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Robustness Assessments of Urban Rail Transit Networks Based on Network Utilization

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ABSTRACT Urban rail transit has become an important traffic mode in large cities and has brought great conveniences to urban residents, however traffic accidents often occur in daily operations, therefore the safety and robustness must be paid more attention. In this paper, the network utilization of urban rail transit networks (*URTNs*) is expressed by the travel time difference between user equilibrium and optimal system, and the travel time difference is considered as the assessment indicator of robustness. Meanwhile, Shanghai metro network is taken as the example to analyze the robustness of *URTNs* under different disturbances, such as node attack, link-demand attack and overall-demand attack. The results show that *URTNs* have better robustness subjected to the random link-demand attack than node attack, and the robustness decreases with the increase of the attack intensity when the network suffers overall-demand attack. Moreover, we find that the critical stations and links can be identified, and *URTNs* can be paralyzed when the attack intensity exceeds the critical threshold.

INDEX TERMS Robustness, user equilibrium, optimal system, urban rail transit networks.

I. INTRODUCTION

With the expansion of the scale of urban rail transit networks (*URTNs*), people pay more and more attention to the research on the robustness and safety of network. Natural disasters, human errors, equipment failures and other reasons will have an impact on the *URTNs*, and even damage people's property and lives. In the transportation network, the changes in passenger flow caused by different disturbances can intuitively reflect the changes in the transportation performance of the transportation system. This paper mainly analyzes the robustness of *URTNs* from the perspective of network resource utilization.

Recent years, more and more researchers focused on characteristics and robustness from the structure and function of *URTNs*, and the corresponding researches directly or indirectly improved the security and robustness of *URTNs*. Guidotti *et al.* proposed a probabilistic methodology to quantify the network reliability based on topological network and complex network theory and this method was applied to analyze a highway transportation network reliability [1]. Jing *et al.* proposed the mean-excess criticality probability as a risk measure to identify the critical stations in metro network considering the routing redundancy and the result show that the critical stations are not necessarily transferred stations or those with a high degree, and the important stations based on betweenness, passenger flow and network efficiency are not necessarily critical for the network redundancy [2]. Zhang and Wang studied the functional vulnerability of *URTNs* based on the moving-block technology which resolves the control problems of safe distance between the forward and the following trains and it can improve the reliability of *URTNs* [3]. Zhou *et al.* developed a hybrid cascading failure model to analyze the robustness of interdependent networks considering loads, group effects, and coupling preferences and the mean size of dependency groups is the key to the robustness of interdependent networks [4]. Wandelt *et al.* systematically evaluated the robustness of transportation networks by analyzing community structure and identified the important roles of intercommunity nodes/edges for robustness improvements [5]. Hong *et al.* introduced a vulnerability analysis method to investigate the complementary bus and subway systems in Wuhan [6] and proposed three types of accessibility metrics based on departure time to investigate the vulnerability of the integrated metro and high-speed rail system in China [7].

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Yu *et al.* [8] and Wang *et al.* [9] constructed the super network model of *URTNs* and proposed a multi-source least transferred algorithm to study the robustness of the network under different attacks. And the results show that the protection of traffic hub stations and trunk lines could enhance the anti-fault ability of the super metro network.

Meanwhile, many scholars also studied the network characteristics from the perspective of passenger flow. Xu *et al.* analyzed the layered multi-center structure formed by the passenger flow data of the Beijing subway network and summarized the urban human mobility pattern within a large subway network [10]. Fan *et al.* combined the passenger flow dynamics and network topological structure to build a temporal network of the Shanghai metro network and used the linear threshold model to analyze the impact of cascading failure on network robustness, and the results show that the large volume of passenger flow can increase the impact of failure on the Shanghai temporal subway network robustness [11]. Yuan *et al.* developed effective flow control strategies by considering the time-dependent passenger demands to reduce the total waiting time in *URTNs* [12]. Zhang *et al.* build a weighted network model based on passenger flow and path distance to identify the critical lines of *URTNs* and provided suggestions for recovery strategies [13]. Liu *et al.* proposed a passenger flow delay reallocation algorithm to analyze the impacts of different station interruption time and the number of passenger flow redistribution on the vulnerability of *URTNs* and the result show that the interruption that lasts more than 30 minutes exerts a great impact on the traffic efficiency of the *URTN*s and exhibits a wide scope [14]. Some improved coupled mapping lattice methods are used to discuss the features of cascading failure of *URTNs* [15]–[17].

