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An MDP Model for Integrated Operational Decision Making Under Uncertainty in a Gas and Oil Separation Plant

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ABSTRACT Oil and gas supply chain (SC) plays key role in the global economy. Gas and oil separation plants (GOSP) belong to the upper stream of the oil and gas SC. In this article we study a GOSP that pumps oil to a stabilizing plant through a 600 KM pipeline. The purpose of this research is to develop an integrated mathematical model to support operational decision making regarding the optimal pumps scheduling and the dosage level of a drag reduction chemical, which is injected in the oil pipeline to stabilize the pressure and achieve higher oil flow rate. To the best of our knowledge, there is no MDP model in the literature that jointly considers pumps scheduling and oil flow control. A novel Markov decision process (MDP) model and two intuitive heuristic polices are proposed and simulated based on historical data. The heuristic policies are to operate the oil pumps in a cyclic weekly or biweekly patterns and to use a maximum likelihood rule to select the dosage level. Compared to the heuristic policies, our results demonstrated that MDP can lead to a substantial amount of savings in terms of the total system operating and maintenance costs. It also provides some practical insights on the interaction ways between the frequency of operating the pumps, the maintenance costs and the costs of a chemical that is used to control the pressure and flow rate of oil in a 600 KM oil pipeline.

INDEX TERMS Decision making under uncertainty, gas and oil separation plant (GOSP), Markov decision Process (MDP), scheduling.

I. INTRODUCTION

Over the past several decades, optimal pump scheduling has gained the interest of researchers. Many studies report substantial amount of savings due to optimal pump scheduling in water distribution applications, where the main savings is due to the reduction in the electricity and maintenance costs.

In the oil and gas industry, the objective of minimizing the deviation between the actual and target flow rates of oil is important too. This article addresses a practical case observed in the oil and gas industry, where a gas and oil separation plant (GOSP) pumps oil to a stabilizing plant. The problem addressed in this work is operational one, which is controlling the flow of oil from a GOSP to a stabilizing plant, through a pipeline, by selecting the combination of pumps in operation (among a group of available pumps) and the required chemical dosage. The chemical dosage is a drag reducing agent that is continuously injected in the oil pipeline to stabilize the pressure of oil and achieve the appropriate oil flow rate, without affecting its inherent properties.

The cost trade–offs in this work include: pumps maintenance cost, chemicals cost and the cost of the deviation between the actual and target oil flow rates. The later cost element is to add a penalty on any gap between the actual and target/demand oil flow rates. The weekly target oil flow rate is based on a quarterly plan that is set by the stabilizing plant. This can be viewed as an integrated production and inventory planning problem [1], where a balance is needed between the production costs (maintenance and chemical dosage), and the inventory costs (the costs of not meeting the exact demand i.e. the holding and shortage costs).

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The current practice in the plant under study is based on human interventions to schedule the pumps and decide on the required dosage level. This is usually done by the process engineer. In this work we propose a novel replacement of the old process by introducing a new pump scheduling and flow rate control model.

The GOSP under consideration has four pumps that pump the crude oil through a 600 KM pipeline, and due to the length of the pipeline, the effect of the chemical dosage appears after a few days. Furthermore, there are some operational restrictions, namely:

- Operate two pumps at a time, to ensure operational continuity and prolonged life of the pump.
- If a pump exceeds two weeks in operation or one week offline, then maintenance will be needed.

The rest of this article is structured as follows: Section II presents the literature review, Section III provides the proposed mathematical model, Section IV demonstrates the proposed mathematical model with numerical examples and Section V presents the concluding remarks of this research work.

II. LITERATURE REVIEW

Since the 70s, pump scheduling have gained a special interest of the researchers and practitioners [2]. The majority of the pump scheduling literature addresses water pumps that are used in water distribution through water networks [2]–[8]. Other applications of pump scheduling include water treatment [9] and oil transportation [8].

A significant part of the expenditures in the pumping stations is related to the consumption of energy [4], [10]–[12] and the most common objective function in the pump scheduling literature is the minimization of the energy/electricity cost [2], [10], [13], [14]. On the other hand, some researchers addressed the problem of minimizing pump maintenance related costs [15], [16], where the maintenance cost is minimized by limiting the number of pump switches. In a liquefied natural gas plant, reference [17] considered the two conflicting objectives of minimizing cost and improving reliability.

The literature spans plethora of techniques that are used for optimal pump scheduling. This includes linear programming [18]–[20], nonlinear programming [21], deterministic dynamic programming (DP) [7], [22], stochastic dynamic programming using Markov decision process (MDP) [2], [5], [9], [10], artificial intelligence (AI) [9], hybrid techniques such as MDP–based AI techniques [6], [9], and discrete event simulation [23].

MDP is a stochastic dynamic programming technique, which is used for decision making under uncertainty [24] with very wide range of real-life applications and efficient solution techniques [24], [25]. The main advantages of MDP include: 1) its ability to obtain optimal policies over finite and infinite planning horizons, where the latter is used to model stationary systems, 2) MDP can have different cost criteria that can depend on the initial system state, the next system state or both [24], 3) MDP is used in real–life for decision making under uncertainty [25], [26]. The main disadvantages of MDP are: 1) the difficulty to obtain their state transition probabilities [27], [28], because they require large number of statistical data [29], 2) establishing the cost/reward criteria and 3) the lack of standard software packages to explicitly solve for individual modeling problems (i.e. a programming effort is mostly needed) [30].

An MDP model consists of system states, a set of actions, transition probabilities between system states, and reward/cost functions that, in their basic form, depend on system state and the action taken [24] and can be constant or time–varying [24], [31]. In MDP, the decision maker observes the state of the system at discrete points in time, then he takes actions that alter the system state probabilistically (otherwise the model becomes deterministic dynamic program), which then leads to reward/loss modeling scenario. The objective is to maximize/minimize the reward/loss over a finite or infinite planning horizon [24].

Reference [7] used DP to solve the problem of pump operation scheduling in a water distribution system. Using historical data, the objective was to determine the water pump combination and the amount of water to be pumped to distribute water to all specified demand points under the pump and water supply capacity constraints to minimize pumping costs, which mainly includes electricity costs. Reference [11] used deterministic dynamic programming to obtain the optimal strategy to operate a set of pumps in a water supply system. The model included a complicated energy tariff, size and configuration differences among water distribution systems and limitations on the number of times pumps are turned on and off. Long and short term models were developed by the authors to obtain monthly and daily operating polices. DP was also used by [32], where operating conditions, variations in water demand, and energy costs were included in their model; the study reported energy cost savings of up to 20.9%.

Many studies reported the usefulness of MDP in operating water pumping stations. Reference [22] developed an MDP model to schedule the operation of water pumps in a water distribution system, which consisted of one water tower and a boosting station. The model included system characteristics such as the efficiency of pumps vs. the number of operating pumps and the pumping rate as a function of water tank levels. An MDP–based framework to optimize the consumption of electricity for water utility companies has been proposed by [2], [33]. The framework was based on detailed information about tank level, pressure measures in some of the network points, power consumption of pumps, energy price schema and water demand by the final consumers. The authors reported the advantages of minimizing the cost of electricity by 39.3% and minimizing supply outage. Later on, reference [10] discussed the advantage of the MDP model by [2] in terms of the solution quality. As compared to other tools like genetic algorithms and pollutant emission pump station optimization (PEPSO) [34], MDP had the minimum energy consumption, while PEPSO took the least computation time. Computation costs of MDPs in pump scheduling applications was also reported by [13]. To overcome the long computation time of the MDP, reference [5] proposed a distributed MDP modelling, however, this leads to degradation of the solution quality, according to the authors.

In other applications, reference [8] studied the operations and maintenance scheduling of pumps used in a wastewater treatment process. The objective was to minimize the pumps' energy consumption and maintenance costs, while maintaining the desired hydraulic workload of the pumps. A neural network algorithm was used to model the performance of pumps and MDP was used to obtain the maintenance schedule. Reference [35] developed a model for the optimal repair of a motor driven sea water centrifugal pump, the author determined an optimal maintenance level of the addressed system.

