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Comparative Analysis Four Different Ways of Calculating Yield Index SSS_{pk} Based on Information of Control Chart, and Six Sigma, to Measuring the Process Performance in Industries: Case Study in Aden's Oil Refinery, Yemen

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ABSTRACT Process capability index PCIs have long been used to assess process efficiency in the manufacturing. While PCIs assess the process stability and efficiency, real measurements of product quality depended on estimate the standard deviation for a short-term variance was calculated as the base approach in several previous studies for measuring PCIs. Studies in recent years seek to estimate shift of the process mean from target. In today's smart manufacturing production climate, manufacturers keep track of production performance. Since the deviance of process mean comparative to the target is different. This allows a more robust norm to be implemented by estimate the relative shift to describe the variance of process in Six Sigma (SS). For that purpose, this paper suggests an evaluation method by comparative analysis to four different ways of calculating yield indexes S_{pk} based on six sigma and control information, of $\bar{X}_S$ chart. Moreover, this paper extends the univariate process index yield SSS_{pk} to a multi- generalized yield index is called $SSMS_{pk}$. An established an updated approach to assess univariate and multi-characteristic, based on understanding variation which is critical to quality (CTQ) and focusing on the position and actions of natural process tolerance between defined tolerances limits. To demonstrate the trueness verification proficiency of this approach, this paper includes an industrial case study to assess the process efficiency in Aden's oil refinery, Yemen. The findings of this study indicated that the indexes which are calculating based on six sigma outperformed existing indices and that this study has important implications for industrial practitioners, researchers and quality control experts interested in the evaluation of process performance.

INDEX TERMS Critical to quality, process yield, six sigma, quality engineering, information of the control charts, oil refinery.

I. INTRODUCTION

In today's dynamic and globalized markets the Industries must deliver high-quality, cost-effective goods that consistently meet customer and engineering design specifications [1]–[3]. thereafter, capability process and quality level have become key issues among manufacturers and essential attributes in order to gain a competitive advantage, especially in the world of intelligent consumers [3]–[5]. Manufacturers have continuously attempted to classify the causes of

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variations in order to establish control measures for removing or mitigating process variations over the years [6]–[8]. Statistical Quality Control (SQC), and Statistical Process Control (SPC). Methods and techniques have been used by different industries to enhance the process efficiency of their manufactured goods and ensure that the outcomes of a process are within the required quality standards [2], [7], [9], [10].

Since the introduction of process charts in the 1924s, (SPC) has become an important tool for monitoring and improving product quality in industrial sectors. SQC approaches have been widely used in a variety of industries and organizations to raise quality standards, increase process efficiency, and minimize product defects and variations [2], [9]–[12]. Furthermore, various approaches have been developed to facilitate better quality control during production processes, with notable contributions from, Ishikawa, Deming, Juran, Montgomery, and others.

Control charts and process capability analysis (PCA) are two tools to apply in SPC [13]. The control charts are employed to observe whether the process is in control where each product or service has an individual characteristic or not. If these characteristics are measurable, variable control charts may be used, otherwise, attribute control charts are useable. Commonly, SPC monitors processes whether it is "in control" or "out of control" [14].

PCA is the other methodology that can be defined as the ability of a process to deal with customer requirements which are defined as specification limits (SLS). Process capability indices (PCIs) are the main outputs of PCA which give a numerical measure of whether a production process is capable of producing items within the SLS predetermined by the consumer. If the certain minimum values of PCIs have been got, the process is named as "capable process" which meaning that it has succeeded in meeting SLs. If these minimum values cannot be met, the process is named as "incapable process" [15].

Capability analysis is a collection of indices and equations used to control whether a production process is statistically capable of producing goods within pre-defined parameters [16], [17]. Capacity analysis is more precisely summarized in indices and techniques that are used to measure, recognize uncertainty, and report the system's process capability and performance. For the past two decades, process capacity analysis has been used in a wide range of industries and organizations, including defining process requirements for new products, vendor selection, predicting a process' ability to hold tolerance, formulating quality assurance plans, and assisting designers in selecting and adjusting processes and maintaining product quality [2], [18]–[23].

The process capability index (PCI) is a flexible statistical instrument for evaluating a process' ability to generate products within predetermined specification limits [24]. PCIs have gotten a lot of attention from control researchers and engineers in the industry since 1980 [25]–[30].

Among different PCIs, C_p, C_{pk}, C_{pm}, *and* C_{pmk} are the more well-widely and known utilized in the industries [14], [24], [28], [31]–[34]. index C_p indicates the potential capability of a process, that is, the capability level reached by a process once it is ideally centred on target; the index C_{pk} takes into consideration both process centring and process dispersion with consideration to the concerned consistency characteristic; C_{pm} tests the process's proximity to the target T, while C_{pmk} is a third-generation PCI that is combined with C_{pk} and C_{pm} [35]. Regrettably, C_{pk} and C_{pm} only provides limits on yield process for normally distributed processes with two-sided specification limits, in which C_{pm} should be more than $\sqrt{1/3}$ [35]. Of the different PCIs, the yield index S_{pk} introduced by Boyles (1994) not only directly reflects yield process but also obtains an accurate measure of yield process for normally distributed processes with two-sided specification bounds. The index S_{pk} is defined as follows:

$$S_{pk} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi(\frac{USL-\mu}{\sigma}) + \frac{1}{2}\Phi(\frac{\mu-LSL}{\sigma})\right]$$

where the $\Phi(\cdot)$ (CDF) of a standard normally distributed is denoted $N(0, 1)$,. The inverse function of standard CDF is Φ^{-1}; both lower and upper specification limits are denoted by LSL & USL, respectively; the process mean is denoted by μ, and the process standard deviation is denoted by σ, the S_{pk} index establishes a relationship between manufacturing requirements and actual process yield, in an exact measurement of process yield. S_{pk}, clearly suggests, for a one-to-one transition to process yield or fraction nonconformance.

Although the estimates of the parameters μ, σ can be derived directly from these control charts for a statistically stable method, they are not used in the subsequent stages of capability assessment in practice. Instead, new samples are drawn to approximate the same parameters, rendering the method extremely inefficient see [36], [37] devised C_p estimators based on $\bar{X}_R$ and $\bar{X}_S$ control chart details, as well as the corresponding testing process. They've also proposed a step-by-step approach to help practitioners for determine whether the process is capable or not [38], argued in favor of estimating C_{pk} using control chart info. With, followed a Bayesian method, which necessitates awareness of the probabilities $P_r = \mu < T$ and $P_r = \mu > T$, which are not always accessible. Despite the fact that Six Sigma techniques can be applied to a variety of organizations, previous research suggests that employing Six Sigma during the manufacturing process to increase product quality and reduce defects and variations requires more effort and concentration [4], [22], [39], [40]. Aside from that, previous PCI research hasn't taken into account a systematic approach to all possible configurations under the probability density curve and tolerance limits [41]–[45]. Furthermore, the literature indicates that implementing process capability indices based on Six Sigma for evaluating the process efficiency of an oil refinery has not been adequately explored in the sense of PCIs based on Six Sigma implementation in industries [46], [47]. In Yemen, in particular, Six Sigma implementation in the petroleum industry is still missing [14], [16], [39], [47]–[51] are examples of studies in the field of oil in various countries, especially in the field of quality control measurement. However, control charts were used in those tests such as $\bar{X}, \bar{R}$ and $\bar{S}$ charts and tools of Ishikawa [47] without concentration the shift the mean process of the target. The aim of this paper was to clarify the origins of the 1.5 sigma change in relation to quality engineering methods by evaluate the process quality in industries, we first use the process capability index S_{pk} that can provide a strict measure of yield process.

Combining these ideas, this paper aims to provide an estimator of C_{pk} and S_{pk} based on $\bar{X}_S$ information control chart and six sigma idea taken into account a systematic approach to all possible configurations under the probability density curve and tolerance limits by proposing four different estimation ways (1, 2, 3, and 4) to the assessment of the six sigma yield (SSY) process and SSC_{pk}, SSS_{pk} yield indexes centered on distribution equivalence by comparing distribution with mean transition, and tolerance analysis-based approach by focusing on the position and actions of natural process tolerance between defined tolerances limits TL and Plug-in estimator of SSY and (SSC_{pk}, SSS_{pk}) based on the information of $\bar{X}_S$ charts. Mathematical proof is also given in each case. This paper introduces the extension of the process index yield SSS_{pk} to a multi- generalized yield index is called $SSMS_{pk}$ that can be used to assess processes with multivariate characteristics in industries. Thus, this research anticipates that the proposed statistical approach leads to ensure that the industrial process is capable of producing products according to the specification limits, improving industrial processes performance, and reducing variations and defects. Also, this research contributes to the existing literature by providing comprehensive theoretical knowledge for developing and extending new process capability indices based on Six Sigma. In addition, this study considers the shift the mean of the target process, and the use of levels in the $4\sigma, 5\sigma, 6\sigma$ to the amount of improvement the current process performance required to produce a capable process, to reduce defects and variations. This paper introduces a case study to determine the process efficiency of oil refinery process in Aden, refineries in order to illustrate applicability of this approach. The findings of this study indicated that the estimation indices based on six sigma outperformed existing indices and that this study has important implications for industrial practitioners, researchers, and quality control experts interested in the evaluation of process performance.

II. PROCESSCHARACTERISTICS: GUIDELINES

Process characterization gives objective evidence that important product parameters and related process parameters are capable of meeting specification limitations on a consistent basis (customer requirements). (L, T, U, μ, σ) $(\mu f', d', D, D_l, D_u)$ are the five independent and dependent characteristics needed to characterize a process given normally distributed data. Any process can be defined provided sufficient information about these parameters is available to plot the probability density curve and specification limits on top of it as shown in Figure (1).

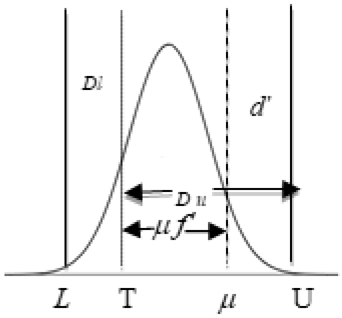

FIGURE 1. Process characterization.

The process characterisation is shown in Figure 1 in terms of an inherent effect and internal engineered specified constraints. It also depicts the process features that are desirable and possible (to be reached).

The following is an explanation of the process characterization notation: $|\mu - T| = Bias(\mu - T) = \mu f', \hat{\mu} f' = |\mu - T|/d$ $D_u = U - T$ part of interval tolerance between specification limits, $D_l = T - L$ Part of interval tolerance between specification limits; $d' = \min|U - \mu|, |\mu - L|$. Here, it should be noted that the previous figure 1 may take different forms that fit the cases referred to in Table No. 1. Thus it clear in the meanwhile, Figure 1 and Table 1 show all potential tolerance limit configurations as well as probability density curves.

TABLE 1. Displays the tolerance limits as well as the probability density curve.

Case	Descriptions
Case A_1	μ close to L T is-close to U
Case A_2	μ close to U T is-close to L
Case B_1	Both μ and T are close to U $\mu < T$
Case B_2	Both μ and T are close to L $\mu > T$
Case C_1	Both, μ and T are close to L $\mu <$ T
Case C_2	Both, μ and T are close to U $\mu > T$

III. ESTIMATION FOR SSY POTENTIAL AND ACTUAL YIELD BASED ON 6σ

This section introduces the related capacity indices, provides a brief description for each, and doing for their use in describing processes. This information will serve as a basis for the parts that follow.

