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Trigonometric Similarity Measures for Neutrosophic Hypersoft Sets With Application to Renewable Energy Source Selection

MUHAMMAD NAVEED JAFAR[1], MUHAMMAD SAEED[1], MUHAMMAD SAQLAIN[2,3], AND MIIN-SHEN YANG[4]

[1]Department of Mathematics, University of Management and Technology, Lahore 54000, Pakistan
[2]Department of Mathematics, Lahore Garrison University, Lahore 54000, Pakistan
[3]School of Mathematics, Northwest University, Xi'an 710069, China
[4]Department of Applied Mathematics, Chung Yuan Christian University, Chung-li, Taoyuan 32023, Taiwan

Corresponding author: Miin-Shen Yang (msyang@math.cycu.edu.tw)

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ABSTRACT Cosine and cotangent similarity measurements are critical in applications for determining degrees of difference and similarity between objects. In the literature, numerous similarity measures for various extensions of fuzzy set, soft set, Intuitionistic Fuzzy Sets (IFSs), Pythagorean Fuzzy Sets (PFSs) and HyperSoft Sets (HSSs) have been explored. Neutrosophic HyperSoft Sets (NHSSs), on the other hand, has fewer cosine and cotangent similarity measures. In this paper, we propose the trigonometric similarity measures of NHSSs. We further investigate the basic operators, theorems, and propositions for the proposed similarity measures. We know that global warming causes environmental problems. One of applications for solving global warming is the concept of renewable energy. To show the effectiveness of the proposed similarity measures, we apply them to renewable energy source selection problems. The study reveals the best geographical area to install the energy production units, under some technical attributive factors. To check the validity and superiority of the proposed work, it is compared with some existing techniques which reveal that, decision-making problems with further bifurcated attributes, have more accurate and precise results and can only be solved with this technique. In the future, the proposed techniques can be applied to case studies, in which attributes are more than one and further bifurcated along with more than one decision-maker. Also, this proposed work can be extended for several existing hybrids of hypersoft sets, intuitionistic hypersoft, neutrosophic hypersoft set, bi-polar hypersoft, m-polar hypersoft sets, and Pythagorean hypersoft set to solve Multi-Criteria Decision Making (MCDM) problems.

INDEX TERMS Cosine, cotangent, neutrosophic hypersoft matrices (NHSMs), neutrosophic hypersoft sets (NHSSs), renewable energy source, similarity measures, MCDM.

I. INTRODUCTION

Multi-criteria decision making is a sub-branch of decision science which is the process of determining alternatives according to multiple criteria and choosing the best among these alternatives and is used at every level of life. However, it is possible to encounter a lot of uncertain information during this process. For example, most of the real-life problems which contain a lot of uncertain information must be modeled to solve these problems. The concept of vague data is one of the most crucial factors that make the decision-making process difficult. To overcome this difficulty, many mathematical theories for modeling uncertain information have been developed. Some of these are fuzzy sets (FSs) [1], intuitionistic FSs (IFSs) [2], complex FSs [3], Pythagorean FSs (PFSs) [4], [5], and q-rung orthopair FSs (q-ROFSs) [6], etc. However, in these sets, the vagueness depends on the concept of belonging and non-belonging functions. Smarandache [7] proposed neutrosophic sets (NSs) that have three free parts, which are truth, indeterminacy, and falsity belonging degree to get rid of the limitation provided by this dependency. The notion has been improved to stand for vague, deficient,

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and incoherent data that exists in real-life problems. In the concept of NSs, belonging degrees are a subset of a real standard or non-standard unit interval] $-0, +1$[, Moreover, no limitation to both belonging functions and the sum of belonging functions. However, set-theoretical operators cannot be defined on a non-standard unit interval and so it is very difficult to deal with real-life problems by using non-standard intervals. For this reason, Wang *et al.* [8] proposed the notion of single-valued NSs (SVNSs). Since it applies to real life, many studies have been done to solve real-world problems by using SVNSs [9]–[11]. However, for all fuzzy set theories, it is in an uncertain state how to set the belonging functions in each special case. To overcome vagueness which is free from the limitations, Molodtsov [12] proposed the notion of soft sets as a new mathematical way.

The concept of soft sets is a parametrized family of subsets of the universal set and it is proposed to solve problems that are arising due to not using adequate parameterization in FSs. Later, Maji *et al.* [13] introduced several operations of soft sets in more detail. Roy and Maji [14] proposed the notion of fuzzy soft sets by combining the notion of soft sets with FSs. Then, Smarandache [15] extended soft sets to hypersoft sets (HSSs) by using a multi-decision function to overcome uncertainty. Furthermore, Saqlain *et al.* [16] converted HSSs to neutrosophic HSSs (NHSSs) to overcome uncertainty problems and proposed the TOPSIS method by using the accuracy function for NHSSs. Khalil *et al.* [17] combined single-valued NHSSs and soft set with applications in decision-making. Saqlain *et al.* [18] constructed the NHSS-TOPSIS system based on their proposed distance and similarity measures for NHSSs. Saeed *et al.* [19] introduce NHSS mappings and used these mappings to diagnose hepatitis and Saeed *et al.* [20] proposed complex NHSSs. Saqlain *et al.* [21] considered single-valued NHSSs and multi-valued NHSSs with tangent similarity measures. Saqlain and Xin [22] further developed interval-valued, m-polar, and interval valued m-polar NHSSs. Rahman *et al.* [23] developed parameterized NHSS theory with the application in decision making.

Similarity measures are a crucial way for measuring the grade of similarity between two sets (objects). Various versions of similarity measures for extensions of fuzzy sets have been proposed and applied in various fields such as database acquisition, pattern recognition, medical diagnose, economic, and multi-criteria decision making [24]–[28]. The trigonometric similarity measure is a type of similarity measure. The notion of trigonometric similarity measures can be defined by using trigonometric functions such as cosine, cotangent, and tangent that had been applied in various fields [39]–[31]. Furthermore, Wang and Garg [32] designed an algorithm for multi-attributive problems with interactive Archimedean norm under Pythagorean fuzzy systems. Verma [33], [35] and Verma and Merigo [34] generalized similarity measures for PFSs and linguistic q-rung orthopair fuzzy environment, and also define cosine similarity for FSs. Wei [36] gave cosine similarity measures and applied them in strategic decision-making. Ye [37] presented a cotangent similarity measure of single-valued NSs and applied it in fault diagnosis of the steam turbine. Khan *et al.* [38] improved cosine and cotangent functions on q-ROFSs and presented TOPSIS techniques in the q-ROFS environment. Ahsan *et al.* [39] proposed complex fuzzy hypersoft mappings and applied them in the diagnosis HIV with its treatment.

In this paper, we first propose five trigonometric similarity measures for NHSSs based on inner product, two based on cosine functions, and two based on cotangent functions, respectively. We then investigate some basic operators, theorems, and propositions for the proposed similarity measure. In order to show the effectiveness of our similarity measures, we apply them to the renewable energy source selection problem, which is commonly referred to global warming problem.

