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Quantized Multi-Tracking in the Multiagent System With Sampled Position Data via Intermittent Control

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ABSTRACT This paper studies the multi-tracking problem of multiagent systems with second-order dynamic. A pulse-modulation intermittent control protocol is proposed with only sampled position data of each agent. To deal with the bandwidth limitation, a stochastic quantization scheme is introduced. And the data of agents is quantified before transmission. Moreover, the proposed control protocol is improved under communication delay. Based on algebraic graph theory, stochastic quantization and stability theory, some necessary and sufficient conditions are obtained to ensure multi-tracking of the controlled system. Finally, simulation results are presented to demonstrate the effectiveness of the proposed control protocol. Agents in each subnetwork asymptotically converge to the same desired trajectory, while there is no consensus among different subnetworks.

INDEX TERMS Multi-tracking, intermittent control, sampled position data, stochastic quantization, communication delay.

I. INTRODUCTION

Cooperative control of the multiagent system has attracted significant interest from various scientific communities in recent years. Consensus problems are one kind of critical problem of cooperative control. The aim of consensus is to reach an agreement among agents under appropriate control based on neighbor information [1]–[3]. Recently, consensus-tracking has received considerable attention as a special consensus problem with a dynamic leader, in which only small part of agents can get the information from the leader. Through information exchange among agents, all agents in the network can track the dynamic leader finally [4]–[6]. Reference [4] shows, each follower can track the leader with different high-dimensions through the information exchange of the local observer. Reference [5] shows, fuzzy logic system and neural networks are employed to approximate the unknown nonlinear dynamics, the consensus tracking can achieve in the second-order multiagent under the proposed new adaptive fuzzy distributed controller. Reference [6] shows, an observer-based adaptive consensus tracking control strategy is developed to solve the unmeasurable state problem of high-order nonlinear multiagent systems.

In practice, the information exchange is discontinuous because of the unreliable communication channels, the limited transmission capability and the constrained total cost. Thus, sampled-data control can be well used in practical situations. Based on the sampled communication, many results on sampled consensus control are reported.

Reference [7] shows, a new sampled-data consensus control protocol is proposed. A distributed linear consensus protocol is designed for multiagent systems with second-order dynamics and sampled data [8]. Therefore, the communication between a group of agents may occur at some disconnected time intervals due to the failure of limited sensing ranges. Hence, it is significant to study the consensus problem with intermittent control. Reference [9] shows, some necessary and sufficient conditions are obtained for consensus of the second-order multiagent system with pulse-modulated intermittent control, all agents are convergence to a common constant. Notions of completely and partly intermittent communication are proposed for achieving consensus tracking [10]. References [11] presented the models of intermittently

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coupled complex-valued networks (ICCVNs) to reveal the mechanism of intermittent coupling.

In the study of the consensus problem in second-order multiagent system, most consensus protocol relies on the full information of agents. However, compared to position information of agents, velocity information can be hard obtained for the reason of technology limitations and environmental disturbances [12]–[15]. Thus, it is meaningful to investigate the consensus-tracking problem of multiagent system without velocity measurement. A new class of distributed finite-time protocol based on the position information is proposed [16]. Two robust controllers are proposed for the second-order tracking without velocity measurements on directed communication topologies [17].

Early efforts on distributed consensus tracking problems mainly focused on the assumption of unlimited communication channels capacity, which is may not be true in practical systems. Thus, information of agents should be quantized before transmission to subject to the channels with bandwidth constraints. Consensus problem with quantized communication has attracted much attention in the past few years [18]. Reference [19] introduced the scheme based on encoding-decoding to impulsive control protocol, the simulation results present that the proposed control protocol is the key to achieve the goal. Reference [20] studied two types of cases with and without leader agent, proposed partial state constraint and full state constraint impulsive control protocols to cut down the cost of communication. Reference [21] considered the general liner system models with and without finite time-varying time delay, introduced an offset only containing desired formation information. Stochastic quantization is adopted to the information transmission among agents for the consensus problem of multiagent systems [22]–[24]. Inspired by the above discussion, stochastic quantization scheme will be used to overcome the limited bandwidth of channels in this paper.

Most of the aforementioned works are focused on a common consensus objective. This demand that all agents of a second-order system must be in agreement so as to respond to unanticipated situations. However, the agreements are different due to the change of environments, situations or cooperative tasks. As a result, a big complex network can be regarded as a combination of some subnetworks, each subnetwork achieves consensus-tracking. The corresponding consensus problem is defined as multi-tracking problem, in which different consistent states among agents in a network can be reached instead of the fact that a consistent state is reached by all agents. A multi-tracking problem was investigated by using a distributed impulsive protocol [25]. The critical problem of multi-tracking is to design appropriate algorithms such that the agents in each subnetwork asymptotically converge to the same desired trajectory while there is no consensus among different subnetworks. The multi-tracking can be achieved by the information exchange among agents and subnetworks. Time delay is inevitable in real systems, reference [26] considered the influences of asynchronous behavior that caused by nonuniform delays, and designed a new distributed observer to eliminate the asynchronous behavior. Reference [27] addresses the regulation control problem of heterogeneous uncertain chaotic systems with nonlinear dynamic and time delay. Reference [28] investigated a class of neural networks with time-varying delays.

Enlightened by all the analysis above, multi-tracking problems of second-order multiagent systems without and with communication delay are studied. Considering technology limitations, only position data of agents is used in the pulse-modulation intermittent control. Stochastic quantization scheme is adopted before information transmission of agents for the bandwidth limitations. Based on algebraic graph theory, stochastic quantization and stability theory, some necessary and sufficient conditions are obtained to ensure multi-tracking of the controlled system. The rest of this paper is organized as follows. Section II describes the graph theory and quantization mechanism, formulate the multi-tracking problem. Quantized intermittent control based on sampled position data without and with delay are proposed in Section III and Section IV, respectively. Numerical examples are given in Section V. Finally, Section VI concludes the whole work.

The notion used in this paper is fairly standard. For convenience, denote the identify and zero matrix of order N by $I \in \mathbb{R}^{N\times N}$ and $\mathbf{0} \in \mathbb{R}^{N\times N}$, respectively. Specially, $I_3 \in \mathbb{R}^{3N\times 3N}$ and $I_5 \in \mathbb{R}^{5N\times 5N}$ are the identify matrix of order $3N$ and $5N$. $\mathbf{1}_{N_l} \in \mathbb{R}^{N_l\times 1}$ is a column vector of all ones. Let $Re(\cdot)$ and $Im(\cdot)$ be the real part and imaginary part of the complex number. The symbol $\otimes$ is the Kronecker product operator.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. GRAPH THEORY

Let $G = \{V, E, A\}$ be a weighted directed graph with a set of vertices $V = \{1, 2, \cdots, N\}$, a set of edges $E \subseteq V \times V$ and a weighted adjacency matrix $A = [a_{ij}]_{N\times N}$. An edge $(j, i) \in E$ of G means that jth agent is transport information to ith agent. The adjacency elements a_{ij} are non-zero if an only if $(j, i) \in E$, we assume $a_{ii} = 0$ for all $i \in V$. For each node $i \in V$, denoted the set of in-neighbors by $N_i^+ = \{j \in V, |(j, i) \in E\}$ with in-degree $d_i^+ = \left|N_i^+\right|$ (where $|\cdot|$ is the number of a set), and the set of out-neighbors by $N_i^- = \{j \in V|(i, j) \in E\}$. Denote $D = diag\{d_1^+, d_2^+, \ldots, d_m^+\}$ and the Laplacian matrix of the digraph G is defined as $L = D - A$. A directed path is a finite sequence of ordered edges with the form of $(i_1, i_2), (i_2, i_3), \ldots,$ where $(i_j, i_j + 1) \in E$. A node is called root such that it has a directed path to any other node of the graph. We say that a directed graph has a spanning tree, in which there are only one root and all other nodes of the graph.

