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Kernel Minimum Error Entropy Based Estimator for MIMO Radar in Non-Gaussian Clutter

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ABSTRACT In this paper, a kernel minimum error entropy (KMEE) based estimator is proposed for the estimation of multiple targets' direction of departure (DOD), the direction of arrival (DOA), and the Doppler shift with multiple input multiple output radar in the presence of non-Gaussian clutter. Most existing estimation approaches are based on optimization of a complex cost function which often leads to a sub-optimum solution. Therefore, for the accurate estimation of DOD, DOA and Doppler shift, an efficient, kernel adaptive filter (KAF) based estimation approach is proposed. The proposed estimator utilizes the minimum error entropy (MEE) criterion and minimizes the error entropy function. The MEE, being an information-theoretic criterion, optimizes the higher-order statistics of error and thus makes the proposed estimator robust against the effects of outliers like clutter. The KMEE based estimator without any sparsification suffers from a linear increase in computational complexity. Thus, subsequently, the computational complexity of the proposed KMEE based estimator is reduced by incorporation of novelty criterion (NC) based sparsification technique, and the resulting estimator is called KMEE-NC. The performance of the proposed KMEE-NC based estimator is compared with the recently introduced sparse estimators based on kernel maximum correntropy criterion, and kernel minimum mean square error criterion. Additionally, KMEE-NC based estimator is also compared with other existing conventional estimators. Further, for assessing the accuracy of the proposed estimator, the modified Cramer-Rao lower bound is derived using the modified Fisher information matrix.

INDEX TERMS Cramer-Rao, KLMS-NC, KMC-NC, KMEE, KMEE-NC.

I. INTRODUCTION

Accurate estimation of range and velocity of multiple targets is a crucial problem in radar systems [1], [2]. Multiple input multiple output (MIMO) radar was introduced with the intent of improving performance of the radar systems [3], [4]. Diversity in transmitting the orthogonal waveforms from transmit antennas and collecting the superposition of echoes at each receiving antenna individually, provides improvement in the performance of MIMO radar over single antenna radar system [5], [6]. For instance, in MIMO radar, if the transmitter has N antennas and receiver has M antennas, then, contrary to a single antenna radar system, NM signals are processed at the receiver for detection of targets and estimation of their parameters. Fundamentally, MIMO radar is used for the estimation of targets' parameters which describes the targets' position and velocity: the direction of departure (DOD), the direction of arrival (DOA), and Doppler shift. Accurate estimate of a target's position and velocity is essential for applications like target tracking and navigation. Most conventional estimation techniques for MIMO radar assume the absence of clutter and thermal noise as Gaussian distributed [7]–[10]. Also, the previously proposed subspace-based estimation algorithms (ESPRIT [11] and MUSIC [12]) mostly deal with an additive noise which is assumed to be Gaussian distributed, and most often with known (up to a scaling factor) covariance matrix, considered to be the identity matrix. However, the assumption of clutter being absent

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does not hold for practical radar systems. Hence, estimation techniques in [7]–[12], when employed in the presence of non-Gaussian clutter, yield inaccurate estimates of parameters with a very high variance.

For estimation of the aforementioned targets' parameters in the non-Gaussian clutter environment, various variants of maximum likelihood estimator (MLE) are proposed in [13], [14] and [15]. However, the estimation of DOD, DOA, and Doppler shift, in the presence of non-Gaussian clutter, does not have a closed-form solution for optimization of MLE cost function [16]. Therefore, in [13] and [14], estimators based on iterative conditional MLE, and iterative joint MLE are proposed, respectively. The MLE based solution, proposed in [13] and [14] are based on the conditional likelihood, and the joint likelihood of the observations, respectively. Later, in [15], it is mentioned and shown that the estimators proposed in [13] and [14] are prone to yield suboptimum estimates of required parameters. Therefore in [15], an iterative ML estimator has been proposed. The proposed estimator is based on the marginal likelihood of the observations and claimed to perform better than the estimators proposed in [13] and [14]. However, in [15], as the marginal likelihood of the observation is considered, the final estimate depends on the numerical evaluation of integrals, which is computationally demanding. In addition to need for evaluation of integrals, as mentioned in Section 3, Remark 4 of [15], for say, P targets, the estimate of DOD/DOA, is obtained by performing the $2P$ dimensional grid search algorithm of a highly non-convex function. Implementing a grid search algorithm over $2P$ dimension is computationally complex.

Therefore, in this paper, a kernel minimum error entropy (KMEE) based estimator is proposed for estimation of multiple targets' DOD, DOA, and Doppler shift. Estimation of targets' parameters is pursued in the presence of non-Gaussian clutter and Gaussian distributed thermal noise. The KMEE is an kernel adaptive filtering (KAF) algorithm which uses an iterative stochastic gradient descent algorithm for estimation in a high dimensional reproducing kernel Hilbert space (RKHS) [17]–[19]. In recent literature, estimation of target's range and velocity has been efficiently handled by the other KAF based estimation algorithm called kernel least mean square (KLMS) algorithm [20], [21]. However, since KLMS utilizes mean square error (MSE) criterion [22]–[24], and is optimum for Gaussian noise, the estimation algorithm proposed in [20] and [21] cannot be used for the estimation of parameters in MIMO radar system perturbed by non-Gaussian clutter. In a later attempt to deal with the effects of non-Gaussian clutter, kernel maximum correntropy (KMC) based estimator proves to be better than estimator based on KLMS [25]. Nevertheless, performance of KMC based estimator may not be good when facing more complicated non-Gaussian clutter [26]. The proposed KMEE based estimator is found to yield similar MSE as KMC but with considerably lower computational complexity. The KMEE based proposed estimator utilizes the minimum error entropy (MEE) criterion, which being an information-theoretic criterion (ITC), optimizes the higher-order statistics of error between the desired and the estimated parameter. For instance, the Shannon entropy function $-y\log(y)$ contains the higher order term of y, the same can be made explicit by expanding the $\log(\cdot)$ term in $-y\log(y)$ with Taylor's series as $\log(y) = (y-1) - \frac{1}{2}(y-1)^2 + \frac{1}{3}(y-1)^3 - \cdots$. Therefore, $-y\log(y)$ contains higher order statistics, hence, this makes the estimator based on KMEE criterion robust against the effects of heavy-tailed non-Gaussian clutter.

The proposed adaptive estimator, unlike conventional non-adaptive estimators, learns the unknown function and yields an accurate estimate of the desired parameters. For learning, the proposed estimator utilizes samples (radar observations). Therefore, without any sparsification criterion, the computational complexity of the estimators increases linearly and restricts the practical applicability of the proposed estimators [27]. In this paper, for reducing the computational complexity of the proposed estimator based on KMEE, we use a sparsification technique based on Platt's novelty criterion (NC) [28]. Main contributions of the paper are:

- Adaptive estimator based on KMEE is proposed for the estimation of DOD, DOA, and Doppler shift using MIMO radar system perturbed by non-Gaussian clutter.
- The computational complexity of the proposed estimator is reduced via sparsification based on NC.
- Performance of the proposed estimator is compared with the derived analytical expression for modified Cramer-Rao lower bound (MCRLB) of DOD, DOA, and Doppler shift.
- The presented computer simulations indicate that the proposed estimator in addition to outperforming the other ITC based kernel estimators like KLMS-NC [21] and KMC-NC [25], also outperforms the estimators proposed in [13]–[15] and [16].

Rest of the paper is organized as follows: Section II describes the signal model for MIMO radar. The proposed KMEE-NC based estimator is described next in Section III. Section IV describes the analytical expressions for MCRLB for estimation of DOD, DOA and Doppler shift. In Section V, the generalized analytical expression for the variance in the estimation of DOD, DOA, and Doppler shift of multiple targets is derived. In Section VI, computational complexity of the proposed estimator is analyzed. Simulation results are discussed in Section VII. Finally, Section VIII concludes the work.

Notations: Scalar variables (constants) are denoted by lower (upper) case letters. Vectors (matrices) are denoted by boldface lower (upper) case letters. Superscripts $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ denote matrix transpose, matrix complex conjugate transpose and scalar complex conjugate operation, respectively. $tr(\cdot)$ is the trace of a matrix. $\mathbb{E}[\cdot]$ denotes statistical expectation, and $\mathbb{E}_\alpha[\cdot]$ denotes statistical expectation with respect to the distribution of α, $\otimes$ denotes Kronecker product. $\mathbb{C}$ and $\mathbb{R}$ denotes a set of complex and real numbers, respectively. **Re** and **Im** denotes the real and imaginary part of the complex number, respectively.

