IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received August 21, 2021, accepted September 5, 2021, date of publication September 13, 2021, date of current version September 21, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3112179

Adaptive Finite-Time Tracking Control for Nonlinear Time-Varying Delay Systems With Full State Constraints and Input Saturation

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This work was supported by Guangdong Science and Technology Project under Grant 2015B010133002 and Grant 2017B090910011.

ABSTRACT This paper investigates the adaptive finite-time tracking control problem for a class of nonlinear time-varying delay systems subject to full state constraints and input saturation. The nonlinear state-dependent functions (NSDFs) are introduced to handle the asymmetric time-varying full state constraints without feasibility conditions. To cope with the unknown time-varying delays, the radial basis function neural networks (RBF NNs) and the finite covering lemma are utilized. Avoiding the use of the Lyapunov-Krasovskii functionals (LKFs), no restriction on the derivative of the time-varying delays is needed. Meanwhile, the effect of the input saturation is eliminated by the augmented function with an auxiliary control signal. The adaptive finite-time tracking controller with only one adaptive parameter is constructed by applying the command filter approach and backstepping technique. It is proved that the proposed controller can ensure that all signals in the closed-loop system are bounded and the tracking error converges to a small neighborhood of the origin in a finite time. Finally, the effectiveness of the proposed scheme is demonstrated by two simulation examples.

INDEX TERMS Nonlinear systems, time delays, full state constraints, neural networks, finite-time.

I. INTRODUCTION

In control theory, the existence of time delays may degrade the stability of the systems and give rise to the difficulties of control design. In practical applications, many real systems often suffer from the effect of various constraints, such as output constraints [1], error constraints [2], state constraints [3], and input saturation [4]. Driven by theoretical challenges and application demands, it is necessary to research the control design for nonlinear time-delay systems with full state constraints and input saturation.

Over the past decades, a great deal of attention has been paid to the development of adaptive control. As universal approximators, fuzzy logic systems (FLSs) and NNs have been widely used to approximate the uncertain functions in the adaptive control design [5]–[7]. For example, the adaptive fuzzy output feedback control problem for a class of switched nontriangular structure nonlinear systems was addressed in [5]. Liang *et al.* [6] proposed an adaptive event-triggered neural control strategy for nonaffine pure-feedback nonlinear multiagent systems. In [7], a novel adaptive fault-tolerant controller has been constructed for stochastic discrete-time nonlinear systems. On the other hand, the LKFs have become a popular tool to solve time delays [8]. By combining NNs or FLSs with LKFs, several adaptive tracking control schemes were proposed for nonlinear strict-feedback systems with constant time delays in [9]–[11]. To solve time-varying delays, the LKFs and backstepping technique were utilized in [12]. It should be noted that the LKFs-based methods usually require the time derivative of time-varying delays to be less than one. This drawback can be overcome effectively by the Lyapunov–Razumikhin approach [13]. Based on Lyapunov–Razumikhin approach and backstepping technique, an adaptive state-feedback controller was designed for stochastic nonlinear time-delay systems with perturbations [14]. Nevertheless, the developed controllers for

The associate editor coordinating the review of this manuscript and approving it for publication was Di He.

nonlinear time-varying delay systems in [12]–[14] were memoryless and conservative. To solve this restriction the adaptive memory control approach has been developed in [15]. It is noted that the above works are mainly concerned with the infinite-time stabilization, which implies that the desired system performance is guaranteed as time goes infinity. However, the control objective is required to be achieved in a finite time for many practical applications. Therefore, the finite-time control is worth being further studied.

The finite-time stability was first proposed in [16]. Compared with infinite-time control, finite-time control owns the property of faster convergence, good robustness, and better anti-disturbance performance. Therefore, adaptive finite-time tracking control has been an active topic and considerable results in this field have been obtained for various nonlinear systems [17]–[19]. However, these works are not able to solve the explosion of complexity problem, which is caused by the repeated derivatives of the virtual control signals in the conventional backstepping procedure. To address the above issue, dynamic surface control (DSC) was proposed for a class of nonlinear strict-feedback systems in [20]. Liu *et al.* [21] extended this work to the nonlinear non-strictfeedback systems. It is worth mentioning that the filtering errors caused by the first-order filter are not concerned in the DSC-based schemes, which may deteriorate the control accuracy. This drawback can be overcome by the command filter method. In [22], Sheng *et al.* proposed a command filter-based adaptive fuzzy control scheme for parametric uncertain nonlinear systems with nonlinear faults. By employing NNs and command filter approach, an adaptive control strategy for uncertain nonlinear systems with unknown disturbances was presented in [23]. Furthermore, the finite-time command filter backstepping control problem for a class of nonlinear stochastic systems was addressed in [24]. However, the constraint problem is not considered in the above studies.

It is well known that constraints widely exist in practical systems. In the past pears, notable constraint-handling methods such as barrier Lyapunov functions (BLFs) [1], set invariance [25], reference governors [26], and model predictive control [27] have been well studied. Among these methods, the BLFs-based control has received increasing attention [3], [28]–[31]. With the help of BLFs and command filter method, the adaptive output feedback control problem of full-state constrained nonlinear systems was addressed in [3]. The adaptive DSC design of a class of stochastic nonlinear systems with full state constraints was proposed by employing the BLFs in [28]. For the nonlinear constrained switched system, an adaptive output feedback control scheme based on BLFs was proposed in [29]. Moreover, adaptive finite-time tracking control has been investigated for nonlinear time-varying full stateconstrained systems in strict-feedback [30] and pure-feedback [31] form, respectively. It is worth pointing out that the BLFs-based methods require virtual control signals to satisfy the feasibility conditions. Such restrictive feasibility conditions may bring significant difficulty for the controller design and implementation. To remove the feasibility conditions, the NSDFs were proposed in [32] Recently, the adaptive finite-time fuzzy control for full-state constrained high-order nonlinear systems has been investigated by integrating NSDFs into control design [33]. On the other hand, many practical systems suffer from the input saturation problem. By using the auxiliary dynamic systems, the input saturation problem was solved for marine surface vessels [4] and spacecraft systems [34]. Based on the designed auxiliary system and BLFs, a fuzzy two-bits-triggered control method for nonlinear uncertain systems with input saturation and output constraint has been proposed in [35]. Furthermore, an adaptive NNs-based tracking control approach was investigated for uncertain nonlinear systems with full state constraints and input saturation in [36]. Zhu *et al.* [37] extended this result to the stochastic nonlinear systems. However, the control methods in [4], [33]–[37] are not suitable for the controller design of nonlinear time-delay systems. To the best of our knowledge, the adaptive finite-time tracking control problem has not been fully studied for nonlinear time-varying delay systems with full state constraints and input saturation.

Motivated by the above discussion, this paper aims to design an adaptive finite-time tracking control method for the nonlinear time-varying delay systems in presence of full state constraints and input saturation. The main contributions of this paper are summarized as follows:

1) The finite-time tracking control is applied for nonlinear full state-constrained systems subject to time-varying delays and input saturation without feasibility conditions for the first time. Compared with the works [28], [30], [36]–[40] on the full state-constrained nonlinear systems, the feasibility conditions are removed by introducing the NSDFs.
2) The finite covering lemma combined with the RBF NNs is employed to eliminate the effect of unknown time-varying delays, where the restriction that the time derivative of time-varying delays is less than one in traditional LKFs [41]–[43] is no more needed. Though the above-mentioned restriction can also be solved by the Lyapunov-Razumikhin approach [44], [45], Assumption 3 in [44] and Assumption 4 in [45] are not demanded in our proposed control method, which makes the proposed method less demanding and more flexible in practical applications.
3) By adopting the RBF NNs approximation method, the linear growth conditions of unknown nonlinear functions are eliminated. Moreover, only one adaptive parameter needs to be updated online, greatly reducing the computational burden.

The rest of this paper is organized as follows. The problem statement and preliminaries are given in Section II. The controller design and stability analysis are presented in Section III. The simulation examples are shown in Section IV to illustrate the effectiveness of the proposed method. Finally, Section V concludes this paper.

II. PROBLEM STATEMENT AND PRELIMINARIES

Consider the nonlinear time-varying delay system as follows:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)\, x_{i+1} + h_i(\bar{x}_i(t-\tau_i(t))) + d_i(t) \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)\, u(v) + h_n(\bar{x}_n(t-\tau_n(t))) + d_n(t) \\ y = x_1, \quad i = 1, \ldots, n-1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^T \in R^i, i = 1, \ldots, n-1$ are the system state vectors. $f_i(\cdot) : R^i \to R$ and $h_i(\cdot) : R^i \to R$ are the unknown nonlinear smooth functions, $g_i(\cdot) : R^i \to R$ is the smooth function, $\tau_i(\cdot) \in R$ denotes the unknown time-varying delay, and $d_i(\cdot) \in R$ denotes the uncertain external disturbance. $y \in R$ is the system output. $u(v) \in R$ is the system input subject to saturation nonlinearity, which is described as

$$u(v) = \mathrm{sat}(v) = \begin{cases} u_{\max}, & v > u_{\max} \\ v, & u_{\min} \le v \le u_{\max} \\ u_{\min}, & v < u_{\min} \end{cases} \tag{2}$$

where v is the controller to be designed. $u_{\max} > 0$ and $u_{\min} < 0$ are known constants.

A smooth piecewise function $p(v)$ is adopted to denote the approximation of the input saturation function and $p(v)$ is given as follows:

$$p(v) = \begin{cases} u_{\max} \tanh\left(\dfrac{v}{u_{\max}}\right), & v \ge 0 \\ u_{\min} \tanh\left(\dfrac{v}{u_{\min}}\right), & v < 0 \end{cases} \tag{3}$$

Then, sat(v) can be rewritten as sat$(v) = p(v) + q(v)$, and we have

$$\begin{aligned} |q(v)| &= |\mathrm{sat}(v) - p(v)| \\ &\le \max\{|U_{\max}(1-\tanh(1))|, |U_{\min}(1-\tanh(1))|\} \\ &= F_1 \end{aligned} \tag{4}$$

In this paper, all states are required to satisfy

$$x_i \in \Omega_i := \left\{x_i \in R : -k_{a_i}(t) < x_i < k_{b_i}(t) \quad i = 1, \ldots, n\right\} \tag{5}$$

where $k_{a_i}(t) : R_+ \to R$ and $k_{b_i}(t) : R_+ \to R$ are satisfying $k_{b_i}(t) > k_{a_i}(t) > 0, \forall t \in R_+$.

The control objective is to make the system output y follow the desired signal y_d in a finite time while ensuring that the asymmetric time-varying full state constraints are not transgressed and all signals in the closed-loop system are bounded.

Assumption 1: There exist unknown positive constants $D_1, \ldots, D_n$ such that $|d_i(\cdot)| \le D_i, i = 1, \ldots, n$.

Assumption 2: The desired signal y_d and its first-order time derivative $\dot{y}_d$ are available.

