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A q-Rung Orthopair Cloud-Based Multi-Attribute Decision-Making Algorithm: Considering the Information Error and Multilayer Heterogeneous Relationship of Attributes

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ABSTRACT The representation and aggregation of attribute information is the key to address the multi-attribute decision-making (MADM) problems. Based on the error problem of attribute information given by experts and the heterogeneous relationship between attributes, a new MADM method based on q-rung orthopair cloud interaction weighted Maclaurin symmetric mean (q-ROCIWMSM) operator is proposed. In it, this method firstly considers the randomness of evaluation information given by experts, integrates the q-rung orthopair fuzzy (q-ROF) set and cloud model and define the new concept of q-rung orthopair cloud (q-ROC), so as to betray the error distribution characteristics of membership and non-membership information caused by randomness. Then, to investigate the multi-layer heterogeneous relationship among membership functions and different quantitative attributes, the interaction operator and Maclaurin symmetric mean (MSM) operator are introduced into q-ROC information, and the q-ROCIWMSM operator is proposed to aggregate q-ROC information with multi-dimensional parameter characteristics. Thirdly, a framework of MADM based on q-ROCIWMSM operator is established. Finally, a cross-border e-commerce consumption decision-making case is used to test the effectiveness of the proposed method. At the same time, robustness analysis and method comparison analysis further show the advantages of our approach. The study results show that the proposed method can accurately describe the error distribution characteristics of attribute information and eliminate the adverse effects of extreme values in the evaluation information on the decision-making results. In addition, the method proposed in this paper can flexibly reflect the multi-layer heterogeneous relationship between attributes, and has strong flexibility and applicability.

INDEX TERMS q-rung orthopair cloud, interaction operator, Maclaurin symmetric mean operator, information error, multilayer heterogeneous relationship.

I. INTRODUCTION

Throughout the ages, decision-making has always been a necessary skill for managers. Reasonable decision-making can help decision-makers get out of the current trouble. However, with the rapid development of economy and society, the decision-making environment and situation we are facing are becoming more and more complex, and decision-making has become a very difficult task [1]–[5]. In practical decision-making problems, the values of some attributes can be given with certain values, such as ‘‘temperature and humidity’’, ‘‘vehicle mileage’’, etc., but in more problems, due to the complexity of the problem, the uncertainty of the environment and the limitations of decision-makers' knowledge, it is difficult to give quantitative measures for each decision-making factor, such as ‘‘the impact of weather on site selection’’, ‘‘impact of investors' education on corporate performance’’, etc. For these problems, there are many uncertain

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factors, so it is difficult to give the best decision-making solution based on the traditional accurate numerical measurement method. In view of these phenomena, Zadeh, an American cyberneticist, proposed fuzzy set theory in 1965 to deal with fuzzy and uncertain information in decision-making problems [6]. Zadeh extended the classical set to fuzzy set, and extended the characteristic function to membership function. For the first time, Zadeh used mathematical means to describe the fuzzy concept, and pioneered the study of uncertain phenomena from the perspective of fuzzy sets [6]. In the theory of fuzzy sets, an element is given a concept of membership, which represents the degree to which the element belongs to a set. Compared with the ''either/or'' thought of classical set theory, the concept of membership degree in fuzzy set is more suitable for describing various fuzzy phenomena in real life. Because of these advantages, the fuzzy set theory is widely used in multi-attribute decision-making (MADM) theory. However, only a single membership degree u of fuzzy sets can express the support and opposition states at the same time, i.e. agree as u and oppose as $1 - u$, which cannot represent the neutral state that neither supports nor opposes.

Therefore, Atanassov put forward the concept of intuitionistic fuzzy set (IFS) [7]. IFS includes two dimensions ''membership degree'' and ''non-membership degree'', which are represented by u and v respectively. u is membership functions and v is non membership functions, but they must satisfy the conditions of $u \in[0,1]$, $v \in[0,1]$ and $u+v \leq 1$. In addition, the concept of hesitancy degree is introduced into IFSs. The hesitancy degree is defined as $\pi = 1-u-v$, which represents the hesitancy degree over a decision. IFSs are widely used in disease diagnosis [8]–[12], pattern recognition [13]–[17] and other fields of MADM.

However, it is noticeable that the sum of membership degree and non-membership degree less than or equal to 1 cannot be strictly realized in many cases in daily life. When solving practical problems, we often encounter the situation that the sum of membership degree and non-membership degree is greater than 1. For example, the membership degree u given by an expert for a certain information is 0.7, and the non membership degree v given by him is 0.5. In this case, IFSs cannot be applied for description. In view of this situation, Yager proposed the concept of Pythagorean fuzzy set (PFS) [18], that is, the sum of u and v is allowed to be greater than 1, but the sum of squares of them is not greater than 1. Therefore, it can be seen that PFS has a wider application range. The appearance of PFS effectively broadens the limitations of IFS and brings less information distortion, which provides more possibilities for decision-making. In the follow-up extension studies, Yager introduced the concept of q-rung orthopair fuzzy set (q-ROFS) [19]. The constraint condition of q-ROFS is that the sum of the q^{th} power of membership degree and the q^{th} power of non-membership degree is less than or equal to 1 ($q \geq 1$), that is, $u^q + v^q \leq 1$. It can be seen that when $q = 1$, the q-ROFS evolves into IFS; when $q = 2$, the q-ROFS evolves into PFS. The q-ROFS broadens the boundary conditions of decision-making information wider, so it can express fuzzy information more completely and accurately and it is more in line with the decision-making problems in real life. Moreover, it covers the special cases of $q = 1$ and $q = 2$, so it has stronger generality and wider application range. It is a kind of very effective tool to describe uncertain phenomena. The q-ROFS has been extended by many scholars, and rich achievements have been made in aspects of fundamental property [20]–[24], operation rules [25]–[27], distance and similarity measure [28] and information aggregation operator [29]–[34], etc.

In addition, the randomness of information is an important feature of uncertain decision making [35]. For the quantitative characterization of randomness, Li *et al.* proposed a cloud model to transform the qualitative concept of randomness in uncertain decision-making problems into quantitative data [36]. It is mainly characterized by three digital characteristics, namely, expectation Ex, entropy En and excess entropy He, which realizes the transformation between these fuzzy concepts and specific data and is represented by intuitive cloud images. Compared with the traditional uncertain knowledge representation method for fuzzy information, it is more specific and clear. Cloud model is widely used in the fields of knowledge representation [37]–[39], multi-attribute decision-making [40], comprehensive evaluation [41]–[42], etc. In addition, cloud model is combined with traditional fuzzy set, IFS and PFS, and some new concepts of cloud fuzzy set are proposed [38], [40]. However, these new concepts do not essentially touch the random characteristics of membership function and non-membership function of fuzzy set, but simply combine the overall evaluation value of attribute and its random characteristics with the traditional membership function. For example, in the literatures [38], [40], if an IFS $A=<u, v>$ and cloud model $A=<\text{Ex, En, He}>$, then the intuitionistic fuzzy cloud $A=<u, v, \text{Ex, En, He} >$. In other words, the literatures [38], [40] uses two different knowledge representation models to evaluate two aspects of A independently, but not combines two methods to represent the same attribute.

Furthermore, the information aggregation operator is another key step to handle the MADM problems. Information aggregation operators are mainly reflected in the level of membership function and attribute. In the existing studies, the information aggregation at membership function level is mainly based on Archimedean t-norms and t-conorms (ATT) paradigm, that is, different membership functions are regarded as independent relations [26]. In literatures [43]–[44], the irrationality of the operation rules based on ATT paradigm is proved by an example, and then interaction operator is proposed by He *et al.* [43]. The interaction operator examines the internal relations among the membership functions, which avoids the irrationality of traditional algorithms [45]. Similarly, at the attribute level, different attributes also have internal relations. Therefore, many scholars have introduced Heronian mean (HM), Bonferroni means (BM), Maclaurin symmetric mean (MSM) and

Muirhead mean (MM) operators to investigate the intrinsic relationship of attributes, and build the information aggregation model and MADM methods based on the above operators [26], [29], [31], [44], [46]. However, the heterogeneity of membership function and attribute is seldom considered simultaneously. In fact, the essence of attribute relationship is the internal relationship among different membership functions. Therefore, it is necessary to conduct interactive operations on information at different levels at the same time, and construct the information aggregation operators with multi-layer heterogeneous relationship.

In conclusion, the existing researches on uncertain information presentation do not organically combine the fuzziness and randomness of information, thus reducing the comprehensiveness and accuracy of characterization of uncertain information. In addition, the multi-layer heterogeneity relationships of membership function level and attribute level are not considered simultaneously in uncertain information aggregation, which will aggravate the information distortion degree in the process of information aggregation. For these, this paper combines the q-ROFS with cloud model from the error perspective caused by the random characteristics of uncertain information, and proposes a q-rung orthopair cloud (q-ROC) model to describe the fuzziness and randomness of membership functions. Meanwhile, considering the internal interaction relationship among different membership functions and attributes, the interaction operator is firstly introduced to define the interaction operation laws between q-ROCs, and then MSM operator is introduced to propose the q-rung orthopair cloud interaction weighted Maclaurin symmetric mean (q-ROCIWMSM) operator. Finally, a MADM method based on the proposed aggregation operator is constructed. The contributions of this paper are as follows:

(1) A new concept of q-rung orthopair cloud is proposed to describe the error distribution characteristics of membership function and membership function. The q-rung orthopair cloud can accurately depict the evaluation information of experts and reduce the influence of uncertainty and fuzziness on the decision results.

(2) Some new interactive operational rules between q-ROCs are defined to investigate the internal relationship between membership functions, which can be applied to deal with the extreme values in the information aggregation process and avoid the adverse effects caused by the them.

(3) A q-ROCIWMSM operator is proposed to investigate the internal interactive relationship among membership functions and attributes. Since the proposed operator can investigate the influence of the relationship among any number of attributes on the decision result, it has stronger environmental adaptability.

