IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received August 30, 2021, accepted September 9, 2021, date of publication September 13, 2021, date of current version September 22, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3112600

A Mathematical Programming Model and a Firefly-Based Heuristic for Real-Time Traffic Signal Scheduling With Physical Constraints

HANAA ABOHASHIMA[1,2], **AMR ELTAWIL**[1,3], **AND MOHAMED GHEITH**[1,3]

[1]Department of Industrial and Manufacturing Engineering, Egypt-Japan University of Science and Technology, Alexandria 21934, Egypt
[2]Mechanical Engineering Department, Faculty of Engineering, Fayoum University, Fayoum 63514, Egypt
[3]Production Engineering Department, Faculty of Engineering, Alexandria University, Alexandria 21544, Egypt

Corresponding author: Hanaa Abohashima (hanaa.soliman@ejust.edu.eg)

This work was supported by the Egyptian Ministry of Higher Education Grant and the Japanese International Cooperation Agency (JICA) in the scope of Egypt-Japan University of Science and Technology.

ABSTRACT Traffic congestion is one of the challenges that face urban cities' planners. It affects the environment as it increases the emissions of CO2 and affects the logistics systems as it may increase the travel time of different vehicles. Scheduling traffic signals is one of the ways to solve this problem. In the urban traffic signal scheduling problem, it is desired to get the optimum schedule for each considered traffic signal to maximize or minimize a specific objective function(s); these schedules determine the active and inactive traffic phases during each cycle time. In this paper, a mathematical programming model for solving the urban traffic signal scheduling problem is presented, the proposed mathematical model captures the physical constraints of the problem. Furthermore, a firefly-based rolling horizon approach is proposed to solve the problem. Both methods are used to solve a traffic-responsive system, which is considered the future of traffic control systems. The performance of both methods has been simulated using the SUMO traffic simulator to verify the solutions. The performance of the solutions was measured using the average queue length of the roads, the average waiting time, and the average travel time. The proposed methods have been applied to a real case study, and the results were remarkable.

INDEX TERMS Firefly heuristic, mathematical programming, rolling horizon, traffic signal scheduling, transportation.

I. INTRODUCTION

Traffic congestion is one of the most critical problems that face almost all metropolitan cities in the world. According to the organization of motor vehicle manufacturers, the sales of different types of vehicles show an upward trend with about 3% increase in sales each year, this increases the traffic congestion on different roads. In turn, it affects the traveling time of vehicles and has a negative impact on the environment as it increases the emissions of Co2. Furthermore,

One of the possible ways to decrease the impacts of traffic congestion is to control the timings of traffic signals at the intersections. Typically, there are two types of systems used to control the operation of traffic signals, the fixed-time control system and the traffic-responsive control system [1]. In the fixed-time control system, the system parameters such as the cycle length, the phase splits, and the phases sequence are specified in advance based on historical data. This type of control system is suitable when the flow of an intersection shows no or low changes over time. On the other hand, in the traffic-responsive control system, all the previously mentioned parameters are not fixed, and they are subject to changes according to the changes in the flow rates and patterns. Hence, the traffic-responsive control system can deal with the instantaneous flow of vehicles [2].

Figure 1 shows an illustration of an intersection with the locations of the different traffic signals. The green time duration is the period of time within which vehicles are allowed to move from one road to another, while the red time duration is the period of time that vehicles are not allowed to move from their current road. A phase is a configuration that determines the directions that the vehicles are allowed to move to and from, and the cycle length is the time required to perform a cycle of phases [2]. To control the timings of traffic signals,

The associate editor coordinating the review of this manuscript and approving it for publication was Seyedali Mirjalili.

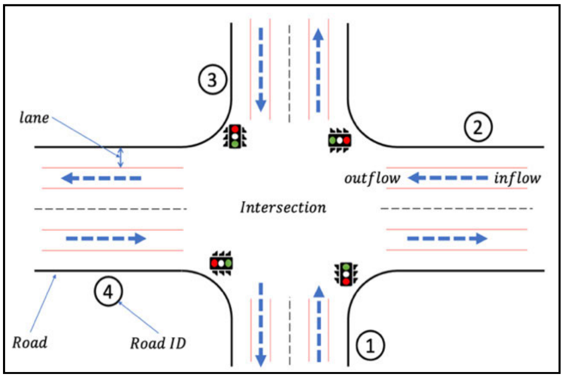

FIGURE 1. Illustration of an intersection.

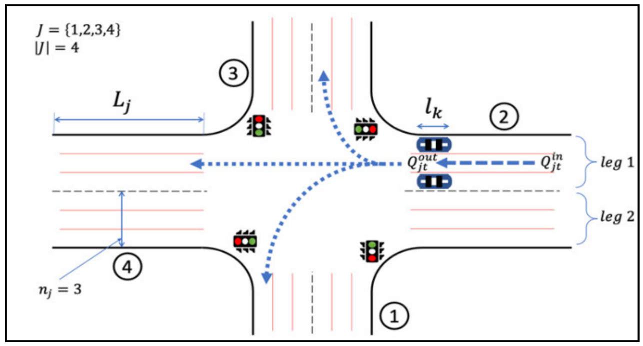

FIGURE 2. A representation for an intersection.

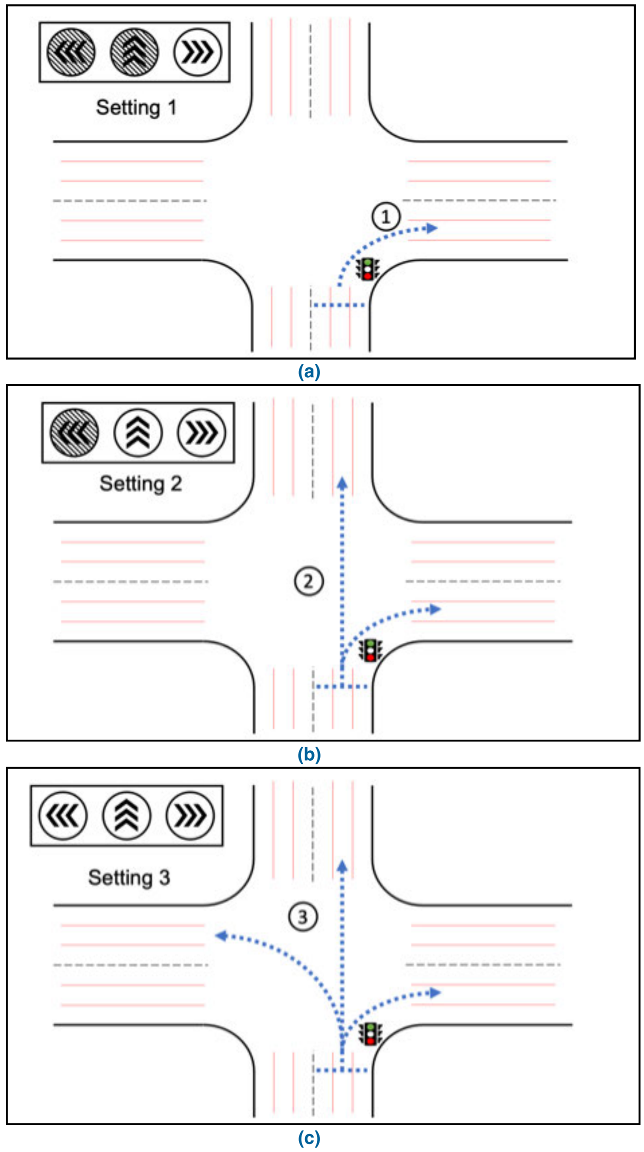

FIGURE 3. Different types of settings for a traffic signal (a) setting 1 (b) setting 2 (c) setting 3.

there is a need for a tool that is effective and efficient in such a dynamic traffic environment. Discrete event simulation is one of the tools used to simulate traffic systems and develop different scenarios to select the best scenario among them. However, the generated solution is not optimum and better solutions may exist [3]. The contributions of this work are:

1. The introduction of a mathematical model for the traffic signal scheduling problem, the model considers the physical constraints related to the problem which have not been investigated before. The model can solve small instances to optimality.
2. The introduction of a new firefly-based constructive heuristic to solve the problem efficiently. This heuristic is used to solve realistic problem instances, that are not tractable by the closed form solution approach.

After this introduction, the problem description is presented in the second section. The related work is discussed in the third section, the proposed new mathematical model is presented in the fourth section, and the proposed new firefly constructive heuristic-based rolling horizon technique is presented in the fifth section. The results and discussion are presented in the sixth section. Finally, the conclusions and the directions for future work are presented.

II. PROBLEM DESCRIPTION

In this work, the traffic signal scheduling problem is investigated considering the traffic-response control system. The problem aims to determine the optimum duration for the green and red times for each traffic signal at each road in a given intersection to achieve a specific objective(s). The start and the duration of these timings (i.e. green and red times) are considered as a schedule for each traffic signal. The schedules of these traffic signals determine the phases which will be activated.

The objective in the presented case is the minimization of the summation of the queue lengths of all the roads of a given intersection.