A. ESTIMATION FOR POTENTIAL PROCESS YIELD SSY BASED ON 6σ

C_p is an index that calculates a process's potential to fulfill requirements. It's defined as following:

$$C_p = \frac{U - L}{6\sigma} = \frac{D}{6\sigma} \tag{1}$$

The relative positions of μ and T are not taken into account in the description of C_p (given by Eq.1). The average of the process distribution and the defined target could shift to any location, but the value of C_p will not change as long as the variability of the distribution σ and the specification tolerance T do not change. As a result, it only assesses a process's potential to generate suitable products or services and provides no detail about the process's actual yield. The advent of C_{pk} is motivated by this. The actual yield of a process is the probability of generating a component within requirement limits. If the process distribution is oriented at the target value, i.e. $\mu = T$, the capacity of a process is the probability of producing a product within specification limits.

The word "six sigma process" refers to the fact that the upper and lower specification limits are separated by two times six standard deviations, i.e. $U - L = 12$ represented by X presumptuous that there are possibility events x_1 and x_2. That mean the variable of $\hat{X}$ between upper and lower specification limits is represents the yield process here the area under the probability density function calculated from $X = L$ to $X = U$ when $\mu = T$ is the potential of a process. In this scenario, there are two possibilities: μ is closer to U or μ is closer to L, thus the potential of the process, can be expressed directly in terms of C_p. The following is the derivation for the first possibility:

$L \leq \hat{X} \leq U$, The event of $x_1 = X \leq U \quad x_1 = \max\left(\frac{D_u}{D}, \frac{D_l}{D}\right)$ and $x_2 = X \geq L$, thus $x_2 = \min\left(\frac{D_u}{D}, \frac{D_l}{D}\right)$ When μ is close to L this implies that $x_1 = \left(\frac{D_u}{D}\right) = x_1 D = D_u$ and $x_2 D = D_l$ this suggests that potential yield of process based on six sigma idea SSY can be estimate as following:

$$\begin{aligned}
SS\hat{Y} &= \Phi\left[\frac{U-L}{\sigma}\right] - \Phi\left[\frac{L-\mu}{\sigma}\right] \\
&= \Phi\left[\frac{U-L}{\sigma}\right] - \left[1 - \Phi(\frac{L-\mu}{\sigma})\right] \\
&= \Phi\left[\frac{x_1 D}{\sigma}\right] - \left[1 - \Phi(\frac{x_2 D}{\sigma})\right] \\
&= \Phi\left[\frac{6x_1 D}{6\sigma}\right] + \left[\Phi(\frac{6x_2 D}{6\sigma})\right] - 1 \\
SS\hat{Y} &= \Phi(6x_1 C_p) + \Phi(6x_2 C_p) - 1 \qquad (2)
\end{aligned}$$

where $\Phi(\cdot)$ is the CDF of the standard-normal distribution. When μ is closer to U, it is easy to verify that potential is the same. Also the potential yield can be calculated as Eq as (2). Hence $LSL \leq X \leq USL$, the event of x_1 and x_2 is an occurrence that can be presented as $x_1 \cap x_2$ and the potential yield process p_r(L, $\leq X \leq$ U), thus

$$\begin{aligned}
p(x_1) &= \frac{(X-\mu)}{\sigma} \leq \frac{(USL-\mu)}{\sigma} = \Phi(6x_1 \times C_p), \\
p(x_2) &= \frac{(X-\mu)}{\sigma} \geq \frac{(LSL-\mu)}{\sigma} = 1 - \Phi(-6x_1 \mathrm{C}_p), \\
&= \frac{(X-\mu)}{\sigma} \geq \frac{(\mu-LSL)}{\sigma} = \Phi(6x_1 \mathrm{C}_p), \qquad (3)
\end{aligned}$$

Thus $SS\hat{Y} = \Phi(6x_1 C_p) + \Phi(6x_2 C_p) - 1$.

B. ESTIMATION FOR SSY ACTUAL YIELD BASED ON 6σ AND CONTROL CHART INFORMATIONS

Kane (1986) proposed the C_{pk} index as a way to assess actual yield process capability C_{pl}, which expresses the effect of the mean process on the overall capacity process, as follows:

$$\begin{aligned}
C_{pl} &= \frac{(\mu - L)}{3\sigma}, \quad C_{pu} = \frac{(U-\mu)}{3\sigma} \\
C_{pk} &= \min\left(C_{pl}, C_{pu}\right) \qquad (4)
\end{aligned}$$

Although the most commonly used tool point estimates of PCIs are (C_p, C_{pk} and S_{pk}). The base approach for estimates the CPIs indexes was to calculate standard deviation for a short-term variance. Variation between subgroups is not taken into account when calculating these indices. For this reason, we use a combination of estimation of parameters to measure yield process with keep track of production performance and the gap between the process's mean change and the target value at all times. Following Pearn et al (2007) [58] the C_{pk} can be outlined in the following way:

$$C_{pk} = C_p\left(1 - \frac{|T-\mu|}{U-L/2}\right) = \frac{U-L}{6\sigma}\left[1 - \frac{|U-\mu|}{U-L/2}\right] \qquad (5)$$

where U *and* L denotes the (upper and lower) specification limits, $\hat{\mu} = \overline{\overline{X}}$ denotes the mean process, that $\overline{\overline{X}} \sim N\left(\mu, \hat{\sigma}^2\right)$ and, and $\hat{\sigma}$ denotes standard process, $\hat{\sigma} = \sigma^2/N$ The target value is $(U+L)/2 = T$, $|\mu - T|$ tests the degree of centering of process. Denoting $|\mu - T| = \mu f'\sigma \ \mu f' \geq 0$, Assume the specification is $U - L = D = D_u + D_l = 2\,k\sigma$. Thus, based on the Eq (5), C_{pk} will become as follows:

$$SSC_{pk} = \frac{2k\sigma}{6\sigma}(1 - \frac{2\mu f'\sigma}{2k\sigma}) = \frac{k-1.5}{3} \qquad (6)$$

Here the k is sigma process level SPL follow (Chen et al 2017), we can collated k based control chart info as following:

$$L\sigma = k = \min\left((\frac{1-\mu f'/d}{\sqrt{(\hat{\sigma}_{ST})^2}/d} + 1.5), \frac{1+\mu f'/d}{\sqrt{(\hat{\sigma}_{ST})^2}/d} + 1.5\right) \qquad (7)$$

where $\hat{\sigma}_{ST} = \sqrt{\left(\bar{S}/C_4\right)^2}$ An unbiased estimate for $\hat{\sigma}$, the $\hat{\mu}f'$ is estimation to $|\overline{\overline{X}} - T|$ and $d = (U-L)/2$, follows a folded normal distribution with mean, as suggested by Leone et al [52]

$$\mu f' = \frac{\sigma\sqrt{2}}{\sqrt{N\pi}} e^{\frac{N(\bar{\bar{X}}-\mu)}{2\sigma^2}} + (\mu - T \times \left[1 - 2\Phi\left(\frac{(\mu-T)}{\sigma}\right)\right] \qquad (8)$$

and $\sigma f' = (\mu - T)^2 + \frac{\sigma^2}{N} - \mu_f^{*2}, \sigma f'$

Notationally $|\overline{\overline{X}} - T| \sim FN\left(\mu f', \sigma f'^2\right)$ Hence

$$E\left(\left|\bar{\bar{X}} - T\right|\right) = \mu f' \qquad (9)$$

The yield of process is

$$SS\hat{Y} = \int_{LSL}^{USL} N(\mu, \sigma^2)dx \qquad (10)$$

By replacement $u = \mu f', \sigma^2 = \sigma f'^2$ $USL = \mu f' + 1.5\sigma f' + k$ and $LSL = \mu f' + 1.5\sigma f' - k$

That is

$$SS\hat{Y} = \left[\Phi((1.5 + (3SSC_{pk} + 1.5)) - \Phi((1.5 - (3SSC_{pk} + 1.5))\right],$$
$$SS\hat{Y} = [\Phi(1.5 + k) - \Phi(1.5 - k)] \tag{11}$$

Motorola developed the Six Sigma Approach, in 1986. The word "six sigma operation" refers to the fact that the upper and lower specification limits are separated by two times of six deviation, i.e. $U - L = 12\sigma$ [53]., Here the assumption a value of $\hat{\mu}f'$, $\hat{\mu}f' = |\mu - T| = 1.5$. Nevertheless-experience has been shown that processes don't always work in the long run like they do in the short run. As a consequence, the process mean μ and the target T are out of sync. As compared to a short-term sample, the value $|\mu - T|$ can increase over time. In fact, 1.5 sigma shift is commonly used to computation to account for this real-life rise in process variance over time. By this concept, a process that suits six sigma would have a long-term yield process is 0.999996 with PPM non-conformities is 3.4. According to this idea of six sigma, the k is level of process a fixed value 1.5. According to Eq (5) we have

$$SSC_{pk}(|\mu f'| = 1.5\sigma f) = \frac{k}{3} - 0.5 \tag{12}$$

From Eq (10) and Eq (12), establish the relationship between the value of K, the yield process SSY and PCI $SSC_{pk}(|\mu - \mathrm{T}| = 1.5\sigma)$ The result indicates there relationship one to one between SSC_{pk} and SSY when the difference $|\mu - \mathrm{T}|$ is taken as a constant value of 1.5. Hence for the calculation of SSC_{pk} should by know value of level sigma process k thus the process level can by collected as Eq (6) that lead to estimation level sigma if we know the value of index SSC_{pk} then the $k = 3\ SSC_{pk} + 1.5$ also the index SSC_{pk} can assess the efficiency of a one-sided process. If the variable only has (L) a lower specification, the process yield can be determined by taking the USL as $+\infty$. On the other hand, it defines for processes that only have an upper specification limit. While the traditional index C_{pk} was alone is insufficient to determine the processes actual yield. Hence the estimation processes yield, distinguish between potential and actual yield, the process's actual yield is same as the potential yield, but if $(\mu \neq T,)$ the yield potential is higher of the actual yield. As a result, it is possible to conclude that: (The process's potential, $\geq$ actual yield). Although C_p, calculates the process' potential, (which may not be equal to the process' actual yield). If $(\mu = T)$ The region between U and L under the probability density curve of quality characteristic X when $(\mu = T)$ is the process' potential. The region if $(\mu \neq T)$ between L and U beneath the PDC of performance characteristic X is the process's actual yield. The lower bound denotes the area in the regions under the PDC of the quality function $X \geq U$ if μ. is closer to U or $X \geq L$ if the μ. is nearer to L Meanwhile, in regions $\mu \leq X \leq U$, the L is double the range under the D-curve of X if μ is near to U or $X \geq L$ and if μ. is close to L.

