Received August 10, 2021, accepted September 6, 2021, date of publication September 23, 2021, date of current version October 4, 2021.

*Digital Object Identifier 10.1109/ACCESS.2021.3115400*

# Monoaxial Electrodynamic Stabilization of an Artificial Earth Satellite in the Orbital Coordinate System via Control With Distributed Delay

**ALEXANDER ALEKSANDROV[1] AND ALEXEY A. TIKHONOV[2,3]**

[1]Department of Medical and Biological Systems Control, St. Petersburg State University, 199034 Saint Petersburg, Russia
[2]Department of Theoretical and Applied Mechanics, St. Petersburg State University, 199034 Saint Petersburg, Russia
[3]Department of Mechanics, Saint Petersburg Mining University, 199106 Saint Petersburg, Russia

Corresponding author: Alexey A. Tikhonov (a.tikhonov@spbu.ru)

**ABSTRACT** An artificial Earth satellite (AES) with three different principal central moments of inertia is under consideration. The AES moves along a Keplerian circular equatorial near-Earth orbit. The AES is equipped with electrodynamic attitude control system that simultaneously generates Lorentz and magnetic control torques. The possibility of using an electrodynamic attitude control system for monoaxial attitude stabilization of AES in the orbital coordinate system is analyzed. The development of the concept of electrodynamic attitude control, including the use of a restoring torque with a distributed delay (integral term), is proposed. The conditions are found under which the electromagnetic attitude control system with distributed delay solves the problem of AES monoaxial stabilization in the presence of the disturbing gravitational torque. In a nonlinear formulation, sufficient conditions for the asymptotic stability of the AES equilibrium position are obtained. A theorem on the asymptotic stability of the AES programmed attitude motion is proved. The effectiveness of the constructed attitude control with a distributed delay is confirmed by numerical modeling.

**INDEX TERMS** Asymptotic stability, attitude control, delay systems, electric variables control, low earth orbit satellites, magnetic variables control.

## I. INTRODUCTION

The electromagnetic interaction of the artificial Earth satellite (AES) with the Earth's magnetic field has a significant effect on the AES attitude dynamics and can be used in AES attitude control system. The magnetic attitude control systems based on this interaction, their advantages and disadvantages are discussed in a variety of works. For example, see [1]–[3] and papers cited therein. The concept of AES attitude control using the Lorentz torque was proposed in [4] and papers cited therein. Later this concept was addressed in [5] and [6] and developed in accordance with modern trends in control theory. The electrodynamic attitude control strategy based on simultaneous usage of the magnetic and Lorentz torques occurred to be potent [7] since it can relief some natural constraints that are inherent to both magnetic and Lorentz attitude control systems.

The associate editor coordinating the review of this manuscript and approving it for publication was Halil Ersin Soken.

A parametric approach for damping the AES oscillations in the orbital frame was proposed in [7] and developed in [8], where consideration was given to AES attitude stabilization in the direct equilibrium position in the orbital frame. The mathematical background of this approach relies on the differential equations of the linear approximation. The permanent stability of the AES direct equilibrium position under the perturbing action of the gravitational torque was proved with the use of numerical analysis of the roots of the characteristic polynomial. The analytical proof of the asymptotic stability of the AES direct equilibrium position was presented in [9] where nonlinear analysis of the differential equations of motion was performed on the basis of the Lyapunov direct method [10]–[13] and on the special approach to constructing Lyapunov functions [14], [15].

The problem of controllability in electrodynamic attitude control system was addressed in [16], where it was shown that the linearized differential system can be reduced to time-invariant one of larger order. For obtained time-invariant

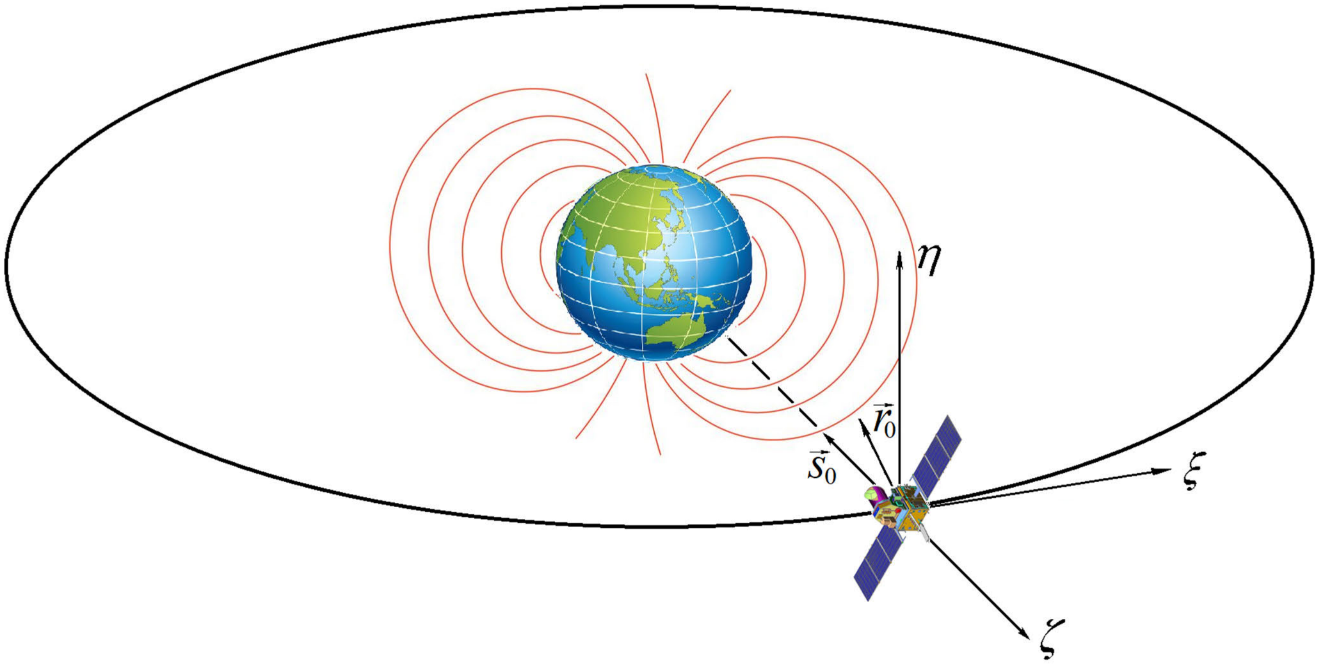



**FIGURE 1.** AES in the orbital coordinate system.

system, controllability is analyzed, and the optimal stabilization algorithm based on the LQR method is constructed.

This paper analyzes the applicability of the electrodynamic attitude control system for monoaxial AES stabilization in the arbitrary angular position in the orbital frame. This problem was previously considered in [17], where it was solved using a completely different control, and the stability analysis was based on approaches that are fundamentally different from those used in this article.

