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Improving Stock Price Prediction Using Combining Forecasts Methods

MOHAMMAD RAQUIBUL HOSSAIN [1,2], MOHD TAHIR ISMAIL [1], AND SAMSUL ARIFFIN BIN ABDUL KARIM [3]

[1] School of Mathematical Sciences, Universiti Sains Malaysia, Pulau Pinang 11800, Malaysia
[2] Department of Applied Mathematics, Noakhali Science and Technology University, Noakhali 3814, Bangladesh
[3] Fundamental and Applied Sciences Department, Centre for Systems Engineering (CSE), Institute of Autonomous System, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, Seri Iskandar, Perak 32610, Malaysia

Corresponding author: Samsul Ariffin Bin Abdul Karim (samsul_ariffin@utp.edu.my)

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ABSTRACT This study presents an outcome of pursuing better and effective forecasting methods. The study primarily focuses on the effective use of divide-and-conquer strategy with Empirical Mode Decomposition or briefly EMD algorithm. We used two different statistical methods to forecast the high-frequency EMD components and the low-frequency EMD components. With two statistical forecasting methods, ARIMA (Autoregressive Integrated Moving Average) and EWMA (Exponentially Weighted Moving Average), we investigated two possible and potential hybrid methods: EMD-ARIMA-EWMA, EMD-EWMA-ARIMA based on high and low-frequency components. We experimented with these methods and compared their empirical results with four other forecasting methods using five stock market daily closing prices from the S&P/TSX 60 Index of Toronto Stock Exchange. This study found better forecasting accuracy from EMD-ARIMA-EWMA than ARIMA, EWMA base methods and EMD-ARIMA as well as EMD-EWMA hybrid methods. Therefore, we believe frequency-based effective method selection in EMD-based hybridization deserves more research investigation for better forecasting accuracy.

INDEX TERMS EMD, combining forecasts, time-series analytics, ARIMA, EWMA.

I. INTRODUCTION

In many actual events, individuals or organizations require making decisions that involve some certainty and uncertainty. That is why we investigate deterministic, probabilistic, or mixed scientific methods for a better decision. Some events are related to time series (data series indexed with time order). Avid practitioners search for effective and robust methods to make a better decision for important time series based on future events. Nevertheless, projecting future reality or forecasting future events is quite challenging due to methodological limitations and unexpected uncertainty. Therefore, the research study's scope of the present paper is to find improved or better forecasting methods to outperform existing or benchmark methods.

Indeed, there are effective benchmark methods with their different variants. The Autoregressive Integrated Moving Average (ARIMA) is still widely used for its satisfactory effectiveness. Some recent ARIMA based works encompass [1]–[4]. Smoothing methods are also efficient and much used till today, and some recent research studies include [5]–[7]. These comparatively basic and traditionally useful methods serve up to general-purpose expectation level of forecasting accuracy. However, many researchers are continually investigating to find better and more effective methods due to the limitations of benchmark methods and expectation for more accuracy. Some of them involve fuzzy theory-based modeling [8], [9]; some are hybridization with neural networks and machine learning methods [10]–[15]. With the current popularity in non-time-series applications, machine learning or artificial neural networks methods got adequate practical attention and research interest.

However, these methods fail to win the race of forecasting competitions [16], [17]; they also have some drawbacks, including sophistication and high computational cost. In quest of better forecasting methods, some research studies

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focus on hybrid forecasting methods based on Empirical Mode Decomposition (EMD), a locally adaptive data decomposition algorithm which was originally devised for signal processing. Thus, this research study focuses on EMD based hybrid forecasting with two prominently useful statistical forecasting methods ARIMA and Exponentially Weighted Moving Average (EWMA). The EMD-ARIMA-EWMA or EMD-EWMA-ARIMA or both methods have high potentiality to be very useful because of the theoretical combination from EMD components.

The concept of EMD is deeply rooted from signal processing and, therefore, widely used in that field. Two crucial original contributions of EMD in time series were the works of [18] (the basic and foundational resource) and [19] (application in the financial domain). Besides signal processing, EMD based research has broad application areas for analyzing and forecasting time series data in various fields. Some recent EMD based forecasting studies include [11]–[15], [20], [21]. Among others, these studies reflect the effectiveness of EMD-based hybridization methods.

EMD is efficient in adaptive decomposition, while statistical forecasting methods ARIMA and EWMA also are simple but effective. We hypothesized that all these three approaches' effective synergy could produce better forecasting results for nonlinear nonstationary mixed-frequency time series. Using statistical forecasting methods ARIMA and EWMA with EMD based on high-frequency and low-frequency EMD components, this research study investigates forecasting strategy improvement, which is the aim of the study. We employed five stock data sets of Toronto Stock Exchange based S&P/TSX 60 Index to experiment with the effectiveness of proposed EMD-ARIMA-EWMA and EMD-EWMA-ARIMA methods. Also, their forecasting performances were compared with benchmark and hybrid methods using both absolute and relative error measures.

The following section presents statistical benchmark methods and their EMD-based hybridizations. Other sections subsequently convey proposed experimental methods, experimental results, discussion for study outcome and conclusion.

II. THE TWO STATISTICAL BENCHMARK METHODS AND THEIR EMD-BASED HYBRIDIZATIONS

This section mainly and concisely describes two selected statistical benchmark methods ARIMA and EWMA, and their EMD-based hybridizations EMD-ARIMA and EMD-EWMA.

A. ARIMA

The concept of Autoregressive Moving Average (ARMA) [2], [4], which is applicable for stationary time series, comes before ARIMA concept of nonstationary series. Generally, a nonstationary series $X = \{x_1, x_2, x_3, \ldots, x_t, \ldots\}$ can be transformed into stationary $Y = \{y_1, y_2, y_3, \ldots, y_t, \ldots\}$ by taking lag difference once or twice, known as the order of differencing. An $ARMA\,(p, q)$ method has the following form:

$$y_t = l + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \ldots + \theta_p y_{t-p} + s_t + \varphi_1 s_{t-1} + \varphi_2 s_{t-2} + \ldots + \varphi_q s_{t-q} \quad (1)$$

where l is constant; y_i's are past p autoregressive terms; s_j's are past q error terms or random shocks; θ_i's and φ_j's are parameters. If the order of differencing (also equivalent to the order of integration) d is required for nonstationary series X to make it stationary Y, ARIMA is better written with notation $ARIMA\,(p, d, q)$.