Assumption 3: The control gain functions $g_i(\cdot), i = 1, \ldots, n$ are known. There exist positive unknown constants g_0 and $\bar{g}_0$ such that $g_0 \le |g_i(\cdot)| \le \bar{g}_0$. Without loss generality, we assume that all $g_i(\cdot)$ are positive.

Remark 1: Numerous physical plants can be converted into the special cases of the nonlinear system (1), such as robotic manipulators [46], chemical processes [47], and helicopter systems [48].

Remark 2: It should be mentioned that the desired signal y_d and its $ith, i = 1, \ldots, n$ order time derivatives are required in traditional backstepping [17], [42], [45], [49]. Besides, the exact information of $y_d, \dot{y}_d, \ddot{y}_d$ is required in DSC-based design methods [20], [21], [32]. Our proposed method merely needs the information of y_d and $\dot{y}_d$, which is less stringent.

Lemma 1 [15]: Suppose $h(x) : \Omega_x \to R$ is a smooth function with Ω_x being a compact set. Let $x = x(t - \tau(t))$ be uniformly continuous about t, where $\tau(t) \in [0, \tau_M]$ is an unknown time-varying delay. The constant τ_M is known. Then for any constant $\gamma > 0$, there exists a finite partition of $[0, \tau_M]$, independent of t

$$0 \le t_1 < t_2 < \cdots < t_m \le \tau_M \tag{6}$$

from which time-varying point $\bar{\tau}_{\sigma(t)} \in \{t_1, \ldots, t_m\}, \sigma(t) \in \{1, \ldots, m\}$ can be extracted, such that

$$\left|h(x(t-\tau(t))) - h\left(x\left(t - \bar{\tau}_{\sigma(t)}\right)\right)\right| < \gamma, \quad \forall t \ge 0 \tag{7}$$

Lemma 2 [50]: For any real numbers $x_1, \ldots, x_n$ and $0 < \rho < 1$, the following inequality holds:

$$(|x_1| + \cdots + |x_n|)^\rho \le |x_1|^\rho + \cdots + |x_n|^\rho \tag{8}$$

Lemma 3 [51]: For any real numbers $\lambda_1 > 0, \lambda_2 > 0$, and $0 < \rho < 1$, an extended Lyapunov condition of finite-time stability can be given in the form $\dot{V}(x) \le -\lambda_1 V(x) - \lambda_2 V^\rho(x)$, where the setting time can be estimated by

$$T_0 \le \frac{1}{\lambda_1(1-\rho)} \ln \frac{\lambda_1 V^{1-\rho}(x_0) + \lambda_2}{\lambda_2}.$$

Remark 3: Note that it is difficult to know the time delays precisely and the time delays are often variant in practical engineering systems. In this paper, there is no limitation on the bound of the time-delay derivative and the time-varying delays are unknown. In addition, the proposed method is useful even when the derivative of time-varying delays may not exist. Without imposing any assumptions on the derivative of time-varying delays, an adaptive finite-time tracking controller is designed for nonlinear full-state constrained systems with unknown time-varying delays and input saturation.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

To ensure the asymmetric time-varying full state constraints are not violated, the following NSDFs are introduced:

$$\xi_i = \frac{x_i}{\beta_i}, \quad \beta_i = \left(k_{a_i}(t) + x_i\right)\left(k_{b_i}(t) - x_i\right), \quad i = 1, \ldots, n \tag{9}$$

Obviously, $\xi_i \to \pm\infty$ only if $x_i \to -k_{a_i}(t)$ or $x_i \to k_{b_i}(t)$. For any initial condition satisfying $-k_{a_i}(t) < x_i(0) < k_{b_i}(t)$, the asymmetric time-varying full state constraints (5) are not violated if $\xi_i \in L_\infty, \forall t \geq 0$.

Remark 4: It is known that current BLFs-based control methods [1], [3], [28], [38]–[40] always involve the feasibility conditions on virtual controllers, which is demanding for the design and implementation of the corresponding control schemes. Not only are such demanding conditions removed, but also the extra efforts keeping the continuity and differentiability of the stabilizing functions are not needed in our method. Hence, the proposed control method is simpler and more applicable than the BLFs-based methods in [1], [3], [28], [38]–[40].

Taking the derivative of ξ_i to time, based on (1) and (9), we have

$$\dot{\xi}_i = \mu_{i1}\dot{x}_i + \mu_{i2}, \quad i = 1, \ldots, n \tag{10}$$

with

$$\mu_{i1} = \frac{k_{a_i}(t)\, k_{b_i}(t) + x_i^2}{\beta_i^2} \tag{11}$$

$$\mu_{i2} = -\frac{\left(\dot{k}_{a_i}(t)\, k_{b_i}(t) + k_{a_i}(t)\, \dot{k}_{b_i}(t) + \left(\dot{k}_{b_i} - \dot{k}_{a_i}\right) x_i\right) x_i}{\beta_i^2} \tag{12}$$

To solve the input saturation, an auxiliary design signal ψ is introduced. Define the dynamic system as

$$\dot{\psi} = \mu_{n1} g_n \left(-\psi + p(v) - v\right) \tag{13}$$

Choose the following coordinate transformation:

$$\begin{cases} e_1 = \xi_1 - \xi_d \\ e_i = \xi_i - \xi_{i,c}, \quad i = 2, \ldots, n-1 \\ e_n = \xi_n - \xi_{n,c} - \psi \end{cases} \tag{14}$$

where $\xi_d = y_d/\beta_d$ with $\beta_d = \left(k_{a_1}(t) + y_d\right)\left(k_{b_1}(t) - y_d\right)$, and $\xi_{i,c}, i = 2, \ldots, n$ are the outputs of the *ith* command filter which will be defined later. Like (10)-(12), it can be obtained that $\dot{\xi}_d = \mu_{d1}\dot{y}_d + \mu_{d2}$ with $\mu_{d1} = k_{a_1}(t)\, k_{b_1}(t)/\beta_d^2 + y_d^2/\beta_d^2$ and $\mu_{d2} = -\left(\dot{k}_{a_1}(t)\, k_{b_1}(t) + k_{a_1}(t)\, \dot{k}_{b_1}(t)\right) y_d/\beta_d^2 - \left(\dot{k}_{b_1} - \dot{k}_{a_1}\right) y_d^2/\beta_d^2$.

The command filters are designed as

$$\begin{cases} \dot{\xi}_{i+1,c} = \omega_n \varphi_{i,2}, \quad i = 1, \ldots, n-1 \\ \dot{\varphi}_{i,2} = -2\varsigma\omega_n\varphi_{i,2} - \omega_n\left(\xi_{i+1,c} - \dfrac{\alpha_i}{\beta_{i+1}}\right) \end{cases} \tag{15}$$

where $\xi_{i+1,c}$ and $\dot{\xi}_{i+1,c}$ are the outputs of each filter. $\varsigma \in (0, 1]$ and $\omega_n > 0$ are the design constants. The initial value $\xi_{i+1,c}$ is equal to $\alpha_i(0)/\beta_{i+1}(0)$ and $\varphi_{i,2}(0) = 0$. $\alpha_i, i = 1, \ldots, n-1$ are the virtual controllers which will be given later.

By designing the following error compensation mechanism, the filtering errors $\left(\xi_{i+1,c} - \alpha_i/\beta_{i+1}\right), i = 1, \ldots, n-1$ caused by the command filters are eliminated.

$$\begin{cases} \dot{s}_1 = -k_1 s_1 + \left(\mu_{11} g_1 \beta_2\right) s_2 + \left(\mu_{11} g_1 \beta_2\right)\left(\xi_{2,c} - \dfrac{\alpha_1}{\beta_2}\right) \\ \dot{s}_i = -k_i s_i + \left(\mu_{i1} g_i \beta_{i+1}\right) s_{i+1} - \left(\mu_{(i-1)1} g_{i-1} \beta_i\right) s_{i-1} \\ + \left(\mu_{i1} g_i \beta_{i+1}\right)\left(\xi_{i+1,c} - \dfrac{\alpha_i}{\beta_{i+1}}\right), i = 2, \ldots, n-1 \\ \dot{s}_n = -k_n s_n - \left(\mu_{(n-1)1} g_{n-1} \beta_n\right) s_{n-1} \end{cases} \tag{16}$$

where $k_i, i = 1, \ldots, n$ are positive design constants.

Step 1: Define $z_1 = e_1 - s_1$. Noting (1), (9), (10), (14), and (16), the time derivative of z_1 is

$$\begin{aligned} \dot{z}_1 = {} & \mu_{11}\left(f_1 + h_1\left(\bar{x}_1\left(t - \tau_1(t)\right)\right)\right) + \mu_{11} g_1 \beta_2 z_2 + \mu_{11} g_1 \alpha_1 \\ & + \mu_{11} d_1 + \mu_{12} - \mu_{d1}\dot{y}_d - \mu_{d2} + k_1 s_1 \end{aligned} \tag{17}$$

According to Lemma 1, we have

$$h_1\left(\bar{x}_1\left(t - \tau_1(t)\right)\right) = h_1\left(\bar{x}_1\left(t - \bar{\tau}_{1,\sigma(t)}\right)\right) + r_1 \tag{18}$$

where $|r_1| \leq R_1$, R_1 is an unknown constant.

Choose the Lyapunov function as $V_1 = \frac{1}{2}z_1^2$. Based on (17) and (18), the time derivative of V_1 is given as

$$\begin{aligned} \dot{V}_1 = {} & z_1\dot{z}_1 = \mu_{11} z_1\left(f_1 + h_1\left(\bar{x}_1\left(t - \bar{\tau}_{1,\sigma(t)}\right)\right) + r_1\right) \\ & + \mu_{11} g_1 \beta_2 z_1 z_2 + \mu_{11} g_1 z_1 \alpha_1 + \mu_{11} d_1 z_1 \\ & + \left(\mu_{12} - \mu_{d2} - \mu_{d1}\dot{y}_d\right) z_1 + k_1 s_1 z_1 \end{aligned} \tag{19}$$

Using Young's inequality, we can obtain

$$\mu_{11} d_1 z_1 \leq \mu_{11}^2 z_1^2 + \frac{1}{4}D_1^2 \tag{20}$$

$$\mu_{11} z_1 r_1 \leq \mu_{11}^2 z_1^2 + \frac{1}{4}R_1^2 \tag{21}$$

$$\left(\mu_{12} - \mu_{d1}\dot{y}_d - \mu_{d2}\right) z_1 \leq \left(\mu_{12} - \mu_{d1}\dot{y}_d - \mu_{d2}\right)^2 z_1^2 + \frac{1}{4} \tag{22}$$

Substituting (20)-(22) into (19), one gets

$$\begin{aligned} \dot{V}_1 \leq {} & \mu_{11} z_1 \bar{f}_1 + \mu_{11} g_1 \beta_2 z_1 z_2 + \mu_{11} g_1 z_1 \alpha_1 + k_1 s_1 z_1 \\ & + \left(2\mu_{11}^2 + \left(\mu_{12} - \mu_{d1}\dot{y}_d - \mu_{d2}\right)^2\right) z_1^2 \\ & + \frac{1}{4}D_1^2 + \frac{1}{4}R_1^2 + \frac{1}{4} \end{aligned} \tag{23}$$

The uncertain term $\bar{f}_1$ in (23) is defined as

$$\bar{f}_1 = f_1 + h_1\left(\bar{x}_1\left(t - \bar{\tau}_{1,\sigma(t)}\right)\right) \tag{24}$$

Inspired by the work in [52], the RBF NNs are used to approximate $\bar{f}_1$ as follows:

$$\bar{f}_1 = W_1^{*T} S_1(Z_1) + \varepsilon_1(Z_1) \tag{25}$$

with $Z_1 = \left[x_1, x_1\left(t - \tau_{1,1}\right) \ldots, x_1\left(t - \tau_{1,m}\right)\right]^T$. $S_1(Z_1) \in R^{l_1}$ is a Gaussian basis function vector with $l_1 > 1$ being the NN node number, $W_1^* \in R^{l_1}$ denotes the optimal weight vector, $\varepsilon_1(Z_1) \in R$ is the approximation error. $\left\|W_1^*\right\| \leq \bar{W}_1$, $|\varepsilon_1(Z_1)| \leq \bar{\varepsilon}_1$ with the constants $\bar{W}_1 > 0, \bar{\varepsilon}_1 > 0$.