(4) A MADM method based on the proposed operator is constructed to support the related decision-making problems. Our method contains multiple parameters, it can adjust the parameters according to the environmental changes to obtain the corresponding decision results, so this method has stronger flexibility.

The other sections of this paper are arranged as follows: Section 2 reviews the basic concepts of q-ROFS and cloud model, and defines the new concept of q-ROC and its interactive operation laws. Section 3 proposes the q-ROCIWMSM operator and analyzes its related properties. In section 4, a MADM method based on q-ROCIWMSM operator is established. Section 5 gives an example to verify the effectiveness of the proposed method and performs the method comparison and sensitivity analysis. Section 6 gives some conclusions.

II. Q-ROC AND ITS INTERACTION OPERATIONAL RULES

Definition 1 [19]: Let X be a ordinal set, then the q-ROF collection A on X can be betrayed as

$$A = \{< x, u_A(x), v_A(x) > | x \in X\} \tag{1}$$

where $u_A(x)$: $X \rightarrow [0, 1]$ and $v_A(x)$: $X \rightarrow [0, 1]$ are the membership degree (MED) and non-membership degree (NMED) of A, respectively, and $\forall x \in X, 0 \leq u_A(x)^q + v_A(x)^q \leq 1 (q \geq 1)$. The hesitation degree can be calculated using $\pi(x) = \sqrt[q]{1 - u_A(x)^q - v_A(x)^q}$. In addition, $\alpha = (u_a, v_a)$ can be recorded as the q-ROF number.

Definition 2 [36]: Let R be a normal set and G be a qualitative concept of R. If $x \in R$ is a stochastic exemplum of G, and it satisfies $x \sim N\left(Ex, En^2\right), En \sim N(En, He^2)$, then the certainty degree y of x belonging to G can be recorded as:

$$y = e^{-\frac{(x-En)^2}{2En^2}} \tag{2}$$

formula (2) can be deemed as a normal cloud of x in R, and (x, y) can be regarded as cloud droplet.

The meaning of the qualitative concept R can be described with three parameters: mathematical expectation Ex, entropy En, and hyper entropy He, respectively. Suppose a cloud be the H, then we defined the cloud H as $H = (Ex, En, He)$, which consists of thousands of cloud droplets (x, y). Where, Ex is the center value of R, En is used to depict the divergence rate of R or the dispersion degree of the cloud droplets (x, y), and He indicates as fuzziness of En or the thickness of the cloud H.

Combing the concepts of q-ROF set and cloud model, we define a new notion of q-rung orthopair cloud as shown in Definition 3.

Definition 3: Let X be a nonempty set, then the q-ROC A on X can be defined as

$$A = \langle (\vartheta_{Ex}, \vartheta_{En}, \vartheta_{He}), (\delta_{Ex}, \delta_{En}, \delta_{He}) \rangle \tag{3}$$

wherein, ϑ_{Ex} and δ_{Ex} are the mathematical expectations of MED and NMED, ϑ_{En} and δ_{En} are the entropies of MED and NMED, ϑ_{He} and δ_{He} are the hyper entropies of them, respectively. In reality decision making problems, the ϑ_{Ex} and δ_{Ex} are the decision information given by experts or decision makers with random and fuzzy characteristics, there must be errors in the decision information. Therefore, ϑ_{En} and δ_{En} can be used to describe the errors of MED and NMED, and ϑ_{He} and δ_{He} can be applied to portray the errors or fuzziness of ϑ_{En} and δ_{En}.

Definition 4: Let $a = \langle(\vartheta_{Ex}, \vartheta_{En}, \vartheta_{He}), (\delta_{Ex}, \delta_{En}, \delta_{He})\rangle$, $a_1 = \langle(\vartheta_{Ex_1}, \vartheta_{En_1}, \vartheta_{He_1}), (\delta_{Ex_1}, \delta_{En_1}, \delta_{He_1})\rangle$ and $a_2 = \langle(\vartheta_{Ex_2}, \vartheta_{En_2}, \vartheta_{He_2}), (\delta_{Ex_2}, \delta_{En_2}, \delta_{He_2})\rangle$ be three q-ROCs, λ is an non-zero real number. Further, considering the interrelationship between q-ROCs, then we can define the interaction operational laws between q-ROCs, as follows, $(1) a_1 \oplus' a_2$, $(2) a_1 \otimes a_2$, $(3) \lambda a$, and $(4) a^\lambda$, as shown at the bottom of the page.

There are some properties on interaction operational laws of q-ROCs as follows:

(1) $\alpha_1 \oplus \alpha_2 = \alpha_2 \oplus \alpha_1$
(2) $\alpha_1 \otimes \alpha_2 = \alpha_2 \otimes \alpha_1$
(3) $\lambda(\alpha_1 \oplus \alpha_2) = \lambda\alpha_1 \oplus \lambda\alpha_2$
(4) $(\alpha_1 \otimes \alpha_2)^\lambda = \alpha_1^\lambda \otimes \alpha_2^\lambda$
(5) $\lambda_1\alpha + \lambda_2\alpha = (\lambda_1 + \lambda_2)\alpha$
(6) $\alpha^{\lambda 1} \otimes \alpha^{\lambda 2} = \alpha^{\lambda 1+\lambda 2}$

We can easily prove the above properties are true according to Definition 4.

Definition 5 [38]: Let $a = \langle(\vartheta_{Ex}, \vartheta_{En}, \vartheta_{He}), (\delta_{Ex}, \delta_{En}, \delta_{He})\rangle$ be a q-ROC, and $(x_{\vartheta_1}, y_{\vartheta_1}), (x_{\vartheta_2}, y_{\vartheta_2}), \ldots, (x_{\vartheta_n}, y_{\vartheta_n})\}$ be n cloud droplets of the MED cloud $(Ex_\vartheta, En_\vartheta, He_\vartheta)$, and $(x_{\delta_1}, y_{\delta_1}), (x_{\delta_2}, y_{\delta_2}), \ldots, (x_{\delta_n}, y_{\delta_n})\}$ be n cloud droplets of the NMED cloud $(Ex_\delta, En_\delta, He_\delta)$. Then, the score function of a can be expressed as

$$G(a) = \frac{1}{n^2}\sum_{i=1}^{n} x_{\vartheta_i} \cdot \sum_{i=1}^{n} y_{\vartheta_i} - \frac{1}{n^2}\sum_{i=1}^{n} x_{\delta_i} \cdot \sum_{i=1}^{n} y_{\delta_i} \tag{4}$$

where $\frac{1}{n^2}\sum_{i=1}^{n} x_{\vartheta_i} \cdot \sum_{i=1}^{n} y_{\vartheta_i}$ and $\frac{1}{n^2}\sum_{i=1}^{n} x_{\delta_i} \cdot \sum_{i=1}^{n} y_{\delta_i}$ denote the comprehensive expectations of clouds $(Ex_\vartheta, En_\vartheta, He_\vartheta)$ and $(Ex_\delta, En_\delta, He_\delta)$, respectively. Hence, the higher the value G(a), the better the corresponding q-ROC a is.

Definition 6 [47]: Let $p \geq 0$, $q \geq 0$ with $p + q > 0$, a_{i_j} be a normal real numbers set, then the expression

$$MSM(a_1, a_2, \ldots, a_n) = \left(\frac{\bigoplus_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \otimes_{j=1}^{k} a_{i_j}}{C_n^k}\right)^{\frac{1}{k}} \tag{5}$$

is referred to as MSM operator.

III. THE Q-ROCIWMSM OPERATOR

We integrate the interaction operator and MSM operator under q-ROC information and present a q-ROCIWMSM operator

Definition 7: Let $a_i = \langle(\vartheta_{Ex_i}, \vartheta_{En_i}, \vartheta_{He_i}), (\delta_{Ex_i}, \delta_{En_i}, \delta_{He_i})\rangle$ be a q-ROC set, $w_i (i = 1, 2, \ldots, n)$ is the weight vector of corresponding a_i, which is constraints into $0 \leq a_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$. The following formula

$$q - ROCIWMSM(a_1, a_2, \ldots, a_n) = \left(\frac{\bigoplus_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \otimes_{j=1}^{k} n w_{i_j} a_{i_j}}{C_n^k}\right)^{\frac{1}{k}} \tag{6}$$

is called as the q-ROCIWMSM operator, where, n is coefficient of balance, $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$

Theorem 1: Let $a_i = \langle(\vartheta_{Ex_i}, \vartheta_{En_i}, \vartheta_{He_i}), (\delta_{Ex_i}, \delta_{En_i}, \delta_{He_i})\rangle$ be a q-ROC set, then the value calculated by the ROCIWMSM operator is still a q-ROC form,

$$\begin{aligned} q - ROCIWMSM(a_1, a_2, \ldots, a_n) &= \left(\frac{\bigoplus_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \bigotimes_{j=1}^{k} n w_{i_j} a_{i_j}}{C_n^k}\right)^{\frac{1}{k}} \\ &= \langle(\vartheta_{Ex}, \vartheta_{En}, \vartheta_{He}), (\delta_{Ex}, \delta_{En}, \delta_{He})\rangle \end{aligned}$$

where, $\vartheta_{Ex}, \vartheta_{En}, \vartheta_{He}$ and $\delta_{Ex}, \delta_{En}, \delta_{He}$, as shown at the bottom of the next page.

Proof: Firstly, we use the interaction operation laws between q-ROCs in **Definition 4** to get, $a_{i_j} n w_{i_j}$, as shown at the bottom of the page 6.

Then, we follow the principles of the mathematical induction theory to obtain $\otimes_{j=1}^{k} n w_{i_j} a_{i_j}$ and $\bigoplus_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \otimes_{j=1}^{k} n w_{i_j} a_{i_j}$, as shown at the bottom of the page 6.