B. EWMA

EWMA (also known as single or simple exponential smoothing) is presented in [6] and [7]. EWMA produces smoothing series $S = \{S_2, S_3, \ldots, S_t, \ldots\}$ from an original series $X = \{x_1, x_2, x_3, \ldots, x_t, \ldots\}$. Started with $S_2 = x_1$ as a seed value, recursive relation of EWMA has the following form:

$$S_2 = x_1 \quad (2)$$

$$S_t = \omega x_{t-1} + (1-\omega)\, S_{t-1}, \quad 0 < \omega < 1,\ t \geq 3, \quad (3)$$

where ω is smoothing parameter, t is time order and S_t is present smoothing term found from the convex combination of most recent original data x_{t-1} and most recent smoothing data S_{t-1}. EWMA's abbreviated form is:

$$S_t = \omega \sum_{i=1}^{t-2} (1-\omega)^{i-1} x_{t-i} + (1-\omega)^{t-2} S_2, \quad t \geq 3 \quad (4)$$

The weights $\omega\,(1-\omega)^t$ for past data geometrically or exponentially diminishes as $(1-\omega)^t$ decrease with the increase of t.

C. EMD-BASED HYBRIDIZATIONS

[18] proposed that EMD is an integral part of Hilbert-Huang transforms for signal processing analysis purposes. From the practical viewpoint, EMD is an adaptive decomposition algorithm which upholds local features. Analysis or decomposition of EMD signal data reserves time domain. The principal procedure of EMD, known as the sifting process, uses local modal values of a time series to produce an orthogonal set of sub-signals or equivalently sub-series of different amplitude and frequency. These orthogonal subseries commonly called Intrinsic Mode Functions (IMFs) must qualify some criteria. The EMD sifting process is repeated until all potential IMFs extraction is completed. The final remnants of the whole process are called the residual. An IMF is defined with two qualifying criteria. One criterion should have a zero mean value, i.e., it is an oscillatory function that oscillates around zero. Another criterion is the difference between the number of extrema, and the number of zero-crossings will be zero or one but not beyond that. In the sifting process, EMD first produces comparatively high frequency or rapidly oscillatory IMFs and gradually extracts comparatively low frequency or slowly oscillating IMFs and residue finally. If N is the total quantity of data, the maximum quantity of IMFs n is less than $\log_2(N)$.

EMD sifting process applied in IMFs extraction from a series or signal data set $x(t)$ includes the following steps:

Step 1: To find local extrema (i.e., minima and maxima) of $x(t)$ or temporary remainder (remainder found immediately after each IMF collection).

Step 2: To fit and formulate two cubic spline envelopes with local extrema-based two sub-datasets. One envelope namely upper envelope (u_E) is for data above local mean, i.e., almost half of all data. Another envelope known as lower envelope (l_E) fits data below local mean, i.e., remaining half of dataset.

Step 3: To obtain arithmetic mean of u_E and l_E as mean envelope m_E.

$$m_E = \frac{u_E + l_E}{2} \tag{5}$$

Step 4: m_E is subtracted from $x(t)$ and 1st temporary remainder H_1 is found.

$$x(t) - m_{E1} = H_1 \tag{6}$$

Step 5: Check if H_1 follows IMF characteristic definition. If it does, then it is an IMF and follows step 6. If H_1 does not satisfy IMF definition, the process for next mean envelope and temporary remainder finding is done for it following steps 2 to 4 which subsequently produces other mean envelopes (from 2nd to higher) and relevant temporary remainders.

$$H_1 - m_{E2} = H_2, \quad H_2 - m_{E3} = H_3, \ \ldots, H_{k-1} - m_{Ek} = H_k \tag{7}$$

Stopping criterion like Cauchy Convergence Criterion (CCC) involving standard deviation is applied in IMF sifting process, which is defined as:

$$CCC_k = \sum_{t=0}^{T} \frac{(H_{k-1}(t) - H_k(t))^2}{H_{k-1}^2(t)} \tag{8}$$

IMF sifting process is terminated when CCC_k becomes smaller than a pre-set minimum value.

Step 6: If an H_k satisfies stopping criterion, it is considered and selected as an IMF. Let 1st IMF be $F_1 = H_k$. Now, a continual remainder is calculated by subtracting F_1 from $x(t)$. Thus, 1st continual remainder is found as $R_1 = x(t) - F_1$. Likewise, following steps 1 to 5 current continual remainder R_i, other next continual remainders R_{i+1}'s are calculated during subsequent IMFs extraction.

$$R_2 = R_1 - F_2, \quad R_3 = R_2 - F_3, \ \ldots, R_n = R_{n-1} - F_n \tag{9}$$

Continual remainders are repeatedly obtained until the last one represents a monotonic function or a monic of first-degree function. The last or final continual remainder R_n of the process is known as residual.

Even though EMD or Hilbert-Huang transforms were initially developed for signal analysis purposes, many research studies of other fields, including economics and finance, used it widely to analyze and forecast time series. Accordingly, EMD based hybridizations involve a significant part in time series-based research domains. Both EMD-ARIMA (focused in [22] and [23]) and EMD-EWMA hybrid methods have similar algorithms. The first step of these methods involves decomposing the given series or signal using EMD sifting process to extract EMD components. The second step involves fitting and forecasting the already obtained EMD component using a method (here ARIMA or EWMA). In the third step, all the forecasted point data or subseries are aggregated to produce final results, and errors are calculated from this final resultant series data and test data.

III. DESCRIPTION OF PROPOSED EXPERIMENTAL METHODS

This section describes two experimental methods and error measures used for performance comparison.