The world currently faces global warming that causes environmental problems. One of the applications for solving global warming is the concept of renewable energy, which has entered our lives and has been important to prevent the negative effects of global warming. Therefore, countries have taken various measures. One of these measures is that countries regulate their energy policies in a way that minimizes the negative impacts of global warming. For this purpose, many countries have determined as a policy to realize energy production by using renewable energy sources instead of fossil fuels. The most used renewable energy sources in the world are wind, solar, hydraulic, and geothermal energy. There are many studies in the literature on the relationship between renewable energy and global warming [40]–[42]. Thus, we use the trigonometric similarity measures to examine the renewable energy source selection problem as a mathematical model. The results reveal the best geographical area to install the energy production units, under some technical attributive factors.

The remainder of the study is arranged as follows. In Section II, we recall some basic concepts of soft sets, HSSs, and NHSSs. In Section III, we propose five trigonometric similarity measures for NHSSs via trigonometric functions which are cosine and cotangent. We give operators, theorems, and propositions relative to these trigonometric similarity measures. Moreover, we also give weighted versions of them. In Section IV, we apply them on renewable energy source selection problem to express the effectiveness of proposed similarity measures. Also, we interpret the results obtained by transferring the obtained data to line and pie charts. Section V is reserved for the conclusion section and future studies are also discussed.

II. PRELIMANARIES

In this section, we review the definitions of soft sets, hypersoft sets (HSSs), and neutrosophic hypersoft sets (NHSSs).

Definition 1 ([12]): The notion of soft sets proposed by Molodtsov [12] to model vague data is defined as follows. *Let* $\mathbb{Y} = \{y_1, y_2, y_3, \ldots y_s\}$ be a finite set and P be a set of parameters. Let $P(\mathbb{Y})$ denotes the power set of $\mathbb{Y}$ and $A \subset P$. A pair $(\wp, A)$ is called a soft set over $\mathbb{Y}$, where the mapping

$\wp$ is given by

$$\wp : A \to P(\mathbb{Y}) \tag{2.1}$$

Definition 2 ([15]): The notion of HSSs proposed by Smarandache [15] which is the expansion of soft sets to model real life problems including uncertainty situations more precisely with higher accuracy is defined as follows. *Let* $\mathbb{Y} = \{y_1, y_2, y_3, \ldots y_s\}$ be a finite set and P be a set of parameters. Let $P(\mathbb{Y})$ denote the power set of $\mathbb{Y}$. Let $\mathcal{V}^1, \mathcal{V}^2, \mathcal{V}^3 \ldots \mathcal{V}^n$ for $n \geq 1$ be different features, whose corresponding feature values are the sets $'\Upsilon^1, '\Upsilon^2, '\Upsilon^3, \ldots '\Upsilon^n$ with $'\Upsilon^l \cap '\Upsilon^m = \emptyset$ for $l \neq m$, $l, m = 1, 2 \ldots n$, respectively. Then, the pair $\left(\wp,\ '\Upsilon^1 \times '\Upsilon^2 \times '\Upsilon^3 \times \ldots \times '\Upsilon^n\right)$ is called hypersoft set over $\mathbb{Y}$, where

$$\wp :\ '\Upsilon^1 \times '\Upsilon^2 \times '\Upsilon^3 \times \ldots \times '\Upsilon^n \to P(\mathbb{Y}) \tag{2.2}$$

Definition 3 ([16]): The notion of NHSSs proposed by Saqlain *et al.* [16] by combining the notion of neutrosophy with HSSs is defined as follows. *Let* $\mathbb{Y} = \{y_1, y_2, y_3, \ldots y_s\}$ be a finite set and P be a set of parameters. Let $P(\mathbb{Y})$ denote the power set of $\mathbb{Y}$. Let $\mathcal{V}^1, \mathcal{V}^2, \mathcal{V}^3 \ldots \mathcal{V}^n$ for $n \geq 1$ be n well defined features, whose corresponding feature values are the sets $'\Upsilon^1, '\Upsilon^2, '\Upsilon^3, \ldots '\Upsilon^n$ with $'\Upsilon^l \cap '\Upsilon^m = \emptyset$ for $l \neq m$, $l, m = 1, 2 \ldots n$, respectively, and let their relation be $\mathfrak{I} = '\Upsilon^1 \times '\Upsilon^2 \times '\Upsilon^3 \times \cdots \times '\Upsilon^n$. Then the pair $(\wp, \mathfrak{I})$ is called an NHSS over $\mathbb{Y}$, where

$$\wp :\ '\Upsilon^1 \times '\Upsilon^2 \times '\Upsilon^3 \times \cdots \times '\Upsilon^n \to P(\mathbb{Y}) \tag{2.3}$$

and

$$\wp\left('\Upsilon^1 \times '\Upsilon^2 \times '\Upsilon^3 \times \ldots \times '\Upsilon^n\right) = \wp(\mathfrak{I}) = \{\prec y, T(\wp(\mathfrak{I})), I(\wp(\mathfrak{I})),\ F(\wp(\mathfrak{I})), y \in \mathbb{Y} \succ\} \tag{2.4}$$

where T, I and F are the belonging values of truthiness, indeterminacy and falsity respectively such that $T, I, F : \mathbb{Y} \to [0, 1]$ with $0 \leq T(\wp(\mathfrak{I})) + I(\wp(\mathfrak{I})) + F(\wp(\mathfrak{I})) \leq 3$.

III. TRIGONOMETRIC SIMILARITY MEASURES FOR NEUTROSOPHIC HYPERSOFT SETS

In this section, we use cosine and cotangent functions to construct five new similarity measures for NHSSs motivating from the tangent similarity measures defined by Saqlain *et al.* [21].

Definition 4: Let $\mathbb{Y}$ be a finite set and let $D = \{\prec y,\ T_D(\wp(\mathfrak{I})), I_D(\wp(\mathfrak{I})),\ F_D(\wp(\mathfrak{I})),\ y \in \mathbb{Y} \succ\}$ and $E = \{\prec y,\ T_E(\wp(\mathfrak{I})), I_E(\wp(\mathfrak{I})),\ F_E(\wp(\mathfrak{I})),\ y \in \mathbb{Y} \succ\}$ be the two NHSSs for $\wp(\mathfrak{I})$. The cosine similarity measures between D and E by using arithmetic mean is given by (3.1), as shown at the bottom of the page.

Proposition 1: The cosine similarity measure C^1_{NHSS} satisfies the following properties:

$$(\boldsymbol{P_1})\ \ \boldsymbol{0 \leq C^1_{NHSS}(D, E) \leq 1}$$
$$(\boldsymbol{P_2})\ \ \boldsymbol{C^1_{NHSS}(D, E) = C^1_{NHSS}(E, D)}$$
$$(\boldsymbol{P_3})\ \ \text{If } \boldsymbol{D = E} \text{ then } \boldsymbol{C^1_{NHSS}(D, E) = 1}$$

Proof: $(\boldsymbol{P_1})$, as shown at the bottom of the page. Since cosine values and truthiness, indeterminacy and falseness of NHSSs are in the interval [0, 1], we have $\boldsymbol{0 \leq C^1_{NHSS}(D, E) \leq 1}$.