As is known, the problem of monoaxial AES stabilization is relevant in connection with many applications, for example, Earth-pointing satellites, remote sensing, scientific space missions with a telescope. In some problems of AES stabilization, it is important not only to meet stabilization conditions but also avoid unacceptable vibrations due to attitude control system. For example, large space telescope needs in vibrations suppression in the process of its pointing [18]. One of the popular ways for smoothing transients and vibration suppression is based on the usage of PID controller. However, this method is not universal, since the integral term introduced into the control process, can play a negative role with time increasing. In particular, this drawback restricts the use of PID controllers for highly precise tracking. To overcome this disadvantage, it was proposed in [19] to use control with distributed delay for some types of mechanical systems.

In this connection, the problem is raised in this paper about the possibility of implementing such a system of electrodynamic AES attitude control by the type of a PID-like controller, in which (in contrast with [17]) the control torque contains a distributed delay (integral component). A theorem on the asymptotic stability of the AES angular position is proved with the use of the original construction for the Lyapunov-Krasovsky functional. The theorem substantiates the possibility of constructing the desired control system. The effectiveness of the constructed control with a distributed delay is confirmed by a numerical simulation.

## II. PROGRAM AES ATTITUDE

We consider an AES whose center of mass (point C) moves in a circular equatorial near-Earth orbit of a radius $R$ as shown in Fig. 1. The AES attitude motion in the orbital coordinate system $C\xi\eta\zeta$ with the unit vectors $\vec{\xi}_0$, $\vec{\eta}_0$, $\vec{\zeta}_0$ is under investigation. The axis $C\xi$ of this coordinate system is directed along the tangent to the orbit in the direction of motion, the axis $C\eta$ is orthogonal to the orbit plane, and the axis $C\zeta$ is directed along the radius vector $R = \overrightarrow{O_E C} = R\vec{\zeta}_0$ of the AES center of mass relative to the Earth center $O_E$. The angular velocity of the orbital coordinate system relative to the inertial system is denoted by $\vec{\omega}_0$. The system of the main central axes of inertia $Cxyz$ (unit vectors $\vec{i}, \vec{j}, \vec{k}$) of the AES is rigidly connected with the AES.

The angular position of the axes $Cxyz$ relative to the axes $C\xi\eta\zeta$ is defined by the matrix $\mathbf{A}$ of direction cosines $\alpha_i$, $\beta_i$, $\gamma_i$ $(i = 1, 2, 3)$ so that the equalities

$$\begin{aligned}\vec{\xi}_0 &= \alpha_1\vec{i} + \alpha_2\vec{j} + \alpha_3\vec{k},\\ \vec{\eta}_0 &= \beta_1\vec{i} + \beta_2\vec{j} + \beta_3\vec{k},\\ \vec{\zeta}_0 &= \gamma_1\vec{i} + \gamma_2\vec{j} + \gamma_3\vec{k}\end{aligned}$$

hold true. Regarding orbital frame as the basic one, we define the program angular position of AES in the orbital frame by matrix $\mathbf{A}_0$ of direction cosines. The unit vectors

$$\mathbf{A}^\top \vec{\xi}_0 = (\alpha_1, \alpha_2, \alpha_3)^\top \risingdotseq \vec{s}_1,$$
$$\mathbf{A}^\top \vec{\eta}_0 = (\beta_1, \beta_2, \beta_3)^\top \risingdotseq \vec{s}_2,$$
$$\mathbf{A}^\top \vec{\zeta}_0 = (\gamma_1, \gamma_2, \gamma_3)^\top \risingdotseq \vec{s}_3$$

are invariable in the basic coordinate system $C\xi\eta\zeta$. Analogously, the unit vectors

$$\mathbf{A}_0^\top \vec{\xi}_0 = (\alpha_{10}, \alpha_{20}, \alpha_{30})^\top \risingdotseq \vec{r}_1,$$
$$\mathbf{A}_0^\top \vec{\eta}_0 = (\beta_{10}, \beta_{20}, \beta_{30})^\top \risingdotseq \vec{r}_2,$$
$$\mathbf{A}_0^\top \vec{\zeta}_0 = (\gamma_{10}, \gamma_{20}, \gamma_{30})^\top \risingdotseq \vec{r}_3$$

rigidly connected with the AES, are invariable in the coordinate system $Cxyz$.

Let the unit vector $\vec{s}_0 = c_1\vec{s}_1 + c_2\vec{s}_2 + c_3\vec{s}_3$, where $c_i = \text{const}$, $i = 1, 2, 3$ be invariable in the basic coordinate system $C\xi\eta\zeta$ (Fig. 1). Let some axis with the unit vector $\vec{r}_0 = x_0\vec{i} + y_0\vec{j} + z_0\vec{k}$, where $x_0 = \text{const}$, $y_0 = \text{const}$, $z_0 = \text{const}$, be fixed in AES, that is invariable in the coordinate system $Cxyz$.

The AES angular position corresponding the equality $\vec{r}_0 = \vec{s}_0$ will be referred to as a program angular position.

We consider the following control problem: it is required to design the control torques providing existence and asymptotic stability of AES programmed motion in the orbital frame such that

$$\vec{r}_0 = \vec{s}_0, \quad \vec{\omega}' = \vec{0}. \tag{1}$$

Here and in what follows $\vec{\omega}'$ is the AES angular velocity relative to the orbital frame in the projections on the axes $Cxyz$. The attitude motion (1) is called the program attitude motion of the AES.

It should be noted that the absolute angular velocity of the AES is not zero in the program attitude motion. So, the stabilization of program attitude motion (1) of the AES in the rotating orbital frame is equivalent to the stabilization of permanent rotation [14], [20], [21] of the AES in the inertial frame.

## III. CONTROL DESIGN

We consider electrodynamic attitude control system based on the simultaneous usage of Lorentz and magnetic control torques [7], [22]

$$\vec{M}_L = \vec{P} \times \vec{T}, \quad \vec{M}_M = \vec{I} \times \vec{B}, \tag{2}$$

where $\vec{B}$ is the Earth's magnetic field induction, calculated at the AES center of mass, $\vec{P} = Q\vec{\rho}_0$, $Q$ is the total charge of the AES, $\vec{\rho}_0 = Q^{-1}\int_V \mu\vec{\rho}dV$ is the radius vector of the AES center of charge relative to its center of mass, $\mu$ is the density of charge distribution over the AES volume $V$, $\vec{\rho}$ is the radius vector of the AES element $dV$ with respect to its center of mass, $\vec{T} = \vec{v}_C \times \vec{B}$, $\vec{v}_C$ is the velocity of the AES center of mass relative to the greenwich coordinate system, vector $\vec{I}$ is the intrinsic magnetic moment of the AES.