A. EXPERIMENTAL METHODS

This study investigated the frequency-based method selection for EMD components. Firstly, EMD sifting algorithm was applied to obtain EMD components, i.e., IMFs along with residual. These components were classified into two categories. One category, say category I, is a high-frequency category that involves rapidly oscillating stationary IMFs; another category, category II, is the low-frequency category involving the IMFs from slowly oscillating nonstationary to residual (refer to Fig. 2). For considering EMD components in these frequency categories, the Augmented Dickey-Fuller (ADF) test was used. For any EMD component, if the p-value of ADF test is less than 5% or 0.05, it fails to accept the null hypothesis that the EMD component possesses unit root (or in other words it is non-stationary). Also, it cannot reject the null hypothesis if the p-value is more significant than 0.05. Therefore, when the p-value of any EMD component is smaller than 0.05, it is stationary or category I EMD component. Two statistical methods ARIMA and EWMA were used to fit and forecast EMD components in these categories, which led to two experimental methods. One method is EMD-ARIMA-EWMA, where stationary components were fitted and forecasted using ARIMA while remaining non-stationary components were fitted and forecasted with EWMA. Another method EMD-EWMA-ARIMA applies EWMA for stationary and ARIMA for nonstationary components.

The experimental method I: EMD-ARIMA-EWMA

Let $EMDcom_i \in \{IMF_1, IMF_2, \ldots, IMF_i, \ldots, IMF_n, residual\}$, $i \in \{1, 2, 3, \ldots, n+1\}$ and $i \in N$.

$$forecast1(EMDcom_i) = \begin{cases} ARIMA(EMDcom_i), & if\ p-value(EMDcom_i) \le 0.05 \\ EWMA(EMDcom_i), & if\ p-value(EMDcom_i) > 0.05 \end{cases} \tag{10}$$

$$EMD-ARIMA-EWMA_{forecast} = \sum forecast1(EMDcomp_i) \tag{11}$$

Experimental method II: EMD-EWMA-ARIMA

$$forecast2\,(EMDcom_i) = \begin{cases} EWMA\,(EMDcom_i)\,, & \\ \quad if\ p-value\,(EMDcom_i) \leq 0.05 \\ ARIMA\,(EMDcom_i)\,, & \\ \quad if\ p-value\,(EMDcom_i) > 0.05 \end{cases} \quad (12)$$

$$EMD-EWMA-ARIMA_{forecast} = \sum forecast2\,(EMDcom_i) \quad (13)$$

Theoretically, it is expected that suitable model selection and parameter value computation of any method (here ARIMA and EWMA) can fit data optimally to produce better forecast data. Hence, if two different suitable and effective methods are used for different types or categories of EMD components, the new combined method is expected to improve the forecast results of two individual methods. Since EMD is a locally adaptive data-driven approach, if EMD components are efficiently fitted and forecasted with optimal models, the final aggregate result is expected to be superior to its constituents. In EMD-ARIMA-EWMA, ARIMA and EWMA were optimized by information criterion, e.g., AIC and optimal parameter selection (also parameter values come from optimization approaches with the underlying methods), which is in support of principle of parsimony and optimum model selection to avoid under-fitting or over-fitting. However, the out-sample forecast errors or accuracy of a method are better indicators for its outperformance than compared methods.

B. ALGORITHMS OF EXPERIMENTAL METHODS

Algorithms of the two experimental methods EMD-ARIMA-EWMA and EMD-EWMA-ARIMA are very similar. Therefore, below is presented algorithm only for EMD-ARIMA-EWMA method.

A brief explanation of EMD-ARIMA-EWMA algorithm is presented here. In step 2, input data X is read and stored (here 1416 data in each dataset). Then step 3 X is split into training subset X_{train} (here 1400 data) and test subset X_{test} (16 data). In step 4, EMD algorithm is applied to the training subset X_{train} and EMD components are computed and stored (here 9 IMFs and residual). In step 5, EMD components are tested for whether it is stationary or nonstationary using ADF test. An EMD component is fitted and forecasted using ARIMA method if it is stationary; while an EMD component is nonstationary, it is fitted and forecasted using EWMA. In contrast, it is in the opposite direction for EMD-EWMA-ARIMA. In step 6, all component forecasts are aggregated to obtain complete final forecast data. In step 7, an error function is defined with arbitrary variable M and N which will respectively take test data and forecasted data as input and produce three types of error or accuracy results which are defined in next sub-section. In step 8, the errors of the method are computed using the error function with test data X_{test} and forecasted data $aggr_{forecast}$. In step 9, error results and forecasted data are given as output.

Algorithm EMD-ARIMA-EWMA

Input: X, a pre-processed stock price dataset.
Output: Error results from EMD-ARIMA-EWMA forecast.
Step 1: Begin
Step 2: Read X
Step 3: Split X into X_{train} and X_{test} and store; set future horizon h
Step 4: Apply EMD sifting process on X_{train}; store IMFs $IMF_1, IMF_2, \ldots, IMF_i, \ldots, IMF_n$ and $residual$
Step 5: If IMF_i is stationary or category I component, fit and forecast with ARIMA method, i.e., $ARIMA\,(IMF_i)$ and $forecast\,(ARIMA\,(IMF_i)\,, h)$; if IMF_i or $residual$ is nonstationary, fit and forecast with EWMA method, i.e., $EWMA\,(IMF_i)$ and $forecast\,(EWMA\,(IMF_i)\,, h)$
Step 6: Aggregate all forecast data, i.e.,

$$aggr_{forecast} = \sum_{1}^{i} \begin{pmatrix} forecast\,(ARIMA\,(IMF_i)\,, h) \\ +forecast\,(EWMA\,(IMF_i)\,, h) \\ +forecast\,(EWMA\,(residual)\,, h) \end{pmatrix}$$

Step 7: Define $error\,(M, N)$
Step 8: Compute error, $err = error\left(X_{test}, aggr_{forecast}\right)$
Step 9: Print error err and output $aggr_{forecast}$ for EMD-ARIMA-EWMA method
Step 10: End

C. ERROR TOOLS FOR COMPARING FORECASTING PERFORMANCE

This research study used two absolute error measuring tools and one relative error tool to compare the forecast performance of employed methods. The absolute error tools are Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). Also, the relative error tool is Mean Absolute Percentage Error (MAPE). These error formulae are defined in (14-16):

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{1}^{n} D_t^2}{n}} \quad (14)$$

$$\mathrm{MAE} = \frac{\sum_{1}^{n} |D_t|}{n} \quad (15)$$

$$\mathrm{MAPE} = \frac{\sum_{1}^{n} \left|\frac{D_t}{y_t}\right|}{n}, \quad (16)$$

where $D_t = y_t - f_t$, the difference of forecast data from original test data; t is time sequence, and n is the quantity of test data (or forecast data). If the value of error tools for any method is smaller, then the forecasting method is considered better, i.e., when a method produces immense error value, its forecasting performance is poor.