II. MIMO RADAR SIGNAL MODEL

In this section, the generalized signal model for MIMO radar is briefly discussed. The considered MIMO radar is assumed to consist of N transmit antennas, and M receiving antennas. Let the surveillance environment consist of P targets (identified by index p) with unknown DODs and DOAs be $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_p, \ldots, \theta_P]$, and $\boldsymbol{\phi} = [\phi_1, \phi_2, \ldots, \phi_p, \ldots, \phi_P]$, respectively. The p^{th} target is assumed to be moving with a velocity causing a Doppler shift f_p. Further, if orthogonal waveforms are transmitted, then the $N \times M$ MIMO radar signal matrix $\mathbf{R}(q)$ for q^{th} pulse after matched filtering in one coherent pulse interval at the receiver is given by [29].

$$\mathbf{R}(q) = \sum_{p=1}^{P} \exp(j2\pi f_p q)\mathbf{a}(\theta_p)\mathbf{a}^T(\phi_p) + \mathbf{W}(q) + \mathbf{C}(q), \quad \text{for } q \in [1, 2, \ldots, Q] \tag{1}$$

where f_p is the Doppler shift for the p^{th} target normalized to the MIMO radar pulse repetition frequency. $\mathbf{a}(\theta_p) = [\exp(j\frac{2\pi \sin(\theta_p)}{\lambda}d_1^t), \ldots, \exp(j\frac{2\pi \sin(\theta_p)}{\lambda}d_N^t)]^T$ is the transmit steering vector, and

$$\mathbf{a}(\phi_p) = [\exp(j\frac{2\pi \sin(\phi_p)}{\lambda}d_1^r), \ldots, \exp(j\frac{2\pi \sin(\phi_p)}{\lambda}d_M^r))]$$

is the receiving steering vector in which d_n^t, and d_m^r is distance of n^{th} and m^{th} antenna from the reference transmit and reference receive antenna, respectively. $\mathbf{W}(q)$ is the $N \times M$ matrix of samples of thermal noise, and $\mathbf{C}(q)$ is the $N \times M$ matrix of samples of non-Gaussian clutter.

Concatenating $\mathbf{R}(q)$ from (1) into $NM \times 1$ vector yields

$$\mathbf{r}(q) = \mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v}(q) + \mathbf{w}(q) + \mathbf{c}(q). \quad \text{for } q \in [1, 2, \ldots, Q] \tag{2}$$

For simplicity, after dropping index q, (2) is given by

$$\mathbf{r} = \mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v}(\mathbf{f}) + \mathbf{w} + \mathbf{c}, \tag{3}$$

where $\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi}) = [\mathbf{a}(\theta_1, \phi_1), \mathbf{a}(\theta_2, \phi_2), \ldots, \mathbf{a}(\theta_P, \phi_P)]$, $\mathbf{a}(\theta_p, \phi_p) = vec(\mathbf{a}(\theta_p)\mathbf{a}^T(\phi_p))$, $\mathbf{v}(\mathbf{f}) = [\exp(j2\pi f_1 q), \ldots, \exp(j2\pi f_p q), \ldots, \exp(j2\pi f_P q)]^T$, $\mathbf{w} = [\mathbf{W}(1,1), \mathbf{W}(1,2), \ldots, \mathbf{W}(N,M)]^T$, and $\mathbf{c} = [\mathbf{C}(1,1), \mathbf{C}(1,2), \ldots, \mathbf{C}(N,M)]^T$.

From (1) and (3), the unknown parameters of interest θ_p, ϕ_p, and f_p of p^{th} target are exponentially related to $\mathbf{r}$. Additionally, from (1) and (3), it is explicit that, the unknown parameters are easily estimated if the unknown inverse relationship between θ_p, ϕ_p, f_p and $\mathbf{r}$ is known. Therefore, in the following section, estimation of DOD, DOA, and Doppler shift is performed using a KMEE based adaptive estimator. The proposed estimator is based on the optimization of entropy of the error between the true and estimated parameter set. Moreover, as optimization of entropy leads to the minimization of higher-order statistics of error, the proposed estimator provides robustness against the non-Gaussian clutter, which in turn reduces the effect of outliers introduced by the clutter non-Gaussianity.

III. PROPOSED ESTIMATOR BASED ON KMEE-NC

This section discusses the proposed estimation technique based on KMEE along-with the sparsification technique incorporated to reduce the computational complexity of the proposed estimator. In this work, estimation of the set of DODs ($\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_p, \ldots, \theta_P]$), DOAs ($\boldsymbol{\phi} = [\phi_1, \phi_2, \ldots, \phi_p, \ldots, \phi_P]$), and Doppler shifts $\mathbf{f} = [f_1, f_2, \ldots, f_p, \ldots, f_P]$ for P different targets are done individually and individual parameter set is represented by $\boldsymbol{\Theta}$. $\boldsymbol{\Theta}$ can be either $\boldsymbol{\theta}$, $\boldsymbol{\phi}$ or $\mathbf{f}$ depending upon which set of parameter have to be estimated. This makes the considered estimation problem P dimensional i.e $\boldsymbol{\Theta} \in \mathbb{R}^P$. As the proposed estimation algorithm is adaptive and works in two phases: training and testing, the training and testing data is obtained by measuring MIMO radar return given in (3) for K (number of iterations) different values of $\boldsymbol{\theta}$, $\boldsymbol{\phi}$, and $\mathbf{f}$. For this, the range in which unknown $\theta_p \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $\phi_p \in (-\frac{\pi}{2}, \frac{\pi}{2})$, and $f_p \in (-0.5, 0.5)$ [15] are expected to take the value, are uniformly divided into K different values. Consequently, the MIMO radar measurements for any k^{th} value of unknown parameter set given the value of other two parameter set are given by

$$\mathbf{r}_{\boldsymbol{\theta}_k|\boldsymbol{\phi},\mathbf{f}} = \mathbf{A}(\boldsymbol{\theta}_k, \boldsymbol{\phi})\mathbf{v}(\mathbf{f}) + \mathbf{w}_k + \mathbf{c}_k, \tag{4}$$

$$\mathbf{r}_{\boldsymbol{\phi}_k|\boldsymbol{\theta},\mathbf{f}} = \mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi}_k)\mathbf{v}(\mathbf{f}) + \mathbf{w}_k + \mathbf{c}_k, \tag{5}$$

$$\mathbf{r}_{\mathbf{f}_k|\boldsymbol{\theta},\boldsymbol{\phi}} = \mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v}(\mathbf{f}_k) + \mathbf{w}_k + \mathbf{c}_k. \tag{6}$$

After measuring the MIMO radar return, the MIMO radar return $\mathbf{x}_k$ (which can be either $\mathbf{r}_{\boldsymbol{\theta}_k|\boldsymbol{\phi},\mathbf{f}}$, $\mathbf{r}_{\boldsymbol{\phi}_k|\boldsymbol{\theta},\mathbf{f}}$ or $\mathbf{r}_{\mathbf{f}_k|\boldsymbol{\theta},\boldsymbol{\phi}}$ at k^{th} instant, corresponding to the estimation of $\boldsymbol{\theta}_k$, $\boldsymbol{\phi}_k$ or $\mathbf{f}_k$, respectively) is mapped into a high dimensional RKHS ($\mathscr{H}$), via an implicit mapping function $\boldsymbol{\Phi}(\cdot) : \mathbb{C}^N \longrightarrow \mathscr{H}$, such that $\mathbf{x}_k$ is mapped in $\mathscr{H}$ as $\boldsymbol{\Phi}(\mathbf{x}_k)$. If $\boldsymbol{\Omega}_{k-1}$ is an unknown explicit weight vector in $\mathscr{H}$, then the unbiased estimate of the unknown parameter set $\mathbf{g}_k = \hat{\boldsymbol{\Theta}}_k$ is given by

$$\mathbf{g}_k = \langle \boldsymbol{\Omega}_{k-1}, \boldsymbol{\Phi}(\mathbf{x}_k) \rangle_{\mathscr{H}} = \boldsymbol{\Phi}^H(\mathbf{x}_k)\boldsymbol{\Omega}_{k-1}, \tag{7}$$

where $\langle \cdot, \cdot \rangle_{\mathscr{H}}$ is the inner product operator in $\mathscr{H}$.

