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# Formation Static Output Control of Linear Multi-Agent Systems With Hidden Markov Switching Network Topologies

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**ABSTRACT** This paper addresses the $\mathcal{H}_2$, $\mathcal{H}_\infty$ and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ formation static output control of continuous-time linear multi-agent systems with Markovian switching network topologies. It is assumed that the operation mode of the network topology cannot be directly measured but, instead, can be estimated by an imperfect detector. To model this problem, we consider a continuous-time hidden Markov model, in which the hidden component represents the real operation mode of the network topology while the observed component represents the information emitted from the detector and available for the controller. It is also assumed that only a partial information from the state variables of the multi-agent systems is available. By using an LMI (linear matrix inequality) formulation, a distributed static output controller which switches according to the detector information is designed to guarantee the stability in the mean square sense of the closed loop system, as well as an upper-bound for an index performance. Three situations are considered for the performance criteria: the $\mathcal{H}_2$ norm, the $\mathcal{H}_\infty$ norm, and the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ norms. The paper is concluded with a numerical example to illustrate the effectiveness of the theoretical results.

**INDEX TERMS** $\mathcal{H}_2$, $\mathcal{H}_\infty$ and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ formation control, hidden Markov switching topologies, linear matrix inequalities, linear multi-agent system, static output control.

## I. INTRODUCTION

The necessity to optimize the operation of network systems, as those found in power systems, unmanned vehicles or sensor networks [1], has lately drawn a great deal of attention in the control community. Several approaches have been proposed to address this cooperative control problem, referred to in the literature as multi-agent systems (MAS) control, distributed systems control, networks control, swarm systems control, etc. [2]. Among the vast field of study represented by cooperative systems, one can highlight the so-called formation control, which can be described by three factors: the agents, the communication among them and their geographical position. Considering these factors, it is possible to develop formation strategies that allow agents to follow trajectories while maintaining predefined geometric patterns, also known as topologies [3].

Due to changes in environmental conditions or transmission failures, it is reasonable to consider scenarios where formations may vary over time. With this in mind, the control law has to be designed to maintain stability as well as performance even when subjected to these changes. This problem, known as time-varying formation (TVF) control, has lately attracted considerable attention from the academic community [4]–[6]. In practice, the aforementioned events can happen randomly so that, to deal with this case, a possible approach would be to model these changes as following a Markov chain so that the whole dynamic system could be seen as a Markov jump linear system (MJLS) [7]. In the case of MAS formations, the states of the Markov chain would represent configurations in the formation topology.

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Several authors have been using this Markov chain formulation for the MAS topology. As a sample of these works we can mention [8], which considers the problem of leader-following consensus stability and stabilization for multi-agent systems with interval time-varying delays and Markovian switching interconnection information among agents, [9] which analyzes the leader-following consensus of MAS with random switching topologies, where the dwell time in each topology consists of a fixed part and random part, and the topology switching signal in the random part is modeled by a semi-Markov process. The work [10] addresses the distributed formation control for a group of quadrotor unmanned aerial vehicles (UAVs) under Markovian switching topologies with partially unknown transition rates, [11] deals with the sampled-data leader-following consensus of nonlinear multi-agent systems with Markovian switching network topologies and communication delay, [12] tackles the $\mathcal{H}_\infty$ leader-following consensus problem for nonlinear MAS under semi-Markovian switching topologies. In [13] the authors deal with the stochastic consensus problem for MAS over Markovian switching networks with time-varying delays and topology uncertainties, [14] addresses the consensus problem for a MAS with Markovian network topologies and external disturbance, and partial observation of system states, [15] investigates the $\mathcal{H}_\infty$ consensus of MAS with semi-Markov switching network topologies and measurement noises. In [16] the authors study the distributed formation problem of MAS with Markovian switching topologies and time-varying delay, considering the non-linear dynamics of each agent, [17] analyzes the TVF control of MAS, where the communication topology switches from several different topologies following a Markov chain, [18] deals with the problem of leader-following consensus for MAS with an asynchronous control mode of the Markov parameter. In [19] it is investigated the $\mathcal{H}_\infty$ output consensus problem for MAS with Markov jumps and external disturbance in both continuous-time and discrete-time domains, by considering an output feedback controller based on a hidden Markov model. Within the discrete-time set up, [20] studies the $\mathcal{H}_\infty$ consensus control problem for MAS with Markovian switching topology, under the hypothesis of partial information exchange among neighbor agents, and [21] addresses the velocity-constrained mean-square consensus problem of heterogeneous MAS with Markovian switching topologies and time-delay, which consist of first-order and second-order agents.

From the practical point of view, the controller may not always have access to the mode of operation of the system (the Markov parameter $\theta(t)$), so that it is important to consider the case of partial observations. For continuous-time MJLS this has been analyzed by considering an exponential hidden Markov approach in [22]–[25] for the $\mathcal{H}_2$ state-feedback, $\mathcal{H}_\infty$ static output feedback control problems, and for the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ dynamic control problem. In these cases the controller relies only on the information coming from a detector device (represented by $\widehat{\theta}(t)$), and that the joint process $Z(t) = (\theta(t), \widehat{\theta}(t))$ is an exponential hidden Markov chain, with $\widehat{\theta}(t)$ being the observable part. It is important to point out that a key difference with respect to [18], [19], which deal only with the $\mathcal{H}_\infty$ case, lies in the model representing the detector. The formulation considered in these papers is based on a conditional probability condition that must hold for each time $t$ (equation (4) in [19]) which can be hard to be checked, while in our formulation the model $Z(t)$ is assumed to be an exponential hidden Markov process, so that the time evolution of the process $Z(t)$ is well defined and can be easily simulated. Notice also that this formulation encompasses the so-called mode-dependent case, mode-independent case, and cluster case (see Remark 2).

The problem of synthesizing controllers that fulfill multiple performance criteria has drawn a great deal of attention in the literature. An useful framework in this direction is the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problem, which combines the minimization of a quadratic functional, related to the $\mathcal{H}_2$ control, while ensuring some degree of robustness to the closed-loop system, the $\mathcal{H}_\infty$ control problem (see, for instance [26], [27], and [28]–[30] for the stochastic case). Another interesting formulation for the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem is based on game theory associated to a Nash equilibrium between two performance indexes, as presented, for instance in [31], in which it is desired to minimize the output energy for the given control law whenever the worst-case disturbance is applied to the system.

In this work, we tackle the challenging problem of TVF control of MAS with partial observations on the Markov parameter that characterizes the communication topology as well as the state variables of the MAS, under the $\mathcal{H}_2$, $\mathcal{H}_\infty$, and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance criteria. The design technique is based on LMI optimization problems, so that the powerful toolboxes available for this class of problems can be used. As far as the authors are aware of, there are no results in the literature concerning the $\mathcal{H}_2$, $\mathcal{H}_\infty$ and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ TVF control of MAS considering an static output feedback and hidden exponential Markov model for the formation topology. Bearing this in mind, the main contributions of this paper are summarized as follows:

- We propose design conditions in terms of LMI for the synthesis of an static output TVF controller for MAS *that depends only on* $\widehat{\theta}(t)$ and such that the closed-loop MAS is mean square stable with $\mathcal{H}_\infty$ norm less than a given $\gamma > 0$.
- Similarly as above, we also treat the $\mathcal{H}_2$ case, and propose design conditions such that the $\mathcal{H}_2$ norm of the closed-loop MAS is less than a given $\varphi > 0$.
- By combining the previous results we tackle the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ case.
- We illustrate our results by means of numerical examples of a TVF control of a MAS consisting of six agents.

This work is organized as follows. In section II we introduce the notation that is used throughout our work. In section III some basic results related to exponential hidden MJLS are presented as well as the concepts of mean square stability and results for the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms. In section IV it is presented the hidden Markov switching topologies and the definition of the TVF control for MAS. The main results are derived in section V, in which conditions for obtaining static-output controllers that depend only on $\widehat{\theta}(t)$ and satisfy $\mathcal{H}_2$, $\mathcal{H}_\infty$, and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ upper-bound norm values are presented. Numerical examples are provided in section VI to illustrate our results. The paper is concluded in section VII with some final remarks.

## II. NOTATION

The $n$-dimensional real Euclidian space is denoted by $\mathbb{R}^n$, and $\mathbb{R}^+$ represents the positive real numbers. The identity matrix of size $n \times n$ is given by $I_n$ and $\mathbf{1}_n$ denotes the $n$-dimensional column vector with all ones. The transpose operator is represented by $(\cdot)'$, $\bullet$ represents blocks induced by symmetry in a square matrix, $Tr(\cdot)$ is the trace operator, $A \otimes B$ denotes the Kronecker product of matrices $A$ and $B$ and, for a square matrix $G$ we define $Her(G) \triangleq G + G'$.

For $N$ and $M$ positive integers we set $\mathcal{N} \triangleq \{1, \ldots, N\}$, $\mathcal{M} \triangleq \{1, \ldots, M\}$ and $\mathcal{V} \subseteq \mathcal{N} \times \mathcal{M}$. The probability space is defined by $(\Omega, \mathcal{F}, P)$, with a right-continuous filtration $\mathcal{F}_t$. $\mathbf{E}(\cdot)$ denotes the mathematical expectation with respect to $P$ and $L_2^r(\Omega, \mathcal{F}, P)$ (or just $L_2^r$ for simplicity) the set of square integrable stochastic processes $z = \{z(t) \in \mathbb{R}^r, t \in \mathbb{R}^+\}$ with $z(t)$ being $\mathcal{F}_t$-measurable for each $t \in \mathbb{R}^+$. In this case we set $\|z\|_2^2 = \int_0^\infty \mathbf{E}(\|z(t)\|^2)dt$. Finally, $o(h)$ denotes a function where $lim_{h\to 0} o(h)/h = 0$.

In what follows let $\widetilde{B}$ be the orthogonal complement matrix of the row space of a matrix $B$, so that $B\widetilde{B} = 0$. We conclude this section with the following version of the Finsler's lemma (see [32]) that will be needed later in the paper.

*Lemma 1: The following statements are equivalent*:

a) $\widetilde{B}' A \widetilde{B} > 0$.

b) $A + XB + B'X' > 0$ *for some matrix* $X$.