Definition 1: Define the network $G_l = \{V_l, E_l, A_l\}$ to be the subnetwork of the network $G = \{V, E, A\}$ if $V_l \subseteq V$, $E_l \subseteq E$ and $A_l \subseteq A$.

Lemma 1 [29]: G contains a spanning tree, if and only if zero is a simple eigenvalue and all other eigenvalue have positive real parts of the Laplacian matrix L.

The Laplacian matrix $L = [l_{ij}]_{N\times N}$ of graph G is defined as $l_{ii} = \sum_{j\in N_i} a_{ij}$ and $l_{ij} = -a_{ij}$ for $j \neq i$.

B. QUANTIZATION MECHANISM

In this section, some brief review of the quantization scheme is introduced. Assume the value $R \in [-\mathfrak{U}, \mathfrak{U}]$ is a scalar to be quantized. We wish to obtain the quantized message $Q(R)$ with length κ bits before transmit to neighbors. Then, we will have $s = 2^{\kappa}$ quantization points given by the set $\eta = \eta_1, \eta_2, \ldots, \eta_s$. The quantization interval is $\theta = \eta_{j+1} - \eta_j$ for $j \in \ 1, 2, \ldots, s-1$. Suppose $R \in [\eta_j, \eta_{j+1}]$ and R is quantized in the following manner:

$$P\{Q(R)) = \eta_j\} = 1-\delta, \quad P\{Q(R)) = \eta_{j+1}\} = \delta, \tag{1}$$

where $\delta = (R-\eta_j)/\theta$. Thus, we can get an important lemma from [29].

Lemma 2: Assume $R \in [\eta_j, \eta_{j+1}]$ and $Q(R)$ is an κ-bite quantized message. $Q(R)$ can unbiased representation of R, which implies

$$E\{Q(R)\} = R, \quad E\{Q(R) - R)^2\} \leq \frac{\theta^2}{4} \tag{2}$$

Define $\epsilon = Q(R) - R$ as the quantization error, we have $E(\epsilon^2) \leq \theta^2/4$.

C. PROBLEM FORMULATION

Suppose that the network $G = \{V, E, A\}$ is composed of N vertices and $M \geq 2$ subnetworks, each subnetwork has $N_l, \sum_{l=1}^{M} N_l = N$ vertices, which is represented as $G_l = \{V_l, E_l, A_l\}$, $l = 1, 2, \ldots, M$, $V_l \neq \emptyset$, $\cup_{l=1}^{M} V_l = V$. For any $l' \in l, l'' \in l, l' \neq l''$, define $V_l \cap V_{l'} = \emptyset$. Labeled the vertex indexes of the lth subnetworks by $V_l = \{1 + \sum_{i=1}^{l} N_{i-1}, \ldots, \sum_{i=1}^{l} N_i\}$ and $N_0 = 0$.

Consider a group of agents with second-order dynamic described as

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = \mu_i(t), \end{cases} \tag{3}$$

where $\mu_i(t)$ is the control information to agent i. $x_i(t) \in \mathbb{R}$ and $v_i(t) \in \mathbb{R}$ represent the position and velocity value of agent i, respectively.

Specifically, the second-order dynamic of the lth subnetworks desired trajectory is given by

$$\begin{cases} \dot{x}_l^d(t) = v_l^d(t), \\ \dot{v}_l^d(t) = f_l(t), \quad l = 1, 2, \ldots, M \end{cases} \tag{4}$$

where $x_l^d(t) \in \mathbb{R}$ and $v_l^d(t) \in \mathbb{R}$ is the desired position and velocity of agent lth subnetwork, respectively. The function $f_l \geq 0 \in [0, +\infty)$ is the piecewise continuous in time t. The topology of leader and followers denoted by $\tilde{\mathcal{G}}$.

Assumption 1: The desired trajectories are controlled by a common virtual pinner which labeled by vertex 0.

Remark 1: In the case of large multi-agent system, it is difficult to control all agents simultaneously. The pinning control scheme is introduced, where the vertex 0 and the vertexes in each subnetwork should form a directed spanning tree. Therefore, vertex 0 is the only root and a small portion of the vertexes in each subnetwork is pinned. The pinning matrix is denoted by $B = diag(b_1, b_2, \ldots, b_N)$. Moreover, the vertices with zero in-degree must be pinned, i.e., $d_i^+ = 0 \Rightarrow b_i > 0$.

Definition 2: The multi-tracking of the second-order multi-agent system (3) can be reached if for any initial value,

$$\lim_{t\to\infty} E(|x_i(t) - x_l^d(t)|^2) \leq g_x(\theta), \quad \forall i \in V_l$$
$$\lim_{t\to\infty} E(|v_i(t) - v_l^d(t)|^2) \leq g_v(\theta), \quad l = 1, 2, \ldots, M.$$

where g_x and g_v are two monotonically increasing functions to θ which satisfied:

$$\lim_{\theta\to 0} g_x(\theta) = 0, \quad \lim_{\theta\to 0} g_v(\theta) = 0.$$

Let $x(t) = [x_1(t), x_2(t), \ldots, x_N(t)]^T$, $v(t) = [v_1(t), v_2(t), \ldots, v_N(t)]^T$ and $\mu(t) = [\mu_1(t), \mu_2(t), \ldots, \mu_N(t)]^T$. Under the control input $\mu_i(t)$, the multi-tacking of second-order multi-agent systems (3) can be rewritten as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & I \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mu(t) \end{bmatrix}. \tag{5}$$

Next, quantized intermittent control schemes $\mu(t)$ are proposed to solve the multi-tracking problem under the situation of without and with communication delay, respectively.

III. QUANTIZED INTERMITTENT CONTROL BASED ON SAMPLED POSITION DATA WITHOUT DELAY

In the following, a distributed intermittent control based quantization sampled position data is proposed for each subnetwork reaching a desired trajectory. The information exchange among agents occurs at sampling instant, the sampling time sequence satisfies $0 \leq t_0 < t_1 < \cdots < t_k < \cdots$ and $\tau = t_k - t_{k-1}$ represent the sampling interval.

$$\begin{aligned} \mu_i(t) &= -\phi(t-t_k)(\alpha\varphi_i(t_k) + \beta\psi_i(t_k)), \\ \varphi_i(t_k) &= b_i(x_i(t_k) + \epsilon_{x_i}(t_k) - (x_l^d(t_k) + \epsilon_{x_l^d}(t_k))) \\ &\quad + \sum_{j\in N_i} l_{ij}(x_j(t_k) + \epsilon_{x_j}(t_k) - (x_i(t_k) + \epsilon_{x_i}(t_k))), \\ \psi_i(t_k) &= b_i\zeta_i(k) + \sum_{j\in N_i} l_{ij}(\zeta_j(k) - \zeta_i(k)), \\ \zeta_i(k+1) &= -\gamma\varphi_i(t_k), \quad i \in V_l \end{aligned} \tag{6}$$

where $t_k < t \leq t_{k+1}$, $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\epsilon_{x_l^d}(t_k) = Q(x_l^d(t_k)) - x_l^d(t_k)$ is the desired position quantized error of lth subnetwork and $\epsilon_{x_i}(t_k) = Q(x_i(t_k)) - x_i(t_k)$ is the position quantized error of the ith agents. $\phi(t)$ is the pulse function which is defined below:

$$\phi(t) = \begin{cases} \hat{\phi}(t), & t \in (0, \varrho] \\ 0, & t \notin (0, \varrho] \end{cases}$$

where $\varrho \in (0, \tau]$ is the control duration, the function $\hat{\phi}(t)$ can be chosen depending on different control constraints. From the Lemma 2, we can get some properties of the position quantized error:

$$E\{\epsilon_{x_j}(t_k)\}=0,\quad E\{\epsilon_{x_j}^T(t_k)\epsilon_{x_j}(t_k)\}\le\frac{\theta_{x_j}^2}{4},$$
$$E\{\epsilon_{x_l^d}(t_k)\}=0,\quad E\{\epsilon_{x_l^d}^T(t_k)\epsilon_{x_l^d}(t_k)\}\le\frac{\theta_{x_l^d}^2}{4},$$

where θ_{x_j} and $\theta_{x_l^d}$ are constants and denoted as the quantization intervals of the position and desired position, respectively.

