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Sampled-Data Control of Distributed Parameter Systems via an Event-Based Communication Scheme

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ABSTRACT This paper studies the problem of event-triggered control for a class of networked distributed parameter systems with Markov jump parameters. To reduce the number of packages transmitted over the communication network, an adaptive event-triggered mechanism is introduced. The Galerkin method is employed to obtain the nonlinear ordinary differential equation systems, which can accurately describe the dynamics of the dominant modes of the considered distributed parameter systems. The systems are subsequently parameterized by a multilayer neural network with one-hidden layer and zero bias terms, and the linear ordinary differential equation systems are derived. Then, Lyapunov approach is used to analyze stability of the considered systems, and by employing the strong law of large numbers and Gronwall inequality technique, almost surely exponential stability condition is derived. Moreover, a linear sampled-data-based controller is designed to stabilize the closed-loop systems. Finally, a practical example is shown to demonstrate the effectiveness of the achieved theoretical results.

INDEX TERMS Distributed parameter systems, Markov jump parameters, Galerkin method, neural model, adaptive event-triggered networked control, almost surely exponential stability.

I. INTRODUCTION

Distributed parameter systems (DPSs) have been listed as one of the most popular subjects due to their effective and successful modeling of chemical reactor processes, transport-reaction processes and curing oven [1]. Recent years, significant efforts have been devoted into different control problems for DPSs, such as approximation [2], estimation [3], consensus [4], optimization [5].

In view of the complicated system structure and nonlinear terms of the most practical engineering DPSs, the conventional control approach is invalid. In this context, Christofides [6] introduced Galerkin method for nonlinear DPSs to derive low dimensional approximation system. Based on the method proposed by [6], Wu and Li [7] developed Takagi-Sugeno fuzzy control approach to design a fuzzy observer-based controller for nonlinear DPSs. These methods are proposed for DPSs with nonlinear functions completely known, while few approaches are available for DPSs with unknown nonlinearities. Considering the applicability of neural network (NN) control design for systems with unknown nonlinearities [8], Wu and Li [9] solved the guaranteed cost control problem for DPSs with unknown nonlinearities by using Galerkin method and NN technique. This is one motivation of our study.

It is well known that sampling is an inevitable part in working out the control signals. The traditional sampling data transmitting scheme is periodical which may be beneficial to analysis and design. Nevertheless, this may lead to unnecessary waste of computation and communication resources. To overcome this drawback, the event-triggered control scheme is proposed to manage the information data release only when the given condition is triggered, in other words, only necessary data will be transmitted [10]. The event-triggered control problem of finite dimensional systems were studied widely [11]–[16], [34]–[37], while Yao and El-Farra [17] recently investigated event-triggered control problem for DPSs as the first attempt. Based on mobile sensor and actuator, Jiang *et al.* [18] considered event-driven observer-based control problem for DPSs by employing operator semigroup method. Moreover, Selivanov and Fridman [19] discussed distributed event-triggered network control problem for DPSs

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by using Lyapunov control method. Whereas, the aforementioned event-triggered control methods are available for the system with fixed topology structure. To solve the event-triggered control problem of the system with random switching structure, Zhang *et al.* [20] studied finite-time event-triggered H_∞ boundedness for nonlinear system with Markov switching parameters by using discontinuous Lyapunov control method. Very recently, Shen *et al.* [21] proposed an event-triggered scheme based controller for the system with Markov switching parameters.

The threshold of the trigger condition plays the crucial role in the event-triggered control issue. Different value of the threshold may produce unsatisfactory data releasing performance, for example, when the threshold selected as zero, then the data releasing scheme becomes time-triggered. Moreover, the pre-set threshold may be not suitable for varying complex structure of systems. Therefore, it is necessary to develop an adaptive event-triggered law to adapt such changing condition. Recently, Ran *et al.* [36] and Liu *et al.* [37] proposed event-triggered communication scheme to analyze problems of both dissipative control and finite-time filtering of interval type-2 fuzzy systems, respectively. However, there are few references published on adaptive event-triggered control for DPSs with Markov jump parameters. This is another motivation of this paper.

To compensate for the above discussion, the adaptive event-triggered networked control of distributed parameter systems with Markov jump parameters is discussed in this paper. An adaptive event-triggered mechanism is introduced to reduce the number of the package transmission in the networked system. Based on the spatial operator, the distributed parameter systems can be divided into a slow finite dimensional system and a fast infinite dimensional system. The Galerkin method is employed to obtain the ordinary differential equation systems, which can accurately describe the dynamics of the dominant (slow) modes of the considered systems. Subsequently, a multilayer neural network with one-hidden layer and zero bias terms is used to parameterize the resulting nonlinear systems, and the linear systems can be derived. Then, a new Lyapunov functional is established for the stability of the linear Markov jump systems. To deal with the influence of Markov jump, some efforts have been made recently, for example, the weak infinitesimal generator technology is employed in [28]–[32]. Different from the above methods, this paper employs the strong law of large numbers and Gronwall inequality technology to prove most surely exponential stability of the closed-loop system. Furthermore, a linear adaptive event-triggered networked feedback controller is derived to stabilize the distributed parameter systems. Finally, a practical example is shown to demonstrate the validity of the achieved theoretical results. In this paper, the main contributions are listed as follows: (1) An event-triggered communication scheme with an adaptive event-triggered threshold is proposed for the distributed parameter systems, which performs better than conventional event-triggered communication scheme with a constant threshold. (2) To reduce the conservation, a Wirtinger's inequality-dependent Lyapunov functional is constructed to analyze the stability problem for the closed-loop finite-dimensional slow system. (3) Different from the traditional stochastic analysis approach, such as infinitesimal generator, this paper employs the strong law of large numbers and inequality techniques to obtain almost surely exponentially stable criteria.

Notations: The notation used here is fairly standard except where otherwise stated.

Notation	Illustration
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^n$	the n-dimensional Euclidean space
$\mathbb{R}^{n\times m}$	the set of all real $n \times m$ matrices
Superscript "$'$"	if A is a matrix, A' is the transpose of A
$*$	$\begin{pmatrix} A & B \\ * & C \end{pmatrix} = \begin{pmatrix} A & B \\ B' & C \end{pmatrix}$
$diag\{\cdot\}$	$diag\,(A,\ B,\ C) = \begin{pmatrix} A & 0 & 0 \\ * & B & 0 \\ * & * & C \end{pmatrix}$
$\mathcal{E}\{\cdot\}$	Mathematical expectation
$\mathcal{E}_\pi\{\cdot\}$	Expectation under probability distribution π

II. PRELIMINARIES

Set $\{\sigma(t),\ t \geq 0\}$ be a Markov chain with right continuity on the probability space. The form process $\sigma(t)$ is time homogeneous and takes values on a finite state space (denoted by $\mathcal{S} = \{1,\ 2,\ \cdots,\ \ell\}$) with stationary distribution. For $i, j \in \mathcal{S}$, the transition probability from mode $\sigma(t) = i$ to mode $\sigma(t) = j$ is given as following

$$Prob\{\sigma(t+h) = j \,|\, \sigma(t) = i\} = \begin{cases} \mathfrak{p}_{ij}h + o(h), & i \neq j, \\ 1 + \mathfrak{p}_{ii}h + o(h), & i = j, \end{cases}$$

where $h \geq 0$ (h is a small time interval if t stands for time) and $\lim_{h\to 0} \frac{o(h)}{h} = 0$; for $i \neq j$, $\mathfrak{p}_{ij} \geq 0$ stands for the transition rate from mode $\sigma(t) = i$ at time t to mode $\sigma(t) = j$ at time $t + h$; else, $\mathfrak{p}_{ii} = -\sum_{j\in\mathcal{S},\, j\neq i} \mathfrak{p}_{ij}$. The transition rate matrix of Markov process $\sigma(t)$ is denoted by $\mathbb{P} = [\mathfrak{p}_{ij}]$. Definite the stationary probability distribution of the ith mode $\pi_i = Prob\{\sigma(t) = i\}$ and $\pi = [\pi_1,\ \pi_2,\ \cdots,\ \pi_n]'$.