## III. AUXILIARY RESULTS

In this subsection we present some definitions and auxiliary results that will be needed along the paper. On the probability space $(\Omega, \mathcal{F}, P)$, with $\mathcal{F}_t$ a right-continuous filtration, we consider a continuous-time Markov jump linear system (MJLS) given by:

$$\mathcal{G}: \begin{cases} \dot{x}(t) = A_{\theta(t)}x(t) + B_{\theta(t)}u(t) + E_{\theta(t)}w(t) \\ z(t) = C_{\theta(t)}x(t) + D_{\theta(t)}u(t) + L_{\theta(t)}w(t) \\ y(t) = F_{\theta(t)}x(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ denotes the vector of states, $u(t) \in \mathbb{R}^{n_u}$ denotes the vector of control, $w(t) \in \mathbb{R}^{n_w}$ an external disturbance, $z(t) \in \mathbb{R}^{n_z}$ the vector of output variables, $y(t) \in \mathbb{R}^{n_y}$ the measured output, and $\{\theta(t)\}$ is a Markov chain taking values in the set $\mathcal{N}$ and with transition rates $\lambda_{pr}$, with $\lambda_{pr} \geq 0$ for all $p \neq r$. All matrices are considered to be of compatible dimensions.

It is assumed that $\theta(t)$ is not known but, instead, there is an estimation $\widehat{\theta}(t)$ for this variable, and that $Z(t) = (\theta(t), \widehat{\theta}(t))$, $t \in \mathbb{R}^+$, is a continuous-time hidden Markov model, with the hidden state $\theta(t)$ taking values in $\mathcal{N}$, and the observation state $\widehat{\theta}(t)$ taking values in $\mathcal{M}$. It is assumed that $Z(t)$ is a homogeneous Markov process having transition rates $\nu_{(pk)(r\ell)}$, with $\nu_{(pk)(r\ell)} \geq 0$ for $(r, \ell) \neq (p, k)$ and $-\nu_{(pk)(pk)} = \sum_{(r\ell)\neq(pk)} \nu_{(pk)(r\ell)}$. The transition rates $\nu_{(pk)(r\ell)}$ of $Z(t) = (\theta(t), \widehat{\theta}(t))$, are given by

$$\begin{aligned} &P(Z(t+h) = (r, \ell) \mid Z(t) = (p, k)) \\ &\quad = \begin{cases} \nu_{(pk)(r\ell)}h + o(h), & (r, \ell) \neq (p, k) \\ 1 + \nu_{(pk)(pk)}h + o(h), & (r, \ell) = (p, k), \end{cases} \end{aligned} \quad (2)$$

where

$$\nu_{(pk)(r\ell)} = \begin{cases} \alpha_{r\ell}^k \lambda_{pr}, & p \neq r, \ell \in \mathcal{M}, \\ q_{k\ell}^p, & r = p, \ell \neq k, p \in \mathcal{N}, \\ \lambda_{pp} + q_{kk}^p, & r = p, \ell = k, \\ 0, & \text{otherwise} \end{cases}$$

and $\alpha_{r\ell} \geq 0$, $\sum_{\ell\in\mathcal{M}} \alpha_{r\ell}^k = 1$, $q_{k\ell}^p \geq 0$, $\ell \neq k$, $\lambda_{pp} = -\sum_{r\in\mathcal{N}} \lambda_{pr}$, $q_{kk}^p = -\sum_{\ell\in\mathcal{M}} q_{k\ell}^p$. We denote by $\mathcal{V} \subseteq \mathcal{N} \times \mathcal{M}$ an invariant set of $Z(t)$, that is, $P(Z(t) \in \mathcal{V}) = 1$ whenever $Z(0) \in \mathcal{V}$.

*Remark 1: Notice that, for the observed state $\widehat{\theta}(t)$, simultaneous or spontaneous jumps with respect to $\theta(t)$ are modeled by the parameters $\alpha_{r\ell}^k$ and $q_{k\ell}^p$ respectively. Indeed, recalling that $\lambda_{pr}$ represents the transition rate of $\theta(t)$, we get that $\alpha_{rl}^k$ and $q_{kl}^p$ models simultaneous and spontaneous jumps of $\widehat{\theta}(t)$, that is, for small $h > 0$, $P(\widehat{\theta}(t+h) = \ell \mid \theta(t+h) = r, Z(t) = (p, k)) = \alpha_{r\ell}^k + r(h)$ for some function such that $\lim_{h\to 0} r(h) = 0$, and $P(\widehat{\theta}(t+h) = \ell \mid \theta(t+h) = p, Z(t) = (r, k)) = q_{k\ell}^r h + o(h)$. See [24] for more details.*

*Remark 2: The above approach allows modeling the following cases (see [23]):*

- *Mode-dependent case: $\mathcal{M} = \mathcal{N}$, $q_{k\ell}^p = 0$, $\alpha_{rr}^k = 1$, and $\alpha_{r\ell}^k = 0$ for $r \neq \ell$, with invariant set $\mathcal{V} = \{(p, p) \in \mathcal{N} \times \mathcal{N}\}$. Note that in this case $\theta(t)$ and $\widehat{\theta}(t)$ will be equal.*
- *Mode-independent case: $\mathcal{M} = \{1\}$, $q_{kl}^p = 0$, and $\alpha_{r1}^1 = 1$. In this setting, the detector would be always equal to 1.*
- *The Cluster Case: In this case the Markov chain states can be written as the union of $M \leq N$ disjoint sets (clusters) $\mathcal{N}_\ell$ so that $\mathcal{N} = \cup_{\ell\in\mathcal{M}} \mathcal{N}_\ell$. By defining $g : \mathcal{N} \to \mathcal{M}$ such that $g(p) = \ell$ we have that this function represents the cluster where the Markov state belongs to, and thus the controller would have access to $g(p)$. This would be equivalent to take $q_{k\ell}^p = 0$ and $\alpha_{pg(p)}^k = 1$, so that whenever $\theta(t)$ jumps to $p$, $\widehat{\theta}(t)$ would jump simultaneously to $g(p)$.*

The feedback control law depends only on the observable variables $y(t)$ and $\widehat{\theta}(t)$, so that it takes the form

$$u(t) = K_{\widehat{\theta}(t)} y(t). \quad (3)$$

By applying (3) into (1), we get that the closed loop system is given by

$$\mathcal{G}_{cl}: \begin{cases} \dot{x}(t) = \widetilde{A}_{\theta(t)\widehat{\theta}(t)}x(t) + E_{\theta(t)}w(t) \\ z(t) = \widetilde{C}_{\theta(t)\widehat{\theta}(t)}x(t) + L_{\theta(t)}w(t) \end{cases} \tag{4}$$

where, for $(r, \ell) \in \mathcal{N} \times \mathcal{M}$,

$$\widetilde{A}_{r\ell} = A_r + B_r F_r K_\ell, \quad \widetilde{C}_{r\ell} = C_r + D_r F_r K_\ell. \tag{5}$$

We present now the definition of mean square stability (MSS) and the $\mathcal{H}_\infty$ norm for system (4).

*Definition 1 (Mean Square Stability (MSS) [7]): Consider $w(t) = 0$ in (1). System (1) is said to be mean square stable if for arbitrary initial conditions $(\theta_0, \widehat{\theta}_0) \in \mathcal{V}$, and second order initial state vector $x_0$, we have that*

$$\lim_{t\to\infty} \mathbf{E}(\|x(t)\|^2) = 0.$$

*Definition 2 ($\mathcal{H}_\infty$ Norm [7]): Suppose that system (1) is MSS. The $\mathcal{H}_\infty$-norm for system (4) is defined as*:

$$\|\mathcal{G}_{cl}\|_\infty = \sup\left\{\frac{\|z\|_2}{\|w\|_2};\ w \in L_2^{n_w},\ w \neq 0\right\}.$$

Notice that the norm defined above represents a measure for the worst-case effect of finite-energy disturbances on the output. We will need the following result along the paper (the proof can be found in [7]).

*Lemma 2 (Bounded Real Lemma): System (4) is MSS with an $\mathcal{H}_\infty$ cost smaller than $\gamma$ if, for all $(p, k) \in \mathcal{V}$, there exist $R_{pk} > 0$ such that*

$$\begin{bmatrix} R_{pk}\widetilde{A}_{pk} + \widetilde{A}'_{pk}R_{pk} + \mathcal{R}_{pk}(R) & \bullet & \bullet \\ E'_p R_{pk} & -\gamma^2 I_{n_w} & \bullet \\ \widetilde{C}_{pk} & L_p & -I_{n_z} \end{bmatrix} < 0, \tag{6}$$

*where $\mathcal{R}_{pk}(R) = \sum_{(r,\ell)\in\mathcal{V}} \nu_{(pk)(r\ell)}R_{r\ell}$.*

For the definition of the $\mathcal{H}_2$ norm we consider in system (4) that $L_p = 0$, since the output $z(t)$ in this case is related to the quadratic cost of the state and control variables, and not to the external input. In what follows we set $P(Z(0) = (p, k)) = \mu_{pk} \geq 0$, $(p, k) \in \mathcal{V}$.