The structure of a typical intersection is presented in figure 2. An intersection is composed of a number of roads ($|J|$), each road has a length of (L_j), a number of lanes (n_j), and two legs. For each of the roads in a given intersection, there are two important parameters, the inflow from road (j) at each time period (t) to the intersection, and it is represented by (Q_{jt}^{in}), and the outflow from that road at each time period to the other roads and hence outside the intersection, and it is represented by (Q_{jt}^{out}).

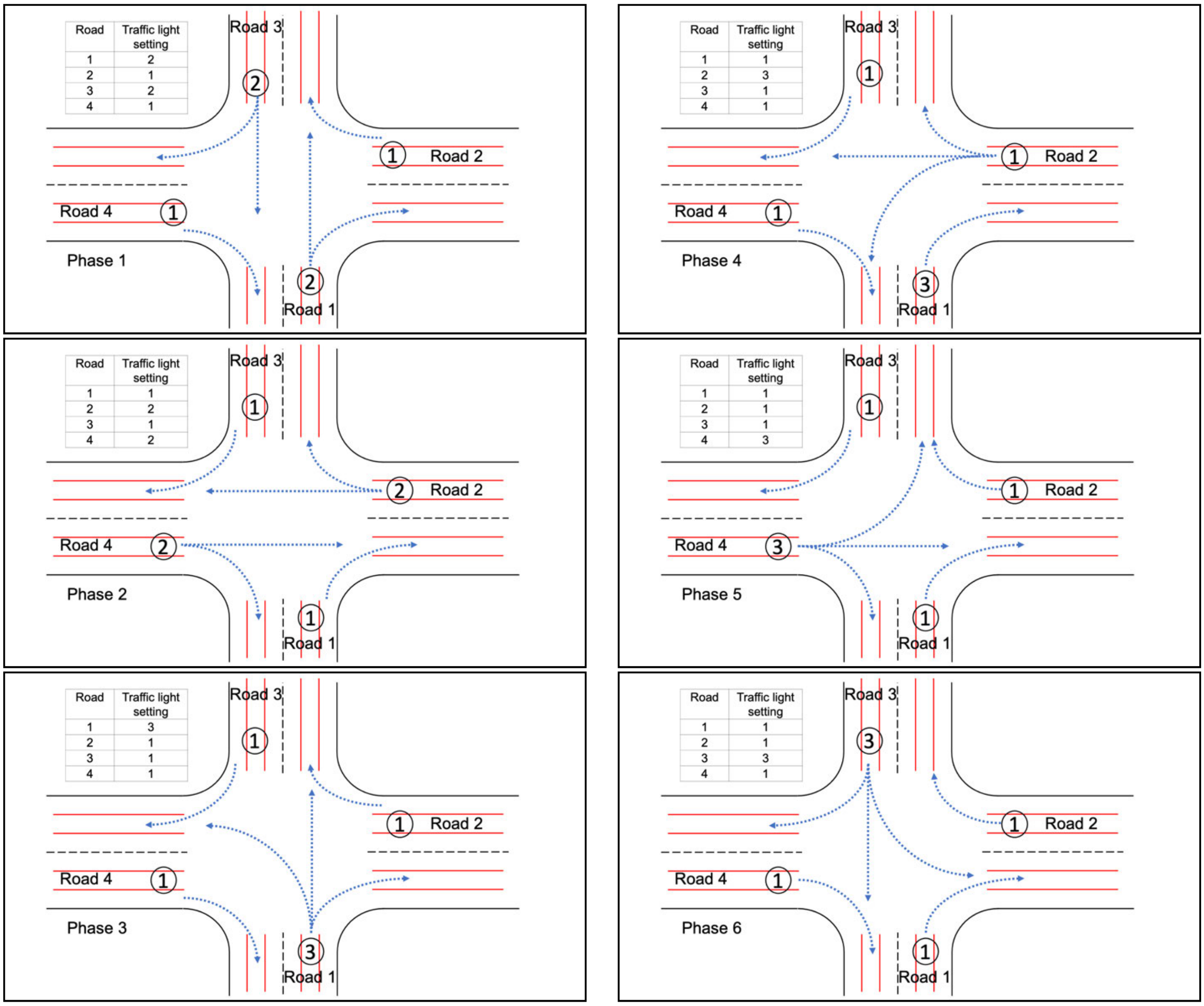

FIGURE 4. Different types of traffic phases.

The total flow to road (j) at time period (t)(TF_{jt}^{in}) is affected by two components; the queue that is accumulated from the previous period ($W_{j(t-1)}$), and the inflow to the road at this time period (Q_{jt}^{in}). Different types of vehicles are passing through the intersection, each vehicle of type (k) has a length of (l_k), and the total number of types of vehicles is (K).

Each road has a Traffic Signal (TS) that regulates the flow from this road to the other roads in the intersection. Each TS has three different settings that restrict the direction of flow from the road to the rest of the roads in an intersection as shown in figure 3. According to the settings of the different (TSs) in an intersection, a phase is formed. The different types of phases are shown in figure 4.

III. RELATED WORK

The literature is reviewed using these keywords: urban traffic light, traffic signals, traffic signals scheduling, and traffic control systems. The reviewed research articles are identified by searching on various search engines such as google search and google scholar. In addition, different databases are surfed to obtain the most relevant research studies. These databases are, for example, Elsevier, and Springer.

Andrea Villagra *et al.*, proposed two versions of a Cellular Genetic Algorithm (CGA) to solve the traffic signal scheduling problem, namely the synchronous and asynchronous versions. They applied their approach to real cases in Spain and France [4], and the results show the superiority of their method. Yousef *et al.*, proposed a history-based traffic management algorithm to tackle the traffic signal scheduling problem. The algorithm depends on the history of traffic data to predict the green/red times for each TL at a given intersection. Their proposed approach has been tested using SUMO (i.e. traffic simulation environment) [5]. Matias Peres *et al.*, presented a methodology based on combining discrete event simulation with a multi-objective evolutionary method to solve the traffic signal scheduling problem. They applied their proposed methodology to three real cases in Uruguay [6].

Kaizhou Gao *et al.*, solved the traffic signals scheduling problem using a discrete harmony search algorithm. The algorithm was tested in a case study in Singapore, and the results show the superiority of the used algorithm [7].

Javier Ferrer *et al.*, proposed an iterated racing algorithm that adjusts the number of traffic scenarios. They applied their proposed algorithm to a real case study in Spain [8]. Peirong (Slade) Wang *et al.*, proposed a linear integer programming formulation for joint optimization of vehicle space-time trajectories and traffic control. Lagrangian decomposition and dynamic programming were used to solve the problem understudy [9].

Sabar *et al.*, provided an adaptive memetic algorithm to find the optimum schedule for each traffic signal, the data was collected using induction loop detectors [10]. Gao *et al.*, studied a large-scale urban traffic signal scheduling problem, they proposed three different metaheuristics to solve the problem. The metaheuristics: Jaya algorithm, harmony search algorithm, and water cycle algorithm were used to minimize the total network delay time [7]. Khooban *et al.*, combined the general type-2 fuzzy logic sets and the Modified Back tracking Search Algorithm (MBSA) to control the traffic signals and phase succession. The solution method guarantees a smooth flow of traffic with minimum waiting times and average queue length [11].

Jiajia et al, used an ant colony optimization algorithm to minimize the time delay of vehicles [12]. Huan *et al.* proposed a genetic optimization algorithm for optimizing the timing durations of traffic signals at a. The authors summarized their model that the coordination of traffic signals at intersections mimic the coordination cells in cellular space. Their experimental results were generated with the aid of urban simulation software [13].

Jin *et al.*, introduced an intelligent control system called Fuzzy Intelligent Traffic Signal (FITS) to control the cycle time of traffic signals. They also developed a computational framework to evaluate the FITS system using microscopic traffic simulation [14]. Computer simulation is a widely used tool to represent traffic systems. Jaime *et al.*, performed different experiments using the PTV-VISSIMTM simulation software to find the best combination of factors that minimizes the average time in the system, fuel consumption, and CO2 emissions [15]. Adriana *et al.*, addressed a multi-objective simulation-based signal control problem through an eco-neighbourhood case study to find the optimal traffic control plan to reduce congestion during peak hours [16].

Optimal scheduling of traffic signals is of importance nowadays to tackle the traffic congestion problem. The literature that considers the traffic-responsive control system is very limited, and most of these studies used approximate solution methods to solve the problem. Therefore, it is crucial to investigate new methods and models that can get the optimum scheduling of traffic signals according to the changes in the flow rates and patterns.

In this work, a mathematical formulation has been proposed to solve the traffic signal scheduling problem. The model considers the minimization of the total queue length that accumulates at an intersection, which is the summation of the total queue lengths of all the roads at a given intersection. To the best of the author's knowledge, this model is the first to capture the physical characteristics of an intersection such as, the length of the roads in the given intersection, the maximum queue length of each road, the number of lanes of each road, the length of different types of vehicles passing through the intersection, and the maximum queue of vehicles which a road can accommodate. In addition, it considers the main aspects of the traffic signals scheduling problem.

In the era of Industry 4.0 and the growth of information and communication technology, the importance of transportation systems increases as a main key in a successful Industry 4.0 system. Travel times need to be decreased to improve the performance measure of the system.