Consider the following DPSs in one spatial dimension with a state-space description of the form

$$\frac{\partial w(x,t)}{\partial t} = A_1^\sigma \frac{\partial w(x,t)}{\partial x} + A_2^\sigma \frac{\partial^2 w(x,t)}{\partial x^2} + g(w(x,t)) + A_3^\sigma f(w(x,t))u(t), \tag{1}$$

subject to the boundary conditions

$$\begin{cases} B_1 w(\underline{x}, t) + C_1 \dfrac{\partial w}{\partial x}(\underline{x}, t) = a_1, \\ B_2 w(\overline{x}, t) + C_2 \dfrac{\partial w}{\partial x}(\overline{x}, t) = a_2, \end{cases} \tag{2}$$

and the initial condition

$$w(x,0) = w_0(x), \tag{3}$$

where $[w_1(x,t),\ w_2(x,t),\ \cdots,\ w_n(x,t)]'$ is the vector of state variables and denoted by $w(x,t)$; $[\underline{x},\ \overline{x}] \subset \mathbb{R}$ is the domain of definition of the process; $x \in [\underline{x},\ \overline{x}]$ is the spatial coordinate; $t \geq 0$ is the time; $u(t)$ is the manipulated input vector; $\frac{\partial w(x,t)}{\partial x}$ and $\frac{\partial^2 w(x,t)}{\partial x^2}$ are the first- and second-order spatial derivatives of $w(x, t)$, respectively; $g(w(x, t))$ is an unknown nonlinear vector function satisfying $g(0) = 0$ and locally Lipschitz continuous; $f(w(x, t)) = [f_1(w(x, t)),\ f_2(w(x, t)),\ \cdots,\ f_n(w(x, t))]'$ is a known smooth vector function of $w(x, t)$, where each $f_i(w(x, t))$ $(i = 1,\ 2,\ \cdots,\ p)$ describes how the control action $u_i(t)$ is distributed in interval $[\underline{x},\ \overline{x}]$; $A_l^\sigma = A_l(\sigma(t))$ for $l = 1, 2, 3$, and A_l^i are known constant matrices $i \in \mathcal{S}$; B_1, B_2, C_1 and C_2 are constant matrices; a_1 and a_2 are column vectors; and $w_0(x)$ is the given initial condition.

Remark 1: Due to the spatially distributed nature and the existence of unknown nonlinearities in parabolic DPS (1)-(3), it is difficult to design a state feedback controller for the system. Motivated by this, in the following study, Galerkin method is developed to reduce the model (1)-(3) to an ODE system in low dimension with unknown nonlinearities; and a NN is applied to approximate the unknown nonlinearities in the ODE system.

A. A LOW DIMENSIONAL APPROXIMATION MODEL

Let $\mathcal{H}$ stands for a Hilbert space of one-dimensional functions defined on $[\underline{x},\ \overline{x}]$ that satisfy the boundary conditions shown in (2), with inner product and norm

$$< y_1,\ y_2 > = \int_{\underline{x}}^{\overline{x}} < y_1(x),\ y_2(x) >_{\mathbb{R}^N} dx,$$
$$||y_1||_2 = \sqrt{< y_1,\ y_1 >},$$

in which y_1, y_2 are two elements of $\mathcal{H}$ and $< \cdot,\ \cdot >_{\mathbb{R}^N}$ denotes the standard inner product in $\mathbb{R}^N$. Define $\mathscr{D}$ as the spatial operator in $\mathcal{H}$ in the form

$$\mathscr{D}w = A_1^\sigma \frac{\partial w}{\partial x} + A_2^\sigma \frac{\partial^2 w}{\partial x^2},$$
$$w \in \mathcal{D}(\mathscr{D}) \doteq \Big\{ w \in \mathcal{H},\ B_1 w(\underline{x}, t) + C_1 \frac{\partial w}{\partial x}(\underline{x}, t) = a_1,$$
$$B_2 w(\overline{x}, t) + C_2 \frac{\partial w}{\partial x}(\overline{x}, t) = a_2 \Big\}.$$

For the eigenvalue problem, define $\mathscr{D}v_j(x) = \lambda_j v_j(x)$ $(j = 1,\ 2,\ \cdots,\ \infty)$, in which λ_j represents j-th eigenvalue and $v_j(x)$ represents the corresponding eigenfunction. For simplicity, the eigenfunctions $v_j(x)$ $(j = 1,\ 2,\ \cdots,\ \infty)$ are considered orthonormalized.

Assumption 1: a) $Re\{\lambda_1\} \geq Re\{\lambda_2\} \geq \cdots Re\{\lambda_j\} \geq \cdots$, *in which* $Re\{\lambda_j\}$ *represents the real part of* λ_j; *b)* $\zeta(\mathscr{D})$ *can be rewritten as* $\zeta(\mathscr{D}) = \zeta_1(\mathscr{D}) + \zeta_2(\mathscr{D})$, *in which* $\zeta_1(\mathscr{D})$ *consists of the first n finite eigenvalues, i.e.,* $\zeta_1(\mathscr{D}) = \{\lambda_1,\ \lambda_2,\ \cdots,\ \lambda_n\}$, *and* $\frac{|Re\{\lambda_1\}|}{|Re\{\lambda_n\}|} = O(1)$; *c)* $Re\{\lambda_{n+1}\} < 0$ *and* $\frac{|Re\{\lambda_1\}|}{|Re\{\lambda_{n+1}\}|} = O(\epsilon)$, *in which* $\epsilon \doteq \frac{|Re\{\lambda_1\}|}{|Re\{\lambda_{n+1}\}|} < 1$ *is a small positive number.*

According to [6], [9], [24] and Assumption 1, Galerkin method can be applied to the system (1)-(3) to obtain an approximate nonlinear ODE system with finite dimension. Here, assume $w(x, t) \in \mathbb{R}$ be the one dimensional state variable. First, by the separation of time and spatial variables [9], [24], a nonlinear ODE system can be given as follows

$$\begin{cases} \dot{w}_s(t) = A_s^\sigma w_s(t) + g_s(w_s(t), w_f(t)) + A_{us}^\sigma u(t), \\ \dot{w}_f(t) = A_f^\sigma w_f(t) + g_f(w_s(t), w_f(t)) + A_{fs}^\sigma u(t), \\ w_s(0) = w_{s,0}, \quad w_f(0) = w_{f,0}, \end{cases} \tag{4}$$

in which

$$\begin{aligned} w_s(t) &= [w_1(t),\ w_2(t),\ \cdots,\ w_n(t)]' \in \mathbb{R}^n, \\ w_f(t) &= [w_{n+1}(t),\ w_{n+2}(t),\ \cdots,\ w_\infty(t)]' \in \mathbb{R}^\infty, \\ A_s^\sigma &= diag\{\lambda_1,\ \lambda_2,\ \cdots,\ \lambda_n\}, \\ A_f^\sigma &= diag\{\lambda_{n+1},\ \lambda_{n+2},\ \cdots,\ \lambda_\infty\}, \\ g_s(w_s(t), w_f(t)) &= < \phi_s(x),\ g(w) >, \\ g_f(w_s(t), w_f(t)) &= < \phi_f(x),\ g(w) >, \\ A_{us}^\sigma &= < \phi_s(x),\ A_3^\sigma >, \\ A_{uf}^\sigma &= < \phi_f(x),\ A_3^\sigma >, \\ w_{s,0} &= < \phi_s(x),\ w_0(x) >, \\ w_{f,0} &= < \phi_f(x),\ w_0(x) >, \\ \phi_s(x) &= [\phi_1(x),\ \phi_2(x),\ \cdots,\ \phi_n(x)]', \\ \phi_f(x) &= [\phi_{n+1}(x),\ \phi_{n+2}(x),\ \cdots,\ \phi_\infty(x)]'. \end{aligned}$$

Thus, a finite dimensional slow system in the following form can be obtained by neglecting the fast modes

$$\begin{cases} \dot{w}_s(t) = A_s^\sigma w_s(t) + g_s(w_s(t),\ w_f(t)) + A_{us}^\sigma u(t), \\ w_s(0) = w_{s,0}. \end{cases} \tag{5}$$

B. NEURAL APPROXIMATION OF UNKNOWN NONLINEAR LOW DIMENSIONAL ODE SYSTEM

In this paper, the following task is to analyze the related properties of the finite dimensional slow system (5), which is equivalent to derive the properties of the system (4). However, the study cannot be processing effectively due to the unknown nonlinear vector function $g_s(w_s(t), 0)$. To deal with the nonlinear function, multilayer neural networks (MNNs) with one or more hidden layers are applied in this paper to approximate the continuous nonlinear function $g_s(w_s(t), 0)$. In the following, the unknown nonlinear vector function $g_s(w_s(t), 0)$ in (5) will be parameterized by an multilayer neural network (MNN).