Let $p(t)=x(t)-x^d(t)$ and $y(t)=v(t)-v^d(t)$, where $x^d(t)=[x_1^d(t)\cdot\mathbf{1}_{N_1}^T,x_2^d(t)\cdot\mathbf{1}_{N_2}^T,\ldots,x_M^d(t)\cdot\mathbf{1}_{N_M}^T]^T$, $v^d(t)=[v_1^d(t)\cdot\mathbf{1}_{N_1}^T,v_2^d(t)\cdot\mathbf{1}_{N_2}^T,\ldots,v_M^d(t)\cdot\mathbf{1}_{N_M}^T]^T$. Denote $\omega=\omega=\int_0^{\varrho}\phi(t)dt$, $\varpi=\int_0^{\varrho}t\phi(t)dt$, $\varphi(t_k)=[\varphi_1(t_k),\varphi_2(t_k),\ldots,\varphi_N(t_k)]^T$ and $\psi(t_k)=[\psi_1(t_k),\psi_2(t_k),\ldots,\psi_N(t_k)]^T$.

For $t\in(t_k,t_k+\varrho]$, one has

$$\begin{aligned}y(t_k+\varrho)&=y(t_k)-(\alpha\varphi(t_k)+\beta\psi(t_k))\omega,\\ p(t_k+\varrho)&=p(t_k)+\int_{t_k}^{t_k+\varrho}y(t)dt\\ &=p(t_k)+\varrho y(t_k)-(\alpha\varphi(t_k)+\beta\psi(t_k))\\ &\quad\times\int_{t_k}^{t_k+\varrho}\int_0^{t-t_k}\hat{\phi}(s)dsdt\\ &=p(t_k)+\varrho y(t_k)-(\alpha\varphi(t_k)+\beta\psi(t_k))(\varrho\omega-\varpi).\end{aligned}$$

For $t\in(t_k+\varrho,t_{k+1}]$, $\mu(t)=0$ then

$$\begin{aligned}y(t_{k+1})&=y(t_k+\varrho)\\ &=y(t_k)-(\alpha\varphi(t_k)+\beta\psi(t_k))\omega\\ &=y(t_k)-\omega(\alpha\epsilon(k)+(L+B)(\alpha p(t_k)+\beta\zeta(k))),\\ p(t_{k+1})&=p(t_k+\varrho)+(\tau-\varrho)y(t_k+\varrho)\\ &=p(t_k)+\tau y(t_k)+\alpha(\varpi-\tau\omega)\epsilon(k)+(\varpi-\tau\omega)(L\\ &\quad+B)(\alpha p(t_k)+\beta\zeta(k)),\\ \zeta(k+1)&=-\gamma((L+B)p(t_k)+\epsilon(k)),\end{aligned}\tag{7}$$

where $\epsilon_i(k)=b_i(\epsilon_{x_i}(t_k)-\epsilon_{x_l^d}(t_k))+\sum_{j\in N_i}l_{ij}(\epsilon_{x_j}(t_k)-\epsilon_{x_i}(t_k))$, $\epsilon(k)=[\epsilon_1(k),\epsilon_2(k),\ldots,\epsilon_N(k)]^T$, and $\zeta(k)=[\zeta_1(t_k),\zeta_2(t_k),\ldots,\zeta_N(t_k)]^T$.

Let $\mathfrak{D}(t_k)=[p^T(t_k),y^T(t_k),\zeta^T(k)]^T$ and $\vartheta=\varpi-\tau\omega$, from (6) and (7), we have

$$\begin{aligned}\mathfrak{D}(t_{k+1})&=(W(\tau)+Z(\tau))\mathfrak{D}(t_k)+C(\tau)\epsilon(k),\\ W(t)&=\begin{pmatrix}I&tI&\mathbf{0}\\ \mathbf{0}&I&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}\end{pmatrix}\\ Z(t)&=\begin{pmatrix}\alpha L(\varpi-t\omega)&\mathbf{0}&\beta L(\varpi-t\omega)\\ -\omega\alpha L&\mathbf{0}&-\omega\beta L\\ -\gamma L&\mathbf{0}&\mathbf{0}\end{pmatrix},\\ C(t)&=\begin{pmatrix}(\varpi-t\omega)\alpha I\\ -\omega\alpha I\\ -\gamma I\end{pmatrix},\end{aligned}\tag{8}$$

where $\mathcal{L}=L+B$. Motivated by [30] $\aleph=W(\tau)+Z(\tau)$, the Lemma below can be obtained.

Lemma 3: Under the control scheme (6), the multi-tracking in the second-order multi-agent system (3) can be reached if and only if $\rho(\aleph)<1$, where $\rho(\cdot)$ is the matrix spectral radius of $\aleph$.

Proof: Let $\eta(k)=E(\mathfrak{D}(t_k)\otimes\mathfrak{D}(t_k))$, from (8),

$$\eta(k+1)=(\aleph\otimes\aleph)\eta(k)+(C\otimes C)E(\epsilon(k)\otimes\epsilon(k)).\tag{9}$$

Obviously, $\|\eta(k)\|_1=E(\|\mathfrak{D}(t_k)\|_1^2)$. And $\|x\|_2^2\le\|x\|_1\le\sqrt{3N}(\|x\|_2^2)$, then

$$E(\|\mathfrak{D}(t_k)\|_2^2)\le\|\eta(k)\|_1\le 3NE(\|\mathfrak{D}(t_k)\|_2^2).$$

where $\|\cdot\|_1$ and $\|\cdot\|_2$ denote the 1-norm and 2-norm, respectively. Thus, the consensus problem of the multi-agent system (3) under the control (6) can be studies by analyzing $\eta(k)$.

Necessity: If $\rho(\aleph)\ge 1$, then $\rho(\aleph\otimes\aleph)\ge 1$, the multi-agent system (3) under the control (6) must be diverging and cannot reach multi-tracking.

Sufficient: If $\rho(\aleph)<1$, then $\rho(\aleph\otimes\aleph)<1$. There must exist a small enough constant ℓ, such that $\|\aleph\otimes\aleph\|_2=\ell<1$. Combined with (9), we can conclude that

$$\|\eta(k+1)\|_2\le\ell^{k+1}\|\eta(0)\|_2+\sum_{i=0}^{k}\ell^i f(\theta_z),$$

where $f(\theta_z)=\sqrt{27N^3\|Q\otimes Q\|_2}\theta_z$ and $\theta_z=max(\theta_{x_j},\theta_{x_l^d})$.