*Definition 3 ($\mathcal{H}_2$ Norm [22]): Suppose that system (1) is MSS. The $\mathcal{H}_2$-norm for system (4) is defined as follows: for $x_0 = 0$,*

$$\|\mathcal{G}_{cl}\|_2^2 = \sum_{s=1}^{n_w} \sum_{(p,k)\in\mathcal{V}} \mu_{pk}\|z_{s,(p,k)}\|_2^2,$$

*where $z_{s,(p,k)}(t)$ is the controlled output of (4) for $w(t) = v_s\delta(t)$, $\delta(t)$ is the unitary impulse, $v_s$ is the $s^{th}$ element of the canonical basis of $\mathbb{R}^{n_w}$ and $\theta(0) = p$, $\widehat{\theta}(0) = k$.*

For obtaining conditions for an upper-bound for the $\mathcal{H}_2$ norm of (4), we can resort to the following lemma (see [22]):

*Lemma 3: System (4) is MSS with an $\mathcal{H}_2$ cost smaller than $\varphi$ if, for all $(p, k) \in \mathcal{V}$, there exist $R_{pk} > 0$ such that*

$$\sum_{(r,\ell)\in\mathcal{V}} \mu_{r\ell} Tr(E'_r R_{r\ell} E_r) < \varphi^2 \tag{7}$$

$$Her(R_{pk}\widetilde{A}_{pk}) + \sum_{(r,\ell)\in\mathcal{V}} \nu_{(pk)(r\ell)}R_{r\ell} + \widetilde{C}'_{pk}\widetilde{C}_{pk} < 0. \tag{8}$$

## IV. PRELIMINARIES AND PROBLEM FORMULATION

### A. MARKOV SWITCHING TOPOLOGIES

For the continuous-time Markov process $\theta(t)$ taking values in $\mathcal{N}$ as described in Section III, we consider a multi-agent time-varying topology represented by the undirected graph $\mathcal{G}_{\theta(t)} = \mathcal{G}(\mathbb{V}, \mathcal{E}_{\theta(t)}, \mathcal{A}_{\theta(t)})$, where $\theta(t) \in \mathcal{N}$ denotes the network topology mode, $\mathbb{V} = \{1, \ldots, v\}$ and $\mathcal{E}_{\theta(t)} \subseteq \{(i, j)|i, j \in \mathbb{V}, i \neq j\}$ are the set of nodes and edges respectively. An edge $(i, j) \in \mathcal{E}_{\theta(t)}$ represents a connection of node $i$ and $j$. $\mathcal{A}_{\theta(t)} = [a_{ij,\theta(t)}] \in \mathbb{R}^{v\times v}$ is the adjacency matrix, with $a_{ij,\theta(t)} = 1$ if $(i, j) \in \mathcal{E}_{\theta(t)}$ and $a_{ij,\theta(t)} = 0$ otherwise. Let $\mathcal{D}_{\theta(t)} = diag(d_{1,\theta(t)}, \ldots, d_{v,\theta(t)}) \in \mathbb{R}^{v\times v}$ be the degree matrix with $d_{i,\theta(t)} = \sum_{j=1}^{v} a_{ij,\theta(t)}$. The Laplacian matrix of graph $\mathcal{G}_{\theta(t)}$ is defined as $\mathcal{L}_{\theta(t)} = \mathcal{D}_{\theta(t)} - \mathcal{A}_{\theta(t)}$.

As described in Section III, we assume that is not observable for the controller, but it can be estimated by an imperfect detector $\widehat{\theta}(t)$ taking values in $\mathcal{M}$, with $Z(t) = (\theta(t), \widehat{\theta}(t))$, $t \in \mathbb{R}^+$ being a homogeneous hidden Markov model with transition rate matrix given by (2).

### B. PROBLEM FORMULATION

Consider the linear multi-agent system

$$\mathcal{G}: \begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Ew_i(t) \\ y_i(t) = Fx_i(t) \\ x_i(0) = x_i^0, \quad i = 1, 2, \ldots, v. \end{cases} \tag{9}$$

Let $x(t) = [x'_1(t), \ldots, x'_v(t)]'$, $u(t) = [u'_1(t), \ldots, u'_v(t)]'$ and $y(t) = [y'_1(t), \ldots, y'_v(t)]'$ be the aggregate vectors of the states $x_i(t) \in \mathbb{R}^n$, control inputs $u_i(t) \in \mathbb{R}^{n_u}$ and measured outputs $y_i(t) \in \mathbb{R}^{n_y}$ respectively. We define $\{w_i(t)\} \in L_2^{n_w}$ as the $i$-th external disturbance with aggregate vector $w(t) = [w'_1(t), \ldots, w'_v(t)]'$. For the $i$-th agent, we consider the following distributed control protocol

$$u_i(t) = K_h F h_i(t) + K_{\widehat{\theta}(t)} \sum_{j\in\mathbb{V}} a_{ij,\theta(t)}\Big((y_j(t) - Fh_j(t)) - (y_i(t) - Fh_i(t))\Big), \tag{10}$$

where $K_h$ is set to manage the formation vector $h_i(t)$ and $K_{\widehat{\theta}(t)}$ will be designed to drive the states of the MAS (9) to achieve the desired time-varying formation under switching topologies. Notice that the feedback gain matrix $K_{\widehat{\theta}(t)}$ depends only on the observed mode of operation $\widehat{\theta}(t)$ while the adjacency matrix $a_{ij,\theta(t)}$ depends on the Markov process $\theta(t)$, with the joint process $Z(t) = (\theta(t), \widehat{\theta}(t))$ defined as in (2). The formation vectors $h_i(t)$ satisfy the following dynamic equations:

$$\dot{h}_i(t) = (A + BK_h F)h_i(t). \tag{11}$$

Substituting the control protocol (10) into the multi-agent system (9) we get that

$$\dot{x}_i(t) = Ax_i(t) + BK_hFh_i(t) + BK_{\widehat{\theta}(t)}F\Big(\sum_{j\in\mathbb{V}} a_{ij,\theta(t)}((x_j(t) - h_j(t)) - (x_i(t) - h_i(t)))\Big) + Ew_i(t). \quad (12)$$

The output controlled variable $z_i(t) \in \mathbb{R}^{n_z}$, $i = 1, \ldots, v$, is defined as

$$z_i(t) = C(x_i(t) - h_i(t)) + D(u_i(t) - u_{h_i}(t)) + Lw_i(t),$$
$$u_{h_i}(t) = K_hFh_i(t), \quad (13)$$

where $C$, $D$ and $L$ are weighting matrices related to the state error, control effort, and external disturbance. Let $\varepsilon_i(t) = x_i(t) - h_i(t)$. From (11) and (12) we get that

$$\dot{\varepsilon}_i(t) = A\varepsilon_i(t) + BK_{\widehat{\theta}(t)}F\Big(\sum_{j\in\mathbb{V}} a_{ij,\theta(t)}(\varepsilon_j(t) - \varepsilon_i(t))\Big) + Ew_i(t), \quad (14)$$

which, in a compact way, can be re-written as

$$\dot{\varepsilon}(t) = (I_v \otimes A)\varepsilon(t) - (\mathcal{L}_{\theta(t)} \otimes BK_{\widehat{\theta}(t)}F)\varepsilon(t) + (I_v \otimes E)w(t). \quad (15)$$

We define the average error $\delta_i(t)$ as

$$\delta_i(t) = (x_i(t) - h_i(t)) - \frac{1}{v}\sum_{j=1}^{v}(x_j(t) - h_j(t)), \quad (16)$$

which can be re-written in a compact form as

$$\delta(t) = [(I_v - J_v) \otimes I_n](x(t) - h(t)) = [(I_v - J_v) \otimes I_n]\varepsilon(t), \quad (17)$$

where $J_v = \frac{1}{v}\mathbf{1}_v\mathbf{1}_v'$. Notice that for any symmetric $v \times v$ matrix $S$ and any $n \times n$ matrix $Z$, we have that $(J_v \otimes I_n)(S \otimes Z) = (S \otimes Z)(J_v \otimes I_n)$. Thus we get that

$$[(I_v - J_v) \otimes I_n](I_v \otimes A) = (I_v \otimes A)[(I_v - J_v) \otimes I_n],$$
$$[(I_v - J_v) \otimes I_n](\mathcal{L}_p \otimes BK_kF) = (\mathcal{L}_p \otimes BK_kF)[(I_v - J_v) \otimes I_n].$$

The output averaged controlled variable $z_i^a(t)$ is defined as

$$z_i^a(t) = z_i(t) - \frac{1}{v}\sum_{j=1}^{v} z_j(t) \quad (18)$$

and $z(t) = [z_1^{a\prime}(t), \ldots, z_v^{a\prime}(t)]'$ is the aggregate vector of the averaged controlled outputs, with $z(t) \in \mathbb{R}^{n_z v}$. Considering the average error $\delta(t)$ in (17) and system (15), and the controlled output variables (13), (18), we get the dynamical system for the error $\delta(t)$ and $z(t)$ as

$$\mathscr{G}_{cl} : \begin{cases} \dot{\delta}(t) = \widetilde{A}_{\theta(t)\widehat{\theta}(t)}\delta(t) + \widetilde{E}w(t), \\ z(t) = \widetilde{C}_{\theta(t)\widehat{\theta}(t)}\delta(t) + \widetilde{L}w(t), \end{cases} \quad (19)$$

where

$$\widetilde{A}_{pk} = I_v \otimes A - \mathcal{L}_p \otimes BK_kF, \quad \widetilde{E} = (I_v - J_v) \otimes E$$
$$\widetilde{C}_{pk} = I_v \otimes C - \mathcal{L}_p \otimes DK_kF, \quad \widetilde{L} = (I_v - J_v) \otimes L.$$

The goal is to obtain $K_k$ in (10), $k \in \mathscr{M}$, such that we have TVF mean square stability and either an $\mathcal{H}_\infty$ or $\mathcal{H}_2$ performance (or both), as described next:

1) TVF mean square stability: system $\mathscr{G}_{cl}$ in (19) is MSS, that is, with $w(t) = 0$,
$$\lim_{t\to\infty} \mathbf{E}(\|\delta(t)\|^2) = 0, \quad (20)$$
for any initial conditions $\delta(0)$ and $(\theta(0), \widehat{\theta}(0)) \in \mathscr{V}$.
2) $\mathcal{H}_\infty$ performance: for some performance level $\gamma$, we have that $\|\mathscr{G}_{cl}\|_\infty < \gamma$, that is,
$$\mathbf{E}\int_0^\infty ||z(t)||^2\, dt < \gamma^2\mathbf{E}\int_0^\infty ||w(t)||^2 dt, \quad (21)$$
for any $w \in L_2^{n_w v}$, $w \neq 0$.
3) $\mathcal{H}_2$ performance: for some performance level $\varphi$, we have that
$$\|\mathscr{G}_{cl}\|_2 < \varphi. \quad (22)$$

## V. MAIN RESULTS

### A. PRELIMINARIES

In this section we present LMI conditions to obtain $K_k$ in (10), $k \in \mathscr{M}$, such that (20) and either (21) or (22) (or both) are satisfied. For this we need to make the following assumption.