IV. EXPERIMENTAL RESULTS

This section presents experimental data sets (with summary statistics and graphs) and forecast accuracy results of all methods used in this study (with brief description on computation time).

A. DATASETS

Data sets used for experimenting the proposed EMD-ARIMA-EWMA method are closing price data of five stock listed companies namely Bank of Montreal (BMO), Bank of Nova Scotia (BNS), Imperial Oil Limited (IMO), Sun Life Financial Inc. (SLF) and TC Energy Corporation (TRP). These companies are included in the S&P/TSX 60 Index of Toronto Stock Exchange. Each set has data of 1416 days (from December 31, 2013 to August 21, 2019), presenting the closing price for each day. For method implementation, 1400 data (from December 31, 2013 to July 29, 2019) were separated for training purposes. The remaining 16 data (from July 30, 2019 to August 21, 2019) were kept for testing purposes. Furthermore, four subsets from testing data were taken at four future horizons, i.e., h = 4, 8, 12 and 16. Training data of all five sets are graphically presented in Fig. 1. These data are also concisely presented in Table 1 with summary statistics, including mean, median, minimum, maximum, Coefficient of Variation (or briefly COV), skewness, and kurtosis. Furthermore, typical EMD components (for BMO data set) are shown in Fig. 2, reflecting the price dynamics from a granular point of view.

Fig.1 portrays that all the series are fluctuating oscillations of upward and downward. It can also be seen that all the stock price effect by mini-recession on 2015-2016 before moving upward again. Meanwhile, Table 1 shows that the mean of BMO closing stock prices is the highest, but all stocks have similar COV around 0.1, which indicate a similar investment risk. Besides, the five stocks' distribution are not symmetric, where some stocks are positively skewed, and some are negatively skewed. Moreover, the distributions are also platykurtic. Also, Fig. 2 displays EMD decomposed components of the BMO data set from high frequency to low frequency with the residual to capture the data's remaining noise and essential features.

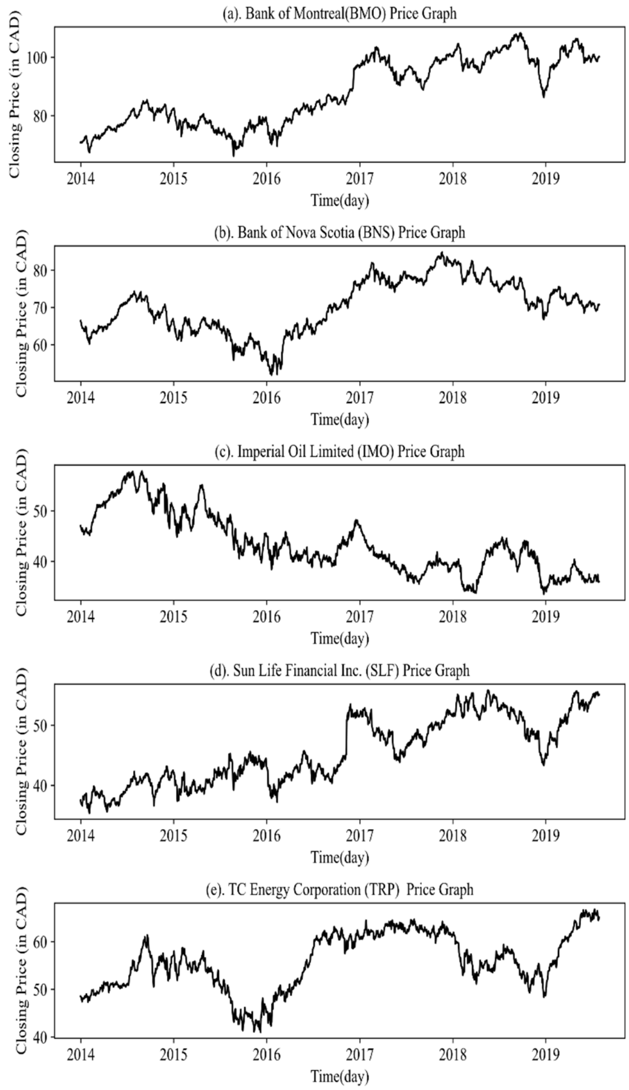

FIGURE 1. Closing price graphs for five stocks (a) BMO, (b) BNS, (c) IMO, (d) SLF and (e) TRP.

B. RESULTS

Table 2 and Table 3 present the empirical results as found in this study. These tables contain forecast accuracy tools RMSE and MAE for measuring absolute errors and MAPE as relative error measure. A small error value indicates better accuracy or better performance. They also contain five data sets (BMO, BNS, IMO, SLF and TRP) and six methods (i.e., two benchmark methods – ARIMA and EWMA; two EMD-based hybrid methods EMD-ARIMA and EMD-EWMA; two experimental methods EMD-ARIMA-EWMA and EMD-EWMA-ARIMA). Table 2 presents the methods' performance results at horizon $h = 4$ and 8, while Table 3 contains results related to horizon $h = 12$ and 16. Between two experimental methods, only EMD-ARIMA-EWMA outperformed all other methods considering all error values and all datasets. In Fig. 3, forecasted data and test data are presented for all six methods and five datasets where ARIMA and EWMA results are very close which seem overlapped in the graph. Table 4 presents computation time for each dataset for h = 16 for each method (along with EMD algorithm). The computation time of EWMA was the lowest among all methods and the highest time required for EMD-ARIMA. Hybrid methods require more time than single methods. In the following section, performance accuracies and overall results regarding methods and datasets are discussed, along with computation time (in second).

V. DISCUSSION AND OUTCOME OF THE STUDY

Experimental results (Table 2 and Table 3) found for five sets of stock price data reveal the superiority of proposed

TABLE 1. Summary statistics for five daily closing stock prices.

	Mean	Median	Min	Max	COV	Skew	Kurt	Count
BMO	88.061	85.98	66.18	108.32	0.128	0.028	-1.458	1400
BNS	70.364	70.79	51.87	84.87	0.102	-0.186	-0.751	1400
IMO	43.411	42.055	33.54	57.86	0.135	0.643	-0.46	1400
SLF	45.673	44.57	35.39	55.83	0.121	0.159	-1.296	1400
TRP	55.845	55.825	41.02	66.81	0.106	-0.319	-0.708	1400

TABLE 2. Forecasting accuracy of five stock data sets for forecast horizon $h = 4$ and $h = 8$.