Due to $\ell<1$, we have

$$\lim_{k\to\infty}\|\eta(k)\|_2\le f(\theta_z)/(1-\ell).$$

Then

$$E(\|\mathfrak{D}(t_k)\|_2^2)\le\lim_{k\to\infty}\|\eta(k)\|_1\le\sqrt{3N}f(\theta_z)/(1-\iota).$$

Thus, there exist positive constants $a>0$ and $b>0$, such that $\lim_{t\to\infty}E(\|p(t)\|_2^2)\le af(\theta_z)/(1-\ell)$ and $\lim_{t\to\infty}E(\|y(t)\|_2^2)\le bf(\theta_z)/(1-\ell)$. That is mean

$$\begin{aligned}&\lim_{t\to\infty}E(|x_i(t)-x_l^d(t)|^2)\le a\theta_z,\\ &\lim_{t\to\infty}E(|v_i(t)-v_l^d(t)|^2)\le b\theta_z,\quad i\in v_l,\ l=1,2,\ldots,M.\end{aligned}$$

Obviously, the right hand of the inequality is the monotonously increasing function of θ_z. Based on Definition 2, the multi-tracking of the system (3) is said to be achieved.

Lemma 4 [31]: The complex polynomial $\mathcal{R}(\sigma)=\sigma^3+(a_2+b_2i)\sigma^2+(a_1+b_1i)\sigma+a_0+b_0$. $\mathcal{R}(\sigma)$ is stable if and only if $a_2>0$, $a_2b_2b_1+a_2^2a_1-b_1^2-a_2a_0>0$ and

$$a_2det\begin{pmatrix}b_2&-a_1&-b_0&0\\ a_2&b_1&-b_1&0\\ 1&b_2&-a_1&-b_0\\ 0&a_2&b_1&-a_0\end{pmatrix}-det\begin{pmatrix}b_1&-a_0&0&0\\ a_2&b_1&-a_0&0\\ 1&b_2&-a_1&-b_0\\ 0&a_2&b_1&-a_0\end{pmatrix}>0.\tag{10}$$

Theorem 1: Suppose the direct graph $\tilde{\mathcal{G}}$ contains a spanning tree, $\kappa_i, i = 1, 2, \ldots, N$ is the ith eigenvalue of $L + B$, define the parameters below:

$$c_1 = Re\left(\frac{1}{\tau\omega\alpha - \tau\omega\gamma\beta\kappa_i}\right), \quad c_2 = Im\left(\frac{1}{\tau\omega\alpha - \tau\omega\gamma\beta\kappa_i}\right),$$
$$c_3 = Re\left(\frac{\kappa_i}{\tau\omega\alpha - \tau\omega\gamma\beta\kappa_i}\right), \quad c_4 = Im\left(\frac{\kappa_i}{\tau\omega\alpha - \tau\omega\gamma\beta\kappa_i}\right),$$
$$c_5 = Re\left(\frac{1}{(\tau\omega\alpha - \tau\omega\gamma\beta\kappa_i)\kappa_i}\right),$$
$$c_6 = Im\left(\frac{1}{(\tau\omega\alpha - \tau\omega\gamma\beta\kappa_i)\kappa_i}\right),$$
$$a_2 = (\tau\omega + 2\varpi)\gamma\beta c_3 - \vartheta\alpha c_1,$$
$$b_2 = (\tau\omega + 2\varpi)\gamma\beta c_4 - \vartheta\alpha c_2,$$
$$a_1 = 4c_5 - 4\varpi\beta\gamma c_3, \quad b_1 = 4c_6 - 4\varpi\beta\gamma c_4,$$
$$a_0 = 4c_5 - a_2, b_0 = 4c_6 - b_2.$$

Under the distributed intermittent control (6), the uniform multi-tracking of the second-order system (3) can be asymptotically reached if and only if inequalities (10) are satisfied.

Proof: Inspired by [32], the characteristic polynomial of $\aleph$ is given by

$$\begin{aligned} &det(\lambda I_3 - \aleph) \\ &= det\left(\begin{bmatrix} (\lambda-1)I - \alpha\vartheta\mathcal{L} & -\tau I & -\beta\vartheta\mathcal{L} \\ \omega\alpha\mathcal{L} & (\lambda-1)I & \omega\beta\mathcal{L} \\ \gamma\mathcal{L} & \mathbf{0} & \lambda I \end{bmatrix}\right) \\ &= det\Big(\lambda(\lambda-1)^2 I - (\lambda-1)\lambda\vartheta\alpha\mathcal{L} \\ &\quad + \lambda(\omega\alpha\mathcal{L} + \gamma\beta\vartheta\mathcal{L}^2) - \varpi\gamma\beta\kappa_i^2\Big) \\ &= \prod_{i=1}^{N}\{\lambda(\lambda-1)^2 - (\lambda-1)\lambda\vartheta\alpha\kappa_i \\ &\quad + \lambda(\omega\alpha\kappa_i + \gamma\beta\vartheta\kappa_i^2) - \varpi\gamma\beta\kappa_i^2\}, \end{aligned} \tag{11}$$

where κ_i is the eigenvalue of $\mathcal{L}$. Thus, three eigenvalues of H can be obtained for each κ_i. The roots of characteristic polynomial $det(\lambda I_3 - \aleph) = 0$ can be denoted as

$$\lambda(\lambda-1)^2 - (\lambda-1)\lambda\alpha\kappa_i + \lambda(\omega\alpha\kappa_i\tau + \gamma\beta\vartheta\kappa_i^2) - \varpi\gamma\vartheta\kappa_i^2 = 0. \tag{12}$$

Let $\lambda = z + 1/z - 1$, applying a bilinear transformation, (12) can be cast to

$$R_i(z) = z^3 + (a_2 + ib_2)z^2 + (a_1 + ib_1)z + a_0 + ib_0. \tag{13}$$

From Lemma 3 we know that the multi-tracking in the second-order multi-agent system (3) can be achieved if and only if $\rho(\aleph) < 1$, which is mean the (13) is Hurwitz stable. From Lemma 4, polynomial (13) is Hurwitz stable if and only if the inequalities in (10) holds. Thus, the proof is completed.

IV. QUANTIZED INTERMITTENT CONTROL BASED ON SAMPLED POSITION DATA WITH DELAY

As we know, communication delay always exists in practical application. And it has great impact on consensus performance of multi-agent system. In this section, the communication delay $\iota < \tau$ is considered in the multi-agent system (3). A quantized intermittent control with communication delay for each subnetwork reaching a desired trajectory is proposed below:

$$\begin{aligned} \mu_i(t) &= -\phi(t - t_k)(\alpha\varphi_i(t_k - \iota) + \beta\psi_i(t_k)), \\ \varphi_i(t_k - \iota) &= b_i(x_i(t_k - \iota) + \epsilon_{x_i}(t_k - \iota) - (x_l^d(t_k - \iota) \\ &\quad + \epsilon_{x_l^d}(t_k - \iota))) \\ &\quad + \sum_{j\in N_i} l_{ij}(x_j(t_k - \iota) + \epsilon_{x_j}(t_k - \iota) - (x_i(t_k \\ &\quad - \iota) + \epsilon_{x_i}(t_k - \iota))), \\ \psi_i(t_k) &= b_i\zeta_i(k) + \sum_{j\in N_i} l_{ij}(\zeta_j(k) - \zeta_i(k)), \\ \zeta_i(k+1) &= -\gamma\varphi_i(t_k), \quad i \in V_l \end{aligned} \tag{14}$$