AES attitude stabilization in the orbital coordinate system is analyzed in terms of the nonlinear differential equations of the AES attitude motion based on the Euler–Poisson scheme [20]:

$$\frac{d}{dt}(\mathbf{J}\vec{\omega}) + \vec{\omega} \times (\mathbf{J}\vec{\omega}) = \vec{M}_G + \vec{M}_L + \vec{M}_M, \tag{3}$$
$$\frac{d\vec{s}_i}{dt} + \vec{\omega}' \times \vec{s}_i = 0, \quad i = 1, 2, 3. \tag{4}$$

Here $\vec{\omega} = \vec{\omega}_0 + \vec{\omega}'$ is the absolute angular velocity of the AES.

The procedure of the control design is in accordance with [17], where it was shown that each control parameter $\vec{P}$ and $\vec{I}$ can be chosen as a sum of restoring term, dissipative term and compensating one. As is known, AES that moves in the Earth's gravitational and magnetic fields [20], [23], [24] is subjected to a lot of disturbing torques [3], [20], [25]. In this paper we consider gravitational torque $\vec{M}_G$ as the most significant disturbing torque. Compensating terms of control torques $\vec{M}_L$ and $\vec{M}_M$ allows the suppression of disturbing gravitational torque $\vec{M}_G = 3\omega_0^2\vec{s}_3 \times (\mathbf{J}\vec{s}_3)$, where $\mathbf{J} = \text{diag}(A, B, C)$ is the inertia tensor of AES in the coordinate system $Cxyz$. Corresponding components of control vectors $\vec{P}$ and $\vec{I}$ are constructed in [17].

The dissipative component of control torque we take in the form

$$h_L(\vec{\omega}' \times (\mathbf{A}^\top\vec{T})) \times (\mathbf{A}^\top\vec{T}) + h_M(\vec{\omega}' \times (\mathbf{A}^\top\vec{B})) \times (\mathbf{A}^\top\vec{B}),$$
$$\mathbf{A}^\top\vec{T} = T_\xi\vec{s}_1 + T_\eta\vec{s}_2 + T_\zeta\vec{s}_3,$$
$$\mathbf{A}^\top\vec{B} = B_\xi\vec{s}_1 + B_\eta\vec{s}_2 + B_\zeta\vec{s}_3. \tag{5}$$

This approach is rather natural since the dissipative control torque is linear with respect to the relative angular velocity $\vec{\omega}'$ [14], [26], [27].

The restoring component of control torque is constructed in the form

$$a\vec{r}_0 \times \vec{s}_0 + b\int_{t-\tau}^{t} \vec{r}_0 \times \vec{s}_0(\sigma)d\sigma, \tag{6}$$

where $a, b$ are constants with $a > 0$. Unlike [17], the restoring torque contains distributed delay (integral term).

Substitution of the control torque in the right hand side of the equations (3) results in the following equations:

$$\frac{d}{dt}(\mathbf{J}(\vec{\omega}' + \omega_0\vec{s}_2)) + \vec{\omega}' \times (\mathbf{J}\vec{\omega}') + \omega_0\vec{s}_2 \times (\mathbf{J}\vec{\omega}')$$
$$+\omega_0\vec{\omega}' \times (\mathbf{J}\vec{s}_2) = a\vec{r}_0 \times \vec{s}_0 + b\int_{t-\tau}^{t} \vec{r}_0 \times \vec{s}_0(\sigma)d\sigma$$
$$+h_L(\vec{\omega}' \times (\mathbf{A}^\top\vec{T})) \times (\mathbf{A}^\top\vec{T}) + h_M(\vec{\omega}' \times (\mathbf{A}^\top\vec{B})) \times (\mathbf{A}^\top\vec{B}). \tag{7}$$

The system (7), (4) admits the programmed motion (1).

In dipole approximation of the Earth's magnetic field ("straight magnetic dipole") [20], [23], [24] $\mathbf{A}^\top\vec{B} = B_\eta\vec{s}_2$,

$B_\eta = -(R_E/R)^3 g_1^0 = \text{const}$, $\mathbf{A}^\top \vec{T} = T_\zeta \vec{s}_3$, $T_\zeta = R(\omega_0 - \omega_E) B_\eta = \text{const}$ and equations (7) take on the form

$$\frac{d}{dt}(\mathbf{J}(\vec{\omega}' + \omega_0 \vec{s}_2)) + \vec{\omega}' \times (\mathbf{J}\vec{\omega}') + \omega_0 \vec{s}_2 \times (\mathbf{J}\vec{\omega}') + \omega_0 \vec{\omega}' \times (\mathbf{J}\vec{s}_2) = a\vec{r}_0 \times \vec{s}_0 + b\int_{t-\tau}^{t} \vec{r}_0 \times \vec{s}_0(\sigma)d\sigma - h_L(\vec{\omega}' - (\vec{\omega}'\vec{s}_3)\vec{s}_3) - h_M(\vec{\omega}' - (\vec{\omega}'\vec{s}_2)\vec{s}_2). \tag{8}$$

We assume that initial functions for the system (8), (4) belong to the space $C([-\tau, 0], \mathbb{R}^{12})$ of continuous functions $\vec{\varphi}(\xi) : [-\tau, 0] \to \mathbb{R}^{12}$ with the uniform norm

$$\|\vec{\varphi}\|_\tau = \max_{\xi \in [-\tau, 0]} \|\vec{\varphi}(\xi)\|,$$

where $\|\cdot\|$ denotes the Euclidean norm of a vector. Let $(\vec{\omega}_t^\top, \vec{s}_{1t}^\top, \vec{s}_{2t}^\top, \vec{s}_{3t}^\top)^\top$ be the restriction of a solution $(\vec{\omega}^\top(t), \vec{s}_1^\top(t), \vec{s}_2^\top(t), \vec{s}_3^\top(t))^\top$ to the segment $[t-\tau, t]$ [28].

## IV. STABILITY ANALYSIS

Let $h_L = h\tilde{h}_L$, $h_M = h\tilde{h}_M$. Here $\tilde{h}_L, \tilde{h}_M$ are fixed positive numbers, $h$ is a positive parameter. Then

$$h_L(\vec{\omega}' - (\vec{\omega}'\vec{s}_3)\vec{s}_3) + h_M(\vec{\omega}' - (\vec{\omega}'\vec{s}_2)\vec{s}_2) = h\mathbf{D}(\vec{s}_2, \vec{s}_3)\vec{\omega}',$$

where

$$\mathbf{D}(\vec{s}_2, \vec{s}_3) = \tilde{h}_L(\mathbf{I} - \vec{s}_3^\top \vec{s}_3) + \tilde{h}_M(\mathbf{I} - \vec{s}_2^\top \vec{s}_2)$$

and $\mathbf{I}$ is the $(3 \times 3)$–identity matrix. It is worth noting that the matrix $\mathbf{D}(\vec{s}_2, \vec{s}_3)$ is symmetric and positive definite.