	Forecast horizons →	h=4			h=8		
	↓ Methods, Errors →	RMSE	MAE	MAPE	RMSE	MAE	MAPE
BMO	ARIMA	2.414	2.134	2.188	3.881	3.529	3.677
	EWMA	2.388	2.105	2.158	3.855	3.500	3.647
	EMD-ARIMA	4.100	4.017	4.104	5.385	5.216	5.417
	EMD-EWMA	2.811	2.574	2.637	4.285	3.969	4.132
	EMD-EWMA-ARIMA	5.549	5.429	5.547	7.055	6.859	7.120
	EMD-ARIMA-EWMA	1.411	1.162	1.194	2.666	2.327	2.428
BNS	ARIMA	0.783	0.689	0.986	1.424	1.260	1.820
	EWMA	0.752	0.655	0.937	1.393	1.225	1.770
	EMD-ARIMA	0.453	0.402	0.574	0.735	0.578	0.835
	EMD-EWMA	1.057	0.990	1.415	1.695	1.560	2.252
	EMD-EWMA-ARIMA	1.129	1.056	1.508	1.830	1.684	2.431
	EMD-ARIMA-EWMA	0.381	0.336	0.481	0.638	0.454	0.656
IMO	ARIMA	0.986	0.725	2.086	2.201	1.837	5.507
	EWMA	0.986	0.725	2.086	2.201	1.837	5.506
	EMD-ARIMA	0.664	0.395	1.145	1.808	1.430	4.302
	EMD-EWMA	1.495	1.214	3.478	2.826	2.459	7.343
	EMD-EWMA-ARIMA	1.513	1.231	3.527	2.859	2.489	7.434
	EMD-ARIMA-EWMA	0.651	0.384	1.116	1.775	1.402	4.220
SLF	ARIMA	1.591	1.220	2.302	2.162	1.904	3.617
	EWMA	1.591	1.220	2.302	2.162	1.904	3.617
	EMD-ARIMA	1.485	1.109	2.094	1.930	1.683	3.199
	EMD-EWMA	1.506	1.108	2.094	2.064	1.792	3.407
	EMD-EWMA-ARIMA	1.574	1.174	2.217	2.190	1.910	3.632
	EMD-ARIMA-EWMA	1.417	1.043	1.970	1.806	1.564	2.974
TRP	ARIMA	0.544	0.465	0.722	1.092	0.952	1.493
	EWMA	0.544	0.465	0.722	1.092	0.952	1.493
	EMD-ARIMA	0.299	0.296	0.458	0.819	0.691	1.083
	EMD-EWMA	0.375	0.316	0.490	0.908	0.769	1.205
	EMD-EWMA-ARIMA	0.461	0.356	0.553	1.106	0.921	1.444
	EMD-ARIMA-EWMA	0.265	0.255	0.395	0.631	0.539	0.845

EMD-ARIMA-EWMA among all the six methods in all the forecast horizons h = 4, 8, 12 and 16. The smaller the errors (RMSE and MAE for absolute; MAPE for relative measure), the better the method. Statistical methods effectively synergized with EMD components, i.e., ARIMA and EWMA contributed better performance in high-frequency IMFs and low-frequency IMFs along with residual respectively as expected.

Each dataset is intrinsically different, and therefore forecasting performance of different methods on them usually vary. For first dataset BMO, the best performing EMD-ARIMA-EWMA produced relative MAPE errors 1.194, 2.428, 3.590 and 4.256 at h = 4, 8, 12 and 16 respectively. The second-best performing method is EWMA with MAPE 2.158, 3.647, 4.834 and 5.502 at the same h values in the same order. Furthermore, for other datasets (i.e., BNS, IMO, SLF and TRP), EMD-ARIMA-EWMA has the smallest error result of RMSE, MAPE, or MAE value all h locations. However, at all h values, the second-best performer was EMD-ARIMA for BNS data. Also, for IMO and SLF

TABLE 3. **Forecasting accuracy of five stock data sets for forecast horizon $h = 12$ and $h = 16$.**

	Forecast horizons →	h=12			h=16		
	↓ Methods, Errors →	RMSE	MAE	MAPE	RMSE	MAE	MAPE
BMO	ARIMA	5.06	4.61	4.865	5.622	5.210	5.532
	EWMA	5.03	4.58	4.834	5.595	5.181	5.502
	EMD-ARIMA	6.62	6.32	6.650	7.260	6.959	7.372
	EMD-EWMA	5.47	5.05	5.325	6.032	5.651	5.996
	EMD-EWMA-ARIMA	8.26	7.97	8.385	8.883	8.610	9.112
	EMD-ARIMA-EWMA	3.90	3.39	3.590	4.470	4.000	4.256
BNS	ARIMA	1.94	1.70	2.480	2.173	1.964	2.870
	EWMA	1.90	1.67	2.429	2.141	1.929	2.819
	EMD-ARIMA	0.77	0.62	0.899	0.810	0.680	0.993
	EMD-EWMA	2.20	2.00	2.914	2.447	2.264	3.306
	EMD-EWMA-ARIMA	2.41	2.18	3.180	2.715	2.505	3.659
	EMD-ARIMA-EWMA	0.64	0.48	0.702	0.614	0.479	0.699
IMO	ARIMA	2.62	2.32	7.011	2.780	2.541	7.711
	EWMA	2.62	2.32	7.010	2.779	2.541	7.710
	EMD-ARIMA	2.00	1.73	5.241	1.827	1.572	4.771
	EMD-EWMA	3.28	2.98	9.003	3.468	3.229	9.779
	EMD-EWMA-ARIMA	3.33	3.03	9.136	3.531	3.286	9.954
	EMD-ARIMA-EWMA	1.95	1.69	5.114	1.774	1.516	4.601
SLF	ARIMA	2.43	2.22	4.240	2.348	2.186	4.158
	EWMA	2.43	2.22	4.240	2.348	2.186	4.158
	EMD-ARIMA	2.20	2.00	3.808	2.171	2.014	3.832
	EMD-EWMA	2.33	2.11	4.028	2.244	2.074	3.946
	EMD-EWMA-ARIMA	2.51	2.28	4.355	2.473	2.298	4.372
	EMD-ARIMA-EWMA	2.02	1.83	3.481	1.943	1.790	3.407
TRP	ARIMA	1.14	1.01	1.589	1.036	0.896	1.402
	EWMA	1.14	1.01	1.589	1.036	0.896	1.402
	EMD-ARIMA	0.98	0.85	1.325	0.930	0.802	1.254
	EMD-EWMA	0.96	0.86	1.339	0.867	0.738	1.155
	EMD-EWMA-ARIMA	1.21	1.03	1.617	1.155	1.000	1.565
	EMD-ARIMA-EWMA	0.75	0.67	1.048	0.676	0.593	0.926