By substituting (25) into (23), we have

$$\begin{aligned}\dot{V}_1 \leq\ & \mu_{11} z_1 \left(W_1^{*T} S_1(Z_1) + \varepsilon_1(Z_1) \right) + \mu_{11} g_1 \beta_2 z_1 z_2 \\ & + \mu_{11} g_1 z_1 \alpha_1 + k_1 s_1 z_1 \\ & + \left(2\mu_{11}^2 + (\mu_{12} - \mu_{d1} \dot{y}_d - \mu_{d2})^2 \right) z_1^2 \\ & + \frac{1}{4} D_1^2 + \frac{1}{4} R_1^2 + \frac{1}{4} \end{aligned} \tag{26}$$

Using Young's inequality, we get

$$\mu_{11} z_1 W_1^{*T} S_1(Z_1) \leq \frac{\theta}{2a_1^2} \mu_{11}^2 z_1^2 \| S_1(Z_1) \|^2 + \frac{1}{2} a_1^2 \tag{27}$$

$$\mu_{11} z_1 \varepsilon_1(Z_1) \leq \frac{1}{2} \mu_{11}^2 z_1^2 + \frac{1}{2} \bar{\varepsilon}_1^2 \tag{28}$$

where a_1 in (27) is a positive design constant, $\theta = \max\left\{ \|W_1^*\|^2, \ldots, \|W_n^*\|^2 \right\}$, and $\|W_2^*\|, \ldots, \|W_n^*\|$ will be defined later. $\tilde{\theta} = \hat{\theta} - \theta$ is the estimated error. $\hat{\theta}$ is the estimate of θ and $\hat{\theta}(0)$ is the initial value of $\hat{\theta}$ such that $\hat{\theta}(0) \geq 0$.

Design the virtual controller as

$$\begin{aligned}\alpha_1 =\ & \frac{1}{g_1} \left(-\frac{1}{\mu_{11}} k_1 e_1 - \frac{1}{\mu_{11}} k_{\rho_1} z_1^{2\rho-1} \right. \\ & \left. - \frac{\hat{\theta}}{2a_1^2} \mu_{11} z_1 \| S_1(Z_1) \|^2 \right) \\ & - \frac{1}{g_1 \mu_{11}} \left(2\mu_{11}^2 + (\mu_{12} - \mu_{d1} \dot{y}_d - \mu_{d2})^2 \right) z_1 \end{aligned} \tag{29}$$

where $0 < \rho < 1$ and k_{ρ_1} are positive design constants.

Substituting (27)-(29) into (26) leads to

$$\begin{aligned}\dot{V}_1 \leq\ & -k_1 z_1^2 - k_{\rho_1} z_1^{2\rho} - \frac{\tilde{\theta}}{2a_1^2} \mu_{11}^2 z_1^2 \| S_1(Z_1) \|^2 \\ & + \mu_{11} g_1 \beta_2 z_1 z_2 \\ & + \left(\frac{1}{4} D_1^2 + \frac{1}{4} R_1^2 + \frac{1}{2} \bar{\varepsilon}_1^2 + \frac{1}{2} a_1^2 + \frac{1}{4} \right) \end{aligned} \tag{30}$$

Step i $(2 \leq i \leq n-1)$: Define $z_i = e_i - s_i$. Based on (1), (9), (10), (14), and (16), the time derivative of z_i is

$$\begin{aligned}\dot{z}_i =\ & \mu_{i1} \left(f_i + h_i(\bar{x}_i(t - \tau_i(t))) \right) + \mu_{i1} g_i \beta_{i+1} z_{i+1} + \mu_{i1} g_i \alpha_i \\ & + \mu_{i1} d_i + \mu_{i2} - \dot{\xi}_{i,c} + k_i s_i + \mu_{(i-1)1} g_{i-1} \beta_i s_{i-1} \end{aligned} \tag{31}$$

According to Lemma 1, we have

$$h_i(\bar{x}_i(t - \tau_i(t))) = h_i\left(\bar{x}_i\left(t - \bar{\tau}_{i,\sigma(t)} \right) \right) + r_i \tag{32}$$

where $|r_i| \leq R_i$, R_i is an unknown constant.

Define the Lyapunov function as $V_i = \frac{1}{2} z_i^2 + V_{i-1}$. From (31) and (32), the time derivative of V_i is given by

$$\begin{aligned}\dot{V}_i =\ & z_i \dot{z}_i + \dot{V}_{i-1} \\ =\ & \mu_{i1} z_i \left(f_i + h_i\left(\bar{x}_i\left(t - \bar{\tau}_{i,\sigma(t)} \right) \right) + r_i \right) + \mu_{i1} g_i \beta_{i+1} z_i z_{i+1} \\ & + \mu_{i1} g_i z_i \alpha_i + \mu_{i1} d_i z_i + \left(\mu_{i2} - \dot{\xi}_{i,c} \right) z_i + k_i s_i z_i \\ & + \dot{V}_{i-1} + \mu_{(i-1)1} g_{i-1} \beta_i z_i s_{i-1} \end{aligned} \tag{33}$$

Using Young's inequality, we obtain

$$\mu_{i1} d_i z_i \leq \mu_{i1}^2 z_i^2 + \frac{1}{4} D_i^2 \tag{34}$$

$$\mu_{i1} z_i r_i \leq \mu_{i1}^2 z_i^2 + \frac{1}{4} R_i^2 \tag{35}$$

$$\left(\mu_{i2} - \dot{\xi}_{i,c} \right) z_i \leq \left(\mu_{i2} - \dot{\xi}_{i,c} \right)^2 z_i^2 + \frac{1}{4} \tag{36}$$

Substituting (34)-(36) into (33), one has

$$\begin{aligned}\dot{V}_i \leq\ & \mu_{i1} z_i \bar{f}_i + \mu_{i1} g_i \beta_{i+1} z_i z_{i+1} + \mu_{i1} g_i z_i \alpha_i + k_i s_i z_i \\ & + \left(2\mu_{i1}^2 + \left(\mu_{i2} - \dot{\xi}_{i,c} \right)^2 \right) z_i^2 + \tilde{\theta}_i \dot{\hat{\theta}}_i + \frac{1}{4} D_i^2 + \frac{1}{4} R_i^2 + \frac{1}{4} \\ & + \dot{V}_{i-1} + \mu_{(i-1)1} g_{i-1} \beta_i z_i s_{i-1} \end{aligned} \tag{37}$$

The uncertain term $\bar{f}_i$ in (37) is defined as

$$\bar{f}_i = f_i + h_i\left(\bar{x}_i\left(t - \bar{\tau}_{i,\sigma(t)} \right) \right) \tag{38}$$

The RBF NNs are used to approximate $\bar{f}_i$ as

$$\bar{f}_i = W_i^{*T} S_i(Z_i) + \varepsilon_i(Z_i) \tag{39}$$

where $Z_i = \left[x_1, \ldots, x_i, x_i\left(t - \tau_{i,1} \right), \ldots, x_i\left(t - \tau_{i,m} \right) \right]^T$. $S_i(Z_i) \in R^{l_i}$ is a Gaussian basis function vector with $l_i > 1$ being the NN node number, $W_i^* \in R^{l_i}$ denotes the optimal weight vector, $\varepsilon_i(Z_i) \in R$ is the approximation error. $\|W_i^*\| \leq \bar{W}_i$, $|\varepsilon_i(Z_i)| \leq \bar{\varepsilon}_i$ with the constants $\bar{W}_i > 0$, $\bar{\varepsilon}_i > 0$.

By substituting (39) into (37), one gets

$$\begin{aligned}\dot{V}_i \leq\ & \mu_{i1} z_i \left(W_i^{*T} S_i(Z_i) + \varepsilon_i(Z_i) \right) + \mu_{i1} g_i \beta_{i+1} z_i z_{i+1} \\ & + \mu_{i1} g_i z_i \alpha_i + k_i s_i z_i + \left(\mu_{i1}^2 + \left(\mu_{i2} - \dot{\xi}_{i,c} \right)^2 \right) z_i^2 + \frac{1}{4} D_i^2 \\ & + \frac{1}{4} R_i^2 + \frac{1}{4} + \dot{V}_{i-1} + \mu_{(i-1)1} g_{i-1} \beta_i z_i s_{i-1} \end{aligned} \tag{40}$$

Using Young's inequality, we have

$$\mu_{i1} z_i W_i^{*T} S_i(Z_i) \leq \frac{\theta}{2a_i^2} \mu_{i1}^2 z_i^2 \| S_i(Z_i) \|^2 + \frac{1}{2} a_i^2 \tag{41}$$

$$\mu_{i1} z_i \varepsilon_i(Z_i) \leq \frac{1}{2} \mu_{i1}^2 z_i^2 + \frac{1}{2} \bar{\varepsilon}_i^2 \tag{42}$$

where a_i in (41) is a positive design constant.

Design the virtual controller as

$$\begin{aligned}\alpha_i =\ & \frac{1}{g_i} \left(-\frac{1}{\mu_{i1}} k_i e_i - \frac{1}{\mu_{i1}} k_{\rho_i} z_i^{2\rho-1} - \frac{\hat{\theta}}{2a_i^2} \mu_{i1} z_i \| S_i(Z_i) \|^2 \right) \\ & - \frac{1}{g_i \mu_{i1}} \left(2\mu_{i1}^2 + \left(\mu_{i2} - \dot{\xi}_{i,c} \right)^2 \right) z_i - \frac{\mu_{(i-1)1}}{g_i \mu_{i1}} g_{i-1} \beta_i e_{i-1} \end{aligned} \tag{43}$$

where k_{ρ_i} is a positive design constant.

