\mathrm{UA}}{\mathrm{V}\rho C_\mathrm{p}} + \frac{(-\Delta H)}{\rho C_\mathrm{p}}\varphi_2^0 \end{bmatrix},$$

$$\tilde{A}_3 = \begin{bmatrix} -\frac{\mathrm{q}}{\mathrm{V}} - \varphi_1^0 & -\varphi_2^0 - 2g_2(\bar{x}_2) \\ \frac{(-\Delta H)}{\rho C_\mathrm{p}}\varphi_1^0 & -\frac{\mathrm{q}}{\mathrm{V}} - \frac{\mathrm{UA}}{\mathrm{V}\rho C_\mathrm{p}} + \frac{(-\Delta H)}{\rho C_\mathrm{p}}\varphi_2^0 + 2\frac{(-\Delta H)}{\rho C_\mathrm{p}} g_2(\bar{x}_2) \end{bmatrix},$$

$$\tilde{A}_4 = \begin{bmatrix} -\frac{\mathrm{q}}{\mathrm{V}} - \varphi_1^0 & -\varphi_2^0 - 2g_2(\underline{x}_2) \\ \frac{(-\Delta H)}{\rho C_\mathrm{p}}\varphi_1^0 & -\frac{\mathrm{q}}{\mathrm{V}} - \frac{\mathrm{UA}}{\mathrm{V}\rho C_\mathrm{p}} + \frac{(-\Delta H)}{\rho C_\mathrm{p}}\varphi_2^0 + 2\frac{(-\Delta H)}{\rho C_\mathrm{p}} g_2(\underline{x}_2) \end{bmatrix},$$

$$\tilde{B}_1 = \tilde{B}_2 = \tilde{B}_3 = \tilde{B}_4 = \begin{bmatrix} 0 \\ \frac{\mathrm{UA}}{\mathrm{V}\rho C_\mathrm{p}} \end{bmatrix}$$

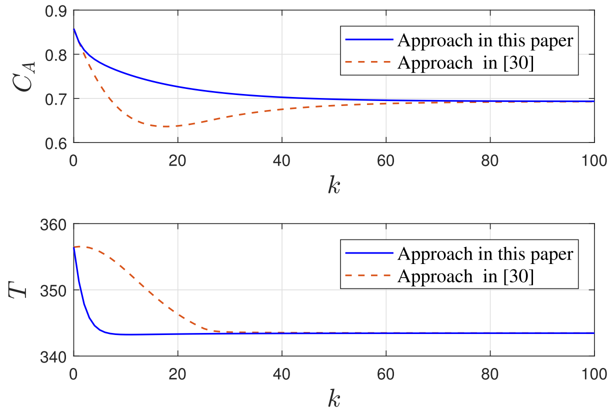

FIGURE 2. State responses.

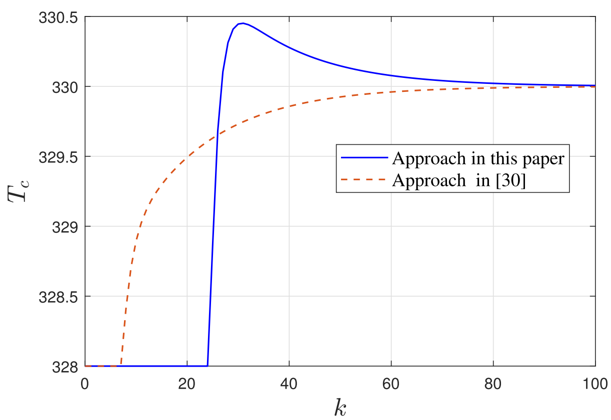

FIGURE 3. Control input.

TABLE 1. Computational time.

	Total time for 100 simulation steps (seconds)
Approach in this paper	230.4209
Approach in [30]	884.2202

$$A_3 = \begin{bmatrix} 0.9202 & -0.0018 \\ 3.1152 & 1.0368 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0.9202 & -0.0005 \\ 3.1152 & 0.8922 \end{bmatrix},$$

$$B_1 = B_2 = B_3 = B_4 = \begin{bmatrix} 0 \\ 0.1046 \end{bmatrix}.$$

Base on (16), letting $\dot{C}_A = 0$ and $\dot{T} = 0$, we obtain the steady-state model $g(x_{ss}, u_{ss})$ of (16). Choose the input steady-state setpoint $T_c^{ss} = 330$, and find C_A^{ss}, T^{ss} satisfying (13), i.e., $C_A^{ss} = 0.693$, $T^{ss} = 343.465$. Thus, we have $x_{ss} = [0.135; -9.982]$, $u_{ss} = -8$. Choose the initial state $x(0) = [0.3; 3]$, $N = 5$.

To illustrate the effectiveness of the proposed approach, we take the synthesis MPC in [30] which also uses the tree trajectory approach for comparison. The simulation results are shown in Figures 2 and 3. From the figures we find that the values of the input and state finally converge to the steady-state setpoints, while the deviations between the actual steady-state values and the steady-state setpoints are zero. This reveals that the offset-free property is achieved. However, since [30] needs to guarantee the stability and considers all possible realization of the system model in the robust worst-case manner, optimization inevitably suffers from high computational burden (see Table 1).

VI. CONCLUSION

In this paper, a parameter-dependent open-loop MPC for LPV model is investigated. The controller utilizes a tree trajectory to predict the vertices of future state predictions. Based on these predictions, the optimization problem is transformed into a QP involving all vertices of state predictions, input predictions and steady-state targets. Under this scheme, the offset-free control is achieved with a low computational burden.

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XIN ZAN was born in Heilongjiang, China. He received the M.S. and Ph.D. degrees from Xi'an Jiaotong University, Xi'an, China, in 2001 and 2011, respectively. From July 2013 to August 2014, he was as a Visiting Scholar at the CE-CERT Laboratory, University of Riverside, CA, USA. Since September 2002, he has been a Lecture with Xi'an Jiaotong University. His research interests include predictive control, robotics navigation and control, and computer vision.

LU JIN received the B.S. degree from Anyang Normal University, Henan, China, in 2020. She is currently pursuing the M.S. degree in control engineering with the School of Automation, Chongqing University of Posts and Telecommunications. Her main research interest includes multi-energy flow systems.

JUN WANG (Member, IEEE) was born in Hubei, China. He received the Ph.D. degree from Xi'an Jiaotong University, Xi'an, China, in 2020. He is currently a Lecturer with the College of Automation, Chongqing University of Posts and Telecommunications, Chongqing. His main research interests include model predictive control, cyber-physical systems, and multi-energy flow systems.