IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received August 19, 2021, accepted September 4, 2021, date of publication September 20, 2021, date of current version September 24, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3112592

Adaptive Tracking Control for a Class of Uncertain Systems With Output Disturbance

JIANPING CAI [1], **WEI WANG** [2], **AND DONG GUO** [1]

[1] Key Laboratory for Technology in Rural Water Management of Zhejiang Province, Zhejiang University of Water Resources and Electric Power, Hangzhou 310018, China

[2] School of Mathematics and Physics, Hebei GEO University, Shijiazhuang 050031, China

Corresponding author: Jianping Cai (caijianping2001@163.com)

This work was supported in part by Zhejiang Provincial Natural Science Foundation of China under Grant LY17F30015, and in part by the National Natural Science Foundation of China (NSFC) under Grant 61573322.

ABSTRACT In this paper, an adaptive tracking control scheme is proposed for a class of uncertain systems with external disturbances existing in every state equation and output equation. The output disturbance considered here does not need to be generated by a known different equation. It leads to the system about coordinate changes variables used for virtual controls design no longer meet the triangular requirement of the backstepping approach. Thus the traditional backstepping techniques cannot be used in the controller design. This problem has been solved by the proposed robust adaptive control scheme. In addition, the effect caused by the disturbance in state equation can be compensated for by selecting appropriate design parameters in every step. It is shown that the proposed adaptive control scheme can ensure all signals in the closed-loop system bounded.

INDEX TERMS Adaptive control, output disturbance, backstepping, tracking control.

I. INTRODUCTION

Backstepping technique is a popular approach for the controller design and stability analysis of the strict feedback systems [1], [2]. The system control law can be established by the recursive design of a series of virtual control signals. Especially for the strict feedback systems with uncertainties, some problem solving skills has been constantly proposed to improve the backstepping method. As we all know, uncertainties widely exist in practical systems. There are many reasons for the uncertainty of the system, such as unknown parameters, non-smooth nonlinearities, actuator failures, external disturbance, modelling error and so on. For the linearized unknown parameters in the state equations, update laws can be constructed by tuning functions. To the uncertainties caused by non-smooth nonlinearities input, the approximation or smooth inverse are usually constructed. A backlash-like model was built to approximate backlash input. By estimating the linearized parameters in backlash-like model and the unknown upper bound of the disturbance-like term, an adaptive backstepping control scheme had been proposed in [3], [4]. To make full use of the structural information of dead-zone actuator nonlinearity, an smooth inverse of dead-zone was built and an output feedback adaptive control scheme was developed by backstepping approach [5]. In [4]–[6], the unknown external disturbance was considered. This disturbance is only allowed to exist in the last state equation and bounded by an unknown constant. An estimator was constructed to realize the online estimation of this unknown upper bound. Thus the effect caused by external disturbance can be compensated for. Two kinds of actuator failures including partial loss and total loss of effectiveness were considered in [7], [8]. Different faults were represented as changes of linearized parameters. Then the adaptive failure compensation control scheme was developed by estimating these unknown parameters. The unmodeled dynamics were considered in the controller design of nonlinear systems by backstepping. The output feedback control schemes were proposed in [9]–[12]. It is worth noting that these unmodeled dynamics exist in the output signal and are assumed to be generated by differential equation. The system stability analysis relies on such an assumption. In [13], [15], unknown modelling errors were considered in the controller design. An output feedback control scheme were proposed by backstepping in [15]. Compared to [15], modelling errors considered in [13] can exist every state equation and is non-triangular. This leads to the triangle requirement of the backstepping method no longer being met. A new method to overcome this problem

The associate editor coordinating the review of this manuscript and approving it for publication was Mou Chen.

was proposed. Based on this method, the backstepping is improved and an adaptive robust control scheme is developed.

Throughout the control problem of uncertain systems, the available results for strict feedback system with unknown output disturbance are still limited. The main difficulty is that the triangle structure of controlled system will be destroyed when the output disturbance is introduced in coordinate changes. In other words, the system about coordinate changes variables z_i used for virtual controls design no longer meet the requirement of the backstepping approach. To this problem, we address the controller design by backstepping technique for a class of strict feedback systems with the output disturbance in this paper. To solve the above main difficulty, the uncertainties caused by the derivative of the unknown output disturbance are not considered in every step of the controller design. Instead, their effects will be separated two different parts by using Young's inequality in every step. The first part can be dealt with by changing feedback gain parameters. The other part will be accumulated to the last step for compensation by selecting appropriate design parameters. A sufficient condition composed by a series of inequalities have been established to judge the design parameters being appropriate or not. The main contribution of this paper can be summarized as follows:

- Different from the existing results [9]–[12], the output disturbance considered in this paper is more general. It does not need to be generated by a known different equation.
- The recursive algorithm for the selection of design parameters has been established. It is also a sufficient condition to maintain the stability of the system.

The paper is organized as follows. In section 2, our control problem is introduced. In section 3, our control scheme is proposed. Finally, the simulation is given in section 5 and the paper is concluded in section 6.