Consider an MNN with one hidden layer and zero bias terms which is described in matrix-vector notation as

$$g_{nn}(w_s(t),\ M_1,\ M_2) = M_2 \mu(M_1 w_s(t)), \tag{6}$$

in which $g_{nn}(w_s(t),\ M_1,\ M_2) \in \mathbb{R}^n$ denotes the output of the network, $M_1 \in \mathbb{R}^{n_1 \times n_2}$ and $M_2 \in \mathbb{R}^{n \times n_1}$ represent the first-to-second layer interconnection weight matrix and the second-to-third layer interconnection weight matrix, respectively. n_1 denotes the number of hidden neurons. $\mu(\cdot) : \mathbb{R}^{n_1} \to \mathbb{R}^{n_1}$ is the activation function vector of the network and defined by

$$\mu(\ell) \doteq \left[\mu_1(\ell_1)\ \mu_2(\ell_2)\ \cdots\ \mu_{n_1}(\ell_{n_1})\right]', \tag{7}$$

in which $\ell = [\ell_1 \, \ell_2 \, \cdots \, \ell_{n_1}]' \in \mathbb{R}^{n_1}$, $\mu_i(\cdot)$ $(i = 1, 2, \cdots, n_1)$ represents the activation function of the i-th hidden neuron. Without loss of generality, assume that $\mu_i(\cdot)$ satisfies the following conditions for all $i = 1, 2, \cdots, n_1$.

Assumption 2: a) $\mu_i(\cdot)$ is differentiable; b) $\mu_i(0) = 0$ and $\mu_i(\ell_i) \in [-q_i, q_i]$, $q_i > 0$, $\forall \ell_i \in \mathbb{R}$.

In this section, consider the following bipolar sigmoid function which is symmetric with respect to the origin and satisfies Assumption 2

$$\mu_i(\ell_i) = \frac{q_i[1 - exp(-\ell_i/d_i)]}{1 + exp(-\ell_i/d_i)}, \tag{8}$$

in which $q_i > 0$ and $d_i > 0$ are parameters of the function. In the following, to train the NN (7) to approximate $g_s(w_s(t), 0)$. Similar to the backpropagation procedure, all connecting weights are supposed to be determined via a learning rule for MNNs.

According to the approximation theorem for MNNs in [25], if $\delta > 0$ is given accurately, there must exist ideal matrices M_1^* and M_2^* defined as

$$(M_1^*, M_2^*) \doteq \arg \min_{(M_1, M_2)} \Big\{ \max_{w_s \in \mathbf{D}} ||g_s(w_s, 0) - g_s(w_s, M_1, M_2)|| \Big\}$$

such that

$$\max_{w_s \in \mathbf{D}} ||g_s(w_s, 0) - g_s(w_s, M_1, M_2)|| \le \delta ||w_s||, \tag{9}$$

where $\mathbf{D}$ is a compact subset of $\mathbb{R}^n$.

Here, the state vector $w(x, t)$ of the system (1) is assumed completely available as an output. Then, $w_s(t)$ can be derived immediately by employing the modal analyzer [26]. More specifically, if $w(x, t) \in \mathbb{R}$ is one-dimensional, then the j-th element of $w_s(t)$ can be obtained

$$w_j(t) = < w(x, t), \phi_j(x) > . \tag{10}$$

The minimum and maximum values of $\dot{\mu}_i(\ell_i) \doteq \frac{d\mu_i(\ell_i)}{d\ell_i}$ are respectively denoted by

$$\xi_{i,min} \doteq \min_{\ell_i} \dot{\mu}_i(\ell_i) \quad \xi_{i,max} \doteq \max_{\ell_i} \dot{\mu}_i(\ell_i), \quad i \in \mathcal{H}. \tag{11}$$

Further, we have

$$\mu_i(\ell_i)/\ell_i \in \left[\xi_{i,min}, \xi_{i,max}\right], \tag{12}$$

which implies that for any ℓ_i, the following condition is satisfied

$$\left[\mu_i(\ell_i) - \xi_{i,min}\ell_i\right]\left[\xi_{i,max}\ell_i - \mu_i(\ell_i)\right] \ge 0, \quad i \in \mathcal{H}. \tag{13}$$

In view of the function (8), it is obvious that $\xi_{i,min} = 0$ and $\xi_{i,max} = \frac{q_i}{2d_i}$ $(i \in \mathcal{H})$.

Let $\ell(t) = M_1^* w_s(t)$. Based on the approximating NN $g_{nn}(w_s, M_1^*, M_2^*)$, the slow system (5) can be obtained as follows

$$\dot{w}_s(t) = A_s^\sigma w_s(t) + M_2^* \mu(\ell(t)) + e(w_s(t)) + A_{us}^\sigma u(t), \tag{14}$$

in which $e(w_s(t)) = g_s(w_s(t), 0) - M_2^* \mu(\ell(t))$ is the approximation error of the network.

C. ADAPTIVE EVENT-TRIGGERED COMMUNICATION SCHEME

To save the network bandwidth and reduce the number of package transmission, an adaptive event-triggered mechanism is introduced in Fig. 1. It is used to determine whether the newly sampled state will be sent out to the controller or not, which compensates the disadvantages of traditional periodic sampling method. In view of the adaptive event-triggered mechanism (AETC), the data only transmitted when the triggering condition is violated, where the condition is depending on both the states at the latest releasing instant and the current sampling instant. Generally, the following assumptions are necessary for Fig. 1: the sensors are clock driven; the controller and the actuator are event driven; the ZOH is used to hold the control signal before the new data arrived at the actuator; and in every control period, the data is transmitted over the network by a single packet.

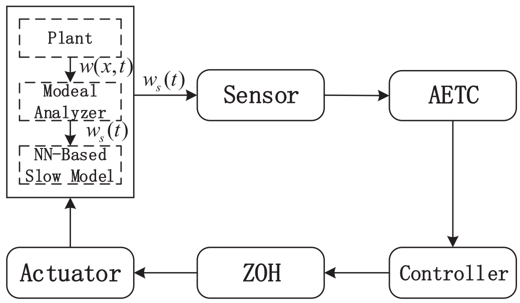

FIGURE 1. Distributed parameter model feedback controller.

In view of the neural model (14), the modal feedback control law with the following form is considered for the DPS (1) - (3)

$$u(t) = Kw_s(r_k h), \qquad t \in [r_k h + \tau_k, \; r_{k+1} h + \tau_{k+1}), \tag{15}$$

where $K \in \mathbb{R}^{p \times n}$ stands for the control gain matrix and will be determined later; $r_k (k = 1, 2, 3, \cdots)$ are some integers such that $\{r_1, r_2, r_3, \cdots\} \subset \{0, 1, 2, \cdots\}$; h is a sampling period; $r_k h$ is the k-th releasing instant of the system; τ_k is a transmission delay at the k-th releasing instant; and $[r_k h + \tau_k, \; r_{k+1} h + \tau_{k+1})$ is the hold interval of zero order hold.