$(\boldsymbol{P_2})$: Proof is straightforward.

$(\boldsymbol{P_3})$: If $\boldsymbol{D = E}$, then $\boldsymbol{T_D(\wp(\mathfrak{I}))_r = T_E(\wp(\mathfrak{I}))_r}$, $\boldsymbol{I_D(\wp(\mathfrak{I}))_r = I_E(\wp(\mathfrak{I}))_r}$, and $\boldsymbol{F_D(\wp(\mathfrak{I}))_r = F_E(\wp(\mathfrak{I}))_r}$ for $\boldsymbol{r = 1, 2, \ldots n}$. Thus, we obtain $\boldsymbol{C^1_{NHSS}(D, E) = 1}$. ■

Definition 5: Let $\mathbb{Y}$ be a finite set and let $\boldsymbol{D} = \{\prec \boldsymbol{y}, \boldsymbol{T_D}(\wp(\mathfrak{I})), \boldsymbol{I_D}(\wp(\mathfrak{I})), \boldsymbol{F_D}(\wp(\mathfrak{I})), \boldsymbol{y} \in \mathbb{Y} \succ\}$ and $\boldsymbol{E} = \{\prec \boldsymbol{y}, \boldsymbol{T_E}(\wp(\mathfrak{I})), \boldsymbol{I_E}(\wp(\mathfrak{I})), \boldsymbol{F_E}(\wp(\mathfrak{I})), \boldsymbol{y} \in \mathbb{Y} \succ\}$ be the two NHSSs for $\wp(\mathfrak{I})$. The cosine similarity measures between $\boldsymbol{D}$ and $\boldsymbol{E}$ based on the cosine function is given by

$$C^2_{NHSS}(D, E) = \frac{1}{n}\sum\nolimits_{r=1}^{n} \cos\left[\frac{\pi}{2}\left(\left|T_D(\wp(\mathfrak{I}))_r - T_E(\wp(\mathfrak{I}))_r\right| \vee \left|I_D(\wp(\mathfrak{I}))_r - I_E(\wp(\mathfrak{I}))_r\right| \vee \left|F_D(\wp(\mathfrak{I}))_r - F_E(\wp(\mathfrak{I}))_r\right|\right)\right] \tag{3.2}$$

and

$$C^3_{NHSS}(D, E) = \frac{1}{n}\sum\nolimits_{r=1}^{n} \cos\left[\frac{\pi}{6}\left(\left|T_D(\wp(\mathfrak{I}))_r - T_E(\wp(\mathfrak{I}))_r\right| \vee \left|I_D(\wp(\mathfrak{I}))_r - I_E(\wp(\mathfrak{I}))_r\right| \vee \left|F_D(\wp(\mathfrak{I}))_r - F_E(\wp(\mathfrak{I}))_r\right|\right)\right] \tag{3.3}$$

Proposition 2: The cosine similarity measures $C^k_{NHSS}(D, E)$, $(k = 2, 3)$ satisfies P_1, P_2 and the following properties:

$$C^1_{NHSS}(D, E) = \frac{1}{n}\sum\nolimits_{r=1}^{n} \frac{\left(T_D(\wp(\mathfrak{I}))_r T_E(\wp(\mathfrak{I}))_r + I_D(\wp(\mathfrak{I}))_r I_E(\wp(\mathfrak{I}))_r + F_D(\wp(\mathfrak{I}))_r F_E(\wp(\mathfrak{I}))_r\right)}{\sqrt{T_D^2(\wp(\mathfrak{I}))_r + I_D^2(\wp(\mathfrak{I}))_r + F_D^2(\wp(\mathfrak{I}))_r}\sqrt{T_E^2(\wp(\mathfrak{I}))_r + I_E^2(\wp(\mathfrak{I}))_r + F_E^2(\wp(\mathfrak{I}))_r}} \tag{3.1}$$

$$(\boldsymbol{P_1}): \boldsymbol{\frac{1}{n}\sum\nolimits_{r=1}^{n} \frac{\left(T_D(\wp(\mathfrak{I}))_r T_E(\wp(\mathfrak{I}))_r + I_D(\wp(\mathfrak{I}))_r I_E(\wp(\mathfrak{I}))_r + F_D(\wp(\mathfrak{I}))_r F_E(\wp(\mathfrak{I}))_r\right)}{\sqrt{T_D^2(\wp(\mathfrak{I}))_r + I_D^2(\wp(\mathfrak{I}))_r + F_D^2(\wp(\mathfrak{I}))_r}\sqrt{T_E^2(\wp(\mathfrak{I}))_r + I_E^2(\wp(\mathfrak{I}))_r + F_E^2(\wp(\mathfrak{I}))_r}} = \cos\theta_r}.$$

($\mathbf{P'_3}$) $D = E$if and only if$C^k_{NHSS}(D, E) = 1,\ (k = 2, 3)$.

($\mathbf{P_4}$) If F is a NHSS in$\mathbb{Y}$and$D \subset E \subset F$, then$C^k_{NHSS}(D, F) \leq C^k_{NHSS}(D, E)$ and $C^k_{NHSS}(D, F) \leq C^k_{NHSS}(E, F)$.

Proof:

$(\boldsymbol{P_1})$ **:** Since the value of cosine function and the truthiness, indeterminacy and falseness of NHSSs are in the interval [0, 1], the similarity measures based on the cosine function which is arithmetic mean of these cosine functions, are also in [0, 1]. Therefore, $\mathbf{0} \leq C^k_{NHSS}(D, E) \leq 1$ for $k = 2, 3$.

$(\boldsymbol{P_2})$ Proof is straightforward.

$(\boldsymbol{P'_3})$ For any two NHSSs $\boldsymbol{D}$ and $\boldsymbol{E}$ in $\mathbb{Y}$, if $\boldsymbol{D} = \boldsymbol{E}$, then $\boldsymbol{T_D(\wp(\Im))_r = T_E(\wp(\Im))_r,\ I_D(\wp(\Im))_r = I_E(\wp(\Im))_r}$ and $F_D(\wp(\Im))_r = F_E(\wp(\Im))_r for_r = 1, 2, \ldots n$. Thus, we obtain

$$\left|T_D(\wp(\Im))_r - T_E(\wp(\Im))_r\right| = 0;$$
$$\left|I_D(\wp(\Im))_r - I_E(\wp(\Im))_r\right| = 0;$$
$$\left|F_D(\wp(\Im))_r - F_E(\wp(\Im))_r\right| = 0.$$

And so the cosine similarity measure $C^k_{NHSS}(D, E) = 1$, for $k = 2, 3$. Conversely, let $C^k_{NHSS}(D, E) = 1$, for $k = 2, 3$. Since $\cos 0 = 1$, this implies that

$$\left|T_D(\wp(\Im))_r - T_E(\wp(\Im))_r\right| = 0;$$
$$\left|I_D(\wp(\Im))_r - I_E(\wp(\Im))_r\right| = 0;$$
$$\left|F_D(\wp(\Im))_r - F_E(\wp(\Im))_r\right| = 0.$$

Therefore, we obtain $T_D(\wp(\Im))_r = T_E(\wp(\Im))_r$, $I_D(\wp(\Im))_r = I_E(\wp(\Im))_r$, $F_D(\wp(\Im))_r = F_E(\wp(\Im))_r$ for $_r = 1, 2, 3 \ldots n$. Hence, $D = E$.