Moreover, many studies used the analytical method of traffic network flow to study the dynamic characteristics of the traffic network. Among them, the more classic ones are user equilibrium and system optimization models [18], [19]. The user equilibrium model is to find the path with the shortest travel time from the perspective of each user. The optimal system model may harm the interests of a small number of users to achieve the shortest total travel time for all users. And these different models will form different network flow states according to different passenger flow distribution mechanisms. The analysis of traffic network performance under different network flow states can combine with the dynamic fluctuation of passenger flow for more accurate traffic network analysis. Çolak *et al.* found that the congested cities benefit more from incorporating social good considerations into routing behavior by comparing the user equilibrium and system optimal [20]. Sumalee and Xu analyzed marginal cost pricing in traffic networks and compared network performance under different toll regimes [21]. Almotahari and Yazici paid attention to the passenger flow and developed the link criticality index as a measure to identify critical components of traffic networks using the convex combinations solution algorithm [22]. He *et al.* comprehensively analyzed the robustness of intermodal transportation by combining user equilibrium and optimal system methods [23]. Nogal *et al.* evaluated the resilience of *URTNs* by analyzing the time changes when the network reaches a new equilibrium state according to a dynamic restricted equilibrium model [24]. However, the influence of different passenger flow distributions on network robustness has rarely been examined directly, and this paper mainly analyzes the network robustness which can be reflected by the different passenger flow states under different disturbances.

In this paper, the robustness of *URTNs* is investigated by comparing the user equilibrium and optimal system, and Shanghai metro network is considered as an example to indicate the feasibility and effectiveness of the research scheme. The robustness of the network means the ability of the network to maintain stable performance when it is subjected to external disturbances and the robustness of the network is mainly reflected in the perspective of resource utilization in this paper. The remainder of this paper is organized as follows. Section 2 introduces the mechanism and solution algorithm of user equilibrium and optimal system. Section 3 presents an assessment indicator and research model on network robustness. Section 4 analyzes the robustness of *URTNs* under different attacks and conclusions are given in Section 5.

II. TRAFFIC NETWORK FLOW

In this section, we construct the topological structure graph of *URTNs* and set the capacity of each link by integrating complex network theory and graph theory. According to the characteristics of the rail transit system, we build the Space L model to analyze the network performance. Among them, the node represents the station of the transportation system, and the edge represents that there is a track directly connected between the two stations. Then, different network flow configurations in the network can be obtained according to the different user selection mechanisms. In this article, different traffic models will form different passenger flow configuration states according to different configuration principles, namely system optimal and user equilibrium. Therefore, the robustness of network can be assessed according to different network flow states.

A. PRIVATE MARGINAL COST AND MARGINAL SOCIAL COST

In the public transportation service, the entering of an additional user x will increase the additional user's own cost and also affect the normal travel of the users being on the link l. And the incremental cost borne by the additional user x is the private marginal cost of the user x, which is defined as $PMC(x)$, and the incremental cost caused by the additional user x to the other users $(0, x)$ on the link is the marginal external cost, which is defined as $MEC(x)$. The sum of the private cost marginal and marginal external cost of these x users is defined as the marginal social cost $MSC(x)$. This part explains the cost calculation methods of different passenger flow distribution methods from the perspective of economics.

For simplicity, we assume that the cost of a link for user x is equal to the travel time, $c(x) = t(x)$, where $t(x)$ is the travel time of user x. The related calculation formulas and relations are given as follows,

$$PMC(x) = \mathrm{d}\left[\int_0^x c(\omega)\,\mathrm{d}\omega\right]/\mathrm{d}x \tag{1}$$

$$MEC(x) = \int_0^x [c(x) - c(\omega)]\,\mathrm{d}\omega \tag{2}$$

$$MSC(x) = PMC(x) + MEC(x) \tag{3}$$

B. OPTIMAL SYSTEM

The optimal system is a flow configuration in which the average travel time of the network is minimized through the cooperation of all users in the network. The average travel time under the optimal system can be regarded as the shortest average travel time in theory and used as a benchmark when calculating network utilization. The optimal system problem can be formulated as a convex optimization problem,