Remark 5: Using the command filter technique, the proposed controller can be implemented without differentiation of the virtual controllers $\alpha_i, i = 2, \ldots, n-1$. Removing analytic computation of $\dot{\alpha}_i$ avoids the appearance of $z_i^{2\rho-2}$. Therefore, the controller singularity problems will not occur. This makes our proposed method more easily derived and implemented.

In step $(i-1)$, it can be obtained that

$$\begin{aligned}\dot{V}_{i-1} \leq & -\sum_{j=1}^{i-1} k_j z_j^2 - \sum_{j=1}^{i-1} k_{\rho_j} z_j^{2\rho} - \sum_{j=1}^{i-1} \frac{\tilde{\theta}}{2a_j^2} \mu_{j1}^2 z_j^2 \left\| S_j\left(Z_j\right)\right\|^2 \\ & +\mu_{(i-1)1} g_{i-1} \beta_i z_{i-1} z_i \\ & +\sum_{j=1}^{i-1}\left(\frac{1}{4} D_j^2+\frac{1}{4} R_j^2+\frac{1}{2} \bar{\varepsilon}_j^2+\frac{1}{2} a_j^2+\frac{1}{4}\right) \end{aligned} \tag{44}$$

Substituting (41)-(44) into (40) results in

$$\begin{aligned}\dot{V}_{i} \leq & -\sum_{j=1}^{i} k_j z_j^2 - \sum_{j=1}^{i} k_{\rho_j} z_j^{2\rho} - \sum_{j=1}^{i} \frac{\tilde{\theta}}{2a_j^2} \mu_{j1}^2 z_j^2 \left\| S_j\left(Z_j\right)\right\|^2 \\ & +\mu_{i1} g_{i} \beta_{i+1} z_{i} z_{i+1} \\ & +\sum_{j=1}^{i}\left(\frac{1}{4} D_j^2+\frac{1}{4} R_j^2+\frac{1}{2} \bar{\varepsilon}_j^2+\frac{1}{2} a_j^2+\frac{1}{4}\right) \end{aligned} \tag{45}$$

Step n : Define $z_n = e_n - s_n$. From (1), (9), (10), (13) (14), and (16), the time derivative of z_n is

$$\begin{aligned}\dot{z}_n = & \mu_{n1}\left(f_n + h_n\left(\bar{x}_n\left(t-\tau_n(t)\right)\right)\right) + \mu_{n1} g_n u + \mu_{n1} d_n \\ & +\mu_{n2} - \dot{\xi}_{n,c} + k_n s_n - \dot{\psi} + \mu_{(n-1)1} g_{n-1} \beta_n s_{n-1} \end{aligned} \tag{46}$$

According to Lemma 1, we have

$$h_n\left(\bar{x}_n\left(t-\tau_n(t)\right)\right) = h_n\left(\bar{x}_n\left(t-\bar{\tau}_{n,\sigma(t)}\right)\right) + r_n \tag{47}$$

where $|r_n| \leq R_n$, R_n is an unknown constant.

Design the Lyapunov function as $V_n = \frac{1}{2} z_n^2 + \frac{1}{2}\tilde{\theta}^2 + V_{n-1}$. Based on (46) and (47), the time derivative of V_n is given as

$$\begin{aligned}\dot{V}_n = & z_n \dot{z}_n + \tilde{\theta}\dot{\hat{\theta}} + \dot{V}_{n-1} \\ \leq & \mu_{n1} z_n\left(f_n + h_n\left(\bar{x}_n\left(t-\bar{\tau}_{n,\sigma(t)}\right)\right) + r_n\right) \\ & +\mu_{n1} g_n\left(v+\psi+F_1\right) + \mu_{n1} d_n z_n + \left(\mu_{n2} - \dot{\xi}_{n,c}\right) z_n \\ & +k_n s_n z_n + \tilde{\theta}\dot{\hat{\theta}} + \dot{V}_{n-1} + \mu_{(n-1)1} g_{n-1} \beta_n z_n s_{n-1} \end{aligned} \tag{48}$$

Using Young's inequality, we can obtain

$$\mu_{n1} d_n z_n \leq \mu_{n1}^2 z_n^2 + \frac{1}{4} D_n^2 \tag{49}$$

$$\mu_{n1} z_n r_n \leq \mu_{n1}^2 z_n^2 + \frac{1}{4} R_n^2 \tag{50}$$

$$\left(\mu_{n2} - \dot{\xi}_{n,c}\right) z_n \leq \left(\mu_{n2} - \dot{\xi}_{n,c}\right)^2 z_n^2 + \frac{1}{4} \tag{51}$$

Remark 6: The purpose of inequalities (36) and (51) is to cope with the terms $\left(\mu_{i2} - \dot{\xi}_{i,c}\right) z_i$ in (33) and $\left(\mu_{n2} - \dot{\xi}_{n,c}\right) z_n$ in (48).

Substituting (49)-(51) into (48) yields

$$\begin{aligned}\dot{V}_n \leq & \mu_{n1} z_n \bar{f}_n + \mu_{n1} g_n\left(v+\psi+F_1\right) + k_n s_n z_n \\ & +\left(2\mu_{n1}^2 + \left(\mu_{n2} - \dot{\xi}_{n,c}\right)^2\right) z_n^2 + \tilde{\theta}\dot{\hat{\theta}} + \frac{1}{4} D_n^2 \\ & +\frac{1}{4} R_n^2 + \frac{1}{4} + \dot{V}_{n-1} + \mu_{(n-1)1} g_{n-1} \beta_n z_n s_{n-1} \end{aligned} \tag{52}$$

The uncertain term $\bar{f}_n$ in (52) is defined as

$$\bar{f}_n = f_n + h_n\left(\bar{x}_n\left(t-\bar{\tau}_{n,\sigma(t)}\right)\right) \tag{53}$$

The RBF NNs are used to approximate $\bar{f}_n$ as

$$\bar{f}_n = W_n^{*T} S_n\left(Z_n\right) + \varepsilon_n\left(Z_n\right) \tag{54}$$

with $Z_n = \left[x_1, \ldots, x_n, x_n\left(t-\tau_{n,1}\right), \ldots, x_n\left(t-\tau_{n,m}\right)\right]^T$. $S_n\left(Z_n\right) \in R^{l_n}$ is a Gaussian basis function vector with $l_n > 1$ being the NN node number, $W_n^* \in R^{l_n}$ denotes the optimal weight vector, $\varepsilon_n\left(Z_n\right) \in R$ is the approximation error. $\left\|W_n^*\right\| \leq \bar{W}_n$, $\left|\varepsilon_n\left(Z_n\right)\right| \leq \bar{\varepsilon}_n$ with the constants $\bar{W}_n > 0$, $\bar{\varepsilon}_n > 0$.

By substituting (54) into (52), one can get

$$\begin{aligned}\dot{V}_n \leq & \mu_{n1} z_n\left(W_n^{*T} S_n\left(Z_n\right) + \varepsilon_n\left(Z_n\right)\right) + \mu_{n1} g_n\left(v+\psi+F_1\right) \\ & +k_n s_n z_n + \left(\mu_{n1}^2 + \left(\mu_{n2} - \dot{\xi}_{n,c}\right)^2\right) z_n^2 \\ & +\tilde{\theta}\dot{\hat{\theta}} + \frac{1}{4} D_n^2 + \frac{1}{4} R_n^2 + \frac{1}{4} + \dot{V}_{n-1} \\ & +\mu_{(n-1)1} g_{n-1} \beta_n z_n s_{n-1} \end{aligned} \tag{55}$$

Using Young's inequality, we have

$$\mu_{n1} z_n W_n^{*T} S_n\left(Z_n\right) \leq \frac{\theta}{2a_n^2} \mu_{n1}^2 z_n^2 \left\|S_n\left(Z_n\right)\right\|^2 + \frac{1}{2} a_n^2 \tag{56}$$

$$\mu_{n1} z_n \varepsilon_n\left(Z_n\right) \leq \frac{1}{2} \mu_{n1}^2 z_n^2 + \frac{1}{2} \bar{\varepsilon}_n^2 \tag{57}$$

where a_n in (56) is a positive design constant.

Design the actual controller as

$$\begin{aligned} v = & \frac{1}{g_n}\left(-\frac{1}{\mu_{n1}} k_n e_n - \frac{1}{\mu_{n1}} k_{\rho_n} z_n^{2\rho-1} - \frac{\hat{\theta}}{2a_n^2} \mu_{n1} z_n \left\|S_n\left(Z_n\right)\right\|^2\right. \\ & \left. -\frac{\mu_{(n-1)1}}{\mu_{n1}} g_{n-1} \beta_n e_{n-1} - \frac{1}{\mu_{n1}}\left(2\mu_{n1}^2 + \left(\mu_{n2} - \dot{\xi}_{n,c}\right)^2\right) z_n\right) \\ & -\psi - F_1 \end{aligned} \tag{58}$$

where k_{ρ_n} is a positive design constant.

Remark 7: Compared with the infinite-time control design approaches [39, 42, 46, 49], our designed finite-time controller has the advantages of high tracking precision and fast transient performances.

The adaptive law is chosen as

$$\dot{\hat{\theta}} = \sum_{j=1}^{n} \frac{\tilde{\theta}}{2a_j^2} \mu_{j1}^2 z_j^2 \left\|S_j\left(Z_j\right)\right\|^2 - \delta\hat{\theta} \tag{59}$$

where δ is a positive design constant.