Table 1 shows that cases (A_2 B_2, and C_2) are mirror-imageries of cases (A_1, B_1, and C_1) respectively. As a consequence, the actual yield reckoning for a group a case would be the same as for the corresponding group '(A_1, B_1, and C_1)' cases. As a result, by Table 1, the actual yield terms for cases (A_1, B_1, and C_1) are as follows:

$$\begin{aligned}
&\text{Actual yield}(\mathrm{A}_1)\\
&= \Phi\,[U - \mu/\sigma] - \Phi\,[L - \mu/\sigma]\\
&= \Phi\left[D_u + \mu f'/\sigma\right] - [1 - (\Phi(\mu - L/\sigma)]\\
&= \Phi\left[D_u + \mu f'/\sigma\right] - \left[1 - (\Phi(d'/\sigma)\right]\\
&= \Phi\left[(D_u + Dl)/\sigma) - d'/\sigma\right] - \left[1 - \Phi(3d'/3\sigma)\right]\\
&= \Phi\left[6(D_u + Dl)/6\sigma) - 3d'/3\sigma\right] + \left[\Phi(3d'/3\sigma) - 1\right]\\
&= \Phi(6 \times C_p - 3 \times C_{pk}) + \Phi(3 \times C_{pk}) - 1
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\text{Actual yield }(\mathrm{B}_1)\\
&= \Phi\,[U - \mu/\sigma] - \Phi\,[L - \mu/\sigma]\\
&= [d/\sigma] - [1 - \Phi(\mu - L/\sigma]\\
&= \Phi\left[\mathrm{D}_u + \mu f'\right] - \left[1 - \Phi(D_u + D_l) - d')/\sigma\right]\\
&= \Phi\left[3d'/3\sigma\right] - \left[1 - \Phi(D_u + D_l)/\sigma) - d'/\sigma\right]\\
&= \Phi\left[(D_u + Dl)/\sigma) - d'/\sigma\right] - \left[1 - \Phi(3d'/3\sigma)\right]\\
&= \Phi\left[6(D_u + Dl)/6\sigma) - 3d'/3\sigma\right] + \left[\Phi(3d'/3\sigma) - 1\right]\\
&= \Phi(6 \times C_p - 3 \times C_{pk}) + \Phi(3 \times C_{pk}) - 1
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\text{Actual yield }(\mathrm{C}_1)\\
&= \Phi\,[U - \mu/\sigma] - \Phi\,[\mu - L/\sigma]\\
&= \Phi\left[D_u + D_l - d'/\sigma\right] - [1 - \Phi(\mu - L)/\sigma]\\
&= \Phi\left[((D_u + Dl)/\sigma) - (d'/\sigma\right] - \left[1 - \Phi(d')/\sigma\right]\\
&= \Phi\left[(6D_u + Dl)/6\sigma) - d'/\sigma\right] - \left[1 - \Phi(d'/\sigma)\right]\\
&= \Phi\left[6(D_u + Dl)/6\sigma) - 3d'/3\sigma\right] - \left[1 - \Phi(3d'/3\sigma) - 1\right]\\
&= \Phi(6 \times C_p - 3 \times C_{pk}) + \Phi(3 \times C_{pk}) - 1
\end{aligned} \tag{15}$$

By the above, the general expression of a process's actual yield can be determined for all cases as follows:

$$SSY = \Phi(6 \times C_p - 3 \times C_{pk}) + \Phi(3 \times C_{pk}) - 1 \tag{16}$$

Also a control chart may be used to collect information related to process output when the process is typically distributed and reaches a steady state condition (i.e. a well-controlled process) and allows the easy identification of assignable triggers for process adjustments Montgomery, 2009). For this reason, we use a combination of $\bar{X}$ *and* σ charts to estimation processes with two-sided specifications, follow (Chen et al 2017), the process yield can be rewritten as functions of and as follows:

$$SS\hat{Y} = \left[\Phi\left(\frac{(1 + \mu f')/d}{(\bar{s}/c_4)/d}\right) + \Phi\left(\frac{(1 - \mu f')/d}{(\bar{s}/c_4)/d}\right)\right] - 1 \tag{17}$$

As previously mentioned, there are (4) four ways to estimation the yield process by using Six Sigma and information control chart with analyzing the components of tolerance. First way for estimation SSY is according to the possibility of yield as show at equation (2). The second way for estimated the yield process is based on collected the index SSC_{pk} and level sigma of process as shown in Eqs (5 to 12). The third way for estimation yield process is by the potential and actual yield process as shown in Eqs 13 to 15 and then the a general expression of a process's actual yield can be determined the samurais for that as Eq (16). Finally the forth way for estimator yield process by the information control chart as shown at Eq (17). All cases can be arranged as follows:

$$\begin{aligned}
\text{Case (1) } SS\hat{Y} &= \Phi(6x_1C_p) + \Phi(6x_2C_p) - 1 \\
\text{Case (2) } SS\hat{Y} &= \big[\Phi((1.5 + (3SSC_{pk} + 1.5)) \\
&\qquad - \Phi((1.5 - (3SSC_{pk} + 1.5))\big] \\
&= [\Phi(1.5 + k) - \Phi(1.5 - k)] \\
\text{Case (3) } SS\hat{Y} &= \Phi(6 \times C_p - 3 \times C_{pk}) + \Phi(3 \times C_{pk}) - 1 \\
\text{Case (4) } SS\hat{Y} &= \left[\Phi\left(\frac{(1+\mu f')/d}{(\bar{s}/c_4)/d}\right) + \Phi\left(\frac{(1-\mu f')/d}{(\bar{s}/c_4)/d}\right)\right] - 1
\end{aligned} \tag{18}$$

All cases estimation focused on keep track of yield process and the gap between the process's mean change and the target value at all times.

IV. ESTIMATION OF THE SIX SIGMA YIELD INDEX SSSPK ACCORDING TO THE INFORMATION CONTROL CHART and 6σ

As a result of the above, we've already defined the six sigma yield process by deferent ways as shown at Eq (18) by use any one of that equations, we can estimation the process yield index SSS_{pk} based on, six sigma concept and control chart information follows normal distribution with two sided that the same way as S_{pk} index shown Eq (1) was suggested by (Boyles, 1994).

$$SS\hat{S}_{pk} = \frac{1}{3}\Phi^{-1}\left[(SS\hat{Y} + 1)/2\right] \tag{19}$$

The index SSS_{pk} have a one-to-one correspondence, the higher the SSS_{pk}, lead to the better yield process SSY, and the smaller the SSS_{pk}, lead to the worse yield process. As a result, SSS_{pk} is able to present an accurate representation of process yield from which manufacturers can derive benchmarks and serve as a product quality guide for third parties. Then, according to Equations (18 and 19), can get the correspondence between process yield SSY and SSS_{pk} index for different $|\bar{\bar{X}} - T|\hat{\mu}f'$ can see some of the values as shown that at table (3) it's come of analysis data of our case study. Obviously, SSS_{pk} provide an exact measure of the process yield without consider the estimator the $\hat{\mu}f'$. Thus the $|\bar{\bar{X}} - T|$ follows a folded normal distribution, already defined at Eq 7 and Eq 8. By replacing SSY in the equation 19 the index SSS_{pk} can by collected or can be rewritten as follows:

Case (1) $SS\hat{S}_{pk}$

$$= \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}(\Phi(6x_1C_p) + \Phi(6x_2C_p)) - 1 + 1\right] \tag{20}$$

Case (2): $SS\hat{S}_{pk}$

$$\begin{aligned}
&= \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\big((\Phi(1.5 + (3SSC_{pk} + 1.5))\right. \\
&\qquad \left. - \Phi(1.5 - (3SSC_{pk} + 1.5))\big) + 1\right],
\end{aligned}$$

$SS\hat{S}_{pk}$

$$= \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}(\Phi(1.5 + k) - \Phi(1.5 - k)) + 1\right], \tag{21}$$

Case (3) $SS\hat{S}_{pk}$

$$= \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\left[\Phi(6 \times C_p - 3 \times C_{pk}) + \Phi(3 \times C_{pk}) - 1\right] + 1\right] \tag{22}$$

Case (4) $SS\hat{S}_{pk}$

$$\begin{aligned}
&= \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\left[\left(\Phi\left(\frac{(1+\mu f')/d}{(\bar{s}/c_4)/d} + \Phi\left(\frac{(1-\mu f')/d}{(\bar{s}/c_4)/d}\right)\right) - 1\right)\right]\right. \\
&\qquad \left. + 1\right\}
\end{aligned} \tag{23}$$

Follow Boyles, (1994), The univariate yield index S_{pki} has been expanded to include processes with multiple quality characteristics $X_i = x_1, x_2, x_3 \ldots\ldots x_v$ The S_{pk} index can be used to define relationships between both the process's actual output and its tolerance limits for normal distributed processes with the i^{th} characteristic of $i = 1, 2, 3 \ldots\ldots v$, according (Boyles, 1994) the index for multivariate characteristic, can be define as follows:

$$S_{pki} = \frac{1}{3}\Phi^{-1}\left[\prod_{i=1}^{v}\frac{(U_i - \mu_i)}{\sigma_i} + \frac{(\mu_i - L_i)}{\sigma_i}\right] \tag{24}$$

where the L_i, U_i are lower, upper limits of $\mu_i = \bar{X}_i$, the i^{th} of the quality characteristics. However, the aim of this research is to look at multivariate industrial processes with two-sided specification limits. Follow Chen *et al.*, (2003) TS_{pk} was suggested total yield index for multiple quality characteristics. Thus, the SSS_{pki} can be expanded to a multivariate generalized total yield index SSS_{pki} based on six sigma using the following formula:

$$\begin{aligned}
SSMS_{pk} &= 1/3\Phi^{-1}\prod_{i=1}^{v}(2\Phi(3 \times SS\hat{S}pk_i) - 1) + 1/2 \\
&= 1/3\Phi^{-1}\prod_{i=1}^{v}(SST\hat{Y}_i) + 1/2
\end{aligned} \tag{25}$$

Distribution of $SSM\hat{S}_{pk}$

$$SSM\hat{S}_{pk} = \frac{1}{3}\Phi^{-1}\left[\frac{\prod_{j=1}^{v}(2\Phi(3 \times SS\hat{S}_{pki}) - 1) + 1}{2}\right] \tag{26}$$

where $SS\hat{S}_{pki}$ denotes the estimator of $SSS_{pki}, SS\hat{S}_{pki} \sim N\left(SSMS_{pt,}\, (a_i^2, b_i^2)\,/36\, n\,(\phi\,(3\, SSS_{pki}))^2\right.$ and all $SS\hat{S}_{pki}$ are mutually independent then $SS\hat{S}_{pki}$ has the mean of the asymptotic normal distribution $SSMS_{pk}$ and the asymptotic normal distribution

$$\frac{1}{36n[\phi(3 \times SSMS_{pk})]^2} \times \left[\sum_{i=1}^{v}\left(\left(a_i^2 + b_i^2\right)\left(\frac{\prod_{i=1}^{v}(2\Phi(3 \times SSS_{pki}) - 1)^2}{2\Phi(3 \times SSS_{pki}) - 1)^2}\right)\right)\right] \tag{27}$$

$$SSM\hat{S}_{pk} \sim N\left\{\begin{array}{l} SSMS_{pk}\left[1/36n \times (\phi(3 \times SSMS_{pk}\right] \\ \times \left[\sum_{i=1}^{v} a_i^2 + b_{i=1}^2\left(\frac{\prod\limits_{i=1}^{v}(2\Phi(3 \times SSS_{pki}) - 1)^2}{2\Phi(3 \times SSS_{pki}) - 1)^2}\right)\right]\end{array}\right\} \tag{28}$$

Proof: Using v-variate Taylor's first order expansion

$$\Rightarrow f(x) = f(x_0) + \sum_{i=1}^{v}\frac{\partial f(x_0)}{\partial x_i}(x_i - x_{i0}),$$

where $X = x_1, x_2 \ldots\ldots X_v$ we take $v = 2$ to derive the asymptotic distribution of $SSM\widehat{S}_{pk}$, for example

$$\begin{aligned} E\left(SSM\hat{S}_{pk}\right) &= SS\hat{S}_{pki}, \operatorname{Var} SS\hat{S}_{pki} \\ &= \frac{(a_i^2 + b_i^2)}{36n(\phi(3 \times SSS_{pki}))^2} \quad \forall = 1, 2 \end{aligned} \tag{29}$$

From the explanation we have (30), as shown at the bottom of the page.

Then, (32) as shown at the bottom of the page.

Similarly, (33) as shown at the bottom of the page.