As for the oil and gas industry, energy and chemical consumption are among the major contributors in the operating costs of a GOSP. For large scale companies, a saving of 1% in energy costs can reflect as a 7–digits figure cost–wise [36]. On the other hand, maintaining operational efficiency and planning production is important as well [37].

The oil supply chain literature spans a large number of publications on planning and managing the supply of oil [37]; decisions include strategic, tactical and operational levels. At the tactical level, Reference [36] developed an integer programming model for optimal operation of a network of GOSPs. At the operational level, Reference [9] presented a multi–agent approach for the dynamic maintenance task scheduling for a petroleum industry production system. A reinforcement learning technique is used to generate optimal maintenance and production schedules. At the operational–tactical levels reference [38] used mixed integer programming to plan upper stream logistics of an oil SC.

Based on our literature review it was observed that the majority of the pump scheduling models are related to water distribution systems. Furthermore, it was found that MDP is an efficient tool for decision making under uncertainty for the addressed problem. To the best of our knowledge, there is no MDP model in the literature that jointly considers pumps scheduling and oil flow control. Therefore, in this article, we have addressed both decisions in a GOSP and demonstrated our proposed model using historical data. The closest research works to our study were introduced in [2], [33] and [22]. The novel contributions of our work are:

1) To the best of our knowledge, this work is among the first to jointly address pump scheduling and incorporate the drag reduction agent. In this article, we have addressed both decisions in a GOSP and demonstrated our proposed MDP model using historical data.
2) Maintenance–requirement is included in the proposed model.
3) The optimal MDP–based solution is compared to two intuitive heuristics (i.e., weekly and biweekly alternating operation of pumps).

Due to the absence of appropriate data in our case study, we did not consider the pressure of oil in the oil pipeline. Furthermore, in this study the cost of electricity is not addressed because the pumps operate continuously at a constant speed.

III. MATHEMATICAL MODEL

This section presents an MDP model to minimize the overall costs of operating and maintaining the pumps and the cost of chemical consumption in a GOSP. Furthermore, we demonstrate the model elements by using the case presented in Section I. The main notations used in this paper are given in Table 1.

TABLE 1. Notations.

Parameter	Description
a_t^b	The action of selecting pump combination b at week t, $a_t^b \in \{1, 2, \ldots 6\}$
a_t^d	The action of selecting dosage level d_t at week t, i.e. this gives the volume of the chemical that is consumed during week t
b_t	The combination of pumps in operation at the beginning of the week t
$c^{dev(+)}$	The cost of positive deviation from the target oil flow rate
$c^{dev(-)}$	The cost of negative deviation from the target oil flow rate
c^{ma}	the cost of performing maintenance on a pump
c^g	The cost per gallon of the drag reduction agent
f_t	Oil flow rate at the beginning of week t
f_t^T	The target oil flow rate at beginning of week t
i_t^l	The idleness history of pump l at the beginning of week t
i_t^L	The idleness history pumps (a vector) at the beginning of the week t, $i_t^L \in \{i_t^1, \ldots, i_t^4\}$. Four pumps exist in the system on hand but the model applies to any number of pumps
S_t	The system state at time t
α	Discount factor to account for time value of money

The MDP model is formulated as follows:

1) *Decision/Time Epochs* ($t \in T$)*:* Decision points in time where actions are applied to the system. We assume a finite horizon of T time epochs. In this model, without loss of generality, one week is the time epoch.
2) *State Space* ($S_t = b_t, f_t, i_t^L$)*:* At each decision epoch, the system is fully described by its state, S, the state of the system is comprised of three elements:
   - The combination of pumps in operation at the beginning of a given week t is b_t, where $b_t \in \{1, 2, \ldots 6\}$. Since there are four pumps and two of them must be selected every week, then there are six possible pump combinations.
   - The oil flow rate at the beginning of the week t is f_t, where $f_t \in \{1, 2, 3, 4\}$ is defined as follows:
     - 1: the oil flow rate in the pipeline is less than 470 thousand barrels of oil per day (MBOD)

TABLE 2. Demonstrative example on idleness history calculation.

Pump	Pump idleness at the Beginning of Week (t)	Pump idleness right after taking the action at week (t)	Pump idleness at the end of the week (t) or beginning of week ($t+1$)
1	-2	-2	3
2	-2	0	1
3	2	0	-1
4	2	2	3

– 2: the oil flow rate in the pipeline is ranging between 471 – 520 MBOD
– 3: the oil flow rate in the pipeline is ranging between 521 – 570 MBOD
– 4: the oil flow rate in the pipeline is more than 571 MBOD

These flow rate intervals are defined after consulting with the process engineer. Furthermore, the developed MDP model applies to any number of flow rate intervals.

- The idleness history of pump l at the beginning of a given week t is i_t^l. The subscript l indicates the pump number and i indicates the idleness history and can take a value between $[-2, 3]$ in our case. The negative value of i means working since i weeks, and the positive value of i means idle since i weeks. If a pump is turned on or idled, the idleness history will be reset to zero. To demonstrate how the idleness history of a pump is updated, assume an example in which pumps 1 and 2 have been in operation since the last two weeks until the beginning of week t. Also, assume that the remaining pumps have been idle. If a decision is made, at the beginning of t, to use the pump combination 1 and 3, then Table 2 demonstrates updating the idleness history of each pump right after taking this decision.

At each time epoch, the process engineer has all the information regarding the system state (b_t, f_t, i_t^l); this information is utilized to decide on the appropriate actions (decisions). The model has a finite state space; because there are finite number of pumps, finite number of possible discretized flow rates, and the finite range of i_t^l.

3) *Action space* (A)*:* At every time epoch, the process engineer observes the system state and select an action, $a \in A$, that takes the system to a new state given the associated costs for that action. At the beginning of the week $t \in T$, the process engineer selects a pump combination (a_t^b) and a chemical dosage level (a_t^d). The chemical dosage should be applied uniformly to ensure no upset to the process as well as to facilitate the planning for chemical inventory. The chemical dosage is normally done in multiples of 5 gallons per hour (GPH), which is the size of the container. As part of the process guidelines, the maximum chemical dosage allowed is 25 GPH. Since its effect on the oil flow rate starts to appear after few days along the 600 KM pipeline, the required dosage (a_t^d) is continuously applied to the oil pipeline. From operational perspective, the chemical dosage level is usually adjusted on weekly basis. In our example, the possible chemical dosage actions are: $\{1, 2, \ldots, 5\}$, which represents $\{5, 10, \ldots, 25\}$ GPH respectively.

4) *Cost Criteria,* $R_t(S_t, S_{t+1}, a_t^b, a_t^d)$*:* At each decision epoch (t), when the actions are taken, the system incurs costs that depend on the current and future system states and the actions taken by the decision maker $(S_t, S_{t+1}, a_t^b, a_t^d) = (b_t, f_t, i_t^L, b_{t+1}, f_{t+1}, i_{t+1}^L, a_t^b, a_t^d)$. System costs can be fixed or time–varying [31]. The cost structure of our proposed model is calculated as follows:

- The cost of operating a pump for one week $c^{op}(a_t^b)$
- The cost of the chemical dosage that is consumed during week $t = d_t \times c^g$
- The cost of performing maintenance for pump l, which has been operating since more than two weeks: $\alpha \times c^{ma} \times H[i_{t+1}^l]$, where $H[i_{t+1}^l] = 1$, if $i_{t+1}^l < -2$ and zero otherwise.
- The cost of performing maintenance for pump l, which has been offline since more than one week: $\alpha \times c^{ma} \times G[i_{t+1}^l]$; where $G[i_{t+1}^l] = 1$, if $(i_{t+1}^l) > 1$ and zero otherwise.
- The cost of deviating from the target flow rate $\alpha \times c^{dev(+)} \times (f_{t+1} - f_{t+1}^T)^+ + \alpha \times c^{dev(-)} \times (f_{t+1}^T - f_{t+1})^+$

The value of the term $(a-b)^+$ should be equal to $a-b$ only if $a > b$ and it is equal to zero otherwise. Note that the last three cost items have the time subscript $t+1$; this is to indicate that these costs are realized at the end of time t. Or in other words the beginning of time $t+1$ according to the notations we defined in this article. The value of the target oil flow rate is updated on quarterly basis by the stabilizing plant. The current plan for the next quarter is shown in Table 3. Each time epoch any deviation (positive or negative) from the target is penalized with the respective cost. The costs of positive and negative deviations are mainly calculated based on the holding and shortage costs per 50 thousand barrels per week respectively. These costs are estimated

TABLE 3. Target oil flow rates over the planning horizon of 12 weeks.