TABLE 4. **Computation time (in second) of six methods (and EMD algorithm) for five stock data sets for forecast horizon $h = 16$.**

Methods	BMO	BNS	IMO	SLF	TRP
ARIMA	0.106	0.216	0.134	0.108	0.070
EWMA	0.010	0.008	0.004	0.004	0.008
EMD-ARIMA	5.975	8.986	5.516	10.367	2.270
EMD-EWMA	0.025	0.077	0.025	0.068	0.033
EMD-EWMA-ARIMA	1.041	1.522	1.580	2.407	0.850
					1.454
EMD-ARIMA-EWMA	1.960	1.541	1.961	2.027	
EMD (algorithm)	**0.005**	**0.004**	**0.004**	**0.009**	**0.004**

datasets, EMD-ARIMA performed as a second-best method at all h values. Finally, for TRP dataset considering MAPE results, EMD-ARIMA was second-best performing method at h = 4, 8 and 12; but EMD-EWMA was a second-best performer at h = 16. From Table 2-3, on average, the worst-performing method was EMD-EWMA-ARIMA, except for SLF data at h = 4 and TRP data at h = 4 and 8.

All datasets are not equally forecastable due to their characteristic features. Forecastability rank or order can be found by comparing the least MAPE value for each data at each horizon. Therefore, with the MAPE results of the outperforming method EMD-ARIMA-EWMA, the best-forecasted datasets at different forecast horizons are TRP at horizon h = 4 (MAPE 0.395), BNS at other three horizons h = 8

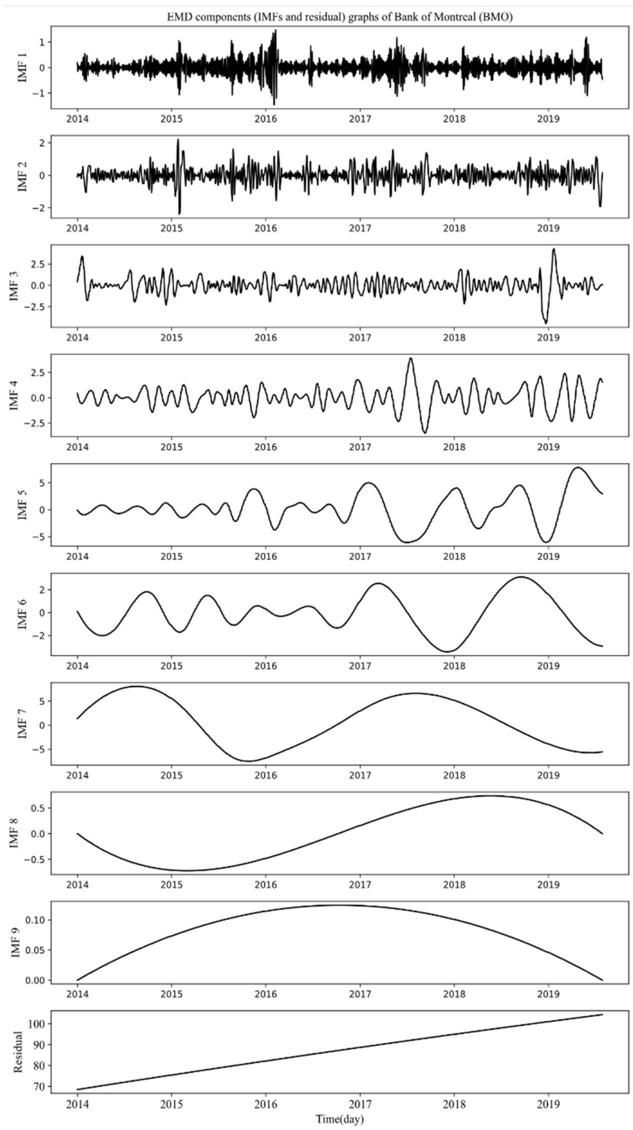

FIGURE 2. EMD component (IMFs and residual) graphs for Bank of Montreal (BMO).