A according to the central limit theorem $\geq SSM\hat{S}_{pk}$ has an asymptotic normal distribution with mean and variance.

$$\left.\begin{array}{l}\left(\dfrac{1}{36n \times (\phi(3 \times SSMS_{pk}))^2}\right) \\ \times \left(\begin{array}{l}(a_1^2 + b_1^2) \times (2\Phi(3 \times SSS_{pk2}) - 1)^2 \\ + (a_2^2 + b_2^2) \times (2\Phi(3 \times SSS_{pk1}) - 1)^2\end{array}\right)\end{array}\right\} \tag{33}$$

Similarly consider v as a set of variables, and the asymptotic distribution of $SSM\hat{S}_{pk}$ can be calculated as

$$SSM\hat{S}_{pk} \sim N\left(\frac{1}{36n \times [\phi(3 \times SSMS_{pk})} \times \left(\sum_{i=1}^{v}\left[a_i^2 + b_i^2)\frac{\prod\limits_{i=1}^{v}(2\Phi(3 \times SSS_{pki}) - 1)^2}{(2\Phi(3 \times SSS_{pki}) - 1)^2}\right]\right)\right) \tag{34}$$

V. IMPLEMENTATION AND RESULTS, DISCUSSIONS IN ADEN REFINERY OF OIL

A. PROCESS OVERVIEW OF THE ADEN REFINERY

Water and mechanical impurities (salt, sand, and clay) are removed and filtered after crude petroleum processing in

$$\left.\begin{aligned} E(SSM\hat{S}_{pki}) &= E(f(SSS_{pk1}, SSS_{pk2})) + E\left(\frac{\partial f(SSS_{pk1}, SSS_{pk2})}{\partial(SS\hat{S}_{pk1})}\right) \times SS\hat{S}_{pk1} - SSS_{pk1} \\ &\quad + E\left(\frac{\partial f(SSS_{pk2}, SSS_{pk2})}{\partial(SS\hat{S}_{pk2})}\right) SS\hat{S}_{pk2} - SSS_{pk2} \\ &= f(SSS_{pk1}, SSS_{pk2}) = (SSMS_{pki}) = (SSMS_{pki}) = \frac{1}{3}\Phi^{-1} \\ &\quad \times \left\{\left[(2\Phi(3 \times SSS_{pk1}) - 1)(2\Phi(3 \times SSS_{pk2}) - 1) + 1\right]/2\right\} \\ \operatorname{var} SSM\hat{S}_{pki} &= \left(\frac{\partial f(SSS_{pk1}, SSS_{pk2})}{\partial(SS\hat{S}_{pk1})}\right)^2 \operatorname{var} SS\hat{S}_{pk1} \\ &\quad + \left(\frac{\partial f(SSS_{pk1}, SSS_{pk2})}{\partial(SS\hat{S}_{pk2})}\right)^2 \operatorname{var} SS\hat{S}_{pk2} \because f(SS\hat{S}_{pk1}, SS\hat{S}_{pk2}) \\ &= \frac{1}{3}\Phi^{-1}\left(2\Phi(3 \times SS\hat{S}_{pk1}) - 1)(2\Phi(3 \times SS\hat{S}_{pk1}) - 1) + 1\right)/2 \end{aligned}\right\} \tag{30}$$

$$\left.\begin{aligned} \left(\frac{\partial f(SS\hat{S}_{pk1}, SS\hat{S}_{pk2})}{\partial SS\hat{S}_{pk1}}\right) &= \left(\frac{(2\Phi(3 \times SS\hat{S}_{pk1}) - 1)\phi(3 \times SS\hat{S}_{pk1})}{\phi(\Phi)^{-1}((2\Phi(3SS\hat{S}_{pk1}) - 1)(2\Phi(3SS\hat{S}_{pk2}) - 1) + 1)/2}\right) \times \left(\frac{\partial f(SS\hat{S}_{pk1}, SS\hat{S}_{pk2})}{\partial SS\hat{S}_{pk1}}\right) \\ &= \left(\frac{(2\Phi(3SSS_{pk2}) - 1)\phi(3SSS_{pk1})}{\phi(\Phi)^{-1}((2\Phi(3 \times SS\hat{S}_{pk1}) - 1)(2\Phi(3 \times SS\hat{S}_{pk2}) - 1) + 1)/2}\right) \end{aligned}\right\} \tag{31}$$

$$\left.\begin{aligned} \left(\frac{\partial f(SS\hat{S}_{pk1}, SS\hat{S}_{pk2})}{\partial SS\hat{S}_{pk2}}\right) &= \left(\frac{(2\Phi(3 \times SS\hat{S}_{pk1}) - 1)\phi(3 \times SS\hat{S}_{pk2})}{\phi(\Phi)^{-1}((2\Phi(3 \times SS\hat{S}_{pk1}) - 1)(2\Phi(3 \times SS\hat{S}_{pk2}) - 1) + 1)/2}\right) \\ &= \operatorname{var} SS\hat{M}S_{pk} = \frac{1}{36n(\phi(3 \times SSMS_{pk}))^2}\left(\begin{array}{l}(a_1^2 + b_1^2) \times (2\Phi(3 \times SSS_{pk2}) - 1)^2 \\ + (a_2^2 + b_2^2) \times (2\Phi(3 \times SSS_{pk1}) - 1)^2\end{array}\right) \end{aligned}\right\} \tag{32}$$

production centers (petroleum wells). The refined petroleum is kept before being shipped to refineries or sold to other countries. Because crude petrol is a mixture of diverse hydrocarbon components, decomposition of crude petroleum into fundamental components begins at the refinery stage. Refining is required for the processing of crude petrol because it removes undesirable chemicals and turns them into consumable products.

Petroleum is the world's main source of oil. The petroleum refining process involves using a chemically modified device to convert crude oil into usable items such as petroleum gas (LPG), gasoline (petrol), diesel, kerosene, fuel oils, and jet fuel. It is an important source of energy that is used in a range of applications such as transportation, manufacturing, and electric power generation. In Yemen, oil accounts for 30-40% of GDP, 70% of government income, and 63% of export value (Aden Refinery 2016). Specific laboratory tests for the ratio and spillage rate, as well as indirect measures such as viscosity forms and refractive index, are used to accomplish this. Distillation is one of the most widely used methods for processing crude petroleum. The Aden refinery divides petroleum derivatives into two phases using two distillation processes (Aden Refinery 2016):

Controlling oil products (petroleum derivatives) is a highly precise and complex process that involves many key stages. Before performing the necessary tests on each component, make sure the density rate is right, as well as the amount of octane number required to determine the burning phenomenon of oil once used in a engine that uses this petroleum. If the octane is low in this event, it may normally be increased by mixing it with another oil with a higher octane or by addition lead materials, such as 2.5 g/gallon (corresponding to 0.69 grams of lead per liter of oil). Quality control is carried out separately for each product in Aden's oil refinery, and it is normally done in several stages. After the oil products have been segregated, statistical control is normally done after the products have been separated in individual tanks. The statistical regulation can be explained in the following ways:

i. Every 8 hours, a sample of oil is taken from each tank from three separate locations: the top, middle, and bottom.
ii. Sample testing (tests): It is a test procedure
iii. Following the completion and recording of the previous tests, each final value (resulting from the measurement) is compared to the international and domestic standard defined for each test. The procedure is carried out in this case with a minimum and maximum range. Since most tests do not have super or small output values, only values between the min and max are possible. Controlling a particular product involves commanding a collection of physical properties that are then used to evaluate the product's effectiveness. The three gasoline characteristics of octane number, vapor pressure, and density, are used to evaluate the quality of the petroleum product generated A Case Study in Aden's Oil Refinery, Yemen. The following is a description of those characteristics.

1) DENSITY OF CHARACTERISTIC

This feature is crucial because it is impossible to monitor other petrol characteristics without having the same density. We show that each product exhibits its own specified features, some of which are global and others which are limited to the refinery, based on the properties of petroleum derivatives and the boundaries of the standardized requirements in the Aden refinery in Yemen. Petrol has properties such as density that is between the upper and lower limitations based on the refinery's set restrictions (specification limits). The upper and lower specification limits for oil density are respectively 0.73 and 0.70. These are also the local refinery oil specification restrictions in Aden (Source: Aden oil refineries). As a result, the oil is said to be kerosene if the density is less than 0.73. As a consequence, if the density value is greater than 0.73, the oil is kerosene; if it is less than 0.70, the oil is vapors and gases.

2) OCTANE NUMBER OF CHARACTERISTIC

The machines' degree of explosion (combustion) is represented by the octane of fuel, which has a variety of effects on the oil's overall consistency. For example, if the octane number is less than 90, it reasons consumption instability, increases engine temperature, and affects car speeds. The presence of paraffin's and aromatic hydrocarbons (naphthalence), among other things, causes the octane number to be low during the distillation process, resulting in oil instability against the fugitive (its explosive stability).This impact can be altered by addition materials to the liquid, such as lead, or by combining liquids of different octane numbers. For this reason, the Aden refinery's quality control of oil petroleum users tests the oil on a regular basis to ensure that it is processed in accordance with the The appropriate octane number ranges from 90 to 100.

3) VAPOR PRESSURE OF CHARACTERISTIC

At a certain temperature, the vapor pressure is known as steam vapor pressure. Generally, any substance with a higher vapor pressure has a higher chance of being flammable and exploding. When a liquid reaches its boiling point, it begins to evaporate, and Particles start to leave the liquid surface and enter the void above it. However, when there's no vacuum point up the product, these molecules will reach top temperature, the vapor pressure on the vessel's walls will be equivalent to the amount of atmospheric pressure and vapor pressure. As a result, the vapor pressure of gasoline is generally controlled to stay within the upper and lower requirements limits of 7 and 10, respectively. The vapor pressure of oil products varies greatly from one country to the next. It should be noted that lower vapor pressure values (below the lower specification limit) make starting machines or cars difficult. The lower and upper density, octane number, and vapor pressure requirements limits are shown in Table 2.

TABLE 2. **Gasoline specification limits (Aden Refinery 2016).**

Tests/Characteristics	LSL	USL
Density	0.70	0.73
Octane-number	90	100
Vapor-pressure	7	10

The multivariate process characteristics in this study (vapor pressure, density, and octane number) are normally distributed. The target, and the upper and lower specification limits, as shown below:

$$L' = (L_1, L_2 \ldots, L_k) = (90, 7, 0.70)$$
$$U' = (U_1, U_2 \ldots, U_k) = (100, 10, 0.73)$$
$$T' = (T_1, T_2 \ldots, T_k) = (95, 85, 0.715)$$

B. ASSESSMENT OF ADEN REFINERY'S CURRENT PERFORMANCE

The first step in assessing the process efficiency status of every industry is to measure current performance. There are several metrics that can be used to determine the current process efficiency. The majority of these metrics are subjected to a number of estimation techniques, resulting in a variety of results. As a result, it's critical to use the right estimation methods and measurement techniques when evaluating process results. This is the focus of many studies, and the purpose of this research is to develop metrics for evaluating and measuring industrial process production consequence, this paper provides a case analysis to measure process efficiency of Aden's oil refinery, Yemen.