In this paper, the following event-triggered scheme is considered

$$r_{k+1} h = r_k h + \min_{j \in \mathbb{N}} \Big\{ jh \,|\, \oplus(r_j h)' P_0 \oplus(r_j h) - \rho(r_k h) w_s(r_k h)' P_0 w_s(r_k h) > 0 \Big\}, \tag{16}$$

where $\oplus(t) = x(r_k h) - x(r_k h + jh)$ and P_0 is a positive definite matrix to be designed and $\rho(r_k h)$ is an adaptive parameter which satisfies

$$\rho(r_k h) = \begin{cases} 1, & if \; w_s(r_k h) = 0, \\ 0, & if \; w_s(r_{k-1} h)' w_s(r_{k-1} h) \\ & -w_s(r_k h)' w_s(r_k h) \le 0 \text{ and } w_s(r_k h) \ne 0, \\ \frac{2}{\pi} atan(\frac{w_s(r_{k-1} h)' w_s(r_{k-1} h)}{w_s(r_k h)' w_s(r_k h)}), & \text{otherwise}, \end{cases} \tag{17}$$

with $atan(\cdot)$ being the arctangent function.

Based on the proposed transmission scheme (16), the sampled data is not transmitted over the communication network unless the threshold condition is satisfied. The following cases given in [10] are now considered:

1) If $r_k h + h + \overline{\tau} \geq r_{k+1}h + \tau_{k+1}$, where $\overline{\tau} = \max\{\tau_k\}$, the delay function $\tau(t)$ is denoted by $\tau(t) = t - r_k h$, $t \in [r_k h + \tau_k,\ r_{k+1}h + \tau_{k+1})$. It is clear that $\tau_k \leq \tau(t) \leq (r_{k+1} - r_k)h + \tau_{k+1} \leq h + \overline{\tau}$.
2) If $r_k h + h + \overline{\tau} < r_{k+1}h + \tau_{k+1}$, there exists a positive constant d_M such that $r_k h + d_M h + \overline{\tau} < r_{k+1}h + \tau_{k+1} \leq r_k h + d_M h + h + \overline{\tau}$ and $w_s(r_k h)$ and $w_s(r_k h + jh)$ satisfy the proposed transmission scheme (16) with $j = 1, 2, \cdots, d_M - 1$. Define

$$\begin{cases} \Omega_0 = [r_k h + \tau_k,\ r_k h + h + \overline{\tau}), \\ \Omega_j = [r_k h + jh + \overline{\tau},\ r_k h + jh + h + \overline{\tau}), \\ \Omega_{d_M} = [r_k h + d_M h + \overline{\tau},\ r_{k+1}h + \tau_{k+1}). \end{cases} \tag{18}$$

Then, the interval $[r_k h + \tau_k,\ r_{k+1}h + \tau_{k+1}) = \cup_{j=0}^{d_M}\Omega_j$, which means that

$$\tau(t) = \begin{cases} t - r_k h, & t \in \Omega_0, \\ t - r_k h - jh, & t \in \Omega_j,\ j = 1, \cdots, d_M - 1, \\ t - r_k h - d_M h, & t \in \Omega_{d_M}. \end{cases}$$

Therefore, for $t \in [r_k h + \tau_k,\ r_{k+1}h + \tau_{k+1})$ one has

$$0 \leq \tau_k \leq \tau(t) \leq h + \overline{\tau} \doteq \tau_M. \tag{19}$$

In Case 1, $\mathbb{e}(t) = 0$ for $t \in [r_k h + \tau_k,\ r_{k+1}h + \tau_{k+1})$.

In Case 2, one has $\mathbb{e}(t) = \begin{cases} 0, & t \in \Omega_0, \\ e_w(r_k h + jh), & t \in \Omega_j, \\ e_w(r_k h + d_M h), & t \in \Omega_{d_M}. \end{cases}$

where $e_w(r_k h + jh) = w_s(r_k h) - w_s(r_k h + jh)$, $e_w(r_k h + d_M h) = w_s(r_k h) - w_s(r_k h + d_M h)$.

Then, for $t \in [t_k h + \tau_k,\ t_{k+1}h + \tau_{k+1})$, the event-triggered condition (16) can be redescribed as

$$\mathbb{e}(t)'P_0\mathbb{e}(t) - \rho(t - \tau(t))w_s(t - \tau(t))'P_0 w_s(t - \tau(t)) > 0. \tag{20}$$

Combining (14) and (15), the closed-loop neural system in the following form can be derived

$$\begin{aligned} \dot{w}_s(t) = A_s^{\sigma} w_s(t) + M_2^* \mu(\ell(t)) + e(w_s(t)) \\ + A_{us}^{\sigma} K w_s(t - \tau(t)) + A_{us}^{\sigma} K \mathbb{e}(t). \end{aligned} \tag{21}$$

Without loss of generality, we suppose that $k_0, k_1, \cdots, k_{N_t} \in [t_k h + \tau_k,\ t_{k+1}h + \tau_{k+1})$ stand for the jumping moment of Markov modes; the independent and identically distributed random variables $\Delta t_{k_j} = t_{k_j} - t_{k_{j-1}}$ satisfy $\mathcal{E}\{\Delta t_{k_j}\} = \tilde{\Delta}$ and $\mathcal{E}\{\Delta t_{k_j}^2 < \infty\}$, where $\{t_{k_j}\}$ forms a renewal process; the topology switching and switching time are independent; on each finite time interval, switching occurs for only finite times with probability 1 which excludes the possibility of infinitely fast switching and ensures the existence and uniqueness of the standard solution of (21).

III. MAIN RESULTS

In this section, we will state and prove our main results.

Theorem 1: Consider the slow system (5) and the DPS (1) - (3). Given τ_M, if there exist positive definite matrices P_0, P_1 and P_2, such that the matrix inequality

$$\sum_{i_\sigma \in \mathcal{S}} \pi_{i_\sigma} \mathbb{M}_{i_\sigma} < 0 \tag{22}$$

holds, in which

$$\mathbb{M}_{i_\sigma} = \begin{bmatrix} \mathbb{M}_{11} & P_1 M_2^* & P_1 & \mathbb{M}_{14} & \mathbb{M}_{15} & \mathbb{M}_{16} \\ * & -2G_1 & 0 & (P_1 M_2^*)' & (P_1 M_2^*)' & \mathbb{M}_{26} \\ * & * & -G_2 & P_1 & P_1 & \mathbb{M}_{36} \\ * & * & * & \mathbb{M}_{44} & \mathbb{M}_{45} & \mathbb{M}_{46} \\ * & * & * & * & \mathbb{M}_{55} & \mathbb{M}_{56} \\ * & * & * & * & * & -\tau_M^2 P_2 \end{bmatrix};$$

$$\begin{aligned} \mathbb{M}_{11} &= (P_1 A_s^{i_\sigma})' + P_1 A_s^{i_\sigma} - \frac{\pi^2}{4} P_2 \\ &\quad -2(\Xi_{min} M_1^*)' G_1 (\Xi_{max} M_1^*) + \delta^2 G_2; \\ \mathbb{M}_{14} &= (P_1 A_s^{i_\sigma})' + P_1 A_s^{i_\sigma}; \\ \mathbb{M}_{15} &= (P_1 A_s^{i_\sigma})' + P_1 A_s^{i_\sigma} + \frac{\pi^2}{4} P_2; \\ \mathbb{M}_{16} &= \tau_M^2 (P_2 A_s^{i_\sigma})'; \mathbb{M}_{26} = \tau_M^2 (P_2 M_2^*)'; \mathbb{M}_{36} = \tau_M^2 P_2; \\ \mathbb{M}_{44} &= (P_1 A_{us}^{i_\sigma} K)' + P_1 A_{us}^{i_\sigma} K - P_0; \\ \mathbb{M}_{45} &= (P_1 A_{us}^{i_\sigma} K)' + P_1 A_{us}^{i_\sigma} K; \\ \mathbb{M}_{46} &= \tau_M^2 (P_2 A_{us}^{i_\sigma} K)'; \mathbb{M}_{56} = \tau_M^2 (P_2 A_{us}^{i_\sigma} K)'; \\ \mathbb{M}_{55} &= (P_1 A_{us}^{i_\sigma} K)' + P_1 A_{us}^{i_\sigma} K + P_0 - \frac{\pi^2}{4} P_2. \end{aligned}$$

Then, the closed-loop system (21) is almost surely exponentially stable.