Week	1	2	3	4	5	6	7	8	9	10	11	12
Target oil flow rate	2	3	4	3	4	4	2	2	3	3	2	4

based on the average oil price. In Section IV, we use scaled values of these costs.

Adding up these cost elements gives the overall weekly system cost as follows

$$\begin{aligned} &R_t[b_t, f_t, i_t^l, b_{t+1}, f_{t+1}, i_{t+1}^l, a_t^b, a_t^d] \\ &= c^{op}(a_t^b) + d_t c^g + \alpha \times \Big\{ c^{ma} \sum_l (H[i_{t+1}^l] + G[i_{t+1}^l]) \\ &+ c^{dev(+)} \times (f_{t+1} - f_{t+1}^T)^+ \\ &+ c^{dev(-)} \times (f_{t+1}^T - f_{t+1})^+ \Big\}, \end{aligned} \tag{1}$$

To illustrate how the cost for performing maintenance can be calculated, Table 4 will be used as a quick reference to determine the maintenance cost. From Table 4, an expression can be developed to describe the cost structure of performing maintenance based on two conditional scenarios:

If a pump is selected to be included in the operating set (denoted as b_{t+1}) then

$$i_{t+1}^l = \begin{cases} -1 & \text{if } i_t^l = -3 \\ i_t - 1 & \text{if } -2 \le i_t^l \le 0 \\ -1 & \text{if } i_t^l > 0 \end{cases} \tag{2}$$

If a pump is selected to be included in the offline set (denoted as b_t) then

$$i_{t+1}^l = \begin{cases} 1 & \text{if } i_t^l \ge 2 \\ i_t + 1 & \text{if } 0 \le i_t^l \le 1 \\ 1 & \text{if } i_t^l < 0 \end{cases} \tag{3}$$

5) *State Transition Probability* $P[S_{t+1}|S_t, a]$: This is defined as the probability that a system will go to a posterior state $S_{t+1} = b_{t+1}, f_{t+1}, i_{t+1}^L)$, given its current state and the chosen actions. Assuming that the pump combination and dosage selection are independent, then:

$$\begin{aligned} &P[b_{t+1}, f_{t+1}, i_{t+1}^l | b_t, f_t, i_t^l, a_t^b, a_t^d] \\ &\quad = P(f_{t+1}|f_t, a_t^b, a_t^d) \times P(i_{t+1}^l | i_t^l, a_t^b) \\ &\quad \times \Pi_l P(b_{t+1}|i_t^l, a_t^b) \end{aligned} \tag{4}$$

where $P(b_{t+1}|i_t^l, a_t^b)$ equals 1 if b_{t+1} includes the pumps selected by the action a_t^b and 0 otherwise, $P(f_{t+1}|f_t, a_t^b, a_t^d)$ is given in the appendix and estimated based on historical data, and $P(i_{t+1}^l|i_t^l, a_t^b)$ equal 1 if the scenarios in (2) and (3) apply for the selected and non–selected pumps and 0 otherwise. The transition probability matrices in the appendix depend on the pump selection and dosage level actions. Therefore, there will be 5 matrices (5 different dosage levels) for each pump combination (a total of 6 pump combinations), which results in 30 matrices. The matrices for each pump combination are given in the appendix (Tables 11 to 40).

The expected minimum overall cost over a finite planning horizon T can be expressed by Equation 5.

$$\begin{aligned} &\mathrm{V}_t(b_t, f_t, i_t^l) \\ &= min_{a_t^b, a_t^d} \Big\{ \sum_{b_{t+1}} \sum_{f_{t+1}} \sum_{i_{t+1}^l} \{R_t \\ &\quad \times [b_t, f_t, i_t^l, b_{t+1}, f_{t+1}, i_{t+1}^l, a_t^b, a_t^d] \\ &\quad + \alpha V_{t+1}(b_{t+1}, f_{t+1}, i_{t+1}^l)\} \\ &\quad \times P[b_{t+1}, f_{t+1}, i_{t+1}^l | b_t, f_t, i_t^l, a_t^b, a_t^d] \Big\} \quad t \in \{1, \ldots, T\}, \end{aligned} \tag{5}$$

where $R_t[b_t, f_t, i_t^l, b_{t+1}, f_{t+1}, i_{t+1}^l, a_t^b, a_t^d]$ and $P[b_{t+1}, f_{t+1}, i_{t+1}^l | b_t, f_t, i_t^l, a_t^b, a_t^d]$ are given by (1) and (4) respectively. Equation 5 is based on Bellman's optimality principle [24]. The equation gives the minimum expected system cost as a function of the system state (b_t, f_t, i_t^l) at $t \in T$. The first term, $\sum_{b_{t+1}} \sum_{f_{t+1}} \sum_{i_{t+1}^l} R_t[b_t, f_t, i_t^l, b_{t+1}, f_{t+1}, i_{t+1}^l, a_t^b, a_t^d] \times P[b_{t+1}, f_{t+1}, i_{t+1}^l | b_t, f_t, i_t^l, a_t^b, a_t^d]$; represents the initial expected system costs due to selecting the actions a_t^b, a_t^d at $t \in T$. The second term of (5) is the minimum system expected costs at $t + 1 \in T$, where the decision maker is assumed to follow an optimal policy starting from $t + 1$ i.e. $\sum_{b_{t+1}} \sum_{f_{t+1}} \sum_{i_{t+1}^l} \alpha V_{t+1}(b_{t+1}, f_{t+1}, i_{t+1}^l) \times P[b_{t+1}, f_{t+1}, i_{t+1}^l | b_t, f_t, i_t^l, a_t^b, a_t^d]$).

The optimal actions are obtained using the following expression based on (5) as follows

$$a_t^{b*}, a_t^{d*} = argmin_{a_t^b, a_t^d} \left\{ V_t(b_t, f_t, i_t^l) \right\}$$

Fig. 1 demonstrates the dynamics of our proposed pump combination and dosage selection model using MDP. The figure shows the state, actions, reward criterion and the state transitions of the on–hand system in a typical time period. The proposed dynamic programming model starts with an initial system state at time t, followed by two actions by the decision maker i.e., selecting the pump combination and the chemical dosage level; based on the initial system state and the set of actions that were taken, the system moves to a new state, incurs costs and a new time period $(t + 1)$ starts.