Main objective of an estimator based on MEE is to find the optimum $\boldsymbol{\Omega}_o$ such that the cost function (ξ-entropy[1]) of the error minimizes [26], [30] i.e

$$\boldsymbol{\Omega}_o = \arg \min_{\boldsymbol{\Omega}} H_\xi(\mathbf{e}_k),$$

where $H_\xi(\mathbf{e}_k)$ is the ξ-entropy cost function, $\mathbf{e}_k = \mathbf{d}_k - \mathbf{g}_k$, $\mathbf{d}_k$ is the desired or true value of parameter set i.e. $\mathbf{d}_k = \boldsymbol{\theta}_k$, $\boldsymbol{\phi}_k$, or, $\mathbf{f}_k$.

Since, minimizing $H_\xi(\mathbf{e}_k)$ analytically is difficult, $H_\xi(\mathbf{e}_k)$ can be minimized iteratively using the weight update equation as

$$\boldsymbol{\Omega}_k = \boldsymbol{\Omega}_{k-1} - \eta \frac{\partial}{\partial \boldsymbol{\Omega}_{k-1}}(\hat{H}_\xi(\mathbf{e}_k)), \tag{8}$$

[1]The term ξ-entropy is used to generalizes the entropy function. However, in this work the most commonly used Shannon entropy ($-y \log y$) is used, the other types of entropy is described in [26].

where η is the learning parameter,

$$\hat{H}_{\xi}(\mathbf{e}_k) = \frac{1}{L}\sum_{u=k-L+1}^{k} \xi[\hat{p}_{\mathbf{e}}(\mathbf{e}(k,u))]$$

is the sample estimate of $H_{\xi}(\mathbf{e}_k)$. The $\hat{p}_{\mathbf{e}}(\mathbf{e}(k,u))$ is the estimated probability density function (PDF) [31] of $\mathbf{e}_k$ and $\mathbf{e}(k,u)s$ are the L most recent errors at k^{th} instant.

Theoretically, iterative minimization of $\hat{H}_{\xi}(\mathbf{e}_k)$ guarantees the convergence of the estimate $\mathbf{g}_k$ to the actual value of parameters. Hence, the ensemble average of the estimated parameters equates to the actual value of the parameters, and this makes the proposed estimator unbiased [26], [32]. Substituting $\hat{H}_{\xi}(\mathbf{e}_k)$ into (8), and using

$$\hat{p}_{\mathbf{e}}(\mathbf{e}(k,u)) = \frac{1}{L}\sum_{i=k-L+1}^{k} \kappa_d(\mathbf{e}(k,u)-\mathbf{e}(k,i)),$$

(where $\kappa_d(\mathbf{x},\mathbf{y}) = \exp\left(-\frac{\|\mathbf{x}-\mathbf{y}\|^2}{\sigma_d^2}\right)$ and σ_d is the kernel width), yields

$$\mathbf{\Omega}_k = \mathbf{\Omega}_{k-1}\eta\frac{\partial}{\partial\Omega_{k-1}}\left(\frac{1}{L}\sum_{u=k-L+1}^{k} \times\xi\left[\frac{1}{L}\sum_{i=k-L+1}^{k}\kappa_d(\mathbf{e}(k,u)-\mathbf{e}(k,i))\right]\right). \quad (9)$$

Because of the outer summation, evaluating (9) is computationally inefficient. For an on-line adaptation of $\mathbf{\Omega}$, the instantaneous ξ-entropy could be used by dropping the outer summation in (9), this, yields the weight update equation as

$$\begin{aligned}\mathbf{\Omega}_k &= \mathbf{\Omega}_{k-1}-\eta\frac{\partial}{\partial\Omega_{k-1}}\left(\xi\left[\frac{1}{L}\sum_{i=k-L+1}^{k}\kappa_d(\mathbf{e}(k,k)-\mathbf{e}(k,i))\right]\right),\\ &= \mathbf{\Omega}_{k-1} - \frac{\eta}{L}\xi'\left[\frac{1}{L}\sum_{i=k-L+1}^{k}\kappa_d(\mathbf{e}(k,k)-\mathbf{e}(k,i))\right]\\ &\quad\times\sum_{i=k-L+1}^{k}\kappa'_d(\mathbf{e}(k,k)-\mathbf{e}(k,i))\left(\frac{\partial\mathbf{e}(k,k)}{\partial\mathbf{\Omega}_{k-1}}-\frac{\partial\mathbf{e}(k,i)}{\partial\mathbf{\Omega}_{k-1}}\right),\\ &= \mathbf{\Omega}_{k-1} + \frac{\eta}{L}\xi'\left[\frac{1}{L}\sum_{i=k-L+1}^{k}\kappa_d(\mathbf{e}(k,k)-\mathbf{e}(k,i))\right]\\ &\quad\times\sum_{i=k-L+1}^{k}\kappa'_d(\mathbf{e}(k,k)-\mathbf{e}(k,i))(\mathbf{\Phi}(\mathbf{x}_k)-\mathbf{\Phi}(\mathbf{x}_i)).\end{aligned} \quad (10)$$

From (7) and (10), utilizing Mercer's theorem $\left(\kappa_{\sigma}(\mathbf{x}_j,\mathbf{x}_k) = \langle\mathbf{x}_j,\mathbf{x}_k\rangle_{\mathscr{H}}\right)$ [32], estimate of $\mathbf{\Theta}$ at k^{th} instant is given by

$$\mathbf{g}_k = \sum_{j=1}^{k-1}\boldsymbol{\gamma}_j(k)\kappa_{\sigma}(\mathbf{x}_j,\mathbf{x}_k), \quad (11)$$

where $\kappa_{\sigma}(\mathbf{x}_j,\mathbf{x}_k) = \exp\left(-\frac{\|\mathbf{x}_j-\mathbf{x}_k^*\|^2}{\sigma^2}\right)$ is the RKHS Mercer's kernel function in which σ is the kernel width, $\boldsymbol{\gamma}_j(k) = \boldsymbol{\gamma}_j(k-1)+\sum_l \zeta_l(k)$, and

$$\zeta_l(k) = \begin{cases} \frac{\eta}{L}\xi'\left[\frac{1}{L}\sum_{i=k-L+1}^{k}\kappa_d(\mathbf{e}(k,k)-\mathbf{e}(k,i))\right] \\ \times\sum_{i=k-L+1}^{k}\kappa'_d(\mathbf{e}(k,k)-\mathbf{e}(k,i)), \\ \qquad\qquad \text{if } l=k \\ -\frac{\eta}{L}\xi'\left[\frac{1}{L}\sum_{i=k-L+1}^{k}\kappa_d(\mathbf{e}(k,k)-\mathbf{e}(k,i))\right] \\ \times\kappa'_d(\mathbf{e}(k,k)-\mathbf{e}(k,l)), \quad \text{for } k-L<l<k. \end{cases} \quad (12)$$

As shown in (11), in evaluating $\mathbf{g}_k$, there is a temporal increase in MIMO radar observation. Consequently, as k (index for number of radar observation) increases, computational complexity of the estimator increases linearly. This, in turn, restricts the practical viability of the estimator. To circumvent this, we use Platt's NC [28]. According to the criterion the newly arrived MIMO radar observation $\mathbf{x}_k$ [32] will only be used for learning and store in the dictionary $\mathscr{D}_k$ (the $\mathscr{D}_k$ is defined as dictionary in matrix form which stores the MIMO radar observation used for the learning of an estimator) if it satisfies following conditions

$$||\mathbf{e}_k||_2 \geq \delta_e,$$
$$\underbrace{\min}_{0\leq s\leq S} ||\mathscr{D}_k^s - \mathbf{x}_k||_2 \geq \delta_d,$$

where S is the size (number of samples) in the dictionary $\mathscr{D}_k$, $\mathscr{D}_k^s$ represent the past MIMO radar observation, stored in $\mathscr{D}_k$ at s^{th} column. The $\|\cdot\|_2$ is the l_2 norm in Euclidean space, and δ_e and δ_d are the parameters used to make the estimator sparse. A reasonable default value for δ_e is the square root of the steady state MSE and a reasonable δ_d is around one tenth of the kernel width (σ).

The other sparsification techniques available are approximate linear dependency criterion, coherence criterion, surprise criterion, and data quantization. However, in this work, the most common and simple, NC has been used to highlight the importance of NC.

Algorithm-1, describes the pseudo-code of the proposed estimator based on KMEE-NC. In Algorithm-1, $|\cdot|$, is the cardinality (number of elements in a vector), and $\mathscr{D}_k$ represents the dictionary used for training of estimator.