$$\begin{aligned} &\min \sum_{l\in L} x_l \cdot t_l(x_l) \\ &\text{s.t.} \sum_k f_k^{st} = f^{st} \\ &\quad f_k^{st} \geq 0, \quad \forall k \\ &\quad x_l = \sum_s \sum_t \sum_k f_k^{st} \delta_{l,k}^{st} \end{aligned} \tag{4}$$

where L is the set of all link l in a traffic network, $t_l(x_l)$ is the link characteristic function which describes the relationship between the travel time $t_l(x_l)$ and the passenger flow x_l on link l. In this paper, the BPR function [21] is considered as the link characteristic function of *URTNs*. f_k^{st} refers to the passenger flow between the original node s and destination node t on the path k, f^{st} refers to the total flow between the original node s and destination node t, and $\delta_{a,k}^{st} = 1$ when the link l lies on path k. And the shape coefficients of the link characteristic function $\alpha = 0.15$, $\beta = 8$ are set to discuss the relationship between link travel time and link flow of *URTNs* in Eq. (5).

$$t_l(x_l) = t_{0l}\left(1 + \alpha\left(\frac{x_l}{c_l}\right)^\beta\right) \tag{5}$$

The optimal objective function of the optimal system is the sum of the marginal social costs on each link of the network. When the state of the optimal system is reached, no user can unilaterally change the travel path to reduce the total network travel time. Therefore, it is reasonable to use the average travel time under the optimal system configuration of *URTNs* as the calculation benchmark for network utilization.

C. USER EQUILIBRIUM

Each user in the network independently chooses the path with the smallest travel time to form a flow configuration called user equilibrium. The flow distribution under the user balance network can be regarded as the flow distribution state of the natural network without external intervention. The convex programming for the user equilibrium problem can be formulated as follow,

$$\begin{aligned} &\min \sum_{l\in L} \int_0^{x_l} t_l(x)\,\mathrm{d}x \\ &\text{s.t. Constraints in Eq. (4)} \end{aligned} \tag{6}$$

The objective function of user equilibrium is the sum of private marginal costs on each link in the network. When the state of user equilibrium is reached, the travel time of all used paths between the same OD pair is equal and minimum, and the travel time of all unused paths is greater than or equal to the travel time of the used path.

D. SOLVING ALGORITHM

Frank-Wolf algorithm [20] is to explore the feasible direction for solving the equilibrium problem and includes two main aspects: determining the exploration direction and determining the moving step. Therefore, the Frank-Wolf algorithm is adopted to solve the user equilibrium problem of *URTNs* in this paper, and the solving algorithm is summarized as follows,

Algorithm 1 Frank-Wolf Algorithm

Input: link characteristic function $t_l\left(x_l^n\right)$
Output: link flow $\left\{x_l^n\right\}$
1: Initialization $\left\{x_l^n\right\} \leftarrow \{0\}$, $t_l^n = t_l\left(x_l^n\right), \forall\, l \in L$, n=0
2: **while** convergence test **do**
3: n = n + 1
4: direction search $\left\{y_l^n\right\}$ according to all-or-nothing load
5: linear search λ_n
6: move $x_l^{n+1} = x_l^n + \lambda_n\left(y_l^n - x_l^n\right)$
7: **end while**

In this solving algorithm, n is the iteration number, l is the link, x_l^n is the passenger flow on link l at the iteration number n, t_l^n is the travel time passing through the link l at the iteration number n, y_l^n represents another passenger flow on link l at the iteration number n, which indicates the forward direction of iteration under the all-or-nothing load distribution rule. Meanwhile, λ_n is the optimal move step and $\lambda_n \in [0, 1]$ by the linear search.

According to the relationship between the user equilibrium and optimal system, the Frank-Wolf algorithm can be also used to solve the optimal system problem by adjusting the link characteristic function $t_l(x_l)$ in Eq. (6) to $t_l'(x_l)$ which is defined as follows,

$$t_l'(x_l) = t_l(x_l) + x_l\frac{\mathrm{d}t_l(x_l)}{\mathrm{d}x_l} \tag{7}$$

III. ASSESSMENT MODEL

User equilibrium is a balanced state caused by individual users seeking to maximize their interests, and the optimal system is a balanced state that achieves the best system efficiency by sacrificing the interests of some individual users.

