IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received August 29, 2021, accepted September 1, 2021, date of publication September 3, 2021, date of current version September 13, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3110278

Observer-Based Control for Markovian Jump Fuzzy Systems Under Mismatched Fuzzy Basis Functions

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This work was supported by the 2021 Research Fund of the University of Ulsan under Grant 2021-0408.

ABSTRACT This paper investigates the observer-based dissipative control problem for a class of discrete-time Markovian jump fuzzy systems under mismatched fuzzy basis functions. In the practical implementation of the observer-based control scheme, the system state variable can be measured with uncertainties and disturbances, which acts as a factor that prevents accurate measurement of the premise variable. Thus, in this case, it is necessary to explore the phenomenon that the system premise variable cannot be reflected in the design of the fuzzy-basis-dependent observer and controller. In response to this need, this paper proposes a method to deal with the mismatch phenomenon in the observer-based stabilization problem of MJFSs by devising a two-step approach to solve the inherent decoupling problem and by providing a useful relaxation technique for the error of mismatched fuzzy basis functions.

INDEX TERMS Markovian jump fuzzy systems, mismatched fuzzy basis functions, observer-based control, relaxation method.

I. INTRODUCTION

In the early 2000s, the study of Markovian jump fuzzy systems (MJFSs) has received considerable interest from the control community as it has been validated as one of the most suitable mathematical models for describing nonlinear dynamic systems with random abrupt changes due to intrinsic and extrinsic factors (refer to [10]–[13] and the references therein). Thus, over the past few decades, various control synthesis problems for real application systems modeled with MJFSs have been addressed through pioneering work. As representative results, [14] addressed the problem of designing a reliable robust $\mathcal{H}_\infty$ fuzzy control for uncertain nonlinear continuous-time systems with Markovian jumping actuator faults, [15] proposed a method of designing $\mathcal{H}_\infty$ state-feedback control for MJFSs with incomplete knowledge of transition probabilities, and [16] studied the stability and stabilization problem of Markovian chaotic systems via fuzzy sampled-data control. Furthermore, in the 2020s, [17] dealt with the problem of designing a dissipativity-based sampled-data control for MJFSs with incomplete transition rates, and [18] proposed a method of deriving less conservative stabilization conditions for nonhomogeneous MJFSs with higher-level operation modes. However, it is worth pointing out that all of the aforementioned work requires the assumption that full information about the system state variables is always available.

Indeed, from the point of view of designing a more realistic fuzz control, it is necessary to consider cases where the system state variables cannot be directly measured due to the impracticality of installing sensors (see [19]–[21] and the reference therein). For this reason, significant effort has been made to effectively design the output-feedback control for MJFSs (see [22]–[26]). To be specific, [22] presented a method of designing fuzzy dynamic output feedback controllers for MJFSs with interval time-varying delays, [23] addressed the problem of delay-dependent static output-feedback control for MJFSs with actuator faults, [24] studied the non-fragile observer-based control synthesis problem for a class of fractional-order nonlinear systems under the fractional-order fuzzy model with Markovian jump, and [26] addressed the problem of observer-based control synthesis of fuzzy degenerate jump systems with mode-dependent time-varying delays. However, one thing to note is that the premise variables that must be defined when generating a set of fuzzy rules from nonlinear time-varying terms generally depend on

The associate editor coordinating the review of this manuscript and approving it for publication was Engang Tian.

the system state variables. Thus, if the state variables related to the premise variables cannot be directly measured to the output side, then serious problems can arise when implementing the fuzzy-basis-dependent output-feedback controls mentioned above.

To make up for this weakness, the premise variables can be estimated based on the observer state variables, but the estimated variables cannot perfectly match the original ones. In other words, when designing fuzzy observer-based controls for MJFSs, it is essential to consider the occurrence of mismatched premise variables that give rise to mismatched fuzzy basis functions. However, despite the possibility of the appearance of mismatched fuzzy basis functions, little progress has been made in analyzing the differences between fuzzy basis functions and their estimated functions and in investigating the problem of observer-based control synthesis of MJFSs under mismatched fuzzy basis functions. Thus, motivated by the lack of this study, this paper focuses on the problem of designing an observer-based dissipative control governed by the mismatched fuzzy basis functions. To be specific, the main contributions of this paper can be highlighted as follows.