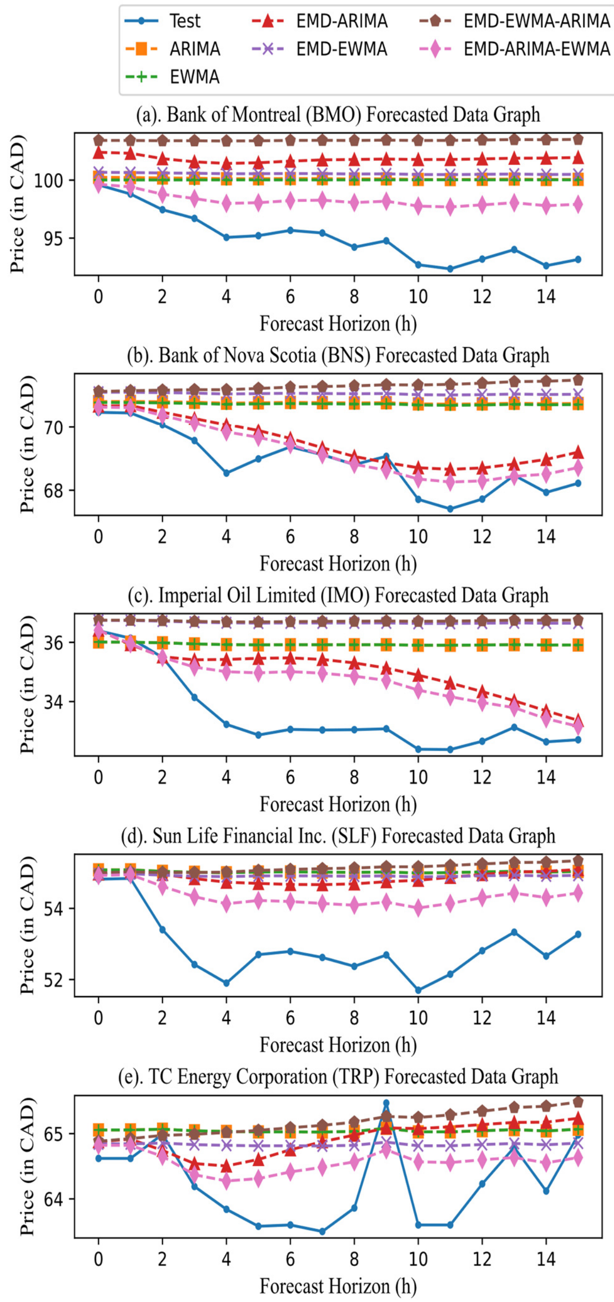

FIGURE 3. Forecast data graphs of six methods for five datasets.

(MAPE 0.656), h = 12 (MAPE 0.702) and h = 16 (MAPE 0.699). Similarly, least forecasted stock datasets with the best performing EMD-ARIMA-EWMA method are SLF at h = 4 (MAPE 1.970), IMO at other horizons h = 8 (MAPE 4.220), h = 12 (MAPE 5.114) and h = 16 (MAPE 4.601).

In any hybrid method, efficient constituent methods can improve the overall performance, and best combination can produce the best results. Likewise, in combining EMD with ARIMA and EWMA, EMD-ARIMA-EWMA is potentially best suited to perform better. However, EMD-EWMA-ARIMA is ill-suited and perform poorly. This was an essential part of our experimental investigation.

It should be expected that due to their additional computation or computational complexity, the hybrid or combination methods require more computation time in comparison to single methods. In an EMD-based hybrid method, a single method (here ARIMA or EWMA) is used for each EMD component which contributes to much computation time. In their hybrid method, [24] found that EWT-Q-BPNN (where EWT stands for Empirical Wavelet Transform, Q stands for Q-learning algorithm and BPNN stands for Back Propagation Neural Network) method required 614.63s while ARIMA required 13.55s for their Series #1 dataset. Here, for BMO dataset, the proposed EMD-ARIMA-EWMA required 1.96s while ARIMA required 0.106s. However, EMD algorithm required only 0.005s. Therefore, constituent ARIMA method contributed more time to the proposed method. Thus, the proposed or other EMD-based similar methods can further be improved using a suitable and effective combination of parametric, non-parametric and semi-parametric methods (e.g., as presented by [25]) with EMD components. However, this study aimed to find an EMD-based better forecasting method which approach can further be extended to develop hybrid methods that will be optimized with forecast accuracy and computation time.

Complementing overall discussion with the empirical results, the study's outcome is that EMD-ARIMA-EWMA method had comparatively high potentiality over other methods to produce better forecast accuracy. Relevantly, we assume that adaptive decomposition EMD-based hybridization with different well-suited statistical methods can forecast better if these methods are applied considering frequencies of the EMD components. Furthermore, although the proposed method is employed on stock market data, this or other potentially similar methods can be practicable in other forecasting related fields of time series application.

VI. CONCLUDING REMARKS

Time series forecasting is challenging, especially when it holds excessive uncertainty and contains less capturable patterns. However, in many cases, efficient methods can capture dominating patterns and features. The local adaptation feature of the EMD algorithm has effective pattern extracting capability, which can play an essential role in serving or boosting other methods' efficiency. This study found that experimented method EMD-ARIMA-EWMA outperformed the results of single ARIMA, EWMA, hybrid EMD-ARIMA and EMD-EWMA. This reflection was found through the results related to all the five data sets and four forecast horizons. Hopefully, this research study will inspire further EMD-based effective method development and will get the attention of researchers and practitioners of time series from different fields beyond the financial or economic domain. In the future, we plan to investigate more on EMD-based methods or their existing problems yet unexplored and contribute to analysis and forecasting of time series empirical data as well as in geophysical events.

REFERENCES

[1] T. Alghamdi, K. Elgazzar, M. Bayoumi, T. Sharaf, and S. Shah, "Forecasting traffic congestion using ARIMA modeling," in *Proc. 15th Int. Wireless Commun. Mobile Comput. Conf. (IWCMC)*, Jun. 2019, pp. 1227–1232.

[2] M. Alsharif, M. Younes, and J. Kim, "Time series ARIMA model for prediction of daily and monthly average global solar radiation: The case study of Seoul, South Korea," *Symmetry*, vol. 11, no. 2, p. 240, Feb. 2019.

[3] A. Hernandez-Matamoros, H. Fujita, T. Hayashi, and H. Perez-Meana, "Forecasting of COVID19 per regions using ARIMA models and polynomial functions," *Appl. Soft Comput.*, vol. 96, Nov. 2020, Art. no. 106610.

[4] J. W. Miller, "ARIMA time series models for full truckload transportation prices," *Forecasting*, vol. 1, no. 1, pp. 121–134, Sep. 2018.

[5] J. Köppelová and A. Jindrová, "Application of exponential smoothing models and arima models in time series analysis from Telco area," *Agris Line Papers Econ. Informat.*, vol. 11, no. 3, pp. 73–84, Sep. 2019.

[6] H. Liu, C. Li, Y. Shao, X. Zhang, Z. Zhai, X. Wang, X. Qi, J. Wang, Y. Hao, Q. Wu, and M. Jiao, "Forecast of the trend in incidence of acute hemorrhagic conjunctivitis in China from 2011–2019 using the seasonal autoregressive integrated moving average (SARIMA) and exponential smoothing (ETS) models," *J. Infection Public Health*, vol. 13, no. 2, pp. 287–294, Feb. 2020.

[7] Q. T. Tran, L. Hao, and Q. K. Trinh, "A comprehensive research on exponential smoothing methods in modeling and forecasting cellular traffic," *Concurrency Comput., Pract. Exper.*, vol. 32, no. 23, Dec. 2020, Art. no. e5602.