IV. MODIFIED CRAMER-RAO LOWER BOUND FOR DOD, DOA, AND DOPPLER SHIFT

In this section, an analytical expression is derived for MCRLBs over the variance of the unbiased estimate of DOD, DOA, and Doppler shift. Conventionally, Cramer Rao lower bound (CRLB) is defined as the lower bound on the variance of an unbiased estimator. However, for non-Gaussianity,

Algorithm 1 Estimation of DOD, DOA, and Doppler Shift Using Sparse Estimator Based on KMEE-NC

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Inputs: $x_k = \mathbf{r}_{\theta_k|\boldsymbol{\phi},\mathbf{f}}, \mathbf{r}_{\phi_k|\boldsymbol{\theta},\mathbf{f}}, \mathbf{r}_{\mathbf{f}_k|\boldsymbol{\theta},\boldsymbol{\phi}} \forall\ k$, $\mathbf{d}_k = \boldsymbol{\theta}_k|\{\boldsymbol{\phi}_k, \mathbf{f}_k\}, \boldsymbol{\phi}_k|\{\boldsymbol{\theta}_k, \mathbf{f}_k\}, \mathbf{f}_k|\{\boldsymbol{\theta}_k, \boldsymbol{\phi}_k\}\ \forall\ k$
Initialize constants $\delta_e$, $\delta_d$, $\mathscr{D}_1 = \{x_1\}$, $\eta$, $\boldsymbol{\gamma}_1(0)$, K (maximum number of iterations), $L$ (error length).
while $k \leq$ K do
    $\mathbf{g}_k = \sum_{j=1}^{|\mathscr{D}_k^j|} \boldsymbol{\gamma}_j(k)\kappa_\sigma(\mathscr{D}_k^j, \mathbf{x}_k)$
    $\mathbf{e}_k = \mathbf{g}_k - \mathbf{d}_k$
    for $l = 1 : L$ do
        Compute $\zeta_l(k)$ according to (12)
    end for
    $\boldsymbol{\gamma}_j(k) = \boldsymbol{\gamma}_j(k-1) + \sum_{\forall l} \zeta_l(k)$
    if $\min_i \|\mathscr{D}_k^{(i)} - x_k\|_2 \geq \delta_e$ and $||\mathbf{e}_k||_2 \geq \delta_d$ then
        $\mathscr{D}_k := \mathscr{D}_k \cup (x_k)$
    end if
    $k = k + 1$
end while
```

the Fisher information matrix and hence the corresponding CRLB is difficult to obtain in close form. Therefore, in this section, the modified form of Fisher information matrix information, which in comparison to the Fisher information matrix, is used. For the modified CRLB (MCRLB), the PDF of the radar observation is assumed to be Gaussian, and subsequently averaged it to obtain the MCRLBs in close form [33]. The obtained MCRLBs are an alternative to CRLBs for the non-Gaussian case and effectively provide a means to assess the performance of the proposed estimator dealing with clutter non-Gaussianity [34].

In this work, the non-Gaussian clutter $\mathbf{c}$ in (3) is modeled by a spherically invariant random process (SIRP) [35]. The SIRP is mathematically described by a local Gaussian distribution with variance modulated by an independent scalar random process [36], [37]. Hence, the clutter $\mathbf{c}$ can be mathematically modeled as [13]–[15]

$$\mathbf{c} \in \mathbb{C}^{NM\times 1} = \sqrt{\alpha}\mathbf{z}, \tag{13}$$

where α is a Gamma random variable with shape and size parameter as ν (reflects the non-Gaussianity of the clutter) and μ_c (the average power of the clutter), respectively. Since, α is a Gamma distributed, $\mathbf{c}$ follows K-distribution with parameters ν and μ_c. The $\mathbf{z}$ is a complex correlated multi-dimensional Gaussian vector with a normalized covariance matrix $\boldsymbol{\Sigma}_\mathbf{z} = \mathbb{E}[\mathbf{z}\mathbf{z}^H] = \rho^{|i-j|}\ \forall\ i, j \in [1, 2, \ldots, NM]$ and zero mean vector, i.e $\mathbf{z} \sim \mathbb{C}\mathscr{N}_{\mathrm{NM}}(\mathbf{0}, \boldsymbol{\Sigma}_\mathbf{z})$.

Subsequently, the thermal noise process vector $\mathbf{w}$, in (3), is modeled by independent and identically distributed (i.i.d) additive white Gaussian noise process with zero mean and Ω_w^2 variance, i.e $\mathbf{w} \sim \mathbb{C}\mathscr{N}_{\mathrm{NM}}(\mathbf{0}, \Omega_w^2\mathbf{I})$, where $\mathbf{I}$ is the identity matrix.

In (3), $\mathbf{z}$ and $\mathbf{w}$ are consider to be independent and it is also assume that they satisfy the circularity condition. Therefore, the composite covarince matrix of equivalent perturbation $(\mathbf{c} + \mathbf{w})$ is given by

$$\boldsymbol{\Sigma} = \mu_c\boldsymbol{\Sigma}_\mathbf{z} + \Omega_w^2\mathbf{I},$$

where, $\mu_c = \mathbb{E}[\alpha]$

Since the estimation of $\boldsymbol{\Theta}$ is performed in a high-dimensional space $\mathscr{H}$, therefore, the analytical expression for MCRLB on the variance of the estimate of $\boldsymbol{\Theta}$ is also derived in $\mathscr{H}$. Subsequently, mapping the MIMO radar signal model given in (3) into $\mathscr{H}$ via $\boldsymbol{\Phi}(\cdot)$, yields

$$\boldsymbol{\Phi}(\mathbf{r}) = \boldsymbol{\Phi}(\mathbf{s} + \mathbf{w} + \mathbf{c}), \tag{14}$$

where $\mathbf{s} = \mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v}(\mathbf{f})$

The first order Taylor series approximation of (14) yields

$$\boldsymbol{\Phi}(\mathbf{r}) = \boldsymbol{\Phi}(\mathbf{s}) + \nabla\boldsymbol{\Phi}(\mathbf{s})(\mathbf{w} + \mathbf{c}), \tag{15}$$

where $\nabla\boldsymbol{\Phi}(\mathbf{s})$ is the Jacobian matrix.

From the set of unknown parameter ($\boldsymbol{\Theta} = [\Theta_1, \Theta_2, \ldots, \Theta_p, \ldots, \Theta_P]$), which is either the set of DODs, DOAs, or Doppler shifts (i.e. $\boldsymbol{\Theta} = [\theta_1, \theta_2, \ldots, \theta_p, \ldots, \theta_P]$, $\boldsymbol{\Theta} = [\phi_1, \phi_2, \ldots, \phi_p, \ldots, \phi_P]$ or $\boldsymbol{\Theta} = [f_1, f_2, \ldots, f_p, \ldots, f_P]$), from (15), the MCRLB for Θ_p is given by

$$MCRLB(\Theta_p) = \nabla\boldsymbol{\Phi}(\mathbf{s})^H[\mathbf{I}^{-1}(\boldsymbol{\Theta})]_{pp}\nabla\boldsymbol{\Phi}(\mathbf{s}), \quad \forall\ p \in [1, 2, \ldots, P] \tag{16}$$

where $\mathbf{I}(\boldsymbol{\Theta})$ is the modified Fisher information matrix for the MIMO radar signal model described in (3), subsequently, $\mathbf{I}(\boldsymbol{\Theta})$ is given by

$$\mathbf{I}(\boldsymbol{\Theta}) = \mathbb{E}_\alpha\Big[\mathbf{I}(\alpha; \boldsymbol{\Theta})\Big]. \tag{17}$$