- As is well known, the premise variables are usually subject to the measurable output or immeasurable state. Thus, if the measurable output is affected by the disturbance, or if the immeasurable state must be estimated, it is impossible to accurately measure the premise variable. In this context, this paper proposes a method of dealing with this mismatch phenomenon in the observer-based stabilization problem of MJFSs.
- The mismatched fuzzy basis functions tend to make the observer-based stabilization problem more complicated because the control target system and the observer cannot share the same fuzzy basis functions. Eventually, the well-known decoupling method cannot be applied as it is. For this reason, this paper introduces a simple method to solve the decoupling problem within two steps.
- To obtain less conservative output-feedback stabilization conditions, the error constraints on mismatched fuzzy basis functions must be incorporated into the control design procedure, which means that a novel relaxation technique for the mismatched fuzzy basis functions needs to be additionally provided. In line with this need, this paper provides a useful relaxation method for the error of mismatched fuzzy basis functions.

A. NOTATIONS

The notations $X \geq Y$ and $X > Y$ mean that $X - Y$ is positive semi-definite and positive definite, respectively. In symmetric block matrices, the asterisk $(*)$ is used as an ellipsis for terms induced by symmetry. $\mathbf{E}\{\cdot\}$ denotes the mathematical expectation; $\mathbf{Pr}(X)$ and $\mathbf{Pr}(X|Y)$ mean the probability of X and the probability of X conditional on Y, respectively; $\mathbf{diag}(\cdot)$ stands for a block-diagonal matrix; $\mathbf{col}(v_1, v_2, \ldots, v_n) = \begin{bmatrix} v_1^T & v_2^T & \cdots & v_n^T \end{bmatrix}^T$ for scalars or vectors v_i; $\mathbf{He}\{\mathcal{Q}\} = \mathcal{Q} + \mathcal{Q}^T$ for any square matrix $\mathcal{Q}$; and $\mathcal{L}_2[0, \infty)$ stands for the space of square summable sequences over $[0, \infty)$. The notations 0_n, $0_{m,n}$ and I_n stand for a zero matrix in $\mathbb{R}^{n\times n}$, a zero matrix in $\mathbb{R}^{m\times n}$ and a identity matrix $\mathbb{R}^{n\times n}$, respectively. For $\mathbf{N}_p = \{1, 2, \cdots, p\}$, it is defined that:

$$\begin{aligned} \left[\mathcal{Q}_i\right]_{i\in\mathbf{N}_p} &= \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 & \cdots & \mathcal{Q}_p \end{bmatrix} \\ \left[\mathcal{Q}_i\right]^{\mathbf{d}}_{i\in\mathbf{N}_p} &= \mathbf{diag}\big(\mathcal{Q}_1, \mathcal{Q}_2, \cdots, \mathcal{Q}_p\big) \\ \left[\mathcal{Q}_{ij}\right]_{i,j\in\mathbf{N}_p} &= \begin{bmatrix} \mathcal{Q}_{11} & \cdots & \mathcal{Q}_{1p} \\ \vdots & \ddots & \vdots \\ \mathcal{Q}_{p1} & \cdots & \mathcal{Q}_{pp} \end{bmatrix} \end{aligned}$$

where $\mathcal{Q}_i$ and $\mathcal{Q}_{ij}$ denote real submatrices with appropriate dimensions or scalar values.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

For a given complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, let us consider a class of Markovian jump fuzzy systems (MJFSs) as follows:

$$\begin{cases} x(k+1) = A(\theta(k), \phi(k))x(k) + B(\theta(k), \phi(k))u(k) \\ \qquad\qquad + E(\theta(k), \phi(k))w(k) \\ y(k) = C(\theta(k), \phi(k))x(k) + D(\theta(k), \phi(k))w(k) \\ z(k) = G(\theta(k), \phi(k))x(k) + H(\theta(k), \phi(k))u(k) \end{cases} \tag{1}$$