*Assumption 1: $F$ has full row rank matrix.*

Note that Assumption 1 is a standard assumption to avoid redundant measurements. From Assumption 1 we have that there exists a non-singular matrix $T$ such that

$$FT = \begin{bmatrix} I_{n_y} & 0 \end{bmatrix}. \quad (23)$$

In what follows we define, for $(p, k) \in \mathscr{V}$,

$$\mathscr{V}_{(p,k)} = \{(r, \ell) \in \mathscr{V}; (r, \ell) \neq (p, k) \text{ and } \nu_{(p,k)(r,l)} \neq 0\}$$
$$= \{r_{(p,k)}(1), \ldots, r_{(p,k)}(\tau_{(p,k)}); r_{(p,k)}(\iota) \in \mathscr{V}, \iota = 1, \ldots, \tau_{(p,k)}\}.$$

Consider $n \times n$ matrices $X_{pk} > 0$, $(p, k) \in \mathscr{V}$, and set

$$\Pi_{pk} = [\sqrt{\nu_{(p,k)r_{(p,k)}(1)}}(I_v \otimes I_n) \ldots \sqrt{\nu_{(p,k)r_{p,k}(\tau_{(p,k)})}}(I_v \otimes I_n)],$$
$$\mathscr{D}_{pk} = diag(I_v \otimes X_{r_{(p,k)}(1)}, \ldots, I_v \otimes X_{r_{(p,k)}(\tau_{(p,k)})}).$$

Notice that

$$\sum_{(r,\ell)\in\mathscr{V}} \nu_{(p,k)(r,\ell)}(I_v \otimes X_{rl}^{-1}) = \sum_{(r,\ell)\in\mathscr{V}_{(p,k)}} \nu_{(p,k)(r,\ell)}(I_v \otimes X_{r\ell}^{-1}) + \nu_{(p,k)(p,k)}(I_v \otimes X_{pk}^{-1})$$
$$= \Pi_{pk}\mathscr{D}_{pk}^{-1}\Pi_{pk}' + \nu_{(p,k)(p,k)}(I_v \otimes X_{pk}^{-1}). \quad (24)$$

### B. $\mathcal{H}_\infty$ CONTROL

The following theorem, based on the results in [23], presents a solution for the $\mathcal{H}_\infty$ problem for the cooperative control of multi-agent system (9) under hidden Markov switching topologies, based on the solution of a set of LMI.

*Theorem 1: Consider a fixed upper-bound $\gamma > 0$ and suppose that for all $(p, k) \in \mathscr{V}$, there exist matrices $X_{pk} > 0$,*

*$G_k$ and $V_k$ and a scalar $\epsilon_\infty > 0$ such that the following set of LMI is satisfied*:

$$\begin{bmatrix} \nu_{(p,k)(p,k)} I_v \otimes X_{ik} & \bullet & \bullet & \bullet & \bullet \\ (I_v - J_v)\otimes E' & -\gamma^2(I_v \otimes I_{n_w}) & \bullet & \bullet & \bullet \\ 0 & (I_v - J_v)\otimes L & -I_v \otimes I_{n_z} & \bullet & \bullet \\ I_v \otimes X_{pk} & 0 & 0 & 0 & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & 0 & 0 & -\mathscr{D}_{pk} \end{bmatrix} + Her\left( \begin{bmatrix} I_v \otimes (ATG_k) - \mathcal{L}_p \otimes B \begin{bmatrix} V_k & 0 \end{bmatrix} \\ 0 \\ I_v \otimes (CTG_k) - \mathcal{L}_p \otimes D \begin{bmatrix} V_k & 0 \end{bmatrix} \\ -I_v \otimes (TG_k) \\ 0 \end{bmatrix} \begin{bmatrix} \epsilon_\infty (I_v \otimes I_n) \\ 0 \\ 0 \\ I_v \otimes I_n \\ 0 \end{bmatrix}' \right) < 0, \tag{25}$$

*with $G_k$ in the following form*:

$$G_k = \begin{bmatrix} G_{k1} & 0 \\ G_{k2} & G_{k3} \end{bmatrix}. \tag{26}$$

*Then the multi-agent system (9) is mean square stable with a closed-loop norm $\|\mathscr{G}_{cl}\|_\infty < \gamma$ whenever the distributed control protocol (10) is applied, with the feedback controller matrices $K_k$ given by*:

$$K_k = V_k G_{k1}^{-1}, \quad k \in \mathscr{M}. \tag{27}$$

*Proof:* From (25) we have that $I_v \otimes (TG_k) + I_v \otimes (G_k' T') > 0 = I_v \otimes (TG_k + G_k' T') > 0$ so that it follows that $TG_k + G_k' T' > 0$, which implies that $G_k$ is non-singular, and thus the inverse in (27) is well defined. From (26) and (27) we have that $V_k = K_k G_{k1}$ and

$$K_k \begin{bmatrix} I_{n_y} & 0 \end{bmatrix} G_k = \begin{bmatrix} K_k & 0 \end{bmatrix} G_k = \begin{bmatrix} V_k & 0 \end{bmatrix}, \tag{28}$$

so that (23) and (28) yields to

$$\begin{aligned} I_v \otimes (ATG_k) - \mathcal{L}_p \otimes B \begin{bmatrix} V_k & 0 \end{bmatrix} &= I_v \otimes (ATG_k) - \mathcal{L}_p \otimes BK_k \begin{bmatrix} I_{n_y} & 0 \end{bmatrix} G_k \\ &= I_v \otimes (ATG_k) - \mathcal{L}_p \otimes BK_k FTG_k \\ &= (I_v \otimes A - \mathcal{L}_p \otimes BK_k F)(I_v \otimes TG_k). \end{aligned}$$

Similarly we have that

$$I_v \otimes (CTG_k) - \mathcal{L}_p \otimes D \begin{bmatrix} V_k & 0 \end{bmatrix} = (I_v \otimes C - \mathcal{L}_p \otimes DK_k F)(I_v \otimes TG_k).$$

Set

$$\widetilde{A}_{pk} = I_v \otimes A - \mathcal{L}_p \otimes BK_k F, \ \widetilde{E} = (I_v - J_v) \otimes E, \tag{29}$$
$$\widetilde{C}_{pk} = I_v \otimes C - \mathcal{L}_p \otimes DK_k F, \ \widetilde{L} = (I_v - J_v) \otimes L, \tag{30}$$

so that (25) can be re-written as

$$\Phi_{pk} + Her\left( \begin{bmatrix} \widetilde{A}_{pk} \\ 0 \\ \widetilde{C}_{pk} \\ -I_v \otimes I_n \\ 0 \end{bmatrix} I_v \otimes (TG_k) \begin{bmatrix} \epsilon_\infty (I_v \otimes I_n) \\ 0 \\ 0 \\ I_v \otimes I_n \\ 0 \end{bmatrix}' \right) < 0, \tag{31}$$

where

$$\Phi_{pk} = \begin{bmatrix} \nu_{(p,k)(p,k)} I_v \otimes X_{pk} & \bullet & \bullet & \bullet & \bullet \\ \widetilde{E}' & -\gamma^2(I_v \otimes I_{n_w}) & \bullet & \bullet & \bullet \\ 0 & \widetilde{L} & -I_v \otimes I_{n_z} & \bullet & \bullet \\ I_v \otimes X_{pk} & 0 & 0 & 0 & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & 0 & 0 & -\mathscr{D}_{pk} \end{bmatrix}.$$

Defining

$$\widetilde{W}_{pk} = \begin{bmatrix} I_v \otimes I_n & 0 & 0 & 0 \\ 0 & I_v \otimes I_{n_w} & 0 & 0 \\ 0 & 0 & I_v \otimes I_{n_z} & 0 \\ \widetilde{A}'_{pk} & 0 & \widetilde{C}'_{pk} & 0 \\ 0 & 0 & 0 & I_v \otimes I_n \end{bmatrix},$$
$$W_{pk} = \begin{bmatrix} \widetilde{A}'_{pk} & 0 & \widetilde{C}'_{pk} & -I_v \otimes I_n & 0 \end{bmatrix} \tag{32}$$

it follows that $\widetilde{W}_{pk}$ has full rank and that $W_{pk}\widetilde{W}_{pk} = 0$, so that from Finsler's lemma (see Lemma 1) and (31) we have that

$$\widetilde{W}'_{pk} \Phi_{pk} \widetilde{W}_{pk} < 0. \tag{33}$$

From (33) we conclude that

$$\begin{bmatrix} \mathcal{Z}_{pk} & \bullet & \bullet & \bullet \\ \widetilde{E}' & -\gamma^2(I_v \otimes I_{n_w}) & \bullet & \bullet \\ \widetilde{C}_{pk}(I_v \otimes X_{pk}) & \widetilde{L} & -I_v \otimes I_{n_z} & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & 0 & -\mathscr{D}_{pk} \end{bmatrix} < 0,$$

where

$$\mathcal{Z}_{pk} = \nu_{(p,k)(p,k)} I_v \otimes X_{ik} + \widetilde{A}_{pk}(I_v \otimes X_{pk}) + (I_v \otimes X_{pk})\widetilde{A}'_{pk}.$$

From Schur's complement we get that

$$\begin{bmatrix} \bar{\mathcal{Z}}_{pk} & \bullet & \bullet \\ \widetilde{E}' & -\gamma^2(I_v \otimes I_{n_w}) & \bullet \\ \widetilde{C}_{pk}(I_v \otimes X_{pk}) & \widetilde{L} & -I_v \otimes I_{n_z} \end{bmatrix} < 0,$$

where

$$\bar{\mathcal{Z}}_{pk} = \mathcal{Z}_{pk} + (I_v \otimes X_{pk})\Pi_{pk}\mathscr{D}_{pk}^{-1}\Pi'_{pk}(I_v \otimes X_{pk}).$$