1) DATA COLLECTION AND ACQUISITION

Three among the most important features of petroleum, namely octane number, density, and vapor pressure, are examined in this study. These features are, without a doubt, the most important characteristics of all oil products. The following steps are used to calculate relative vapor pressure, density, and octane number: This technique is used to obtain relative data about gasoline: To begin, a sample of petroleum is randomly collected of the tanks oil using (hydrometer) at three locations: the bottom, middle and upper, parts of the tank. Since the density values vary at different locations in the tank, the sample is then mixed together. Following the mixing of the sample, it is taken to the laboratory to be checked in order to obtain the results. The density, octane number, and vapor characteristics. 40 samples were acquired, each consisting five items, from the product in even intervals (every 8 h). After the data was collected randomly to 200 samples size. Statistical important tests associated with the effectiveness of the data for moreover analysis has been done. It includes of key statistical tests which are normality, stationary, autocorrelation and process capability tests and heteroscedasticity (autoregressive model) test. The results of tests of the petroleum properties for octane number, density and vapor, stable where results conclude that the tested series do not have a unit root. Also, the data of the density, octane number, and vapor characteristics do not have autoregressive and the results indicate that the process is capable of density, octane and vapor characteristics, therefore, the results for all characteristics concluded that normality, stationary, not have autoregressive and capable that means on the tested the all characteristics are statistically reliable for further analysis we implement the process actual yield proposed indexes based on six sigma concept and information control charts as follows:

2) EVALUATION OF LEVEL PROCESS PERFORMANCE AT YEMEN'S ADEN OIL REFINERY

This section explains how to evaluate and calculate potential and actual yield process by used the traditional indications and then comparison of results with the suggested approach in this study which are indices using the Six Sigma principle, the 1.5 sigma shift of the mean from the target. Standard deviation, and magnitude of variance coefficients, and information control chart. In summary we'll look at the case of petroleum refining process products. Each product is made up of a number of different components. A Gasoline refinery needs to compare the process quality of Oil Characteristics between three Characteristics to decide which Characteristics to choose in this process. All, the Characteristics selection process was assessed a $\propto = 0.05$ significance level. All Characteristics collected sample data (200 of Gasoline, the intervals from a stable process) for a mass-produced Gasoline. The overall sample means X_i pooled sample standard deviations σ_{i} and calculation of yield process SSY for the 3 Characteristics (i.e. I = 1; 2; 3). Then the four different ways to estimate the process yield based on Six Sigma and information of control charts for characteristics, as discussed previously using Eq (2), for the first estimation way, Eqs (4-12) for second way, Eqs (13-16) for the third way, and Eq (17) for the fourth way.

C. DISCUSSION OF THE RESULTS

This part include a detailed explanation of the pre-processing steps where various significant statistical tests results and discussions associated with the validation of the data are performed prior to measure current performance and implementation of the proposed PCIs based on Six Sigma to evaluate the process performance of the oil refinery process in Yemen, to ensure that the data in hand are reliable, follow normal distribution, and statistically sufficient for further analysis. Normality, stationary, autocorrelation, and heteroscedasticity tests (autoregressive model; univariate and multivariate). As well as this part illustrates an elaborates the results and discussions of the proposed approach for evaluating and improving process performance of oil refinery in Aden, Yemen. It contains the results of implementation the theoretical side thus this part demonstrate the trueness verification proficiency of all research objectives of this study.

1) STATISTICAL TESTS RESULTS AND DISCUSSIONS

Table 3 shows the results of the normality test for density, octane number, and vapor pressure. The three properties

TABLE 3. Shown the normality tests for gasoline properties.

	Univariate				**Multivariate**			
	KolmogorovSmirnov^{a}		**Shapiro.Wilk**		**Mardia kurtosis**		**Marda Skewness**	
	Statistic	P. value.	Statistic	P. value.	Statistic	P. value	Statistic	P. value
Density,	0.056	0.201	0.996	0.677	-.031	0.173	-0.153	0.343
Octane-Number.	0.065	0.201	0.985	0.062	-0.159	0.173	-0.688	0.343
Vapor-Pressure.	0.074	0.049	0.989	0.097	0.122	0.173	0.271	0.343

definitely follow a normal distribution, as evidenced by the p-values of the Kolmogorov-Smirnov and Shapiro-Wilk statistical tests, which are both over 5%. It's also worth noting that the density, octane number, and vapor pressure are all statistically significant and so suitable for further research. As a result, the null hypothesis is accepted because the p-value is higher than the significance level of 5%. The skewness is also between $(-1/2)$ and $(+1/2)$, indicating that the distribution is approximate.

a: PROCESS STABILITY TEST (UNIT ROOT)

Table 4 shows the results of the Augmented Dicky-Fuller unit root test. All of the series are tested with constant and trend, and the best lag is chosen based on the frequency of the data. According to Perron, P. (1989) [18] experimenting with a range of values is normal, and the frequency of the data can be used as a criterion for selecting the number of lags of the residuals. The t-statistics were bigger than their critical value, indicating that all of the variables evaluated were integrated of order zero or stationary with the I(0) process at the 5% significant level. The findings show that the tested series do not have unit root.

TABLE 4. Shows the results of the ADF unit root test in terms of level and first difference.

Variable	T-Statistics(Level)	T-Statistics (First Difference)	Order of integration
Vapor P	-6.07(8)**	-7.57(8)	I(0)
Octane N	-9.04(8)**	-9.17(8)	I(0)
Density	-5.69(8)**	-7.14(8)	I(0)

Notes: MacKinnon (1996) [31] calculated critical values of -4.00, -3.43, and -3.14 at the 1%, 5%, and 10% levels, respectively. 2. A 5% level of significance is indicated by a **.

b: TEST OF AUTOREGRESSIVE

The study is expanded to examine the fundamental statistical tests that were linked with the univariate autoregressive model in order to investigate the data's features. Normality, autocorrelation, and heteroskedasticity are the tests that are applied to the underlying series individually. Table 5 summarizes the findings.

The univariate autoregressive model results for density, octane number, and vapor pressure, as given in Table 5 with one degree of freedom, the Jarque- Bera test of normality for residuals was used. The JB-test statistic was found to have values of 0.18, 3.54, and 1.33 for, vapor pressure, octane number, and density respectively. At a 5% level of significance, these results are less than the critical value of 3.84. Because the p-values are bigger than the 5% level (% 0.93, % 0.16%, and 0.52%), these traits are not statistically significant. As a result, the residuals of the octane number, vapor pressure and density series were all found to be regularly distributed.

TABLE 5. Results using a univariate autoregressive model.

	Statistic Tests		
Series	***JB*** Statistic	***LM*** Statistic	***H*** Statistic
Vapor P	0.18(0.93)	0.57(0.76)	2.87(0.08)
Octane N	3.54(0.16)	0.43(0.82)	0.09(0.76)
Density	1.33(0.52)	5.65(0.07)	0.64(0.41)

Notes:[56] JB, LM and H Statistics refer to Jarque- Bera (1990) test of normality, Breusch-Godfrey Serial Correlation Lagrange Multiplier (1978) test and Engle's ARCH (1982) test for heteroskedasticity respectively, and the tests are distributed as Chi-Squared tests with their p-values as in parentheses.

The Breusch-Godfrey-Lagrange Multiplier (LM) was used to test serial correlation with two degrees of freedom for density, octane number, and vapor pressure. For vapor pressure, octane number, density and the resulting LM statistic values are 0.57, 0.43 and 5.65 respectively. These values are statistically insignificant at the 5% level since they are less than the crucial value of 5.98. Furthermore, the test indicated that the residuals of the investigated characteristics are not serially associated because the p-values for vapor pressure, octane number and density are %0.76, %0.82, and %0.07 respectively, which are greater than the 5% level of significance.

The AutoRegressive Conditional Heteroskedasticity (ARCH) test was used to examine the heteroscedasticity of the three attributes with only one degree of freedom. The ARCH statistic values for vapor pressure, octane number and density are 2.85, 0.09 and 0.64 respectively, based on the results. These values are statistically insignificant at the 5% level since they are less than the crucial value of 3.90. Furthermore, the p-values for vapor pressure, octane number, and density with values of %0.08, 0.76, and 0.41, respectively, are greater than the 5% significance level.

The analysis progressed to apply the vector autoregression (VAR) model to perform this conclusion on data validation. The analysis was built on the specified VAR model, which was checked for attributes of interest series. The octane number and vapor pressure series were regressed on themselves, and the results are shown in Table 6.

TABLE 6. **Results test by multivariate model autoregressive.**

Series	Statistic Tests		
	JB **Statistic**	***LM*** **Statistic**	***H*** **Statistic**
Vapor P Octane Density,	1.86(0.35)	5.25(0.07)	0.80(0.26)

The Jarque- Bera residuals normality test was used in conjunction with one degree of freedom. The JB-test statistic had a value of 1.86, which was less than its critical value of 3.53, and was not significant because the p-value (percent 0.35) was more than the 5% level. As a result, the residuals of the calculated multivariate autoregressive model were shown to have a normal distribution.

The Breusch-Godfrey of Lagrange Multiplier (LM) test statistic value (5.25) for serial correlation with two degrees of freedom was less than its critical value of 4.98 and statistically insignificant at the 5% level. The test indicated that the residuals of the VAR model were not serially associated because the p-value (percentage 0.07) was greater than the 5% level of significance.

With only one degree of freedom, the AutoRegressive Conditional Heteroskedasticity (ARCH) test was used to test for heteroscedasticity in the VAR model. The 0.80 value of the ARCH statistic was less than its critical value of 3.53, indicating that it was statistically insignificant at the 5% level. The p-value (percentage 0.26) was higher above the significance threshold of 5%. As a result, the residuals were homoscedastic, indicating that there was no indication of heteroscedasticity on the tested VAR model.

2) RESULTS AND DISCUSSIONS FOR CURRENT PERFORMANCE

The result and discussion of the current process performance for density, octane number, and vapor pressure through estimation to sigma level and process capability indices. For that, the estimation (SD) is an important feature that serves as the foundation for statistical analysis of process capabilities. Where capability indices derived from sample statistics are prone to statistical variability, which has an impact on the indices calculated. In this study improvement in processes is investigated and estimated in terms of sigma levels. From acquired data, can be used different methods to determine standard deviation, as follows:

First method (long-term view):

Standard deviation is estimated using individual data points and is given as:

$$\hat{\sigma}_{LT} = \sqrt{\frac{\sum_{j}^{m}\sum_{i}^{n}(X_{ij} - \overline{\overline{X}})^2}{(mn-1)}}$$

Second models based on control charts (short-term): Here we have different methods for determining standard deviation which are $\hat{\sigma}_R = \bar{R}/d_2(n), \hat{\sigma}_{s_i} = S_i/C_4(v); \hat{\sigma}_S = S/C_4\,(n)\,\hat{\sigma}_{w_i} = \frac{1}{\sum_{i=1}^{N} w_i} \times \sum_{i=1}^{N} \frac{w_i R_i}{d_2(n)}$ and $\hat{\sigma}_{h_i} = \frac{1}{\sum_{i=1}^{N} h_i} \sum_{i=1}^{N} \frac{h_i s_i}{C_4(n)}$.