$(\boldsymbol{P_4})$ If $D \subset E \subset F$, then $T_D(\wp(\Im))_r \leq T_E(\wp(\Im))_r \leq T_F(\wp(\Im))_r$, $I_D(\wp(\Im))_r \geq I_E(\wp(\Im))_r \geq T_F(\wp(\Im))_r$, and $F_D(\wp(\Im))_r \geq F_E(\wp(\Im))_r \geq F_F(\wp(\Im))_r$ for $_r = 1, 2, 3 \ldots n$.

Thus, we have

$$\begin{aligned}
&\left|T_D(\wp(\Im))_r - T_E(\wp(\Im))_r\right| \\
&\qquad \leq \left|T_D(\wp(\Im))_r - T_F(\wp(\Im))_r\right| \\
&\left|T_E(\wp(\Im))_r - T_F(\wp(\Im))_r\right| \\
&\qquad \leq \left|T_D(\wp(\Im))_r - T_F(\wp(\Im))_r\right| \\
&\left|I_D(\wp(\Im))_r - I_E(\wp(\Im))_r\right| \\
&\qquad \geq \left|I_D(\wp(\Im))_r - I_F(\wp(\Im))_r\right| \\
&\left|I_E(\wp(\Im))_r - I_F(\wp(\Im))_r\right| \\
&\qquad \geq \left|I_D(\wp(\Im))_r - I_F(\wp(\Im))_r\right| \\
&\left|F_D(\wp(\Im))_r - F_E(\wp(\Im))_r\right| \\
&\qquad \geq \left|F_D(\wp(\Im))_r - F_F(\wp(\Im))_r\right| \\
&\left|F_E(\wp(\Im))_r - F_F(\wp(\Im))_r\right| \\
&\qquad \geq \left|F_D(\wp(\Im))_r - F_F(\wp(\Im))_r\right|
\end{aligned}$$

Hence,$D \subset E \subset F$. Then,$C^k_{NHSS}(D, F) \leq C^k_{NHSS}(D, E)$ and $C^k_{NHSS}(D, F) \leq C^k_{NHSS}(E, F)$. For $k = 2, 3$, as the cosine function is decreasing with the interval $[0, \frac{\pi}{2}]$, the proof is completed. ■

Definition 6: Let $\mathbb{Y}$ be a finite universal set and let $D = \{\prec y,\ T_D(\wp(\Im)), I_D(\wp(\Im)),\ F_D(\wp(\Im)),\ y \in \mathbb{Y} \succ\}$ and $E = \{\prec y,\ T_E(\wp(\Im)), I_E(\wp(\Im)),\ F_E(\wp(\Im)),\ y \in \mathbb{Y} \succ\}$ be the two NHSSs for $\wp(\Im)$. The cotangent similarity measures based on the cotangent function between D and E are given with

$$\begin{aligned}
&C^4_{NHSS}(D, E) \\
&\quad = \frac{1}{n}\sum\nolimits_{r=1}^{n} \cot\left[\frac{\pi}{4} + \frac{\pi}{4}\left(\left|T_D(\wp(\Im))_r - T_E(\wp(\Im))_r\right|\right.\right. \\
&\quad \vee \left|I_D(\wp(\Im))_r - I_E(\wp(\Im))_r\right| \vee \left|F_D(\wp(\Im))_r\right. \\
&\quad \left.\left.\left. - F_E(\wp(\Im))_r\right|\right)\right] \qquad (3.4)
\end{aligned}$$

and

$$\begin{aligned}
&C^5_{NHSS}(D, E) \\
&\quad = \frac{1}{n}\sum\nolimits_{r=1}^{n} \cot\left[\frac{\pi}{4} + \frac{\pi}{12}\left(\left|T_D(\wp(\Im))_r - T_E(\wp(\Im))_r\right|\right.\right. \\
&\quad \vee \left|I_D(\wp(\Im))_r - I_E(\wp(\Im))_r\right| \vee \left|F_D(\wp(\Im))_r\right. \\
&\quad \left.\left.\left. - F_E(\wp(\Im))_r\right|\right)\right] \qquad (3.5)
\end{aligned}$$

where $\vee$ denotes the maximum operator.

Proposition 3: The cotangent similarity measures C^k_{NHSS}, $(k = 4, 5)$ satisfies P_1, P_2, P'_3 and P_4.

Proof: The proof can be made by similar to Proposition 2.∋

Similarly, the weighted version of similarity measures of equations (3.1)-(3.5) are given as (3.6)-(3.10), shown at the bottom of the next page, where $0 \leq \mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \ldots \mathcal{W}_n \leq 1$with $\sum_r^n \mathcal{W}_r = 1$.

IV. ALGORITHM AND ILUSTRATIVE EXAMPLES

In this section, we give the algorithm based on the proposed similarity measures. We then apply it in the renewable energy source selection problem.

A. ALGORITHM BASED ON NHSS SIMILARITY MEASURES

Let $G^1, G^2, G^3 \ldots G^n$ be the distinct set of geographical regions of a country, $C^1, C^2, C^3 \ldots C^n$ by the set of norms for geographical regions and $P^1, P^2, P^3 \ldots P^n$ be the renewable power set of options for each geographical region. A decision maker can evaluate G regions and P power types under C norms by using a decision-making technique. As a result of this evaluation, one can interpret which renewable energy source should be used in which geographical region. Thus, it can choose the best match between geographic regions and renewable energy sources.

We next give the implementation steps of the proposed algorithm based on trigonometric similarity measures for NHSSs in which the flow chart of the proposed algorithm is shown in Fig. 1.

Step 1: Firstly, geographical regions to be evaluated and renewable power source types that can be used in these regions should be selected. Then, the norms of these regions and energy resources should be determined. The association between geographical regions and the norms should be given by using decision matrix in terms of NHSSs.

Step 2: The association between the norms and the options that is renewable power types should be given by using decision matrix in terms of NHSSs.