with

$$\begin{aligned} A(\theta(k), \phi(k) = g) &= \sum_{i=1}^{r} \theta_i(\eta(k))A_{gi} \\ B(\theta(k), \phi(k) = g) &= \sum_{i=1}^{r} \theta_i(\eta(k))B_{gi} \\ C(\theta(k), \phi(k) = g) &= \sum_{i=1}^{r} \theta_i(\eta(k))C_{gi} \\ D(\theta(k), \phi(k) = g) &= \sum_{i=1}^{r} \theta_i(\eta(k))D_{gi} \\ E(\theta(k), \phi(k) = g) &= \sum_{i=1}^{r} \theta_i(\eta(k))E_{gi} \\ G(\theta(k), \phi(k) = g) &= \sum_{i=1}^{r} \theta_i(\eta(k))G_{gi} \\ H(\theta(k), \phi(k) = g) &= \sum_{i=1}^{r} \theta_i(\eta(k))H_{gi} \end{aligned}$$

where $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$, $y(k) \in \mathbb{R}^{n_y}$, $z(k) \in \mathbb{R}^{n_z}$, $w(k) \in \mathbb{R}^{n_w}$, and $\phi(k) \in \mathbb{N}_\phi = \{1, 2, \cdots, s\}$ denote the state, the control input, the measured output, the performance output, the external disturbance belonging to $\mathcal{L}_2[0, \infty)$, and the system operation mode, respectively; A_{gi}, B_{gi}, C_{gi}, D_{gi}, E_{gi}, G_{gi} and H_{gi} are known system matrices with appropriate dimensions; and r indicates the number of fuzzy rules.

To be specific, $\theta(k) = \mathbf{col}(\theta_1(\eta(k)), \theta_2(\eta(k)), \cdots, \theta_r(\eta(k)))$ stands for the fuzzy basis function vector that depends on the premise variable vector $\eta(k) = \mathbf{col}(\eta_1(k), \cdots, \eta_p(k))$, where $\theta_i(\eta(k))$ indicates the ith normalized fuzzy-basis function that satisfies $0 \le \theta_i(\eta(k)) \le 1$, for all $i \in \mathbb{N}_\theta = \{1, 2, \cdots, r\}$, and $\sum_{i=1}^{r} \theta_i(\eta(k)) = 1$. Furthermore, the process $\{\phi(k), k \ge 0\}$ is characterized by a discrete-time Markov chain subject to the following transition probability:

$$\pi_{gh} = \mathbf{Pr}\big(\phi(k+1) = h \mid \phi(k) = g\big), \ \forall g, h \in \mathbb{N}_\phi$$

which satisfies $0 \le \pi_{gh}(k) \le 1$ and $\sum_{h=1}^{r} \pi_{gh}(k) = 1$. Especially, for brevity, this paper will use the following notations: i) $\theta = \theta(k)$, ii) $\theta_i = \theta_i(\eta(k))$, iii) $\theta^+ = \theta(k+1)$, iv) $\mathcal{O}(\theta(k), \phi(k) = g) = \mathcal{O}_g(\theta)$ for any mode-dependent and fuzzy-basis-dependent matrix $\mathcal{O}(\cdot)$.

Remark 1: In general, the premise variable vector $\eta(k)$ explicitly depends on the measurable output $y(k)$ or the immeasurable state $x(k)$. Thus, if any of the following occurs:

- the measurable output associated with $\eta(k)$ is affected by the disturbance $w(k)$,
- the immeasurable state associated with $\eta(k)$ must be estimated.

it is impossible to share the same premise variable as (1) when designing a fuzzy observer-based control.

Thus, to deal with the actual case mentioned in Remark 1, this paper takes into account the following observer-based fuzzy control law: for $\phi(k) = g$,

$$\begin{cases} \widetilde{x}(k+1) = A_g(\widetilde{\theta})\widetilde{x}(k) + B_g(\widetilde{\theta})u(k) \\ \qquad\qquad + L_g(\widetilde{\theta})\big(y(k) - C_g(\widetilde{\theta})\widetilde{x}(k)\big) \\ u(k) = F_g(\widetilde{\theta})\widetilde{x}(k) \end{cases} \tag{2}$$