Using the approach developed in [29]–[31], choose a Lyapunov function candidate for (8), (4) as follows

$$V = \frac{1}{2}\|\vec{r}_0 - \vec{s}_0\|^2 + \frac{\chi}{2}\vec{\omega}'^\top \mathbf{J}\vec{\omega}' + \frac{1}{h}(\vec{s}_0 \times \vec{r}_0)^\top \mathbf{D}^{-1}(\vec{s}_2, \vec{s}_3)\mathbf{J}\vec{\omega}',$$

where $\chi$ is a positive parameter.

We obtain

$$\frac{1}{2}\|\vec{r}_0 - \vec{s}_0\|^2 + \chi b_1\|\vec{\omega}'\|^2 - \frac{1}{h}b_3\|\vec{s}_0 - \vec{r}_0\|\|\vec{\omega}'\| \le V \le \frac{1}{2}\|\vec{r}_0 - \vec{s}_0\|^2 + \chi b_2\|\vec{\omega}'\|^2 + \frac{1}{h}b_3\|\vec{s}_0 - \vec{r}_0\|\|\vec{\omega}'\|,$$

$$\dot{V} \le \chi\vec{\omega}'^\top\Big(\omega_0\mathbf{J}(\vec{\omega}' \times \vec{s}_2) - \omega_0\vec{s}_2 \times (\mathbf{J}\vec{\omega}') + a\vec{r}_0 \times \vec{s}_0 + b\int_{t-\tau}^{t} \vec{r}_0 \times \vec{s}_0(\sigma)d\sigma - h\mathbf{D}(\vec{s}_2, \vec{s}_3)\vec{\omega}'\Big) + \frac{1}{h}(\vec{s}_0 \times \vec{r}_0)^\top \mathbf{D}^{-1}(\vec{s}_2, \vec{s}_3)\Big(\omega_0\mathbf{J}(\vec{\omega}' \times \vec{s}_2) - \omega_0\vec{s}_2 \times (\mathbf{J}\vec{\omega}') - \omega_0\vec{\omega}' \times (\mathbf{J}\vec{s}_2) + a\vec{r}_0 \times \vec{s}_0 + b\int_{t-\tau}^{t} \vec{r}_0 \times \vec{s}_0(\sigma)d\sigma\Big) + \frac{1}{h}b_4\|\vec{\omega}'\|^2 + \frac{1}{h}b_5\|\vec{r}_0 - \vec{s}_0\|\|\vec{\omega}'\|^2,$$

where $b_j > 0, j = 1, \ldots, 5$.

Taking into account the positive definiteness of $\mathbf{D}(\vec{s}_2, \vec{s}_3)$, we arrive at the estimate

$$\dot{V} \le -\chi h b_6\|\vec{\omega}'\|^2 + \chi b_7\|\vec{\omega}'\|^2 + \frac{1}{h}b_8\|\vec{r}_0 - \vec{s}_0\|\|\vec{\omega}'\| + \chi a\|\vec{r}_0 - \vec{s}_0\|\|\vec{\omega}'\| + \chi|b|\|\vec{\omega}'\|\left\|\int_{t-\tau}^{t} \vec{r}_0 \times \vec{s}_0(\sigma)d\sigma\right\| - \frac{a}{h}(\vec{s}_0 \times \vec{r}_0)^\top \mathbf{D}^{-1}(\vec{s}_2, \vec{s}_3)(\vec{s}_0 \times \vec{r}_0) + \frac{b}{h}(\vec{s}_0 \times \vec{r}_0)^\top \mathbf{D}^{-1}(\vec{s}_2, \vec{s}_3)\int_{t-\tau}^{t} \vec{r}_0 \times \vec{s}_0(\sigma)d\sigma + \frac{1}{h}b_4\|\vec{\omega}'\|^2 + \frac{1}{h}b_5\|\vec{r}_0 - \vec{s}_0\|\|\vec{\omega}'\|^2,$$

where $b_6$, $b_7$ are positive constants.

Next, construct a Lyapunov–Krasovskii functional in the form (see [28], [32], [33])

$$\widetilde{V} = V + \frac{\varepsilon}{h}\int_{t-\tau}^{t} (\sigma - t + \tau)(\vec{s}_0(\sigma) \times \vec{r}_0)^\top \mathbf{D}^{-1}(\sigma)(\vec{s}_0(\sigma) \times \vec{r}_0)d\sigma.$$

Here $\varepsilon = const > 0$, $\mathbf{D}^{-1}(\sigma) = \mathbf{D}^{-1}(\vec{s}_2(\sigma), \vec{s}_3(\sigma))$. Then

$$\dot{\widetilde{V}} \le -\chi h b_6\|\vec{\omega}'\|^2 + \chi b_7\|\vec{\omega}'\|^2 + \frac{1}{h}b_8\|\vec{r}_0 - \vec{s}_0\|\|\vec{\omega}'\| + \chi a\|\vec{r}_0 - \vec{s}_0\|\|\vec{\omega}'\| + \chi|b|\|\vec{\omega}'\|\left\|\int_{t-\tau}^{t} \vec{r}_0 \times \vec{s}_0(\sigma)d\sigma\right\| + \frac{1}{h}b_4\|\vec{\omega}'\|^2 + \frac{1}{h}b_5\|\vec{r}_0 - \vec{s}_0\|\|\vec{\omega}'\|^2 - \frac{a}{h}(\vec{s}_0 \times \vec{r}_0)^\top \mathbf{D}^{-1}(t)(\vec{s}_0 \times \vec{r}_0) + \frac{b}{h}(\vec{s}_0 \times \vec{r}_0)^\top \mathbf{D}^{-1}(t)\int_{t-\tau}^{t} \vec{r}_0 \times \vec{s}_0(\sigma)d\sigma - \frac{\varepsilon}{h}\int_{t-\tau}^{t} (\vec{s}_0(\sigma) \times \vec{r}_0)^\top \mathbf{D}^{-1}(\sigma)(\vec{s}_0(\sigma) \times \vec{r}_0)d\sigma + \frac{\varepsilon\tau}{h}(\vec{s}_0 \times \vec{r}_0)^\top \mathbf{D}^{-1}(t)(\vec{s}_0 \times \vec{r}_0).$$

With the aid of the substitution $\vec{\psi}(t) = \mathbf{D}^{-1/2}(t)(\vec{s}_0 \times \vec{r}_0)$, we obtain

$$a(\vec{s}_0 \times \vec{r}_0)^\top \mathbf{D}^{-1}(t)(\vec{s}_0 \times \vec{r}_0) + b(\vec{s}_0 \times \vec{r}_0)^\top \mathbf{D}^{-1}(t)\int_{t-\tau}^{t} \vec{s}_0(\sigma) \times \vec{r}_0 d\sigma + \varepsilon\int_{t-\tau}^{t} (\vec{s}_0(\sigma) \times \vec{r}_0)^\top \mathbf{D}^{-1}(\sigma)(\vec{s}_0(\sigma) \times \vec{r}_0)d\sigma - \varepsilon\tau(\vec{s}_0 \times \vec{r}_0)^\top \mathbf{D}^{-1}(t)(\vec{s}_0 \times \vec{r}_0) = (a - \varepsilon\tau)\vec{\psi}^\top(t)\vec{\psi}(t) + \varepsilon\int_{t-\tau}^{t} \vec{\psi}^\top(\sigma)\vec{\psi}(\sigma)d\sigma$$