Further, we ca get the following result $\frac{\bigoplus_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \otimes_{j=1}^{k} n w_{i_j} a_{i_j}}{C_n^k}$, as shown at the page 7.

$$(1) a_1 \oplus' a_2 = \left\langle \begin{aligned} &\left(\vartheta_{Ex_1}^q + \vartheta_{Ex_2}^q - \vartheta_{Ex_1}^q \vartheta_{Ex_2}^q\right)^{\frac{1}{q}}, \vartheta_{En_1} + \vartheta_{En_2}, \vartheta_{He_1} + \vartheta_{He_2}; \\ &\left(\delta_{Ex_1}^q + \delta_{Ex_2}^q - \delta_{Ex_1}^q \delta_{Ex_2}^q - \delta_{Ex_1}^q \vartheta_{Ex_2}^q - \delta_{Ex_2}^q \vartheta_{Ex_1}^q\right)^{\frac{1}{q}}, \delta_{En_1} + \delta_{En_2}, \delta_{He_1} + \delta_{He_2} \end{aligned} \right\rangle,$$

$$(2) a_1 \otimes a_2 = \left\langle \begin{aligned} &\left(\vartheta_{Ex_1}^q + \vartheta_{Ex_2}^q - \vartheta_{Ex_1}^q \vartheta_{Ex_2}^q - \vartheta_{Ex_1}^q \delta_{Ex_2}^q - \vartheta_{Ex_2}^q \delta_{Ex_1}^q\right)^{\frac{1}{q}}, \vartheta_{En_1} + \vartheta_{En_2}, \vartheta_{He_1} + \vartheta_{He_2}; \\ &\left(\delta_{Ex_1}^q + \delta_{Ex_2}^q - \delta_{Ex_1}^q \delta_{Ex_2}^q\right)^{\frac{1}{q}}, \delta_{En_1} + \delta_{En_2}, \delta_{He_1} + \delta_{He_2} \end{aligned} \right\rangle,$$

$$(3) \lambda a = \left\langle \begin{aligned} &\left(1 - \left(1 - \vartheta_{Ex}^q\right)^\lambda\right)^{\frac{1}{q}}, \lambda\vartheta_{En}, \lambda\vartheta_{He}; \\ &\left(\left(1 - \vartheta_{Ex}^q\right)^\lambda - \left(1 - \left(\vartheta_{Ex}^q + \delta_{Ex}^q\right)\right)^\lambda\right)^{\frac{1}{q}}, \lambda\delta_{En}, \lambda\delta_{He} \end{aligned} \right\rangle,$$

$$(4) a^\lambda = \left\langle \begin{aligned} &\left(\left(1 - \delta_{Ex}^q\right)^\lambda - \left(1 - \left(\delta_{Ex}^q + \vartheta_{Ex}^q\right)\right)^\lambda\right)^{\frac{1}{q}}, \lambda\vartheta_{En}, \lambda\vartheta_{He}; \\ &\left(1 - \left(1 - \delta_{Ex}^q\right)^\lambda\right)^{\frac{1}{q}}, \lambda\delta_{En}, \lambda\delta_{He} \end{aligned} \right\rangle$$

Finally, the following equation can be obtained $\left(\frac{\underset{1\le i_1\le\cdots\le i_k\le n}{\oplus}\otimes_{j=1}^{k} nw_{i_j}a_{i_j}}{C_n^k}\right)^{\frac{1}{k}}$, as shown at the page 7.

Meanwhile, for the above equation, we set $\vartheta_{Ex}, \vartheta_{En}$ and $\delta_{Ex}, \delta_{En}, \delta_{He}$, as shown at the bottom of the page 8.

Therefore, we get

$$\left(\frac{\underset{1\le i_1\le\cdots\le i_k\le n}{\oplus}\otimes_{j=1}^{k} nw_{i_j}a_{i_j}}{C_n^k}\right)^{\frac{1}{k}} = \langle(\vartheta_{Ex}, \vartheta_{En}, \vartheta_{He}), (\delta_{Ex}, \delta_{En}, \delta_{He})\rangle = q - ROCIWMSM\,(a_1, a_2, \ldots, a_n).$$

End of the proof process.

Subsequently, we can get the Theorem 1 is true.

In addition, some basic properties of q-ROCIWMSM operators can be investigated as follows:

Theorem 2 (Idempotency): Suppose $a_i = \langle(\vartheta_{Ex_i}, \vartheta_{En_i}, \vartheta_{He_i}), (\delta_{Ex_i}, \delta_{En_i}, \delta_{He_i})\rangle$ be the same with a, then $q - ROCIWMSM(a_1, a_2, \ldots, a_n) = a$.

Proof: We let $w_i = \frac{1}{n}$ for any i, since $a_i\langle(\vartheta_{Ex_i}, \vartheta_{En_i}, \vartheta_{He_i}), (\delta_{Ex_i}, \delta_{En_i}, \delta_{He_i})\rangle = a$, then we can get $\left(\frac{\underset{1\le i_1\le\cdots\le i_k\le n}{\oplus}\otimes_{j=1}^{k} nw_{i_j}a_{i_j}}{C_n^k}\right)^{\frac{1}{k}}$, as shown at the bottom of the page 9, then, we can obtain $\left(\frac{\underset{1\le i_1\le\cdots\le i_k\le n}{\oplus}\otimes_{j=1}^{k} nw_{i_j}a_{i_j}}{C_n^k}\right)^{\frac{1}{k}}$, as shown at the bottom of the page 10.

Subsequently, we can get $\left(\frac{\underset{1\le i_1\le\cdots\le i_k\le n}{\oplus}\otimes_{j=1}^{k} nw_{i_j}a_{i_j}}{C_n^k}\right)^{\frac{1}{k}}$, as shown at the bottom of the page 10.

Therefore, $q - ROCIWMSM\,(a_1, a_2, \ldots, a_n) = a$.

$$\vartheta_{Ex} = \left(\left(\begin{array}{l}1-\left(\prod\limits_{1\le i_1\le\cdots\le i_k\le n}\left(\begin{array}{l}1-\prod\limits_{j=1}^{k}\left(1-\left(1-\delta_{Ex_{i_j}}^{q}\right)^{nw_{i_j}}+\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)+\\ \prod\limits_{j=1}^{k}\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\end{array}\right)\right)^{\frac{1}{C_n^k}}+\\ \left(\prod\limits_{1\le i_1\le\cdots\le i_k\le n}\left(\prod\limits_{j=1}^{k}\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\right)^{\frac{1}{C_n^k}}\end{array}\right)^{\frac{1}{k}} - \left(\left(\prod\limits_{1\le i_1\le\cdots\le i_k\le n}\left(\prod\limits_{j=1}^{k}\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}},$$

$$\vartheta_{En} = \left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\vartheta_{En_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}},$$

$$\vartheta_{He} = \left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\vartheta_{He_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}}$$

$$\delta_{Ex} = \left(1-\left(\begin{array}{l}1-\left(\prod\limits_{1\le i_1\le\cdots\le i_k\le n}\left(\begin{array}{l}1-\prod\limits_{j=1}^{k}\left(1-\left(1-\delta_{Ex_{i_j}}^{q}\right)^{nw_{i_j}}+\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)+\\ \prod\limits_{j=1}^{k}\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\end{array}\right)\right)^{\frac{1}{C_n^k}}+\\ \left(\prod\limits_{1\le i_1\le\cdots\le i_k\le n}\left(\prod\limits_{j=1}^{k}\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\right)^{\frac{1}{C_n^k}}\end{array}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}},$$

$$\delta_{En} = \left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\delta_{En_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}},$$

$$\delta_{He} = \left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\delta_{He_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}}$$

End

The Theorem 2 is true.

Theorem 3 (Boundedness): Let $a_i = \langle(\vartheta_{Ex_i}, \vartheta_{En_i}, \vartheta_{He_i}), (\delta_{Ex_i}, \delta_{En_i}, \delta_{He_i})\rangle$ be a q-ROC set, $a^- = \underset{\leq i \leq n}{min}\{a_i\}, a^+ = \underset{\leq i \leq n}{max}\{a_i\}$, then $a^- \leq q - ROCIWMSM(a_1, a_2, \ldots, a_n) \leq a^+$.

Proof: According to Theorem 2, and we suppose $a_i = \langle(\vartheta_{Ex_i}, \vartheta_{En_i}, \vartheta_{He_i}), (\delta_{Ex_i}, \delta_{En_i}, \delta_{He_i})\rangle = a^+$ for any i. Then, we can get $q - ROCIWMSM(a^+, a^+, \ldots, a^+) = a^+$ and

$$q - ROCIWMSM(a_1, a_2, \ldots, a_n) \leq q - ROCIWMSM(a^+, a^+, \ldots, a^+) = a^+.$$

Similarly, we suppose

$$a_i = \langle(\vartheta_{Ex_i}, \vartheta_{En_i}, \vartheta_{He_i}), (\delta_{Ex_i}, \delta_{En_i}, \delta_{He_i})\rangle = a^-$$

then, we can get

$q - ROCIWMSM(a^-, a^-, \ldots, a^-) = a^-$ and

$$q - ROCIWMSM(a_1, a_2, \ldots, a_n) \geq q - ROCIWMSM(a^-, a^-, \ldots, a^-) = a^-$$

As such, $a^- \leq q - ROCIWMSM(a_1, a_2, \ldots, a_n) \leq a^+$.

End

The Theorem 3 is true.

Theorem 4 (Monotonicity): Let $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ be two q-ROC sets, $a_i = \left\langle\left(\vartheta_{Ex_{a_i}}, \vartheta_{En_{a_i}}, \vartheta_{He_{a_i}}\right), \left(\delta_{Ex_{a_i}}, \delta_{En_{a_i}}, \delta_{He_{a_i}}\right)\right\rangle$, and. For $\forall i$, if $\vartheta_{Ex_{a_i}} \geq \delta_{Ex_{a_i}}, \vartheta_{En_{a_i}} \leq \delta_{En_{a_i}}, \vartheta_{He_{a_i}} \leq \delta_{He_{a_i}}$, then

$$q - ROCIWMSM(a_1, a_2, \ldots, a_n) \geq q - ROCIWMSM(b_1, b_2, \ldots, b_n).$$

Proof: Since $l_{a_{\varsigma_i}} \geq l_{b_{\varsigma_i}}, l_{a_{\tau_i}} \geq l_{b_{\tau_i}} \phi_{a_i} \geq \phi_{b_i}, \varphi_{a_i} \leq \varphi_{b_i}$ for any i.