where $\widetilde{x}(k) \in \mathbb{R}^{n_x}$ denotes the estimated system state; $\widetilde{\theta} = \mathbf{col}(\widetilde{\theta}_1, \widetilde{\theta}_2, \cdots, \widetilde{\theta}_r)$ denotes the mismatched fuzzy basis function vector; $\widetilde{\theta}_i = \theta_i(\widetilde{\eta}(k))$ denotes the ith element of $\widetilde{\theta}$; $\widetilde{\eta}(k)$ denotes the estimated premise variable vector; $L_g(\widetilde{\theta})$ and $F_g(\widetilde{\theta})$ are the mode-dependent fuzzy observer and control gains to be obtained later, respectively; and

$$A_g(\widetilde{\theta}) = \sum_{i=1}^{r} \widetilde{\theta}_i A_{gi}, \ B_g(\widetilde{\theta}) = \sum_{i=1}^{r} \widetilde{\theta}_i B_{gi}$$
$$C_g(\widetilde{\theta}) = \sum_{i=1}^{r} \widetilde{\theta}_i C_{gi}.$$

As a result, letting

$$e(k) = x(k) - \widetilde{x}(k)$$
$$\zeta(k) = \mathbf{col}(\widetilde{x}(k), e(k)) \in \mathbb{R}^{2n_x \times 2n_x}$$

the closed-loop control system with (1) and (2) is represented as follows:

$$\zeta(k+1) = \mathbf{A}_g(\theta, \widetilde{\theta})\zeta(k) + \mathbf{E}_g(\theta, \widetilde{\theta})w(k) \tag{3}$$
$$z(k) = \mathbf{G}_g(\theta, \widetilde{\theta})\zeta(k) \tag{4}$$

where

$$\mathbf{A}_g(\theta, \widetilde{\theta}) = \begin{bmatrix} \begin{pmatrix} A_g(\widetilde{\theta}) \\ +B_g(\widetilde{\theta})F_g(\widetilde{\theta}) \\ +\Delta_1 \end{pmatrix} & L_g(\widetilde{\theta})C_g(\theta) \\ \Delta_2 - \Delta_1 & \begin{pmatrix} A_g(\theta) \\ -L_g(\widetilde{\theta})C_g(\theta) \end{pmatrix} \end{bmatrix} \tag{5}$$

$$\mathbf{E}_g(\theta, \widetilde{\theta}) = \begin{bmatrix} L_g(\widetilde{\theta})D_g(\theta) \\ E_g(\theta) - L_g(\widetilde{\theta})D_g(\theta) \end{bmatrix} \tag{6}$$

$$\mathbf{G}_g(\theta, \widetilde{\theta}) = \begin{bmatrix} G_g(\theta) + H_g(\theta)F_g(\widetilde{\theta}) & G_g(\theta) \end{bmatrix}$$
$$\Delta_1 = L_g(\widetilde{\theta})\big(C_g(\theta) - C_g(\widetilde{\theta})\big)$$
$$\Delta_2 = \big(A_g(\theta) - A_g(\widetilde{\theta})\big) + \big(B_g(\theta) - B_g(\widetilde{\theta})\big)F_g(\widetilde{\theta}). \tag{7}$$

Remark 2: If the fuzzy basis function of (2) is matched with that of (1), it holds that $\widetilde{\theta} = \theta$, which leads to $\Delta_1 = 0$ and $\Delta_2 = 0$. However, if (2) fails to share the premise variables of (1), the following mismatched components appear in the closed-loop system:

$$\Delta_1 = L_g(\widetilde{\theta})C_g(\nu), \quad \Delta_2 = A_g(\nu) + B_g(\nu)F_g(\widetilde{\theta})$$

where $\nu = \mathbf{col}(\nu_1, \nu_2, \cdots, \nu_r)$ and $\nu_i = \theta_i - \widetilde{\theta}_i$ (i.e., $\theta_i = \widetilde{\theta}_i + \nu_i$) denotes the error of the ith mismatched fuzzy basis function such that $\sum_{i=1}^{r} \nu_i = 0$ holds. Furthermore, the appearance of Δ_1 and Δ_2 causes a nonconvex problem that prevents the stabilization condition from being expressed in the form of linear matrix inequalities (LMIs). Thus, when dealing with the problem of observer-based control for T-S fuzzy systems, we need to pay more attention to handling the nonconvex problem as well as performing a congruent transformation that can separate the control gain and the observer gain in the stabilization condition.