Due to the complex non-Gaussian mathematical representation of the PDF of $\mathbf{c}$, the PDF of $\mathbf{r}$ is difficult to obtain in the close form. Alternatively, for given α, the PDF of $\mathbf{r}$ ($\mathscr{P}(\mathbf{r}|\alpha; \boldsymbol{\Theta})$) would be Gaussian. Consequently, the elements of Fisher information, for given α is given by

$$[\mathbf{I}(\alpha; \boldsymbol{\Theta})]_{ij} = -\mathbb{E}_{\mathbf{r}|\alpha}\left[\frac{\partial^2 \ln \mathscr{P}(\mathbf{r}|\alpha; \boldsymbol{\Theta})}{\partial\Theta_i\partial\Theta_j}\right], \quad \forall\ i, j \in [1, 2, \ldots, P] \tag{18}$$

where

$$\mathscr{P}(\mathbf{r}|\alpha; \boldsymbol{\Theta}) = \frac{1}{\pi^{NM}|\boldsymbol{\Sigma}_\alpha|}\exp\Big(-(\mathbf{r} - \mathbf{s})^H\boldsymbol{\Sigma}_\alpha{}^{-1}(\mathbf{r} - \mathbf{s})\Big)$$

and $\boldsymbol{\Sigma}_\alpha = \alpha\boldsymbol{\Sigma}_\mathbf{z} + \Omega_w^2\mathbf{I}$

As shown in (18), $\mathbf{I}(\alpha; \boldsymbol{\Theta})$ is the Fisher information of $\mathbf{r}$ for a particular case (when α is considered to be deterministic). Therefore, to find the Fisher information matrix of $\mathbf{r}$ for the generalized case (when α is random), the $\mathbf{I}(\alpha; \boldsymbol{\Theta})$ needs to be statistically averaged over α. The Fisher information matrix, obtained by statistical averaging of $\mathbf{I}(\alpha; \boldsymbol{\Theta})$ over α is termed as modified Fisher information matrix and is given by (17). Consequently, the CRLB corresponding to the modified Fisher information matrix ($\mathbf{I}(\boldsymbol{\Theta})$) is termed as modified CRLB.

Solving (18) for $\mathbf{s} = \mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v}$, yields

$$[\mathbf{I}(\alpha; \boldsymbol{\Theta})]_{ij} = 2\left[\frac{\partial(\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v})^H}{\partial\Theta_i}\boldsymbol{\Sigma}_\alpha{}^{-1}\frac{\partial(\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v})}{\partial\Theta_j}\right]. \quad (19)$$

With the composite form of $\boldsymbol{\Sigma}_\alpha$ ($\boldsymbol{\Sigma}_\alpha = \alpha\boldsymbol{\Sigma}_\mathbf{z} + \Omega_w^2\mathbf{I}$), solving (19) is hard, as $\boldsymbol{\Sigma}_\alpha{}^{-1}$ is difficult to obtain. Therefore, in this work to derive the MCRLB, as per [33], the hypothesis that the clutter power is much greater than the thermal noise i.e. ($\mathbb{E}[\alpha] \gg> \Omega_w^2$) is considered. The aforementioned assumption simplifies $\boldsymbol{\Sigma}_\alpha$ as $\boldsymbol{\Sigma}_\alpha = \alpha\boldsymbol{\Sigma}_\mathbf{z}$. Hence, (19) is given by

$$[\mathbf{I}(\alpha; \boldsymbol{\Theta})]_{ij} = \frac{2}{\alpha}\left[\frac{\partial(\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v})^H}{\partial\Theta_i}\boldsymbol{\Sigma}_\mathbf{z}^{-1}\frac{\partial(\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v})}{\partial\Theta_j}\right]. \quad (20)$$

Invoking the trace identity, (20) is expressed as

$$[\mathbf{I}(\alpha; \boldsymbol{\Theta})]_{ij} = \frac{2}{\alpha}tr\left[\frac{\partial(\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v})}{\partial\Theta_j}\frac{\partial(\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v})^H}{\partial\Theta_i}\boldsymbol{\Sigma}_\mathbf{z}^{-1}\right]. \quad (21)$$

As mentioned in Section II, for $\boldsymbol{\Sigma}_\mathbf{z} = \rho^{|k-l|}$, using

$$\boldsymbol{\Sigma}_\mathbf{z}^{-1} = \begin{Bmatrix} 0, & \text{if } |k-l| > 1 \\ 1/(1-\rho^2), & \text{if } k=l=1 \text{ or } k=l=NM \\ (1+\rho^2)/(1-\rho^2), & \text{if } k=l \text{ and } 2 \le k \le NM-1 \\ -\rho/(1-\rho^2), & \text{if } |k-l| = 1 \end{Bmatrix}$$

the elements of $\mathbf{I}(\alpha; \boldsymbol{\Theta})$ after using $tr[\mathbf{BC}] = \sum_{k=1}^{NM}\sum_{l=1}^{NM}[\mathbf{B}]_{k,l}[\mathbf{C}]_{l,k}$, is given by

$$[\mathbf{I}(\alpha; \boldsymbol{\Theta})]_{ij} = \frac{2}{\alpha}\sum_{k=1}^{NM}\sum_{l=1}^{NM}[\mathbf{D}]_{k,l}[\boldsymbol{\Sigma}_\mathbf{z}^{-1}]_{l,k}, \quad (22)$$

where $\mathbf{D} = \frac{\partial(\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v})}{\partial\Theta_j}\frac{\partial(\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{v})^H}{\partial\Theta_i}$.

Substituting (22) in (17), yields

$$[\mathbf{I}(\boldsymbol{\Theta})]_{ij} = \mathbb{E}_\alpha\left[\frac{2}{\alpha}\right]\sum_{k=1}^{NM}\sum_{l=1}^{NM}[\mathbf{D}]_{k,l}[\boldsymbol{\Sigma}_\mathbf{z}^{-1}]_{l,k}. \quad (23)$$

In (23), since α is Gamma distributed, $\mathbb{E}_\alpha\left[\frac{2}{\alpha}\right] = \frac{2\nu}{\mu_c(\nu-1)}$. Therefore, (23) is given by

$$[\mathbf{I}(\boldsymbol{\Theta})]_{ij} = \frac{2\nu}{\mu_c(\nu-1)}\sum_{k=1}^{NM}\sum_{l=1}^{NM}[\mathbf{D}]_{k,l}[\boldsymbol{\Sigma}_\mathbf{z}^{-1}]_{l,k}. \quad (24)$$

Solving (24) for $\boldsymbol{\Theta} = [\theta_1, \theta_2, \ldots, \theta_p, \ldots, \theta_P]$, $\boldsymbol{\Theta} = [\phi_1, \phi_2, \ldots, \phi_p, \ldots, \phi_P]$ or $\boldsymbol{\Theta} = [f_1, f_2, \ldots, f_p, \ldots, f_P]$, yields the element of $\mathbf{I}(\boldsymbol{\Theta})$ for DOD, DOA, or Doppler shift, respectively. Subsequently, $\mathbf{I}^{-1}(\boldsymbol{\Theta})$ is obtained and from (16), using $\boldsymbol{\Phi}(\mathbf{s})^H\boldsymbol{\Phi}(\mathbf{s}) = \langle\boldsymbol{\Phi}(\mathbf{s}), \boldsymbol{\Phi}(\mathbf{s})\rangle_{\mathscr{H}} = NM$, the MCRLB for p^{th} target on the variance of DOD, DOA, or Doppler shift is given by

$$MCRLB(\Theta_p) = NM[\mathbf{I}^{-1}(\boldsymbol{\Theta})]_{pp} \quad \forall\, p \in [1, 2, \ldots, P] \quad (25)$$

V. ANALYTICAL EXPRESSION FOR OVERALL VARIANCE OF ESTIMATOR BASED ON KMEE

In this section, we derive the generalized analytical expression for the variance in the estimation of DOD, DOA, and Doppler shift of multiple targets. The variance $\Omega_{\Theta_p}^2$ in the estimation of Θ_p (where Θ_p is either θ_p, ϕ_p, or f_p) of p^{th} target with estimator based on KMEE is given by

$$\Omega_{\Theta_p}^2 = MCRLB(\Theta_p) + S_{EMSE}, \quad (26)$$