II. PROBLEM STATEMENT

The following uncertain nonlinear system is considered to illustrate our design ideas.

$$\begin{aligned}\dot{x}_1 &= x_2 + d_1(t)\\ \dot{x}_2 &= x_3 + d_2(t)\\ &\vdots\\ \dot{x}_n &= bu + \theta f(x) + \phi(x) + d_n(t)\\ y &= x_1 + d_y(t)\end{aligned} \tag{1}$$

where $x = (x_1, x_2, \cdots, x_n)$ is the system state, y is the output, and u is the input. b and θ are unknown parameters, $f(x)$ and $\phi(x)$ are known nonlinear functions. $d_i(t)$ $(i = 1, \cdots, n)$ denotes external disturbance in the differential equation of system states. $d_i(t)$ is bounded by an unknown positive constant D_i. $d_y(t)$ is the unknown output disturbance.

To design the adaptive tracking control scheme by backstepping, the following Assumptions on disturbance are made.

Assumption 1: The output disturbance $d_y(t)$ satisfies

$$|Sd_y(t)| \le D_y \tag{2}$$

where S represents the differential operator $\frac{d}{dt}$ and D_y is an unknown constant.

Assumption 2: The unknown parameter b is not equal to zero and $sign(b)$ is known. Without loss of generality, we assume that b is positive.

Assumption 3: The reference signal $y_r(t)$ and its nth derivative are known and bounded.

Remark 1: Two kinds of external disturbances including output disturbance and disturbance in state equation $\dot{x}_i$ are considered in (1). Different form the disturbance $d_i(t)$, the output disturbance is not required to be bounded by an unknown constant. The only requirement is that its derivative is bounded.

Our control purpose is to design the adaptive control law for system (1) by using backsteppting techniques. Then the tracking performance can be achieved under the effect of the unknown output disturbance.

III. DESIGN OF ADAPTIVE CONTROLLERS

In order to obtain the adaptive control law by backstepping, the following coordinate transformations are made.

$$\begin{aligned}z_1 &= y - y_r = x_1 + d_y(t) - y_r\\ z_i &= x_i - \alpha_{i-1} - y_r^{(i-1)} \quad (i = 2, \cdots, n)\end{aligned} \tag{3}$$

Step 1: The derivative of z_1 is

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1 - y_r^{(1)} + Sd_y(t)\\ &= x_2 + d_1(t) - y_r^{(1)} + Sd_y(t)\\ &= z_2 + \alpha_1 + d_1(t) + Sd_y(t)\end{aligned} \tag{4}$$

The virtual control α_1 can be chosen as

$$\alpha_1 = -k_1 z_1 \tag{5}$$

where design parameter k_1 is a positive constant. The following Lyapunov function V_1 is considered.

$$V_1 = \frac{1}{2} z_1^2 \tag{6}$$

With (1) and (3)-(6), the derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= z_1 \dot{z}_1\\ &= z_1(z_2 + \alpha_1 + d_1(t) + Sd_y(t))\\ &= -k_1 z_1^2 + z_1 z_2 + z_1 d_1(t) + z_1 Sd_y(t)\end{aligned} \tag{7}$$

Notice that

$$\begin{aligned}z_1 d_1(t) &\le \frac{z_1^2}{4\varepsilon_1^2} + \varepsilon_1^2 d_1^2(t)\\ z_1 Sd_y(t) &\le \frac{z_1^2}{4\delta_1^2} + \delta_1^2 (Sd_y(t))^2\end{aligned} \tag{8}$$

where ε_1, δ_1 are positive constants. The derivative of V_1 can be rewritten as

$$\begin{aligned}\dot{V}_1 &\leq -k_1 z_1^2 + z_1 z_2 + \frac{z_1^2}{4\varepsilon_1^2} + \varepsilon_1^2 d_1^2(t) \\ &\quad + \frac{z_1^2}{4\delta_1^2} + \delta_1^2 (Sd_y(t))^2 \\ &= -(k_1 - \frac{1}{4\varepsilon_1^2} - \frac{1}{4\delta_1^2}) z_1^2 + z_1 z_2 \\ &\quad + \varepsilon_1^2 d_1^2(t) + \delta_1^2 (Sd_y(t))^2 \end{aligned} \tag{9}$$

Step 2: The derivative of z_2 is

$$\begin{aligned}\dot{z}_2 &= \dot{x}_2 - y_r^{(2)} - \dot{\alpha}_1 \\ &= x_3 + d_2(t) - y_r^{(2)} - \dot{\alpha}_1 \\ &= z_3 + \alpha_2 + d_2(t) - \frac{\partial \alpha_1}{\partial x_1}(x_2 + d_1(t)) \\ &\quad - \frac{\partial \alpha_1}{\partial y} Sd_y(t) - \frac{\partial \alpha_1}{\partial y_r} y_r^{(1)} \end{aligned} \tag{10}$$

The virtual control α_2 is chosen as

$$\alpha_2 = -k_2 z_2 - z_1 + \frac{\partial \alpha_1}{\partial y_r} y_r^{(1)} + \frac{\partial \alpha_1}{\partial x_1} x_2 \tag{11}$$

where design parameter k_2 is a positive constant. The following Lyapunov function V_2 is considered.