Proof: First, choose a Lyapunov functional candidate for the system (16) as

$$V(w_s(t)) = V_1(w_s(t)) + V_2(w_s(t)), \tag{23}$$

where

$$\begin{aligned} V_1(w_s(t)) &= [w_s(t) + \mathbb{e}(t) + w_s(t - \tau(t))]' P_1 \\ &\quad \times [w_s(t) + \mathbb{e}(t) + w_s(t - \tau(t))], \\ V_2(w_s(t)) &= \tau_M^2 \int_{t-\tau(t)}^{t} \dot{w}_s(y)' P_2 \dot{w}_s(y) dy \\ &\quad - \frac{\pi^2}{4} \int_{t-\tau(t)}^{t} [w_s(y) - w_s(t - \tau(t))]' P_2 \\ &\quad \times [w_s(y) - w_s(t - \tau(t))] dy, \end{aligned}$$

where P_1 and P_2 are positive definite matrices with appropriate dimensional. Based on the developed Wirtinger-type inequality proposed in Lemma 3.1 [27], $V_2(w_s(t))$ is positive definite with $P_2 > 0$, then Lyapunov functional $V(w_s(t))$ is positive definite.

The derivative of $V(w_s(t))$ along the solution $w_s(t)$ of the system (21) is denoted by $\dot{V}(w_s(t))$. Consider $t \in [t_k h + \tau_k,$

$t_{k+1}h+\tau_{k+1})$, then one has

$$\dot{V}_1(w_s(t)) = 2\left[A_s^{\sigma} w_s(t) + M_2^{*}\mu(\ell(t)) + e(w_s(t)) + A_{us}^{\sigma}K\mathbb{e}(t) + A_{us}^{\sigma}Kw_s(t-\tau(t))\right]' P_1 \times \left[w_s(t) + \mathbb{e}(t) + w_s(t-\tau(t))\right]. \quad (24)$$

$$\dot{V}_2(w_s(t)) = \tau_M^2 \dot{w}_s(t)'P_2\dot{w}_s(t) - \frac{\pi^2}{4}\left[w_s(t) - w_s(t-\tau(t))\right]'P_2\left[w_s(t) - w_s(t-\tau(t))\right]. \quad (25)$$

According to (9), (13) and (14), it is derived for any positive diagonal matrices G_1 and G_2

$$\left[\mu(\ell(t)) - \Xi_{min}M_1w_s(t)\right]^T G_1 \left[\Xi_{max}M_1w_s(t) - \mu(\ell(t))\right] \geq 0;$$
$$\delta^2 w_s(t)^T G_2 w_s(t) - e(w_s(t))^T G_2 e(w_s(t)) \geq 0. \quad (26)$$

In view of the event-triggered condition (16), the sampled data are not transmitted over the communication networks for $t \in [t_kh+\tau_k,\ t_{k+1}h+\tau_{k+1})$ because the threshold condition is unsatisfied, that is, the following condition

$$0 \leq \rho(t-\tau(t))w_s(t-\tau(t))'P_0w_s(t-\tau(t)) - \mathbb{e}(t)'P_0\mathbb{e}(t) \leq -\mathbb{e}(t)'P_0\mathbb{e}(t) + w_s(t-\tau(t))'P_0w_s(t-\tau(t)). \quad (27)$$

Thus, from (24) to (27), we can get

$$\dot{V}(w_s(t)) \leq \psi(t)'\mathbb{M}_{\sigma(t)}\psi(t), \quad (28)$$

where

$$\psi(t) = \left[w_s(t)'\ \mu(\ell(t))'\ e(w_s(t))'\ \mathbb{e}(t)'\ w_s(t-\tau(t))'\right]'.$$

Define $\varphi(t) = \left[w_s(t)'\ \mathbb{e}(t)'\ w_s(t-\tau(t))'\right]'$. One has $\varphi(t) = U\psi(t)$, where $U = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$.

In view of (23), one obtains

$$V(w_s(t)) \geq V_1(w_s(t)) = \psi(t)'\widehat{U}\psi(t), \quad (29)$$

where

$$\widehat{U} = \begin{bmatrix} P_1 & 0 & 0 & P_1 & P_1 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & P_1 & P_1 \\ * & * & * & * & P_1 \end{bmatrix}.$$

It is easily derived that at $t \in [t_kh+\tau_k,\ t_{k+1}h+\tau_{k+1})$,

$$V(w_s(t)) = V(w_s(t_kh+\tau_k)) + \int_{t_kh+\tau_k}^{t} \psi(y)'\mathbb{M}_{\sigma_y}\psi(y)dy. \quad (30)$$

Substituting (30) into (29), one has

$$\psi(t)'\widehat{U}\psi(t) \leq V(w_s(t_kh+\tau_k)) + \int_{t_kh+\tau_k}^{t} \psi(y)'\mathbb{M}_{\sigma_y}\psi(y)dy, \quad (31)$$

which implies that there exists an arbitrary positive scalar ϱ such that

$$\begin{aligned}\psi(t)'\widehat{U}\psi(t) &+ \int_{t_kh+\tau_k}^{t} \varrho\chi_t(y)\psi(y)'\psi(y)dy \\ &\leq V(w_s(t_kh+\tau_k)) + \int_{t_kh+\tau_k}^{t} \psi(y)'\mathbb{M}_{\sigma(y)}\psi(y)dy + \int_{t_kh+\tau_k}^{t} \varrho\chi_t(y)\psi(y)'\psi(y)dy \\ &\leq V(w_s(t_kh+\tau_k)) + \int_{t_kh+\tau_k}^{t} \psi(y)'\mathbb{M}_{\sigma(y)}\psi(y)dy + \int_{t_kh+\tau_k}^{t} \varrho\psi(y)'\psi(y)dy \\ &= V(w_s(t_kh+\tau_k)) + \int_{t_kh+\tau_k}^{t} \psi(y)'\widehat{U}_{\varrho}\psi(y)dy, \end{aligned} \quad (32)$$

where $\chi_b(t) = \begin{cases} 0, & t \neq b \\ 1, & t = b \end{cases}$ is an indicative function for $t \in [a,\ b]$; $\widehat{U}_{\varrho}((\sigma(t))) = \mathbb{M}_{\sigma(t)} + \varrho I$.