where S_{EMSE} is the steady state excess mean square error of estimator based on KMEE

As shown in (26), the variance of the proposed estimator is given by the minimum achievable variance (MCRLB) plus excess mean square error yield by the proposed estimator in steady-state defined by S_{EMSE}. In (26), if S_{EMSE} is nill/zero, the estimator variance reaches the MCRLB; however, this is the ideal case. Therefore, the variance for the proposed estimator is given by MCRLB plus S_{EMSE}.

The S_{EMSE} as per [38], is given by

$$S_{EMSE} = \lim_{k\to\infty}\mathbb{E}\big[\,\|\mathbf{e}_a(k)\|_{\mathbf{G}(k)}^2\,\big], \quad (27)$$

where $\|\mathbf{e}_a(k)\|_{\mathbf{G}(k)}^2$ is the l_2 norm in $\mathscr{H}$ given the $L \times L$ Gram matrix $\mathbf{G}(k) = \boldsymbol{\Phi}^H(\mathbf{x}_k)\boldsymbol{\Phi}(\mathbf{x}_k)$ and $\mathbf{e}_a(k) = \boldsymbol{\Phi}^H(\mathbf{x}_k)\tilde{\boldsymbol{\Omega}}_{k-1}$ is the a priori error vector, i.e. $\tilde{\boldsymbol{\Omega}}_{k-1} = \boldsymbol{\Omega}_o - \boldsymbol{\Omega}_{k-1}$ is the weight error vector in $\mathscr{H}$ at k^{th} iteration.

Using the energy conservation relation as per [26], we get

$$\mathbb{E}\Big[\Big\|\tilde{\boldsymbol{\Omega}}_k\Big\|_{\mathbf{G}(k)}^2\Big] = \mathbb{E}\Big[\Big\|\tilde{\boldsymbol{\Omega}}_{k-1}\Big\|_{\mathbf{G}(k)}^2\Big] - 2\eta\mathbb{E}[\mathbf{e}_a^H(k)\mathbf{h}_\phi(\mathbf{e}(k))] + \eta^2\mathbb{E}[\mathbf{h}_\phi(\mathbf{e}^H(k))\mathbf{G}(k)\mathbf{h}_\phi(\mathbf{e}(k))], \quad (28)$$

where, from (10), $\mathbf{h}_\phi(\mathbf{e}(k)) = \frac{1}{L}\phi'\Big[\frac{1}{L}\sum_{i=k-L+1}^{k}\kappa_d(\mathbf{e}(k,k) - \mathbf{e}(k,i))\Big]\sum_{i=k-L+1}^{k}\kappa'_d(\mathbf{e}(k,k) - \mathbf{e}(k,i))(\boldsymbol{\Phi}(\mathbf{x}_k) - \boldsymbol{\Phi}(\mathbf{x}_i))$

In (28), $\mathbb{E}[\mathbf{e}_a^H(k)\mathbf{h}_\phi(\mathbf{e}(k))]$ can be define as

$$\mathbb{E}[\mathbf{e}_a^H(k)\mathbf{h}_\phi(\mathbf{e}(k))] = \gamma_k^2 h_G(\gamma_k^2)$$

where $\gamma_k^2 = \mathbb{E}[\|\mathbf{e}_a(k)\|_{\mathbf{G}(k)}^2]$

Further, in (28), $\mathbb{E}[\mathbf{h}_\phi(\mathbf{e}^H(k))\mathbf{G}(k)\mathbf{h}_\phi(\mathbf{e}(k))]$ can be simplified as

$$\mathbb{E}[\mathbf{h}_\phi(\mathbf{e}^H(k))\mathbf{G}(k)\mathbf{h}_\phi(\mathbf{e}(k))] = h_I(\gamma_k^2)\mathbb{E}[\|\boldsymbol{\Phi}(\mathbf{x}_k)\|_{\mathbf{G}(k)}^2], \quad (29)$$

$h_I(\gamma_k^2) = \sum_{i=1}^{L}\mathbb{E}[h_\phi^i(\mathbf{e}(k))^2]$

The γ_k^2 can be further simplified as

$$\begin{aligned}\gamma_k^2 &= \mathbb{E}[\|\mathbf{e}_a(k)\|_{\mathbf{G}(k)}^2] = \mathbb{E}[\mathbf{e}_a^H(k)\mathbf{G}(k)\mathbf{e}_a(k)] \\ &= \mathbb{E}[\tilde{\boldsymbol{\Omega}}_{k-1}^H\boldsymbol{\Phi}(\mathbf{x}_k)\mathbf{G}(k)\boldsymbol{\Phi}^H(\mathbf{x}_k)\tilde{\boldsymbol{\Omega}}_{k-1}] \\ &= \mathbb{E}[\tilde{\boldsymbol{\Omega}}_{k-1}^H\|\boldsymbol{\Phi}(\mathbf{x}_k)\|_{\mathbf{G}(k)}^2\tilde{\boldsymbol{\Omega}}_{k-1}] \\ &= NML(\mu_c + \Omega_w^2)\mathbb{E}\Big[\Big\|\tilde{\boldsymbol{\Omega}}_{k-1}\Big\|_{\mathbf{G}(k)}^2\Big], \end{aligned} \quad (30)$$

where $\mathbb{E}[\|\boldsymbol{\Phi}(\mathbf{x}_k)\|_{\mathbf{G}(k)}^2] = NML(\mu_c + \Omega_w^2)$

Solving for (30), yields

$$\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_k\right\|^2_{\mathbf{G}(k)}] = \mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}] - 2\eta NML(\mu_c+\Omega_w^2) \times\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}]h_G\big(NML(\mu_c+\Omega_w^2) \times\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}]\big) + NML\eta^2 \times h_I\big(NML(\mu_c+\Omega_w^2)\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}]\big) \times(\mu_c+\Omega_w^2) \quad (31)$$

For steady state analysis taking $\lim_{k\to\infty}$ to both sides of (31), yields

$$\lim_{k\to\infty}\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_k\right\|^2_{\mathbf{G}(k)}] = \lim_{k\to\infty}\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}] - 2\eta NML(\mu_c+\Omega_w^2) \times\lim_{k\to\infty}\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}]h_G\big(NML(\mu_c+\Omega_w^2) \times\lim_{k\to\infty}\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}]\big) + NML\eta^2 \times h_I\big(NML(\mu_c+\Omega_w^2)\lim_{k\to\infty}\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}]\big) \times(\mu_c+\Omega_w^2). \quad (32)$$

Since $\lim_{k\to\infty}\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_k\right\|^2_{\mathbf{G}(k)}] = \lim_{k\to\infty}\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}]$, (32) is given by

$$\lim_{k\to\infty}\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}] = \frac{\eta}{2}\frac{h_I\big(NML(\mu_c+\Omega_w^2)\lim_{k\to\infty}\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}]\big)}{h_G\big(NML(\mu_c+\Omega_w^2)\lim_{k\to\infty}\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}]\big)}. \quad (33)$$

Representing $S_{WEP} = \lim_{k\to\infty}\mathbb{E}[\left\|\tilde{\mathbf{\Omega}}_{k-1}\right\|^2_{\mathbf{G}(k)}]$ as the steady state weight error power, (33) is given by

$$S_{WEP} = \frac{\eta}{2}\frac{h_I\big(NML(\mu_c+\Omega_w^2)S_{WEP}\big)}{h_G\big(NML(\mu_c+\Omega_w^2)S_{WEP}\big)}. \quad (34)$$

Using (27) and (30), S_{EMSE} is given by

$$S^2_{EMSE} = NML(\mu_c+\Omega_w^2)S_{WEP}. \quad (35)$$

VI. COMPUTATIONAL COMPLEXITY ANALYSIS OF ESTIMATOR BASED ON KMEE-NC

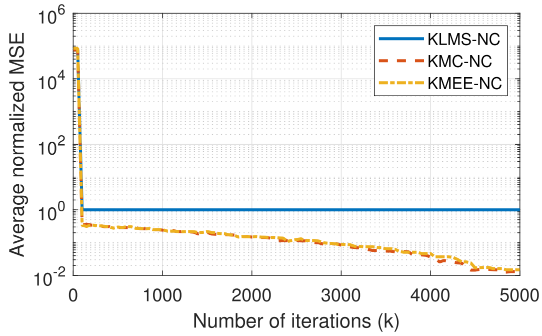