$$V_2 = V_1 + \frac{1}{2} z_2^2 \tag{12}$$

With (1), (3) and (10)-(12), the derivative of V_2 is

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + z_2 \dot{z}_2 \\ &= \dot{V}_1 + z_2(z_3 - k_2 z_2 - z_1 + d_2(t) - \frac{\partial \alpha_1}{\partial x_1} d_1(t) \\ &\quad - \frac{\partial \alpha_1}{\partial y} Sd_y(t)) \\ &= \dot{V}_1 + z_2 z_3 - k_2 z_2^2 - z_1 z_2 + z_2 d_2(t) \\ &\quad - \frac{\partial \alpha_1}{\partial x_1} z_2 d_1(t) - \frac{\partial \alpha_1}{\partial y} z_2 Sd_y(t)) \end{aligned} \tag{13}$$

To positive constants ε_2 and δ_2, notice that

$$\begin{aligned} z_2 d_2(t) &\leq \frac{z_2^2}{4\varepsilon_2^2} + \varepsilon_2^2 d_2^2(t) \\ \frac{\partial \alpha_1}{\partial x_1} z_2 d_1(t) &\leq \frac{(\frac{\partial \alpha_1}{\partial x_1})^2 z_2^2}{4\varepsilon_2^2} + \varepsilon_2^2 d_1^2(t) \\ \frac{\partial \alpha_1}{\partial y} z_2 Sd_y(t) &\leq \frac{(\frac{\partial \alpha_1}{\partial y})^2 z_1^2}{4\delta_2^2} + \delta_2^2 (Sd_y(t))^2 \end{aligned} \tag{14}$$

the derivative of V_2 can be written as

$$\begin{aligned}\dot{V}_2 &\leq -(k_1 - \frac{1}{4\varepsilon_1^2} - \frac{1}{4\delta_1^2}) z_1^2 + z_2 z_3 \\ &\quad - (k_2 - \frac{1}{4\varepsilon_2^2} - \frac{(\frac{\partial \alpha_1}{\partial x_1})^2}{4\varepsilon_2^2} - \frac{(\frac{\partial \alpha_1}{\partial y})^2}{4\delta_2^2}) z_2^2 + \varepsilon_1^2 d_1^2(t) \\ &\quad + \delta_1^2 (Sd_y(t))^2 + \varepsilon_2^2 d_2^2(t) + \varepsilon_2^2 d_1^2(t) + \delta_2^2 (Sd_y(t))^2 \end{aligned} \tag{15}$$

Step 3: The derivative of z_3 is

$$\begin{aligned}\dot{z}_3 &= \dot{x}_3 - y_r^{(3)} - \dot{\alpha}_2 \\ &= z_4 + \alpha_3 + d_3(t) - \frac{\partial \alpha_2}{\partial x_2} d_2(t) - \frac{\partial \alpha_2}{\partial x_1} d_1(t) \\ &\quad - \frac{\partial \alpha_2}{\partial y} Sd_y(t) - (\frac{\partial \alpha_2}{\partial x_2} x_3 + \frac{\partial \alpha_2}{\partial x_1} x_2) \\ &\quad - (\frac{\partial \alpha_2}{\partial y_r^{(1)}} y_r^{(2)} + \frac{\partial \alpha_2}{\partial y_r} y_r^{(1)}) \end{aligned} \tag{16}$$

The virtual control α_3 is chosen as

$$\begin{aligned}\alpha_3 &= -k_3 z_3 - z_2 + (\frac{\partial \alpha_2}{\partial x_2} x_3 + \frac{\partial \alpha_2}{\partial x_1} x_2) \\ &\quad + (\frac{\partial \alpha_2}{\partial y_r^{(1)}} y_r^{(2)} + \frac{\partial \alpha_2}{\partial y_r} y_r^{(1)}) \end{aligned} \tag{17}$$

where design parameter k_3 is a positive constant. The following Lyapunov function V_3 is considered.

$$V_3 = V_2 + \frac{1}{2} z_3^2 \tag{18}$$

Then the derivative of V_3 can be written as

$$\dot{V}_3 = \dot{V}_2 + z_3 \dot{z}_3 \tag{19}$$

To positive constants ε_3 and δ_3, notice that

$$\begin{aligned} z_3 d_3(t) &\leq \frac{z_3^2}{4\varepsilon_3^2} + \varepsilon_3^2 d_3^2(t) \\ \frac{\partial \alpha_2}{\partial x_2} z_3 d_2(t) &\leq \frac{(\frac{\partial \alpha_2}{\partial x_2})^2 z_3^2}{4\varepsilon_3^2} + \varepsilon_3^2 d_2^2(t) \\ \frac{\partial \alpha_2}{\partial x_1} z_3 d_1(t) &\leq \frac{(\frac{\partial \alpha_2}{\partial x_1})^2 z_3^2}{4\varepsilon_3^2} + \varepsilon_3^2 d_1^2(t) \\ \frac{\partial \alpha_2}{\partial y} z_3 Sd_y(t) &\leq \frac{(\frac{\partial \alpha_2}{\partial y})^2 z_3^2}{4\delta_3^2} + \delta_3^2 (Sd_y(t))^2 \end{aligned} \tag{20}$$