Based on the above (32), there exists a positive scalar $\underline{\lambda}$ (e.g., the minimum of eigenvalues of the matrix $\widehat{U} + \varrho I$ can be chosen as $\underline{\lambda}$) such that

$$\psi(t)'\psi(t) \leq \frac{1}{\underline{\lambda}}V(w_s(t_kh+\tau_k)) + \frac{1}{\underline{\lambda}}\int_{t_kh+\tau_k}^{t} \psi(y)'\widehat{U}_{\varrho}((\sigma(y)))\psi(y)dy. \quad (33)$$

Set T is an orthogonal matrix and $T'T = I$ such that $T'\widehat{U}_{\varrho}((\sigma(t)))T = diag\{\lambda_1(\sigma(t)),\ \lambda_2(\sigma(t)),\ \cdots,\ \lambda_n(\sigma(t))\}$. Set $\phi(t) = T'\psi(t)$. Then, one has

$$\phi(t)'\phi(t) \leq \frac{1}{\underline{\lambda}}V(w_s(t_kh+\tau_k)) + \frac{1}{\underline{\lambda}}\int_{t_kh+\tau_k}^{t} \phi(y)'diag\{\lambda_1(\sigma(y)), \lambda_2(\sigma(y)),\ \cdots,\ \lambda_n(\sigma(y))\}\phi(y)dy, \quad (34)$$

which implies that for $j = 1,\ 2,\ \cdots,\ n$,

$$\phi_j(t)^2 \leq \frac{1}{n\underline{\lambda}}V(w_s(t_kh+\tau_k)) + \int_{t_kh+\tau_k}^{t} \frac{\lambda_j(\sigma(y))}{\underline{\lambda}}\phi_j(y)^2dy. \quad (35)$$

By employing the Gronwall inequality, it is derived that

$$\phi_j(t)^2 \leq \frac{1}{n\underline{\lambda}}V(w_s(t_kh+\tau_k))exp\left\{\int_{t_kh+\tau_k}^{t} \frac{\lambda_j(\sigma(t))}{\underline{\lambda}}dy\right\}. \quad (36)$$

According to Theorem 7.3 in [33], one has

$$\lim_{t\to\infty} \frac{N_t}{t-(t_kh+\tau_k)} = \frac{1}{\tilde{\Delta}}. \quad (37)$$

It is clear that $\frac{k_{N_t}}{t} = 1$ as $k_{N_t \to t}$. And one has

$$\begin{aligned}\frac{1}{t-(t_k h+\tau_k)}&\int_{t_k h+\tau_k}^{t}\frac{\lambda_j(\sigma(y))}{\underline{\lambda}}dy\\ &= \lim_{\substack{k_{N_t}\to t,\\ t\to\infty}} \frac{N_t}{t-(t_k h+\tau_k)}\frac{1}{N_t}\sum_{j=1}^{N_t}\int_{k_{j-1}}^{k_j}\frac{\lambda_j(\sigma(y))}{\underline{\lambda}}dy\\ &\quad+\lim_{\substack{k_{N_t}\to t,\\ t\to\infty}}\frac{1}{t-(t_k h+\tau_k)}\int_{k_{N_t}}^{t}\frac{\lambda_j(\sigma(y))}{\underline{\lambda}}dy\\ &= \lim_{\substack{k_{N_t}\to t,\\ t\to\infty}}\frac{1}{\tilde{\Delta}}\frac{1}{N_t}\sum_{j=1}^{N_t}\int_{k_{j-1}}^{k_j}\frac{\lambda_j(\sigma(y))}{\underline{\lambda}}dy. \end{aligned}\tag{38}$$

By employing the strong law of large numbers for Markovian, one has that

$$\begin{aligned}\lim_{\substack{k_{N_t}\to t,\\ t\to\infty}}&\frac{1}{\tilde{\Delta}}\frac{1}{N_t}\sum_{j=1}^{N_t}\int_{k_{j-1}}^{k_j}\frac{\lambda_j(\sigma(y))}{\underline{\lambda}}dy\\ &= \lim_{\substack{k_{N_t}\to t,\\ t\to\infty}}\frac{1}{\tilde{\Delta}}\frac{1}{N_t}\mathcal{E}_\pi\left\{\frac{\lambda_j(\sigma(t))}{\underline{\lambda}}\right\}\mathcal{E}\{\Delta t_{k_j}\}\\ &= \sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}\frac{\lambda_j(i_\sigma)}{\underline{\lambda}} \end{aligned}\tag{39}$$

holds almost surely.

According to the above analysis, (36) is equivalent to

$$\phi_j(t)^2 \le \frac{1}{n\underline{\lambda}}V(w_s(t_k h+\tau_k)) \times exp\left\{\int_{t_k h+\tau_k}^{t}\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}\frac{\lambda_j(i_\sigma)}{\underline{\lambda}}dy\right\} \tag{40}$$

with probability 1.

In view of (21), if $\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}\frac{\lambda_j(i_\sigma)}{\underline{\lambda}} < 0$ hold for $j = 1, 2, \cdots, n$, that is,

$$\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}\widehat{U}_\varrho((i_\sigma)) < 0 \tag{41}$$

holds, then $\phi(t)'\phi(t)$ is almost sure exponential convergence to zero. Based on the inequality $\phi(t)$, $x_s(t)'x_s(t) \le \psi(t)'\psi(t) = \phi(t)'\phi(t)$. Furthermore, it is easily obtained that $x_s(t)'x_s(t)$ is almost sure exponential convergence to zero, which means that the closed-loop system (21) is almost surely exponentially stable.

Since $\varrho > 0$ is arbitrary, it can be arbitrarily close to zero. Then, the condition (41) can be described as

$$\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}\mathbb{M}_{i_\sigma} < 0. \tag{42}$$

Now, this proof is completed. □ □

Remark 2: In view of the condition (22), one has that controller gain K is not directly derived from $P_1 A_{us}^{i_\sigma} K$ because $A_{us}^{i_\sigma}$ in the term $\mathcal{X} = P_1\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}A_{us}^{i_\sigma}K$ is not invertible, usually, it is full column rank. In order to solve this difficulty, define $N\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}A_{us}^{i_\sigma} = \mathcal{X}$ and $M\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}A_{us}^{i_\sigma} = \sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}A_{us}^{i_\sigma}P_1$. then, one has

$$\left(M\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}A_{us}^{i_\sigma}-\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}A_{us}^{i_\sigma}P_1\right)' \times\left(M\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}A_{us}^{i_\sigma}-\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}A_{us}^{i_\sigma}P_1\right) = 0,$$

which implies that there exists an arbitrarily positive scalar γ such that

$$\begin{bmatrix}\gamma I & \left(M\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}A_{us}^{i_\sigma}-\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}A_{us}^{i_\sigma}P_1\right)'\\ * & I\end{bmatrix} > 0$$

holds. Due to full column rank of $\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}A_{us}^{i_\sigma}$, it is therefore clear that there exists an invertible matrix M such that the above LMI holds.

In order to easily design the controller gain K, according to the analysis in Remark 2, we give the following theorem.

Theorem 2: Consider the slow system (5) and the DPS (1) - (3). Given τ_M, if there exist positive definite matrices P_0, P_1, and P_2 such that the matrix inequality

$$\sum_{i_\sigma\in\mathcal{S}}\pi_{i_\sigma}\widehat{\mathbb{M}}_{i_\sigma} < 0 \tag{43}$$

and

$$P_2 - 2P_1 < 0 \tag{44}$$

hold, where

$$\widehat{\mathbb{M}}_{i_\sigma} = \begin{bmatrix}\mathbb{M}_{11} & P_1 M_2^* & P_1 & \mathbb{M}_{14} & \mathbb{M}_{15} & \widehat{\mathbb{M}}_{16}\\ * & -2G_1 & 0 & \widehat{\mathbb{M}}_{24} & \widehat{\mathbb{M}}_{24} & \tau_M^2\widehat{\mathbb{M}}_{24}\\ * & * & -G_2 & P_1 & P_1 & \tau_M^2 P_1\\ * & * & * & \widehat{\mathbb{M}}_{44} & \widehat{\mathbb{M}}_{45} & \tau_M^2\mathcal{X}'\\ * & * & * & * & \widehat{\mathbb{M}}_{55} & \tau_M^2\mathcal{X}'\\ * & * & * & * & * & \widehat{\mathbb{M}}_{66}\end{bmatrix};$$

$\widehat{\mathbb{M}}_{16} = \tau_M^2(P_1 A_s^{i_\sigma})'$; $\widehat{\mathbb{M}}_{24} = (P_1 M_2^*)'$;
$\widehat{\mathbb{M}}_{44} = \mathcal{X}' + \mathcal{X} - P_0$; $\widehat{\mathbb{M}}_{45} = \mathcal{X}' + \mathcal{X}$;
$\widehat{\mathbb{M}}_{55} = \mathcal{X}' + \mathcal{X} + P_0 - \frac{\pi^2}{4}P_2$; $\widehat{\mathbb{M}}_{66} = \tau_M^2(P_2 - 2P_1)$.