Similar to the analysis of Section (III), for $t \in (t_k, t_k + \varrho]$,

$$\begin{aligned} y(t_k + \varrho) &= y(t_k) - (\alpha\varphi(t_k - \iota) + \beta\psi(t_k))\omega, \\ p(t_k + \varrho) &= p(t_k) + \int_{t_k}^{t_k+\varrho} y(t)dt \\ &= p(t_k) + \varrho y(t_k) - (\alpha\varphi(t_k - \iota) + \beta\psi(t_k)) \\ &\quad \times \int_{t_k}^{t_k+\varrho}\int_0^{t-t_k} \hat{\phi}(s)dsdt \\ &= p(t_k) + \varrho y(t_k) - (\alpha\varphi(t_k - \iota) \\ &\quad + \beta\psi(t_k))(\varrho\omega - \varpi). \end{aligned}$$

For $t \in (t_k + \varrho, t_{k+1}]$, $\hat{\mu}_i(t) = 0$, one has

$$\begin{aligned} y(t_{k+1}) &= y(t_k + \varrho) \\ &= y(t_k) - (\alpha\varphi(t_k - \iota) + \beta\psi(t_k))\omega \\ &= y(t_k) - \omega(\alpha\varsigma(k) + \mathcal{L}(\alpha p(t_k - \iota) + \beta\zeta(k))), \\ p(t_{k+1}) &= p(t_k + \varrho) + (\tau - \varrho)y(t_k + \varrho) \\ &= p(t_k) + \tau y(t_k) + \alpha\vartheta\varsigma(k) + \vartheta\mathcal{L}(\alpha p(t_k - \iota) \\ &\quad + \beta\zeta(k)), \\ \zeta(k+1) &= -\gamma(\mathcal{L}p(t_k - \iota) + \varsigma(k)), \end{aligned} \tag{15}$$

where $\varsigma(k) = [\epsilon_1(t_k - \iota), \epsilon_2(t_k - \iota), \ldots, \epsilon_N(t_k - \iota)]^T$ and $\epsilon_i(t_k - \iota) = b_i(\epsilon_{x_i}(t_k - \iota) - \epsilon_{x_l^d}(t_k - \iota)) + \sum_{j\in N_i} l_{ij}(\epsilon_{x_j}(t_k - \iota) - \epsilon_{x_i}(t_k - \iota))$. Let $\mathfrak{F}(t_k) = [p(t_k)^T, y(t_k)^T, \zeta(k)]^T$, under the control (14), the multi-agent system (3) can be expressed as:

$$\mathfrak{F}(t_{k+1}) = W(\tau)\mathfrak{F}(t_k) + Z(\tau)\mathfrak{F}(t_k - \iota) + C(\tau)\varsigma(k). \tag{16}$$

In order to obtain $\mathfrak{F}(t_k - \iota)$, we discuss two cases of ι below: case 1): $\iota < \tau - \varrho$

$$\begin{aligned} y(t_{k+1} - \iota) &= y(t_k + \varrho) \\ &= y(t_k) - \omega(\alpha\varsigma(k) + \mathcal{L}(\alpha p(t_k - \iota) + \beta\zeta(k))), \\ p(t_{k+1} - \iota) &= p(t_k + \varrho) + (\tau - \varrho - \iota)y(t_k + \varrho) \\ &= p(t_k) + (\tau - \iota)y(t_k) + (\varpi - (\tau - \iota)\omega)(\alpha\varsigma(k) \\ &\quad + \mathcal{L}(\alpha p(t_k - \iota) + \beta\zeta(k))). \end{aligned}$$

Then

$$\mathfrak{F}(t_{k+1}-\imath) = W(\tau-\imath)\mathfrak{F}(t_k) + Z(\tau-\imath)\mathfrak{F}(t_k-\imath) + C(\tau-\imath)\varsigma(k). \quad (17)$$

Let $\mathfrak{M}(t_k) = [p(t_k)^T, y(t_k)^T, p(t_k-\imath)^T, y(t_k-\imath)^T, \zeta(k)]^T$ and $\vartheta_\imath = \varpi - (\tau-\imath)\omega$. Combing the equation (16) and (17), one can obtain:

$$\mathfrak{M}(t_{k+1}) = H\mathfrak{M}(t_k) + D\varsigma(k).$$
$$H = \begin{pmatrix} I & \tau I & \alpha\mathcal{L}\vartheta & \mathbf{0} & \beta\mathcal{L}\vartheta \\ \mathbf{0} & I & -\omega\alpha\mathcal{L} & 0 & -\omega\beta\mathcal{L} \\ I & (\tau-\imath)I & \alpha\mathcal{L}\vartheta_\imath & \mathbf{0} & \beta\mathcal{L}\vartheta_\imath \\ \mathbf{0} & I & -\omega\alpha\mathcal{L} & \mathbf{0} & -\omega\beta\mathcal{L} \\ \mathbf{0} & \mathbf{0} & -\gamma\mathcal{L} & \mathbf{0} & \mathbf{0} \end{pmatrix},$$
$$D = \begin{pmatrix} \vartheta\alpha I \\ -\omega\alpha I \\ \vartheta_\imath\alpha I \\ -\omega\alpha I \\ -\gamma I \end{pmatrix}. \quad (18)$$

case 2): $\tau-\varrho \le \imath < \tau$

Let $\omega_\imath = \int_0^{\tau-\imath}\phi(t)dt$, $\varpi_\imath = \int_0^{\tau-\imath} t\phi(t)dt$ and $\hat{\vartheta} = \varpi_\imath - (\tau-\imath)\omega_\imath$.

$$\begin{aligned} y(t_{k+1}-\imath) &= y(t_k) - \int_{t_k}^{t_{k+1}-\imath}\phi(t-t_k) \\ &\quad \times (\alpha\varphi(t_k-\imath) + \beta\psi(t_k))dt \\ &= y(t_k) - \omega_\imath(\alpha\varsigma(k) + \mathcal{L}(\alpha p(t_k-\imath) + \beta\zeta(k))), \\ p(t_{k+1}-\imath) &= p(t_k) + \int_{t_k}^{t_{k+1}-\imath}\int_0^{t-t_k} y(t)dt \\ &= p(t_k) + (\tau-\imath)y(t_k) + \hat{\vartheta}(\alpha\varsigma(k) \\ &\quad + \mathcal{L}(\alpha p(t_k-\imath) + \beta\zeta(k))). \end{aligned}$$

Then

$$\mathfrak{F}(t_{k+1}-\imath) = W(\tau-\imath)\mathfrak{F}(t_k) + \hat{Z}(\tau-\imath)\mathfrak{F}(t_k-\imath) + \hat{C}(\tau-\imath)\varsigma(k). \quad (19)$$

where

$$\hat{Z}(t) = \begin{pmatrix} \alpha\mathcal{L}(\varpi_\imath - t\omega_\imath) & \mathbf{0} & \beta\mathcal{L}(\varpi_\imath - t\omega_\imath) \\ -\omega_\imath\alpha\mathcal{L} & \mathbf{0} & -\omega_\imath\beta\mathcal{L} \\ -\gamma\mathcal{L} & \mathbf{0} & \mathbf{0} \end{pmatrix},$$
$$\hat{C}(t) = \begin{pmatrix} (\varpi_\imath - t\omega_\imath)\alpha I \\ -\omega_\imath\alpha I \\ -\gamma I \end{pmatrix}.$$

Combing the equation (16) and (19), one can obtain:

$$\mathfrak{M}(t_{k+1}) = \hat{H}\mathfrak{M}(t_k) + \hat{D}\varsigma(k).$$
$$\hat{H} = \begin{pmatrix} I & \tau I & \alpha\mathcal{L}\vartheta & \mathbf{0} & \beta\mathcal{L}\vartheta \\ \mathbf{0} & I & -\omega\alpha\mathcal{L} & \mathbf{0} & -\omega\beta\mathcal{L} \\ I & (\tau-\imath)I & \alpha\mathcal{L}\hat{\vartheta} & \mathbf{0} & \beta\mathcal{L}\hat{\vartheta} \\ \mathbf{0} & I & -\omega_\imath\alpha\mathcal{L} & \mathbf{0} & -\omega_\imath\beta\mathcal{L} \\ \mathbf{0} & \mathbf{0} & -\gamma\mathcal{L} & \mathbf{0} & \mathbf{0} \end{pmatrix},$$
$$\hat{D} = \begin{pmatrix} \vartheta\alpha I \\ -\omega\alpha I \\ \hat{\vartheta}\alpha I \\ -\omega_\imath\alpha I \\ -\gamma I \end{pmatrix}. \quad (20)$$

Similar analysis by Lemma 3, for case (1), one can deduce that the multi-agent system (3) with communication delay can be reached multi-tracking if an only if $\rho(H) < 1$. And for case (2), one can obtain that the multi-agent system (3) with communication delay can be reached multi-tracking if an only if $\rho(\hat{H}) < 1$.