IV. NUMERICAL ILLUSTRATION

This section presents examples on the integrated model, which is given in Section III. The system state transition probabilities are based on historical data and given in the

TABLE 4. **Cost of performing maintenance.**

Idleness history (i_t^l) at the beginning of the week	If a pump is put into operation		If pump is put on standby (turned off)	
	i_{t+1}^l beginning of the next week	Cost of Performing Maintenance	i_{t+1}^l beginning of the next week	Cost of Performing Maintenance
-3	-1	c^{ma}	1	c^{ma}
-2	-3	c^{ma}	1	0
-1	-2	0	1	0
1	-1	0	2	c^{ma}
2	-1	c^{ma}	3	c^{ma}
3	-1	c^{ma}	4	c^{ma}
4	-1	c^{ma}	1	c^{ma}

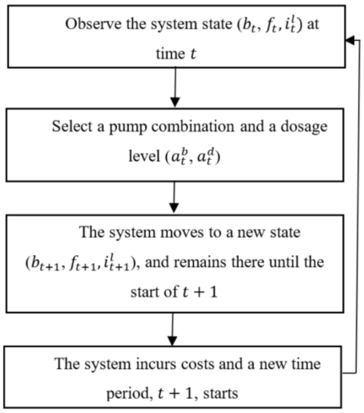

FIGURE 1. **Markov decision process (MDP) for integrated operational decision making.**

Appendix A (Tables 11 to 40). Financial data are scaled by a factor and provided in Table 5. In Section IV(A), we solve the MDP model with no restriction on the duration that a pump can operate, and then we add the constraint that a pump can't operate continuously for more than 1 and 2 weeks respectively. These constraints are added to allow for switching the pumps and perform preventive maintenance. In Section IV(B). We examine two intuitive heuristic polices: I) to operate two pumps for 2 weeks then the other two pumps for the next 2 weeks and so on II) to switch the pumps in operation every 1 week. The results of the MDP and heuristic policies are compared in terms of cost, mean absolute deviation from the target oil flow rate and the number of maintenance instances. In Section IV (C), we perform sensitivity analysis on some of the cost parameters, this is to study their impact on the optimal policy and its corresponding cost. Finally, to demonstrate the ability of the MDP in achieving the target oil flow rates, Section IV(D) presents the solutions for an unconstrained MDP with neglecting the cost of chemical (i.e. chemical cost trade–off is not incorporated in the model).

A. RESULTS OF THE MDP–BASED POLICY

The MDP model was solved for the optimal decision policy, using the backward value iteration Algorithm [24]. The

TABLE 5. **Cost parameters of the proposed model.**

Parameter	**Average scaled cost (based on two–years historical records)**
$c^{dev(+)}$	\$ 12,830
$c^{dev(-)}$	\$ 19,560
c^{ma}	\$ 1,710
c^{op}	\$ 1,340
c^{g}	\$ 17.9
α	0.98

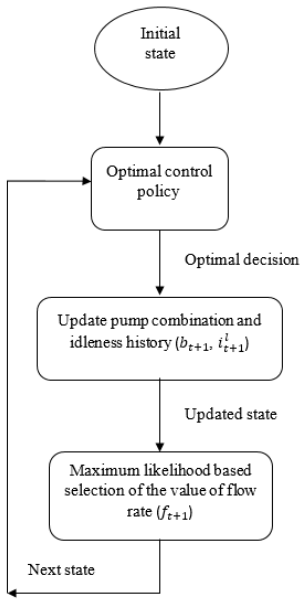

FIGURE 2. **Diagram of the steps involved in the trajectory generation for the different polices.**

resulting optimal control policy provided the optimal decision rule for each state at each of the twelve time epochs.

To simulate the MDP and heuristic polices, a maximum likelihood trajectory of the state variables has been generated. The steps of generating the trajectory are summarized in Fig. 2. A trajectory is generated for both, the MDP and heuristic policies, and then during each decision epoch, decisions will be taken and the states will be updated. In case of

MDP, the decision regarding the dosage level and the pump configuration is obtained from the optimal policy. Then, the values of the pump configuration and the idleness history are updated deterministically (depending on the decision rule that is obtained from the optimal policy). Next, the value of the flow rate is realized based on the maximum likelihood of that flow rate, given the current state and the optimal decision rule, which is obtained using the transition matrices given in Appendix A. Finally, the resulting next state is used to obtain the decision rule at the next decision epoch and so on. Note that through applying this process we are generating the most probable trajectories for a given initial state and a pre-calculated optimal policy.

Regarding the heuristic policies (for comparison with the MDP policies), the heuristic policy based trajectories are generated using the same method as in Fig. 2. The only difference is that the determination of the dosage and the pump configuration is done using the heuristic policy instead of the MDP based optimal policy. The details of the heuristic policy are discussed in Subsection IV(B).

The trajectories due to the MDP optimal policy are shown in Fig. 3 and Fig. 4. As can be seen in Fig. 3, the optimal policy opted for various pump combinations between 1 and 6, while the desirable flow rate has been tracked successfully at eight out of 12 instances (66.66% success). The percentage value of the mean absolute deviation between the optimal and target flow rate levels is given in Table 6 (11.11% in this case), which is calculated as a percentage of the maximum possible total absolute deviation, i.e., maximum possible absolute deviation is 36 units (3 × 12). Utilization of the pumps is demonstrated by Fig. 4, which indicates one instance ($t = 8$) where a pump (pump 1) is utilized for more than two weeks.

To perform preventive maintenance, we also added the constraint that no pump can continuously operate for more than two weeks and recomputed the optimal MDP policy. The results are demonstrated by Figs. 5 and 6. It is evident from Fig. 5 that the added constraint causes imperfect tracking of the flow rate at seven instances (58.33% success and the mean absolute deviations is 16.66%). However, no pump is utilized for more than two weeks as shown in Fig. 6. Similarly, another MDP–based optimal policy was computed after replacing the two–weeks constraint by a one–week constraint, the results are demonstrated by Figs. 7 and 8. Note that the flow rate tracking has worsened (41.66% success and mean of absolute deviations is 27.77%). Next we compare the results of the MDP with some heuristic policies. The summary of the comparison results is presented in Table 6. It is evident that the MDP outperforms the heuristic policies in terms of the total cost (with nominal value of the cost of deviation in the flow rate). The details regarding the heuristic policies are presented in Subsection IV(B).

B. HEURISTIC POLICIES

This section presents the results from two heuristic policies. The first one is to use pump combinations (1, 2) and (3, 4) in a weekly alternation and the dosage is set based on the level

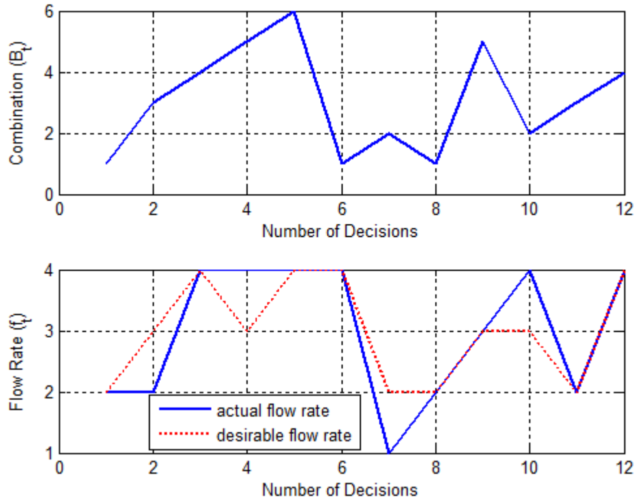

FIGURE 3. Pump configuration and flow rate, an optimal unconstrained MDP policy.