Multiplying on the left hand side and right hand side by $diag((I_v \otimes X_{pk}^{-1}), (I_v \otimes I_{n_w}), I_v \otimes I_{n_z})$ we get that

$$\begin{bmatrix} \widetilde{\mathcal{Z}}_{pk} & \bullet & \bullet \\ \widetilde{E}'(I_v \otimes X_{pk}^{-1}) & -\gamma^2(I_v \otimes I_{n_w}) & \bullet \\ \widetilde{C}_{pk} & \widetilde{L} & -I_v \otimes I_{n_z} \end{bmatrix} < 0 \tag{34}$$

where, from (24),

$$\begin{aligned} \widetilde{\mathcal{Z}}_{pk} &= (I_v \otimes X_{pk}^{-1})\widetilde{A}_{pk} + \widetilde{A}'_{pk}(I_v \otimes X_{pk}^{-1}) \\ &\quad + \nu_{(p,k)(p,k)}(I_v \otimes X_{ik}^{-1}) + \Pi_{pk}\mathscr{D}_{pk}^{-1}\Pi'_{pk} \\ &= (I_v \otimes X_{pk}^{-1})\widetilde{A}_{pk} + \widetilde{A}'_{pk}(I_v \otimes X_{pk}^{-1}) \\ &\quad + \sum_{(r,\ell)\in\mathscr{V}} \nu_{(p,k)(r,\ell)}(I_v \otimes X_{rl}^{-1}). \end{aligned} \tag{35}$$

By combining (34) and (35) we have that (6) is satisfied by taking $R_{pk} = I_v \otimes X_{pk}^{-1}$. From Lemma 2 and considering the representation in (19) for $\mathscr{G}_{cl}$ we get the desired result. □

### C. $\mathcal{H}_2$ CONTROL

We present next a solution for the $\mathcal{H}_2$ problem for the cooperative control of multi-agent system (9) under hidden Markov switching topologies, based on the solution of a set of LMI. We recall that in this case we consider $L = 0$ since the output $z(t)$ is only related to the quadratic cost of the state and control variables.

*Theorem 2: Consider a fixed upper-bound $\varphi > 0$ and suppose that for all $(p, k) \in \mathscr{V}$, there exist matrices $W_{pk} > 0$, $X_{pk} > 0$, $G_k$ and $V_k$ and a scalar $\epsilon_2 > 0$, such that the following set of LMI is satisfied*:

$$\sum_{(p,k)\in\mathscr{V}} \mu_{pk} Tr(W_{pk}) < \varphi^2, \tag{36}$$

$$\begin{bmatrix} W_{pk} & \bullet \\ (I_v - J_v)\otimes E & I_v \otimes X_{pk} \end{bmatrix} > 0, \tag{37}$$

*and*

$$\begin{bmatrix} \nu_{(p,k)(p,k)} I_v \otimes X_{pk} & \bullet & \bullet & \bullet \\ 0 & -I_v \otimes I_{n_z} & \bullet & \bullet \\ I_v \otimes X_{pk} & 0 & 0 & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & 0 & -\mathscr{D}_{pk} \end{bmatrix} + Her\left(\begin{bmatrix} I_v \otimes (ATG_k) - \mathcal{L}_p \otimes B\begin{bmatrix} V_k & 0\end{bmatrix} \\ I_v \otimes (CTG_k) - \mathcal{L}_p \otimes D\begin{bmatrix} V_k & 0\end{bmatrix} \\ -I_v \otimes (TG_k) \\ 0 \end{bmatrix}\begin{bmatrix} \epsilon_2 (I_v \otimes I_n) \\ 0 \\ I_v \otimes I_n \\ 0 \end{bmatrix}'\right) < 0, \tag{38}$$

*with $G_k$ as in (26). Then the multi-agent system (9) is mean square stable with a closed-loop norm $\|\mathscr{G}_{cl}\|_2 < \varphi$ whenever the distributed control protocol (10) is applied, with the feedback controller matrices $K_k$ given by (27).*

*Proof:* As before, from (31) we have that $G_k$ is nonsingular, so that the inverse in (27) is well defined. As in the proof of Theorem 1 and using the same notation as in (29), (30), we have that (38) can be re-written as

$$\Phi_{pk} + Her\left(\begin{bmatrix} \widetilde{A}_{pk} \\ \widetilde{C}_{pk} \\ -I_v \otimes I_n \\ 0 \end{bmatrix} I_v \otimes (TG_k) \begin{bmatrix} \epsilon_2 (I_v \otimes I_n) \\ 0 \\ I_v \otimes I_n \\ 0 \end{bmatrix}'\right) < 0, \tag{39}$$

where

$$\Phi_{pk} = \begin{bmatrix} \nu_{(p,k)(p,k)} I_v \otimes X_{pk} & \bullet & \bullet & \bullet \\ 0 & -I_v \otimes I_{n_z} & \bullet & \bullet \\ I_v \otimes X_{pk} & 0 & 0 & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & 0 & -\mathscr{D}_{pk} \end{bmatrix}.$$

Defining

$$\widetilde{W}_{pk} = \begin{bmatrix} I_v \otimes I_n & 0 & 0 \\ 0 & I_v \otimes I_{n_z} & 0 \\ \widetilde{A}'_{pk} & \widetilde{C}'_{pk} & 0 \\ 0 & 0 & I_v \otimes I_n \end{bmatrix}, \quad W'_{pk} = \begin{bmatrix} \widetilde{A}_{pk} \\ \widetilde{C}_{pk} \\ -I_v \otimes I_n \\ 0 \end{bmatrix} \tag{40}$$

it is easy to see that $\widetilde{W}_{pk}$ has full rank and that $W_{pk}\widetilde{W}_{pk} = 0$, so that from Finsler's lemma (see Lemma 1) and (40) we have that

$$\widetilde{W}'_{pk} \Phi_{pk} \widetilde{W}_{pk} < 0. \tag{41}$$

From (41) we conclude that

$$\begin{bmatrix} \mathcal{Z}_{pk} & \bullet & \bullet \\ \widetilde{C}_{pk}(I_v \otimes X_{pk}) & -I_v \otimes I_{n_z} & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & -\mathscr{D}_{pk} \end{bmatrix} < 0, \tag{42}$$

where

$$\mathcal{Z}_{pk} = \nu_{(p,k)(p,k)} I_v \otimes X_{ik} + \widetilde{A}_{pk}(I_v \otimes X_{pk}) + (I_v \otimes X_{pk})\widetilde{A}'_{pk}.$$

By applying the Schur's complement in (42) we get that

$$\mathcal{Z}_{pk} + (I_v \otimes X_{pk})\Pi_{pk}\mathscr{D}_{pk}^{-1}\Pi'_{pk}(I_v \otimes X_{pk}) + (I_v \otimes X_{pk})\widetilde{C}'_{pk}\widetilde{C}_{pk}(I_v \otimes X_{pk}) < 0.$$

Multiplying on the left and right hand side by $I_v \otimes X_{pk}^{-1}$ we get that

$$\widetilde{\mathcal{Z}}_{pk} + \widetilde{C}'_{pk}\widetilde{C}_{pk} < 0 \tag{43}$$

where $\widetilde{\mathcal{Z}}_{pk}$ is as in (35). By combining (43) and (35) we have that (8) is satisfied by taking $R_{pk} = I_v \otimes X_{pk}^{-1}$. Moreover from Schur's complement in (37) we get that $W_{pk} > (I_v - J_v) \otimes E'(I_v \otimes X_{pk})(I_v - J_v) \otimes E$ so that from (36) we get that $\sum_{(p,k)\in\mathscr{V}} Tr((I_v - J_v) \otimes E'(I_v \otimes X_{pk})(I_v - J_v) \otimes E) < \varphi^2$ showing that (7) is also satisfied. From Lemma 3 we get the desired result. □

### D. MIXED $\mathcal{H}_2/\mathcal{H}_\infty$ CONTROL

The following corollary is straightforward after combining the results from Theorems 1 and 2.

*Corollary 1: Consider fixed upper-bounds $\gamma > 0$ and $\varphi > 0$. If for all $(p, k) \in \mathscr{V}$, there exist matrices $W_{pk} > 0$, $X_{pk} > 0$, $G_k$ and $V_k$ and scalars $\epsilon_\infty > 0$, $\epsilon_2 > 0$, such that the LMI (25), (36), (37), (38) are satisfied, where $G_k$ is as in (26) then the multi-agent system (9) is mean square stable with a closed-loop norm $\|\mathscr{G}_{cl}\|_2 < \varphi$ and $\|\mathscr{G}_{cl}\|_\infty < \gamma$ whenever the distributed control protocol (10) is applied, with the feedback controller matrices $K_k$ given as in (27).*

From the previous results the following LMI optimization problems could be defined:

1) Pure $\mathcal{H}_\infty$ control problem: min $\gamma^2$ such that the LMI (25) is satisfied.
2) Pure $\mathcal{H}_2$ control problem: min $\varphi^2$ such that the LMI (36), (37), (38) are satisfied.
3) Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problems:
   - 3.a) for $\beta_2 \geq 0$, $\beta_\infty \geq 0$, min $\beta_2\varphi^2 + \beta_\infty\gamma^2$ such that the LMI (25), (36), (37), (38) are satisfied.
   - 3.b) for fixed $\varphi > 0$, min $\gamma^2$ such that the LMI (25), (36), (37), (38) are satisfied.
   - 3.c) for fixed $\gamma > 0$, min $\varphi^2$ such that the LMI (25), (36), (37), (38) are satisfied.

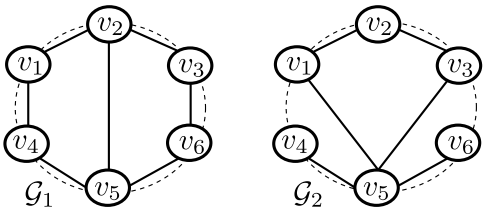



**FIGURE 1.** Network topologies.

## VI. ILLUSTRATIVE EXAMPLE

In this section, numerical examples are presented to illustrate the effectiveness of the proposed method. The first one deals with a comparison between the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ costs for the mode-dependent (synchronous) and mode-independent (asynchronous) cases, by varying the parameter $\alpha^k_{r\ell}$ in (2). Next, the average error responses $\delta_i(t)$ are studied for the controllers $\mathcal{H}_2$, $\mathcal{H}_\infty$ and $\mathcal{H}_2/\mathcal{H}_\infty$. Finally, the ability of the control protocol $\mathcal{H}_2/\mathcal{H}_\infty$ to achieve the TVF in the MAS is verified.