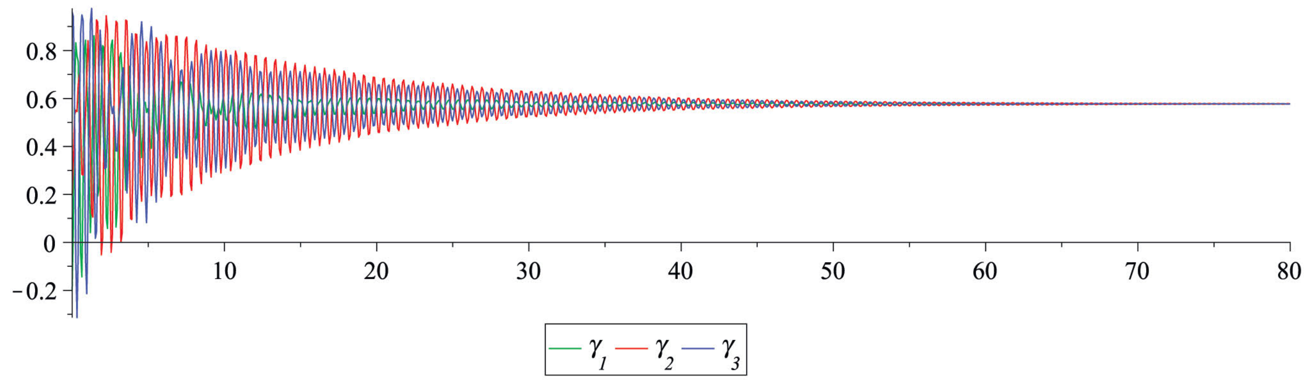



**FIGURE 2. Direction cosines. No delay, $\tau = 0$.**

$$+ b\vec{\psi}^{\top}(t)\mathbf{D}^{-1/2}(t)\int\limits_{t-\tau}^{t}\mathbf{D}^{1/2}(\sigma)\vec{\psi}(\sigma)d\sigma$$

$$= \int\limits_{t-\tau}^{t}\begin{pmatrix}\vec{\psi}(t)\\ \vec{\psi}(\sigma)\end{pmatrix}^{\top}\mathbf{W}\begin{pmatrix}\vec{\psi}(t)\\ \vec{\psi}(\sigma)\end{pmatrix}d\sigma$$

$$+ b\vec{\psi}^{\top}(t)\mathbf{D}^{-1/2}(t)\int\limits_{t-\tau}^{t}(\mathbf{D}^{1/2}(\sigma)-\mathbf{D}^{1/2}(t))\vec{\psi}(\sigma)d\sigma.$$

Here

$$\mathbf{W} = \begin{pmatrix}\left(\frac{a}{\tau}-\varepsilon\tau\right)\mathbf{I} & \frac{b}{2}\mathbf{I}\\ \frac{b}{2}\mathbf{I} & \varepsilon\mathbf{I}\end{pmatrix}$$

and $\mathbf{I}$ is the $(3\times 3)$–identity matrix.

Applying Mean Value Theorem, it is easy to verify that

$$\left\|\int\limits_{t-\tau}^{t}(\mathbf{D}^{1/2}(\sigma)-\mathbf{D}^{1/2}(t))\vec{\psi}(\sigma)d\sigma\right\| \le b_8\|\vec{\omega}'_t\|_\tau\int\limits_{t-\tau}^{t}\|\vec{s}_0(\sigma)\times\vec{r}_0\|d\sigma,$$

where $b_8$ is a positive constant.

Let us define conditions under which there exists $\varepsilon > 0$ such that the matrix $\mathbf{W}$ is positive definite. It can be shown that, to derive less conservative constraint on the parameters $a$, $b$, one should take

$$\varepsilon = \frac{a}{2\tau^2}. \tag{9}$$

As a result, we obtain the following domain of admissible values of $a$, $b$:

$$a > \tau|b|. \tag{10}$$

Thus, if the conditions (9), (10) hold, then

$$\dot{\widetilde{V}} \le -\chi hb_6\|\vec{\omega}'\|^2 + \chi b_7\|\vec{\omega}'\|^2 + \frac{1}{h}b_8\|\vec{r}_0-\vec{s}_0\|\|\vec{\omega}'\|$$

$$+\chi a\|\vec{r}_0-\vec{s}_0\|\|\vec{\omega}'\| + \chi|b|\|\vec{\omega}'\|\left\|\int\limits_{t-\tau}^{t}\vec{r}_0\times\vec{s}_0(\sigma)d\sigma\right\|$$

$$+\frac{1}{h}b_4\|\vec{\omega}'\|^2 + \frac{1}{h}b_5\|\vec{r}_0-\vec{s}_0\|\|\vec{\omega}'\|^2$$

$$+b_8\|\vec{\omega}'_t\|_\tau\|\vec{s}_0\times\vec{r}_0\|\int\limits_{t-\tau}^{t}\|\vec{s}_0(\sigma)\times\vec{r}_0\|d\sigma,$$

$$-\frac{1}{h}b_9(\vec{s}_0\times\vec{r}_0)^{\top}\mathbf{D}^{-1}(t)(\vec{s}_0\times\vec{r}_0)$$

$$-\frac{1}{h}b_{10}\int\limits_{t-\tau}^{t}(\vec{s}_0(\sigma)\times\vec{r}_0)^{\top}\mathbf{D}^{-1}(\sigma)(\vec{s}_0(\sigma)\times\vec{r}_0)d\sigma,$$

where $b_9 > 0$, $b_{10} > 0$.

In addition, there exist positive numbers $\delta, b_{11}, b_{12}$ such that the estimates

$$(\vec{s}_0\times\vec{r}_0)^{\top}\mathbf{D}^{-1}(t)(\vec{s}_0\times\vec{r}_0) \ge b_{11}\|\vec{s}_0\times\vec{r}_0\|^2 \ge b_{12}\|\vec{s}_0-\vec{r}_0\|^2$$

are valid for $\|\vec{s}_0-\vec{r}_0\| < \delta$.