Heuristics are suitable tools to generate efficient and effective solutions to complex problems [17]. In this paper, a firefly-based heuristic is developed to solve the traffic signal scheduling problem efficiently. The firefly algorithm has been specifically used as it shows a very good performance in solving other problems such as the supply chain network design [18], resource allocation [19], vehicle routing problem [20], capacitated vehicle routing problem [19], [20], machine scheduling [21], and layout optimization [22]. All the previous problems can be categorized either in the assignment problems category or in the scheduling and sequencing problems category. Also, the traffic light scheduling problem is categorized in the scheduling problems category. Therefore, the firefly algorithm has been used in the presented problem.

IV. SOLUTION METHODOLOGIES

A. THE MATHEMATICAL PROGRAMMING MODEL

A binary mathematical model has been developed to solve the urban traffic signal scheduling problem. The sets, parameters, decision variables, objective function, and constraints are illustrated next.

1) SETS AND PARAMETERS

J	Set of roads with an index of (j) such that $j \in \{1, 2, \ldots .J\}$.
I	Set of traffic signal settings with an index of (i) such that $i \in \{1, 2, 3\}$.
T	Set of time periods such that $t \in \{1, 2, \ldots, T\}$.
K	Set of vehicles' types such that $k \in \{1, 2, \ldots, K\}$.
R^j_{Comp}	Set of compatible roads of road (j), where $R^j_{Comp} \subset J$.
R^j_{InComp}	Set of incompatible roads of road (j), where $R^j_{InComp} \subset J$.
L_j	Length of road (j).
Q^{in}_{jt}	Inflow to road (j) at time period (t) from outside the intersection (*vehicles*). In other words, it is the total number of vehicles that enter road (j) at time period (t) from outside the interaction.

Q_{jt}^{out} Outflow from road (j) at time period (t), this represents the outflow from road (j) to other roads, and hence outside the intersection – (*vehicles*). In other words, it is the total number of vehicles that leave road (j) at time period (t).

W_{jt} The queue of vehicles, measured by the number of waited vehicles at road (j) at time period (t) due to unserved vehicles – (*vehicles*).

TF_{jt} Total number of vehicles that are accumulated at road (j) at time period (t) . This number considers the inflow to road (j) at time period (t) (Q_{jt}^{in}), and the number of waited vehicles at road (j) from the previous time period ($W_{j(t-1)}$) - (*vehicles*).

C_{max} Maximum cycle length.

C_{red} Minimum red time duration for a setting of a traffic signal.

l_k Average length of type (k) vehicle.

nr_j Number of lanes for road (j).

p_{ibj} Percentage of vehicles that discharged from road (b) to road (j) if traffic signal of setting (i) is set to be "ON".

q_j^{max} Maximum physical queue length of road (j). In other words, it is the maximum number of vehicles that road (j) can accommodate – the queue length is expressed in terms of the number of vehicles.

The set of compatible roads for a specific road (j) can be defined as roads that demonstrate no conflict with road (j) if both roads have either a traffic signal of setting one or setting two. On the other hand, the set of incompatible roads for a specific road (j) can be defined as the roads that demonstrate conflict with road (j) if both roads have a traffic signal of setting two. For example, referring to figure 2 and figure 3, if road one has a traffic signal of setting two sets to be "ON", then the sets of compatible roads and incompatible roads are {3}, and {2,4} respectively. Figure 5 shows the sets of compatible and incompatible roads for each road in a given intersection. The value of one in the figure demonstrates that both roads show no conflict, and the value of zero otherwise. If any road has a traffic signal of setting three is set to be "ON", then all other roads are considered incompatible roads.

2) DECISION VARIABLES

a: IN-DEPENDED DECISION VARIABLES

$x_{ijt} \in \{0, 1\}$, where $x_{ijt} = 1$, if a traffic signal of setting (i) at road (j) is set to be "ON" at time period (t), and $x_{ijt} = 0$, otherwise.

b: DEPENDED DECISION VARIABLES

$y_{ijt} \in \{0, 1\}$, where $y_{ijt} = 1$, if a traffic signal of setting (i) at road (j) is set to be "OFF" at time period (t), and $y_{ijt} = 0$, otherwise. The relationship between x_{ijt} and y_{ijt} is controlled by constraint (2).

Road | Setting		1		2		3		4		Compatable roads	Incompatable roads
		Setting 2	Setting 3	Setting 2	Setting 3	Setting 2	Setting 3	Setting 2	Setting 3		
1	Setting 2	---		0		1	0	0		{3}	{2,4}
	Setting 3					0	0			---	{2,3,4}
2	Setting 2	0		---		0		1	0	{4}	{1,3}
	Setting 3							0	0	---	{1,3,4}
3	Setting 2	1	0	0		---		0		{1}	{2,4}
	Setting 3	0	0							---	{1,2,4}
4	Setting 2	0		1	0	0		---		{2}	{1,3}
	Setting 3			0	0					---	{1,2,3}

FIGURE 5. Compatible and in combatable roads.

3) OBJECTIVE FUNCTION

$$Min \sum_{j=1}^{J} \sum_{t=1}^{T} W_{jt}$$

The objective function of the proposed model is the minimization of the total number of waited vehicles at a given intersection. The total number of waited vehicles is the summation of the waited vehicles of all the roads at a given intersection, for instance, if the number of waited vehicles at roads 1, 2, 3, and 4 are 3, 3, 5, and 6, respectively, then the total number of waited vehicles, in this case, will be 17 vehicles. The mathematical model has been verified using randomly generated instances to ensure that it is working as desired and that all the constraints are necessary and sufficient to reach an optimum solution for the problem.

4) CONSTRAINTS

$$\sum_{i=1}^{I} x_{ijt} \leq 1 \quad \forall j \in J, t \in T \tag{1}$$

$$y_{ijt} = 1 - x_{ijt} \quad \forall i \in I, j \in J, t \in T \tag{2}$$

$$\sum_{t}^{\min(t+C_{max},T)} x_{ijt} + y_{ijt} \leq C_{max} \quad \forall i \in I, j \in J \tag{3}$$

$$\sum_{t}^{\min(t+C_{max},T)} x_{ijt} \leq C_{max} - C_{red} \quad \forall i \in I, j \in J \tag{4}$$

$$q_j^{max} = \frac{KL_j nr_j}{\sum_k (l_k + 1)} \quad \forall j \in J \tag{5}$$

$$x_{ijt} - x_{ij(t+1)} \geq 0 \quad \forall i \in I, j \in J, t \in T \tag{6}$$

$$y_{ijt} - y_{ij(t+1)} \geq 0 \quad \forall i \in I, j \in J, t \in T \tag{7}$$

$$x_{3jt} + \sum_{\substack{b \in J \\ b \neq j}}^{J} \sum_{i=2}^{I} x_{ibt} \leq 1 \quad \forall j \in J, t \in T \tag{8}$$

$$x_{2jt} + \sum_{b \in R^{j}_{InComp}} \sum_{i=2}^{I} x_{ibt} + \sum_{r \in R^{j}_{Comp}} x_{3rt} \leq 1 \quad \forall j \in J, t \in T \tag{9}$$

$$Q^{out}_{jt} = \sum_{\substack{b \in J \\ b \neq j}}^{J} \sum_{i=1}^{I} TF^{in}_{jt} p_{ijb} x_{ijt} \quad \forall j \in J, t \in T \tag{10}$$

$$TF_{jt} = W_{j(t-1)} + Q^{in}_{jt} \quad \forall j \in J, t \in T \tag{11}$$

$$W_{jt} = TF_{jt} - Q^{out}_{jt} \quad \forall j \in J, t \in T \tag{12}$$

$$W_{jt} \leq q^{max}_{j} \quad \forall j \in J, t \in T \tag{13}$$

As mentioned earlier, in the traffic-responsive control system, the traffic system parameters (e.g. green duration, red duration, cycle length…etc.) are not fixed, and they are subjected to changes according to the flow pattern, but these changes are within pre-specified limits. Constraint (1) ensures that at most only one setting for a traffic signal could be "ON" at any time period. The set of constraints (2) is to define the dependent variable (y_{ijt}), which represents the "OFF" status of a traffic signal of setting (i) at road (j) at time period (t). Constraint (3) ensures that for each traffic signal setting at each road, the summation of green time periods and red time periods should not exceed the permitted cycle time.

Constraint (4) ensures the maximum green time duration for a traffic signal setting, for a traffic signal of setting (i) at road (j), the maximum green time is the subtraction of the minimum red time duration from the total cycle time. Constraint (5) determines the maximum queue length for road (j). For each road (j), the maximum queue length is represented by the number of vehicles that the road can absorb. It is affected by the number of lanes of the road (n_j), the number of vehicles' types (K), the length of each type of vehicles (l_k), and the length of the road (L_j). figure 6 illustrates a numerical example of how to calculate the maximum queue for a road.