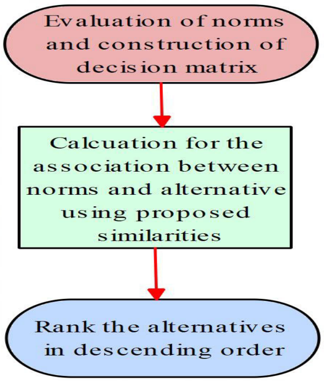

FIGURE 1. Flow chart of the proposed algorithm.

Step 3: The association between geographical regions and the options is determined with the help of proposed trigonometric similarity measures for NHSSs by using equations (3.1)-(3.5). The best option is decided by choosing the highest value in which the highest value represents the best option for the geographical regions. This value is highlighted by typing in bold.

B. APPLICATION IN RENEWABLE ENERGY SOURCE SELECTION

Global warming negatively affects our world. Most countries had taken some measures to minimize the negativity.

One of these measures is a development plan that includes the renewable energy policy of the countries. Thus, renewable energy resources have increased in the world and the countries have started to determine the amount of renewable energy resources that they can own. For this reason, we try to create a mathematical model for handling this renewable energy source problem. First of all, we try to appeal to every country by keeping our criterion scale higher and by keeping the lower criteria ranges wider.

We assume that we have 10 geographic regions indicated by the set

$$\mathbb{G}=\left\{g^1,g^2,g^3,g^4,g^5,g^6,g^7,g^8,g^9,g^{10}\right\}.$$

Then, we identify the most frequently used and most popular renewable power sources in the world and show them with the set $\mathbb{E}$:

$$\mathbb{E}=\{\text{Solar Power, Wind Power, Hydraulic Power, Geothermal Power}\}.$$

We identify the most frequently used criteria and sub-criteria to evaluate these energy resources and geographic regions as $\mathscr{Y}$:

$$\mathscr{Y}=\left\{\begin{array}{c}\mathscr{Y}^1\ (\text{Annual average daily bath time } (h/day))\\ \mathscr{Y}^2\ (\text{Average Flow Intensity of streams } (m^3/sec))\\ \mathscr{Y}^3\ (\text{Annual Average rainfall } (mm))\\ \mathscr{Y}^4\ (\text{Annual Average daily Wind Speed } (km/h))\\ \mathscr{Y}^5(\text{Underground Geothermal Water Density } (\%))\end{array}\right\}$$

$\mathscr{Y}$ is the set of attributes corresponding to $\mathbb{G}$ and $\mathbb{E}$ where $''h/day''$ is the unit that gives the average sunbathing hour in per day; $''m^3/sec''$ is the average volume of water that passes in per second; $''mm''$ is the average height of the amount of water per square meter; $''km/h''$ is the average speed of wind under kilometers in per hour and $''\%''$ is the percentage of the average amount of geothermal water underground.

$$\mathscr{Y}^1=\left\{\begin{array}{c}\text{below}1h,\ 1h\text{–}4h,\ 4h\text{–}6h\\ 6h\text{–}8h,\ 8h\text{–}10h,\ 10h\text{–}14h\\ \text{above } 14\ h\end{array}\right\}$$

$$\mathscr{Y}^2=\left\{\begin{array}{c}\text{below } 500\ m^3,\ 500\ m^3\text{–}2000m^3\\ 2000\ m^3\text{–}4000m^3,\ 4000m^3\text{–}60000m^3\\ 6000m^3\text{–}8000m^3,\ 8000m^3\text{–}10,000m^3\\ 10,000m^3\text{–}20000m^3,\ 20,000m^3\text{–}40,000m^3\\ \text{Above } 40,000m^3\end{array}\right\}$$

$$wC^1_{NHSS}(D,E)=\frac{1}{n}\sum_{r=1}^{n}\mathcal{W}_r\frac{\left(T_D(\wp(\Im))_r T_E(\wp(\Im))_r+I_D(\wp(\Im))_r I_E(\wp(\Im))_r+F_D(\wp(\Im))_r F_E(\wp(\Im))_r\right)}{\sqrt{T_D^2(\wp(\Im))_r+I_D^2(\wp(\Im))_r+F_D^2(\wp(\Im))_r}\sqrt{T_E^2(\wp(\Im))_r+I_E^2(\wp(\Im))_r+F_E^2(\wp(\Im))_r}} \tag{3.6}$$

$$wC^2_{NHSS}(D,E)=\frac{1}{n}\sum_{r=1}^{n}\mathcal{W}_r\cos\left[\frac{\pi}{2}\left(\left|T_D(\wp(\Im))_r-T_E(\wp(\Im))_r\right|\vee\left|I_D(\wp(\Im))_r-I_E(\wp(\Im))_r\right|\vee\left|F_D(\wp(\Im))_r-F_E(\wp(\Im))_r\right|\right)\right] \tag{3.7}$$

$$wC^3_{NHSS}(D,E)=\frac{1}{n}\sum_{r=1}^{n}\mathcal{W}_r\cos\left[\frac{\pi}{6}\left(\left|T_D(\wp(\Im))_r-T_E(\wp(\Im))_r\right|\vee\left|I_D(\wp(\Im))_r-I_E(\wp(\Im))_r\right|\vee\left|F_D(\wp(\Im))_r-F_E(\wp(\Im))_r\right|\right)\right] \tag{3.8}$$

$$wC^4_{NHSS}(D,E)=\frac{1}{n}\sum_{r=1}^{n}\mathcal{W}_r\cot\left[\frac{\pi}{4}+\frac{\pi}{4}\left(\left|T_D(\wp(\Im))_r-T_E(\wp(\Im))_r\right|\vee\left|I_D(\wp(\Im))_r-I_E(\wp(\Im))_r\right|\vee\left|F_D(\wp(\Im))_r-F_E(\wp(\Im))_r\right|\right)\right] \tag{3.9}$$

$$wC^4_{NHSS}(D,E)=\frac{1}{n}\sum_{r=1}^{n}\mathcal{W}_r\cot\left[\frac{\pi}{4}+\frac{\pi}{12}\left(\left|T_D(\wp(\Im))_r-T_E(\wp(\Im))_r\right|\vee\left|I_D(\wp(\Im))_r-I_E(\wp(\Im))_r\right|\vee\left|F_D(\wp(\Im))_r-F_E(\wp(\Im))_r\right|\right)\right] \tag{3.10}$$

TABLE 1. Decision matrix between the relation of geographical regions and criteria.

Geographical Regions	$6h-8h$	$500\,m^3-2000m^3$	$500mm-1000mm$	$10km-20km$	$30\%-40\%$,
g^2	(0.5,0.7,0.6)	(0.3,0.1,0.2)	(0.1,0.1,0.3)	(0.5,0.8,0.6)	(0.9,0.1,0.1)
g^3	(0.0,0.2,0.9)	(0.4,0.0,0.6)	(0.5,0.3,0.2)	(0.5,0.3,0.6)	(0.1,0.7,0.4)
g^6	(0.8,0.1,0.2)	(0.0,0.1,0.6)	(0.0,0.4,0.5)	(0.5,0.4,0.6)	(0.1,0.7,0.1)
g^8	(0.3,0.7,0.6)	(0.4,0.0,0.5)	(0.5,0.1,0.3)	(0.3,0.7,0.8)	(0.9,0.2,0.1)
g^{10}	(0.5,0.4,0.0)	(0.9,0.1,0.6)	(0.6,0.2,0.3)	(0.5,0.5,0.0)	(0.1,0.9,0.2)

TABLE 2. Decision matrix between the relation of power sources and criteria.