Hence, if $\chi, \delta$ are sufficiently small and $h$ is sufficiently large, then

$$\frac{1}{4}\left(\|\vec{r}_0-\vec{s}_0\|^2+\chi b_1\|\vec{\omega}'\|^2\right) \le \widetilde{V}$$

$$\le \|\vec{r}_0-\vec{s}_0\|^2 + 2\chi b_2\|\vec{\omega}'\|^2 + b_{13}\int\limits_{t-\tau}^{t}\|\vec{s}_0(\sigma)\times\vec{r}_0\|^2 d\sigma,$$

$$\dot{\widetilde{V}} \le -\frac{1}{2}\chi hb_6\|\vec{\omega}'\|^2 - \frac{1}{2h}b_9\, b_{12}\|\vec{s}_0-\vec{r}_0\|^2$$

$$-\frac{1}{2h}b_{10}b_{11}\int\limits_{t-\tau}^{t}\|\vec{s}_0(\sigma)\times\vec{r}_0\|^2\,d\sigma$$

for $\|\vec{s}_0-\vec{r}_0\| < \delta$, $\|\vec{\omega}'_t\|_\tau < \delta$. The fulfilment of these inequalities implies (see [28]) the asymptotic stability of the programmed motion (1).

As a result, we arrive at the following theorem.

*Theorem 1:* Under the condition (10), there exists $h_0 > 0$ such that the programmed motion (1) is asymptotically stable for all $h \ge h_0$.

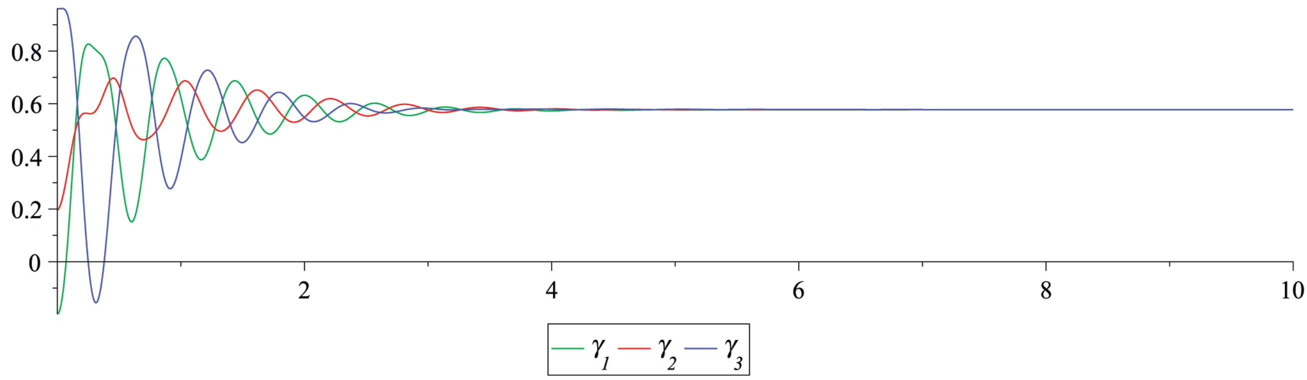



**FIGURE 3.** Direction cosines. Distributed delay, $\tau = 0.3$.

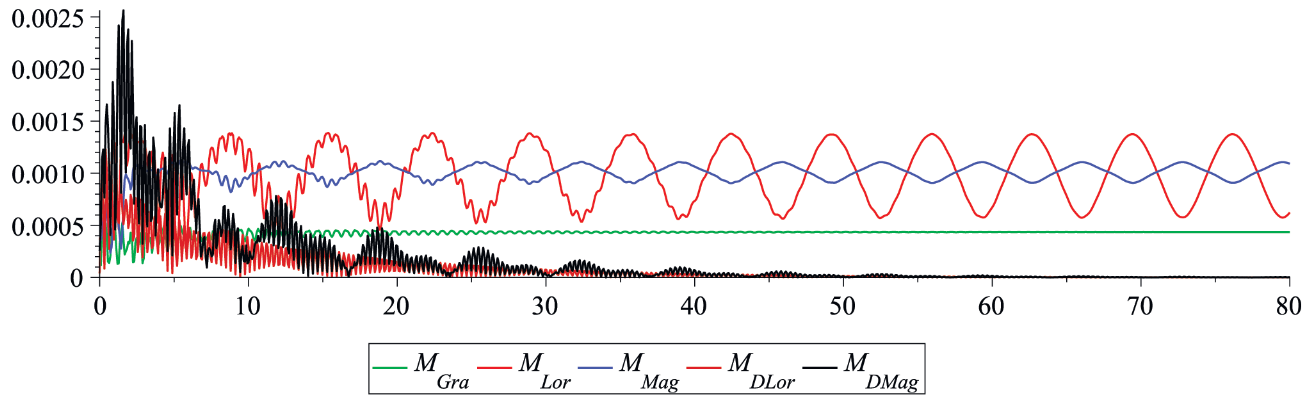



**FIGURE 4.** Gravitational and control torques. No delay, $\tau = 0$.

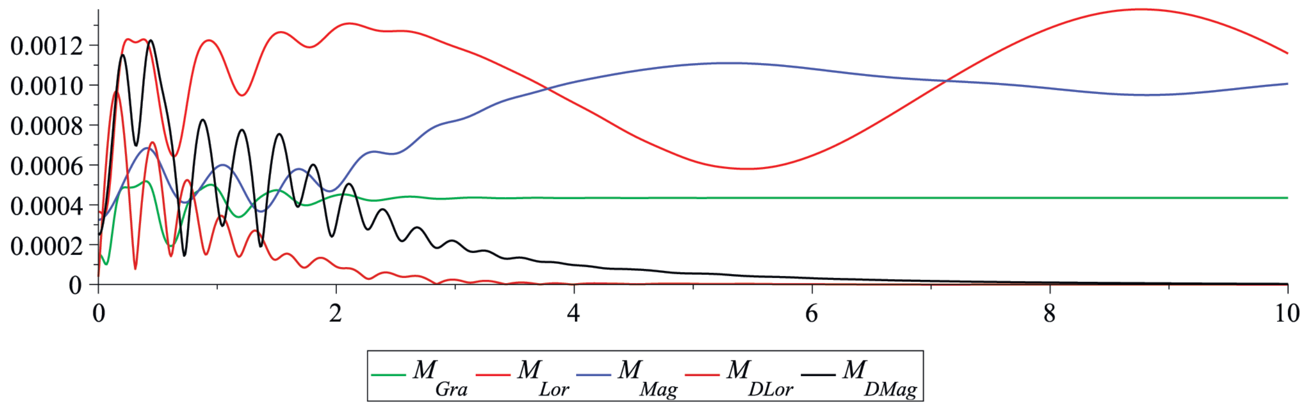



**FIGURE 5.** Gravitational and control torques. Distributed delay, $\tau = 0.3$.

## V. COMPUTER MODELING

Let us consider an AES with tensor of inertia $\mathbf{J} = \mathrm{diag}(1000, 700, 800)$ and the total electric charge $Q = 0.005$. Here and in what follows all dimension values are in the SI units. The program motion is the equilibrium $\vec{r}_0 = \vec{s}_0$ in the orbital frame, where $\vec{r}_0 = (\gamma_1, \gamma_2, \gamma_3)^\top = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})^\top$. The attitude stabilization problem is solved with the use of control torques in accordance with equations (7), (4), where we put $a = 0.1$, $h_L = 0.0983$, $h_M = 0.4966$.