Before going ahead, this paper presents the following definitions that will be used to develop our main results in a stochastic setting.

Definition 1 ([1], [2]): For $w(k) \equiv 0$, system (3) is said to be stochastically stable if for any initial condition $\zeta(0) = \mathbf{col}(\widetilde{x}(0), e(0))$ and $\phi(0)$, the following inequality holds:

$$\mathbf{E}\left\{ \sum_{k=0}^{\infty} ||\zeta(k)||^2 \,\Big|\, \zeta(0), \phi(0) \right\} < \infty. \tag{8}$$

Definition 2 ([3], [4]): Let us consider a quadratic energy supply rate of the following form:

$$\begin{aligned} &\mathcal{W}(z(k), w(k)) \\ &= \begin{bmatrix} z(k) \\ w(k) \end{bmatrix}^T \begin{bmatrix} \mathcal{Q} & \mathcal{S} \\ (*) & \mathcal{R} \end{bmatrix} \begin{bmatrix} z(k) \\ w(k) \end{bmatrix} \\ &= \begin{bmatrix} \zeta(k) \\ w(k) \end{bmatrix}^T \begin{bmatrix} \mathbf{G}_g^T(\theta, \widetilde{\theta})\mathcal{Q}\mathbf{G}_g(\theta, \widetilde{\theta}) & (*) \\ \mathcal{S}^T\mathbf{G}_g(\theta, \widetilde{\theta}) & \mathcal{R} \end{bmatrix} \begin{bmatrix} \zeta(k) \\ w(k) \end{bmatrix} \end{aligned} \tag{9}$$

where $\mathcal{Q} = \mathcal{Q}^T < 0$ (i.e., $-\mathcal{Q} = \mathcal{Q}_1^T\mathcal{Q}_1 \in \mathbb{R}^{n_z \times n_z}$), $\mathcal{S} \in \mathbb{R}^{n_z \times n_w}$ and $\mathcal{R} = \mathcal{R}^T \in \mathbb{R}^{n_w \times n_w}$ are given real matrices. Then, for $x(0) \equiv 0$, system (3) is said to be strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-$\beta$-dissipative if for $\beta > 0$ and $T > 0$, the following energy supply function $\mathcal{J}_T$ satisfies

$$\mathcal{J}_T = \sum_{k=0}^{T} \mathbf{E}\Big\{\mathcal{W}(z(k), w(k))\Big\} > \beta \sum_{k=0}^{T} ||w(k)||^2 \quad (10)$$

where β indicates the dissipativity performance level.

Based on Definitions 1 and 2, this paper aims to design (2) such that closed-loop system (3) is stochastically stable for $w(k) \equiv 0$ and is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-$\beta$-dissipative for $x(0) \equiv 0$.

Throughout this paper, the following lemmas will be used.

Lemma 3 ([5], [6]): For any symmetric matrix $\mathbf{M}_{ij}$, the following condition holds:

$$0 > \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i\theta_j \mathbf{M}_{ij}$$

if it is satisfied that

$$0 > \mathbf{M}_{ii}, \ \forall i \in \mathbb{N}_\theta \quad (11)$$

$$0 > \frac{1}{r-1}\mathbf{M}_{ii} + \frac{1}{2}\Big(\mathbf{M}_{ij} + \mathbf{M}_{ji}\Big), \ \forall i, j (\neq i) \in \mathbb{N}_\theta. \quad (12)$$

Lemma 4 [7]: For any matrices $\Phi \in \mathbb{R}^{n\times n}$ and $0 < P = P^T \in \mathbb{R}^{n\times n}$, it holds that $-\Phi P \Phi^T \leq -\mathbf{He}\{\Phi\} + P^{-1}$.

Remark 3: As is well known, semi-Markovian jump systems [27] can cover a broad range of stochastic hybrid systems by overcoming the limits of homogeneous Markov process with the aid of sojourn-time-dependent transition rates. However, to better demonstrate the relaxation technique for the error of mismatched fuzzy basis functions, this paper excludes the setting to include the time variability of the transition rates in MJFSs, which is a drawback of this paper. Instead, based on [25] and [28], the proposed method can be extended to the problem of observer-based control for semi-Markovian jump fuzzy systems.