In KAF based on-line learning algorithms, most of the computational time consuming part is the calculation of the prediction error ($\mathbf{e}_k = \mathbf{g}_k - \mathbf{d}_k$). As the calculation of prediction error depends on the evaluation of $\mathbf{g}_k = \sum_{j=1}^{k-1}\boldsymbol{\gamma}_j(k)\kappa_\sigma(\mathbf{x}_j,\mathbf{x}_k)$, the time taken to compute $\mathbf{g}_k$, mostly affect the computational complexity of KMEE algorithm. Consequently, the computational complexity of estimator based on KMEE is given by $O(k)$. Thus, it can be observed that the complexity grows unboundedly, which is an undesirable feature of estimator based on KMEE. Furthermore, since no sparsification criterion is applied, the number of terms under the summation for calculating $\mathbf{g}_k$ is equal to iteration number k, which increases linearly upto K. However, as mentioned in Section III, with KMEE-NC, $\mathbf{g}_k$ is given by $\mathbf{g}_k = \sum_{j=1}^{|\mathscr{D}_k^j|}\boldsymbol{\gamma}_j(k)\kappa_\sigma(\mathscr{D}_k^j), \mathbf{x}_k$, where $|\mathscr{D}_{k-1}|$ is the cardinality of $\mathscr{D}_{k-1}$ (dictionary containing input observations satisfying NC criterion at iteration k). In comparison to KMEE, at each iteration the number of terms ($|\mathscr{D}_{k-1}|$) under summation is less than k, this reduces the computational complexity of KMEE-NC and is given by $O(|\mathscr{D}_{k-1}^{KMME-NC}|)$.

Similarly, the computaional complexity of the other existing KAF based estimators KLMS-NC and KMC-NC is given by $O(|\mathscr{D}_{k-1}^{KLMS-NC}|)$ and $O(|\mathscr{D}_{k-1}^{KMC-NC}|)$, respectively. Furthermore, since $\max|\mathscr{D}_{k-1}^{KMEE-NC}| < K$, employing NC for sparsification, reduces the number of computations as compared to KMEE. The computational complexity of KMEE-NC based estimator in comparison to KLMS-NC and KMC-NC is further reduced by incorporating the MEE criterion. Therefore, for KMEE-NC, KMC-NC, and KLMS-NC, the relation: $\max|\mathscr{D}_{k-1}^{KMME-NC} < \max|\mathscr{D}_{k-1}^{KMC-NC}| < \max|\mathscr{D}_{k-1}^{KLMS-NC}| < K$ holds true.

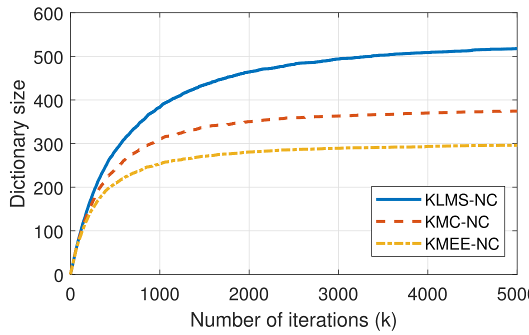
VII. SIMULATION RESULTS AND DISCUSSION

In this section, we present and discuss simulation results performed to validate performance of the proposed estimator based on KMEE-NC. For evaluating the average normalized mean square error $\Big(\text{NMSE} = \frac{1}{K_{te}}\sum_{k=1}^{K_{te}}\frac{\left\|\mathbf{\Theta}^k-\hat{\mathbf{\Theta}}^k\right\|^2}{\left\|\mathbf{\Theta}^k\right\|^2}$, where K_{te} is the number of MIMO radar observations used for testing$\Big)$ performance of the proposed estimator, the signal to clutter ratio $\Big(\text{SCR} = \frac{\mathbf{v}^H\mathbf{A}^H(\boldsymbol{\theta},\boldsymbol{\phi})\mathbf{A}(\boldsymbol{\theta},\boldsymbol{\phi})\mathbf{v}}{\mu_c tr(\mathbf{\Sigma_z})}\Big)$, and clutter to noise ratio $\Big(\text{CNR} = \frac{\mu_c tr(\mathbf{\Sigma_z})}{NM\Omega^2_{\mathbf{w}}}\Big)$, both are fixed at 30 dB. The simulations are performed to estimate the DOD, DOA, and Doppler shift of four different targets (i.e $P = 4$) illuminated by the MIMO radar with $N = 4$, and $M = 3$ [15]. To compare performance of the proposed estimator with other sparsified version of kernel based estimators (KLMS-NC and KMC-NC) and estimators proposed in [13]–[15] and [16], variance in estimation of parameters of first target is evaluated in the SCR range from −30 dB to 30 dB with an increment of 2 dB. The reported simulation results are obtained by ensembeling of 100 Monte Carlo runs. The free parameter values of KMEE-NC, KMC-NC, and KLMS-NC for the estimation of DOD, DOA, and Doppler shift are summarized in Table 1, Table 2, and Table 3, respectively. The value of

[2] Please note, because of the need to evaluate theoretical expectation $\mathbb{E}$, the obtained expression for variance is only for theoretical importance and cannot be simulated.



(a) Average normalized MSE



(b) Dictionary size

FIGURE 1. (a) Average normalized MSE, and (b) Dictionary size in the estimation of DOD using estimators based on KMEE-NC, KMC-NC, and KLMS-NC.

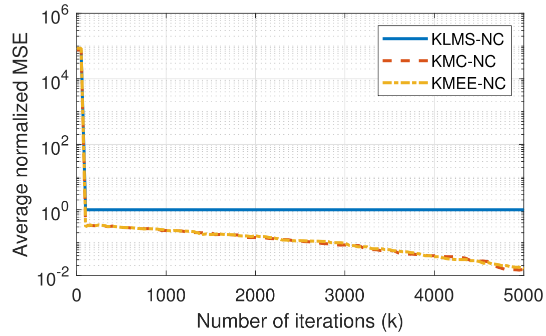
TABLE 1. Parameters values used for estimation algorithms based on KMEE-NC, KMC-NC, and KLMS-NC for estimating Doppler shift via simulations.

Parameters	KMEE-NC	KMC-NC	KLMS-NC
η	0.57	0.57	0.57
σ	10^{-6}	10^{-6}	10^{-6}
σ_d	2×10^{-5}	0.02	Nil
δ_e	$10^{-8.5}$	$10^{-8.5}$	$10^{-8.5}$
δ_d	1.9	1.9	1.9
L	25	Nil	Nil

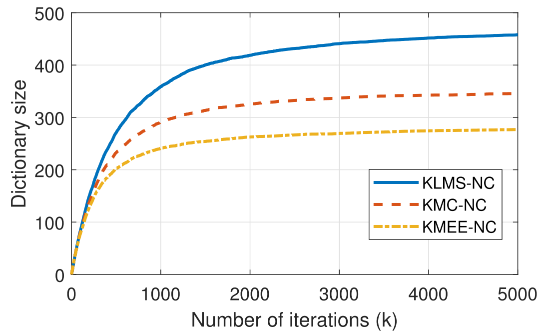
TABLE 2. Parameters values used for estimation algorithms based on KMEE-NC, KMC-NC, and KLMS-NC for estimating DOA via simulations.

Parameters	KMEE-NC	KMC-NC	KLMS-NC
η	0.57	0.57	0.57
σ	10^{-6}	10^{-6}	10^{-6}
σ_d	2×10^{-5}	0.02	Nil
δ_e	$10^{-8.5}$	$10^{-8.5}$	$10^{-8.5}$
δ_d	2	2	2
L	20	Nil	Nil

free parameters mentioned in Table 1, Table 2, and Table 3 are obtained by cross-validation [32], [39], to achieve a desirable average NMSE convergence speed with minimum average NMSE value.



(a) Average normalized MSE



(b) Dictionary size

FIGURE 2. (a) Average normalized MSE, and (b) Dictionary size in the estimation of DOA using estimators based on KMEE-NC, KMC-NC, and KLMS-NC.

TABLE 3. Parameters values used for estimation algorithms based on KMEE-NC, KMC-NC, and KLMS-NC for estimating Doppler shift via simulations.

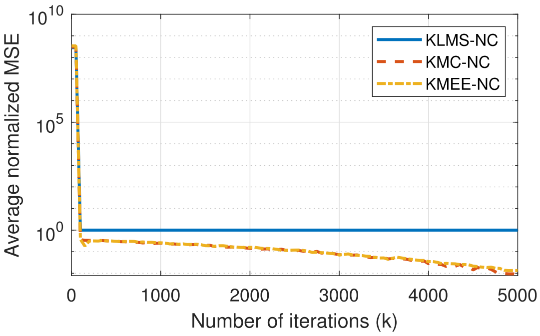
Parameters	KMEE-NC	KMC-NC	KLMS-NC
η	0.55	0.55	0.55
σ	$10^{-7.4}$	$10^{-7.4}$	$10^{-7.4}$
σ_d	2×10^{-5}	0.02	Nil
δ_e	$10^{-8.5}$	$10^{-8.5}$	$10^{-8.5}$
δ_d	6.4	6.4	6.4
L	20	Nil	Nil

A. ESTIMATION OF DOD AND DOA

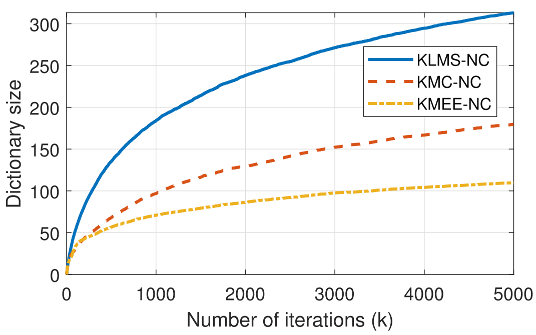
Simulations using estimators based on KMEE-NC, KMC-NC, and KLMS-NC to estimate DOD and DOA of four different targets are performed by dividing the interval of true values of DOD and DOA, i.e., $(\frac{-\pi}{2}, \frac{\pi}{2})$ into $K = 5550$[3] parts. Subsequently, the MIMO radar observations corresponding to $K = 5550$ true values of DOD and DOA are obtained by using (4) and (5), respectively. The starting $K_{tr} = 5000$ pair of DOD and DOA true values and respective MIMO radar observations i.e. $(\mathbf{d}_k, \mathbf{r}_{\theta_k|\boldsymbol{\phi},\mathbf{f}})$ or $(\mathbf{d}_k, \mathbf{r}_{\phi_k|\boldsymbol{\theta},\mathbf{f}})$ are used to train the estimators. The rest $K_{te} = 550$ pairs are used for testing performance of the estimators and evaluating the average NMSE.