Power sources	$6h-8h$	$500\,m^3-2000m^3$	$500mm-1000mm$	$10km-20km$	$30\%-40\%$,
Solar	(0.3,0.1,0.6)	(0.7,0.0,0.2)	(0.4,0.1,0.5)	(0.7,0.8,0.0)	(0.6,0.2,0.1)
Wind	(0.1,0.9,0.8)	(0.5,0.1,0.2)	(0.0,0.1,0.3)	(06,0.5,0.6)	(0.9,0.1,0.1)
Hydraulic	(0.0,0.2,0.9)	(0.8,0.1,0.3)	(0.1,0.7,0.5)	(0.5,0.3,0.6)	(0.1,0.3,0.6)
Geothermal	(0.5,0.1,0.6)	(0.4,0.4,0.5)	(0.0,0.1,0.1)	(0.4,0.6,0.8)	(0.2,0.1,0.1)

TABLE 3. Similarity measures using $C^1_{NHSS}(D,E)$.

Similarity measures	*geographical regions*	*solar power*	Wind power	Hydraulic power	Geothermal power
C^1_{NHSS}	g^2	0.8946	**0.9614**	0.7333	0.8912
	g^3	0.7219	0.6665	**0.8630**	0.7594
	g^6	0.5607	0.5826	0.6304	**0.8178**
	g^8	0.8388	**0.8571**	0.6767	0.8004
	g^{10}	**0.7450**	0.6056	0.5997	0.6324

$$\Upsilon^3 = \left\{ \begin{array}{c} \text{below}250mm,\ 250mm{-}500mm \\ 500mm{-}1000mm,\ 1000mm{-}2000mm \\ 2000mm{-}4000mm,\ 4000mm{-}6000mm \\ 6000mm{-}8000mm,\ 8000mm{-}10,000mm \\ \text{above } 10,000mm \end{array} \right\}$$

$$\Upsilon^4 = \left\{ \begin{array}{c} \text{below}10km,\ 10km{-}20km,\ 20km{-}35km \\ 35km{-}55km,\ 55km{-}70km,\ 70km{-}100km \\ \text{above}100km \end{array} \right\}$$

$$\Upsilon^5 = \left\{ \begin{array}{c} \text{below}5\%\ ,\ 5\%{-}10\%,\ 10\%{-}20\% \\ 20\%{-}30\%,\ 30\%{-}40\%,\ 40\%{-}50\% \\ \text{above}50\% \end{array} \right\}$$

The NHSSs are given as $\wp : \left(\Upsilon^1\times\Upsilon^2\times\Upsilon^3\times\Upsilon^4\times\Upsilon^5\right) \to P(\mathbb{G})$ and $\mathcal{L} : \left(\Upsilon^1\times\Upsilon^2\times\Upsilon^3\times\Upsilon^4\times\Upsilon^5\right) \to P(\mathbb{E})$. Let us assume that

$$\wp\,(\mathfrak{I}) = \left\{ \begin{array}{c} 6h{-}8h,\ 500m^3{-}2000m^3,\ 500mm{-}1000mm \\ 10km{-}20km,\ 30\%{-}40\% \end{array} \right\}$$

We evaluate $\{g^2, g^3, g^6, g^8, g^{10}\}$ and Hydraulic Power, Wind Power, Solar Power, Geothermal Power. Under this relationship, we should first determine the association between $\{g^2, g^3, g^6, g^8, g^{10}\}$ and $\{6h{-}8h,\ 500m^3{-}2000m^3,\ 500mm{-}1000mm,\ 10km{-}20km,\ 30\%{-}40\%\}$. According to Step 1, the association is hypothetically given by the decision matrix in terms of NHSSs, as shown in Table 1. Then, we should determine the association between Hydraulic Power, wind power, Solar Power, Solar Power, Geothermal Power and$\{6h{-}8h,\ 500m^3{-}2000m^3, 500mm{-}1000mm,\ 10km{-}20km,\ 30\%{-}40\%\}$. According to Step 2, the association is given by the decision matrix in terms of NHSSs, as shown in Table 2. Now, we should determine the association between $\{g^2, g^3, g^6, g^8, g^{10}\}$ and Hydraulic Power, wind power, Solar Power, Solar Power, Geothermal Power. According to Step 3, the association is determined with the proposed trigonometric similarity measures for NHSSs by using equations (3.1)-(3.5), as shown in tables Table 3-Table 7.

V. RESULT DISCUSSION AND COMPARISONS

In this section, we compare the proposed technique of similarity measures with some existing methods. We mention that Saqlain *et al.* [21] proposed similarity measures in single-valued neutrosophic hypersoft sets (SVNHSSs). Jafar *et al.* [31] gave similarity measures in neutrosophic sets (NSs). Verma [33], and Verma and Merigo [34] established cosine similarity measures in fuzzy sets (FSs) and Pythagorean FSs (PFSs). Wei [36] gives cosine similarity measures for picture fuzzy sets (PicFSs). Ye [37] considered

TABLE 4. Similarity measures using $C^2_{NHSS}(D,E)$.

Similarity measures	geographical regions	solar power	Wind power	Hydraulic power	Geothermal power
C^2_{NHSS}	g^2	0.7533	**0.9277**	0.6036	0.7660
	g^3	0.7771	0.6460	**0.8854**	0.7404
	g^6	0.6530	0.6407	0.6407	**0.7931**
	g^8	0.7259	**0.8855**	0.6443	0.7091
	g^{10}	**0.7385**	0.5205	0.5860	0.5001

TABLE 5. Similarity measures using $C^3_{NHSS}(D,E)$.

Similarity measures	geographical regions	solar power	Wind power	Hydraulic power	Geothermal power
C^3_{NHSS}	g^2	0.9474	**0.9769**	0.8629	0.9543
	g^3	0.8704	0.8745	**0.9406**	0.9175
	g^6	0.8617	0.8097	0.8751	**0.9360**
	g^8	0.9309	**0.9630**	0.8475	0.9453
	g^{10}	**0.8962**	0.8078	0.8471	0.8810

TABLE 6. Similarity measures using $C^4_{NHSS}(D,E)$.

Similarity measures	geographical regions	solar power	Wind power	Hydraulic power	Geothermal power
C^4_{NHSS}	g^2	0.4769	**0.7405**	0.3451	0.4686
	g^3	0.4948	0.4097	**0.7057**	0.4551
	g^6	0.3805	0.4395	0.4395	**0.5160**
	g^8	0.4870	**0.6440**	0.3833	0.4685
	g^{10}	**0.4827**	0.2952	0.3511	0.2761

TABLE 7. Similarity measures using $C^5_{NHSS}(D,E)$.