In step $(n-1)$, it can be obtained that

$$\begin{aligned}\dot{V}_{n-1} \leq & -\sum_{j=1}^{n-1} k_j z_j^2 - \sum_{j=1}^{n-1} k_{\rho_j} z_j^{2\rho} - \sum_{j=1}^{n-1} \frac{\tilde{\theta}}{2a_j^2} \mu_{j1}^2 z_j^2 \\ & \times\left\|S_j\left(Z_j\right)\right\|^2 + \mu_{(n-1)1} g_{n-1} \beta_n z_{n-1} z_n \\ & +\sum_{j=1}^{n-1}\left(\frac{1}{4} D_j^2+\frac{1}{4} R_j^2+\frac{1}{2} \bar{\varepsilon}_j^2+\frac{1}{2} a_j^2+\frac{1}{4}\right) \end{aligned} \tag{60}$$

By substituting (56)-(60) into (55), one deduces

$$\dot{V}_n \leq -\sum_{j=1}^{n} k_j z_j^2 - \sum_{j=1}^{n} k_{\rho_j} z_j^{2\rho} - \delta\tilde{\theta}\hat{\theta} + \sum_{j=1}^{n}\left(\frac{1}{4}D_j^2 + \frac{1}{4}R_j^2 + \frac{1}{2}\bar{\varepsilon}_j^2 + \frac{1}{2}a_j^2 + \frac{1}{4}\right) \tag{61}$$

Using Young's inequality, one has

$$-\tilde{\theta}\hat{\theta} \leq -\frac{(1-\rho)\,\tilde{\theta}^2}{2} + \frac{\theta^2}{2} - \frac{\tilde{\theta}^{2\rho}}{2} + \frac{1-\rho}{2} \tag{62}$$

Substituting (62) into (61) provides

$$\begin{aligned}\dot{V}_n \leq& -\sum_{j=1}^{n} k_j z_j^2 - \frac{(1-\rho)\,\delta\tilde{\theta}^2}{2} - \sum_{j=1}^{n} k_{\rho_j} z_j^{2\rho} - \frac{\delta\tilde{\theta}^{2\rho}}{2}\\ &+\sum_{j=1}^{n}\left(\frac{1}{4}D_j^2 + \frac{1}{4}R_j^2 + \frac{1}{2}\bar{\varepsilon}_j^2 + \frac{1}{2}a_j^2 + \frac{1}{4}\right)\\ &+\frac{1}{2}\delta\theta^2 + \frac{1}{2}(1-\rho)\,\delta\\ \leq& -\lambda_1 V_n - \lambda_2 V_n^{\rho} + C\end{aligned} \tag{63}$$

where $\lambda_1 = \min\left\{2k_j, \delta(1-\rho), j = 1, \ldots, n\right\}$, $\lambda_2 = \min\left\{2 \times k_{\rho_j}, 2^{\rho-1}\delta, j = 1, \ldots, n\right\}$ and

$$C = \sum_{j=1}^{n}\left(\frac{1}{4}D_j^2 + \frac{1}{4}R_j^2 + \frac{1}{2}\bar{\varepsilon}_j^2 + \frac{1}{2}a_j^2 + \frac{1}{4}\right) + \frac{1}{2}\delta\theta^2 + \frac{1}{2}(1-\rho)\,\delta.$$

Theorem 1. Consider the closed-loop system consisting of the nonlinear system (1), the virtual controllers (29), (43), the actual controller (58), and the adaptation law (59). Suppose that Assumptions 1-3 hold and the initial states satisfy $x_i(0) \in \Omega_i$, then all signals in the closed-loop system are bounded, the tracking error converges to a small bound around the origin in a finite time, and the asymmetric time-varying full state constraints are never violated.

Proof. According to (63), we can obtain

$$\dot{V}_n \leq -\lambda_1 V_n + C \tag{64}$$

Multiplying both sides by $e^{\lambda_1 t}$ and integrating it over $[0, t]$, one has

$$V_n \leq \left(V_n(0) - \frac{C}{\lambda_1}\right)e^{-\lambda_1 t} + \frac{C}{\lambda_1} \leq V_n(0)\,e^{-\lambda_1 t} + \frac{C}{\lambda_1} \tag{65}$$

Therefore, $V_n \in L_\infty$ for any initial conditions satisfying $x_i(0) \in \Omega_i$, which indicates that $z_i \in L_\infty$ and $\tilde{\theta} \in L_\infty$. From (59) and the boundedness of $\tilde{\theta}$, we know that $\hat{\theta}$ is bounded. Based on the singular perturbation theory [53, 54] and (15), it can be proved that $\varphi_{i,2}, \xi_{i+1,c}, i = 1, \ldots, n-1$ are bounded. Based on the work in [55] and (16), we know that $s_i, i = 1, \ldots, n$ are bounded. From $z_1 = e_1 - s_1$ and the boundedness of z_1, s_1, we can infer that e_1 is bounded. Based on Assumption 2 and the definition of ξ_d, we can obtain ξ_d is bounded. Combined with $e_1 = \xi_1 - \xi_d$, we have $\xi_1 \in L_\infty$. Then, it can be obtained that the state x_1 never violates the predefined constrained state space $\Omega_1 = \left\{x_1 \in R: -k_{a_1}(t) < x_1 < k_{b_1}(t)\right\}$. From (29), we can further have α_1 is bounded. Similarly, we can obtain $\xi_i \in L_\infty$, the state x_i remains in the space $\Omega_i = \left\{x_i \in R: -k_{a_i}(t) < x_i < k_{b_i}(t)\right\}$, α_i and v are bounded for $i = 2, \ldots, n-1$. Furthermore, we can obtain u is bounded from (2). Thus, all closed-loop signals are bounded, and the asymmetric time-varying full state constraints are not violated.

According to Corollary 1 in [51], Lemma 3, and (63), we have $V_n \leq (C/\lambda_2(1-\chi))^{1/\rho}, 0 < \chi < 1$ in a finite time. Based on the above discussion, one has

$$\frac{1}{2}z_i^2 \leq V_n \leq \left(\frac{C}{\lambda_2(1-\chi)}\right)^{\frac{1}{\rho}} \tag{66}$$

Then, we have

$$|z_i| \leq \left(\frac{C}{\lambda_2(1-\chi)}\right)^{\frac{1}{2\rho}} \tag{67}$$

in the settling time

$$T_0 \leq \frac{1}{\lambda_1(1-\rho)}\ln\frac{\lambda_1 V^{1-\rho}(x_0) + \chi\lambda_2}{\lambda_1\left(\frac{C}{\lambda_2(1-\chi)}\right)^{\frac{1-\rho}{\rho}} + \chi\lambda_2}.$$

Define the Lyapunov function as

$$V_s = \sum_{i=1}^{n}\frac{1}{2}s_i^2 \tag{68}$$

Based on (16), the time derivative of V_s is

$$\dot{V}_s = -\sum_{i=1}^{n} k_i s_i^2 + \sum_{i=1}^{n-1} s_i\left(\mu_{i1} g_i \beta_{i+1}\right)\left(\xi_{i+1,c} - \frac{\alpha_i}{\beta_{i+1}}\right) \tag{69}$$

From Assumption 3, we can obtain that g_i is bounded. From (9) and the boundedness of x_i, it can be obtained that μ_{i1} and β_{i+1} are bounded. In [55], we know that $\left(\xi_{i+1,c} - \alpha_i/\beta_{i+1}\right)$ is bounded. Then, it can be inferred that $\left(\mu_{i1} g_i \beta_{i+1}\right)\left(\xi_{i+1,c} - \alpha_i/\beta_{i+1}\right)$ is bounded. It is assumed that $\left|\left(\mu_{i1} g_i \beta_{i+1}\right)\left(\xi_{i+1,c} - \alpha_i/\beta_{i+1}\right)\right| \leq \psi_i$, where ψ_i is a positive constant. Together with (69), one obtains

$$\dot{V}_s \leq -\sum_{i=1}^{n} k_i s_i^2 + \sum_{i=1}^{n-1} s_i \psi_i \tag{70}$$

By employing Young's inequality, we get

$$\sum_{i=1}^{n-1} s_i \psi_i \leq \sum_{i=1}^{n-1}\frac{1}{4}s_i^2 + \sum_{i=1}^{n-1}\psi_i^2 \tag{71}$$

Substituting (71) into (70) leads to

$$
\begin{aligned}
\dot{V}_s &\leq -\sum_{i=1}^{n} k_i s_i^2 + \sum_{i=1}^{n-1} \frac{1}{4} s_i^2 + \sum_{i=1}^{n-1} \psi_i^2 \\
&\leq -\sum_{i=1}^{n} \left(\frac{1}{2} k_i s_i^2\right)^{\rho} - \sum_{i=1}^{n} k_i s_i^2 + \sum_{i=1}^{n} \frac{1}{4} s_i^2 \\
&\quad + \sum_{i=1}^{n-1} \psi_i^2 + \sum_{i=1}^{n} \left(\frac{1}{2} k_i s_i^2\right)^{\rho} \qquad (72)
\end{aligned}
$$

By means of Lemma 3 in [56], we can obtain

$$
\left(\frac{1}{2} k_i s_i^2\right)^{\rho} \leq (1-\rho)\, e^{\frac{\rho}{1-\rho} \ln \rho} + \frac{1}{2} k_i s_i^2 \qquad (73)
$$

Substituting (73) into (72), we have

$$
\begin{aligned}
\dot{V}_s &\leq -\sum_{i=1}^{n} \left(\frac{1}{2} k_i s_i^2\right)^{\rho} - \sum_{i=1}^{n} \frac{1}{4} k_i s_i^2 + \sum_{i=1}^{n-1} \psi_i^2 \\
&\quad + n(1-\rho)\, e^{\frac{\rho}{1-\rho} \ln \rho} \\
&\leq -\bar{\lambda}_1 V_s^{\rho} - \bar{\lambda}_2 V_s + C_s \qquad (74)
\end{aligned}
$$

where $\bar{\lambda}_1 = \min\left\{k_i^{\rho}, i = 1, \ldots, n\right\}$, $\bar{\lambda}_2 = \min\left\{\frac{k_i}{2}, i = 1, \ldots, n\right\}$ and $C_s = \sum_{i=1}^{n-1} \psi_i^2 + n(1-\rho)\, e^{\frac{\rho}{1-\rho} \ln \rho}$. Like (66) and (67), we can obtain that $|s_i| < \left(C_s / \left(\bar{\lambda}_2 (1-\vartheta)\right)\right)^{1/(2\rho)}$, $i = 1, \ldots, n$ in a finite time. The convergence time T_s can be estimated as

$$
T_s \leq \frac{1}{\bar{\lambda}_1 (1-\rho)} \ln \frac{\lambda_1 V_s^{1-\rho}(s_0) + \upsilon \bar{\lambda}_2}{\bar{\lambda}_1 \left(\frac{C_s}{\bar{\lambda}_2 (1-\upsilon)}\right)^{\frac{1-\rho}{\rho}} + \upsilon \bar{\lambda}_2}, 0 < \upsilon < 1.
$$

Thus, it can be inferred that the tracking error converges to a small bound around the origin in a finite time by choosing appropriate parameters.

The proof is completed.

Remark 8: Compared with the command-filter-based adaptive fuzzy finite-time control method in [57], the advantages and merits of the proposed method are twofold. First, our proposed method can handle the full state constraints and time-varying delays. However, the control method in [57] cannot be applied to the systems with full state constraints and time-varying delays. Second, unlike multiple adaptive laws in [57], fewer adaptive laws are introduced in this paper. Thus, the computational burden is greatly reduced, which makes our proposed method more suitable and effective for practical systems.

Remark 9: Although there are some meaningful studies on nonlinear full state-constrained [28], [37]–[40] and time-varying delay systems [12]–[14], there are still some problems that need to be improved. For example, the feasibility conditions [28], [37]–[40], the restriction on time delays [12]–[14], and the ignorance of input saturation [12]–[14], [28], [38]–[40] remain to be solved. In this paper, the above-mentioned problems have been solved by the proposed adaptive finite-time controller for the nonlinear time-varying delay systems with full state constraints and input saturation.