For numerical simplicity, let us consider that the multi-agent system (9) consists of six agents with $x_i(t) = [x_{i1}(t)\ x_{i2}(t)\ x_{i3}(t)\ x_{i4}(t)\ x_{i5}(t)\ x_{i6}(t)]'$ $(i = 1, 2, \ldots, 6)$ and state matrices adopted from [17], defined as

$$A = \begin{bmatrix} 0_{3\times3} & I_3 \\ -I_3 & 0_{3\times3} \end{bmatrix},\quad B = \begin{bmatrix} 0_{3\times3} \\ I_3 \end{bmatrix},\quad E = \begin{bmatrix} 0.8 \\ 0.5 \\ 1 \\ 0_{3\times1} \end{bmatrix},$$

$$C^{(\infty)} = C^{(2)} = I_6, \qquad F = \begin{bmatrix} I_3 & 0_{3\times3} \end{bmatrix}.$$

The Markovian mode-dependent network topologies, represented by the undirected graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ in Fig. 1, are described by the following Laplacian matrices $\mathcal{L}_p$ with $p \in \mathcal{N} \triangleq \{1, 2\}$

$$L_1 = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & -1 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix},$$

and the transition rate matrix is given by

$$\Lambda = \begin{bmatrix} -0.3 & 0.3 \\ 0.5 & -0.5 \end{bmatrix}.$$

The observed mode $\widehat{\theta}(t)$ is set with $M = 2$ along with

$$[\alpha^k_{r\ell}] = \begin{bmatrix} \varsigma_1 & 1-\varsigma_1 \\ 1-\varsigma_2 & \varsigma_2 \end{bmatrix}, \quad \forall k \in \mathcal{M},$$

**TABLE 1.** $\mathcal{H}_2$ and $\mathcal{H}_\infty$ costs.

| Operation mode | $\varphi$ | $\gamma$ |
|---|---|---|
| Synchronous | 0.1189 | 0.0249 |
| Asynchronous | 0.1368 | 0.0295 |

for $\varsigma_1, \varsigma_2 \in [0, 1]$ and $[q^p_{k\ell}] = 0$. The desired time-varying formation for the six agents is a periodic rotation parallel hexagon, where the formation vector $h_i(t)$ $(i = 1, 2, \ldots, 6)$ is specified by

$$h_i(t) = \begin{bmatrix} 2sin(2t + \frac{(i-1)\pi}{3}) - 2cos(2t + \frac{(i-1)\pi}{3}) \\ sin(2t + \frac{(i-1)\pi}{3}) + cos(2t + \frac{(i-1)\pi}{3}) \\ 4cos(2t + \frac{(i-1)\pi}{3}) \\ 4sin(2t + \frac{(i-1)\pi}{3}) + 4cos(2t + \frac{(i-1)\pi}{3}) \\ 2cos(2t + \frac{(i-1)\pi}{3}) - 2sin(2t + \frac{(i-1)\pi}{3}) \\ 8sin(2t + \frac{(i-1)\pi}{3}) \end{bmatrix}.$$

Let the initial states $x_i(0) = [x_{i1}(0)\, x_{i2}(0)\, x_{i3}(0)\, x_{i4}(0)\, x_{i5}(0)\, x_{i6}(0)]'$ $(i = 1, 2, \ldots, 6)$ be random values uniformly chosen between -10 and 10. With the purpose of determining the performance of the proposed solutions, we set the gain matrix $K_h = [-I_3\ \ 0_{3\times3}]$ and the external disturbance input $w_i(t)$ as follows

$$w_{i1}(t) = \begin{cases} 2, & \text{for } t \in [0, 10) \cup [20, 30) \\ -2, & \text{for } t \in [10, 20) \cup [30, 40) \\ 0, & \text{otherwise,} \end{cases}$$

$$w_{i2}(t) = \begin{cases} 1, & \text{for } t \in [0, 10) \cup [20, 30) \\ -1, & \text{for } t \in [10, 20) \cup [30, 40) \\ 0, & \text{otherwise,} \end{cases}$$

$$w_{i3}(t) = \begin{cases} 2, & \text{for } t \in [0, 10) \cup [20, 30) \\ -2, & \text{for } t \in [10, 20) \cup [30, 40) \\ 0, & \text{otherwise.} \end{cases}$$

In order to compare the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ costs for the synchronous and asynchronous modes, we set $\varsigma_1 = \varsigma_2 = 1$ for the synchronous case $(\theta(t) = \widehat{\theta}(t))$ and $\varsigma_1 = \varsigma_2 = 0.6$ for the asynchronous case, which indicates the imperfect information case (we could have $\theta(t) \neq \widehat{\theta}(t)$). The $\mathcal{H}_2$ controller is obtained by solving the LMI (36), (37), (38) in Theorem 2 with $\epsilon_2 = 20$, which yields the performance values shown in Table 1. Notice that the values $\varphi = 0.1189$ for the synchronous mode and $\varphi = 0.1368$ for the asynchronous mode differ by only 13.1%. Similarly, for the $\mathcal{H}_\infty$ controller obtained from Theorem 1 by fixing $\epsilon_\infty = 10$ and solving the LMI (25), the values between the synchronous mode $(\gamma = 0.0249)$ and the asynchronous mode $(\gamma = 0.0295)$ differ only by 15.6%. These results indicate that the performance and robustness are maintained even if the detector emits mismatching signals with respect to the network mode of operation.