the derivative of V_3 can be written as

$$\begin{aligned}\dot{V}_3 &\leq -(k_1 - \frac{1}{4\varepsilon_1^2} - \frac{1}{4\delta_1^2}) z_1^2 + z_3 z_4 \\ &\quad - (k_2 - \frac{1}{4\varepsilon_2^2} - \frac{(\frac{\partial \alpha_1}{\partial x_1})^2}{4\varepsilon_2^2} - \frac{(\frac{\partial \alpha_1}{\partial y})^2}{4\delta_2^2}) z_2^2 \\ &\quad - (k_3 - \frac{1}{4\varepsilon_3^2} - \frac{(\frac{\partial \alpha_2}{\partial x_2})^2}{4\varepsilon_3^2} - \frac{(\frac{\partial \alpha_2}{\partial x_1})^2}{4\varepsilon_3^2} - \frac{(\frac{\partial \alpha_2}{\partial y})^2}{4\delta_3^2}) z_3^2 \\ &\quad + (\varepsilon_1^2 + \varepsilon_2^2) d_1^2(t) + \varepsilon_2^2 d_2^2(t) + (\delta_1^2 + \delta_2^2)(Sd_y(t))^2 \\ &\quad + \varepsilon_3^2 d_3^2(t) + \varepsilon_3^2 d_2^2(t) + \varepsilon_3^2 d_1^2(t) + \delta_3^2 (Sd_y(t))^2 \\ &= -\sum_{l=1}^{3} [k_l - \frac{1}{4\varepsilon_l^2}(1 + \sum_{m=1}^{l-1} (\frac{\partial \alpha_{l-1}}{\partial x_m})^2) \\ &\quad - \frac{1}{4\delta_l^2} (\frac{\partial \alpha_{l-1}}{\partial y})^2] z_l^2 + z_3 z_4 \\ &\quad + \sum_{m=1}^{3} \sum_{l=m}^{3} \varepsilon_l^2 d_m^2(t) + \sum_{l=1}^{3} \delta_l^2 (Sd_y(t))^2 \end{aligned} \tag{21}$$

Remark 2: The virtual control $\alpha_i (i = 1, 2, 3)$ shown in (5), (11) and (17) is linear function of variables x_j and $y_r^{(j-1)} (j = 1, \cdots, i)$. Thus we can get that $\frac{\partial \alpha_i}{\partial x_j}$ and $\frac{\partial \alpha_i}{\partial y_r^{j-1}}$ are all constants. This conclusion is correct for all virtual controls.

Step n: The derivative of z_n is

$$\begin{aligned}\dot{z}_n &= \dot{x}_n - y_r^{(n)} - \dot{\alpha}_{n-1} \\ &= bu + \theta f(x) + \varphi(x) + d_n(t) - y_r^{(n)} \\ &\quad - \sum_{m=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_m}(x_{m+1} + d_m(t)) - \frac{\partial \alpha_{n-1}}{\partial y} Sd_y(t) \\ &\quad - \sum_{m=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(m-1)}} y_r^{(m)} \end{aligned} \tag{22}$$

The control input u can be designed as

$$\begin{aligned} u &= \hat{e}\bar{u} \\ \bar{u} &= -k_n z_n - z_{n-1} - \hat{\theta} f(x) - \varphi(x) \\ &\quad - \sum_{m=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_m} x_{m+1} + \frac{\partial \alpha_{n-1}}{\partial y_r^{(m-1)}} y_r^{(m)}\right) + y_r^{(n)} \end{aligned} \tag{23}$$

where design parameter k_n is a positive constant. $\hat{\theta}$ is the estimation of θ. $\hat{e}$ is the estimation of $e = \frac{1}{b}$ and $\tilde{e} = e - \hat{e}$ is the estimation error. Notice that

$$bu = b\hat{e}\bar{u} = b(e - \tilde{e})\bar{u} = \bar{u} - b\tilde{e}\bar{u} \tag{24}$$

The derivative of $\frac{z_n^2}{2}$ can be written as

$$\begin{aligned} z_n \dot{z}_n &= -k_n z_n^2 - z_{n-1} z_n + \tilde{\theta} f(x) z_n - b\tilde{e}\bar{u} z_n \\ &\quad + z_n\left(d_n(t) - \sum_{m=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_m} d_m(t)\right) \\ &\quad - z_n \frac{\partial \alpha_{n-1}}{\partial y} Sd_y(t) \end{aligned} \tag{25}$$

The following Lyapunov function V_n is considered.

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{b}{2\gamma_e}\tilde{e}^2 + \frac{1}{2\gamma_\theta}\tilde{\theta}^2 \tag{26}$$

where γ_e and γ_θ are positive constants. $\tilde{e}$ and $\tilde{\theta}$ are estimation errors of e and θ, respectively. Then the derivative of V_n is

$$\begin{aligned} \dot{V}_n &\le -\sum_{l=1}^{n-1}\Big[k_l - \frac{1}{4\varepsilon_l^2}\Big(1 + \sum_{m=1}^{l-1}\Big(\frac{\partial \alpha_{l-1}}{\partial x_m}\Big)^2\Big) \\ &\quad - \frac{1}{4\delta_l^2}\Big(\frac{\partial \alpha_{l-1}}{\partial y}\Big)^2\Big] z_l^2 + \sum_{m=1}^{n-1}\sum_{l=m}^{n-1} \varepsilon_l^2 d_m^2(t) \\ &\quad + \sum_{l=1}^{n-1} \delta_l^2 (Sd_y(t))^2 - \frac{b}{\gamma_e}\tilde{e}\dot{\hat{e}} - \frac{1}{\gamma_\theta}\tilde{\theta}\dot{\hat{\theta}} \\ &\quad - k_n z_n^2 + \tilde{\theta} f(x) z_n - b\tilde{e}\bar{u} z_n \\ &\quad + z_n\Big(d_n(t) - \sum_{m=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_m} d_m(t)\Big) \\ &\quad - z_n \frac{\partial \alpha_{n-1}}{\partial y} Sd_y(t) \end{aligned} \tag{27}$$