Theorem 2: Suppose the direct graph $\tilde{\mathcal{G}}$ contains a spanning tree, $\kappa_i, i = 1, 2, \ldots, N$ is the ith eigenvalue of $L + B$. Under the distributed intermittent control (14), the multi-tracking of the second-order system (3) can be asymptotically reached if and only if the polynomial:

$$\begin{aligned} \hat{R}(z) = &(1+d_3+d_2+d_1+d_0)z^4 + (4+2d_3-2d_1-4d_0)z^3 \\ &+ (6-2d_2+6d_0)z^2 + (4-2d_3+2d_2-4d_0)z + (1 \\ &- d_3+d_2-d_1+d_0) \end{aligned} \quad (21)$$

is Hurwitz stable, where

$$\begin{aligned} d_3 &= -(2+\alpha\kappa_i\hbar), \\ d_2 &= 1 + 2\alpha\kappa_i\hbar + \alpha\kappa_i\vartheta + \gamma\kappa_i^2\beta\hbar + \omega\alpha\kappa_i(\tau-\imath), \\ d_1 &= \omega\beta\kappa_i^2\gamma(\tau-\imath) + \gamma\beta\kappa_i^2\vartheta + \alpha\kappa_i\vartheta + \alpha\kappa_i\hbar + \omega\alpha\imath\kappa_i, \\ d_0 &= \omega\beta\kappa_i^2\gamma(2\tau-\imath) + \gamma\kappa_i^2\beta\hbar + \gamma\kappa_i^2\beta\vartheta \end{aligned} \quad (22)$$

1). when $\imath < \tau-\varrho$, $\hbar = \vartheta_\imath$;

2). when $\tau-\varrho \le \imath < \tau$, $\hbar = \hat{\vartheta}$.

Proof: For the case (1), the characteristic polynomial of H is given by

$$\begin{aligned} &det(\lambda I_5 - H) \\ &= det\left(\begin{bmatrix} (\lambda-1)I & -\tau I & -\alpha\mathcal{L}\vartheta & \mathbf{0} & -\beta\mathcal{L}\vartheta \\ \mathbf{0} & (\lambda-1)I & \omega\alpha\mathcal{L} & \mathbf{0} & \omega\beta\mathcal{L} \\ -I & -(\tau-\imath)I & \lambda I - \alpha\mathcal{L}\vartheta_\imath & \mathbf{0} & -\beta\mathcal{L}\vartheta_\imath \\ \mathbf{0} & -I & \omega\alpha\mathcal{L} & \lambda I & \omega\beta\mathcal{L} \\ \mathbf{0} & \mathbf{0} & \gamma\mathcal{L} & \mathbf{0} & \lambda I \end{bmatrix}\right) \\ &= det(\lambda((\lambda^4 - 2\lambda^3 + \lambda^2)I - (\alpha\vartheta_\imath\lambda^3 - (2\alpha\vartheta_\imath + \alpha\vartheta - \omega\alpha(\tau \\ &\quad -\imath))\lambda^2 - (\alpha\vartheta + \alpha\vartheta_\imath + \omega\alpha\imath)\lambda)\mathcal{L} + (\gamma\beta\vartheta_\imath\lambda^2 - (\omega\beta\gamma(\tau-\imath) \\ &\quad + \gamma\beta\vartheta)\lambda + \omega\beta\gamma(2\tau-\imath) + \gamma\beta\vartheta_\imath + \gamma\beta\vartheta)\mathcal{L}^2)) \\ &= \prod_{i=1}^N \{\lambda((\lambda^4 - 2\lambda^3 + \lambda^2) - (\alpha\vartheta_\imath\lambda^3 - (2\alpha\vartheta_\imath + \alpha\vartheta - \omega\alpha(\tau \\ &\quad -\imath))\lambda^2 - (\alpha\vartheta + \alpha\vartheta_\imath + \omega\alpha\imath)\lambda)\kappa_i + (\gamma\beta\vartheta_\imath\lambda^2 - (\omega\beta\gamma(\tau-\imath) \\ &\quad + \gamma\beta\vartheta)\lambda + \omega\beta\gamma(2\tau-\imath) + \gamma\beta\vartheta_\imath + \gamma\beta\vartheta)\kappa_i^2)\}. \end{aligned} \quad (23)$$

The eigenvalues of H satisfy

$$\begin{aligned} &\lambda(\lambda^4 - (2+\alpha\kappa_i\vartheta_\imath)\lambda^3 + (1 + 2\alpha\kappa_i\vartheta_\imath + \alpha\kappa_i\vartheta + \gamma\kappa_i^2\beta\vartheta_\imath \\ &\quad + \omega\alpha\kappa_i(\tau-\imath))\lambda^2 - (\omega\beta\kappa_i^2\gamma(\tau-\imath) + \gamma\beta\kappa_i^2\vartheta + \alpha\kappa_i\vartheta \\ &\quad + \alpha\kappa_i\vartheta_\imath + \omega\alpha\imath\kappa_i)\lambda + \omega\beta\kappa_i^2\gamma(2\tau-\imath) + \gamma\kappa_i^2\beta\vartheta_\imath \\ &\quad + \gamma\kappa_i^2\beta\vartheta) = 0. \end{aligned}$$

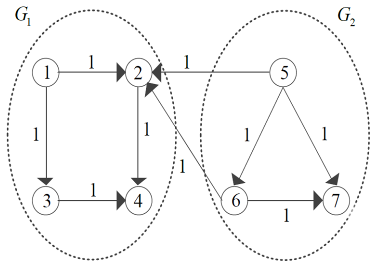

FIGURE 1. Topology graphs of a multi-agent system with seven agents.

It not difficult to see, for each given κ_i, five eigenvalues of H can be obtained where 0 is a special eigenvalue. For simplified calculation, let

$$\lambda = \frac{z+1}{z-1}$$

and

$$\begin{aligned} d_0 &= \omega\beta\kappa_i^2\gamma(2\tau - \iota) + \gamma\kappa_i^2\beta\vartheta_\iota + \gamma\kappa_i^2\beta\vartheta, \\ d_1 &= \omega\beta\kappa_i^2\gamma(\tau - \iota) + \gamma\beta\kappa_i^2\vartheta + \alpha\kappa_i\vartheta + \alpha\kappa_i\vartheta_\iota + \omega\alpha\iota\kappa_i, \\ d_2 &= 1 + 2\alpha\kappa_i\vartheta_\iota + \alpha\kappa_i\vartheta + \gamma\kappa_i^2\beta\vartheta_\iota + \omega\alpha\kappa_i(\tau - \iota), \\ d_3 &= -(2 + \alpha\kappa_i\vartheta_\iota). \end{aligned}$$

One has

$$\begin{aligned} &(1 + d_3 + d_2 + d_1 + d_0)z^4 + (4 + 2d_3 - 2d_1 - 4d_0)z^3 \\ &\quad + (6 - 2d_2 + 6d_0)z^2 + (4 - 2d_3 + 2d_1 - 4d_0)z + 1 \\ &\quad - d_3 + d_2 - d_1 + d_0 = 0 \end{aligned} \tag{24}$$

It is not difficult to know that $\rho(H) < 1$ if and only if $R(z)$ is Hurwitz stable. It implies that the multi-tracking of the second-order system (3) can be asymptotically achieved if and only if the polynomial (21) is Hurwitz stable.