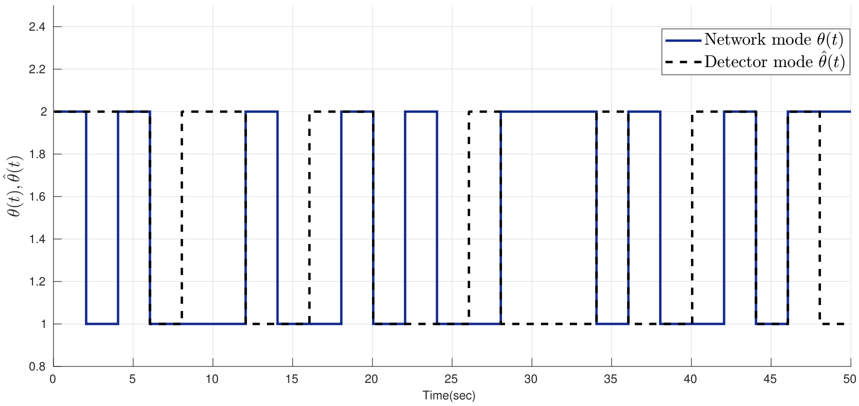



**FIGURE 2.** Asynchronous switching signal.

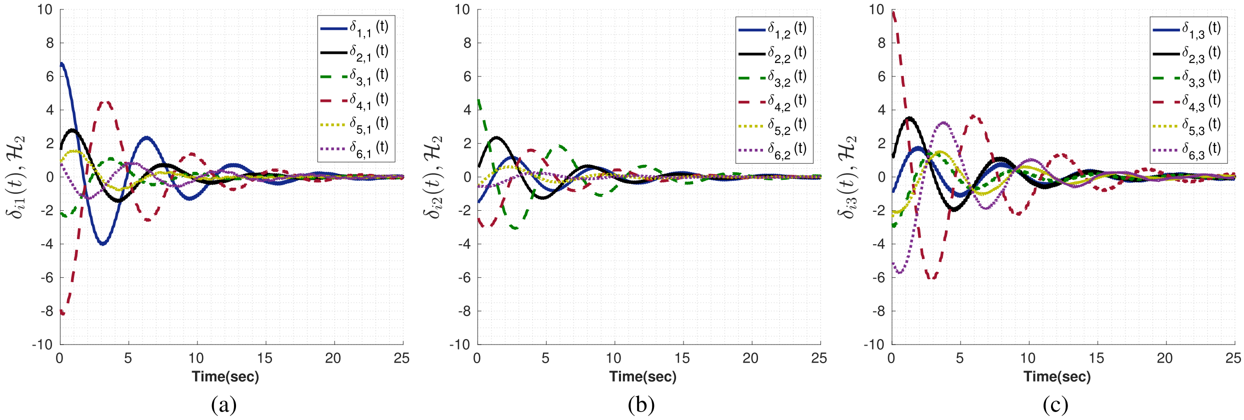



**FIGURE 3.** $\mathcal{H}_2$ **average TVF error response** $\delta_i(t)$**.**

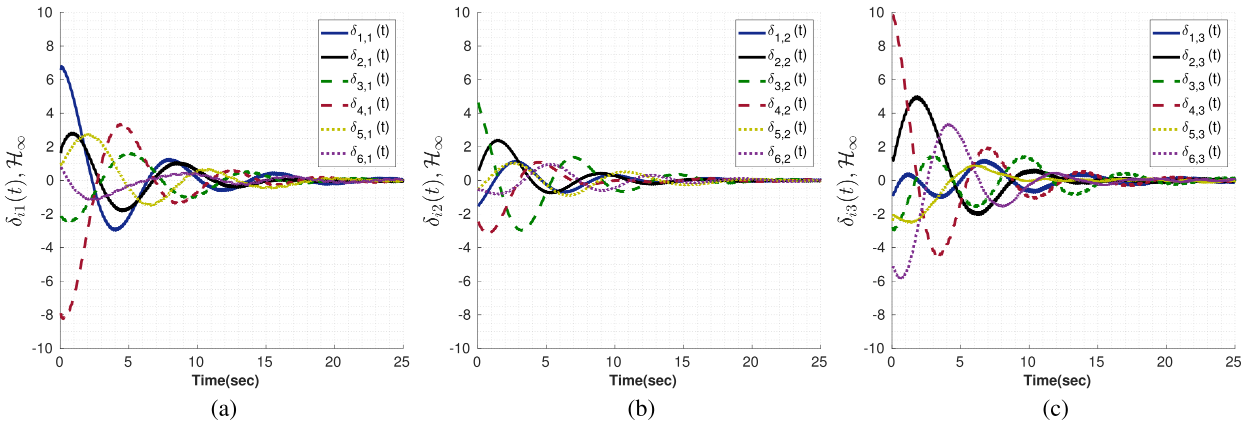



**FIGURE 4.** $\mathcal{H}_\infty$ **average TVF error response** $\delta_i(t)$**.**

Based on the asynchronous mode control gains, Fig. 2 shows the evolution of the network and detector modes performed in the simulation. We notice that there are mismatches between the modes of the network $\theta(t)$ and the detector $\widehat{\theta}(t)$ at some times during the simulation. Fig. 3 and Fig. 4 show the time-varying formation average error of each agent, denoted by $\delta_i(t) = [\delta_{i1}(t)\ \delta_{i2}(t)\ \delta_{i3}(t)\ \delta_{i4}(t)\ \delta_{i5}(t)\ \delta_{i6}(t)]'$ $(i = 1, 2, \ldots, 6)$. In order to study the average error responses of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ solutions, we consider two parameters: the velocity of the response, characterized by the time at which the signal reaches a value very close to zero, denoted by $\zeta_i = [\zeta_{i1}\ \zeta_{i2}\ \zeta_{i3}\ \zeta_{i4}\ \zeta_{i5}\ \zeta_{i6}]'$ $(i = 1, 2, \ldots, 6)$ and the maximum overshoot magnitude, denoted by $\vartheta_i = [\vartheta_{i1}\ \vartheta_{i2}\ \vartheta_{i3}\ \vartheta_{i4}\ \vartheta_{i5}\ \vartheta_{i6}]'$ $(i = 1, 2, \ldots, 6)$. The results for some selected values, which illustrate the differences between the control strategies, are summarized in Table 2. In general, the $\mathcal{H}_2$ control shows

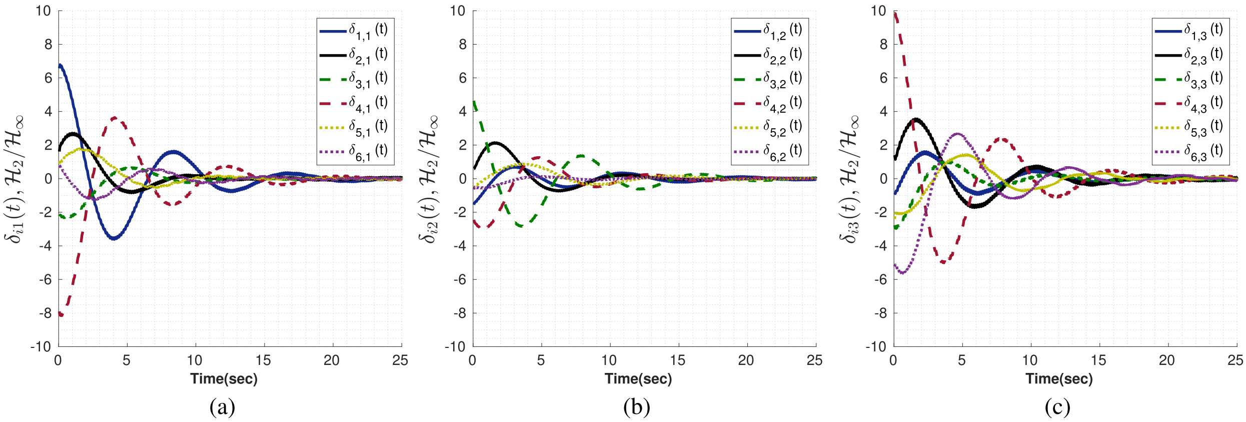



**FIGURE 5.** $\mathcal{H}_2/\mathcal{H}_\infty$ **TVF average error response** $\delta_i(t)$**.**

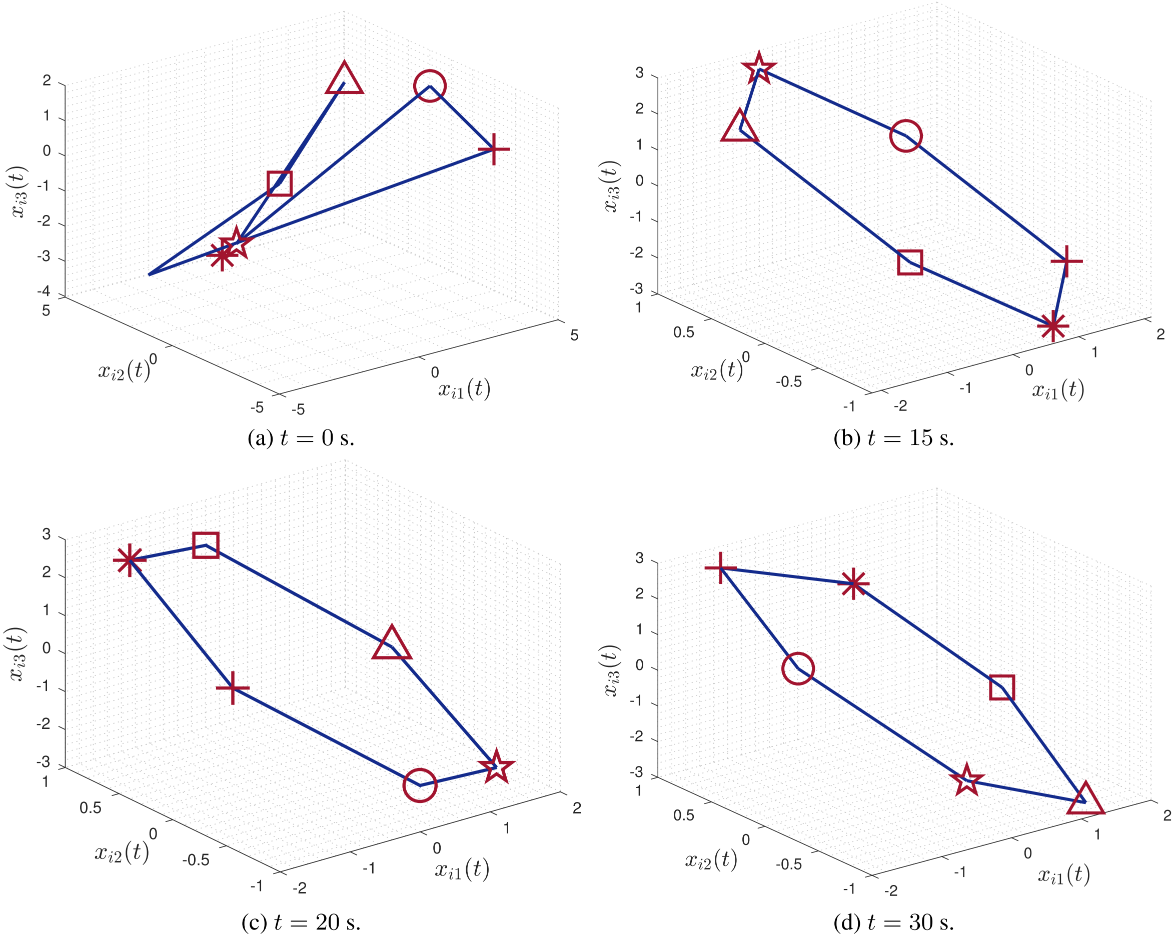



**FIGURE 6.** **States of the six agents at different time instants** $t$**.**

faster times in the parameter $\zeta_i$ than the $\mathcal{H}_\infty$ control as, for instance, the value $\zeta_{41}^2 = 1.8$ s (Fig. 3a) which is 35.7% lower than the value $\zeta_{41}^\infty = 2.8$ s (Fig. 4a). In contrast, the $\mathcal{H}_\infty$ control shows lower values $\vartheta_i$ than the $\mathcal{H}_2$ control. This effect becomes evident in states with high initial values such as $\vartheta_{43}^2 = -6$ (Fig. 3c) in comparison to $\vartheta_{43}^\infty = -4.5$ (Fig. 4c), with a 25% difference between them. These results suggest a robust response to a worst-case situation. In addition to these improvements, in both solutions $\mathcal{H}_2$ and $\mathcal{H}_\infty$ the TVF average error $\delta_i(t)$ converges to zero, showing that the presented method is capable to stabilize the multi-agent system even in the presence of uncertainties concerning to the mode of operation $\theta(t)$.

We return to Corollary 1 in order to investigate the $\mathcal{H}_2/\mathcal{H}_\infty$ control problem considering case 3.a (min $\beta_2\varphi^2 + \beta_\infty\gamma^2$). By setting $\beta_2 = \frac{1}{3}$, $\beta_\infty = \frac{2}{3}$ and $\epsilon_\infty = \epsilon_2 = 15$, the

**TABLE 2.** Selected $\zeta_i$ and $\vartheta_i$ values from Fig. 3, Fig. 4 and Fig. 5.

| | $\zeta_i$(sec) | | | $\vartheta_i$ | | |
|---|---|---|---|---|---|---|
| $\delta_{iv}(t)$ | $\mathcal{H}_2$ | $\mathcal{H}_\infty$ | $\mathcal{H}_2/\mathcal{H}_\infty$ | $\mathcal{H}_2$ | $\mathcal{H}_\infty$ | $\mathcal{H}_2/\mathcal{H}_\infty$ |
| $\delta_{11}(t)$ | 1.8 | 2.4 | 2 | -4.1 | -3 | -3.4 |
| $\delta_{41}(t)$ | 1.8 | 2.8 | 2.1 | 4.5 | 3.4 | 3.5 |
| $\delta_{12}(t)$ | 0.6 | 0.7 | 0.7 | 1.2 | 1.1 | 0.8 |
| $\delta_{62}(t)$ | 2.5 | 3.2 | 2.9 | 0.4 | 0.8 | 0.1 |
| $\delta_{43}(t)$ | 1.3 | 2 | 1.8 | -6 | -4.5 | -5 |
| $\delta_{63}(t)$ | 2.2 | 2.5 | 2.4 | 3.4 | 3.2 | 2.5 |

LMI (25), (36), (37), (38) are solved for the case of asynchronous mode operation in Fig. 2. This method achieves values $\gamma = 0.1068$ and $\varphi = 0.2188$. It is worth pointing out that the optimal values for the cost are influenced by the scalars $\epsilon_2$ and $\epsilon_\infty$. Figure 5 shows that the TVF average error $\delta_i(t)$ converges to zero despite the topology network changes and divergences between the mode of operation $\theta(t)$ and the detector $\widehat{\theta}(t)$. The $\mathcal{H}_2/\mathcal{H}_\infty$ control also combines the fast response and overshoot attenuation of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control respectively (Table 2), for instance, the value $\zeta_{11}^{2/\infty} = 2$ s (Fig. 5a) differs only in 10% with respect to $\zeta_{11}^{2} = 1.8$ s (Fig. 3a), and the value $\vartheta_{63}^{2/\infty} = 2.5$ (Fig. 5c) is even 28.12% lower than $\vartheta_{63}^{\infty} = 3.2$ (Fig. 4c).