TABLE 7. **Shows the PCIs obtained using various variance estimation approaches.**

Characteristics	Indices	$\hat{\sigma}_T$	$\hat{\sigma}_R$	$\hat{\sigma}_s$	$\hat{\sigma}_{Si}$	$\hat{\sigma}_{wi}$	$\hat{\sigma}_{hi}$
Density	$\hat{C}_p$	0.46	1.04	1.01	1.09	1.040	1.01
	$\hat{C}_{pk}$	0.37	0.83	0.81	0.87	0.830	0.81
Octane	$\hat{C}_p$	0.27	0.93	0.94	0.96	0.917	0.90
	$\hat{C}_{pk}$	0.23	0.59	0.60	0.64	0.594	0.596
Vapor	$\hat{C}_p$	0.58	1.51	1.59	1.57	1.509	1.474
	$\hat{C}_{pk}$	0.36	1.37	1.39	1.37	1.307	1.271

Table 7 shows the current process performance data for the density, octane and vapor pressure characteristics. The reported results are based on estimation of traditional capability process indices C_p and C_{pk}. It can be seen that the oil gasoline refinery's process performance for the density, octane and vapor pressure characteristics don't meet the predefined standards. This conclusion is based on the values of C_p and C_{pk} in Table 3 For example, the long-term value of C_p are 0.464, 0.27 and 0.58 which are less than 1. The value of C_{pk}, which is smaller than 1 for all estimations, is consistent with this. In addition, sigma level for density, octane and vapor pressure characteristics can be obtained using the relationship between process capability and level sigma as the following:

For density $L\sigma = 3 \times C_p = 1.09 \times 3 = 3.27, L\sigma = 3 \times C_p = 0.46 \times 3 = 1.40$, for octane $L\sigma = 3 \times C_p = 0.96 \times 3 = 2.88, L\sigma = 3 \times C_p = 0.27 \times 3 = 0.81$

For Vapor $L\sigma = 3 \times C_p = 1.58 \times 3 = 4.74 L\sigma = 3 \times C_p = 0.58 \times 3 = 1.74$.

Overall, the sigma level varies between 1.59 and 3.20 for density, also the sigma level for octane number varies between 0.81 and 2.88 and the sigma level for vapor varies between 1.7 and 4.74. Based on previous equations, the sigma level used in Aden refinery oil is equal to 3.20, 2.88 and 4.74 for density, octane and vapor respectively. It's also worth noting that the estimated by traditional indexes are done collected for gasoline characteristics, use the S_{pk} index the results 1.78, 1.03 and 0.77, for vapor pressures, density, and octane number respectively. For evaluation of total yield process performance at Yemen's Aden oil we use the TS_{pk}, S_{sp}^{T} and MS_{pk} indexes [54], [56]–[59]. The results are (0.7114, 0.72 and 0.734). Table 8 shows the guide to interpreting the contribution of the capability and yield process based on the traditional indexes. From the above, it can be said that the use of traditional indicators shows that the sigma level in Aden refineries is less than 3 sigma, where the values of capability indices less than 1.

Based on the results described earlier, the following has been highlighted both in practice and in theory. In theory, the traditional indexes can measure process yield with respect to specification limits and cannot measure potential or shift

TABLE 8. S_{pk} **interpretive guide for traditional indexes CPIs.**

Value for Capability	Grading	Levels Sigma	Target distribution (yield)
$S_{pk} <1$	Inadequate.	$LS <3$	$Y < 0.9973$
$1. \leq S_{pk}< 1.33$,	Capable	$3 \leq LS < 4$	$0.9973 \leq Y \leq 0.99994$
$1.33 \leq S_{pk}< 1.5$,	Satisfactory	$4 \leq LS < 5$	$0.999994 \leq Y < 0.9999994$
$1.50 \leq S_{pk} < 2$,	Excellent	$5 \leq LS < 6$	$0.\ 9999994 \leq Y < 0.999999998$
$S_{pk} \geq 2.$	"Super".	$LS \geq 6$	$Y \geq 0.9999999998$

thus the traditional indexes lack effective and sufficient to measure the process performance based on Six Sigma idea and information control charts, also traditional indexes do not have an index to measure yield process withe considered the comprehensive to all different probable configurations under the probability density curve and tolerance limits. In practice, the traditional indexes lack effective and sufficient implementation of measure potential or shift the mean of the target in process control, yield process.This has necessitated the expansion of the research to include new performance indicators based on the Six Sigma concept for evaluating univariate and multivariate product quality attributes. As a result, can be the users, professional engineers, and statistical data analysts who are interested in assessing and enhancing the performance of industrial processes will benefit from this research. This study practically can be used as a guide to measure and evaluate the performance of yemen's oil refinery process in order to comply with quality specifications and international standards for petroleum products, resulting in high-quality and environmentally friendly petroleum products through implementation as follows:

3) SIX SIGMA EVALUATION FOR PROCESS EFFICIENCY OF ADEN'S OIL REFINERY

By evaluation and measurement process efficiency of Aden's oil refinery, Yemen based on six sigma idea there are variances in the rate of SSY between characteristics because the difference of level sigma, for each characteristics and the degree shift the mean of the target at every characteristics the summarized for parameters of that in Table (9), also the industrial process level assessment chart suggested by (Chen & Chang, 2017) can be used as shown in Figure (2). As well as the summarized for different ways (1, 2, 3, and 4) to estimate the process yield in Table 10.

The process yield index SSS_{pk} calculated for the vapor pressure, density and octane number, using Eqs (20 to 23) based on Six Sigma concept with TL and information control charts the summarized for that in Table (11). In addition, we hypothetical that the satisfactory quality level of each characteristic petroleum for the three attributes vapor pressure, density and octane number must meet the 4σ level particular by Aden oil refinery. We used $(\hat{\mu}f'/d)_i$ and $(\hat{\sigma}_{ST}/d)_i$ estimation by control chart info as a standard in the assessment of quality level to ensure quality assurance and improve assessment reliability. Use figure 2 chart suggested by (Chen & Chang, 2017) the outcomes for each characteristic. Figure 2 shows that point 3σ is located within zone 3σ. This means that the sprocket process efficiency associated with octane number characteristic number 3 does not fulfil the purchasing user's needs. Inadequate process precision (excessive process variance) is the root cause of the octane number characteristic's inability to deliver the 4σ level. The underlying explanation for the characteristic 3 is insufficient process accuracy (immoderate process mean shift). The Aden oil refinery's respective factors are both inadequate process reliability and inadequate process precision. The management of the Aden oil refinery should devise steps to improve process efficiency. Experts with extensive experience can help improve process accuracy by reducing the incidence of incorrect characteristic parameter settings, improving staff training and selection processes to avoid unnecessary processing, and reviewing laboratory work. Procedures and establishing standard operating procedures to prevent inappropriate operation. For the purpose of improving the process accuracy of insufficient processes.

The principle of interchangeability, which focuses on interchangeability over four different types of estimation, is where tolerance analysis evolved. A multivariate process is a set of different attributes with a correlation structure that simultaneously influence the performance of the process. By Consider three quality attributes provided by the process performance at Yemen's Aden oil refinery, density, octane, and vapor pressure. To demonstrate the application of the ($SSMS_{pk}$) multivariate total yield index and ($SSTY$) total yield process by used the Eq 33 for that. For the third case (3) of estimation, which is based on potential yield and actual yield, with assumptions the tolerance limits equal 12 the SSY

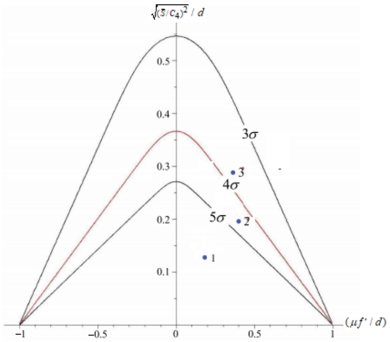

FIGURE 2. **Chart for evaluating process performance.**

TABLE 9. Quality characteristics of the process $(\hat{\mu} f'/d)_i$ and $(\hat{\mu} f'/d)_i$.

Characteristics		$\hat{\mu}_i$	$\hat{\sigma}_i$	$(\hat{\mu} f'/d)_i$	$(\hat{\sigma}_{STi}/d_i)_i$	SSC_{pki}	$L\hat{\sigma}$
1	Vapor pressures	8.2935	0.32118	0.13767	0.202305	1.420847	5.76254
2	Density	0.71796	0.00415	0.1970	0.278369	0.961554	4.38466
3	Octane number	93.4735	1.16041	0.3013	0.317969	0.73246	3.69738

TABLE 10. Process yield for density, octane number and vapor pressure by analysis tolerance and SS.

No	Type of cases calculated	SSY_i to density	SSY_i to octane	SSY_i to V-Pressure
1	$Case(1)SS\hat{Y} = \left[\Phi(6x_1 \times C_p) + \Phi(6\times_2 \times C_p)\right] - 1$	0.998331772	0.980053184	0.999988946
2	$Case(2)SS\hat{Y} = \left[\Phi((1.5 + (3SSC_{pk} + 1.5)) - \Phi((1.5 - (3SSC_{pk} + 1.5))\right], =$	0.998915699	0.986003458	0.999990
	$= \left[\Phi(1.5 + k) - \Phi(1.5 - k)\right]$	0.998915699	0.986003458	0.999990
3	$Case(3)SS\hat{Y} = \left[\Phi(6 \times C_p - 3 \times C_{pk}) + \Phi(3 \times C_{pk})\right] - 1$	0.998331772	0.980053184	0.999988946
4	$Casw(4)SS\hat{Y} = \left[\Phi\left(\frac{(1+\mu f')/d}{(\bar{S}/C_4)/d}\right) + \Phi\left(\frac{(1-\mu f')/d}{(\bar{S}/C_4)/d}\right)\right] - 1$	0.998915699	0.986003458	0.999990

TABLE 11. SSS_{pk} process yield indices estimation.

Case	cases of estimate	Density $SS\hat{S}_{pk}$	Octane $SS\hat{S}_{pk}$	Vapor $SS\hat{S}_{pk}$
1	$Case(1)SS\hat{S}_{pk} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\left(\Phi(6x_1 C_p) + \Phi(6\times_2 C_p)) - 1\right) + 1\right]$	1.044	0.778	1.465
2	$Case(2)SS\hat{S}_{pk} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\left(\Phi(1.5 + (3SSC_{pk} + 1.5)) - \Phi(1.5 - (3SSC_{pk} + 1.5))\right) + 1\right], =$ $= \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\left(\Phi(1.5 + k) - \Phi(1.5 - k)\right) + 1\right]$	1.047	0.819	1.474
3	$Case(3)SS\hat{S}_{pk} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\left[\left(\Phi(6 \times C_p - 3 \times C_{pk}) + \Phi(3 \times C_{pk})\right) - 1\right] + 1\right]$	1.044	0.778	1.465
4	$Casw(4)SS\hat{S}_{pk} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\left[\left(\Phi\left(\frac{(1+\mu f')/d}{(\bar{S}/C_4)/d}\right) + \Phi\left(\frac{(1-\mu f')/d}{(\bar{S}/C_4)/d}\right)\right) - 1\right] + 1\right\}$	1.047	0.819	1.474

results for, vapor pressure, density and octane number are, 0.999988946, 0.998331772, and 0.980053184 respectively. It can be shown that the cases 1 and 3 have different ways estimations but the result is same, also can see the vapor pressure characteristic yielded the highest percentage of SSY, while the octane number characteristic yielded the lowest. For the fourth case (4), which is based on estimation by control chart info as a standard in the assessment of quality level. To ensure quality assurance and improve assessment within the tolerance interval based on Six Sigma. The yields SSY for Vapor pressure, density, and octane number, are 0.998915699, 0.986003458 and 0.999990 respectively.

Meanwhile, Table 11 shows the results of the performance index for petroleum characteristics, vapor pressures, density, and octane number, which are divided into four separate estimation cases: For the first case, (1) and third case (3) the SSS_{pk} yields index values have same result which are 1.465, 1.044 and 0.778 for, vapor pressure, density and octane number, respectively. For the second (2) and four cases, (4) it's have same result, the maximum value of SSS_{pk} yield was got for the vapor characteristic by value index is 1.474. The density characteristic had the second highest percentage of SSS_{pk} with 1.047, and the octane number characteristic had the lowest SSS_{pk} With, 0.819 value. When the SSS_{pk} value is 1.5, the third and fourth cases the process with the highest yield value. As a result, if the SSS_{pk} is 1.5, the yield percent is 0.999996 and the sigma level is 6. Table 9, provide a guide to view the yield process index output based on the calculated cases. Estimation outperforms because the yield index value SSS_{pk} is1.1474, which is close to 1.5.