For the case (2), the characteristic polynomial of $\hat{H}$ is given by $det(\lambda I_5 - \hat{H})$. By an analysis similar to proof of case 1), one can easily get the Hurwitz stable of (21) in case 2).

V. NUMERICAL EXAMPLES

In this section, several simulations are shown to demonstrate the effectiveness of the proposed control protocol. A directed multi-agent system with seven agents is considered, and the topology graph is given in Figure. 1. From Figure. 1, it is easy to known the network is divided into two subnetworks: $V_1 = \{1, 2, 3, 4\}$ and $V_2 = \{5, 6, 7\}$. Only the agent 1 and 5 are pinned, and the matrix $B = diag(1, 0, 0, 0, 0, 1, 0)$. Next, we will evaluate the performance of the intermittent control without and with communication delay, perspective.

A. MULTI-TRACKING WITHOUT COMMUNICATION DELAY

In this section, the multi-tracking problem of second-order multi-agent system without communication delay is considered. In particular, let the function $\tilde{\phi} = 1/\varrho$, one has $\omega = 1$ and $\varpi = \varrho/2$, in which the control scheme (eq6) is reduced to an impulsive control. We denote the control parameters $\alpha = 3$, $\beta = 0.4$, $\gamma = 0.1$ and the control duration $\varrho = 0.001$.

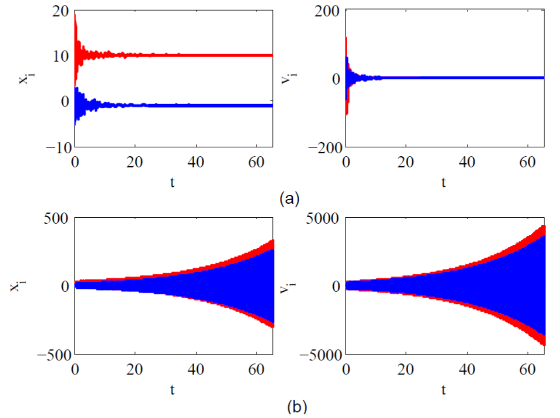

FIGURE 2. Static multi-tracking. (a) $\tau = 0.14$. (b) $\tau = 0.1476$.

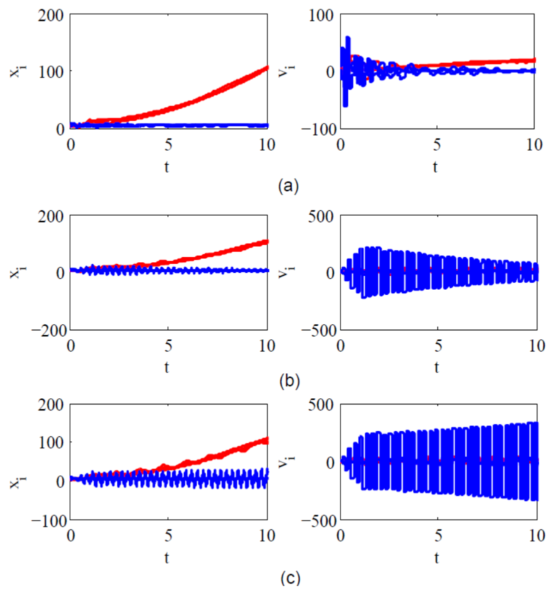

FIGURE 3. Dynamic multi-tracking. (a) $\tau = 0.5$. (b) $\tau = 0.58$. (c) $\tau = 0.59$.

Let the desired positions are $x_1^d = 10$ and $x_2^d = -1$. Then, it is not difficult to get that $0.001 < \tau < 0.1476$ from Lemma 1. Simulation results are shown in Figure. 2, one can see that the multi-tracking can be achieved with $\tau = 0.14$ form Figure. 2(a), while the multi-tracking cannot be achieved with $\tau = 0.1476$ form Figure.2 (b).

B. MULTI-TRACKING WITH COMMUNICATION DELAY

In this section, the multi-tracking problem of second-order multi-agent system with communication delay is considered. We denote the control parameters $\alpha = 4.5$, $\beta = 3$ and $\gamma = 0.3$. In particular, let the function $\tilde{\phi} = 1$, in which the control scheme (14) is reduced to a sample control. Let the desired positions are $x_1^d = t^2$ and $x_2^d = 5$.

Case 1): $\iota < \tau - \varrho$. Choose $\iota = 0.05$ and $\varrho = 0.1$. By Theorem 2, one has that multi-tracking of system (3) under protocol (14) can be achieved if and only if $0.1 < \tau < 0.58$. From Fig. 3, one has the multi-tracking can be reached

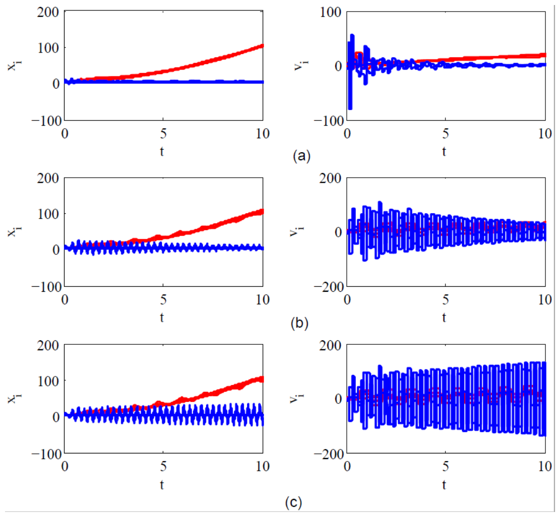

FIGURE 4. Dynamic multi-tracking. (a) $\tau = 0.65$. (b) $\tau = 0.7$. (c) $\tau = 0.71$.

if $\tau = 0.5$ and $\tau = 0.58$, while it can note be reached if $\tau = 0.59$.

Case 2): $\tau - \varrho < \iota < \tau$. Choose $\iota = 0.1$ and $\varrho = 0.2$. By Theorem 2, one has that multi-tracking of system (3) under protocol (14) can be achieved if and only if $0.2 < \tau < 0.7$. From Fig. 4, one has the multi-tracking can be reached if $\tau = 0.65$ and $\tau = 0.7$, while it can note be reached if $\tau = 0.71$.

VI. CONCLUSION

In this paper, we investigated the multi-tracking problem of the second-order multi-agent system with directed communication topology. A pulse-modulated intermittent control protocol is proposed, which only use the sampled position data of each agent. To overcome the limitation of communication bandwidth, the stochastic quantization scheme is introduced. And the information of agents is quantified before transmission. Moreover, an improved control protocol is proposed, which consider the influences of communication delay. Finally, under some necessary and sufficient conditions, the effectiveness of proposed multi-tracking control protocols is demonstrated.

REFERENCES

[1] M. Hu, T. Shen, J. Men, Z. Yu, and Y. Liu, "CRSM: An effective blockchain consensus resource slicing model for real-time distributed energy trading," *IEEE Access*, vol. 8, pp. 206876–206887, Nov. 2020.