Constraints (6) and (7) ensure that if a traffic signal of any setting being set to "ON" or "OFF" at any time period, then it should remain "ON" or "OFF" for a specific number of successive time periods which is at most equal to ($C_{max} - C_{red}$) according to constraints (4). In practice, a traffic signal cannot set to be "ON" or "OFF" for a single time period, it should set to be "ON" or "OFF" for a specific number of successive time periods which is determined by constraint (4)

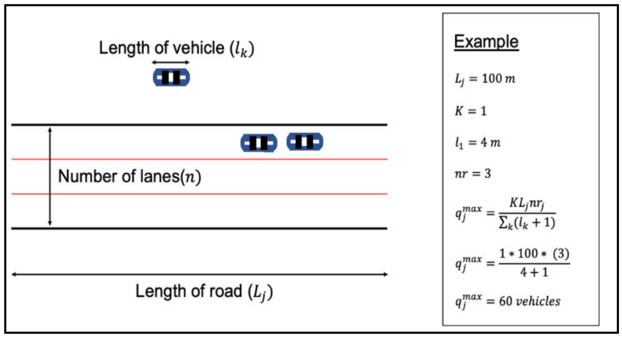

FIGURE 6. Maximum queue for a road.

The sets of constraints (8) and (9) are used to avoid the conflict that may exist due to the different types of traffic signals settings. For example, constraint (8) ensures that if a traffic signal of setting three at any road has set to be "ON" at any time period, then all other traffic signals of settings two or three at the intersection must set to be "OFF" at the same time period. Also, the set of constraints (9) ensures that, if a traffic signal of setting two at any road has been set to be "ON" at any time period, then only the traffic signals of setting two of the compatible roads could set to be "ON".

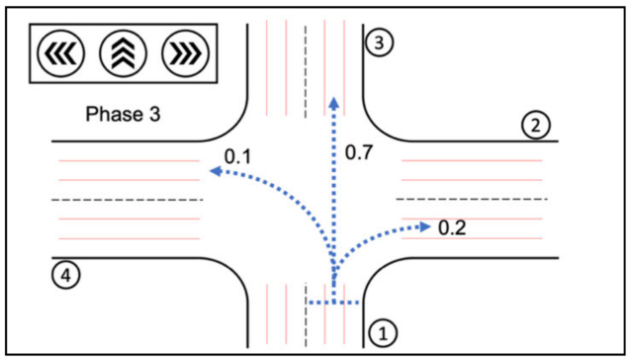

FIGURE 7. An example for the outflow of a road given a traffic signal setting.

Constraint (10) determines the outflow from each road at each time period. According to the traffic signal setting, there is a percentage that determines the flow to the other roads. For example, and as shown in figure 7 road (1) has a traffic signal of setting three which is set to be "ON" at a given time period. For instance, the percentage of vehicles which will be transferred from road (1) to roads (2), (3), and (4) is 0.2, 0.7, and 0.1, respectively. These values could be adapted according to the nature of the intersection understudy.

Constraint (11) calculates the total number of vehicles that accumulated at road (j) at time period (t), this number is equal to the inflow to road (j) at time period (t) and the waiting vehicles at road (j) from the previous time period ($t - 1$). For instance, as shown in figure 8, if the inflow to road (j) at time period (t) is 2 vehicles, and the total number of waiting vehicles at the road from the previous time period ($t - 1$) is 4, then the total number of accumulated vehicles at road (j) is 5. According to the outflow from this road, some of

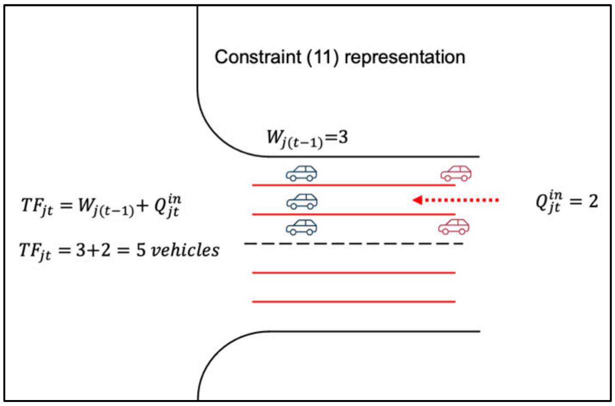

FIGURE 8. Constraint (11) explanation.

these vehicles will be discharged to other roads and others will wait for the next time period ($t + 1$). This process is repeated for each time period to capture the changes in the number of accumulated vehicles.

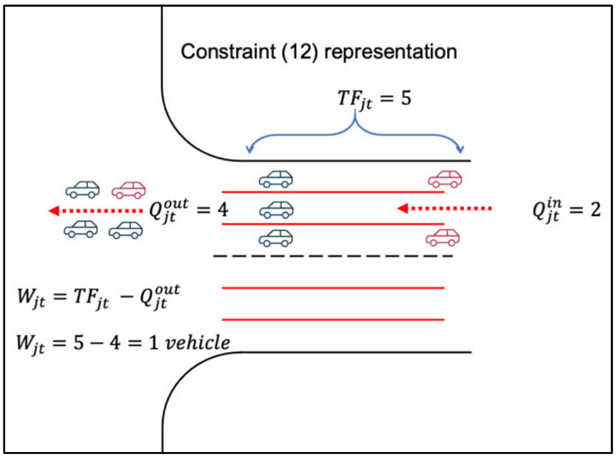

FIGURE 9. Constraint (12) explanation.

Furthermore, the set of constraints (12) determines the queue length of waiting vehicles at road (j) at time period (t), the number of wating vehicles is equal to the total number of accumulated vehicles at road (j) at time period (t) minus the number of vehicles that leave road (j) at time period (t). For example, and as shown in figure 9, if the total number of accumulated vehicles at road (j) at time period (t) is 5 vehicles, and from these 5 vehicles only four vehicles are discharged to other roads, then the number of waiting vehicles, in this case, is only 1 vehicle. In the same manner as constraint (11), this process is repeated for each time period to capture the changes in the number of waiting vehicles. Constraint (13) ensures that the queue length of waiting vehicles at road (j) at time period (t) should not exceed the maximum queue length of road (j).

Although the mathematical model will generate optimum solutions when solving small instances, applying such a model to a real case study is impractical as the computational time is very high. The results, which will be illustrated in section 6, show that getting an optimum solution to the case study will take a very high computational time. Therefore, there is a need for another method that could provide an effective and efficient solution. Hence, the ''firefly constructive-based rolling horizon approach'' heuristic has been proposed.

The firefly algorithm is one of the naturally inspired algorithms, it mimics the behaviour of fireflies to attract mating partners or preys. Fireflies are communicating together through a pattern of flashes, these flashes, in addition to their main function, act as a defence mechanism against predators. The brighter the firefly, the more attractive it is. The brightness (i.e. intensity of the signal) obeys the inverse square law, which states that the signal intensity decreases as the distance between the source and the receiver increases ($I \propto \frac{1}{r^2}$), where (r^2) is the distance between the source of the signal and the receiver [23].

In an optimization context, a firefly could be considered a solution for an optimization problem, and the brightness of the firefly could be considered as the value of the objective function to be optimized. For a maximization problem, the brighter the firefly, the better the solution. The fireflies (i.e. solutions) are attracted to the brightest firefly (i.e. best solution) according to the distance between them. Each firefly has a position, and its attractiveness to the brightest firefly depends on its position and the position of the brightest firefly. For more details regarding the firefly algorithm for continuous optimization problems, the reader is referred to [23].

In this work, the firefly algorithm is adopted to solve the traffic signal scheduling problem. Furthermore, the firefly algorithm is combined with the rolling horizon approach to improve the quality of the constructed solutions as will be explained later. This heuristic will be called ''FBH''.

TABLE 1. The best combination of the heuristic's parameters.

Variable	Description
N	The number of iterations.
n	The population size.
c	The number of cycles in the planning horizon.
z_i^{hk}	The (k^{th}) iteration of generating the schedule of cycle (h) of solution (i).
$B(z_i^{ck})$	The value of the fitness function of the (k^{th}) iteration of generating the schedule of cycle (h) of solution (i).
z_i^h	The schedule of cycle (h) of solution (i) after (g) iterations.
z_i	The complete schedule of firefly (i) (solution (i))
$B(z_i)$	The brightness of firefly (i), or the fitness function of solution (i)
$A(z_i)$	The attractiveness matrix for solution (i)
z^*	The best-recorded solution
p_g	The generated probability
p_s	The pre-specified probability

B. THE FIREFLY-BASED HEURISTIC (FBH)

The nomenclature and the Pseudo code of the implemented (FBH) heuristic are as shown in table 1 and FIGURE 10. The details of the proposed FBH are illustrated in the next subsections.

```
Initialization
Initial population generation
    for i = 1:n
        for h = 1:c
            for k = 1:g
                calculate the brightness (i.e. fitness function) for each cycle (B(z_i^{hk}))
            end for k
            z_i^h ← min{B(z_i^1), B(z_i^2), ... B(z_i^g)}
        end for h
        z_i = [z_i^1, z_i^2, z_i^3, ... z_i^c]
        calculate the brigthness (i.e. fitness function) for each solution (B(z_i))
    end for i

Firefly algorithm
    for j = 1:N
        z* ← min{B(z_1), B(z_2), ... B(z_n),}
        for i = 1:n
            calculate the attractiveness matrix (A(z_i))
            if p_g > p_s
                move z_i towards z* (attractiveness effect)
                update (A(z_i))
            end if
        end for i
    end for j
```

FIGURE 10. **Pseudo code of the firefly constructive heuristic-based rolling horizon technique.**

1) THE WORKING PROCESS OF THE FBH

The following steps describe the working process of the FBH:

1) The FBH starts with generating an initial population that contains a number of solutions equal to the population size (n). each of these solutions is formed using the ''all possible phases'' matrix, which is described in detail in subsection 4. The main objective of the matrix is to ensure that any generated solution is feasible. (Subsections 2, 4, and 5)
2) The brightness (e.g. fitness function) of each solution is calculated, and the solution that has the best fitness function value is labeled as the ''Best solution'' of the ''brightest firefly''. (Subsection 3)
3) For the rest of the solutions in the population, which are not labeled as the ''best solution'', calculate the ''attractiveness matrix''. (Subsection 4 and 6)
4) Improve the solutions according to the ''attractiveness matrix''. Each solution is attracted (e.g. modified) to the ''best solution'' using a set of equations. (Subsection 4 and 6)
5) Repeat the steps from 2 to 4 for (N) number of iterations.