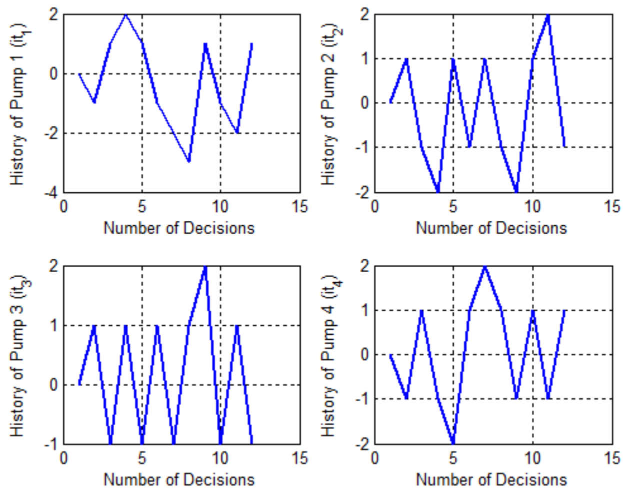

FIGURE 4. Pump utilization, an optimal unconstrained MDP policy.

that yields the maximum likelihood of achieving the target flow rate. The second policy is similar to the first one but the pumps are operated in a biweekly alternation. The resulting state trajectories of the first policy are shown in Figs. 9 and 10. Based on Fig. 9 it is evident that the desirable flow rate is reasonably well tracked (the mean absolute deviation is 13.88%). Based on the values provided in Table 6, the dosage level is too high and hence the overall cost is higher compared to the MDP–based policies. The results of the second policy are presented in Figs. 11 and 12. The flow rate in Fig. 11 indicates that the tracking of the desirable flow rate is almost perfect. Note that this is at the cost of high dosage and hence higher cost as compared to the MDP–based policies (see Table 6).

A more detailed comparison between the heuristic and the MDP policies is provided in Table 6. It is evident that the expected cost incurred by optimal policy due to

TABLE 6. Comparison between various decision policies with nominal cost of deviation in the flow rate.

Performance Parameter (single trajectory)	Unconstrained MDP	Constrained MDP (two week)	Constrained MDP (one weeks)	Heuristic (two week)	Heuristic (one week)
Total dosage	11	12	12	30	37
Total Cost	238,726	277,712	308,732	481,010	620,480
Maintenance incidents	5	5	0	12	0
Mean of Absolute Deviation	11.11%	16.66%	27.77 %	2.77%	13.88%

TABLE 7. Comparison between various decision policies with cost of deviation in the flow rate scaled by 0.5.

Performance Parameter (single trajectory)	Unconstrained MDP	Constrained MDP (two week)	Constrained MDP (one weeks)	Heuristic (two week)	Heuristic (one week)
Total dosage	11	11	11	30	37
Total Cost	235,731	209,331	229,546	474,590	588,407
Maintenance incidents	7	3	0	12	0
Mean of Absolute Deviation	19.44%	13.88%	27.77 %	2.77%	13.88%

TABLE 8. Comparison between various decision policies with cost of deviation in the flow rate scaled by 1.5.

Performance Parameter (single trajectory)	Unconstrained MDP	Constrained MDP (two week)	Constrained MDP (one weeks)	Heuristic (two week)	Heuristic (one week)
Total dosage	11	13	14	30	37
Total Cost	215,856	310,338	316,824	487,420	652,560
Maintenance incidents	7	6	0	12	0
Mean of Absolute Deviation	5.55%	13.88%	13.88 %	2.77%	13.88%

TABLE 9. Comparison between various decision policies with cost of deviation in the flow rate scaled by 2.

Performance Parameter (single trajectory)	Unconstrained MDP	Constrained MDP (two week)	Constrained MDP (one weeks)	Heuristic (two week)	Heuristic (one week)
Total dosage	12	12	14	30	37
Total Cost	247,142	329,032	352,264	493,840	684,630
Maintenance incidents	9	5	0	12	0
Mean of Absolute Deviation	5.55%	13.88%	13.88 %	2.77%	13.88%

TABLE 10. Comparison between various decision policies with cost of deviation in the flow rate scaled by 2.5.

Performance Parameter (single trajectory)	Unconstrained MDP	Constrained MDP (two weeks)	Constrained MDP (one week)	Heuristic (two week)	Heuristic (one week)
Total dosage	13	12	17	30	37
Total Cost	275,008	251,422	400,737	500,250	716,710
Maintenance incidents	9	4	0	12	0
Mean of Absolute Deviation	5.55%	5.55%	11.11 %	2.77%	13.88%

simple MDP (238,726 USD) is the lowest whereas the expected cost incurred by the weekly heuristic policy is the highest (620,480 USD). In terms of total chemical dosage, the two–week constrained and one–week constrained MDPs resulted in a slightly higher value (12). Obviously, the expected cost due to the two–week constrained MDP

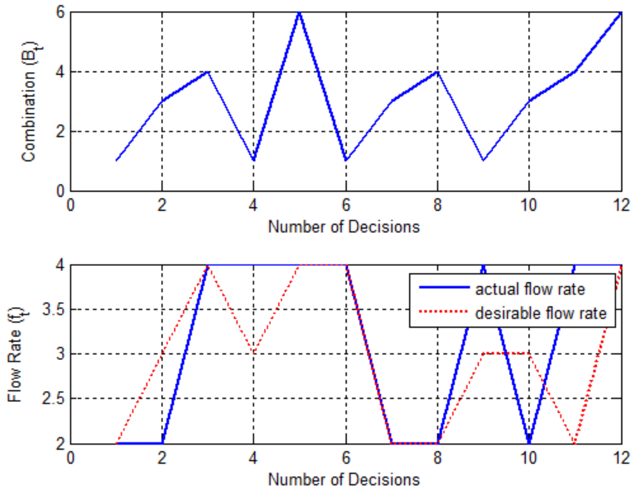

FIGURE 5. Pump configuration and flow rate with optimal (finite horizon) constrained (two–week operation) MDP policy.

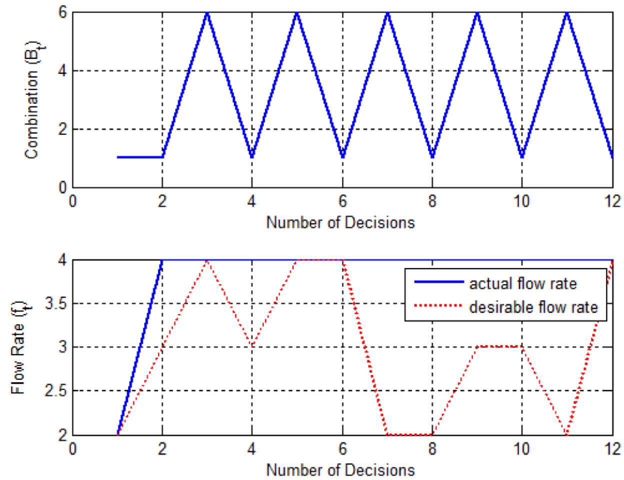

FIGURE 7. Pump configuration and flow rate with optimal (finite horizon) constrained (one–week operation) MDP policy.

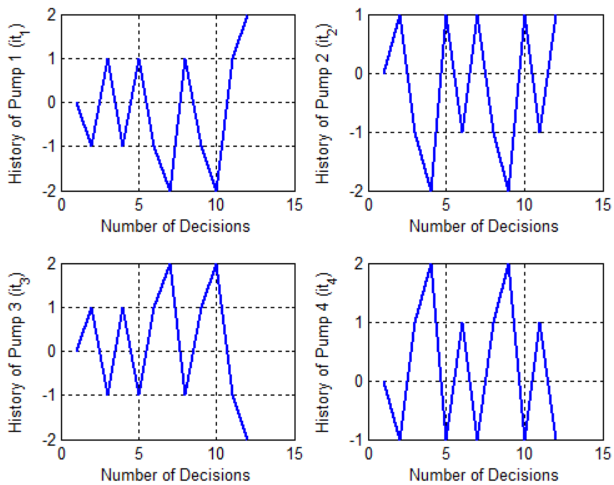

FIGURE 6. Pump utilization with optimal (finite horizon) constrained (two–week operation) MDP policy.