Then, we can find that $\tilde{a}_i \geq \tilde{b}_i$ for all i.

Meanwhile, on the basis of the Theorem 2, 3, then we can obtain $q - ROCIWMSM(a_1, a_2, \ldots, a_n) \geq q - ROCIWMSM(b_1, b_2, \ldots, b_n)$.

End

The Theorem 4 is true.

$$a_{i_j} nw_{i_j} = \left(\begin{array}{l} \left(1 - \left(1 - \vartheta^q_{Ex_{i_j}}\right)^{nw_{i_j}}\right)^{\frac{1}{q}}, nw_i \vartheta_{En_{i_j}}, nw_i \vartheta_{He_{i_j}}; \\ \left(\left(1 - \vartheta^q_{Ex_{i_j}}\right)^{nw_{i_j}} - \left(1 - \left(\vartheta^q_{Ex_{i_j}} + \delta^q_{Ex_{i_j}}\right)\right)^{nw_{i_j}}\right)^{\frac{1}{q}}, nw_i \delta_{En_{i_j}}, nw_i \delta_{He_{i_j}} \end{array} \right)$$

$$\otimes_{j=1}^k nw_{i_j} a_{i_j} = \left(\begin{array}{l} \left(\prod_{j=1}^k \left(1 - \left(1 - \vartheta^q_{Ex_{i_j}}\right)^{nw_{i_j}} + \left(1 - \left(\vartheta^q_{Ex_{i_j}} + \delta^q_{Ex_{i_j}}\right)\right)^{nw_{i_j}}\right) - \prod_{j=1}^k \left(\left(1 - \left(\vartheta^q_{Ex_{i_j}} + \delta^q_{Ex_{i_j}}\right)\right)^{nw_{i_j}}\right)\right)^{\frac{1}{q}}, \\ \prod_{j=1}^k \left(nw_i \vartheta_{En_{i_j}}\right), \prod_{j=1}^k \left(nw_i \vartheta_{He_{i_j}}\right); \\ \left(1 - \prod_{j=1}^k \left(1 - \left(1 - \vartheta^q_{Ex_{i_j}}\right)^{nw_{i_j}} + \left(1 - \left(\vartheta^q_{Ex_{i_j}} + \delta^q_{Ex_{i_j}}\right)\right)^{nw_{i_j}}\right)\right)^{\frac{1}{q}}, \\ \prod_{j=1}^k \left(nw_i \delta_{En_{i_j}}\right), \prod_{j=1}^k \left(nw_i \delta_{He_{i_j}}\right) \end{array} \right)$$

$$\underset{1 \leq i_1 \leq \cdots \leq i_k \leq n}{\oplus} \otimes_{j=1}^k nw_{i_j} a_{i_j} = \left(\begin{array}{l} \left(1 - \prod_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \left(1 - \left(\begin{array}{l} \prod_{j=1}^k \left(1 - \left(1 - \vartheta^q_{Ex_{i_j}}\right)^{nw_{i_j}} + \left(1 - \left(\vartheta^q_{Ex_{i_j}} + \delta^q_{Ex_{i_j}}\right)\right)^{nw_{i_j}}\right) \\ - \prod_{j=1}^k \left(\left(1 - \left(\vartheta^q_{Ex_{i_j}} + \delta^q_{Ex_{i_j}}\right)\right)^{nw_{i_j}}\right) \end{array} \right)\right)\right)^{\frac{1}{q}}, \\ \sum_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \prod_{j=1}^k \left(nw_i \vartheta_{En_{i_j}}\right), \sum_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \prod_{j=1}^k \left(nw_i \vartheta_{He_{i_j}}\right); \\ \left(\begin{array}{l} \prod_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \left(1 - \left(\left(\begin{array}{l} \prod_{j=1}^k \left(1 - \left(1 - \vartheta^q_{Ex_{i_j}}\right)^{nw_{i_j}} + \left(1 - \left(\vartheta^q_{Ex_{i_j}} + \delta^q_{Ex_{i_j}}\right)\right)^{nw_{i_j}}\right) \\ - \prod_{j=1}^k \left(\left(1 - \left(\vartheta^q_{Ex_{i_j}} + \delta^q_{Ex_{i_j}}\right)\right)^{nw_{i_j}}\right) \end{array} \right)\right)\right) \\ - \prod_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \left(\prod_{j=1}^k \left(\left(1 - \left(\vartheta^q_{Ex_{i_j}} + \delta^q_{Ex_{i_j}}\right)\right)^{nw_{i_j}}\right)\right) \end{array} \right)^{\frac{1}{q}}, \\ \sum_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \prod_{j=1}^k \left(nw_i \delta_{En_{i_j}}\right), \sum_{1 \leq i_1 \leq \cdots \leq i_k \leq n} \prod_{j=1}^k \left(nw_i \delta_{He_{i_j}}\right) \end{array} \right)$$

$$
\begin{aligned}
&\frac{\underset{1\le i_1\le\cdots\le i_k\le n}{\bigoplus}\otimes_{j=1}^{k} nw_{i_j}a_{i_j}}{C_n^k}\\
&=\left(\begin{array}{l}
\left(1-\left(\prod_{1\le i_1\le\cdots\le i_k\le n}\left(1-\left(\begin{array}{l}\prod_{j=1}^{k}\left(1-\left(1-\vartheta_{Ex_{i_j}}^{q}\right)^{nw_{i_j}}+\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\\ -\prod_{j=1}^{k}\left(\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\end{array}\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}},\\
\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\vartheta_{En_{i_j}}\right)}{C_n^k},\ \frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\vartheta_{He_{i_j}}\right)}{C_n^k};\\
\left(\begin{array}{l}\left(\prod_{1\le i_1\le\cdots\le i_k\le n}\left(1-\left(\begin{array}{l}\prod_{j=1}^{k}\left(1-\left(1-\vartheta_{Ex_{i_j}}^{q}\right)^{nw_{i_j}}+\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\\ -\prod_{j=1}^{k}\left(\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\end{array}\right)\right)\right)^{\frac{1}{C_n^k}}-\\
\left(\prod_{1\le i_1\le\cdots\le i_k\le n}\left(\prod_{j=1}^{k}\left(\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\right)\right)^{\frac{1}{C_n^k}}\end{array}\right)^{\frac{1}{q}},\\
\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\delta_{En_{i_j}}\right)}{C_n^k},\ \frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\delta_{He_{i_j}}\right)}{C_n^k}
\end{array}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\underset{1\le i_1\le\cdots\le i_k\le n}{\bigoplus}\otimes_{j=1}^{k} nw_{i_j}a_{i_j}}{C_n^k}\right)^{\frac{1}{k}}\\
&=\left(\begin{array}{l}
\left(\begin{array}{l}\left(\begin{array}{l}\left(1-\left(\prod_{1\le i_1\le\cdots\le i_k\le n}\left(1-\left(\begin{array}{l}\prod_{j=1}^{k}\left(1-\left(1-\vartheta_{Ex_{i_j}}^{q}\right)^{nw_{i_j}}+\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)-\\ \prod_{j=1}^{k}\left(\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\end{array}\right)\right)\right)\right)^{\frac{1}{C_n^k}}+\\
\left(\prod_{1\le i_1\le\cdots\le i_k\le n}\left(\prod_{j=1}^{k}\left(\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\right)\right)^{\frac{1}{C_n^k}}\end{array}\right)^{\frac{1}{k}}-\\
\left(\left(\prod_{1\le i_1\le\cdots\le i_k\le n}\left(\prod_{j=1}^{k}\left(\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\end{array}\right)^{\frac{1}{q}},\\
\left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\vartheta_{En_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}},\ \left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\vartheta_{He_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}};\\
\left(1-\left(\begin{array}{l}1-\left(\prod_{1\le i_1\le\cdots\le i_k\le n}\left(1-\left(\begin{array}{l}\prod_{j=1}^{k}\left(1-\left(1-\vartheta_{Ex_{i_j}}^{q}\right)^{nw_{i_j}}+\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)-\\ \prod_{j=1}^{k}\left(\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\end{array}\right)\right)\right)^{\frac{1}{C_n^k}}+\\
\left(\prod_{1\le i_1\le\cdots\le i_k\le n}\left(\prod_{j=1}^{k}\left(\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\right)\right)^{\frac{1}{C_n^k}}\end{array}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}},\\
\left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\delta_{En_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}},\ \left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(nw_i\delta_{He_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}}
\end{array}\right)
\end{aligned}
$$

IV. A MADM APPROACH USING THE Q-ROCIWMSM OPERATOR

The q-ROCIWMSM operator proposed in this paper is applied to the field of decision-making to construct a new MADM method. The framework is shown in Fig. 1, which is described as follows:

In view of the MADM problem in the context of q-ROC information, the decision-making object is to select the best alternative from n alternatives, and the alternative set is S = (S1, S2, S3, S4, S5); then, the alternatives are evaluated from m aspects to form an attribute set C = (C1, C2, C3, C4), and the attribute information is of q-ROC type. According to the above information, the decision matrix $R = \left[a_{ij}\right]_{n\times m}$ is formed, in which $a_{ij} = \left\langle\left(\vartheta_{Ex_{a_{ij}}}, \vartheta_{En_{a_{ij}}}, \vartheta_{He_{a_{ij}}}\right), \left(\delta_{Ex_{a_{ij}}}, \delta_{En_{a_{ij}}}, \delta_{He_{a_{ij}}}\right)\right\rangle$ and the corresponding attribute weight vector is $w = (w_1, w_2, \ldots, w_n)$ with $0 \leq w \leq 1$ and $\sum_{i=1}^{n} w_i = 1$. According to the above information, the steps of the MADM method in this study can be sorted out as follows:

Step 1: Construct the decision matrix according to the decision information.