Then, the closed-loop system (21) is almost surely exponentially stable.

Further, according to Remark 2, the controller gain is derived that

$$K = M^{-1}N. \tag{45}$$

Proof: Let us start with pre- and post-multiplying both sides of $\mathbb{M}_{i_\sigma}$ given in (22) with the following matrices

$$\begin{aligned}&diag\left\{I \quad I \quad I \quad I \quad I \quad P_1 P_2^{-1}\right\},\\ &diag\left\{I \quad I \quad I \quad I \quad I \quad P_2^{-1}P_1\right\},\end{aligned} \tag{46}$$

respectively. Then, one has

$$\sum_{i_\sigma \in \mathcal{S}} \pi_{i_\sigma} \overline{\mathbb{M}}_{i_\sigma} < 0, \quad (47)$$

in which

$$\overline{\mathbb{M}}_{i_\sigma} = \begin{bmatrix} \mathbb{M}_{11} & P_1 M_2^* & P_1 & \mathbb{M}_{14} & \mathbb{M}_{15} & \widehat{\mathbb{M}}_{16} \\ * & -2G_1 & 0 & \widehat{\mathbb{M}}_{24} & \widehat{\mathbb{M}}_{24} & \tau_M^2 \widehat{\mathbb{M}}_{24} \\ * & * & -G_2 & P_1 & P_1 & \tau_M^2 P_1 \\ * & * & * & \mathbb{M}_{44} & \mathbb{M}_{45} & \mathbb{M}_{46} \\ * & * & * & * & \mathbb{M}_{55} & \mathbb{M}_{46} \\ * & * & * & * & * & \mathbb{M}_{66} \end{bmatrix};$$

$$\begin{aligned} \mathbb{M}_{11} &= (P_1 A_s^{i_\sigma})' + P_1 A_s^{i_\sigma} - \frac{\pi^2}{4} P_2 \\ &\quad -2(\Xi_{min} M_1^*)' G_1 (\Xi_{max} M_1^*) + \delta^2 G_2; \\ \mathbb{M}_{14} &= (P_1 A_s^{i_\sigma})' + P_1 A_s^{i_\sigma}; \\ \mathbb{M}_{15} &= (P_1 A_s^{i_\sigma})' + P_1 A_s^{i_\sigma} + \frac{\pi^2}{4} P_2; \\ \mathbb{M}_{44} &= (P_1 A_{us}^{i_\sigma} K)' + P_1 A_{us}^{i_\sigma} K - P_0; \\ \mathbb{M}_{45} &= (P_1 A_{us}^{i_\sigma} K)' + P_1 A_{us}^{i_\sigma} K; \\ \mathbb{M}_{46} &= \tau_M^2 (P_1 A_{us}^{i_\sigma} K)'; \\ \mathbb{M}_{55} &= (P_1 A_{us}^{i_\sigma} K)' + P_1 A_{us}^{i_\sigma} K + P_0 - \frac{\pi^2}{4} P_2; \\ \mathbb{M}_{66} &= -\tau_M^2 P_1 P_2^{-1} P_1. \end{aligned}$$

In view of the above condition, the controller gain matrix K cannot be directly derived by $\overline{\mathbb{M}}_{i_\sigma}$ because $P_1 P_2^{-1} P_1$ is nonlinear term. Thanks to $(P_1 - P_2) P_2^{-1} (P_1 - P_2) > 0$, it is easily derived that

$$P_1 P_2^{-1} P_1 > 2P_1 - P_2. \quad (48)$$

$2P_1 - P_2$ is used to replace $P_1 P_2^{-1} P_1$, then, the proof is completed. □

□

Remark 3: In this paper, the relaxed stability condition is derived based on free weight matrices, which implies that there are a large number of calculations if the chosen free weight matrices are with large dimensions.

IV. APPLICATION TO A CATALYTIC ROD

In this section, to demonstrate the effectiveness of the achieved results, the control problem of the temperature profile of a catalytic rod [6] will be studied by employing the proposed design method. Consider a long, thin rod in a furnace, and the DPS with the following form is assumed to represent the spatio-temporal evolution of the rod temperature

$$\begin{aligned} &\frac{\partial w(x,t)}{\partial t} \\ &= A_2^\sigma \frac{\partial^2 w(x,t)}{\partial x^2} \\ &\quad + \left[50 \left(exp(\frac{-4}{1+w(x,t)}) - exp(-4) \right) - 2w(x,t) \right] \\ &\quad + A_3^\sigma \left(f_1(x) u_1(t) + f_2(x) u_2(t) \right), \end{aligned} \quad (49)$$

which is subject to the Dirichlet boundary conditions

$$w(0,t) = 0, \quad w(\pi,t) = 0, \quad (50)$$

where $w(x,t)$ is the temperature in the reactor; $u_1(t)$ and $u_2(t)$ are the manipulated inputs. We consider that every state of the Markov chain$\{\sigma(t), t \geq 0\}$ is same, that is, both A_2^σ and A_3^σ are constants and take $A_2^\sigma = 1, A_3^\sigma = 2$. It was verified by Christofides in [6] that the operating steady state $w(x,t) = 0$ is an unstable one. The actuator distribution functions are taken to be $f_1(x) = \sqrt{\frac{2}{\pi}} sin(x)$ and $f_2(x) = \sqrt{\frac{2}{\pi}} cos(x)$. By Galerkin method, the following 2-D ODE system is derived

$$\dot{w}_s(t) = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix} w_s(t) + \begin{bmatrix} g_{s1}(w_s(t),0) \\ g_{s2}(w_s(t),0) \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & \frac{16}{3\pi} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \quad (51)$$

where

$$\begin{aligned} w_s(t) &= \left[w_{s1}(t) \; w_{s2}(t) \right]', \\ g_{s1}(w_s(t),0) &= -2w_{s1}(t) + g_3(w_s(t)), \\ g_3(w_s(t)) &= 50 \int_0^\pi \phi_1(s)[-exp(-4) \\ &\quad + exp(-\frac{4}{1 + w_{s1}(t)\phi_1(s) + w_{s2}(t)\phi_2(s)})]ds, \\ g_{s2}(w_s(t),0) &= -2w_{s2}(t) + g_4(w_s(t)), \\ g_4(w_s(t)) &= 50 \int_0^\pi \phi_2(s)[-exp(-4) \\ &\quad + exp(-\frac{4}{1 + w_{s1}(t)\phi_1(s) + w_{s2}(t)\phi_2(s)})]ds. \end{aligned}$$

Furthermore, we adopt the NN (10) with 12 hidden neurons to approximate the nonlinear function vector $\begin{bmatrix} g_{s1}(w_s(t),0) \\ g_{s2}(w_s(t),0) \end{bmatrix}$. To train the network, several different values of w_1 and w_2 are selected from [0.01, 1] at intervals of 0.01 and [0.005, 0.5] at intervals of 0.005, respectively. After 860 epochs, the weight matrices respectively converge to M_1^* and $(M_2^*)'$ as follows:

$$\begin{bmatrix} 2.8233 & 1.0179 \\ -0.9784 & 4.7651 \\ -0.6783 & 4.3396 \\ 4.9217 & 2.1880 \\ 3.2167 & 0.3723 \\ 3.8538 & -2.0794 \\ 2.9649 & -2.9006 \\ 3.6666 & -3.1006 \\ -3.4923 & -3.3663 \\ -3.8808 & 2.8746 \\ 3.4125 & -3.4461 \\ -3.2918 & -2.9350 \end{bmatrix}, \begin{bmatrix} -0.4523 & -0.4316 \\ 0.8151 & 0.4514 \\ 0.8696 & 1.3484 \\ -0.0427 & -0.0532 \\ -0.3278 & -0.4010 \\ -0.3609 & -0.4906 \\ 0.5065 & -0.4211 \\ -0.2057 & -0.7495 \\ -0.0278 & -0.0326 \\ 1.0362 & -0.2083 \\ -0.1232 & -0.0492 \\ 0.0677 & 0.0658 \end{bmatrix}.$$

Consider now the initial value of the system (49) shown as follows

$$w(x,0) = 0.05\sqrt{2/\pi} sin(x) + 0.6\sqrt{2/\pi} sin(2x).$$

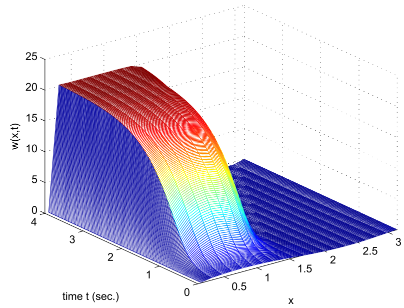

FIGURE 2. The spatio-temporal evolution of the rod temperature of the system (49) without input control.