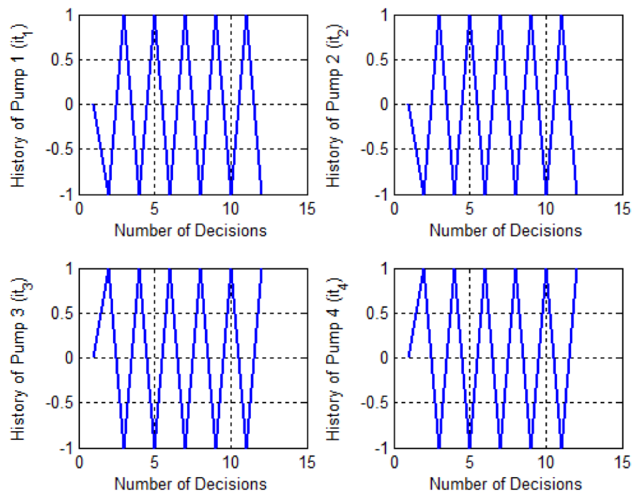

FIGURE 8. Pump utilization with optimal (finite horizon) constrained (one–week operation) MDP policy.

(277,712 USD) is less than that of one–week constrained MDP (308,732 USD). The two–week MDP policy incurs the cost of pump maintenance due to more than one week of idleness (note that the idleness history of the pumps in Fig. 6 reaching the value 2 at multiple occasions for different pumps). The third row of Table 6 presents the comparison of maintenance incidents, where the biweekly heuristic policy has the highest number of maintenance incidents. Note that the costs due to the heuristic policies has been calculated for a particular trajectory that was generated using the maximum likely state transitions (Fig. 2), whereas the MDP policy takes into account all possible transitions. For example, the one–week constrained MDP policy chooses the dosage level 2 (1680 gallons per week) at t = 6 (Fig. 13) because it has 20% chance of achieving the desirable flow rate, i.e., 2 (Table 37, last row). Since that transition is ruled out in the trajectory generation (the trajectory selects the maximum likely transition, which results in the flow rate remains equal to 4), the dosage cost due to MDP policy appears to be wasted. Fig. 13 depicts the comparison of step by step dosage rate due to biweekly heuristic policy and the constrained (one–week) MDP policy. We can conclude (based on Table 6) that the MDP–based policies outperform heuristic/intuitive policies in terms of cost.

C. THE EFFECT OF SCALING THE FLOW RATE DEVIATION COSTS

The deviation costs used in our model are function of the oil price, which is variable. Therefore, this section describes how does the change in the deviation costs (i.e., the cost parameters $c^{dev(+)}$ and $c^{dev(-)}$) affect the optimal solution. We have scaled the cost of deviation using a range of scale values to study their impact on the obtained solutions. These values of scale were in the range of [0.5 to 2.5] with and increment of

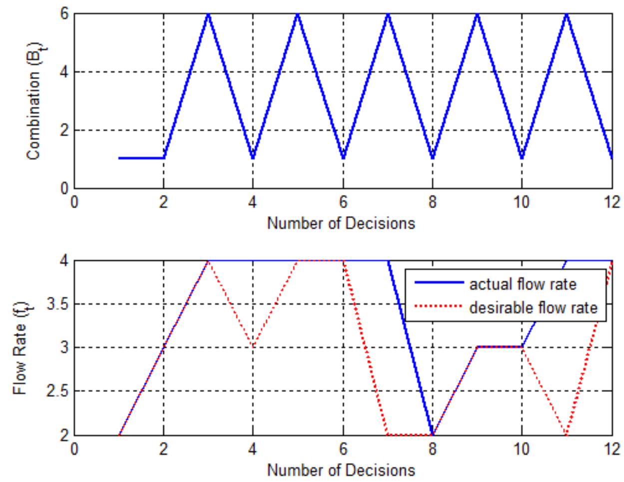

FIGURE 9. Pump configuration and flow rate with weekly shift heuristic policy.

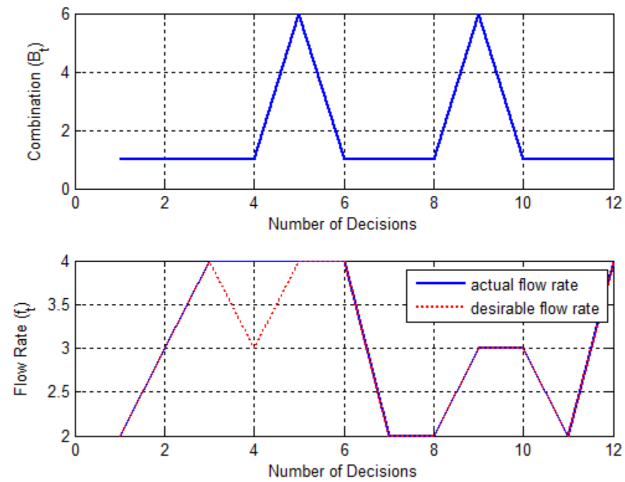

FIGURE 11. Pump configuration and flow rate with biweekly shift heuristic policy.

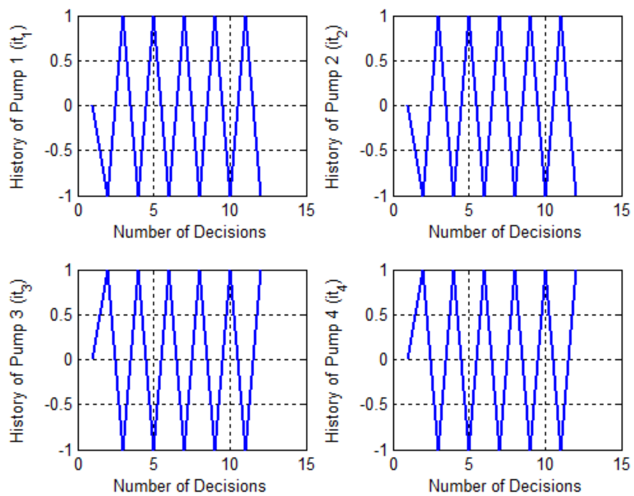

FIGURE 10. Pumps utilization with weekly shift heuristic policy.

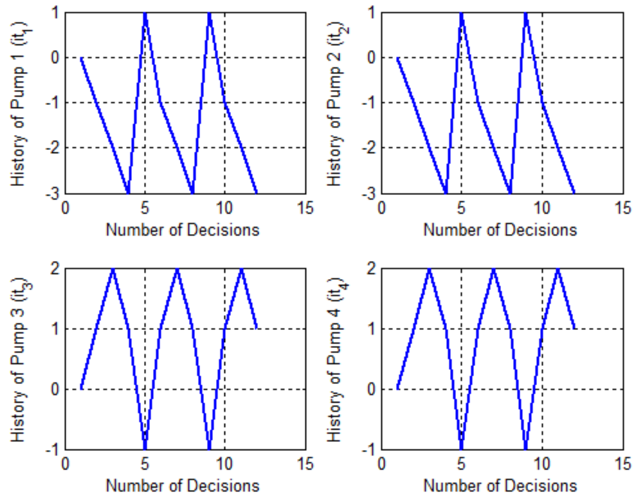

FIGURE 12. Pumps utilization with biweekly shift heuristic policy.

0.5 between each value. Tables 7 to 10 provide the results along with the values of scale. The results presented here are for a single sample trajectory (Fig. 2). Observing the behavior of the unconstrained MDP policy, the increase in the cost of deviation results in the improvement in the accuracy of flow rate tracking but the mean absolute deviation does not go below 5.55 % in the presented results. Note that it does not mean that perfect tracking is impossible. In fact, perfect tracking is achieved if the cost is high enough. Similar trend can be observed in the other MDP policies, i.e., improvement in tracking accuracy as the cost of deviation is increased. However, the impact of the change in cost is not visible in the heuristic policies since the heuristics do not depend on the deviation costs. The dosage and the maintenance incidents in general increase with the cost. However, for the two–week (constrained) MDP policy, there is a nonlinear trend in the maintenance incidents with respect to the cost of deviation, or in other words, the maintenance incidents first increase with the cost and then decrease as the cost is further increased.