Step 2: Use the q-ROCIWMSM operator to aggregate the decision information $a_{ij} = \left\langle\left(\vartheta_{Ex_{a_{ij}}}, \vartheta_{En_{a_{ij}}}, \vartheta_{He_{a_{ij}}}\right), \left(\delta_{Ex_{a_{ij}}}, \delta_{En_{a_{ij}}}, \delta_{He_{a_{ij}}}\right)\right\rangle$ into $a_i = \left\langle\left(\vartheta_{Ex_{a_i}}, \vartheta_{En_{a_i}}, \vartheta_{He_{a_i}}\right), \left(\delta_{Ex_{a_i}}, \delta_{En_{a_i}}, \delta_{He_{a_i}}\right)\right\rangle$.

Step 3: Utilize the Matlab software to compute $\frac{1}{n^2}\sum_{i=1}^{n} x_{\vartheta_i} \cdot \sum_{i=1}^{n} y_{\vartheta_i}$ and $\frac{1}{n^2}\sum_{i=1}^{n} x_{\delta_i} \cdot \sum_{i=1}^{n} y_{\delta_i}$, which represent the comprehensive expectations of clouds $(Ex_\vartheta, En_\vartheta, He_\vartheta)$ and $(Ex_\delta, En_\delta, He_\delta)$.

Step 4: Calculate the score value G(Si) of each alternative.

Step 5: Rank the alternative according to S(Si) to obtain the best decision making solution.

V. CASE ANALYSIS

A. DESCRIPTION OF DECISION MAKING PROBLEM

In recent years, cross-border e-commerce has gradually become a new trend of e-commerce development. Although

$$\vartheta_{Ex} = \left(\begin{array}{l} \left(\begin{array}{l} \left(1 - \left(\prod\limits_{1\leq i_1\leq\cdots\leq i_k\leq n}\left(\begin{array}{l} 1 - \prod\limits_{j=1}^{k}\left(1-\left(1-\delta_{Ex_{i_j}}^{q}\right)^{nw_{i_j}} + \left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right) + \\ \prod\limits_{j=1}^{k}\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}} \end{array}\right)\right)\right)^{\frac{1}{C_n^k}} + \\ \left(\prod\limits_{1\leq i_1\leq\cdots\leq i_k\leq n}\left(\prod\limits_{j=1}^{k}\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\right)^{\frac{1}{C_n^k}} \end{array}\right)^{\frac{1}{k}} - \\ \left(\left(\prod\limits_{1\leq i_1\leq\cdots\leq i_k\leq n}\left(\prod\limits_{j=1}^{k}\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}} \end{array}\right)^{\frac{1}{q}},$$

$$\vartheta_{En} = \left(\frac{\sum_{1\leq i_1\leq\cdots\leq i_k\leq n}\prod_{j=1}^{k}\left(nw_i\vartheta_{En_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}}, \vartheta_{He} = \left(\frac{\sum_{1\leq i_1\leq\cdots\leq i_k\leq n}\prod_{j=1}^{k}\left(nw_i\vartheta_{He_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}}.$$

$$\delta_{Ex} = \left(1 - \left(\begin{array}{l} \left(1 - \left(\prod\limits_{1\leq i_1\leq\cdots\leq i_k\leq n}\left(\begin{array}{l} 1 - \prod\limits_{j=1}^{k}\left(1-\left(1-\delta_{Ex_{i_j}}^{q}\right)^{nw_{i_j}} + \left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right) + \\ \prod\limits_{j=1}^{k}\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}} \end{array}\right)\right)\right)^{\frac{1}{C_n^k}} + \\ \left(\prod\limits_{1\leq i_1\leq\cdots\leq i_k\leq n}\left(\prod\limits_{j=1}^{k}\left(1-\left(\vartheta_{Ex_{i_j}}^{q}+\delta_{Ex_{i_j}}^{q}\right)\right)^{nw_{i_j}}\right)\right)^{\frac{1}{C_n^k}} \end{array}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}},$$

$$\delta_{En} = \left(\frac{\sum_{1\leq i_1\leq\cdots\leq i_k\leq n}\prod_{j=1}^{k}\left(nw_i\delta_{En_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}},$$

$$\delta_{He} = \left(\frac{\sum_{1\leq i_1\leq\cdots\leq i_k\leq n}\prod_{j=1}^{k}\left(nw_i\delta_{He_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}}.$$

the development time of cross-border e-commerce mode is relatively short, it has gradually attracted a large number of consumer groups and merchants due to its rapid development. According to the data of iiMedia Research, the scale of global B2C cross-border e-commerce transactions in 2018 increased by 27.5% year-on-year and it is expected to exceed \$800 billion in 2019, and the popularizing rate of global cross-border online shopping will reach 51.2%. In 2020, the number of users of China's cross-border online shopping service will reach 221 million, showing an astonishing growth compared with that in 2019. On the one hand, the prosperity and development of cross-border e-commerce has brought consumers rich products from all over the world, but it has also caused certain difficulties for consumers' decision-making. In the face of a wide range of similar products, how to select a suitable product is the key link for consumers to make decisions. In view of the above problems, this study uses the proposed multi-attribute decision-making method to simulate and analyze a specific cross-border e-commerce consumption decision-making problem, and the specific description is as follows:

B. DECISION-MAKING PROCESS

Example 1: The consumer wants to purchase a shirt online, and select five alternative brands around the world through a cross-border e-commerce platform to form an alternative set S = (S1, S2, S3, S4, S5); then the above five candidate brands are evaluated from four aspects: shirt material (C1), shirt appearance (C2), price preference (C3), and logistics service level (C4). At the same time, the weight vector of the above four attributes is $w = \{0.25, 0.2, 0.2, 0.35\}^T$. Finally, the following decision information matrix is formed through the evaluation of experts as shown in Table 1.

Step 1: Establish the decision information matrix $R = [a_{ij}]_{5\times 4}$ as shown in Table 1.

Step 2: Obtain the comprehensive decision information using the q-ROCIWMSM operator with k = 2, q = 3 as follows:

S1=<(0.767, 0.133, 0.02), (0.677, 0.124, 0.0185)>,
S2=<(0.602, 0.1, 0.015), (0.722, 0.12, 0.0181)>,
S3=<(0.748, 0.125, 0.0188), (0.664, 0.118, 0.0177)>,
S4=<(0.764, 0.13, 0.0195), (0.554, 0.097, 0.0145)>,
S5=<(0.738, 0.119, 0.0178), (0.745, 0.133, 0.02)>.

$$
\begin{aligned}
&\left(\frac{\bigoplus_{1\le i_1\le\cdots\le i_k\le n}\otimes_{j=1}^{k} n w_{i_j} a_{i_j}}{C_n^k}\right)^{\frac{1}{k}}\\
&= \Bigg(\Bigg(\Bigg(1-\Bigg(\prod_{1\le i_1\le\cdots\le i_k\le n}\Bigg(1-\Bigg(\prod_{j=1}^{k}\left(1-\left(1-\vartheta^q_{Ex_{i_j}}\right)^{n\frac{1}{n}}+\left(1-\left(\vartheta^q_{Ex_{i_j}}+\delta^q_{Ex_{i_j}}\right)\right)^{n\frac{1}{n}}\right)-\prod_{j=1}^{k}\left(\left(1-\left(\vartheta^q_{Ex_{i_j}}+\delta^q_{Ex_{i_j}}\right)\right)^{n\frac{1}{n}}\right)\Bigg)\Bigg)\Bigg)^{\frac{1}{C_n^k}}+\\
&\quad\left(\prod_{1\le i_1\le\cdots\le i_k\le n}\left(\prod_{j=1}^{k}\left(\left(1-\left(\vartheta^q_{Ex_{i_j}}+\delta^q_{Ex_{i_j}}\right)\right)^{n\frac{1}{n}}\right)\right)\right)^{\frac{1}{C_n^k}}\Bigg)^{\frac{1}{k}}-\\
&\quad\left(\left(\prod_{1\le i_1\le\cdots\le i_k\le n}\left(\prod_{j=1}^{k}\left(\left(1-\left(\vartheta^q_{Ex_{i_j}}+\delta^q_{Ex_{i_j}}\right)\right)^{n\frac{1}{n}}\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\Bigg)^{\frac{1}{q}},\\
&\quad\left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(n\frac{1}{n}\vartheta_{En_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}},\left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(n\frac{1}{n}\vartheta_{He_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}};\\
&\quad\Bigg(1-\Bigg(1-\Bigg(\prod_{1\le i_1\le\cdots\le i_k\le n}\Bigg(1-\Bigg(\prod_{j=1}^{k}\left(1-\left(1-\vartheta^q_{Ex_{i_j}}\right)^{n\frac{1}{n}}+\left(1-\left(\vartheta^q_{Ex_{i_j}}+\delta^q_{Ex_{i_j}}\right)\right)^{n\frac{1}{n}}\right)-\prod_{j=1}^{k}\left(\left(1-\left(\vartheta^q_{Ex_{i_j}}+\delta^q_{Ex_{i_j}}\right)\right)^{n\frac{1}{n}}\right)\Bigg)\Bigg)\Bigg)^{\frac{1}{C_n^k}}+\\
&\quad\left(\prod_{1\le i_1\le\cdots\le i_k\le n}\left(\prod_{j=1}^{k}\left(\left(1-\left(\vartheta^q_{Ex_{i_j}}+\delta^q_{Ex_{i_j}}\right)\right)^{n\frac{1}{n}}\right)\right)\right)^{\frac{1}{C_n^k}}\Bigg)^{\frac{1}{k}}\Bigg)^{\frac{1}{q}},\\
&\quad\left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(n\frac{1}{n}\delta_{En_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}},\left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}\left(n\frac{1}{n}\delta_{He_{i_j}}\right)}{C_n^k}\right)^{\frac{1}{k}}\Bigg)
\end{aligned}
$$

Furthermore, based on MATLAB software, the comprehensive attribute values of the five alternatives are generated into an attribute information distribution map composed of 2000 cloud drops. Taking S1 as an example, the program for generating cloud droplet distribution visualization map is shown in Table 2:

Therefore, the cloud droplet distribution map of S1 is obtained as shown in Fig. 2 according to the program in Table 2, and the cloud droplet distribution maps of S2, S3, S4, S5 can be obtained in the same way, as shown in Fig. 3-Fig. 6.