Apart from that, the yield process SSY and SSS_{pk} yield indices have a one-to-one relationship in the four cases of yield calculation, as shown in the above discussion and analysis.

For the comparison of process efficiency indices, It's can be see the result come by the estimated traditional indexes are shows at Table 13 also the result come by the estimated new indices based on six sigma and info control chart indices

TABLE 12. SSS_{pk} **interpretive guide for all analysis cases in the study.**

Value of Capability	Grading	Levels Sigma	Six Sigma Yield
$SSS_{pk} < 0.60$	"Inadequate"	$LS < 3$	$SSY < 0.93328$,
$0.60 \leq SSS_{pk} < 1$	"Capable"	$3 \leq LS \leq 4$	$0.9332895 \leq SSY \leq 0.9937904$
$1 \leq SSS_{pk} < 1.33$	"Satisfactory"	$4 \leq LS < 5$	$0.9937904 \leq SSY < 0.9997675$
$1.33 \leq SSS_{pk} < 1.50$	"Excellent"	$5 \leq LS < 6$	$0.9997675 \leq SSY < 0.9999966$
$SSS_{pk} \geq 1.54$	"Super".	$LS \geq 6$	$SSY \geq 0.9999966$

TABLE 13. **Shows the results of a comparison of process efficiency indices.**

Characteristic of Oil		Traditional Indexes						Indices Based on six sigma			
	Juran 1976 C_p	Kane 1986 C_{pk}	(Boyles , 1994 S_{pk}	$L\sigma$	Chen et al. (2003) TS_{pk}	Pearn , 2006 S_{sp}^{T}	Wang 2010 MS_{pk}	SSC_{pk}	$L\sigma$	SSS_{pk} with four different cases	$SSMS_{pk}$
Octane	1.04	0.69	0.77	2.88	0.71	0.72	0.73	0.73	3.70	See the Table (11)	0.76
Density	1.19	0.95	1.03	3.27				0.96	4.38		
Vapor	1.64	1.41	1.78	4.74				1.42	5.75		

are shown in Table 13. According to the explanatory guide interpreting the contribution of the yield index based on the traditional index, is clear as shows in Table (8). As well as the explanatory guide interpreting the contribution of the yield index based on the estimated new indices based on six sigma and info control chart indices is clear as shows in Table (12). the suggested SSS_{pk} and $SSMS_{pk}$ indexes produces better outcomes than the indexes S_{pk}, TS_{pk} S_{sp}^{T} and MS_{pk} as shows at Table 13. that the proposed indexes outperformed the conventional traditional indexes in four separate estimation cases for the three attributes octane number, density, vapor pressure and overall yield process. Thus, the level sigma are improve that the proposed statistical approach leads to ensure that the industrial process is capable of producing products according to the specification limits, improving industrial processes performance, and reducing variations and defects. Also, this study contributes to the existing literature by providing comprehensive theoretical knowledge for developing and extending new process capability indices based on Six Sigma. In addition, this study considers the shift the mean of the target process, and the use of levels 4σ, 5σ, 6σ to the amount of improvement the current process performance required to produce a capable process, to reduce defects and variations.

The results of the all-scenario cases in this paper, the first- and third-ways estimators for yield process which is determined based on the level of sigma at a super process capacity that is equal on the right, 6 sigma, and on the left, 6 sigma. And the second and fourth ways estimators for yield process which is determined based on the level of sigma at a super process capacity that is equal on the right, 4.5 sigma, and on the left, 4.5 sigma consider shift the mean of the target in the process are the best estimating to overall yield process index and evaluation process performance. The overall yield process $SSTY$ for gasoline characteristics and the multivariate index yield $SSMS_{pk}$ are collected by using Eq (40). The results revealed for that where the total yield $SSTY$ is 0.989503204 and the total yield index $SSMS_{pk}$ is 0.76859. That mean the level of proses in Aden oil refinery by evolution three gasoline characteristics vapor, density and octane is less then 4 sigma. For the comparison of multivariate yield indexes according to by (Chen *et al.* 2003; Pearn & Cheng, 2006; and Wang 2010) the indexes of overall method yield are 0.71,0.72 and 0.7314 (less than one) and its follow the guide indexes CPIs, as show in table (5). The overall process yield $SSMS_{pk}$ is 0.76. The proposed yield indexes are significant for measuring the yield process and provided superior results when compared to its counterpart indices in previous studies such as C_p, C_{pk}, S_{pk} TS_{pk} S_{sp}^{T} *and* MS_{pk}. The all cases, in particular, provide a specific understanding of the yield process index results using 1.5 sigma. The estimation cases supply clarification of the yield index process results based on control charts information. Thus, based on all cases scenario cases of estimating $SSSM_{pk}$, the index $SSMS_{pk}$ can be used to assess process capability in general as follows: If the $SSMS_{pk}$ is less than < 0.6, the process is incapable capable, if the $0.6 \leq SSMS_{pk} < 1$. The process is marginally, if the $1 \leq SSMS_{pk} < 1.33$ process is satisfactory if $1.33 \leq SSMS_{pk} < 1.5$ the process is outstanding, and if the $SSMS_{pk}$ is greater than 1.5, if the $SSMS_{pk} \geq 1.5$ the process is super.

VI. CONCLUSION

Calculating the variance of the process in terms of the idea of Six Sigma is critical to quality (CTQ). There is also a need to use the constants in the control chart to estimate the yield process indicators according to the idea of six sigma.

This is the main objective that was achieved in this study by comparative analysis to four different ways of calculating yield index SSS_{pk} based on six sigma idea, and control chart information, to measuring the process performance in an industrial case study is presented in order to assess the production processes of Aden's Oil Refinery, Yemen., to illustrate the applicability of this approach using density, octane number, and vapor pressure properties of petrol. The findings of this study the estimated yield index by the four separate estimation scenarios as shown in Table 11. If the process distribution is centred at the mean value, the first case (1) is focused on the potential yield estimation, mean equals the target that expresses for possible of generating products within the sensitivity interval based on Six Sigma. The second case of estimation, the process yield was determined using the level sigma and assume a shift the mean process of the target fixed value is 1.5 for collected the index SSC_{pk} and then use it to estimations the process yield. For the third case (3) of estimation, which is based on potential yield and actual yield, with idea of six sigma the tolerance limits equals 12. It is a generalized equation to comprehensive for all possible cases under the probability density curve as shown in Table (1). For the fourth case (4), which is based on level sigma and estimation the different ratio of a change in the mean process from the target. It can be summary the cases 1 and 3 have different ways estimations but the result is same, also the cases 2 and 4 have different ways estimations but the result is same. The second and fourth cases of estimation are better than the first and third case of estimation, as shown for the SSY results in Table 10 and Table 11 for the SSS_{pk} results. The results revealed for that where the total yield $SSTY$ is 0.989503204 and the total yield index $SSMS_{pk}$ is 0.76859. That mean the level of proses in Aden oil refinery by evolution three gasoline characteristics vapor, density and octane is less then 4 sigma. For the comparison of multivariate yield indexes according to traditional indexes and new indexes the advantage for traditional indexes S_{pk} $TS_{pk}\ S^{T}_{sp}\ and\ MS_{pk}$. can measure process yield with respect to specification limits but the disadvantage for traditional indexes cannot measure potential or shift in mean. While in this article, we have different ways of estimating those indicators and making it have measure potential or shift in mean with using a comprehensive approach under the probability density function, demonstrating an extensive analysis for the actual and potential process yield. The result for traditional indexes of overall process yield are 0.71, 0.72 and 0.73 are (less than one) and its follow the interpretive guide indexes CPIs, as show in table (8). And the overall process yield new index $SSMS_{pk}$ is 0.76 with follow the interpretive guide indexes as shown in table 12.

The proposed yield indexes are significant for measuring the yield process and provided superior results when compared to its counterpart indices in previous studies such as $S_{pk}\ TS_{pk}\ S^{T}_{sp}\ and\ MS_{pk}$. The all cases, in particular, provide a specific understanding of the yield process index results using 1.5 sigma. The all cases, in particular, provide a specific understanding of the yield process index results using 1.5 sigma. The estimation cases supply clarification of the yield index process results based on control charts information. Thus, based on all cases scenario cases of estimating $SSSM_{pk}$, the index $SSMS_{pk}$ can be used to assess process capability in general as follows: If the $SSMS_{pk}$ is less than < 0.6, the process is incapable capable, if the $0.6 \leq SSMS_{pk} < 1$. The process is marginally, if the $1 \leq SSMS_{pk} < 1.33$ process is satisfactory if $1.33 \leq SSMS_{pk} < 1.5$ the process is outstanding, and if the $SSMS_{pk}$ is greater than 1.5, if the $SSMS_{pk} \geq 1.5$ the process is super. This research provided a novel approach based on the Six Sigma idea and information control $\mathrm{X}_{\mathrm{bar}}$-s chart, to improve and measure process performance in industries. This study has important implications for industrial practitioners, researchers and quality control experts interested in the evaluation of process performance. especially in the oil field at a developing country such as Yemen, where the Aden's Oil Refinery, Yemen is still dependent on the laboratory examination to assess petroleum characteristics. Finally, the proposed PCIs based on the SS definition are a promising methodology that can be expanded and/or used by other industries and practitioners to evaluate process efficiency in terms of precision and quality control.

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REFERENCES

[1] A. Goswami and H. N. Dutta, "Some studies on normal and non-normal process capability indices," *Int. J. Math. Stat. Invent.*, vol. 1, no. 2, pp. 31–40, 2013.

[2] D. de-Felipe and E. Benedito, "A review of univariate and multivariate process capability indices," *Int. J. Adv. Manuf. Technol.*, vol. 92, nos. 5–8, pp. 1687–1705, Sep. 2017.

[3] J. B. Krolczyk, G. M. Krolczyk, S. Legutko, J. Napiorkowski, S. Hloch, J. Foltys, and E. Tama, "Material flow optimization—A case study in automotive industry," *Tehnicki Vjesnik/Tech. Gazette*, vol. 22, no. 6, pp. 1447–1456, 2015.

[4] V. Leiva, C. Marchant, H. Saulo, M. Aslam, and F. Rojas, "Capability indices for Birnbaum–Saunders processes applied to electronic and food industries," *J. Appl. Statist.*, vol. 41, no. 9, pp. 1881–1902, Sep. 2014.

[5] T. Lupo, "The new nino capability index for dynamic process capability analysis," *Qual. Rel. Eng. Int.*, vol. 31, no. 2, pp. 305–312, Mar. 2015.

[6] R. Joghee, "Control chart for high-quality processes based on six sigma quality," *Int. J. Qual. Rel. Manage.*, vol. 34, no. 1, pp. 2–17, Jan. 2017.

[7] R. Goodwin, *Some Multivariate Process Capability Indices*. Arlington, TX, USA: The Univ. Texas at Arlington, 2015.

[8] Y. T. Tai, W. L. Pearn, and C.-M. Kao, "Measuring the manufacturing yield for processes with multiple manufacturing lines," *IEEE Trans. Semicond. Manuf.*, vol. 25, no. 2, pp. 284–290, May 2012.

[9] Z. Allam, E. Becker, C. Baudouin, R. Bigot, and P. Krumpipe, "Forging process control: Influence of key parameters variation on product specifications deviations," *Proc. Eng.*, vol. 81, pp. 2524–2529, Jan. 2014.