To positive constants ε_n and δ_n, notice that

$$\begin{aligned} z_n d_n(t) &\le \frac{z_n^2}{4\varepsilon_n^2} + \varepsilon_n^2 d_n^2(t) \\ \frac{\partial \alpha_{n-1}}{\partial x_m} z_n d_m(t) &\le \frac{\big(\frac{\partial \alpha_{n-1}}{\partial x_m}\big)^2 z_n^2}{4\varepsilon_n^2} + \varepsilon_n^2 d_m^2(t) \\ m &= 1, 2, \cdots, n-1 \\ \frac{\partial \alpha_{n-1}}{\partial y} z_n Sd_y(t) &\le \frac{\big(\frac{\partial \alpha_{n-1}}{\partial y}\big)^2 z_n^2}{4\delta_n^2} + \delta_n^2 (Sd_y(t))^2 \end{aligned} \tag{28}$$

we have

$$\begin{aligned} \dot{V}_n &\le -\sum_{l=1}^{n}\Big[k_l - \frac{1}{4\varepsilon_l^2}\Big(1 + \sum_{m=1}^{l-1}\Big(\frac{\partial \alpha_{l-1}}{\partial x_m}\Big)^2\Big) \\ &\quad - \frac{1}{4\delta_l^2}\Big(\frac{\partial \alpha_{l-1}}{\partial y}\Big)^2\Big] z_l^2 + \sum_{m=1}^{n}\sum_{l=m}^{n} \varepsilon_l^2 d_m^2(t) \\ &\quad + \sum_{l=1}^{n} \delta_l^2 (Sd_y(t))^2 - \frac{b}{\gamma_e}\tilde{e}(\dot{\hat{e}} + \gamma_e \bar{u} z_n) \\ &\quad - \frac{1}{\gamma_\theta}\tilde{\theta}(\dot{\hat{\theta}} - \gamma_\theta f(x) z_n) \end{aligned} \tag{29}$$

The update laws can be chosen as

$$\begin{aligned} \dot{\hat{e}} &= -\gamma_e \bar{u} z_n - \gamma_e l_e(\hat{e} - e_0) \\ \dot{\hat{\theta}} &= \gamma_\theta f(x) z_n - \gamma_\theta l_\theta(\hat{\theta} - \theta_0) \end{aligned} \tag{30}$$

where $l_e > 0$, $l_\theta > 0$ and e_0, θ_0 are design parameters. Notice that

$$\begin{aligned} l_e \tilde{e}(\hat{e} - e_0) &\le l_e(e - \hat{e})(\hat{e} - e_0) + \frac{1}{2} l_e(\hat{e} - e_0)^2 \\ &= \frac{1}{2} l_e(\hat{e} - e_0)[(\hat{e} - e_0) + 2((e - \hat{e}))] \\ &= \frac{1}{2} l_e(\hat{e} - e_0)[2e - 2\hat{e} + \hat{e} - e_0] \\ &= \frac{1}{2} l_e(\hat{e} - e_0)[2e - \hat{e} - e_0] \\ &= \frac{1}{2} l_e[e - e_0 + e - \hat{e}][e - e_0 - (e - \hat{e})] \\ &= \frac{1}{2} l_e[(e - e_0)^2 - (e - \hat{e})^2] \\ &= -\frac{l_e}{2}\tilde{e}^2 + \frac{1}{2} l_e(e - e_0)^2 \end{aligned} \tag{31}$$

and

$$l_\theta \tilde{\theta}(\hat{\theta} - \theta_0) \le -\frac{l_\theta}{2}\tilde{\theta}^2 + \frac{1}{2} l_\theta(\theta - \theta_0)^2$$

and with (30), we can obtain

$$\begin{aligned} \dot{V}_n &\le -\sum_{l=1}^{n}\Big[k_l - \frac{1}{4\varepsilon_l^2}\Big(1 + \sum_{m=1}^{l-1}\Big(\frac{\partial \alpha_{l-1}}{\partial x_m}\Big)^2\Big) \\ &\quad - \frac{1}{4\delta_l^2}\Big(\frac{\partial \alpha_{l-1}}{\partial y}\Big)^2\Big] z_l^2 + \sum_{m=1}^{n}\sum_{l=m}^{n} \varepsilon_l^2 d_m^2(t) \\ &\quad + \sum_{l=1}^{n} \delta_l^2 (Sd_y(t))^2 - \frac{bl_e}{2}\tilde{e}^2 - \frac{l_\theta}{2}\tilde{\theta}^2 \\ &\quad + \frac{b}{2} l_e(e - e_0)^2 + \frac{1}{2} l_\theta(\theta - \theta_0)^2 \end{aligned} \tag{32}$$

IV. STABILITY ANALYSIS

The following theorem can be established to show the boundedness of all signals in the closed loop system under the proposed control scheme.