Remark 10: The tracking performance depends on the design parameters $k_i, k_{\rho_i}, a_i, \delta$. From (9) and (14), we know that the tracking error $(y - y_d)$ is small if e_1 is small. From $z_1 = e_1 - s_1$, small z_1 and small s_1 results in small e_1. To make z_1 small, we need to choose small C and large λ_2 from (67). From (63), we select small a_i, δ and large k_{ρ_i} to obtain small C and large λ_2. Similarly, we need to choose large k_i to make s_1 small from (74).

The algorithm of the proposed adaptive finite-time tracking control method is given as follows:

Step 1. Calculate $g_i, y_d, \dot{y}_d, k_{a_i}, k_{b_i}, \beta_i$ in (9), and μ_{i1} in (11) for $i = 1, \ldots, n$.

Step 2. Choose appropriate design parameters ρ, δ, ς, ω_n, k_i, k_{ρ_i}, a_i for $i = 1, \ldots, n$.

Step 3. Choose Gaussian basis function vectors $S_i(Z_i)$ for $i = 1, \ldots, n$.

Step 4. Solve adaptive law to find $\hat{\theta}$ in (59).

Step 5. Solve adaptive laws to find ψ in (13), $\xi_{i+1,c}$ and $\dot{\xi}_{i+1,c}$ in (15), s_j in (16) for $i = 1, \ldots, n-1, j = 1, \ldots, n$.

Step 6. Calculate virtual controllers α_1 in (29), α_i in (43) for $i = 2, \ldots, n-1$, and actual control law v in (58).

Step 7. Calculate saturated control input $p(v)$ using (3).

IV. SIMULATION EXAMPLES

In this section, two simulation examples are given to demonstrate the effectiveness of our proposed method

Example 1: Consider the nonlinear time-varying delay system in strict-feedback form as follows:

$$
\begin{cases}
\dot{x}_1 = f_1 + g_1 x_2 + h_1(\bar{x}_1(t - \tau_1(t))) + d_1(t) \\
\dot{x}_2 = f_2 + g_2 u(v) + h_2(\bar{x}_2(t - \tau_2(t))) + d_2(t) \\
y = x_1
\end{cases} \qquad (75)
$$

where $f_1 = x_1 e^{-0.2x_1}, f_2 = x_1 x_2^2, g_1 = 1 + x_1^2, g_2 = 3 + \cos(x_1 x_2)$, $h_1(\bar{x}_1(t - \tau_1(t))) = \cos(x_1(t - \tau_1(t)))$, $h_2(\bar{x}_2(t - \tau_2(t))) = \sin(x_1(t - \tau_2(t)))\, x_2(t - \tau_1(t))$. The external disturbances are $d_1(t) = 0.1 \sin\left(2e^t\right)$ and $d_2(t) = 0.2e^{-t^2}$. The time delays are given as $\tau_1(t) = 1 + \sin(t)$, $\tau_2(t) = 1.2(1 + \cos(t))$. The states are constrained by $-k_{a_i}(t) < x_i < k_{b_i}(t), i = 1, 2$ with $k_{a_1}(t) = 0.8 - 0.1\cos(t), k_{b_1}(t) = 1.4 + 0.5\cos(t), k_{a_2}(t) = 1.5 - 0.5\cos(t)$ and $k_{b_2}(t) = 0.5 - 0.5\sin(t)$. In this simulation, the desired signal is chosen as $y_d = 0.5(\cos(t) + \cos(0.5t))$. The saturation bounds are chosen as: $u_{\max} = 8, u_{\min} = -5$. Choose the initial value $x_1(0) = 1.6, x_2(0) = -0.4$, $\hat{\theta}(0) = 0.6, s_1(0) = s_2(0) = 0$. The design parameters are chosen as: $k_1 = 50, k_2 = 210, k_{\rho_1} = k_{\rho_2} = 1, \rho = 0.5$, $a_1 = 7, a_2 = 2, \delta = 1, \varsigma = 0.8$, and $\omega_2 = 100$.

To further demonstrate the validity of our proposed method, the infinite-time control method proposed by Li *et al.* [42], in which both time delays and full state constraints were taken into account, is also applied to (75) for comparison. The same initial conditions are chosen for the system states. The simulation results are shown

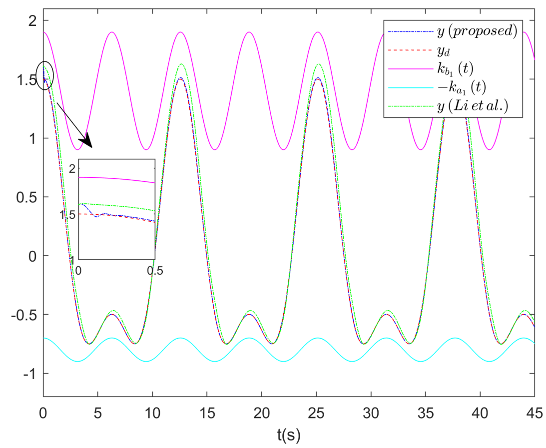

FIGURE 1. Trajectories of the desired signal y_d and the system output y (Example 1).

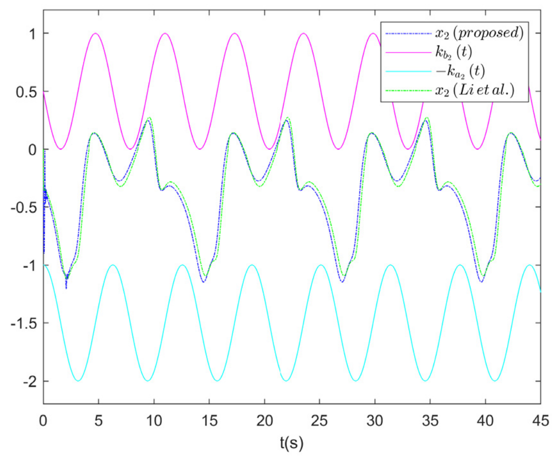

FIGURE 2. Trajectory of the state variable x_2 (Example 1).

in Figure 1- Figure 5. The trajectories of the desired signal y_d and system output y are presented in Figure 1. It is obvious that good tracking performance is achieved by using our proposed controller. Figure 2 shows the trajectory of the state x_2. As observed from Figure 1 and Figure 2, all states satisfy the predefined constraints by using both our proposed method and the infinite-time control approach proposed by Li *et al.* Figure 3 depicts the tracking error $(y - y_d)$. From Figure 1 and Figure 3, it can be seen that our proposed method not only has a faster convergence rate but also achieves better tracking performance than the infinite-time tracking control method in [42]. Figure 4 draws the trajectory of the system input u, from which we can see that the system input under our proposed method never transgresses the saturation bound, while the system input using method in [42] violates the saturation bound. The boundedness of the adaptive parameter can be obtained from Figure 5. Based on the above analysis, it is clearly seen that the proposed control strategy for the nonlinear time-varying delay systems with full state constraints and input saturation is rational and feasible.

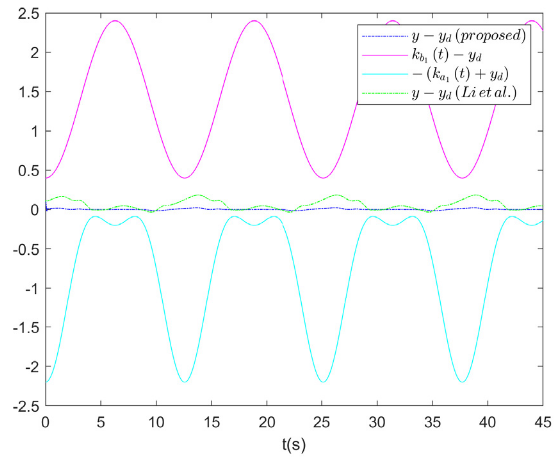

FIGURE 3. Trajectory of the tracking error $(y - y_d)$ (Example 1).

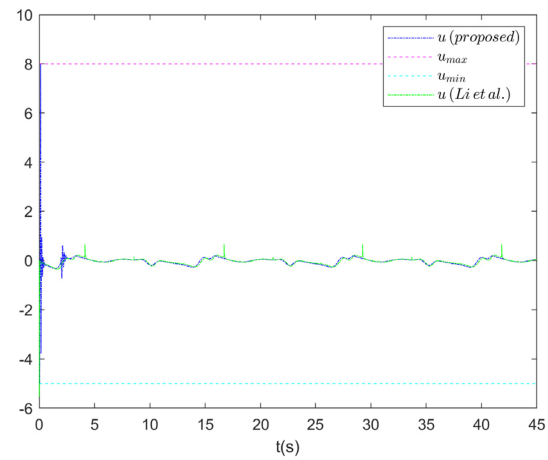

FIGURE 4. Trajectory of the system input **u** (Example 1).

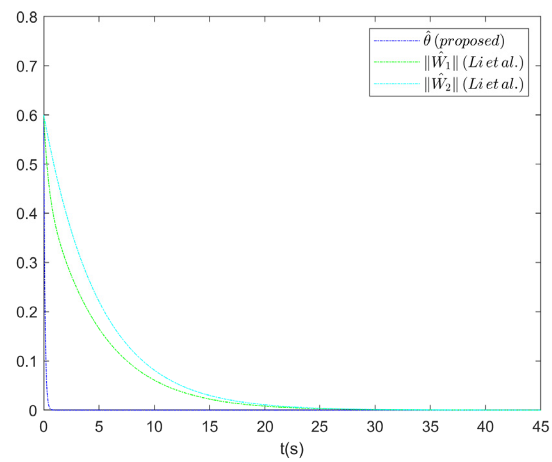

FIGURE 5. The adaptive parameters (Example 1).

Example 2: A practical example of a two-stage chemical reactor system with delayed recycle streams and external perturbations [58] is considered. Tacking time-varying delays

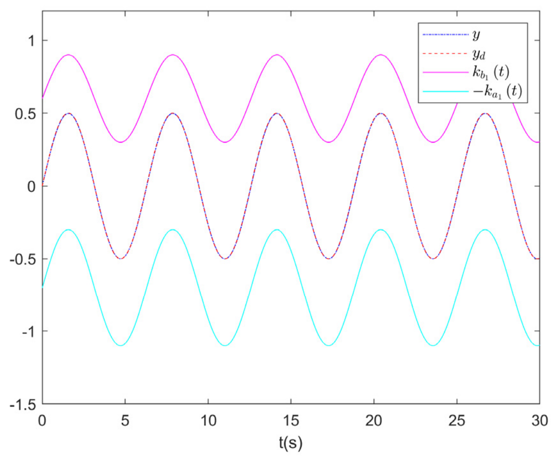

FIGURE 6. Trajectories of the desired signal y_d and the system output y (Example 2).