[10] K. Srinivasan, S. Muthu, S. R. Devadasan, and C. Sugumaran, "Six sigma through DMAIC phases: A literature review," *Int. J. Product. Qual. Manag.*, vol. 17, no. 2, pp. 236–257, 2016.

[11] M. Aslam, R. A. R. Bantan, and N. Khan, "Design of a control chart for gamma distributed variables under the indeterminate environment," *IEEE Access*, vol. 7, pp. 8858–8864, 2019.

[12] I. Kaya and A. Turgut, "Design of variable control charts based on type-2 fuzzy sets with a real case study," *Soft Comput.*, vol. 25, no. 1, pp. 613–633, Jan. 2021.

[13] I. Kaya and C. Kahraman, "Process capability analyses based on fuzzy measurements and fuzzy control charts," *Expert Syst. Appl.*, vol. 38, no. 4, pp. 3172–3184, Apr. 2011.

[14] D. C. Montgomery, *Introduction to Statistical Quality Control*, 7th ed. New York, NY, USA: Wiley, 2009.

[15] S. A. Darestani and M. Nasiri, "Statistical process control: Fuzzy-S control chart and process capability indices in normal data environment," *Int. J. Qual. Rel. Manage.*, vol. 33, no. 1, pp. 2–24, Jan. 2016.

[16] F. A. M. Ali and A. Ahmed, "Six sigma evaluation using process capability and-X control chart for reducing variance in oil density characteristic: A study in Yemen," *Int. J. Qual. Eng. Technol.*, vol. 6, no. 3, pp. 186–209, 2017.

[17] S. Bracke and B. Backes, "Multidimensional analyses of manufacturing processes: Process capability within the case study shape drill manufacturing," *IFAC-PapersOnLine*, vol. 48, no. 3, pp. 2380–2386, 2015.

[18] C.-M. Yu, K.-K. Lai, K.-S. Chen, and T.-C. Chang, "Process-quality evaluation for wire bonding with multiple gold wires," *IEEE Access*, vol. 8, pp. 106075–106082, 2020.

[19] S. Dey, C. Zhang, and M. Saha, "Classical and Bayesian estimation of the index C_{pmk} and its confidence intervals for normally distributed quality characteristic," *J. Stat. Comput. Simul.*, vol. 91, no. 10, pp. 1–24, 2021.

[20] D. Young-Vazquez, R. M. B. Becerra, E. Carrum-Siller, C. Sánchez-Ramírez, and A. Hernández-Hernández, "Multi-process assessment considering the error of measurement systems within the process capacity indices," in *Techniques, Tools and Methodologies Applied to Quality Assurance in Manufacturing*. Cham, Switzerland: Springer, 2021, pp. 175–201.

[21] S. Matsuura, "Bayes estimator of process capability index C_{pk} with a specified prior mean," *Commun. Stat. Methods*, vol. 15, pp. 1–13, 2021.

[22] F. Ali and A. Ahmed, "A review on the statistical concepts of six sigma," *AIP Conf. Proc.*, vol. 1775, no. 1, 2016, Art. no. 30053.

[23] M. Chatterjee and A. K. Chakraborty, "Unification of some multivariate process capability indices for asymmetric specification region," *Statistica Neerlandica*, vol. 71, no. 4, pp. 286–306, Nov. 2017.

[24] A. Parchami, B. Sadeghpour-Gildeh, M. Nourbakhsh, and M. Mashinchi, "A new generation of process capability indices based on fuzzy measurements," *J. Appl. Statist.*, vol. 41, no. 5, pp. 1122–1136, May 2014.

[25] J.-P. Chen, "Re-evaluating the process capability indices for non-normal distributions," *Int. J. Prod. Res.*, vol. 38, no. 6, pp. 1311–1324, Apr. 2000.

[26] C.-W. Wu, W. L. Pearn, and S. Kotz, "An overview of theory and practice on process capability indices for quality assurance," *Int. J. Prod. Econ.*, vol. 117, no. 2, pp. 338–359, Feb. 2009.

[27] S. Kotz and N. L. Johnson, *Process Capability Indices*, no. 2. New York, NY, USA: CRC Press, 1993.

[28] S. Kotz and N. Johnson, "Process capability indices—A review, 1992–2000," *J. Qual. Technol.*, vol. 34, no. 1, pp. 2–19, 2002.

[29] M. Z. Anis, "Basic process capability indices: An expository review," *Int. Stat. Rev.*, vol. 76, no. 3, pp. 347–367, Dec. 2008.

[30] K. S. Chen, K. T. Yu, and S. H. Sheu, "Process capability monitoring chart with an application in the silicon-filler manufacturing process," *Int. J. Prod. Econ.*, vol. 103, no. 2, pp. 565–571, Oct. 2006.

[31] X.-G. Gu, W.-T. Tong, M.-M. Han, Y. Wang, and Z.-Y. Wang, "Parameter and tolerance economic design for multivariate quality characteristics based on the modified process capability index with individual observations," *IEEE Access*, vol. 7, pp. 59249–59262, 2019.

[32] K. G. Khadse and R. L. Shinde, "A new probability-based multivariate process capability index," *Int. J. Qual. Eng. Technol.*, vol. 8, no. 3, pp. 249–267, 2021.

[33] T.-C. Chang and K.-S. Chen, "Statistical test of two taguchi six-sigma quality indices to select the supplier with optimal processing quality," *J. Test. Eval.*, vol. 50, no. 1, pp. 215–228, 2021.

[34] K.-S. Chen, H.-T. Chen, and T.-C. Chang, "The construction and application of six sigma quality indices," *Int. J. Prod. Res.*, vol. 55, no. 8, pp. 2365–2384, Apr. 2017.

[35] I. Ruczinski, "The relation between CPM and the degree of includence," Ph.D. dissertation, Dept. Math, Würzburg Univ., Würzburg, Germany, 1996.

[36] M. Chatterjee and A. K. Chakraborty, "Distributions and process capability control charts forCPUandCPLUsing subgroup information," *Commun. Statist. Theory Methods*, vol. 44, no. 20, pp. 4333–4353, Oct. 2015.

[37] W. L. Pearn, C. C. Wu, and C. H. Wu, "Estimating process capability index C_{pk}: Classical approach versus Bayesian approach," *J. Stat. Comput. Simul.*, vol. 85, no. 10, pp. 2007–2021, Jul. 2015.

[38] H.-C. Lin and G.-J. Sheen, "Practical implementation of the capability index C_{pk} based on the control chart data," *Qual. Eng.*, vol. 17, no. 3, pp. 371–390, Jul. 2005.

[39] F. A. M. Ali, Y. A. Ghaffar, A. Al-Swidi, and A. Ahmad, "Proceedings on engineering six- sigma SSC_{pk} for assessment process performance in industries," *Proc. Eng. Sci.*, vol. 3, no. 2, pp. 139–152, May 2022.

[40] T. T. Allen, *Introduction to Engineering Statistics and Six Sigma: Statistical Quality Control and Design of Experiments and Systems*, 1st ed. London, U.K.: Springer, 2006.

[41] F.-K. Wang, "Measuring the process yield for simple linear profiles with one-sided specification: Yield-profiles-one-sided," *Qual. Rel. Eng. Int.*, vol. 30, no. 8, pp. 1145–1151, Dec. 2014.

[42] W. L. Pearn and C. H. Wu, "Measuring PPM non-conformities for processes with asymmetric tolerances," *Qual. Rel. Eng. Int.*, vol. 29, no. 3, pp. 431–435, Apr. 2013.

[43] C.-W. Wu, T.-H. Wu, and T. Chen, "Developing a variables repetitive group sampling scheme by considering process yield and quality loss," *Int. J. Prod. Res.*, vol. 53, no. 7, pp. 2239–2251, Apr. 2015.

[44] A. Amiri and E. Nikzad, "Statistical analysis of total process capability index in two-stage processes with measurement errors," in *Proc. Int. Conf. Ind. Eng. Oper. Manage. (IEOM)*, Mar. 2015, pp. 1–6.

[45] S. Ali and M. Riaz, "On the generalized process capability under simple and mixture models," *J. Appl. Statist.*, vol. 41, no. 4, pp. 832–852, Apr. 2014.

[46] J. T. Agina-Obu, "Applicability of six sigma in Nigerian fabrication companies: Case of Aveon offshore," *Int. J. Econ. Bus. Manag.*, vol. 1, no. 2, pp. 1–8, 2015.

[47] M. Alkubaisi, "Statistical quality control (SQC) and six sigma methodology: An application of X-bar chart on Kuwait petroleum company," *Int. J. Bus. Admin.*, vol. 4, no. 3, pp. 61–72, May 2013.

[48] L. He, F. Lin, X. Li, H. Sui, and Z. Xu, "Interfacial sciences in unconventional petroleum production: From fundamentals to applications," *Chem. Soc. Rev.*, vol. 44, no. 15, pp. 5446–5494, 2015.

[49] I. J. Dhinakaran, C. M. Ganapathy, T. Kodeswaran, B. M. Kannan, and K. Murugan, "Quality improvement of lubricating oil pump shaft through statistical process control used in automobile industry," *Proc. Eng.*, vol. 38, pp. 2053–2062, Jan. 2012.

[50] A. Bhanpurkar, A. Bangar, S. Goyal, and P. Agrawal, "Implementation of six sigma program for lean manufacturing to reduce the rework waste in transformer manufacturing unit by eliminating defect of leakage from bushings in oil filled transformers," *Int. J. Mech. Ind. Eng.*, vol. 1, no. 3, pp. 6–11, 2012.

[51] M. Al Kindi and M. Al Lawati, "Framework to implement six sigma methodology to oil and gas drilling budget estimation," in *Proc. Int. Conf. Ind. Eng. Oper. Manage.*, nos. 7–9, 2014, pp. 1763–1769.

[52] F. S. Leone, L. S. Nelson, and R. B. Nottingham, "The folded normal distribution," *Technometrics*, vol. 3, no. 4, pp. 543–550, 1961.

[53] K. Gu, X. Jia, H. Liu, and H. You, "Yield-based capability index for evaluating the performance of multivariate manufacturing process," *Qual. Rel. Eng. Int.*, vol. 31, no. 3, pp. 419–430, Apr. 2015.

[54] R. A. Boyles, "Brocess capability with asymmetric tolerances," *Commun. Statist. Simul. Comput.*, vol. 23, no. 3, pp. 615–635, Jan. 1994.

[55] J. Lee and M. C. Strazicich, "Minimum Lagrange multiplier unit root test with two structural breaks," *Rev. Econ. Statist.*, vol. 85, no. 4, pp. 1082–1089, Nov. 2003.

[56] K. S. Chen, W. L. Pearn, and P. C. Lin, "Capability measures for processes with multiple characteristics," *Qual. Rel. Eng. Int.*, vol. 19, no. 2, pp. 101–110, 2003.

[57] W. L. Pearn and C.-W. Wu, "Production quality and yield assurance for processes with multiple independent characteristics," *Eur. J. Oper. Res.*, vol. 173, no. 2, pp. 637–647, Sep. 2006.

[58] W. L. Pearn and Y.-C. Cheng, "Measuring production yield for processes with multiple characteristics," *Int. J. Prod. Res.*, vol. 48, no. 15, pp. 4519–4536, Aug. 2010.

[59] F.-K. Wang, "A general procedure for process yield with multiple characteristics," *IEEE Trans. Semicond. Manuf.*, vol. 23, no. 4, pp. 503–508, Nov. 2010.

[60] W. L. Pearn and S. Kotz, *Encyclopedia and Handbook of Process Capability Indices—Series on Quality, Reliability and Engineering Statistics*. Singapore: World Scientiic; 2007.

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