According to Fig. 2- Fig. 6, the cloud droplets with membership degree in alternative S2 and S5 are mainly distributed on the left side, and the non-membership cloud droplets are mainly distributed on the right side. It can be seen that the non-membership cloud droplets of the two alternatives are

$$
\begin{aligned}
&\left(\frac{\underset{1\le i_1\le\cdots\le i_k\le n}{\bigoplus}\otimes_{j=1}^{k} n w_{i_j} a_{i_j}}{C_n^k}\right)^{\frac{1}{k}} \\
&= \left(\begin{array}{l}
\left(\left(\begin{array}{l}
1-\left(\prod\limits_{1\le i_1\le\cdots\le i_k\le n}\left(1-\left(\left(1-\delta_{Ex_{i_j}}^q\right)^k-\left(1-\left(\vartheta_{Ex_{i_j}}^q+\delta_{Ex_{i_j}}^q\right)\right)^k\right)\right)\right)^{\frac{1}{C_n^k}}+ \\
\left(\prod\limits_{1\le i_1\le\cdots\le i_k\le n}\left(1-\left(\vartheta_{Ex_{i_j}}^q+\delta_{Ex_{i_j}}^q\right)\right)^k\right)^{\frac{1}{C_n^k}}
\end{array}\right)^{\frac{1}{k}}- \\
\left(\left(\prod\limits_{1\le i_1\le\cdots\le i_k\le n}\left(1-\left(\vartheta_{Ex_{i_j}}^q+\delta_{Ex_{i_j}}^q\right)\right)^k\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}
\end{array}\right)^{\frac{1}{q}}, \\
\left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\vartheta_{En_{i_j}}^k}{C_n^k}\right)^{\frac{1}{k}},\left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\vartheta_{He_{i_j}}^k}{C_n^k}\right)^{\frac{1}{k}}; \\
\left(1-\left(\begin{array}{l}
1-\left(\prod\limits_{1\le i_1\le\cdots\le i_k\le n}\left(1-\left(\left(1-\delta_{Ex_{i_j}}^q\right)^k-\left(1-\left(\vartheta_{Ex_{i_j}}^q+\delta_{Ex_{i_j}}^q\right)\right)^k\right)\right)\right)^{\frac{1}{C_n^k}}+ \\
\left(\prod\limits_{1\le i_1\le\cdots\le i_k\le n}\left(1-\left(\vartheta_{Ex_{i_j}}^q+\delta_{Ex_{i_j}}^q\right)\right)^k\right)^{\frac{1}{C_n^k}}
\end{array}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}}, \\
\left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\delta_{En_{i_j}}^k}{C_n^k}\right)^{\frac{1}{k}},\left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\delta_{He_{i_j}}^k}{C_n^k}\right)^{\frac{1}{k}}
\end{array}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\underset{1\le i_1\le\cdots\le i_k\le n}{\bigoplus}\otimes_{j=1}^{k} n w_{i_j} a_{i_j}}{C_n^k}\right)^{\frac{1}{k}} \\
&= \left(\begin{array}{l}
\left(\begin{array}{l}
\left(\begin{array}{l}
1-\left(1-\left(\left(1-\delta_{Ex_{i_j}}^q\right)^k-\left(1-\left(\vartheta_{Ex_{i_j}}^q+\delta_{Ex_{i_j}}^q\right)\right)^k\right)\right)+ \\
\left(1-\left(\vartheta_{Ex_{i_j}}^q+\delta_{Ex_{i_j}}^q\right)\right)^k
\end{array}\right)^{\frac{1}{k}}- \\
\left(\left(1-\left(\vartheta_{Ex_{i_j}}^q+\delta_{Ex_{i_j}}^q\right)\right)^k\right)^{\frac{1}{k}}
\end{array}\right)^{\frac{1}{q}},\left(\vartheta_{En_{i_j}}^k\right)^{\frac{1}{k}},\left(\vartheta_{He_{i_j}}^k\right)^{\frac{1}{k}}; \\
\left(1-\left(\begin{array}{l}
1-\left(1-\left(\left(1-\delta_{Ex_{i_j}}^q\right)^k-\left(1-\left(\vartheta_{Ex_{i_j}}^q+\delta_{Ex_{i_j}}^q\right)\right)^k\right)\right)+ \\
\left(1-\left(\vartheta_{Ex_{i_j}}^q+\delta_{Ex_{i_j}}^q\right)\right)^k
\end{array}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}},\left(\delta_{En_{i_j}}^k\right)^{\frac{1}{k}},\left(\delta_{He_{i_j}}^k\right)^{\frac{1}{k}}
\end{array}\right) \\
&= \left(\begin{array}{l}
\left(\left(1-\delta_{Ex_{i_j}}^q\right)-1+\left(\vartheta_{Ex_{i_j}}^q+\delta_{Ex_{i_j}}^q\right)\right)^{\frac{1}{q}},\vartheta_{En_{i_j}},\vartheta_{He_{i_j}}; \\
\left(1-\left(1-\delta_{Ex_{i_j}}^q\right)\right)^{\frac{1}{q}},\delta_{En_{i_j}},\delta_{He_{i_j}}
\end{array}\right)=\left(\vartheta_{Ex},\vartheta_{En},\vartheta_{He};\delta_{Ex},\delta_{En},\delta_{He}\right)
\end{aligned}
$$

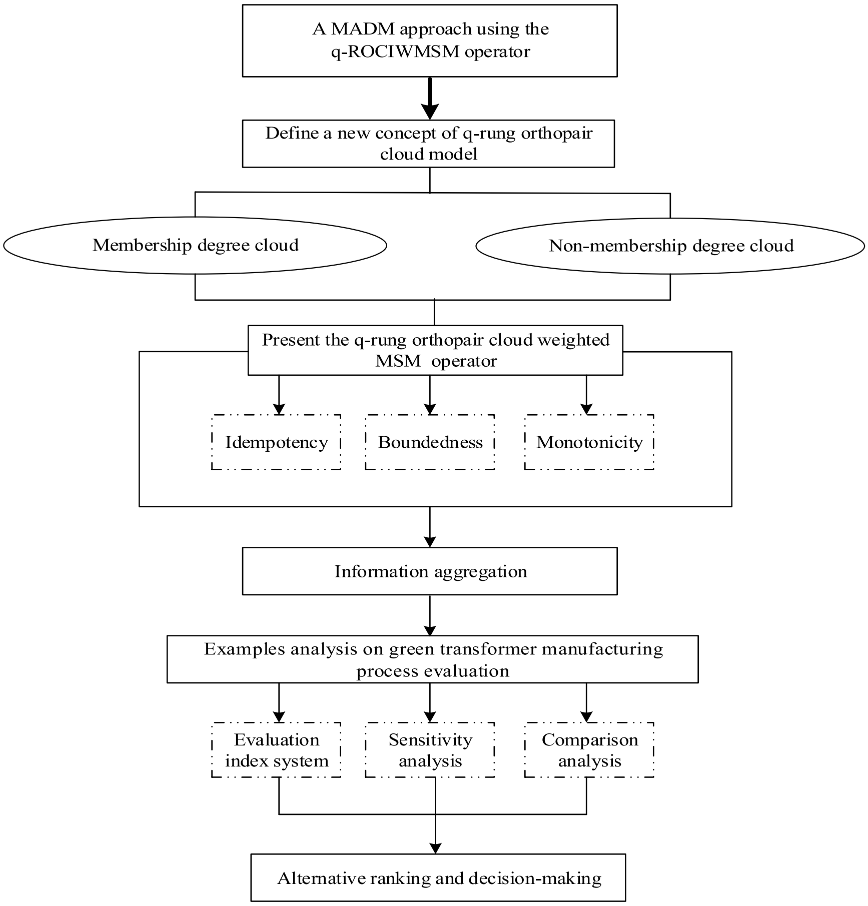

FIGURE 1. **The framework of the proposed method.**

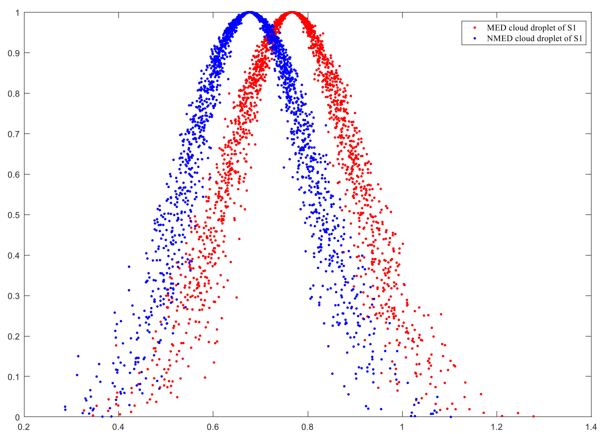

FIGURE 2. **The cloud droplet of S1.**

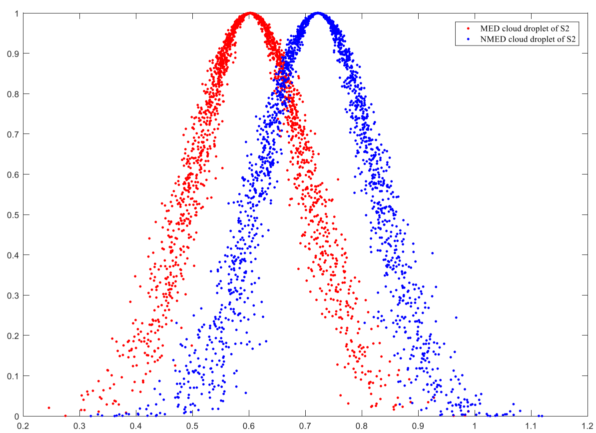

FIGURE 3. **The cloud droplet of S2.**

better than the membership cloud droplets. The distribution of membership and non-membership cloud droplets in other alternatives is opposite to the alternative S2 and S5. In addition, the distribution of membership and non-membership cloud droplets of S5 almost coincide with each other, indicating that the values of both are close to each other.