III. CONTROL SYNTHESIS

Let us choose the following Lyapunov function:

$$V(k) = V(\zeta(k), \phi(k) = g) = \zeta^T(k) P_g(\theta)\zeta(k) \quad (13)$$

where $P_g(\theta) = P_g^T(\theta) > 0$. Then, from (3), we can obtain

$$\begin{aligned} &\mathbf{E}\{\Delta V(k)\} \\ &= \mathbf{E}\Big\{V(\zeta(k+1), \phi(k+1) = h \mid \phi(k) = g)\Big\} \\ &\quad - V(\zeta(k), \phi(k) = g) \\ &= \bar{\zeta}^T(k)\begin{bmatrix} \mathbf{A}_g^T(\theta,\widetilde{\theta}) \\ \mathbf{E}_g^T(\theta,\widetilde{\theta}) \end{bmatrix} \mathbf{P}_g(\theta^+)\begin{bmatrix} \mathbf{A}_g(\theta,\widetilde{\theta}) & \mathbf{E}_g(\theta,\widetilde{\theta}) \end{bmatrix}\bar{\zeta}(k) \\ &\quad -\zeta^T(k)P_g(\theta)\zeta(k) \end{aligned} \quad (14)$$

where $\theta^+ = \theta(\eta(k+1))$, $\bar{\zeta}(k) = \mathbf{col}(\zeta(k), w(k))$, and

$$\mathbf{P}_g(\theta^+) = \sum_{h=1}^{s} \pi_{gh}(k) P_h(\theta^+).$$

The following lemma presents the stochastic stability and strict $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-$\gamma$-dissipativity condition of (3).

Lemma 5: The closed-loop system (3) is stochastically stable and strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-$\beta$-dissipative if the following condition holds:

$$0 > \mathbf{E}\Big\{\Delta V(k)\Big\} + \beta||w(k)||_2^2 - \mathbf{E}\Big\{\mathcal{W}(z(k), w(k))\Big\}. \quad (15)$$

Proof: To begin with, let us consider the case of $w(k) \equiv 0$. Then, by $\mathcal{Q} < 0$, condition (15) reduces to

$$0 > \mathbf{E}\Big\{\Delta V(k)\Big\} - \mathbf{E}\{z^T(k)\mathcal{Q}z(k)\} > \mathbf{E}\Big\{\Delta V(k)\Big\}. \quad (16)$$

Accordingly, we can establish a sufficiently small scalar $\epsilon > 0$ such that $\mathbf{E}\{\Delta V(k)\} \leq -\epsilon||\zeta(k)||^2$. Hence, for $w(k) \equiv 0$, condition (15) ensures

$$\mathbf{E}\{V(T+1)\} - V(0) \leq -\epsilon \mathbf{E}\left\{\sum_{k=0}^{T} ||\zeta(k)||^2 \mid \zeta(0), \phi(0)\right\}$$

that is, $\mathbf{E}\left\{\sum_{k=0}^{T} ||\zeta(k)||_2^2 \mid \zeta(0), \phi(0)\right\} \leq \frac{1}{\epsilon}V(0) < \infty$, which means the closed-loop system (3) is stochastically stable in the absence of disturbances (see Definition 1). Next, let us consider the case where $w(k) \neq 0$ and $\zeta(0) \equiv 0$ (i.e., $V(0) \equiv 0$). Then, from (15), it follows that

$$\begin{aligned} 0 &> \mathbf{E}\Big\{V(T+1)\Big\} + \beta\sum_{k=0}^{T} ||w(k)||^2 - \mathcal{J}_T \\ &> \beta \sum_{k=0}^{T} ||w(k)||^2 - \mathcal{J}_T \end{aligned} \quad (17)$$

which means the closed-loop system (3) is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-$\gamma$-dissipative (see Definition 2). ■

The following lemma presents the stochastic dissipativity-based stabilization conditions for (3), dependent on $\theta(\eta(k))$, $\theta(\widetilde{\eta}(k))$ and $\theta(\eta(k+1))$.