2) SOLUTION REPRESENTATION

Solution representation is crucial to any heuristic. Proper representation could increase the rate of convergence and the quality of the solutions [17]. The proposed representation is as follows, each solution is represented by an $N \times T$ matrix, where N is the number of roads at a given intersection, and T is the number of time periods. The number of time periods could be the cycle length (C_{max}) in the case of the traffic-responsive control system, or it could be the planning horizon in the case of the fixed-time control system. In the case of the fixed-time traffic system, and to maintain a reasonable level of computation time, the planning horizon (T) is divided into cycles each is equal to the cycle length (C_{max}), and the solution is constructed in a sequence based on these cycles while considering the interrelation between these cycles. For instance, if the planning horizon is 100 time periods, and the cycle length is 10 time periods, then the solution is constructed by getting the schedule of the first 10 time periods (time periods from 1 to 10 or the first cycle), then for the second 10 time periods (time periods from 11 to 20 or the second cycle), and so on until constructing the whole solution. On the other hand, in the case of a responsive-traffic control system, the planning horizon is equal to the cycle length.

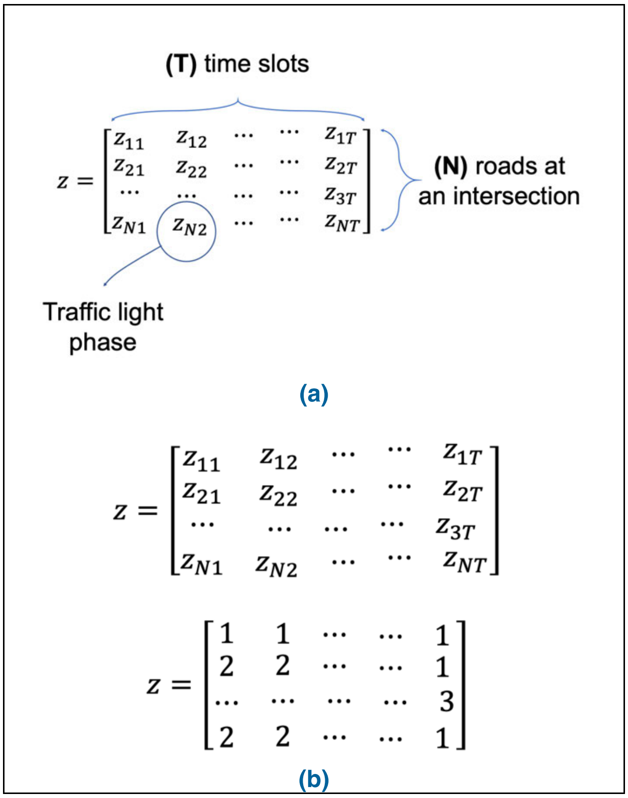

FIGURE 11. **An example of a solution representation.**

The intersection between a row and a column result in a cell representing the traffic signal setting of a road at a given time period. Figure 11(a) shows the proposed representation for a solution. For example, and as shown in figure 11(b), $z_{21} = 2$, which means that the second road at the first time period has an activated traffic signal setting of type 2.

3) SOLUTION QUALITY (BRIGHTNESS)

The quality of each solution is measured by the value of its fitness function (i.e. brightness). For the considered problem, the fitness function minimizes the total number of waited vehicles in the intersection along the planning horizon.

4) THE MATRIX OF THE POSSIBLE PHASES

The possible phases of an intersection are grouped in the matrix of ''all possible phases''. This matrix is developed mainly to avoid any conflict that may arise due to different traffic signal settings, hence it ensures the feasibility of the generated solution. This matrix is used when constructing the solutions as it will be illustrated in the following subsection. The matrix of "all possible phases" is as shown in figure 12. For example, in configuration 2, roads one and three have a traffic signal setting of one, while roads two and four have a traffic signal setting of two, this combination will prevent any conflict that may exist.

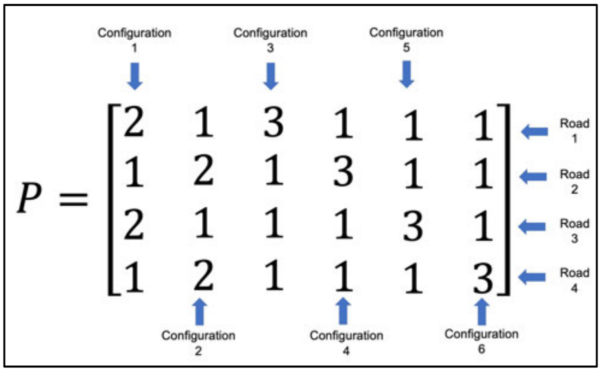

FIGURE 12. The matrix of "all possible phases".

5) SOLUTION FORMULATION BASED ON ROLLING HORIZON APPROACH

Based on the solution representation described earlier, the solution for the traffic signals scheduling problem is based on a constructive heuristic that uses the rolling horizon approach to form an initial solution. Rolling horizon is an approach that splits the whole planning horizon of the main problem into multiple segments, and then it can be solved in sequence as different sub-problems [24]. Rolling horizon has proved to be a worthy choice to deal with different problems such as; airspace sectorization [25], portfolio optimization [26], and scheduling of electric vehicle charging [27]. Based on the rolling horizon approach, the planning horizon is divided into equal segments. For example, in the traffic-responsive control system, the planning horizon is equal to the cycle length (C_{max}), and it is divided into equal segments each is set to (C_{red}).

The constructive heuristic selects from the "matrix of all possible configurations", for each segment, the best combination of phases that generates the minimum fitness function value. The term "best" is based on that, for each segment, a number of iterations (i.e. rolling horizon factor) is performed, and the combination with the best fitness function value is selected to be the solution of this segment. For better illustration, and according to figure 13, three iterations are performed for each segment. For the first segment, the generated schedule in the first iteration is selected as it has the minimum fitness function value. This process is repeated according to the number of segments in the cycle length.

6) SOLUTION IMPROVEMENT USING THE ATTRACTIVENESS EFFECT

The attractiveness effect of the firefly algorithm is used to improve each solution (z_i) by moving it towards the "best" solution (z^*). This is mainly affected by the distance between a firefly and the brighter firefly (r_{z_i,z^*}). Therefore, a solution can be modified according to equation (14), where ($\beta_{r_{z_i,z^*}}$) is the attractiveness factor and it depends on the distance (r_{z_i,z^*}) according to equation (15), where (γ) is the light absorption factor.

$$z^i_{new} = z^i_{old} + \beta_{r_{z^i_{new},z^*}} (z^* - z_i) \tag{14}$$

$$\beta = \frac{1}{1+\gamma r^2} \tag{15}$$

The attractiveness effect is simulated by generating a probability (p_g) for each solution in each iteration, this probability is compared with a pre-specified probability (p_s). If $p_g > p_s$, then this solution is subjected to improvement. The value of (p_s) can be determined using a pilot study of one of the cases related to a considered problem. In the presented problem, a pilot study on one of the cases has been performed to determine the best value of (p_s).

For each solution, an attractiveness matrix $A(z_i)$ is generated. The attractiveness matrix represents the difference between the traffic signal settings of solution (z_i) and the "best" solution (z^*). These differences mimic the distance effect of the firefly algorithm. figure 14 shows an example of an attractiveness matrix. If the value in any cell in the attractiveness matrix is not equal to zero, this means that this cell is a possible chance for improving the solution.

For example, as shown in figure 14, the traffic signal settings of solution (z_i) in the first eight time periods can be modified, while the last four time periods could not. According to a randomly generated position (δ), some time periods can be modified, and others cannot. In the presented example, from the first eight time periods, which can be modified, only the first four are modified, while the other four are not. Therefore, the attractiveness factor is substituted by the attractiveness matrix ($A(z_i)$), and the light absorption factor (γ) is substituted by the randomly generated position (δ). Hence, a solution (z_i) is modified according to equation (16), where the attractiveness matrix is calculated according to equation (17). In the next section, the performance of both methods will be investigated.

$$z^i_{new} = x^i_{old} + A(z_i) \tag{16}$$

$$A(z_i) = \delta(z^i_{old} - z^*) \tag{17}$$

As mentioned earlier, the matrix of "all possible phases" as well as the "attractiveness" matrix guarantee the feasibility of the improved solutions. Any solution subject to improvement is formed using the matrix of "all possible phases", which is already designed to eliminate any conflict that may exist due to unsynchronized settings of the different traffic signals.