Let the initial attitude position of AES is defined by ''aircraft'' angles $\phi(\xi) = 0.2$, $\psi(\xi) = -0.2$, $\theta(\xi) = 0.2$ and angular velocity components $\omega_x(\xi) = 0.1$, $\omega_y(\xi) = 1.1$, $\omega_z(\xi) = 0.1$ for $\xi \in [-\tau, 0]$. First consider the case where $\tau = 0$, that is control system with no integral term ($b = 0$).

In this case the control process converges to the program motion as shown in Fig. 2. Here and on all other graphs, the latitude argument $u = \omega_0\, t$ is plotted along the abscissa.

Now consider the case where $\tau = 0.3$, $b = 1$. In this case the control process converges to the program motion as shown in Fig. 3.

It can be easily seen that the introduction of integral component with distributed delay into the control system can significantly reduce the oscillation of the control process and accelerate its convergence to an asymptotically stable solution. The results obtained indicate that the control method used in this work turns out to be better than the methods that were previously used [17] to solve the problem of AES monoaxial stabilization.

It is worth to note that the problem of AES attitude stabilization is a nonlinear dynamic problem. Therefore, ensuring a higher smoothness of the control process can be fundamentally important when controlling a large space structure, especially when the eigenfrequencies of the system are close to resonance ratios [34], [35].

The control torques were also calculated for the above two cases. The results are shown in Fig. 4 for the case $\tau = 0$ ($b = 0$) and in Fig. 5 for the case $\tau = 0.3$, $b = 1$.

It can be seen that the presence of distributed delay (integral term) makes it possible to halve the values of the control torques at the beginning of the control process.

## VI. CONCLUSION

An artificial Earth satellite (AES) with an arbitrary triaxial ellipsoid of inertia in a circular equatorial orbit is considered. The mode of monoaxial stabilization in the orbital frame is considered as the programmed mode of AES attitude motion. To stabilize the AES axis in the programmed motion mode, an electrodynamic attitude control system is used, which generates the Lorentz torque and the magnetic torque. These two control torques provide compensation of the disturbing gravitational torque, and also implement the restoring and damping components that allow stabilizing the AES axis in the programmed attitude motion mode.

The novelty of the problem statement lies in the fact that, in contrast to the previously known works using an electrodynamic attitude control system, there are no restrictions on the position of the stabilized axis in the orbital frame.

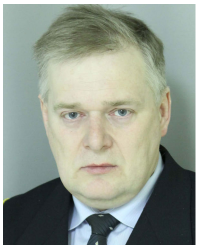
The novelty of the approach to solving the problem lies in the development of the concept of electrodynamic attitude control by using the restoring torque with a distributed delay. A simple and easily verified sufficient condition for the asymptotic stability of the programmed attitude motion of the AES has been obtained in a nonlinear formulation with the use of the approach developed by the authors to constructing Lyapunov functions. Thus, the development of the theory of electrodynamic attitude control is given for solving the practically important problem of monoaxial attitude stabilization of a satellite not only in a straight position, but also in an arbitrary position in the orbital frame. Numerical modeling confirms the conclusion proved in the theorem.

As the result, an extension of the concept of electrodynamic attitude control with electrodynamic compensation of the perturbing torque and distributed delay in integral term was proposed. Conditions were established under which the electromagnetic control supports AES monoaxial attitude stabilization.

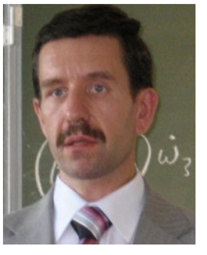
## REFERENCES

[1] J. R. Wertz, *Spacecraft Attitude Determination and Control*. Dordrecht, The Netherlands: D. Reidel Publishing Co., 1985.
[2] P. C. Hughes, *Spacecraft Attitude Dynamics*. New York, NY, USA: Wiley, 1986.
[3] M. L. Battagliere, F. Santoni, F. Piergentili, M. Ovchinnikov, and F. Graziani, ''Passive magnetic attitude stabilization system of the EduSAT microsatellite,'' *Proc. Inst. Mech. Eng., G, J. Aerosp. Eng.*, vol. 224, no. 10, pp. 1097–1106, Oct. 2010.
[4] A. A. Tikhonov, ''A method of semipassive attitude stabilization of a spacecraft in the geomagnetic field,'' *Cosmic Res.*, vol. 41, no. 1, pp. 69–79, 2003.
[5] D. K. Giri and M. Sinha, ''Magnetocoulombic attitude control of earth-pointing satellites,'' *J. Guid., Control, Dyn.*, vol. 37, no. 6, pp. 1946–1960, Nov. 2014.
[6] Y. A. Abdel-Aziz and M. Shoaib, ''Attitude dynamics and control of spacecraft using geomagnetic Lorentz force,'' *Res. Astron. Astrophys.*, vol. 15, no. 1, pp. 127–144, Jan. 2015.
[7] K. A. Antipov and A. A. Tikhonov, ''Parametric control in the problem of spacecraft stabilization in the geomagnetic field,'' *Autom. Remote Control*, vol. 68, no. 8, pp. 1333–1345, 2007.
[8] A. A. Tikhonov, D. T. Spasic, K. A. Antipov, and M. V. Sablina, ''Optimizing the electrodynamical stabilization method for a man-made earth satellite,'' *Autom. Remote Control*, vol. 72, no. 9, pp. 1898–1905, Sep. 2011.
[9] A. Y. Aleksandrov and A. A. Tikhonov, ''Electrodynamic stabilization of earth-orbiting satellites in equatorial orbits,'' *Cosmic Res.*, vol. 50, no. 4, pp. 313–318, Jul. 2012.
[10] A. A. Martynyuk, V. Lakshmikantham, and S. Leela, *Stability Analysis of Nonlinear Systems*. New York, NY, USA: Marcel Dekker, 1989.
[11] P. S. Krasil'nikov and A. Y. Maiorov, ''On the stability of equilibrium of a mechanical system with tracking, potential, and small dissipative forces,'' *Mech. Solids*, vol. 53, no. S2, pp. 52–59, Dec. 2018.
[12] V. V. Grigoriev, S. V. Bystrov, O. K. Mansurova, I. M. Pershin, A. B. Bushuev, and V. A. Petrov, ''Exponential stability regions estimation of nonlinear dynamical systems,'' *Mekhatronika, Avtomatizatsiya, Upravlenie*, vol. 21, no. 3, pp. 131–135, 2020.
[13] G. I. Melnikov, V. G. Melnikov, N. A. Dudarenko, and V. V. Talapov, ''The method of exponential differential inequality in the estimation of solutions of nonlinear systems in the vicinity of the zero of the state space,'' in *Proc. Int. Conf. Mod. Eng., Sci. Educ.*, in Lecture Notes in Mechanical Engineering, 2021, pp. 204–211.
[14] V. I. Zubov, *Theorie de la Commande*. Moscow, Russia: Mir, (in French), 1978.
[15] E. Y. Smirnov, ''Problems of stabilization of nonstationary systems with incomplete feedback,'' *Autom. Remote Control*, vol. 35, no. 11, pp. 1725–1733, 1974.
[16] V. I. Kalenova and V. M. Morozov, ''Novel approach to attitude stabilization of satellite using geomagnetic Lorentz forces,'' *Aerosp. Sci. Technol.*, vol. 106, Nov. 2020, Art. no. 106105.
[17] A. Y. Aleksandrov and A. A. Tikhonov, ''Monoaxial electrodynamic stabilization of earth satellite in the orbital coordinate system,'' *Autom. Remote Control*, vol. 74, no. 8, pp. 1249–1256, Aug. 2013.
[18] B. You, X. Yu, D. Liang, X. Liu, and X. Yang, ''Numerical and experimental investigation on dynamics of deployable space telescope experiencing deployment and attitude adjustment motions coupled with laminated composite shell,'' *Mech. Based Des. Struct. Mach.*, pp. 1–20, Jan. 2020, doi: 10.1080/15397734.2020.1717341.
[19] A. Formal'sky, ''On a modification of the PID controller,'' *Dyn. Control*, vol. 7, no. 3, pp. 269–277, 1997.
[20] V. V. Beletsky, *Motion of an Artificial Satellite About Its Center of Mass*. Jerusalem, Israel: Israel Program for Scientific Translation, 1966.
[21] L. A. Klimina and Y. D. Selyutskiy, ''Method to construct periodic solutions of controlled second-order dynamical systems,'' *J. Comput. Syst. Sci. Int.*, vol. 58, no. 4, pp. 503–514, Jul. 2019.