Step 3: Utilize the Matlab software program in Table 2 to get the expected values of membership cloud and non-membership cloud of each alternative at the same time, as shown in Table 3.

Step 4: Calculate the score G(Si) of each alternative according to Definition 5, as shown in Table 3.

Step 5: Get the ranking of the alternatives is S4 > S3 > S1 > S5 > S2 based on the G(Si).

TABLE 1. Decision matrix of Example 1.

	C1	C2	C3	C4
S1	<0.77, 0.116, 0.0173; 0.59, 0.089, 0.0133>	<0.78, 0.117, 0.0176; 0.68, 0.102, 0.0153>	<0.7, 0.105, 0.0158; 0.68, 0.102, 0.0153>	<0.65, 0.098, 0.0146; 0.71, 0.107, 0.016>
S2	<0.61, 0.092, 0.0137; 0.8, 0.12, 0.018>	<0.56, 0.033, 0.004; 0.75, 0.113, 0.0169>	<0.59, 0.089, 0.0133; 0.43, 0.065, 0.0097>	<0.44, 0.066, 0.0099; 0.62, 0.093, 0.0104>
S3	<0.69, 0.104, 0.0155; 0.59, 0.089, 0.0133>	<0.81, 0.122, 0.0182; 0.62, 0.093, 0.014>	<0.79, 0.119, 0.0178; 0.52, 0.078, 0.0117>	<0.5, 0.075, 0.0113; 0.77, 0.116, 0.0173>
S4	<0.79, 0.119, 0.0178; 0.49, 0.074, 0.011>	<0.7, 0.105, 0.0158; 0.49, 0.074, 0.011>	<0.8, 0.12, 0.018; 0.4, 0.06, 0.009>	<0.57, 0.086, 0.0128; 0.67, 0.101, 0.0151>
S5	<0.57, 0.086, 0.0128; 0.71, 0.107, 0.0106>	<0.61, 0.092, 0.0137; 0.8, 0.12, 0.018>	<0.66, 0.099, 0.0149; 0.53, 0.08, 0.0119>	<0.71, 0.107, 0.016; 0.81, 0.122, 0.0182>

TABLE 2. The matlab code for generating the cloud droplets of Q-ROC S1.

```
N=2000;
  Ex1=0.767; En1=0.133; He1=0.02;
  Ex2=0.677; En2=0.124; He2 =0.0185;
CloDorp = zer (2,N);
For i=1:N
  En_1= norm (En1, He1,1,1);
  En_2= norm (En2, He2,1,1);
    CloDorp (1,i) = norm   (En1, He1,1,1);
    CloDorp (2,i) = exp(-(CloDorp (1,i)- Ex1)^2/(2* En_1^2));
    CloDorp (3,i) = norm ((En2, He2,1,1);
    CloDorp (4,i) = exp(-(CloDorp (3,i)- Ex2)^2/(2* En_2^2));
end
    plot (CloDorp (1,:), CloDorp (2,:),'.')
hold on
    plot (CloDorp (3,:), CloDorp (4,:),'.')
```

Note: Ex1, En1 and He1 represent $Ex_\vartheta, En_\vartheta$, and He_ϑ respectively; Ex2, En2 and He2 represent Ex_δ, En_δ, and He_δ respectively.

C. ROBUSTNESS ANALYSIS OF THE PROPOSED METHOD

The model proposed in this paper mainly involves parameters K and q. Therefore, we further analyze the influence of

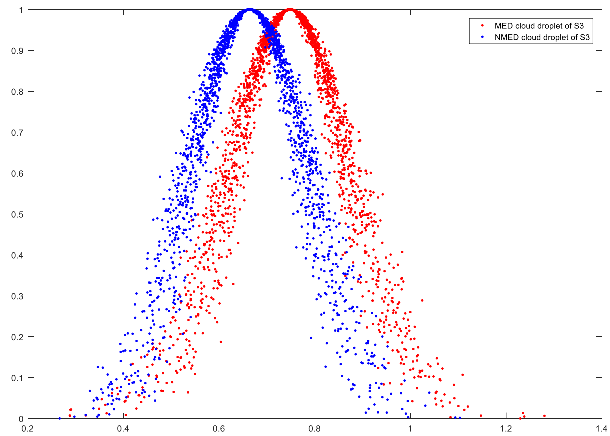

FIGURE 4. The cloud droplet of S3.

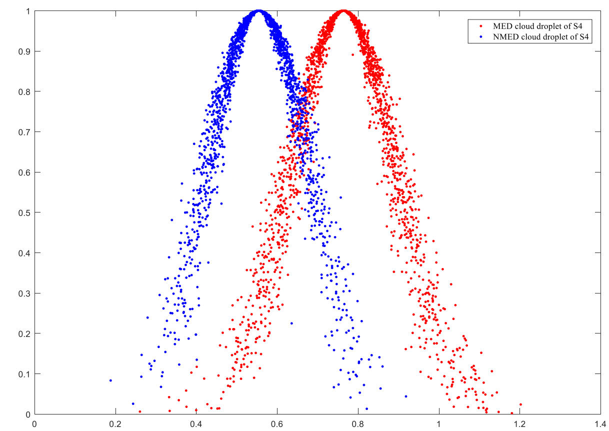

FIGURE 5. The cloud droplet of S4.

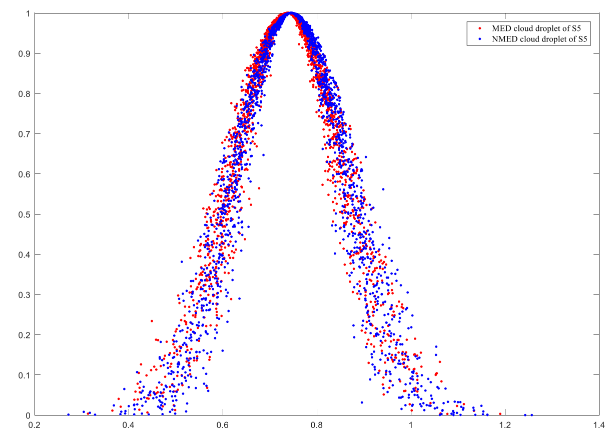

FIGURE 6. The cloud droplet of S5.

parameters on the output of the model to test the robustness of the model.

Firstly, the influence of parameter k is analyzed. k represents the internal relationships among different quantitative attributes and their influence on the output of results. Since the number of attributes in this study is four, the k value can be taken as 1, 2, 3 and 4, respectively. As shown in Table 4,

TABLE 3. The expectation and score value of each scheme.

	$\frac{1}{n}\sum_{i=1}^{n} x_{\vartheta_i}$	$\frac{1}{n}\sum_{i=1}^{n} y_{\vartheta_i}$	$\frac{1}{n}\sum_{i=1}^{n} x_{\delta_i}$	$\frac{1}{n}\sum_{i=1}^{n} y_{\delta_i}$	G(Si)
S1	0.766	0.7103	0.6789	0.7399	0.0418
S2	0.5996	0.711	0.7216	0.636	-0.033
S3	0.7474	0.7127	0.6614	0.7263	0.0523
S4	0.7681	0.7028	0.5518	0.7928	0.1024
S5	0.7338	0.7108	0.7454	0.6461	0.04

TABLE 4. The impact of k value on ranking with q = 3.

k	Ranking
k =1	S4>S1>S3>S5>S2
k =2	S4>S1>S3>S5>S2
k =3	S4>S1>S3>S2>S5
k =4	S4>S1>S3>S5>S2

TABLE 5. The impact of q value on with k = 2.

q	Ranking
q =3	S4>S1>S3>S5>S2
q =5	S4>S1>S3>S2>S5
q =7	S4>S1>S3>S5>S2
q =10	S4>S1>S5>S3>S2
q =15	S4>S1>S5>S3>S2

when $k = 1$ it represents that there is no interaction between attributes, thus the ranking is S4>S1>S3>S5>S2; when $k = 2$, it means the internal relationship between any two attributes in this case, and then the ranking result is S4 > S1 > S3 > S5 > S2; when $k = 3$, it indicates the internal relationship among any three attributes, and then the ranking result is S4 > S1 > S3 > S2 > S5; when $k = 4$, the corresponding ranking result is S4 > S1 > S3 > S5 > S2. No matter how k changes, the optimal solution is always S4.

In the same way, we further analyze the influence of q value changes on decision-making results. q represents the complexity of decision-making information faced by decision-makers. As shown in Table 5, the optimal solution is always S4 when q changes from 3 to 15.

From the above analysis, it can be seen that the parameters K and q have a certain impact on the overall ranking of alternatives, but the optimal solution has not been changed. Therefore, the decision-making results based on this method are less affected by K and q, the proposed method has strong robustness. In practical decision-making, the value of parameters K and q can be changed according to the change of decision-making situation and environment, so as to obtain reasonable decision-making results.

D. COMPARATIVE ANALYSIS BETWEEN DIFFERENT METHODS

Since the information expression structure of original q-ROF is changed by the proposed new concept of q-ROC. In order to compare different methods reasonably, this paper uses the existing methods to only deal with the membership and non-membership information of q-ROC, that is, the entropy and excess entropy of membership and non-membership are not considered. In addition, considering the representativeness of the method, this study selects q-rung orthopair fuzzy weighted arithmetic (q-ROFWA) operator [48], q-rung orthopair fuzzy weighted geometric (q-ROFWG) operator [48], q-rung orthopair fuzzy weighted Heronian mean (q-ROFWHM) operator [46] and q-rung orthopair fuzzy weighted Maclaurin symmetric mean (q-ROFWMSM) operator [29] to compare with the method in this paper, as shown in Table 6.