Under normal circumstances, the balance of the optimal system cannot be achieved unless all users cooperate and follow the command of the dispatching center. Therefore, we take the average travel time of the optimal system as the evaluation benchmark of *URTNs*, which means that the network resources are optimally used and the network benefits are maximized. Meanwhile, traffic planners and managers adjust the network structure and operation plan to make the average travel time of user equilibrium close to the average travel time of the optimal system. However, when the network is disturbed, the difference between these two aspects will change and this model is to analyze the differences in the average travel time between the two configurations when the network is disturbed.

In the model, presented the assumptions of the simulation. Assumption (i): The relationship between link travel time and link flow can be expressed by the BPR function. Assumption (ii): When the transportation system changes, users can compare different paths to form the user equilibrium state. Assumption (iii): The nodes in the network will not return to normal state after being attacked.

A. ROBUSTNESS INDICATOR

Here, we discuss the differences about the average travel time between user equilibrium and optimal system, and we know that the average travel time of the *UTRNs* is the smallest under the optimal system model, therefore the difference of the average travel time between the two models is presented by,

$$\delta = T_{UE} - T_{SO} \tag{8}$$

where T_{UE} and T_{SO} are the average travel time under the user equilibrium state and optimal system state respectively.

This difference combines two different passenger flow configurations of user equilibrium and optimal system, which can reflect the dynamic selection of passenger flow when the network changes and can analyze changes in network performance based on passenger flow fluctuations. This indicator can analyze the changes in the robustness of the network from the perspective of whether the network can make better use of network resources when the network is disturbed. When the difference increases due to disturbance, the network utilization of *URTNs* will change accordingly. Therefore, the difference can be used to analyze the robustness of the network from the perspective of resource utilization. When the network is under a minor attack, the larger the difference of average travel time is, the weaker the network robustness is, and vice versa.

B. NETWORK DISTURBANCES

UTRNs are often affected by sudden accidents in the daily operations, and this paper mainly considers three different disturbances, including node failure, the increase of passenger flow on links and the increase of overall passenger flow, which are called the node attack, link-demand attack and overall-demand attack respectively. These three different attacks simulate scenarios that may change the network from different perspectives, analyze the impact of different types of disturbances on the choice of passenger flow paths in the network, and then analyze the robustness of *UTRNs*.

Node attacks mean that the node is attacked and removed from the network, and the attacked node loses transportation ability. Meanwhile, the removal of the failed node may produce other isolated nodes, which will also lose transportation functions and be removed accordingly. Moreover, when the node is attacked and removed from the network, the passenger flow of the failed node will be allocated to their neighbor nodes. The link-demand attack refers to a sudden increase of passenger flow on a certain link, and the influence to the network caused by the local demand changes can be discussed. To analyze the impact of the increase of passenger flow on the network, we set different link-demand attack intensities as 0.1, 0.2, 0.3, 0.4, and 0.5. The overall-demand attack refers to a sudden increase of the passenger flow on all the OD pairs of the whole network, and the influence to the network caused by the global demand changes can be also analyzed. The overall-demand attack analyzes the changes in network performance when the total passenger flow of the network changes. Similarly, the intensity of the overall-demand attack is also used to analyze the robustness changes of the network. Three different attacks observe the robustness of the network in network utilization from many different aspects.

FIGURE 1. Shanghai metro topology map.

IV. RESULT ANALYSIS

In this section, the model described in Section 3 is applied to assess the robustness of *URTNs*, and Shanghai metro network is taken as an example to illustrate the feasibility and effectiveness of the proposed model. Fig. 1 shows the topological map of the Shanghai metro in 2016, including 288 stations and 640 edges. The Shanghai subway topology data and corresponding OD demand data from 07:30 to 08:30 on July 1, 2016, come from the Shanghai operating company and we can also get the free travel time and capacity of each link according to the actual operation timetable. Therefore, we can combine Eq. (5) to calculate the average travel time of the network under different passenger flow configurations.

This paper uses the difference between the user equilibrium and the optimal average travel time of the network to quantify the robustness. When the network is disturbed, the difference between the average time under the system optimal and the average time under user equilibrium is small, indicating that the network can ensure better network utilization under the disturbance, and the network exhibits high robustness. On the contrary, the robustness of the network is poor. By comparing the difference of the average travel time between user equilibrium and optimal system, we analyze the characteristic changes on robustness of *URTNs* under different attacks.