Theorem 1: Consider the class of uncertain system shown in (1), with external disturbance $d_i(t)$ in state equations, and output disturbance $d_y(t)$ in output y. Under Assumption 1-3 and the controlling of the proposed control scheme shown in (23) and (30) the following results can be achieved:

- All the signals in the closed-loop system are globally bounded.
- The tracking error $|y - y_r|$ satisfies

$$lim_{t\to\infty} sup|y - y_r| \le \sqrt{\frac{2\hbar_2}{\hbar_1}M}$$

where

$$\begin{aligned}
\hbar_1 &= min\{k^*, \frac{bl_e}{2}, \frac{l_\theta}{2}\} \\
\hbar_2 &= max\{\frac{1}{2}, \frac{b}{2\gamma_e}, \frac{1}{2\gamma_\theta}\} \\
k^* &= min\Big\{k_l - \frac{1}{4\varepsilon_l^2}(1 + \sum_{m=1}^{l-1}(\frac{\partial\alpha_{l-1}}{\partial x_m})^2 \\
&\quad - \frac{1}{4\delta_l^2}(\frac{\partial\alpha_{l-1}}{\partial y})^2\Big\} \quad l = 1, \cdots, n \\
M &= M_1 + M_2 \\
M_1 &= \sum_{m=1}^{n}\sum_{l=m}^{n}\varepsilon_l^2 D_m^2 + \sum_{l=1}^{n}\delta_l^2 D_y^2 \\
M_2 &= \frac{b}{2}l_e(e - e_0)^2 + \frac{1}{2}l_\theta(\theta - \theta_0)^2
\end{aligned}$$

Proof: Firstly, the vector χ is defined as

$$\chi^T = (z_1, z_2, \cdots, z_n, \tilde{e}, \tilde{\theta}) \tag{33}$$

From (32), we have

$$\begin{aligned}
\dot{V}_n \le& -\sum_{l=1}^{n}[k_l - \frac{1}{4\varepsilon_l^2}(1 + \sum_{m=1}^{l-1}(\frac{\partial\alpha_{l-1}}{\partial x_m})^2) \\
&- \frac{1}{4\delta_l^2}(\frac{\partial\alpha_{l-1}}{\partial y})^2]z_l^2 + \sum_{m=1}^{n}\sum_{l=m}^{n}\varepsilon_l^2 D_m^2(t) \\
&+ \sum_{l=1}^{n}\delta_l^2 D_y^2 - \frac{bl_e}{2}\tilde{e}^2 - \frac{l_\theta}{2}\tilde{\theta}^2 \\
&+ \frac{b}{2}l_e(e - e_0)^2 + \frac{1}{2}l_\theta(\theta - \theta_0)^2 \\
=& -\sum_{l=1}^{n}[k_l - \frac{1}{4\varepsilon_l^2}(1 + \sum_{m=1}^{l-1}(\frac{\partial\alpha_{l-1}}{\partial x_m})^2) \\
&- \frac{1}{4\delta_l^2}(\frac{\partial\alpha_{l-1}}{\partial y})^2]z_l^2 - \frac{bl_e}{2}\tilde{e}^2 - \frac{l_\theta}{2}\tilde{\theta}^2 \\
&+ M
\end{aligned} \tag{34}$$

Clearly, the design parameter k_l can be chosen to satisfy

$$k_l - \frac{1}{4\varepsilon_l^2}(1 + \sum_{m=1}^{l-1}(\frac{\partial\alpha_{l-1}}{\partial x_m})^2) - \frac{1}{4\delta_l^2}(\frac{\partial\alpha_{l-1}}{\partial y})^2 > 0 \tag{35}$$

Remark 3: Because partial derivative $\frac{\partial\alpha_{l-1}}{\partial x_m}$, $\frac{\partial\alpha_{l-1}}{\partial y}$ depend on $k_1, \cdots, k_{l-1}$, we can set k_l big enough to keep the inequality (35) hold. Thus $k^* > 0$ exists.

Then the derivative of V_n can be rewritten as

$$\begin{aligned}
\dot{V}_n &\le -k^*\sum_{l=1}^{n}z_l^2 - \frac{bl_e}{2}\tilde{e}^2 - \frac{l_\theta}{2}\tilde{\theta}^2 + M \\
&\le -\hbar_1 V_0 + M
\end{aligned} \tag{36}$$

where $V_0 = \chi^T\chi$. With (26), the following inequality can be obtained.

$$V_n \le \hbar_2 V_0 \tag{37}$$

From (37), we know that $V_n(t)$ is bounded. Then signals $z_1, z_2, \cdots, z_n, \tilde{e}, \tilde{\theta}$ are all bounded. Furthermore, virtual control α_i and input u are bounded. Thus all the signals in the closed-loop system are globally bounded.