Similarity measures	geographical regions	solar power	Wind power	Hydraulic power	Geothermal power
C^5_{NHSS}	g^2	0.7286	**0.8617**	0.6068	0.7445
	g^3	0.6034	0.6393	**0.7837**	0.6666
	g^6	0.5792	0.5931	0.6596	**0.6957**
	g^8	0.7276	**0.7794**	0.5668	0.7299
	g^{10}	**0.6483**	0.5464	0.5859	0.2761

similarity measures in single-valued neutrosophic sets (SVNSs). Khan *et al.* [38] gave similarities on q-rung orthopair fuzzy sets (q-ROFSs). All of these similarities were proposed by using aggregate operators. In this proposed work, the similarity measures are established using the inner product, cosine function and cotangent function in NHSS environment which deals with multi-attributive values. NHSS environment generally gives more precise and accurate results. We show these comparisons in Table 8.

The results obtained according to all the proposed trigonometric similarity measures for NHSS are consistent with each other and so the numerical results presented in Table 3 and Fig. 2 show that the g^2, Table 4 with Fig. 3 shows that g^8 region should be selected for wind power, Table 5 and Fig. 4 shows that g^3 region should be selected for hydraulic power, Table 6, Fig. 5 shows that g^6 region should be selected for geothermal power and finally Table 7 and Fig. 6 shows that g^{10} region should be selected for solar power. Moreover, percentages source of wind power in g^2 area is 28, source of hydraulic power in g^3 area is 29, source of geotermal power in g^6 area is 32, source of solar power in g^{10} area is 29, respectively. Only, the percentage of solar and wind energy resources are equal which are 27 in the g^8 region. Thus,

TABLE 8. Comparisons of the proposed method and some existing methods.

Researcher	Similarity Measure	Truthiness	Indeterminacy	Falsity	Sub-Attributes	Weighted Similarity	Similarity Based on Inner Product
Verma [33]	FSs	✓	✕	✕	✕	✕	✕
Verma et al. [34]	PFSs	✓	✕	✓	✕	✕	✕
Khan et al. [38]	q-ROFSs	✓	✕	✓	✕	✕	✕
Wei [36]	PicFSs	✓	✕	✕	✕	✕	✕
Ye [37]	SVNSs	✓	✓	✓	✕	✕	✕
Jafar et al. [31]	NSs	✓	✓	✓	✕	✕	✕
Saqlain et al. [21]	SVNHSSs	✓	✓	✓	✓	✕	✕
Ours	NHSSs	✓	✓	✓	✓	✓	✓

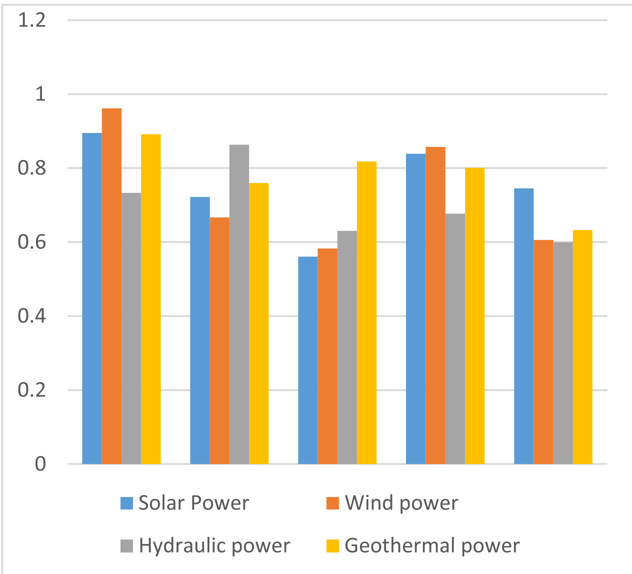

FIGURE 2. Similarity measure using C^1_{NHSS}.

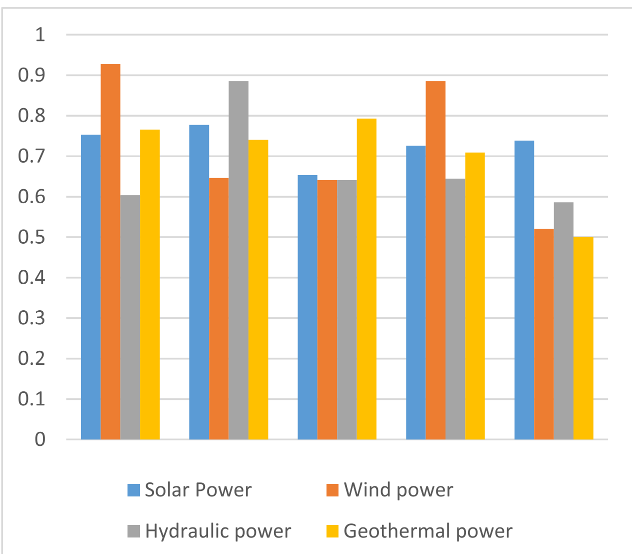

FIGURE 3. Similarity measure using C^2_{NHSS}.

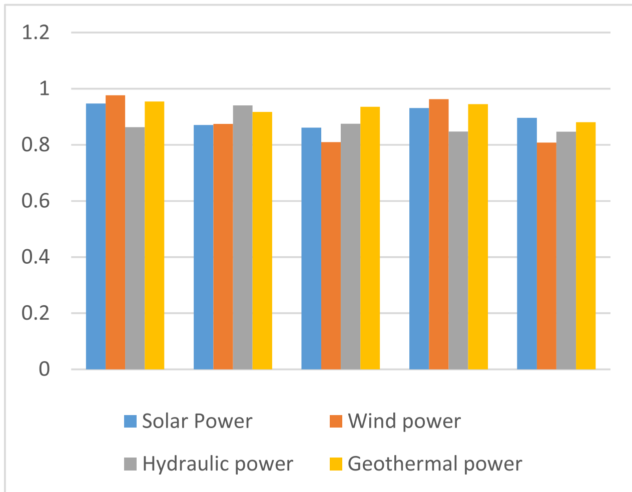

FIGURE 4. Similarity measure using C^3_{NHSS}.