V. RESULTS AND DISCUSSION

A. SOLUTION METHODS VERIFICATION

In order to compare the performance of both methods, it is important first to verify the methods. For the mathematical model, the model has been verified using randomly generated

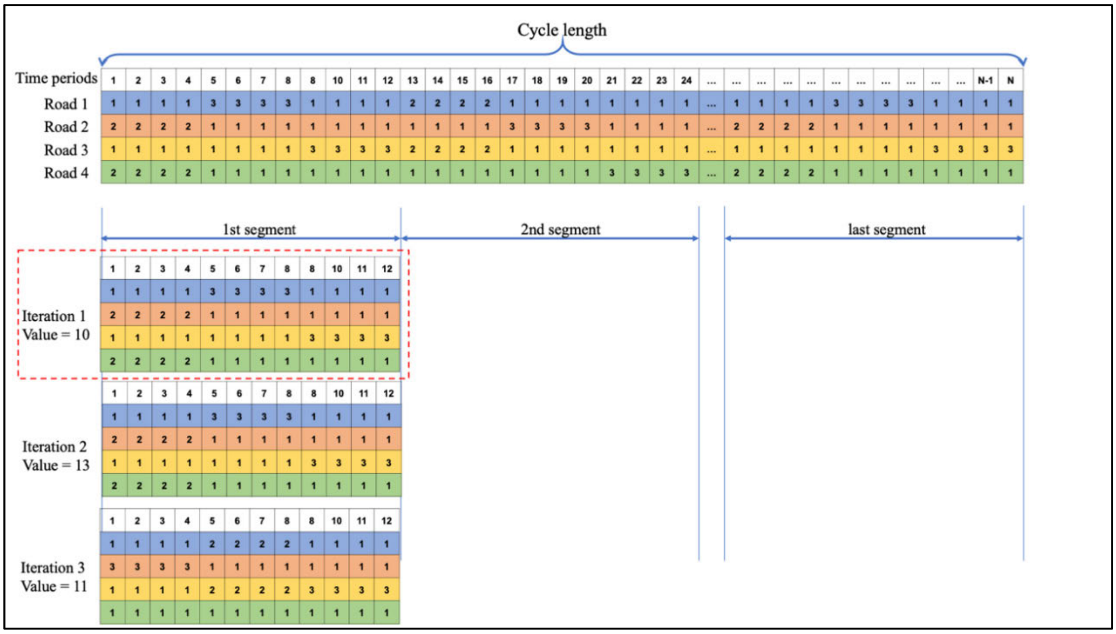

FIGURE 13. Solution formulation based on the rolling horizon technique.

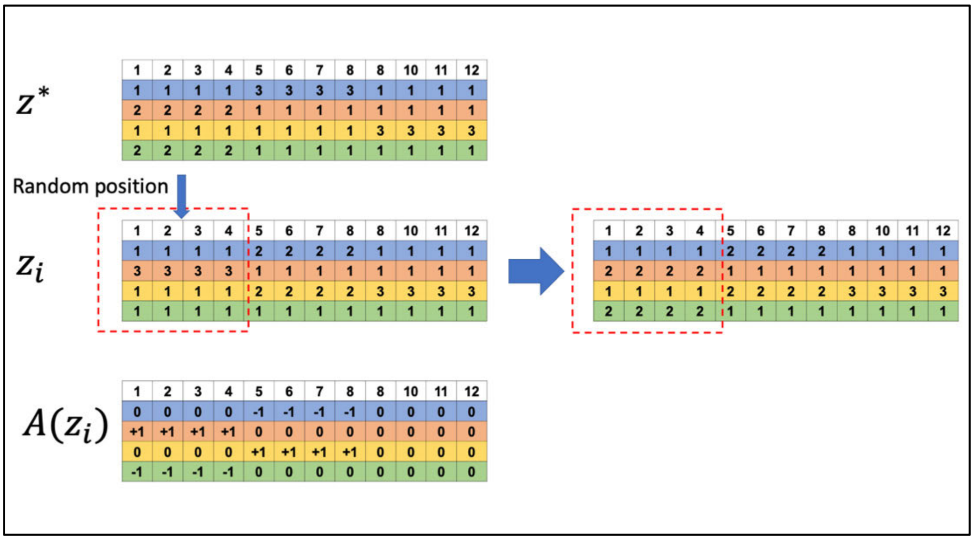

FIGURE 14. Example of an attractiveness matrix.

instances to ensure that all the constraints are necessary and sufficient to generate an optimum solution. On the other hand, the performance of the FBH is affected by four important parameters, which are: the population size (n), the number of iterations (N), the pre-specified probability (p_s), and the rolling horizon factor (g). Hence, it is essential to get the best combination of these parameters.

A pilot experiment has been performed to determine the best combination of these parameters for the given problem. Different values for each parameter have been tested, and the results show that the best combination is shown in Table 2. The details of this experiment are shown in Table 3, for each combination, ten runs have been performed, and the average value has been reported. Each value in the table shows the average gap between the reported solution and the optimum solution which resulted from solving the mathematical model using the branch and bound method. The runs of the best combination are summarized in figure 15. This combination of parameters keeps the average gab at 1.18% from the optimum solution with a standard deviation of 1.2%.

For the number of iterations, and as shown in figure 16, the number of iterations is selected to be 500 as the value of the fitness function already converges starting from the 200$^{\text{th}}$ iteration, hence the number of iterations is set to 500 to make sure that the solution has converged to the minimum value.

B. SOLUTION METHODS VALIDATION

The proposed methods have been applied to one of the most important intersections in the city of Alexandria, the intersection is shown in figure 17 and figure 18. The data for the

TABLE 2. The results of the pilot experiment.

Parameter	N	n	g	(p_s)
Tested range	500 to 10000	{5,10,15,20}	{3,5,7,9}	{0.1,0.3,0.5,0.7,0.9}
Best value	500	15	3	0.9

TABLE 3. Data for the considered intersection.

Population size (n)	Pre-specified probability of attractiveness (p_s)	Rolling horizon factor (g)			
		3	5	7	9
5	0.1	5.36	5.02	5.05	5.74
	0.3	6.51	6.61	6.04	6.37
	0.5	6.09	5.46	6.37	6.37
	0.7	4.47	4.20	5.98	6.37
	0.9	3.35	4.14	6.37	6.37
10	0.1	5.30	4.00	6.37	6.37
	0.3	4.59	4.36	5.89	6.37
	0.5	2.98	5.01	5.89	6.37
	0.7	2.36	3.63	6.37	6.37
	0.9	2.64	3.52	6.37	6.37
15	0.1	5.21	3.48	6.37	6.37
	0.3	2.86	4.62	5.89	6.00
	0.5	3.26	5.08	6.37	6.37
	0.7	2.95	4.49	6.00	6.37
	0.9	1.17	2.92	6.37	6.37
20	0.1	3.53	3.41	6.37	6.37
	0.3	2.84	2.53	6.37	6.37
	0.5	1.18	2.81	6.37	6.37
	0.7	1.26	4.59	6.37	6.37
	0.9	1.68	4.02	6.37	6.37

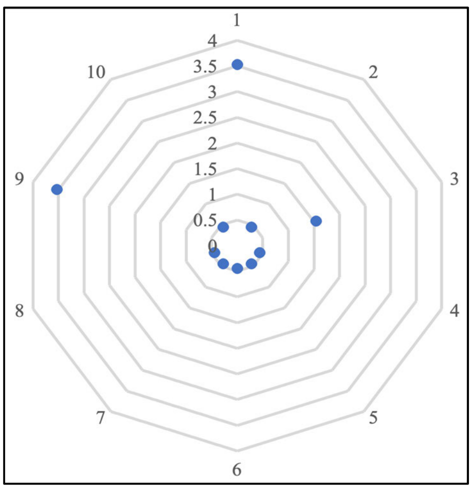

FIGURE 15. The results of the ten runs of the best combination.

intersection understudy is as shown in Table 4. These data have been acquired by field visits and google earth measuring tools.

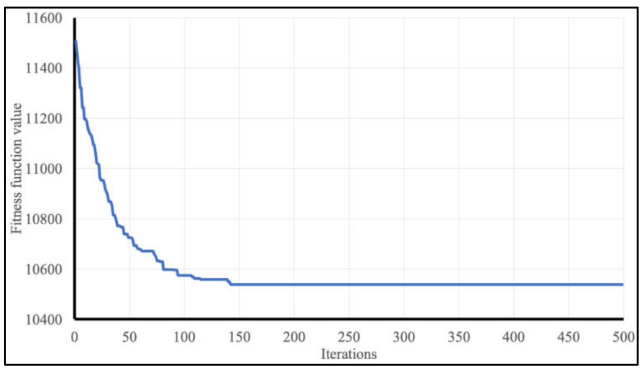

FIGURE 16. The convergence curve for one of the test runs.