Lemma 6: For given $\beta > 0$ and $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$, suppose that there exist matrices $0 < \bar{P}_g(\theta) = \bar{P}_g^T(\theta) \in \mathbb{R}^{2n_x\times 2n_x}$, $0 < \bar{P}_h(\theta^+) = \bar{P}_h^T(\theta^+) \in \mathbb{R}^{2n_x\times 2n_x}$, $Z_{1g} \in \mathbb{R}^{n_x\times n_x}$, $Z_{2g} \in \mathbb{R}^{n_x\times n_x}$, $L_g(\widetilde{\theta}) \in \mathbb{R}^{n_x\times n_y}$ and $\bar{F}_g(\widetilde{\theta}) \in \mathbb{R}^{n_u\times n_x}$ such that the following conditions hold for $g \in \mathbb{N}_\phi$:

$$0 > \left[\begin{array}{cc|c|c} -\bar{P}_g(\theta) & (*) & (*) & (*) \\ -\mathcal{S}^T\bar{\mathbf{G}}_g(\theta,\widetilde{\theta}) & \beta I - \mathcal{R} & 0 & (*) \\ \hline \mathcal{Q}_1\bar{\mathbf{G}}_g(\theta,\widetilde{\theta}) & 0 & -I & 0 \\ \hline \bar{\mathbf{A}}_g(\theta,\widetilde{\theta}) & \bar{\mathbf{E}}_g(\theta,\widetilde{\theta}) & 0 & -\mathbf{He}\left\{\begin{bmatrix} Z_{1g} & 0 \\ 0 & Z_{2g}\end{bmatrix}\right\} + \bar{\mathbf{P}}_g(\theta^+) \end{array}\right] \quad (18)$$

where

$$\bar{\mathbf{A}}_g(\theta,\widetilde{\theta}) = \left[\begin{array}{c|c} A_g(\widetilde{\theta})Z_{1g} + B_g(\widetilde{\theta})\bar{F}_g(\widetilde{\theta}) + \Delta_1 Z_{1g} & \\ \hline Z_{2g}(\Delta_2 - \Delta_1)Z_{1g} & \end{array}\right.$$

$$\left.\begin{array}{|c}L_g(\widetilde{\theta})C_g(\theta)\\ \hline Z_{2g}A_g(\theta)-\bar{L}_g(\widetilde{\theta})C_g(\theta)\end{array}\right] \quad (19)$$

$$\bar{\mathbf{E}}_g(\theta,\widetilde{\theta})=\left[\begin{array}{c}L_g(\widetilde{\theta})D_g(\theta)\\ \hline Z_{2g}E_g(\theta)-\bar{L}_g(\widetilde{\theta})D_g(\theta)\end{array}\right] \quad (20)$$

$$\bar{\mathbf{G}}_g(\theta,\widetilde{\theta})=\left[\,G_g(\theta)Z_{1g}+H_g(\theta)\bar{F}_g(\widetilde{\theta})\,\middle|\,G_g(\theta)\,\right] \quad (21)$$

$$\bar{\mathbf{P}}_g(\theta^+)=\sum_{h=1}^{s}\pi_{gh}(t)\bar{P}_h(\theta^+) \quad (22)$$

in which

$$\bar{F}_g(\widetilde{\theta})=F_g(\widetilde{\theta})Z_{1g},\quad \bar{L}_g(\widetilde{\theta})=Z_{2g}L_g(\widetilde{\theta}).$$

Then, closed-loop system (3) is stochastically stable and strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-$\beta$-dissipative.

Proof: Let us recall (9) as follows:

$$\begin{aligned}&\mathcal{W}(z(k),w(k))\\&\quad=\bar{\zeta}^T(k)\begin{bmatrix}-\mathbf{G}_g^T(\theta,\widetilde{\theta})\mathcal{Q}_1^T\mathcal{Q}_1\mathbf{G}_g(\theta,\widetilde{\theta}) & (*)\\ \mathcal{S}^T\mathbf{G}_g(\theta,\widetilde{\theta}) & \mathcal{R}\end{bmatrix}\bar{\zeta}(k).\end{aligned}$$