Figure 6 displays snapshots of the six agents at $t = 0$ s, $t = 15$ s, $t = 20$ s and $t = 30$ s for the $\mathcal{H}_2/\mathcal{H}_\infty$ asynchronous control, where the states of the agents are denoted by the pentagram, triangle, square, asterisk, cross and circle respectively. From Fig. 6 it can be observed that the states of the six agents keep a parallel hexagon formation and the edge of the parallel hexagon keeps rotating, in a time-varying formation. We believe that these results verify the effectiveness of the proposed method.

## VII. CONCLUSION

In this work, we studied the $\mathcal{H}_2$, $\mathcal{H}_\infty$ and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ time-varying formation control for continuous-time linear multi-agent systems with Markovian switching topologies, under partial information on the Markov parameter. It is considered a hidden Markov model to represent the possible mismatching between the detector and network operation modes. Under this formulation, a set of LMI conditions are provided to design a distributed static output controller that guarantees the closed-loop stability of the MAS in the mean square sense and upper-bounds for each performance case. Notice that the controller relies only on the imperfect information that comes from the detector. Numerical examples were performed to verify the effectiveness of this method, showing that the mixed control $\mathcal{H}_2/\mathcal{H}_\infty$ successfully combines the control properties of the pure $\mathcal{H}_2$ and $\mathcal{H}_\infty$ strategies. As pointed out, for instance, in [13], [15]–[17], communication signals in TVF control of MAS may be affected by transmission noises and/or communication delays. Thus it would be interesting, as future work, to deal with the $\mathcal{H}_2$, $\mathcal{H}_\infty$ and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ TVF control of MAS under the hidden Markov switching topology as proposed in this paper, but also incorporating communication noises and time-delays.

## REFERENCES

[1] S. Butenko, R. Murphey, and P. M. Pardalos, *Cooperative Control: Models, Applications and Algorithms*, vol. 1. Boston, MA, USA: Springer-Verlag, 2013.
[2] J. S. Shamma, *Cooperative Control of Distributed Multi-Agent Systems*. Hoboken, NJ, USA: Wiley, 2007.
[3] H. S. Ahn, *Formation Control: Approaches for Distributed Agents*. Cham, Switzerland: Springer, 2020.
[4] R. Rahimi, F. Abdollahi, and K. Naqshi, "Time-varying formation control of a collaborative heterogeneous multi agent system," *Robot. Auton. Syst.*, vol. 62, no. 12, pp. 1799–1805, Dec. 2014.
[5] G. Antonelli, F. Arrichiello, F. Caccavale, and A. Marino, "Decentralized time-varying formation control for multi-robot systems," *Int. J. Robot. Res.*, vol. 33, no. 7, pp. 1029–1043, 2014.
[6] X. Dong, Y. Zhou, Z. Ren, and Y. Zhong, "Time-varying formation control for unmanned aerial vehicles with switching interaction topologies," *Control Eng. Pract.*, vol. 46, pp. 26–36, Jan. 2016.
[7] O. L. V. Costa, M. D. Fragoso, and M. G. Todorov, *Continuous-Time Markov Jump Linear Systems*. New York, NY, USA: Springer-Verlag, 2013.
[8] M. J. Park, O. M. Kwon, J. H. Park, S. A. Lee, and E. J. Cha, "Randomly changing leader-following consensus control for Markovian switching multi-agent systems with interval time-varying delays," *Nonlinear Anal., Hybrid Syst.*, vol. 12, pp. 117–131, May 2014.
[9] K. Li and X. Mu, "Necessary and sufficient conditions for leader-following consensus of multi-agent systems with random switching topologies," *Nonlinear Anal., Hybrid Syst.*, vol. 37, pp. 1–14, Aug. 2020.
[10] Z. Zhou, H. Wang, Y. Wang, X. Xue, and M. Zhang, "Distributed formation control for multiple quadrotor UAVs under Markovian switching topologies with partially unknown transition rates," *J. Franklin Inst.*, vol. 356, no. 11, pp. 5706–5728, Jul. 2019.
[11] L. Ding and G. Guo, "Sampled-data leader-following consensus for nonlinear multi-agent systems with Markovian switching topologies and communication delay," *J. Franklin Inst.*, vol. 352, no. 1, pp. 369–383, 2015.
[12] M. He, J. Mu, and X. Mu, "$H_\infty$ leader-following consensus of nonlinear multi-agent systems under semi-Markovian switching topologies with partially unknown transition rates," *Inf. Sci.*, vol. 513, pp. 168–179, Mar. 2020.
[13] Y. L. Shang, "Consensus seeking over Markovian switching networks with time-varying delays and uncertain topologies," *Appl. Math. Comput.*, vol. 273, pp. 1234–1245, Jan. 2016.
[14] X. Ge and Q.-L. Han, "Consensus of multiagent systems subject to partially accessible and overlapping Markovian network topologies," *IEEE Trans. Cybern.*, vol. 47, no. 8, pp. 1807–1819, Aug. 2017.
[15] M. Cong and X. Mu, "$\mathcal{H}_\infty$ consensus of linear multi-agent systems with semi-Markov switching network topologies and measurement noises," *IEEE Access*, vol. 7, pp. 156089–156096, 2019.
[16] D. Xue, J. Yao, J. Wang, Y. Guo, and X. Han, "Formation control of multi-agent systems with stochastic switching topology and time-varying communication delays," *IET Control Theory Appl.*, vol. 7, no. 13, pp. 1689–1698, Sep. 2013.
[17] B. Li, G. Wen, Z. Peng, S. Wen, and T. Huang, "Time-varying formation control of general linear multi-agent systems under Markovian switching topologies and communication noises," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 68, no. 4, pp. 1303–1307, Apr. 2021.
[18] N. H. A. Nguyen and S. H. Kim, "Leader-following consensus for multi-agent systems with asynchronous control modes under nonhomogeneous Markovian jump network topology," *IEEE Access*, vol. 8, pp. 203017–203027, 2020.
[19] S. Dong, W. Ren, Z. G. Wu, and H. Su, "$H_\infty$ output consensus for Markov jump multiagent systems with uncertainties," *IEEE Trans. Cybern.*, vol. 50, no. 5, pp. 2264–2273, Dec. 2020.
[20] H. Zhang, R. Yang, H. Yan, and F. Yang, "$H_\infty$ consensus of event-based multi-agent systems with switching topology," *Inf. Sci.*, vols. 370–371, pp. 623–635, 2016.
[21] L. Mo, S. Guo, and Y. Yu, "Mean-square consensus of heterogeneous multi-agent systems with nonconvex constraints, Markovian switching topologies and delays," *Neurocomputing*, vol. 291, pp. 167–174, May 2018.
[22] F. Stadtmann and O. L. V. Costa, "$H_2$-control of continuous-time hidden Markov jump linear systems," *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 4031–4037, Aug. 2017.
[23] F. Stadtmann and O. L. V. Costa, "Exponential hidden Markov models for $H_\infty$ control of jumping systems," *IEEE Control Syst. Lett.*, vol. 2, no. 4, pp. 845–850, Oct. 2018.

[24] A. M. de Oliveira, O. L. V. Costa, M. D. Fragoso, and F. Stadtmann, "Dynamic output feedback control for continuous-time Markov jump linear systems with hidden Markov models," *Int. J. Control*, early access, pp. 1–13, Sep. 2020, doi: 10.1080/00207179.2020.1817566.

[25] A. M. de Oliveira and O. L. V. Costa, "Control of continuous-time Markov jump linear systems with partial information," in *Modern Trends in Controlled Stochastic Processes*, A. Piunovskiy and Y. Zhang, Eds. Cham, Switzerland: Springer, 2021, pp. 87–107.

[26] D. S. Bernstein and W. M. Haddad, "LQG control with an $H_\infty$ performance bound: A Riccati equation approach," *IEEE Trans. Autom. Control*, vol. 34, no. 3, pp. 293–305, Mar. 1989.

[27] P. P. Khargonekar and M. A. Rotea, "Mixed $H_2/H_\infty$ control: A convex optimization approach," *IEEE Trans. Autom. Control*, vol. 36, no. 7, pp. 824–837, Jul. 1991.

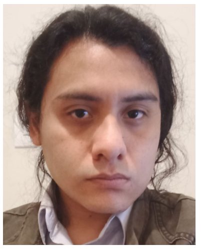
[28] B. S. Chen and W. Zhang, "Stochastic $H_2/H_\infty$ control with state-dependent noise," *IEEE Trans. Autom. Control*, vol. 49, no. 1, pp. 45–57, Jan. 2004.

[29] H. Ma, W. Zhang, and T. Hou, "Infinite horizon $H_2/H_\infty$ control for discrete-time time-varying Markov jump systems with multiplicative noise," *Automatica*, vol. 48, no. 7, pp. 1447–1454, Jul. 2012.

[30] Y. Huang, W. Zhang, and G. Feng, "Infinite horizon $H_2/H_\infty$ control for stochastic systems with Markovian jumps," *Automatica*, vol. 44, no. 3, pp. 857–863, 2008.

[31] D. J. N. Limebeer, B. D. O. Anderson, and B. Hendel, "A Nash game approach to mixed $H_2/H_\infty$ control," *IEEE Trans. Autom. Control*, vol. 39, no. 1, pp. 69–82, Jan. 1994.

[32] P. Finsler, "Über das vorkommen definiter und semidefiniter Formen in Scharen quadratischer Formen," *Commentarii Mathematici Helvetici*, vol. 9, no. 1, pp. 188–192, 1937.

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