The planning horizon was selected to start from 12:00 P.M to 6:00 P.M, which includes the rush hours for this intersection (i.e. between 2:00 P.M to 4:00 P.M). All the experiments were done using 2.4 GHz Quad-Core Intel Core i5 CPU with 8 GB of Ram, and Gurobi®was used as an optimization software to solve the proposed model which uses the branch-and-bound algorithm in solving such types of

TABLE 4. The summary of the results of applying the proposed methods to the case study.

Parameter	Value	Parameter	Value
Length of the first road - L_1 (m)	300	The number of lanes of the second road - nr_2	3
Length of the second road - L_2 (m)	350	The number of lanes of the third road - nr_3	4
Length of the third road - L_3 (m)	180	The number of lanes of the fourth road - nr_4	2
Length of the fourth road - L_4 (m)	275	Road 1 inflow (vehicles/time period)	Uniform (1,3), Uniform (2,4) in the peak period
Number of types of vehicles – K	1	Road 2 inflow (vehicles/time period)	Uniform (0,2), Uniform (1,2) in the peak period
Length of type (k) vehicle - l_k (m)	4	Road 3 inflow (vehicles/time period)	0
The number of lanes of the first road - nr_1	3	Road 4 inflow (vehicles/time period)	Uniform (0,1), Uniform (1,2) in the peak period
Cycle length - C_{max}	160 (S)	Minimum read time - C_{red}	Minimum 0 and maximum 40 (S)

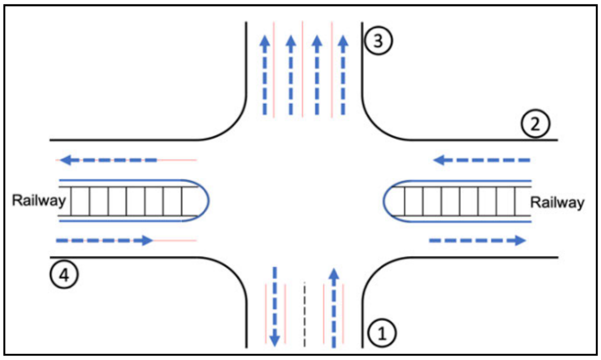

FIGURE 17. Top view for the considered intersection, Alexandria, Egypt.

models. Moreover, the FBH has been developed using Python 3.8. The code of the FBH as well as the data for the case study can be found at (https://github.com/hanaasoliman/-Real-time-traffic-signal-scheduling-using-firefly-heuristic).

FIGURE 18. A google earth photo for the intersection.

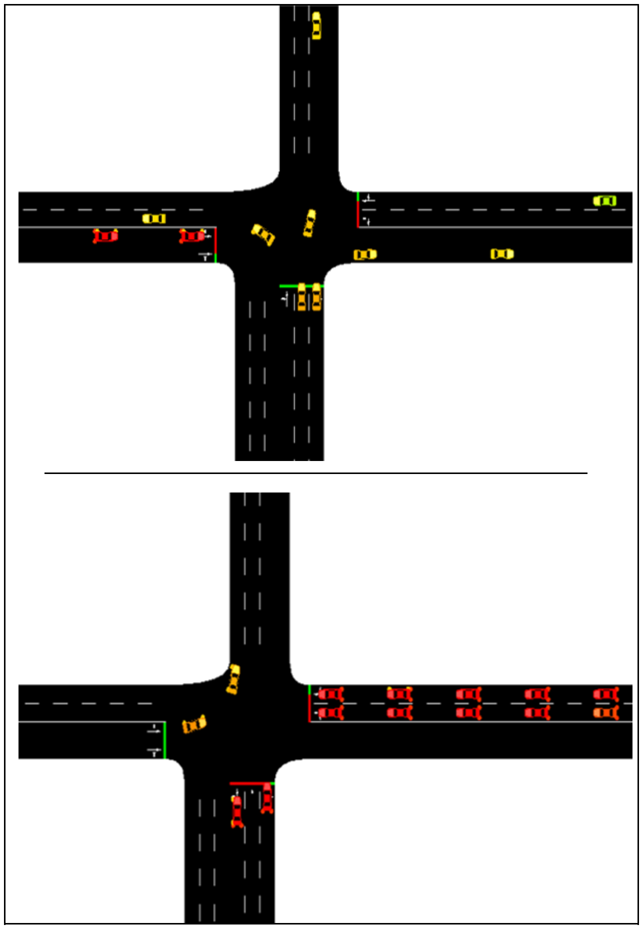

FIGURE 19. Snapshot from SUMO traffic simulator.

The solutions generated by both methods have been simulated using SUMO traffic simulation software to validate the solutions as shown in figure 19. Table 5 shows the summary of the results of applying the proposed methods to the case study. The average values of the performance measure which are stated in the table represent the average during the planning horizon of six hours. The mathematical model failed to get an optimum solution within a reasonable time. Therefore, a time limit of 10 seconds is set for the mathematical model, and the best solution is reported. The time limit of 10 seconds was selected as in the traffic-responsive control systems it is desired to get the schedule of traffic signals in a very short time.

The FBH heuristic can get better performance measures than that of the mathematical model. as for the presentation

TABLE 5. Nomenclature of the proposed (FRHH).

Solution methodology	The average number of waited vehicles	The average waiting time	The average travel time	The maximum number of waited vehicles	The maximum waiting time
Mathematical model	47	600 (S)	141 (S)	114	1288 (S)
FBH	15	478 (S)	92 (S)	67	987 (S)

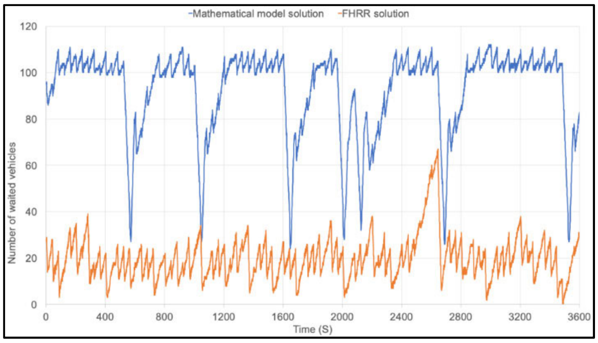

FIGURE 20. Instantaneous number of waiting vehicles for one hour simulation time (2:30 P.M to 3:30 P.M).

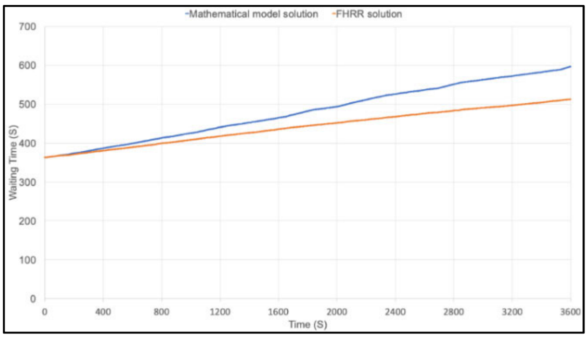

FIGURE 21. Instantaneous waiting time for one hour simulation time (2:30 P.M to 3:30 P.M).

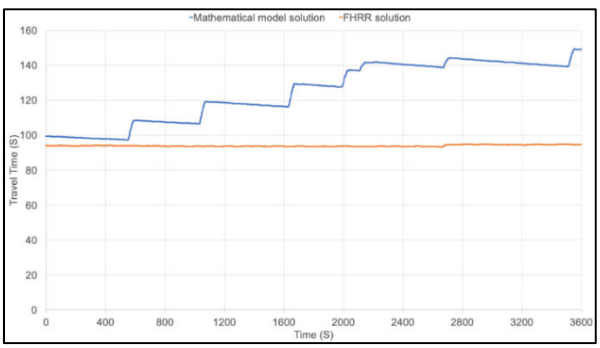

FIGURE 22. Instantaneous travel time for one hour (2:30 P.M to 3:30 P.M).

purposes, figure 20, figure 21, II, show the instantaneous number of waiting vehicles, the instantaneous waiting time, and the instantaneous travel time, respectively for one hour of the simulation run. The interesting remark is that the FBH shows almost a steady behaviour according to the used measures.

VI. CONCLUSION AND FUTURE WORK

In this work, the traffic signal scheduling problem considering the traffic-responsive control system was investigated. A new mathematical model for the traffic signal scheduling problem was proposed, the model captures the physical constraints of the problem as well as the main aspects of the traffic signals scheduling problem. For real-time traffic control, a firefly-based heuristic has been proposed to solve the problem effectively and efficiently. Different parameters' values have been tested and the best combination has been selected for the proposed heuristics. Both methods have been validated using a case study and their performance was compared using three different measures: the average number of waiting vehicles, the average waiting time, and the average travel time. The results show that the performance of the proposed FBH outperforms the performance of the mathematical model. Therefore, it is recommended to apply such approximate solution methods in case of using traffic-responsive control systems as it needs efficient solutions. As for future work, the proposed firefly-based algorithm is to be compared against other metaheuristics, and both methods will be used to investigate the real-time control of a network not only of an intersection.