As shown in Fig. 1a, and Fig. 2a, the estimator based on KMEE-NC converges to a lower average NMSE of the

[3]As convergence is observed with minimum 5000 training samples [32] tested over 550 testing samples, $K = 5550$ is chosen.



(a) Average normalized MSE



(b) Dictionary size

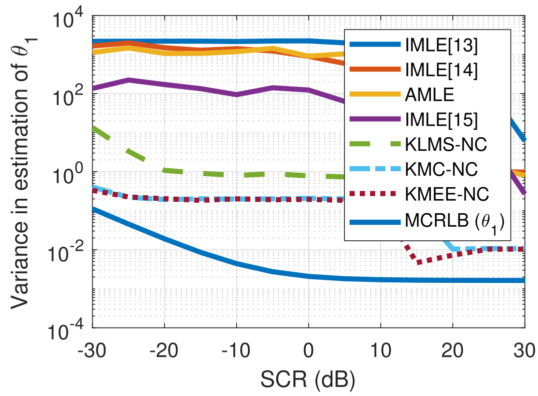
FIGURE 3. (a) Average normalized MSE, and (b) Dictionary size in the estimation of Doppler shift using estimators based on KMEE-NC, KMC-NC, and KLMS-NC.

TABLE 4. Computational complexity comparison of estimators based on KLMS-NC, KMC-NC, and KMEE-NC corresponding to Fig. 1b.

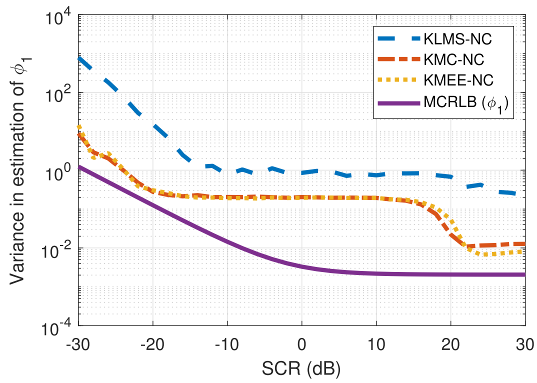
KAF Estimators	Number of multiplication	Complexity order
KLMS-NC	500	$O(\|\mathscr{D}_{k-1}^{KLMS-NC}\|)$
KMC-NC	390	$O(\|\mathscr{D}_{k-1}^{KMC-NC}\|)$
KMEE-NC	300	$O(\|\mathscr{D}_{k-1}^{KMEE-NC}\|)$

where $|\mathscr{D}_{k-1}^{KMEE-NC}| < |\mathscr{D}_{k-1}^{KMC-NC}| < |\mathscr{D}_{k-1}^{KLMS-NC}| < K$.

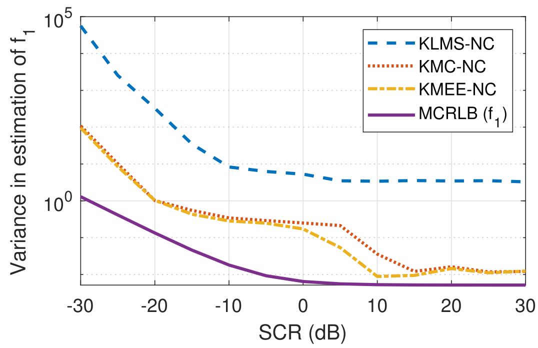
order 10^{-2} as compared to the estimator based on KLMS-NC. Further, as reported in Fig. 1a, and Fig. 2a, the estimator based on KMEE-NC yields estimates with average NMSE equal to the estimator based on KMC-NC. However, as shown in Fig. 1b, and Fig. 2b, and quantified in Table 4, the computational complexity (dictionary size) obtained in the estimation of DOD and DOA, using an estimator based on KMEE-NC is lower than the estimator based on KMC-NC and KLMS-NC. Additionally, from Fig. 1b, and Fig. 2b, we can also observe that the relation: $\max|\mathscr{D}_{k-1}^{KMME-NC} < \max|\mathscr{D}_{k-1}^{KMC-NC}| < \max|\mathscr{D}_{k-1}^{KLMS-NC}| < K$ mentioned in Section VI is verified by the simulations. The lower average NMSE and lower dictionary size obtained by the estimator based on KMEE-NC, validate the superiority of the proposed estimator over other kernel based estimation techniques (KMC-NC and KLMS-NC).



(a) DOD



(b) DOA



(c) Doppler shift

FIGURE 4. Comparative performance of the estimators for estimation of (a) DOD, (b) DOA, and (c) Doppler shift of the first target.

B. ESTIMATION OF DOPPLER SHIFT

Similar to DOD/DOA, simulations to estimate normalized Doppler shift for $P = 4$ using estimators based on KMEE-NC, KMC-NC, and KLMS-NC are performed by dividing the interval of true values of normalized Doppler shift i.e $(-0.5, 0.5)$ into $K = 5550$ parts. The MIMO radar observations corresponding to $K = 5550$ true values of Doppler shift are obtained by using (6). Out of the 5550 pair of Doppler shift and MIMO radar observations i.e. $(\mathbf{d}_k, \mathbf{r}_{\mathbf{f}_k|\boldsymbol{\theta},\boldsymbol{\phi}})$, 5000 are used to train the estimators. Subsequently, the average NMSE in estimating normalized Doppler shift are evaluated using the remaining 550 pairs of Doppler shift and MIMO radar observations $(\mathbf{d}_k, \mathbf{r}_{\mathbf{f}_k|\boldsymbol{\theta},\boldsymbol{\phi}})$.

In the estimation of the Doppler shift, as depicted in Fig. 3a, average NMSE of the estimator based on KMEE-NC converges to the order of 10^{-2} which is lower than average NMSE achieved by the estimators based on KLMS-NC. Further, as reported in Fig. 3a, the performance of the estimator based on KMEE-NC coincides with the performance of estimator based on KMC-NC. However, similar to Doppler shift estimation, estimator based on KMEE-NC offers a gain over KMC-NC and KLMS-NC in terms of computational complexity, as shown in Fig. 3b.

C. COMPARATIVE PERFORMANCE OF ESTIMATORS

For assessing accuracy of the proposed estimation technique compared to existing kernel based estimators and conventional estimators, variance of the estimators is compared with the respective MCRLBs. For this, simulations are performed to evaluate variance in the estimation of DOD, DOA, and Doppler shift of the first target, i.e., $p = 1$.

As shown in Fig. 4a in the estimation of DOD, the variance obtained with KMEE-NC in comparison to KLMS-NC and conventional estimators proposed in [13]–[15] and [16], termed respectively as IMLE [13], IMLE [14], IMLE [15], and AMLE, is closer to the achievable MCRLB. Furthermore, it is also observed that the variance of the estimator based on KMEE-NC and KMC-NC is coinciding. However, as depicted in Fig. 1b, Fig. 2b, and Fig. 3b, KMEE-NC utilizes much lower radar observations than KMC-NC, which results in lower computational complexity. Moreover, as shown in Fig. 4a, particular to DOD estimation, KMEE-NC has lower variance than KMC-NC in the SCR range of 10 dB to 20 dB. Similar to DOD estimation, particular to Doppler shift estimation, as shown in Fig. 4c, KMEE-NC yields lower variance than KMC-NC in the SCR range of 0 dB to 10 dB.

VIII. CONCLUSION

In this paper, an estimator for DOD, DOA, and Doppler shift for multiple targets using MIMO radar in the presence of non-Gaussian clutter is proposed. The effect of non-Gaussian clutter is handled by introducing the adaptive estimator based on KMEE. The KMEE optimizes the MEE criterion in RKHS, which, yields accurate estimates of parameters by compensating the effect of non-Gaussian clutter. The practical viability of the proposed KMEE based estimator is limited by its high computational complexity/dictionary size. Thus, the computational complexity/dictionary size is reduced by incorporation of sparsification technique based on NC. The performance of the proposed algorithm is compared with the derived MCRLB for DOD, DOA, and Doppler shift. Accuracy of the proposed estimator is validated through computer simulations over realistic MIMO radar systems. The obtained simulation results reveal the viability of the proposed KMEE-NC based estimator over other kernel based adaptive estimators and conventional estimators.

In future, the utility of the proposed KMEE-NC based estimator can be explored for nested MIMO radar in the presence of clutter.

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