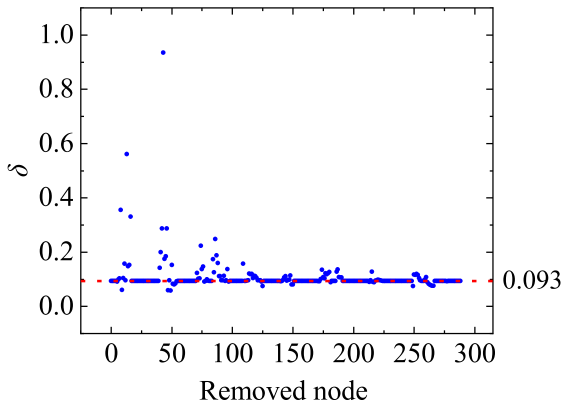

FIGURE 2. Time differences under node attacks.

A. ROBUSTNESS ANALYSIS UNDER THE SINGLE COMPONENT FAILURE

In this subsection, the node attack and the link-demand attack are adopted to study the robustness of *URTNs*, and the difference of the average travel time between user equilibrium and optimal system is taken as the assessment indicator to analyze the characteristic changes of *URTNs*. Fig. 2 describes the difference of the average travel time between user equilibrium and optimal system under the node attacks, and each point in the figure represents the result of a node attack. And we discover that the difference of the average travel time under different node attacks is smaller than 1 minute, and the most difference is 0.935 when the node 43 is attacked. However, most of the differences are smaller than 0.2, and several node attacks can generate small differences which are smaller than the original difference 0.093 (the red dotted line), and the smallest difference is 0.058 when node 49 is removed from the network. In other words, the removal of most nodes has little effect on network utilization of *URNTs*, and only some nodes will reduce the network utilization. Therefore, the results show that *URTNs* have the better robustness subjected to random node attacks.

Fig. 3 shows the difference of the average travel time between user equilibrium and optimal system under the link-demand attacks and different point types are used to indicate the results of different attack intensities. We find that the difference of the average travel time under different link-demand attacks is very small, and the number of differences larger than the original difference (0.093) becomes more and more with the increase of the link-demand attack intensity. Meanwhile, we discover that the largest differences are 0.1065, 0.1315, 0.1375, 0.1375, 0.2030 for different link-demand attack intensities 0.1, 0.2, 0.3, 0.4, 0.5 respectively, and the corresponding attacked links are $l288$, $l288$, $l290$, $l209$, $l113$. However, the smallest differences are 0.0914, 0.0882, 0.0820, 0.0737, 0.0639 for different link-demand attack intensities 0.1, 0.2, 0.3, 0.4, 0.5 respectively, and the corresponding attacked links are the same link $l123$. Moreover, Fig. 3 also illustrates that *URTNs* have the better robustness subjected to random link-demand attacks.

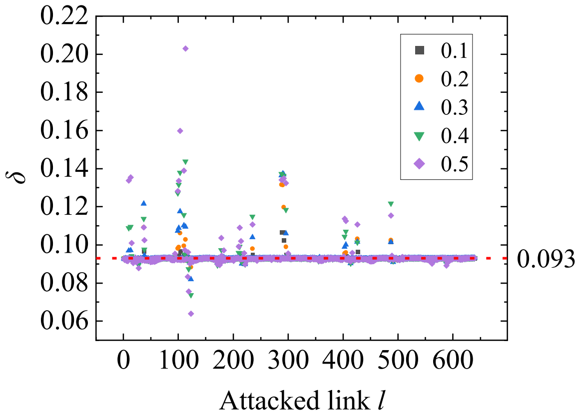

FIGURE 3. Time differences under different intensity link-demand attacks.

Compared Fig. 2 with Fig. 3, we can declare that the overall difference resulted from node attacks is larger than the overall difference generated by the link-demand attacks, therefore we can declare that *URTNs* have better robustness when suffering the random link-demand attacks than random node attacks. Furthermore, the results show *URTNs* have a large network utilization and possess better robustness when they suffer random attacks on a single component.