[2] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[3] C. Hu and H. J. Jiang, "Pinning synchronization for directed networks with node balance via adaptive intermittent control," *Nonlinear Dyn.*, vol. 80, no. 1, pp. 295–307, Apr. 2015.

[4] G. Wen, W. Yu, Z. Li, X. Yu, and J. Cao, "Neuro-adaptive consensus tracking of multiagent systems with a high-dimensional leader," *IEEE Trans. Cybern.*, vol. 47, no. 7, pp. 1730–1742, Jul. 2017.

[5] C. L. P. Chen, C. Ren, and T. Du, "Fuzzy observed-based adaptive consensus tracking control for second-order multiagent systems with heterogeneous nonlinear dynamics," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 4, pp. 906–915, Aug. 2016.

[6] C. L. P. Chen, G. Wen, Y. Liu, and Z. Liu, "Observer-based adaptive backstepping consensus tracking control for high-order nonlinear semi-strict-feedback multiagent systems," *IEEE Trans. Cybern.*, vol. 46, no. 7, pp. 1591–1601, Jul. 2016.

[7] L. Ding and G. Guo, "Sampled-data leader-following consensus for nonlinear multi-agent systems with Markovian switching topologies and communication delay," *J. Franklin Inst.*, vol. 352, no. 1, pp. 369–383, Jan. 2015.

[8] W. Yu, L. Zhou, X. Yu, J. Lü, and R. Lu, "Consensus in multi-agent systems with second-order dynamics and sampled data," *IEEE Trans. Ind. Informat.*, vol. 9, no. 4, pp. 2137–2146, Nov. 2013.

[9] Z. Liu, X. Yu, Z. Guan, B. Hu, and C. Li, "Pulse-modulated intermittent control in consensus of multiagent systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 5, pp. 783–793, May 2017.

[10] H. Na, Z. S. Duan, and Z. Yu, "Leader-following consensus of second-order non-linear multi-agent systems with directed intermittent communication," *IET Control Theory Appl.*, vol. 8, no. 10, pp. 782–795, Jul. 2014.

[11] C. Hu, H. He, and H. Jiang, "Synchronization of complex-valued dynamic networks with intermittently adaptive coupling: A direct error method," *Automatica*, vol. 112, Feb. 2020, Art. no. 108675.

[12] N. Huang and Z. Duan, "Distributed consensus of second-order multi-agent systems with uniquely sampled position data," in *Proc. 34th Chin. Control Conf. (CCC)*, Jul. 2015, pp. 7079–7083.

[13] Y. Liu and H. Su, "Second-order consensus for multiagent systems with switched dynamics and sampled position data," *IEEE Trans. Syst., Man, Cybern., Syst.*, early access, Jun. 23, 2021, doi: 10.1109/TSMC.2021.3089835.

[14] H. Su, Y. Liu, and Z. Zeng, "Second-order consensus for multiagent systems via intermittent sampled position data control," *IEEE Trans. Cybern.*, vol. 50, no. 5, pp. 2063–2072, May 2020, doi: 10.1109/TCYB.2018.2879327.

[15] H. Su, Y. Ye, X. Chen, and H. He, "Necessary and sufficient conditions for consensus in fractional-order multiagent systems via sampled data over directed graph," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 4, pp. 2501–2511, Apr. 2021, doi: 10.1109/TSMC.2019.2915653.

[16] Y. Zhao, Z. Duan, and G. Wen, "Distributed finite-time tracking of multiple Euler-Lagrange dynamics without velocity measurements," in *Proc. 32nd Chin. Control Conf.*, Xi'an, China, Jul. 2013, pp. 6923–6928.

[17] B. Zhu, Z. Li, H. H.-T. Liu, and H. Gao, "Robust second-order tracking of multi-vehicle systems without velocity measurements on directed communication topologies," in *Proc. Amer. Control Conf.*, Portland, OR, USA, Jun. 2014, pp. 5414–5419.

[18] A. Kashyap, T. Başar, and R. Srikant, "Quantized consensus," *Automatica*, vol. 43, no. 7, pp. 1192–1203, 2007.

[19] Y. Han, C. Li, W. Zhang, and H. G. Ahmad, "Impulsive consensus of multiagent systems with limited bandwidth based on encoding–decoding," *IEEE Trans. Cybern.*, vol. 50, no. 1, pp. 36–47, Jan. 2020.

[20] C. Ke, C. Li, and L. You, "Consensus of nonlinear multiagent systems with grouping via state-constraint impulsive protocols," *IEEE Trans. Cybern.*, vol. 51, no. 8, pp. 4162–4172, Aug. 2021.

[21] Y. Han and Z. Zeng, "Asynchronous impulsive protocols with asymmetric feedback saturation on leader-based formation control of multiagent systems," *IEEE Trans. Cybern.*, early access, Dec. 29, 2021, doi: 10.1109/TCYB.2020.3037150.

[22] H. Zhou, Y.-Z. Qin, K. Feng, W.-S. Hu, and Z.-W. Liu, "Impulsive consensus tracking of multiagent systems with quantization and input delays using position-only information," *Math. Problems Eng.*, vol. 2014, Feb. 2014, Art. no. 781048.

[23] S. M. Dibaji, H. Ishii, and R. Tempo, "Resilient randomized quantized consensus," *IEEE Trans. Autom. Control*, vol. 63, no. 8, pp. 2508–2522, Aug. 2018.

[24] Z.-W. Liu, Z.-H. Guan, T. Li, X.-H. Zhang, and J.-W. Xiao, "Quantized consensus of multi-agent systems via broadcast gossip algorithms," *Asian J. Control*, vol. 14, no. 6, pp. 1634–1642, 2012.

[25] G.-S. Han, Z.-H. Guan, J. Li, D.-X. He, and D.-F. Zheng, "Multi-tracking of second-order multi-agent systems using impulsive control," *Nonlinear Dyn.*, vol. 84, no. 3, pp. 1771–1781, May 2016.

[26] C. Deng, W.-W. Che, and Z.-G. Wu, "A dynamic periodic event-triggered approach to consensus of heterogeneous linear multiagent systems with time-varying communication delays," *IEEE Trans. Cybern.*, vol. 51, no. 4, pp. 1812–1821, Apr. 2021.

[27] B. Wang, W. Chen, B. Zhang, and Y. Zhao, "Regulation cooperative control for heterogeneous uncertain chaotic systems with time delay: A synchronization errors estimation framework," *Automatica*, vol. 108, Oct. 2019, Art. no. 108486.

[28] C. Hu, J. Yu, H. Jiang, and Z. Teng, "Exponential stabilization and synchronization of neural networks with time-varying delays via periodically intermittent control," *Nonlinearity*, vol. 23, no. 10, pp. 2369–2391, 2010.

[29] F. Bullo, J. Cortes, and S. Martinez, *Distributed Control of Robotic Networks: A Mathematical Approach to Motion Coordination Algorithms*. Princeton, NJ, USA: Princeton Univ. Press, 2009.

[30] Z.-H. Guan, C. Meng, R.-Q. Liao, and D.-X. Zhang, "Consensus of second-order multi-agent dynamic systems with quantized data," *Phys. Lett. A*, vol. 376, no. 4, pp. 387–393, Jan. 2012.

[31] N. Huang, Z. Duan, and G. R. Chen, "Some necessary and sufficient conditions for consensus of second-order multi-agent systems with sampled position data," *Automatica*, vol. 63, pp. 148–155, Jan. 2016.

[32] I. Kovacs, D. S. Silver, and S. G. Williams, "Determinants of commuting-block matrices," *Amer. Math. Monthly*, vol. 106, no. 10, pp. 950–952, Dec. 1999.

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