REFERENCES

[1] X. Zheng and W. Recker, ''An adaptive control algorithm for traffic-actuated signals,'' *Transp. Res. C, Emerg. Technol.*, vol. 30, pp. 93–115, May 2013, doi: 10.1016/j.trc.2013.02.007.

[2] S. A. Celtek, A. Durdu, and M. E. M. Alı, ''Real-time traffic signal control with swarm optimization methods,'' *Measurement*, vol. 166, Dec. 2020, Art. no. 108206, doi: 10.1016/j.measurement.2020.108206.

[3] H. S. Abohashima, M. Gheith, and A. Eltawil, ''Solving the urban traffic lights scheduling problem using discrete event simulation and design of experiments,'' in *Proc. Int. Conf. Comput. Ind. Eng., (CIE)*, Oct. 2019, pp. 1–10.

[4] A. Villagra, E. Alba, and G. Luque, ''A better understanding on traffic light scheduling: New cellular GAs and new in-depth analysis of solutions,'' *J. Comput. Sci.*, vol. 41, Mar. 2020, Art. no. 101085, doi: 10.1016/j.jocs.2020.101085.

[5] K. M. A. Yousef, A. Shatnawi, and M. Latayfeh, ''Intelligent traffic light scheduling technique using calendar-based history information,'' *Future Gener. Comput. Syst.*, vol. 91, pp. 124–135, Feb. 2019, doi: 10.1016/j.future.2018.08.037.

[6] M. Péres, G. Ruiz, S. Nesmachnow, and A. C. Olivera, ''Multiobjective evolutionary optimization of traffic flow and pollution in montevideo, Uruguay,'' *Appl. Soft Comput.*, vol. 70, pp. 472–485, Sep. 2018, doi: 10.1016/j.asoc.2018.05.044.

[7] K. Gao, Y. Zhang, A. Sadollah, and R. Su, ''Optimizing urban traffic light scheduling problem using harmony search with ensemble of local search,'' *Appl. Soft Comput.*, vol. 48, pp. 359–372, Nov. 2016, doi: 10.1016/j.asoc.2016.07.029.

[8] J. Ferrer, M. López-Ibáñez, and E. Alba, ''Reliable simulation-optimization of traffic lights in a real-world city,'' *Appl. Soft Comput.*, vol. 78, pp. 697–711, May 2019, doi: 10.1016/j.asoc.2019.03.016.

[9] P. Wang, P. Li, F. R. Chowdhury, L. Zhang, and X. Zhou, "A mixed integer programming formulation and scalable solution algorithms for traffic control coordination across multiple intersections based on vehicle space-time trajectories," *Transp. Res. B, Methodol.*, vol. 134, pp. 266–304, Apr. 2020, doi: 10.1016/j.trb.2020.01.006.

[10] N. R. Sabar, L. M. Kieu, E. Chung, T. Tsubota, and P. E. M. de Almeida, "A memetic algorithm for real world multi-intersection traffic signal optimisation problems," *Eng. Appl. Artif. Intell.*, vol. 63, pp. 45–53, Aug. 2017, doi: 10.1016/j.engappai.2017.04.021.

[11] M. H. Khooban, N. Vafamand, A. Liaghat, and T. Dragicevic, "An optimal general type-2 fuzzy controller for urban traffic network," *ISA Trans.*, vol. 66, pp. 335–343, Jan. 2017, doi: 10.1016/j.isatra.2016.10.011.

[12] J. Jin, X. Ma, and I. Kosonen, "An intelligent control system for traffic lights with simulation-based evaluation," *Control Eng. Pract.*, vol. 58, pp. 24–33, Jan. 2017, doi: 10.1016/j.conengprac.2016.09.009.

[13] J. E. Mondragón, J. A. J. García, J. M. M. Flores, J. A. V. López, and S. T. Vázquez, "Experiments simulation and design to set traffic lights' operation rules," *Transp. Policy*, vol. 67, pp. 2–12, Sep. 2018, doi: 10.1016/j.tranpol.2017.09.014.

[14] A. S. Mihăiţă, L. Dupont, and M. Camargo, "Multi-objective traffic signal optimization using 3D mesoscopic simulation and evolutionary algorithms," *Simul. Model. Pract. Theory*, vol. 86, pp. 120–138, Aug. 2018, doi: 10.1016/j.simpat.2018.05.005.

[15] M. Gheith, A. B. Eltawil, and N. A. Harraz, "Solving the container pre-marshalling problem using variable length genetic algorithms," *Eng. Optim.*, vol. 48, no. 4, pp. 687–705, Apr. 2016, doi: 10.1080/0305215X.2015.1031661.

[16] S. Khalifehzadeh and M. B. Fakhrzad, "A modified firefly algorithm for optimizing a multi stage supply chain network with stochastic demand and fuzzy production capacity," *Comput. Ind. Eng.*, vol. 133, pp. 42–56, Jul. 2019, doi: 10.1016/j.cie.2019.04.048.

[17] H.-X. Xing, H. Wu, Y. Chen, and K. Wang, "A cooperative interference resource allocation method based on improved firefly algorithm," *Defence Technol.*, vol. 17, no. 4, pp. 1352–1360, Aug. 2021, doi: 10.1016/j.dt.2020.07.006.

[18] D. Trachanatzi, M. Rigakis, M. Marinaki, and Y. Marinakis, "A firefly algorithm for the environmental prize-collecting vehicle routing problem," *Swarm Evol. Comput.*, vol. 57, Sep. 2020, Art. no. 100712, doi: 10.1016/j.swevo.2020.100712.

[19] A. M. Altabeeb, A. M. Mohsen, L. Abualigah, and A. Ghallab, "Solving capacitated vehicle routing problem using cooperative firefly algorithm," *Appl. Soft Comput.*, vol. 108, Sep. 2021, Art. no. 107403, doi: 10.1016/j.asoc.2021.107403.

[20] A. M. Altabeeb, A. M. Mohsen, and A. Ghallab, "An improved hybrid firefly algorithm for capacitated vehicle routing problem," *Appl. Soft Comput.*, vol. 84, Nov. 2019, Art. no. 105728, doi: 10.1016/j.asoc.2019.105728.

[21] A. A. Ewees, M. A. A. Al-Qaness, and M. A. Elaziz, "Enhanced salp swarm algorithm based on firefly algorithm for unrelated parallel machine scheduling with setup times," *Appl. Math. Model.*, vol. 94, pp. 285–305, Jun. 2021, doi: 10.1016/j.apm.2021.01.017.

[22] A. Kaveh and M. R. Seddighian, "Simultaneously multi-material layout, and connectivity optimization of truss structures via an enriched firefly algorithm," *Structures*, vol. 27, pp. 2217–2231, Oct. 2020, doi: 10.1016/j.istruc.2020.07.029.

[23] X.-S. Yang, *Engineering Optimization: An Introduction With Metaheuristic Applications*. Hoboken, NJ, USA: Wiley, 2009.

[24] L. Tang, J. Guo, B. Zhao, X. Wang, C. Shao, and Y. Wang, "Power generation mix evolution based on rolling horizon optimal approach: A system dynamics analysis," *Energy*, vol. 224, Jun. 2021, Art. no. 120075, doi: 10.1016/j.energy.2021.120075.

[25] C. S. Y. Wong, S. Suresh, and N. Sundararajan, "A rolling horizon optimization approach for dynamic airspace sectorization," *IFAC J. Syst. Control*, vol. 11, Mar. 2020, Art. no. 100076, doi: 10.1016/j.ifacsc.2020.100076.

[26] C. Corinaldesi, D. Schwabeneder, G. Lettner, and H. Auer, "A rolling horizon approach for real-time trading and portfolio optimization of end-user flexibilities," *Sustain. Energy, Grids Netw.*, vol. 24, Dec. 2020, Art. no. 100392, doi: 10.1016/j.segan.2020.100392.

[27] J. Su, T. T. Lie, and R. Zamora, "A rolling horizon scheduling of aggregated electric vehicles charging under the electricity exchange market," *Appl. Energy*, vol. 275, Oct. 2020, Art. no. 115406, doi: 10.1016/j.apenergy.2020.115406.

HANAA ABOHASHIMA received the M.Sc. degree in industrial engineering from Fayoum University, Fayoum, Egypt. She is currently pursuing the Ph.D. degree in industrial engineering with Egypt-Japan University of Science and Technology, Alexandria, Egypt.

She is also an Assistant Lecturer with the Mechanical Engineering Department, Fayoum University. Her current research interests include traffic management and control simulation, optimization, modeling, machine learning, and scheduling.

AMR ELTAWIL is currently the Dean of the School of Innovative Design Engineering, Egypt-Japan University of Science and Technology (E-JUST), Alexandria, Egypt. His research interests include applied operations research, discrete event simulation, supply chain design and management, production planning and scheduling, service systems planning and management especially container terminal operations and management decisions, energy systems optimization, healthcare applications and call centers, as well as transportation and logistics.

MOHAMED GHEITH is currently an Assistant Professor with the Department of Industrial and Manufacturing Engineering, Egypt-Japan University of Science and Technology. He has many publications in the field of services management and combinatorial optimization and performed many professional training for many companies in Egypt. His research interests include applied operations research, combinatorial optimization, and services management.