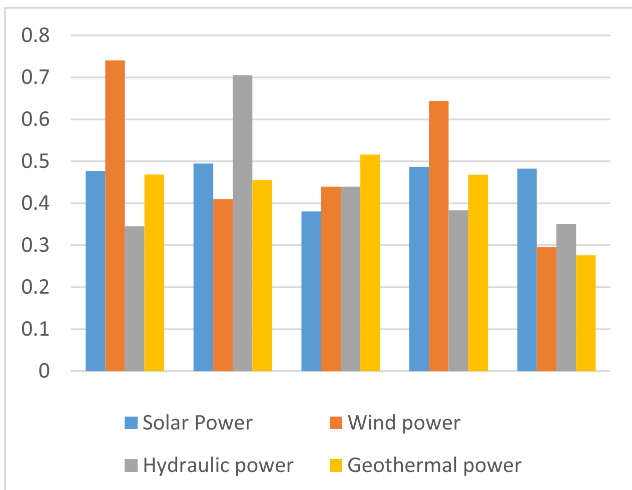

FIGURE 5. Similarity measure using C^4_{NHSS}.

the percentage of renewable energy resources in the regions given according to all the proposed trigonometric measures are found and shown in the above graphical representations. We found this way of finding the best source selection for power energy is very useful tool for selections. Our proposed similarities based on inner product with trigonometric

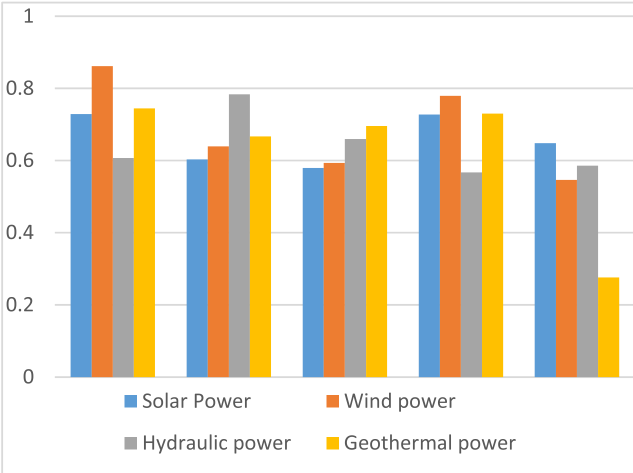

FIGURE 6. Similarity measure using C^5_{NHSS}.

approaches under NHSS structure actually give more accurate and precise.

VI. CONCLUSION

Neutrosophic hypersoft Sets (NHSSs) can be a powerful mathematical paradigm for dealing with data that is incomplete, indeterminate, uncertain, or vague. NHSSs are generally more efficient than fuzzy sets and intuitionistic fuzzy sets at dealing with uncertain and vague information. However, no one had considered cosine and cotangent similarities for NHSSs. We extend the cosine and cotangent techniques to the NHSS environment and constructed an algorithm for the solving of MCDM by employing the proposed similarity measures. We introduced five trigonometric similarity measures for NHSSs with properties. We applied them to renewable power source selection problem by using proposed five trigonometric similarity measures. Thus, we give a useful tool for the renewable energy source selection by using the proposed mathematical model. We also make more comparisons of the proposed method with several existing methods. The suggested NHSS-Similarity measures offer enormous potential for MCDM difficulties in a variety of fields, including supplier selection, manufacturing frameworks, and a variety of other management frameworks. The proposed methodology can be expanded in a variety of ways to cover a wide range of decision-making challenges in various NHSS scenarios. Measurement of NHSS uncertainty/fuzziness, on the other hand, is a key step in NHSS applied systems. In our future works, we will try to extend these similarity measures to hybrids of hypersoft sets (HSS), such as fuzzy HSS, intuitionistic HSS, Pythagorean HSS, and m-polar HSS with applications in real-life examples.

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MUHAMMAD NAVEED JAFAR was born in Pakistan, in 1987. He is currently pursuing the Ph.D. degree with the University of Management and Technology (UMT), Lahore, Pakistan. He taught at Lahore Garrison University, Lahore, and taught mathematics at degree level with exceptional results. He has supervised seven M.S. students and published more than 30 articles in recognized journals. His research interests include fuzzy mathematics, soft set theory, hypersoft set, and neutrosophic sets multi-criteria decision making. He was awarded the Best Teacher Award at college level, in 2014 and 2015, and university level, in 2017 and 2018. He was also awarded the Excellence Performance Award 2020–2021 at Lahore Garrison University.

MUHAMMAD SAEED was born in Pakistan, in July 1970. He received the Ph.D. degree in mathematics from Quaid-i-Azam University, Islamabad, Pakistan, in 2012. He taught mathematics at undergraduate and graduate levels with exceptional results. He was involved as a Teacher Trainer for professional development for more than five years. He worked as the Chairman of the Department of Mathematics, UMT, Lahore, from 2014 to 2021. Under his dynamic leadership, the Mathematics Department has produced ten Ph.D. students. He has supervised 20 M.S. students and four Ph.D. students, and published more than 100 articles in recognized journals. His research interests include fuzzy mathematics, rough sets, soft set theory, hypersoft set, neutrosophic sets, algebraic and hybrid structures of soft sets and hypersoft sets, multicriteria decision making, optimizations, artificial intelligence, pattern recognition and optimization under convex environments, graph theory in fuzzy-like, soft-like, and hypersoft-like environments, and distance measures and their relevant operators in multipolar hybrid structures. He was awarded the Best Teacher in 1999, 2000, 2001, and 2002.

MUHAMMAD SAQLAIN received the M.Phil. degree in mathematics from Lahore Garrison University, Lahore, Pakistan, and the M.Sc. degree from the University of Sargodha, Pakistan. He is currently pursuing the Ph.D. degree with the School of Mathematics, Northwest University, Xi'an, China. He is also a Lecturer with the Department of Mathematics, Lahore Garrison University. He has published more than 35 research articles in different well-known journals, including SCI, SCIE, and ESCI. His research interests include soft set, neutrosophic soft set, neutrosophic hypersoft set (NHSS), and fuzzy logic controllers. He is also an Editor of the book *Theory and Application of Hypersoft Set* and a Section Editor of the journal *Neutrosophic Sets and Systems*.

MIIN-SHEN YANG received the B.S. degree in mathematics from Chung Yuan Christian University (CYCU), Taoyuan, Taiwan, in 1977, the M.S. degree in applied mathematics from National Chiao-Tung University, Hsinchu, Taiwan, in 1980, and the Ph.D. degree in statistics from the University of South Carolina, Columbia, USA, in 1989. In 1989, he joined the faculty of the Department of Mathematics, CYCU, as an Associate Professor, where he has been a Professor, since 1994. From 1997 to 1998, he was a Visiting Professor with the Department of Industrial Engineering, University of Washington, Seattle, USA. From 2001 to 2005, he was the Chairman of the Department of Applied Mathematics, CYCU. Since 2012, he has been a Distinguished Professor with the Department of Applied Mathematics and the Director of Chaplain's Office, and is currently the Dean of the College of Science, CYCU. His research interests include applications of statistics, fuzzy clustering, soft computing, pattern recognition, and machine learning. He was an Associate Editor of the IEEE Transactions on Fuzzy Systems from 2005 to 2011. He is also an Associate Editor of the *Applied Computational Intelligence and Soft Computing* and an Editorial Board Member of Computer Science and Engineering Section in the journal *Electronics* (MDPI).