[8] S.-M. Chen, X.-Y. Zou, and G. C. Gunawan, "Fuzzy time series forecasting based on proportions of intervals and particle swarm optimization techniques," *Inf. Sci.*, vol. 500, pp. 127–139, Oct. 2019.

[9] J. W. Koo, S. W. Wong, G. Selvachandran, H. V. Long, and L. H. Son, "Prediction of air pollution index in Kuala Lumpur using fuzzy time series and statistical models," *Air Qual., Atmos. Health*, vol. 13, no. 1, pp. 77–88, Jan. 2020.

[10] F. M. Khan and R. Gupta, "ARIMA and NAR based prediction model for time series analysis of COVID-19 cases in India," *J. Saf. Sci. Resilience*, vol. 1, no. 1, pp. 12–18, Sep. 2020.

[11] Ü. Ç. Büyükşahin and Ş. Ertekin, "Improving forecasting accuracy of time series data using a new ARIMA-ANN hybrid method and empirical mode decomposition," *Neurocomputing*, vol. 361, pp. 151–163, Oct. 2019.

[12] J. Wang and J. Wang, "Forecasting stochastic neural network based on financial empirical mode decomposition," *Neural Netw.*, vol. 90, pp. 8–20, Jun. 2017.

[13] Z.-X. Wang, Y.-F. Zhao, and L.-Y. He, "Forecasting the monthly iron ore import of China using a model combining empirical mode decomposition, non-linear autoregressive neural network, and autoregressive integrated moving average," *Appl. Soft Comput.*, vol. 94, Sep. 2020, Art. no. 106475.

[14] J. Cao, Z. Li, and J. Li, "Financial time series forecasting model based on CEEMDAN and LSTM," *Phys. A, Stat. Mech. Appl.*, vol. 519, pp. 127–139, Apr. 2019.

[15] N. Nava, T. Matteo, and T. Aste, "Financial time series forecasting using empirical mode decomposition and support vector regression," *Risks*, vol. 6, no. 1, p. 7, Feb. 2018.

[16] S. Makridakis, E. Spiliotis, and V. Assimakopoulos, "The m4 competition: Results, findings, conclusion and way forward," *Int. J. Forecasting*, vol. 34, no. 4, pp. 802–808, Oct. 2018, doi: 10.1016/j.ijforecast.2018.06.001.

[17] S. Makridakis, E. Spiliotis, and V. Assimakopoulos, "Statistical and machine learning forecasting methods: Concerns and ways forward," *PLoS ONE*, vol. 13, no. 3, Mar. 2018, Art. no. e0194889.

[18] N. E. Huang, Z. Shen, S. R. Long, M. C. Wu, H. H. Shih, Q. Zheng, N. C. Yen, C. C. Tung, and H. H. Liu, "The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis," *Proc. Roy. Soc. London Ser. A, Math., Phys. Eng. Sci.*, vol. 454, no. 1971, pp. 903–995, Mar. 1998, doi: 10.1098/rspa.1998.0193.

[19] N. E. Huang, M.-L. Wu, W. Qu, S. R. Long, and S. S. P. Shen, "Applications of Hilbert–Huang transform to non-stationary financial time series analysis," *Appl. Stochastic Models Bus. Ind.*, vol. 19, no. 3, pp. 245–268, Jul. 2003, doi: 10.1002/asmb.501.

[20] Y. Fang, B. Guan, S. Wu, and S. Heravi, "Optimal forecast combination based on ensemble empirical mode decomposition for agricultural commodity futures prices," *J. Forecasting*, vol. 39, no. 6, pp. 877–886, Sep. 2020.

[21] A. M. Awajan, M. T. Ismail, and S. A. Wadi, "Improving forecasting accuracy for stock market data using EMD-HW bagging," *PLoS ONE*, vol. 13, no. 7, Jul. 2018, Art. no. e0199582.

[22] S. Abadan and A. Shabri, "Hybrid empirical mode decomposition-ARIMA for forecasting price of rice," *Appl. Math. Sci.*, vol. 8, pp. 3133–3143, 2014, doi: 10.12988/ams.2014.43189.

[23] M. R. Hossain and M. T. Ismail, "Empirical mode decomposition based on theta method for forecasting daily stock price," *J. Inf. Commun. Technol.*, vol. 19, no. 4, pp. 533–558, Aug. 2020.

[24] H. Liu, C. Yu, C. Yu, C. Chen, and H. Wu, "A novel axle temperature forecasting method based on decomposition, reinforcement learning optimization and neural network," *Adv. Eng. Informat.*, vol. 44, Apr. 2020, Art. no. 101089.

[25] J. Li, X. Xia, W. K. Wong, and D. Nott, "Varying-coefficient semiparametric model averaging prediction," *Biometrics*, vol. 74, no. 4, pp. 1417–1426, Dec. 2018.

MOHAMMAD RAQUIBUL HOSSAIN received the B.S. and M.S. degrees from the Department of Mathematics, University of Dhaka, Bangladesh, in 2006 and 2007, respectively. He is currently pursuing the Ph.D. degree with the School of Mathematical Sciences, Universiti Sains Malaysia, Pulau Pinang, Malaysia. His research interests include time series forecasting, predictive data analytics, data science, and machine learning.

MOHD TAHIR ISMAIL is currently an Associate Professor and a Researcher with the School of Mathematical Sciences, Universiti Sains Malaysia. He has published more than 150 publications in reviewed journals and proceedings (some of them are listed in ISI, Scopus, Zentralblatt, MathSciNet, and other indices). His research interests include financial time series, econometrics, categorical data analysis, and applied statistics. He is currently an Exco Member of the Malaysian Mathematical Sciences Society and an active member of other scientific professional bodies.

SAMSUL ARIFFIN BIN ABDUL KARIM is currently a Senior Lecturer with Universiti Teknologi PETRONAS, Malaysia. He is a Professional Technologist registered with Malaysia Board of Technologists (MBOT). He is a Certified WOLFRAM Technology Associate, Mathematica Student Level. He has published more than 140 papers in journals and conferences, including three edited conferences volume and 60 book chapters. He has also published ten books with springer publishing, including five books with studies in systems, decision and control (SSDC) series, and one book with UTP Press. He was a recipient of the Effective Education Delivery Award and the Publication Award (journal and conference paper), UTP Quality Day 2010, 2011, and 2012.