B. ROBUSTNESS ANALYSIS UNDER OVERALL-DEMAND ATTACKS

In this subsection, the overall-demand attack is involved to analyze the characteristic changes in the robustness of *URTNs* based on the attack intensity belonging to [−0.2, 1.4]. Fig. 4 shows that the difference of the average travel time between user equilibrium and optimal system under the overall-demand attack, and it indicates that when the attack intensity is smaller than 1, the difference of the average travel time between two models increases with the increase of the attack intensity, and the most difference is 0.7 when the attack intensity equals 1, and this phenomenon illustrates that the robustness of the *URTNs* will decrease as the overall passenger flow increases within a certain range. However, we find that when the attack intensity is larger than 1, the difference becomes smaller and smaller with the increase of the attack intensity, this phenomenon indicates that the passenger flow is very congested in the overall network, even has exceeded

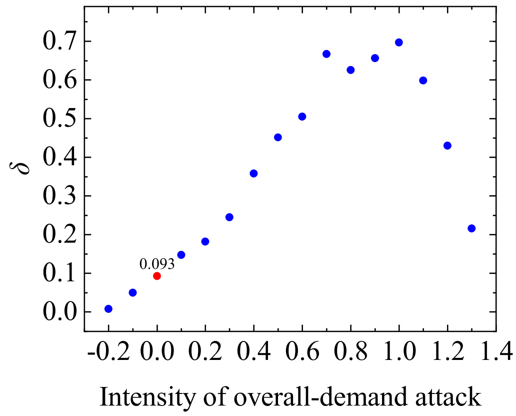

FIGURE 4. Time differences under different intensity overall-demand attacks.

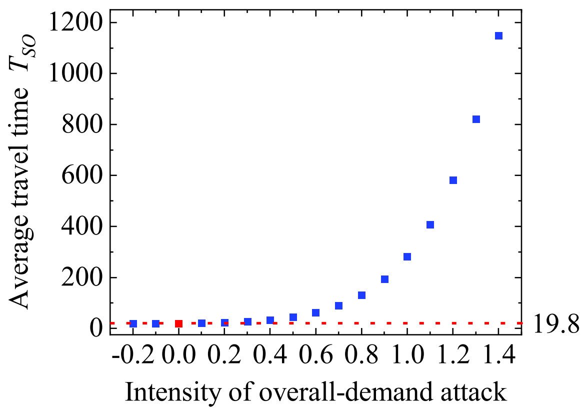

FIGURE 5. Average travel time under different attack intensities.

the network transportation ability, therefore the difference becomes smaller and smaller with the increase of the attack intensity.

Meanwhile, we calculate the average travel time of the optimal system under different attack intensities of overall-demand attacks shown in Fig. 5. The intensity of -0.2 means that the overall network demand, that is, the total passenger flow, is 0.8 times the initial network demand and when the attack intensity is 0, which is the initial demand, the average travel time in the network is 19.8 minutes, and we find that the average travel time becomes larger and larger with the increase of the attack intensity, which indicates that the robustness of *URTNs* to the overall-demand attack will decrease as the attack intensity increases. Moreover, we also find that when the attack intensity exceeds 1, the average travel time is very large and fast rises, which means that the network is already crowded to maintain the normal operation. The phenomenon indicates that the robustness of *URTNs* under the overall-demand attack is very worse when the attack intensity exceeds 1, and shows the same characteristics between user equilibrium and optimal system according to Fig. 4 and Fig. 5.

V. CONCLUSION

This paper investigates the robustness of *URTNs* from the perspective of the network utilization by the travel time difference between user equilibrium and optimal system. Meanwhile, the principles and solving algorithms of user equilibrium and optimal system are detailed described in this paper. Moreover, Shanghai metro network is taken as the example to illustrate the feasibility and effectiveness of the proposed schemes and three aimed attacks called node attack, link-demand attack and overall-demand attack are adopted to imitate the network failures. We find that *URTNs* have the better robustness subjected to random attacks under the node attack and the link-demand attack, and we also discover that *URTNs* have better robustness when suffering the random link-demand attacks than random node attacks. Furthermore, the results also show that the robustness of *URTNs* will decrease with the increase of the attack intensity when suffering the overall-demand attack, and there is a critical threshold about attack intensity that *URTNs* have the worst robustness when the attack intensity exceeds this critical threshold. The robustness analysis framework of *URTNs* applies to a variety of transportation networks and can provide suggestions in transportation planning and design.

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