From (36) and (37), the derivative of V_n satisfy

$$\dot{V}_n \le -\frac{\hbar_1}{\hbar_2}V_n + M \tag{38}$$

By solving (38), we can get

$$\begin{aligned}
V_n(t) &\le V_n(0)e^{-\frac{\hbar_1}{\hbar_2}t} + \frac{\hbar_2}{\hbar_1}M(1 - e^{-\frac{\hbar_1}{\hbar_2}t}) \\
&\le (V_n(0) - \frac{\hbar_2}{\hbar_1}M)e^{-\frac{\hbar_1}{\hbar_2}t} + \frac{\hbar_2}{\hbar_1}M
\end{aligned} \tag{39}$$

So we have

$$lim_{t\to\infty}V_n(t) = \frac{\hbar_2}{\hbar_1}M \tag{40}$$

Because of

$$(y - y_r)^2 = z_1^2 \le 2V_n \tag{41}$$

the upper bound of $|y - y_r|$ can be established as

$$lim_{t\to\infty} sup|y - y_r| \le \sqrt{\frac{2\hbar_2}{\hbar_1}M} \tag{42}$$

■

Remark 4: To guarantee the stability of system (1), the design parameters should be satisfied the following inequalities:

$$\begin{aligned}
&k_1 - \frac{1}{4\varepsilon_1^2} - \frac{1}{4\delta_1^2} \ge 0 \\
&k_2 - \frac{1}{4\varepsilon_2^2} - \frac{(\frac{\partial\alpha_1}{\partial x_1})^2}{4\varepsilon_2^2} - \frac{(\frac{\partial\alpha_1}{\partial y})^2}{4\delta_2^2} \\
&\vdots \\
&k_n - \frac{1}{4\varepsilon_n^2}(1 + \sum_{m=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial x_m})^2) - \frac{1}{4\delta_n^2}(\frac{\partial\alpha_{n-1}}{\partial y})^2
\end{aligned} \tag{43}$$

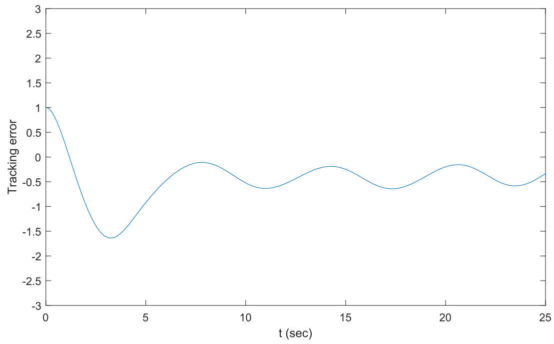

FIGURE 1. Tracking.

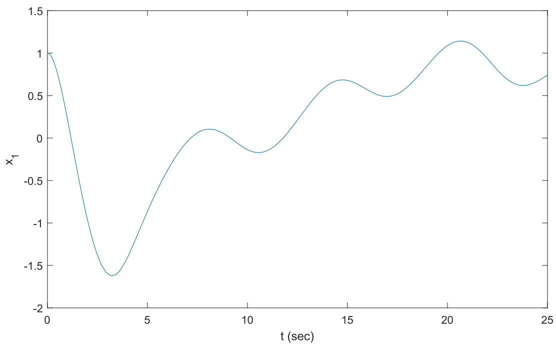

FIGURE 2. State x_2.

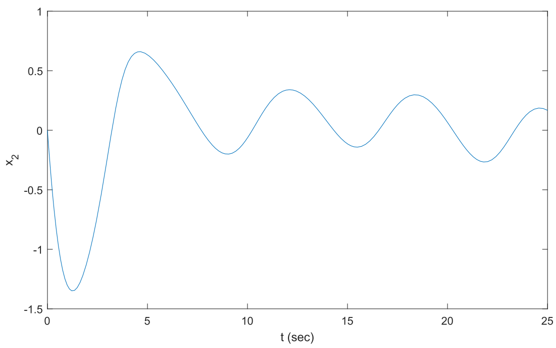

FIGURE 3. Signal v.

V. SIMULATION STUDIES

We now apply the proposed control scheme to the following 2nd-order system.

$$\begin{aligned} \dot{x}_1 &= x_2 + d_1(t) \\ \dot{x}_2 &= u + f(x)\theta + d_2(t) \\ y &= x_1 + d_y(t) \end{aligned} \tag{44}$$

where x_1, x_2 are system states, y is the output signal. u is the input signal. $d_1(t), d_2(t)$ are disturbances in state equations and, $d_y(t)$ represents the output disturbance. $f(x) = 0.2cos(x_2)$ is a known function. θ is an unknown parameter.

(1) Case 1: Firstly, we consider that there are no disturbances in state equations. Namely $d_1(t) = d_2(t) = 0$. In simulation, we take $d_y(t) = sin(0.1t)cos(0.1t)$, and $\theta = 0.2$. The design parameters can be chosen as: $k_1 = 1.2, k_2 = 2, \delta_1 = 0.5, \delta_2 = 0.5, \gamma_\theta = 10, l_\theta = 0.01, \theta_0 = 0.1$.

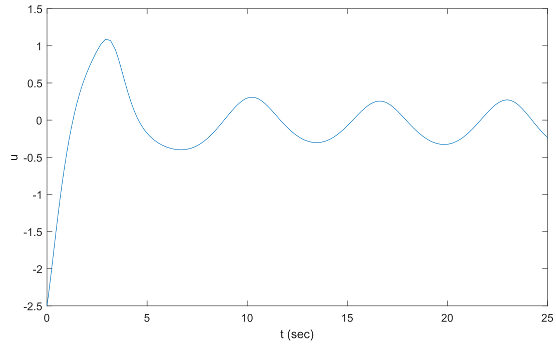

FIGURE 4. Input u.