TABLE 6. **The comparative results based on different methods with q = 3.**

Methods	Ranking
The method using the q-ROFWA operator by[48]	S4>S1>S5>S2>S3
The method using the q-ROFWG operator by [48]	S4>S1>S2>S3>S5
The method using the q-ROFWBM operator by [46]	S4>S1>S3>S5>S2
The method using the q-ROFWMSM operator by [29]	S4>S1>S3>S2>S5
The proposed method	S4>S1>S5>S3>S2

According to Table 6, the optimal solution based on the q-ROFWA [48], q-ROFWG [48], q-ROFWHM [46] and q-ROFWMSM [29] operators and the optimal solution based on our method are both S4. It can be seen that the method in this paper is effective.

In addition, in order to highlight the superiority of the proposed method in this paper for the information aggregation, another simple example is given to compare difference in the ranking results obtained by the MADM methods based on traditional Archimedean t-norms and t-conorms (ATT) paradigm (taking q-ROFWA operator [48] as an example) and the q-ROCIWMSM operator. Details are as follows:

Example 2: A company is looking for a regional investment partner, and there are five candidate enterprises as the object of consideration, forming the alternative set S = (S1, S2, S3, S4, S5). These five candidate enterprises are evaluated from four aspects, including the level of reputation (C1), the level of management (C2), the level of innovation (C3), and the ability to take risks (C4). The weight vector corresponding to the above four attributes is set to $w = \{0.25, 0.2, 0.2, 0.35\}^T$. After data collection, the decision-making information in Table 7 is obtained.

Firstly, based on the decision data in Table 7, we can obtain the synthesize value of each attribute using the q-ROCIWMSM operator (K = 2, q = 3) as follows:

S1=<(0.61, 0.093, 0.0139), (0.592, 0.08, 0.0119)>,
S2=<(0.728, 0.113, 0.0169), (0.691, 0.114, 0.0171)>,
S3=<(0.744, 0.128, 0.0192), (0.554, 0.095, 0.0142)>,
S4=<(0.681, 0.104, 0.0156), (0.542, 0.084, 0.0127)>,
S5=<(0.684, 0.113, 0.017), (0.619, 0.104, 0.0155)>.

Meanwhile, we can get the corresponding synthesize value of each attribute using the q-ROFWA operator (q = 3) as follows:

S1=<0.558, 0>,
S2=<0.627, 0.613>,
S3=<0.699, 0.497>,
S4=<0.64, 0.453>,
S5=<0.637, 0.541>.

Then, calculate the score value of each alternative and ranking result based on the two methods mentioned above respectively, as shown in Table 8.

Table 8 shows the scores of alternatives obtained by using the proposed q-ROCIWMSM operator based on interaction operation rules, and the highest and lowest scores are G(S3) = 0.135 and G(S1) = 0.0121, respectively. Therefore,

TABLE 7. **Decision matrix of Example 2.**

	C1	C2	C3	C4
S1	<0.4, 0.06, 0.009; 0.5, 0.075, 0.0135>	<0.7, 0.105, 0.0158; 0, 0, 0>	<0.3, 0.045, 0.0068; 0.5, 0.075, 0.0135>	<0.6, 0.09, 0.0135; 0.7, 0.105, 0.0158>
S2	<0.6, 0.09, 0.0135; 0.7, 0.105, 0.0158>	<0.6, 0.09, 0.0135; 0.5, 0.075, 0.0135>	<0.5, 0.075, 0.0135; 0.4, 0.06, 0.009>	<0.7, 0.105, 0.0158; 0.8, 0.12, 0.018>
S3	<0.7, 0.105, 0.0158; 0.5, 0.075, 0.0133>	<0.8, 0.12 0.018; 0.6, 0.09, 0.0135>	<0.7, 0.105, 0.0158; 0.6, 0.09, 0.0135>	<0.6, 0.09, 0.0135; 0.4, 0.06, 0.009>
S4	<0.7, 0.105, 0.0158; 0.3, 0.045, 0.0068>	<0.6, 0.09, 0.0135; 0.4, 0.06, 0.009>	<0.8, 0.12, 0.018; 0.4, 0.06, 0.009>	<0.3, 0.045, 0.0068; 0.7, 0.105, 0.0158>
S5	<0.6, 0.09, 0.0135; 0.7, 0.105, 0.0158>	<0.6, 0.09, 0.0135; 0.6, 0.09, 0.0135>	<0.8, 0.12, 0.018; 0.6, 0.09, 0.0135>	<0.5, 0.075, 0.0135; 0.4, 0.06, 0.009>

the corresponding ranking results show that the optimal alternative is S3 and the worst alternative is S1. However, among the scores of alternatives obtained by q-ROFWA operator based on traditional ATT paradigm, the highest score is G(S1) = 0.9117, and the corresponding optimal alternative is S1. It can be seen that the optimal results obtained by using two different methods are in the opposite situation.

Therefore, we further analyze the reasons for the above phenomenon: it is easy to find that in the decision-making information in Table 7, the non-membership value of attribute C2 in alternative S1 is 0, while the non-membership value of other attributes is greater than 0. When the q-ROFWA operator based on ATT paradigm is used for information aggregation, the comprehensive non-membership degree of alternative S1 is also 0. It can be seen that when q-ROFWA operator is used to aggregate all information of alternative S2, ATT operator enlarges the influence of extreme value 0 in attribute C2 on decision-making results, while the influence of other attribute information pairs is completely ignored.

TABLE 8. Comparison of MADM methods based on different algorithms.

Methods	Score values	Ranking
The proposed method	G(S1)=0.0121, G(S2)= 0.0256, G(S3)=0.135, G(S4)=0.0982, G(S5)=0.0459	S3>S4>S5>S2>S1
The method based on the q-ROFWA operator by [48] (merely addressing the membership and non-membership information)	G(S1)= 0.9117, G(S2)= 0.5032, G(S3)= 0.5476, G(S4)=0.5473, G(S5)=0.5229	S1>S3>S4>S5>S2

The comprehensive non-membership degree of alternative S1 calculated by the q-ROCIWMSM operator is 0.592, which weakens the influence of extreme value on decision-making results, avoids the disadvantages of ATT operator completely, and makes the final decision-making result more reasonable.

Subsequently, it can be known that the method proposed in this paper has the following advantages compared with the existing methods:

(1) The concepts of entropy and excess entropy are introduced into the proposed method, that is to say, the errors of membership and non-membership given by experts are considered. This paper improves the expression form of original q-ROF set from the perspective of information error, which makes the information of expert evaluation more accurate, makes the uncertain qualitative information convert into more accurate quantitative information, and makes the expert evaluation information and its decision-making results more objective and reasonable.

(2) The interaction operator is embedded into the q-ROC information environment to explore the internal relationship among different membership functions. The interaction operator avoids the adverse impact of the possible extreme value of the evaluation information given by experts on the decision-making results. For example, if the membership value of one attribute in a q-ROC set is 0, then the non-membership degree in the comprehensive attribute information obtained by the mentioned four methods based on ATT operator [29], [47], [48] is also 0, which completely eliminates the influence of other membership degrees with non-0 values on the comprehensive attribute information. This unreasonable phenomenon is completely avoided by using the interaction operator.

(3) This paper combines MSM operator with q-ROC model, which can fully investigate the relationship among any number of attributes and is conducive to mining the intrinsic characteristics of attributes. In addition, there are many parameters involved in the model, so the values of parameters can be adjusted to adapt to different situations. Therefore, this method has high flexibility and adaptability.

VI. CONCLUSION

In this study, a MADM method based on q-ROCIWMSM operator is proposed. In it, considering the possible errors of the evaluation information given by decision makers, the cloud model is introduced into the q-ROF set, and the new concept of q-ROC is defined to analyze the errors distribution characteristics in the evaluation values, so as to make the evaluation information more accurate. Additionally, in order to investigate the internal relationship between membership functions in q-ROC and the interaction relationship among different attributes, the interaction operator and MSM operator are embedded into q-ROC, and the q-ROCIWMSM operator is proposed. Meanwhile, a MADM method based on q-ROCIWMSM operator is constructed. Finally, a case of cross-border e-commerce shopping decision-making is given to verify the effectiveness of the method proposed in this paper.

The analysis of the parameters in the model shows that the proposed method has strong robustness and flexibility, that is, this method can adapt to the decision-making requirements under different information environment conditions, and the parameters can be adjusted according to the actual needs to obtain reasonable decision-making results. In addition, through the comparative analysis of different methods, it is found that the method in this paper comprehensively considers the error of evaluation information and its error distribution. At the same time, it also investigates the heterogeneity relationships among membership functions and attributes, which makes the decision-making result more reasonable.

This study has the following limitations and needs to be further developed. First, the case data in this paper are simulated data given by expert rather than data from real application scenarios, so future studies can use the proposed method to analyze decision-making problems based on actual data, such as project investment, pattern recognition, medical decision-making, etc. Second, the proposed method does not provide the calculation method of entropy and excess entropy, which prevents us from transforming the accurate real data into multi-dimensional data structure including error and excess error. In future research, we can introduce confidence level into our method to obtain entropy and excess entropy. For example, the value of entropy and excess entropy can be calculated based on the confidence level of word-of-mouth score when making the movie viewing selection decisions. Third, the information environment of this study is limited to the q-rung orthopair fuzzy sets, we can further extend the method to the environment of T-Spherical fuzzy sets by calculating the error characteristics of their support, opposition and hesitation degrees.

AUTHOR CONTRIBUTIONS

Ping He: conceptualization, validation, resources, data curation, writing—original draft preparation, writing—review and editing, funding acquisition.

Chaojun Li: validation, supervision, investigation, data curation, visualization.

Harish Garg: conceptualization, validation, formal analysis, investigation, data curation, writing—review and editing, visualization.

Jian Liu: software, validation, investigation, data curation, writing—original draft preparation, writing—review and editing.

Zaoli Yang: conceptualization, methodology, validation, data curation, writing—original draft preparation, writing—review and editing.

Xudong Guo: validation, investigation, visualization.

DATA AVAILABILITY STATEMENT

Data is contained within the article.

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CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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