D. THE EFFECT OF NEGLECTING THE DOSAGE COST

In this subsection, we study the effect of neglecting the dosage cost on the performance of the MDP policy. We have already seen in the previous sections that the flow rate tracking is not perfect by the MDP and heuristic policies. This is mainly due to the inherent trade–off between the flow rate deviation cost and the cost of chemical dosage that controls the flow rate. If the chemical is cheap (or free), then higher dosage can be used. For demonstration, we solved our proposed MDP model with zero dosage cost. The resulting trajectory is shown in Fig. 14. It is evident that the flow rate is tracked perfectly. However, the total amount of dosage used by the policy is 25 units as compared to that of 11 units used by the MDP based policy from Section IV(A).

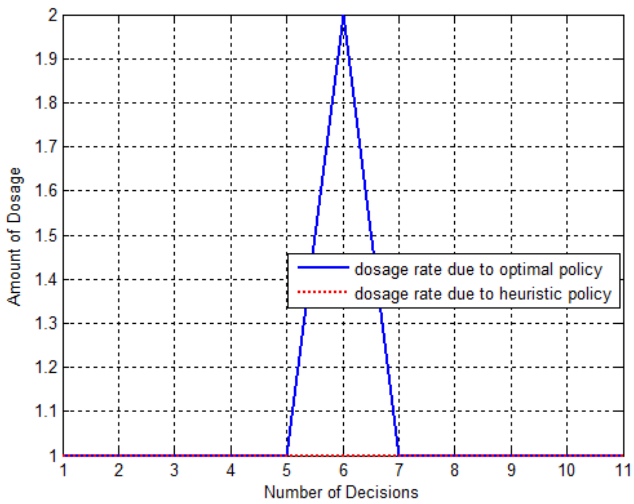

FIGURE 13. Comparison between chemical dosage due to weekly switching heuristic and the one–week constrained MDP policy.

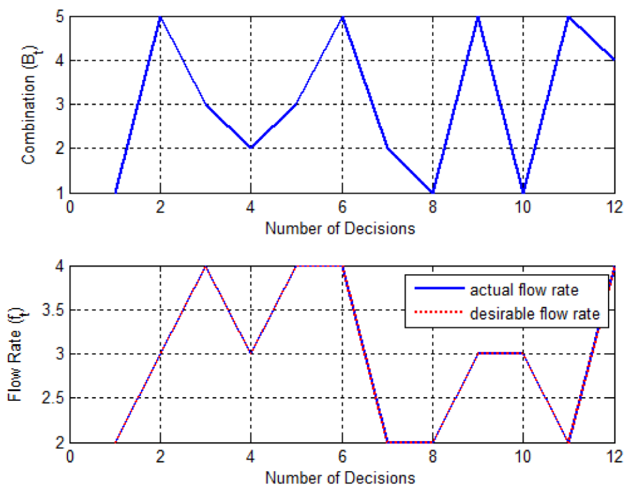

FIGURE 14. Flow rate tracking and pump combinations due MDP policy with zero cost of the chemical dosage.

TABLE 11. Flow rate transition probabilities, when pumps 1&2 are working and the dosage rate is 5 GPH.

P. 1&2 D. 5	F1	F2	F3	F4
F1	0	0.75	0.25	0
F2	0	0.42	0	0.58
F3	0.17	0.50	0	0.33
F4	0.13	0.13	0	0.74

TABLE 12. Flow rate transition probabilities, when pumps 1&2 are working and the dosage rate is 10 GPH.

P. 1&2 D. 10	F1	F2	F3	F4
F1	0.20	0.40	0	0.40
F2	0.06	0.88	0.03	0.03
F3	0.11	0.22	0.11	0.56
F4	0.05	0.45	0.14	0.36

TABLE 13. Flow rate transition probabilities, when pumps 1&2 are working and the dosage rate is 15 GPH.

P. 1&2 D. 15	F1	F2	F3	F4
F1	0.11	0.67	0.22	0
F2	0	0.3	0.5	0.2
F3	0	0	0.71	0.29
F4	0	0.17	0.08	0.75

TABLE 14. Flow rate transition probabilities, when pumps 1&2 are working and the dosage rate is 20 GPH.

P. 1&2 D. 20	F1	F2	F3	F4
F1	0.25	0.5	0	0.25
F2	0.21	0	0.43	0.36
F3	0.15	0.23	0.15	0.46
F4	0.04	0.21	0.29	0.46

TABLE 15. Flow rate transition probabilities, when pumps 1&2 are working and the dosage rate is 25 GPH.

P. 1&2 D. 25	F1	F2	F3	F4
F1	0	0.29	0.71	0
F2	0.45	0.18	0.27	0.09
F3	0	0.08	0.84	0.08
F4	0.2	0.46	0.07	0.27

TABLE 16. Flow rate transition probabilities, when pumps 1&3 are working and the dosage rate is 5 GPH.

P. 1&3 D. 5	F1	F2	F3	F4
F1	0	0.57	0	0.43
F2	0.67	0	0.11	0.22
F3	0	0	0.13	0.88
F4	1	0	0	0

V. CONCLUSION

We proposed a novel dynamic pump scheduling and flow rate control model, formulated as an MDP. Compared to previous research work, we considered a GOSP and developed an integrated model that accounts for pump scheduling and flow rate control of oil from a GOSP to a stabilizing plant. The proposed model accounts for preventive maintenance costs that depend on the scheduling of pumps. Using historical data and scaled cost data, several illustrative examples were solved using the proposed model. This included constrained MDP, unconstrained MDP and heuristic policies (i.e. weekly and biweekly alternating operation of pumps). It was observed that MDP–based polices outperformed the intuitive heuristic policies in terms of cost. Furthermore, the deviations from the target flow rate that resulted from the MDP–based policy were due to the chemical cost trade–off in the model, this was verified by solving an MDP with zero chemical cost. The results due to zero dosage cost indicated perfect tracking of the target oil flow rate.

Numerical illustrations were provided based on realistic state transition data and scaled cost data. Our results demonstrated that MDP can lead to a substantial amount of savings in terms of the total system operating and maintenance costs.

The proposed model has the advantage of enabling integrated decision making under uncertainty in GOSPs. It also

TABLE 17. Flow rate transition probabilities, when pumps 1&3 are working and the dosage rate is 10 GPH.

P. 1&3 D. 10	F1	F2	F3	F4
F1	0	0.33	0.67	0
F2	0	0.71	0	0.29
F3	0.17	0	0.17	0.67
F4	0	0.25	0	0.75

TABLE 18. Flow rate transition probabilities, when pumps 1&3 are working and the dosage rate is 15 GPH.

P. 1&3 D. 15	F1	F2	F3	F4
F1	0.58	0.33	0.08	0
F2	0.13	0.25	0.63	0
F3	0	0.5	0	0.5
F4	0	0.5	0.38	0.13

TABLE 19. Flow rate transition probabilities, when pumps 1&3 are working and the dosage rate is 20 GPH.

P. 1&3 D. 20	F1	F2	F3	F4
F1	0.45	0.24	0	0.31
F2	0	0.17	0.5	0.33
F3	0	0.2	0.2	0.6
F4	0.2	0	0.4	0.4

TABLE 20. Flow rate transition probabilities, when pumps 1&3 are working and the dosage rate is 25 GPH.

P. 1&3 D. 25	F1	F2	F3	F4
F1	0	0.55	0.36	0.09
F2	0	0.25	0.75	0
F3	0	0	0.32	0.68
F4	0	0	0.75	0.25

TABLE 21. Flow rate transition probabilities, when pumps 1&4 are working and the dosage rate is 5 GPH.

P. 1&4 D. 5	F1	F2	F3	F4
F1	0.25	0	0.63	0.13
F2	0.17	0.5	0.33	0
F3	0	0	0	1
F4	0	0.57	0.14	0.29

TABLE 22. Flow rate transition probabilities, when pumps 1&4 are working and the dosage rate is 10 GPH.

P. 1&4 D. 10	F1	F2	F3	F4
F1	0.33	0.44	0.11	0.11
F2	0.05	0.62	0.12	0.21
F3	0.23	0.23	0.15	0.38
F4	0.15	0.3	0.26	0.3

TABLE 23. Flow rate transition probabilities, when pumps 1&4 are working and the dosage rate is 15 GPH.