It is easy to obtain $w_s(0) = [0.05,\ 0.6]'$. Under the conditions $u_1(t) = 0$ and $u_2(t) = 0$, the spatio-temporal evolution of the rod temperature of the system (49) without input control is shown in Fig. 2. It is clear that the trajectory of the rod temperature of the system (49) does not converge to zero.

By employing the Matlab LMI toolbox, one obtains the following parameters based on Theorem 2:

$$P_0 = \begin{bmatrix} 3.6714 & -0.7937 \\ -0.7937 & 3.6714 \end{bmatrix},$$

$$K = \begin{bmatrix} -0.7387 & 0.1169 \\ -0.0552 & -0.1314 \end{bmatrix}.$$

Simulations: In this simulation, we consider cycle sampling style for the adaptive event-triggered communication scheme. Pick up the sampling time interval $t_k - t_{k-1} = 0.1$ and the initial $\rho(0) = 0.8$. Simulation results of the system (49) are shown in Figs. 3-8. In Fig. 3, it shows that the state responses of both w_{s1} and w_{s2} of the slow system (51) with the obtained controller gain. It is easy to see that both w_{s1} and w_{s2} of the slow system (51) converge to zero under the adaptive event-triggered control scheme. Actual trajectory of the adaptive control law $\rho(t)$ is shown in Fig. 4. It is clear to find that the value of the adaptive control law $\rho(t)$ converges to a constant when the slow system (51) reaches stability. The adaptive event-triggered responses of both w_{s1} and w_{s2} of the slow system (51) are shown in Figs. 5-6, respectively. In Fig. 5, the state value w_{s1} of the slow system at the each event-triggered instant $r_k h$ is shown. In Fig. 6, the state value w_{s2} of the slow system at the each event-triggered instant $r_k h$ is shown. It is obvious that the states w_{s1} and w_{s2} of the slow system go to zero as the control time goes to 8. In Fig. 7, actual trajectory of the adaptive event-triggered control input $u(t)$ is given. One can get that the adaptive event-triggered control input $u(t)$ converges to zero as the slow system (51) reaches stability. From Figs. 3-7, the event-triggered control scheme is effective for the slow system (51). In order to illustrate the effectiveness of the event-triggered control scheme for the system (49) with boundary conditions (50), the profile of evolution of rod temperature is given in Fig. 8. It is clear that rod temperature goes to zero as the control time goes to 8, which implies that the proposed control approach in this

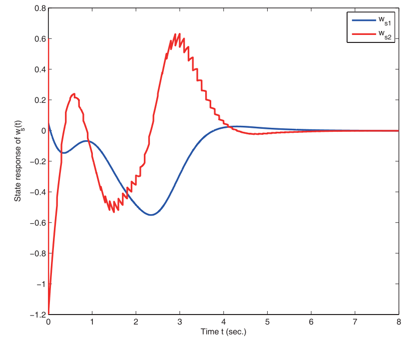

FIGURE 3. Actual trajectories of the states both w_{s1} and w_{s2} of the slow system (51).

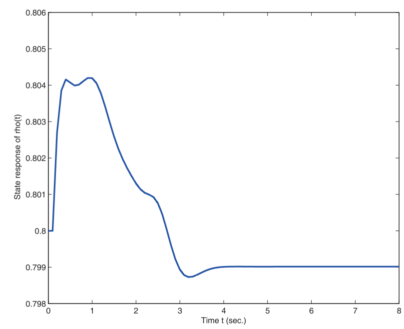

FIGURE 4. Actual trajectory of the variable $\rho(t)$.

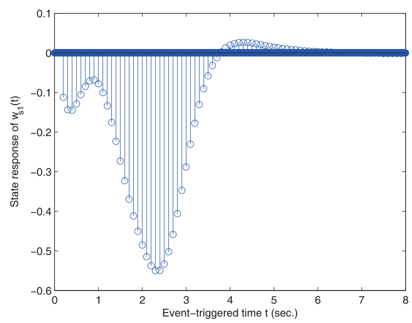

FIGURE 5. Actual trajectory of the state $w_{s1}(t)$.

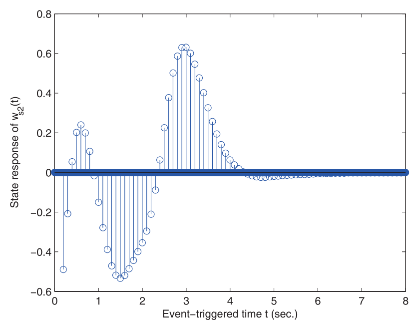

FIGURE 6. Actual trajectory of the state $w_{s2}(t)$.

paper is effective. The simulation results show that the resulting guaranteed cost controller can regulate the temperature profile at the desired steady state $w(x, t) = 0$ under the cost function (17) is satisfied.

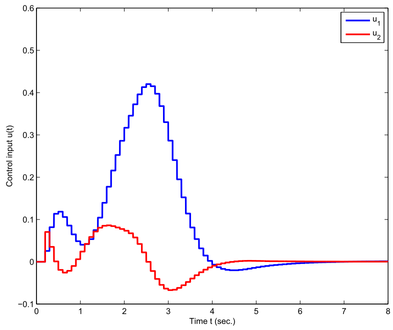

FIGURE 7. Actual trajectory of the adaptive event-triggered control input $u(t)$.

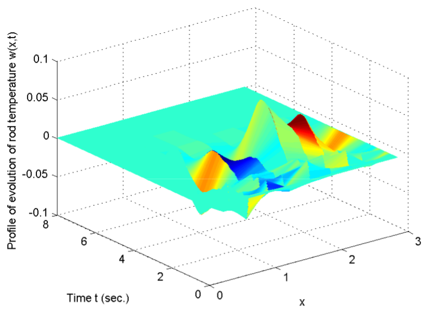

FIGURE 8. Profile of evolution of rod temperature system (49) with the adaptive event-triggered controller.

V. CONCLUSION

In this paper, the problem of adaptive event-triggered networked control is investigated for a class of distributed parameter systems with Markov jump parameters. A proposed adaptive event-triggered control scheme is used to reduce the workload of the network. And the Galerkin method is employed for the distributed parameter systems to derive ordinary differential equation (ODE) systems, which accurately describe the dynamics of the dominant (slow) modes of the considered systems. The resulting nonlinear ODE systems are subsequently parameterized by a multilayer neural network with one-hidden layer and zero bias terms. Furthermore, based on the novel Lyapunov functional and the nontrivial stochastic analysis approach, the stability condition of the systems is derived. Finally, a linear adaptive event-triggered networked feedback controller is designed to stabilize the closed-loop distributed parameter systems.

Notice that the significant stability analysis can be extended to the problem of boundary control. Few techniques were presented to analyze boundary control of distributed parameter systems. Motivated by this, future work will focus on boundary control to keep almost sure stability of the systems with packet losses.

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