[22] A. Y. Aleksandrov and A. A. Tikhonov, "Asymptotic stability of a satellite with electrodynamic attitude control in the orbital frame," *Acta Astronautica*, vol. 139, pp. 122–129, Oct. 2017.
[23] A. A. Tikhonov and K. G. Petrov, "Multipole models of the earth's magnetic field," *Cosmic Res.*, vol. 40, no. 3, pp. 203–212, 2002.
[24] K. A. Antipov and A. A. Tikhonov, "Multipole models of the geomagnetic field: Construction of the nth approximation," *Geomagnetism Aeronomy*, vol. 53, no. 2, pp. 257–267, Apr. 2013.
[25] A. Ajorkar, A. Fazlyab, F. Saberi, and M. Kabganian, "Design of an adaptive-neural network attitude controller of a satellite using reaction wheels," *J. Appl. Comput. Mech.*, vol. 1, no. 2, pp. 67–73, 2015.
[26] M. Dosaev, "Interaction between internal and external friction in rotation of vane with viscous filling," *Appl. Math. Model.*, vol. 68, pp. 21–28, Apr. 2019.
[27] M. Levskii, "Application of complex functional of quality in optimal control of spacecraft motion," *J. Appl. Comput. Mech.*, vol. 7, no. 1, pp. 262–269, 2021.
[28] V. Kharitonov, *Time-Delay Systems: Lyapunov Functionals and Matrices*. Basel, Switzerland: Birkhauser, 2013.
[29] A. Y. Aleksandrov, "The stability of the equilibrium positions of non-linear non-autonomous mechanical systems," *J. Appl. Math. Mech.*, vol. 71, no. 3, pp. 324–338, Jan. 2007.
[30] A. Y. Aleksandrov and A. A. Kosov, "Asymptotic stability of equilibrium positions of mechanical systems with a nonstationary leading parameter," *J. Comput. Syst. Sci. Int.*, vol. 47, no. 3, pp. 332–345, Jun. 2008.
[31] A. Y. Aleksandrov, A. A. Kosov, and Y. Chen, "Stability and stabilization of mechanical systems with switching," *Autom. Remote Control*, vol. 72, no. 6, pp. 1143–1154, Jun. 2011.
[32] A. Aleksandrov and E. Aleksandrova, "Stability analysis of nonlinear mechanical systems with delay in positional forces," *Nonlinear Dyn. Syst. Theory*, vol. 18, no. 3, pp. 225–232, 2018.
[33] D. Efimov and A. Aleksandrov, "Analysis of robustness of homogeneous systems with time delays using Lyapunov–Krasovskii functionals," *Int. J. Robust Nonlinear Control*, vol. 31, no. 9, pp. 3730–3746, Jun. 2021.
[34] A. A. Tikhonov, "Resonance phenomena in oscillations of a gravity-oriented rigid body. Part 4: Multifrequency resonances," *Vestnik Sankt-Peterburgskogo Universiteta. Ser 1 Matematika Mekhanika Astronomiya*, no. 1, pp. 131–137, 2000
[35] E. A. Kosjakov and A. A. Tikhonov, "Differential equations for librational motion of gravity-oriented rigid body," *Int. J. Non-Linear Mech.*, vol. 73, pp. 51–57, Jul. 2015.



**ALEXANDER ALEKSANDROV** received the M.S. degree in applied mathematics and the Ph.D. degree in automatic control from Leningrad State University, Russia, in 1985 and 1989, respectively, and the D.Sc. (higher doctorate) degree in mathematical modeling from St. Petersburg Institute for Informatics and Automation of the Academy of Science of Russia, in 2000. He is currently a Professor with the Faculty of Applied Mathematics and Control Processes, St. Petersburg State University. He is the author of more than 250 scientific articles and books. His current research interests include stability of nonlinear time-varying systems, switched systems, time-delay systems, nonlinear, and robust control.



**ALEXEY A. TIKHONOV** received the M.S. degree in mechanics and the Ph.D. degree in theoretical mechanics from Leningrad State University, Russia, in 1985 and 1989, respectively, and the D.Sc. (higher doctorate) degree in theoretical and applied mechanics from the State Committee of Russian Federation for High Education, in 2002. He is currently a Professor with the Department of Theoretical and Applied Mechanics, Faculty of Mathematics and Mechanics, St. Petersburg State University. He is the author of more than 200 scientific articles and books. His research interests include rigid body dynamics, spacecraft attitude dynamics and control, analytical mechanics, nonlinear vibrations, stability of motion, micro-electro-mechanical systems, mathematical, and computer modeling in mechanics.

He is a member of the Russian National Committee for Theoretical and Applied Mechanics. He is also a member of Expert Staff of the Russian Academy of Sciences. He was a recipient of the Russian Academy of Sciences F. A. Tsander Award, in 2020.