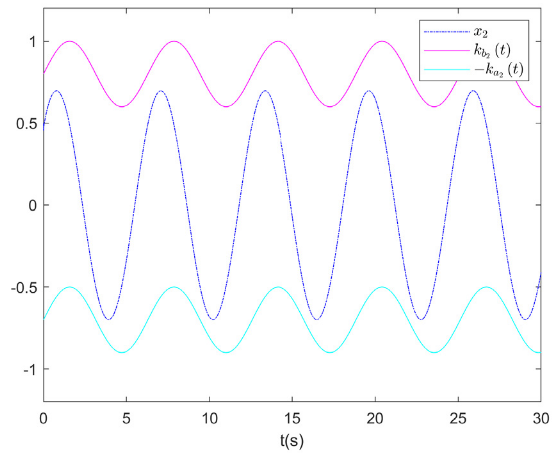

FIGURE 7. Trajectory of the state variable x_2 (Example 2).

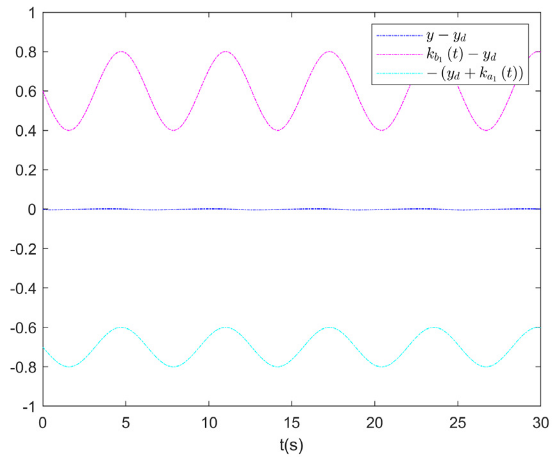

FIGURE 8. Trajectory of the tracking error $(y-y_d)$ (Example 2).

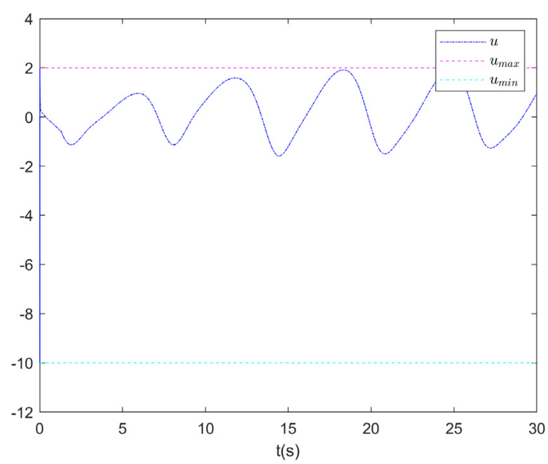

FIGURE 9. Trajectory of the system input u (Example 2).

into account, the balance equation for the above system is given as:

$$\begin{cases} \dot{x}_1 = \dfrac{1-R_2}{V_1}x_2 - \frac{1}{\Theta_1}x_1 - K_1x_1 + d_1(t) \\ \dot{x}_2 = \dfrac{F}{V_2}u - \dfrac{1}{\Theta_2}x_2 - K_2x_2 + \dfrac{R_1}{V_2}x_1(t-\tau_1(t)) \\ \qquad + \dfrac{R_2}{V_2}x_2(t-\tau_2(t)) + d_2(t) \\ y = x_1 \end{cases} \tag{76}$$

where x_1 and x_2 are the compositions, R_1 and R_2 are the recycle flow rates, Θ_1 and Θ_2 are the reactor residence rate, K_1 and K_2 are the reaction constants, F is the feed rate, V_1 and V_2 are the reactor volumes, $d_1(t)$ and $d_2(t)$ are the external perturbations, $\tau_1(t)$ and $\tau_2(t)$ are the time- varying delays, u and y are the input and output of the system.

In this paper, the system parameters are chosen as $R_1 = R_2 = 0.5, \Theta_1 = \Theta_2 = 2, K_1 = K_2 = 0.5, F = 0.5, V_1 = V_2 = 0.5$. The time-varying delays are $\tau_1(t) = 1.2\ln(1+t^2)$ and $\tau_2(t) = 1 + \cos(t)$. The external perturbations are $d_1(t) = 0.01\sin(t)$ and $d_2(t) = 0.01\cos(t)$. The desired signal is $y_d = 0.5\sin(t)$. The state constraint conditions are taken as $k_{a_1}(t) = 0.7 - 0.4\sin(t), k_{b_1}(t) = 0.6 + 0.3sin(t)$, $k_{a_2}(t) = 0.7 - 0.2\sin(t), k_{b_2}(t) = 0.8 + 0.2sin(t)$. The input saturation boundaries are $u_{\max} = 2$ and $u_{\min} = -10$.

In the simulation, the initial conditions are set as $x_1(0) = 0, x_2(0) = 0.5, \hat{\theta}(0) = 0.8, s_1(0) = s_2(0) = 0$. The design parameters are given as $k_1 = 210, k_2 = 300, k_{\rho_1} = k_{\rho_2} = 1, \rho = 0.5, a_1 = 6, a_2 = 3, \delta = 0.5, \varsigma = 0.9, \omega_2 = 1000$.

The simulation results are shown from Figure 6 to Figure 10. The trajectories of the desired signal y_d and the system output y are displayed in Figure 6. It is seen from Figure 6 that the output y can track the desired signal y_d satisfactorily. The trajectory of the state x_2 is given in Figure 7. As observed from Figure 6 and Figure 7, all states never violate the asymmetric time-varying constraints. Figure 8 plots the trajectory of the tracking error $(y-y_d)$. The trajectories of the system input u and the adaptive

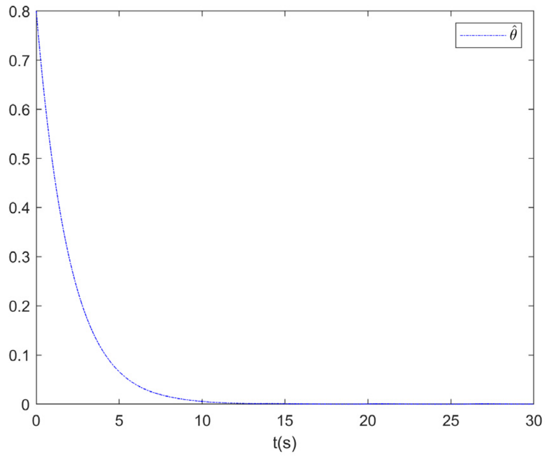

FIGURE 10. The adaptive parameters (Example 2).

parameter $\hat{\theta}$ are plotted in Figure 9 and Figure 10, respectively. Based on the simulation results, it is shown that our proposed method is effective and practicable.

V. CONCLUSION

In this paper, an adaptive finite-time tracking control approach has been proposed for a class of nonlinear time-varying delay systems in strict-feedback form with asymmetric time-varying full state constraints and input saturation. By combining finite covering lemma with RBF NNs, the effect of unknown time-varying delays is eliminated and the assumption that derivative of time delays is less than one is relaxed. The NSDFs and backstepping technique are used to guarantee that all states never violate the asymmetric time-varying constraints, and the feasibility conditions on virtual controllers are not required. The explosion of complexity problem in backstepping is handled by the command filter method. Moreover, the augmented function with an auxiliary control signal is introduced to solve the input saturation. The proposed scheme can guarantee that all signals in the closed-loop system are bounded and the tracking error converges to a small neighborhood around the origin in a finite time. Finally, not only a numerical example but also a practical example is given to verify the effectiveness of the proposed scheme. In the future research, we will employ the Nussbaum gain technique to deal with the unknown control directions and extend this result to the nonlinear pure-feedback systems with completely unknown control gains.

REFERENCES

[1] K. P. Tee, S. S. Ge, and E. H. Tay, "Barrier Lyapunov functions for the control of output-constrained nonlinear systems," *Automatica*, vol. 45, no. 4, pp. 918–927, Apr. 2009.

[2] J. Nie and X. Lin, "Robust nonlinear path following control of underactuated msv with time-varying sideslip compensation in the presence of actuator saturation and error constraint," *IEEE Access*, vol. 6, pp. 71906–71917, 2018.

[3] J. Yu, L. Zhao, H. Yu, and C. Lin, "Barrier Lyapunov functions-based command filtered output feedback control for full-state constrained nonlinear systems," *Automatica*, vol. 105, pp. 71–79, Jul. 2019.

[4] H. Liu and G. Chen, "Robust trajectory tracking control of marine surface vessels with uncertain disturbances and input saturations," *Nonlinear Dyn.*, vol. 100, no. 4, pp. 3513–3528, Jun. 2020.

[5] L. Huang, Y. Li, and S. Tong, "Fuzzy adaptive output feedback control for MIMO switched nontriangular structure nonlinear systems with unknown control directions," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 2, pp. 550–564, Feb. 2020.

[6] H. Liang, G. Liu, H. Zhang, and T. Huang, "Neural-network-based event-triggered adaptive control of nonaffine nonlinear multiagent systems with dynamic uncertainties," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 32, no. 5, pp. 2239–2250, May 2021.

[7] J. Yang, Q. Zhang, Y. Li, and J. Wang, "Robust adaptive control for stochastic discrete-time nonlinear systems and application to gas engine as an electric vehicle extender," *IEEE Access*, vol. 8, pp. 156433–156441, 2020.

[8] V. L. Kharitonov and A. P. Zhabko, "Lyapunov–Krasovskii approach to the robust stability analysis of time-delay systems," *Automatica*, vol. 39, no. 1, pp. 15–20, Jan. 2003.

[9] S. Yin, P. Shi, and H. Yang, "Adaptive fuzzy control of strict-feedback nonlinear time-delay systems with unmodeled dynamics," *IEEE Trans. Cybern.*, vol. 46, no. 8, pp. 1926–1938, Aug. 2016.

[10] B. Li, J. Xia, W. Sun, J. H. Park, and Z. Sun, "Command filter-based event-triggered adaptive neural network control for uncertain nonlinear time-delay systems," *Int. J. Robust Nonlinear Control*, vol. 30, no. 16, pp. 6363–6382, Nov. 2020.

[11] B. Niu, D. Wang, M. Liu, X. Song, H. Wang, and P. Duan, "Adaptive neural output-feedback controller design of switched nonlower triangular nonlinear systems with time delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 10, pp. 4084–4093, Oct. 2020.

[12] S. Zhang, W.-Y. Cui, and F. E. Alsaadi, "Adaptive backstepping control design for uncertain non-smooth strictfeedback nonlinear systems with time-varying delays," *Int. J. Control, Autom. Syst.*, vol. 17, no. 9, pp. 2220–2233, Sep. 2019.

[13] A.-M. Kang and H.-S. Yan, "Stability analysis and dynamic regulation of multi-dimensional Taylor network controller for SISO nonlinear systems with time-varying delay," *ISA Trans.*, vol. 73, pp. 31–39, Feb. 2018.