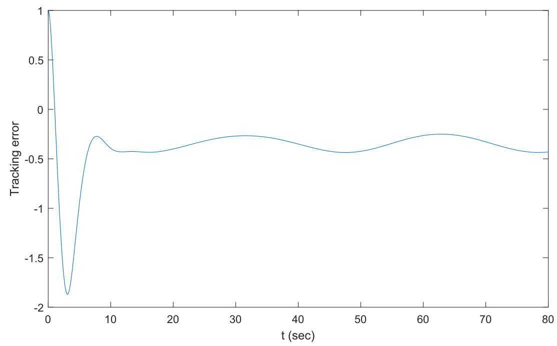

FIGURE 5. Tracking.

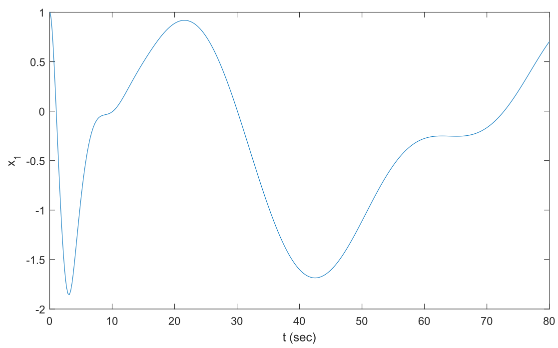

FIGURE 6. State x_2.

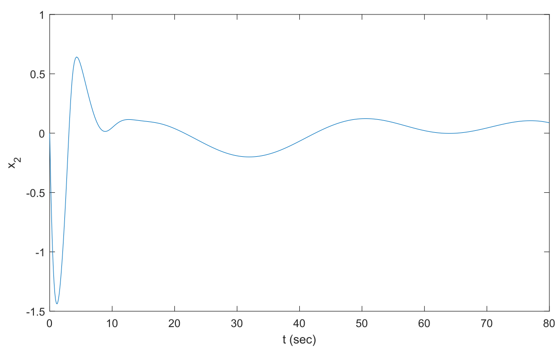

FIGURE 7. Signal v.

The reference signal $y_r = sin(0.1t)$. The initial values are taken as: $x_1(0) = 1, x_2(0) = 0, \hat{\theta}(0) = 0$.

The simulation results are shown in Fig.1-4. Fig.1 represents tracking error and the states x_1, x_2 are shown

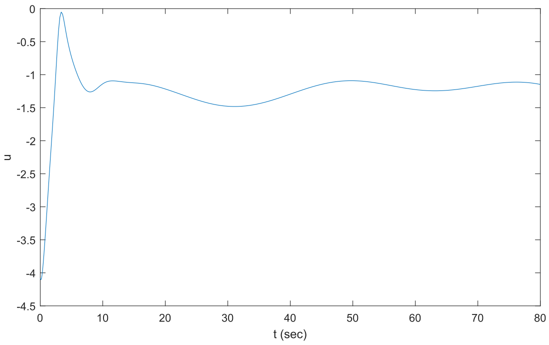

FIGURE 8. **Input *u*.**

in Fig.2 and Fig.3, respectively. Fig.4 shows the input signal u.

(2) Case 2: In this case, we consider that there are disturbances in state equations. In simulation, we take $d_1(t) = 0.1sint, d_2(t) = 0.1cost$, $d_y(t) = sin(0.1t)cos(0.1t)$, and $\theta = 0.2$. The design parameters can be chosen as: $k_1 = 1, k_2 = 1.6, \delta_1 = 0.7, \delta_2 = 0.7, \varepsilon_1 = 0.7, \varepsilon_2 = 0.7$, $\gamma_\theta = 10, l_\theta = 0.01, \theta_0 = 0.1$. The reference signal $y_r = sin(0.1t)$. The initial values are taken as: $x_1(0) = 1$, $x_2(0) = 0, \hat{\theta}(0) = 0$.

The simulation results are shown in Fig.5-8. Fig.5 represents tracking error and the states x_1, x_2 are shown in Fig.6 and Fig.7, respectively. Fig.8 shows the input signal u.

Clearly, we can get that all signals of the systems are bounded under the controlling of the proposed control scheme to the above two cases.

VI. CONCLUSION

In this paper, the control problem is investigated for a class of nonlinear system with unknown external disturbance including disturbance in every state equation and output equation. The uncertainties caused by disturbance in state equation and output equation are all separated two different parts by using Young's inequality in every step. Then they will be compensated by selecting appropriate design parameters. Finally simulation studies are used to verify the effectiveness of the proposed control scheme.

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JIANPING CAI received the Ph.D. degree from Zhejiang University, in 2014. He is currently a Professor with Zhejiang University of Water Resources and Electric Power. His main research interests include nonlinear systems and adaptive control.

WEI WANG is currently a Lecturer with the School of Mathematics and Physics, Hebei GEO University. Her research interests include applied mathematics and nonlinear systems.

DONG GUO received the dual B.S. degrees in electronic and communication engineering from Hangzhou Dianzi University, in 2010 and 2013, where he is currently pursuing the Ph.D. degree. He works with Zhejiang University of Water Resources and Electric Power. His research interests include electronic technology, sensing technology, and automatic control.