P. 1&4 D. 15	F1	F2	F3	F4
F1	0.2	0.4	0.1	0.3
F2	0	0.82	0.18	0
F3	0	0.11	0.78	0.11
F4	0	0	0.14	0.86

TABLE 24. Flow rate transition probabilities, when pumps 1&4 are working and the dosage rate is 20 GPH.

P. 1&4 D. 20	F1	F2	F3	F4
F1	0.25	0	0.25	0.5
F2	0.2	0.2	0.6	0
F3	0.08	0.17	0.25	0.5
F4	0.04	0.04	0.35	0.57

TABLE 25. Flow rate transition probabilities, when pumps 1&4 are working and the dosage rate is 25 GPH.

P. 1&4 D. 25	F1	F2	F3	F4
F1	0	0.38	0.63	0
F2	0.13	0.75	0.13	0
F3	0.09	0	0.87	0.04
F4	0	0.4	0	0.6

TABLE 26. Flow rate transition probabilities, when pumps 2&3 are working and the dosage rate is 5 GPH.

P. 2&3 D. 5	F1	F2	F3	F4
F1	0.43	0.43	0.14	0
F2	0	0.25	0.25	0.5
F3	0	0.58	0.08	0.33
F4	0	0.41	0.09	0.5

TABLE 27. Flow rate transition probabilities, when pumps 2&3 are working and the dosage rate is 10 GPH.

P. 2&3 D. 10	F1	F2	F3	F4
F1	0.08	0.58	0	0.33
F2	0	0.78	0.11	0.11
F3	0.09	0.18	0.27	0.45
F4	0.2	0.33	0.13	0.33

TABLE 28. Flow rate transition probabilities, when pumps 2&3 are working and the dosage rate is 15 GPH.

P. 2&3 D. 15	F1	F2	F3	F4
F1	0.42	0.46	0.12	0
F2	0	0.33	0.07	0.6
F3	0	0.64	0.27	0.09
F4	0	0	0.78	0.22

provides some practical insights on the interaction ways between the frequency of operating the pumps, the maintenance costs and the costs of a chemical that is used to control the pressure and flow rate of oil in a 600 KM oil pipeline.

The limitations of the proposed model are: it takes the expected cost as the decision making criterion, while in real–life risk–related measures are important too. In the demonstrative example, the flow rate intervals were considered to be four with a window of 50 MBDs, this range was selected due to size of the available data set. Yet, the developed model is generic and it can be applied on systems that has hundreds of state variables. For larger systems, it becomes computationally difficult to optimally solve the model, which is often referred to as the "curse of dimensionality." This limitation can be overcome by using approximate algorithms. Finally, future work will require

TABLE 29. Flow rate transition probabilities, when pumps 2&3 are working and the dosage rate is 20 GPH.

P. 2&3 D. 20	F1	F2	F3	F4
F1	0.63	0.25	0	0.13
F2	0	0.33	0.67	0
F3	0.17	0.33	0.17	0.33
F4	0	0.18	0.18	0.64

TABLE 30. Flow rate transition probabilities, when pumps 2&3 are working and the dosage rate is 25 GPH.

P. 2&3 D. 25	F1	F2	F3	F4
F1	0.25	0	0.19	0.56
F2	0	0.22	0.78	0
F3	0	0	0.38	0.62
F4	0	0.06	0.33	0.61

TABLE 31. Flow rate transition probabilities, when pumps 2&4 are working and the dosage rate is 5 GPH.

P. 2&4 D. 5	F1	F2	F3	F4
F1	0	0.25	0.13	0.63
F2	0	0	1	0
F3	0	0.13	0.4	0.47
F4	0	0.14	0	0.86

TABLE 32. Flow rate transition probabilities, when pumps 2&4 are working and the dosage rate is 10 GPH.

P. 2&4 D. 10	F1	F2	F3	F4
F1	0.53	0.24	0.18	0.06
F2	0.25	0.63	0.13	0
F3	0	0	0.36	0.64
F4	0	0	0	1

TABLE 33. Flow rate transition probabilities, when pumps 2&4 are working and the dosage rate is 15 GPH.

P. 2&4 D. 15	F1	F2	F3	F4
F1	0.11	0.56	0.11	0.22
F2	0.14	0.21	0.07	0.57
F3	0.13	0.33	0.07	0.47
F4	0.07	0.39	0.11	0.43

TABLE 34. Flow rate transition probabilities, when pumps 2&4 are working and the dosage rate is 20 GPH.

P. 2&4 D. 20	F1	F2	F3	F4
F1	0	1	0	0
F2	0	0	1	0
F3	0	1	0	0
F4	0.33	0	0	0.67

more data to take into consideration other possible system states, such as the pressure of oil at different points along the oil pipeline and its relation with the flow rate of oil, which can be an interesting future extension. Overall, future research will focus on enlarged state and action spaces, accounting for pumps failures, and including the oil pressure at discrete locations of the oil pipeline in the state space of the model.

TABLE 35. Flow rate transition probabilities, when pumps 2&4 are working and the dosage rate is 25 GPH.

P. 2&4 D. 25	F1	F2	F3	F4
F1	0.71	0.29	0	0
F2	0	0.22	0.78	0
F3	0	0.63	0.37	0
F4	0	0	0.6	0.4

TABLE 36. Flow rate transition probabilities, when pumps 3&4 are working and the dosage rate is 5 GPH.

P. 3&4 D. 5	F1	F2	F3	F4
F1	0.2	0.73	0	0.07
F2	0	0.14	0.57	0.29
F3	0.33	0	0.33	0.33
F4	0	0	0.09	0.91

TABLE 37. Flow rate transition probabilities, when pumps 3&4 are working and the dosage rate is 10 GPH.

P. 3&4 D. 10	F1	F2	F3	F4
F1	0.13	0.27	0.2	0.4
F2	0.02	0.76	0.05	0.17
F3	0.17	0	0.23	0.6
F4	0.18	0.2	0.2	0.43

TABLE 38. Flow rate transition probabilities, when pumps 3&4 are working and the dosage rate is 15 GPH.

P. 3&4 D. 15	F1	F2	F3	F4
F1	0.73	0.09	0.18	0
F2	0	0.25	0.58	0.17
F3	0	0	0.9	0.1
F4	0	0	0.13	0.88

TABLE 39. Flow rate transition probabilities, when pumps 3&4 are working and the dosage rate is 20 GPH.

P. 3&4 D. 20	F1	F2	F3	F4
F1	0.18	0.27	0.09	0.45
F2	0.04	0.22	0.26	0.48
F3	0.17	0.27	0.17	0.4
F4	0.05	0.22	0.22	0.51

TABLE 40. Flow rate transition probabilities, when pumps 3&4 are working and the dosage rate is 25 GPH.

P. 3&4 D. 25	F1	F2	F3	F4
F1	0.6	0	0.35	0.05
F2	0	0.88	0	0.13
F3	0	0	0.93	0.07
F4	0	0	0.33	0.67

APPENDIX A

This appendix provides the state transition probabilities for all possible pump combinations, based on historical data. Transition probabilities are computed for each pump combination and dosage level (e.g. Table 11, pumps 1 and 2 with dosage of 5 GPH). From the records, we have weekly data on the oil flow rate, the dosage level and the pumps that were operated. For each dosage level and pumps combination, we counted the number of times the oil flow rate has transitioned between the four oil flow rate intervals, which were defined in Section III (e.g., F1–F1, F1–F2, F1–F3, F1–F4, F2–F1 …F4–F4). This resulted in a 4 by 4 matrix for each possible pumps combination and dosage level (6 possible pump combinations and 5 different dosage levels), then the row entries of each matrix were normalized using the total of each row to obtain the following Markovian transition matrices.

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