[14] H. F. Min, S. Xu, B. Zhang, and Q. Ma, "Globally adaptive control for stochastic nonlinear time-delay systems with perturbations and its application," *Automatica*, vol. 102, pp. 105–110, Apr. 2019.

[15] D. Zhai, C. Xi, J. Dong, and Q. Zhang, "Adaptive fuzzy fault-tolerant tracking control of uncertain nonlinear time-varying delay systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 5, pp. 1840–1849, May 2020.

[16] L. Weiss and E. F. Infante, "On the stability of systems defined over a finite time interval," *Proc. Nat. Acad. Sci. USA*, vol. 54, no. 1, pp. 44–48, Jul. 1965.

[17] S. Sui, C. L. P. Chen, and S. Tong, "Fuzzy adaptive finite-time control design for nontriangular stochastic nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 1, pp. 172–184, Jan. 2019.

[18] F. Wang and X. Zhang, "Adaptive finite time control of nonlinear systems under time-varying actuator failures," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 9, pp. 1845–1852, Sep. 2019.

[19] P. Zhou, L. Zhang, S. Zhang, and A. F. Alkhateeb, "Observer-based adaptive fuzzy finite-time control design with prescribed performance for switched pure-feedback nonlinear systems," *IEEE Access*, vol. 9, pp. 69481–69491, 2021.

[20] D. Swaroop, J. K. Hedrick, P. P. Yip, and J. C. Gerdes, "Dynamic surface control for a class of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 45, no. 10, pp. 1893–1899, Oct. 2000.

[21] Y. Liu, X. Liu, Y. Jing, and Z. Zhang, "Semi-globally practical finite-time stability for uncertain nonlinear systems based on dynamic surface control," *Int. J. Control*, vol. 94, no. 2, pp. 476–485, Apr. 2019.

[22] N. Sheng, D. Zhang, and Q. Zhang, "Fuzzy command filtered backstepping control for nonlinear system with nonlinear faults," *IEEE Access*, vol. 9, pp. 60409–60418, 2021.

[23] J. Yu, B. Chen, H. Yu, C. Lin, and L. Zhao, "Neural networks-based command filtering control of nonlinear systems with uncertain disturbance," *Inf. Sci.*, vol. 426, pp. 50–60, Feb. 2018.

[24] L. Wang, H. Wang, and P. X. Liu, "Adaptive fuzzy finite-time control of stochastic nonlinear systems with actuator faults," *Nonlinear Dyn.*, vol. 104, no. 1, pp. 523–536, Mar. 2021.

[25] M. Burger and M. Guay, "Robust constraint satisfaction for continuous-time nonlinear systems in strict feedback form," *IEEE Trans. Autom. Control*, vol. 55, no. 11, pp. 2597–2601, Nov. 2010.

[26] A. Bemporad, "Reference governor for constrained nonlinear systems," *IEEE Trans. Autom. Control*, vol. 43, no. 3, pp. 415–419, Mar. 1998.
[27] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert, "Constrained model predictive control: Stability and optimality," *Automatica*, vol. 36, no. 6, pp. 789–814, Jun. 2000.
[28] Q. Zhu, Y. Liu, and G. Wen, "Adaptive neural network output feedback control for stochastic nonlinear systems with full state constraints," *ISA Trans.*, vol. 101, pp. 60–68, Jun. 2020.
[29] M. Wan and Y. Yin, "Adaptive dynamic surface control based on observer for switched non-strict feedback systems with full state constraints," *IEEE Access*, vol. 8, pp. 71008–71020, 2020.
[30] L. Liu, T. Gao, Y.-J. Liu, and S. Tong, "Time-varying asymmetrical BLFs based adaptive finite-time neural control of nonlinear systems with full state constraints," *IEEE/CAA J. Autom. Sinica*, vol. 7, no. 5, pp. 1335–1343, Sep. 2020.
[31] J. Liu, B. Niu, P. Zhao, X. Li, and W. Qi, "Almost fast finite-time adaptive tracking control for a class of full-state constrained pure-feedback nonlinear systems," *Int. J. Robust Nonlinear Control*, vol. 30, no. 17, pp. 7517–7532, Nov. 2020.
[32] K. Zhao and Y. Song, "Removing the feasibility conditions imposed on tracking control designs for state-constrained strict-feedback systems," *IEEE Trans. Autom. Control*, vol. 64, no. 3, pp. 1265–1272, Mar. 2019.
[33] Y. Wu, R. Xie, and X.-J. Xie, "Adaptive finite-time fuzzy control of full-state constrained high-order nonlinear systems without feasibility conditions and its application," *Neurocomputing*, vol. 399, pp. 86–95, Jul. 2020.
[34] B. Zhang, L. Jiang, and J. Bai, "Velocity-free prescribed performance control for spacecraft hovering over an asteroid with input saturation," *J. Franklin Inst.*, vol. 357, no. 11, pp. 6471–6497, Jul. 2020.
[35] Z. Chen, J. Wang, K. Ma, X. Huang, and T. Wang, "Fuzzy adaptive two-bits-triggered control for nonlinear uncertain system with input saturation and output constraint," *Int. J. Adapt. Control Signal Process.*, vol. 34, no. 4, pp. 543–559, Apr. 2020.
[36] C. Xi and J. Dong, "Adaptive neural network-based control of uncertain nonlinear systems with time-varying full-state constraints and input constraint," *Neurocomputing*, vol. 357, pp. 108–115, Sep. 2019.
[37] Q. Zhu, Y. Liu, and G. Wen, "Adaptive neural network control for time-varying state constrained nonlinear stochastic systems with input saturation," *Inf. Sci.*, vol. 527, pp. 191–209, Jul. 2020.
[38] A. Chen, L. Liu, and Y. Liu, "Adaptive control design for MIMO switched nonlinear systems with full state constraints," *Int. J. Adapt. Control Signal Process.*, vol. 33, no. 10, pp. 1583–1600, Oct. 2019.
[39] Y. Wei, P.-F. Zhou, Y.-Y. Wang, D.-P. Duan, and W. Zhou, "Adaptive neural dynamic surface control of MIMO uncertain nonlinear systems with time-varying full state constraints and disturbances," *Neurocomputing*, vol. 364, pp. 16–31, Oct. 2019.
[40] Y. Ji, H. Zhou, and Q. Zong, "Adaptive neural network command filtered backstepping control of pure-feedback systems in presence of full state constraints," *Int. J. Adapt. Control Signal Process.*, vol. 33, no. 5, pp. 829–842, May 2019.
[41] H. Wang, B. Chen, C. Lin, and Y. Sun, "Observer-based adaptive fuzzy tracking control for a class of MIMO nonlinear systems with unknown dead zones and time-varying delays," *Int. J. Syst. Sci.*, vol. 50, no. 3, pp. 546–562, Feb. 2019.
[42] D. Li, C. L. P. Chen, Y.-J. Liu, and S. Tong, "Neural network controller design for a class of nonlinear delayed systems with time-varying full-state constraints," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 9, pp. 2625–2636, Sep. 2019.
[43] X. Shi, C. Lim, S. Xu, and P. Shi, "Robust approximation-based adaptive control of multiple state delayed nonlinear systems with unmodeled dynamics," *Int. J. Robust Nonlinear Control*, vol. 28, no. 9, pp. 3303–3323, Jun. 2018.
[44] C. Zhang, X. Wang, C. Wang, and Z. Liu, "Synchronization in nonlinear complex networks with multiple time-varying delays via adaptive aperiodically intermittent control," *Int. J. Adapt. Control Signal Process.*, vol. 33, no. 1, pp. 39–51, Jan. 2019.
[45] A. Moradvandi, M. Shahrokhi, and S. A. Malek, "Adaptive fuzzy decentralized control for a class of MIMO large-scale nonlinear state delay systems with unmodeled dynamics subject to unknown input saturation and infinite number of actuator failures," *Inf. Sci.*, vol. 475, pp. 121–141, Feb. 2019.
[46] S. Ling, H. Wang, and P. X. Liu, "Adaptive fuzzy tracking control of flexible-joint robots based on command filtering," *IEEE Trans. Ind. Electron.*, vol. 67, no. 5, pp. 4046–4055, May 2020.
[47] M. Wang, B. Chen, X. Liu, and P. Shi, "Adaptive fuzzy tracking control for a class of perturbed strict-feedback nonlinear time-delay systems," *Fuzzy Sets Syst.*, vol. 159, no. 8, pp. 949–967, Apr. 2008.
[48] X. Duan, C. Yue, H. Liu, H. Guo, and F. Zhang, "Attitude tracking control of small-scale unmanned helicopters using quaternion-based adaptive dynamic surface control," *IEEE Access*, vol. 9, pp. 10153–10165, 2021.
[49] H. Wang, Y. Zou, P. X. Liu, X. Zhao, J. Bao, and Y. Zhou, "Neural-network-based tracking control for a class of time-delay nonlinear systems with unmodeled dynamics," *Neurocomputing*, vol. 396, pp. 179–190, Jul. 2020.
[50] Z. Zhu, Y. Xia, and M. Fu, "Attitude stabilization of rigid spacecraft with finite-time convergence," *Int. J. Robust Nonlinear Control*, vol. 21, no. 6, pp. 686–702, Apr. 2011.
[51] J. Xia, J. Zhang, W. Sun, B. Zhang, and Z. Wang, "Finite-time adaptive fuzzy control for nonlinear systems with full state constraints," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 7, pp. 1541–1548, Jul. 2019.
[52] R. M. Sanner and J.-J. E. Slotine, "Gaussian networks for direct adaptive control," *IEEE Trans. Neural Netw.*, vol. 3, no. 6, pp. 837–863, Nov. 1992.
[53] Y. Pan and H. Yu, "Dynamic surface control via singular perturbation analysis," *Automatica*, vol. 57, pp. 29–33, Jul. 2015.
[54] J. A. Farrell, M. Polycarpou, M. Sharma, and W. Dong, "Command filtered backstepping," *IEEE Trans. Autom. Control*, vol. 54, no. 6, pp. 1391–1395, Jun. 2009.
[55] W. Dong, J. A. Farrell, M. M. Polycarpou, V. Djapic, and M. Sharma, "Command filtered adaptive backstepping," *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 3, pp. 566–580, May 2012.
[56] H. Wang, S. Kang, and Z. Feng, "Finite-time adaptive fuzzy command filtered backstepping control for a class of nonlinear systems," *Int. J. Fuzzy Syst.*, vol. 21, no. 8, pp. 2575–2587, Nov. 2019.
[57] S. Li, C. K. Ahn, and Z. Xiang, "Command-filter-based adaptive fuzzy finite-time control for switched nonlinear systems using state-dependent switching method," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 4, pp. 833–845, Apr. 2021.
[58] S. K. Nguang, "Robust stabilization of a class of time-delay nonlinear systems," *IEEE Trans. Autom. Control*, vol. 45, no. 4, pp. 756–762, Apr. 2000.

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