Then, from (14), it follows that

$$\begin{aligned}\mathbf{E}\{\Delta V(k)\}+\beta w^T(k)w(k)-\mathbf{E}\{\mathcal{W}(z(k),w(k))\}\\=\bar{\zeta}^T(k)\Psi\bar{\zeta}(k)\end{aligned} \quad (23)$$

where

$$\begin{aligned}\Psi=&\begin{bmatrix}\mathbf{A}_g^T(\theta,\widetilde{\theta})\\ \mathbf{E}_g^T(\theta,\widetilde{\theta})\end{bmatrix}\mathbf{P}_g(\theta^+)\begin{bmatrix}\mathbf{A}_g(\theta,\widetilde{\theta}) & \mathbf{E}_g(\theta,\widetilde{\theta})\end{bmatrix}\\&+\begin{bmatrix}\mathbf{G}_g(\theta,\widetilde{\theta})^T\mathcal{Q}_1^T\\ 0\end{bmatrix}\begin{bmatrix}\mathcal{Q}_1\mathbf{G}_g(\theta,\widetilde{\theta}) & 0\end{bmatrix}\\&+\begin{bmatrix}-P_g(\theta) & (*)\\ -\mathcal{S}^T\mathbf{G}_g(\theta,\widetilde{\theta}) & \beta I-\mathcal{R}\end{bmatrix}.\end{aligned} \quad (24)$$

As a result, the condition $\Psi<0$ guarantees (15) in Lemma 5, and be transformed by the Schur complement into

$$0>\left[\begin{array}{cc|c|c}-P_g(\theta) & (*) & (*) & (*)\\ -\mathcal{S}^T\mathbf{G}_g(\theta,\widetilde{\theta}) & \beta I-\mathcal{R} & 0 & (*)\\ \hline \mathcal{Q}_1\mathbf{G}_g(\theta,\widetilde{\theta}) & 0 & -I & 0\\ \hline \mathbf{A}_g(\theta,\widetilde{\theta}) & \mathbf{E}_g(\theta,\widetilde{\theta}) & 0 & -\mathbf{P}_g^{-1}(\theta^+)\end{array}\right]. \quad (25)$$

Subsequently, by noting that (18) implies $\mathbf{diag}(\mathbf{He}\{Z_{1g}\},\mathbf{He}\{Z_{2g}\})>0$, we can establish the following nonsingular matrices:

$$\Phi_1=\mathbf{diag}(Z_{1g},I),\quad \Phi_2=\mathbf{diag}(I,Z_{2g}).$$

Then, based on (19), (20) and (21), it is available that

$$\mathbf{A}_g(\theta,\widetilde{\theta})=\Phi_2^{-1}\bar{\mathbf{A}}_g(\theta,\widetilde{\theta})\Phi_1^{-1},\ \mathbf{E}_g(\theta,\widetilde{\theta})=\Phi_2^{-1}\bar{\mathbf{E}}_g(\theta,\widetilde{\theta})$$
$$\mathbf{G}_g(\theta,\widetilde{\theta})=\bar{\mathbf{G}}_g(\theta,\widetilde{\theta})\Phi_1^{-1}.$$

Further, decomposing $P_g(\theta)$ and $\mathbf{P}_g(\theta^+)$ as follows:

$$P_g(\theta)=\Phi_1^{-T}\bar{P}_g(\theta)\Phi_1^{-1}$$
$$\mathbf{P}_g(\theta^+)=\sum_{h=1}^{s}\pi_{gh}(t)\Phi_1^{-T}\bar{P}_h(\theta^+)\Phi_1^{-1}=\Phi_1^{-T}\bar{\mathbf{P}}_g(\theta^+)\Phi_1^{-1}$$

condition (25) can be represented as

$$0>\left[\begin{array}{cc|c|c}-\Phi_1^{-T}\bar{P}_g(\theta)\Phi_1^{-1} & (*) & (*) & (*)\\ -\mathcal{S}^T\bar{\mathbf{G}}_g(\theta,\widetilde{\theta})\Phi_1^{-1} & \beta I-\mathcal{R} & 0 & (*)\\ \hline \mathcal{Q}_1\bar{\mathbf{G}}_g(\theta,\widetilde{\theta})\Phi_1^{-1} & 0 & -I & 0\\ \hline \Phi_2^{-1}\bar{\mathbf{A}}_g(\theta,\widetilde{\theta})\Phi_1^{-1} & \Phi_2^{-1}\bar{\mathbf{E}}_g(\theta,\widetilde{\theta}) & 0 & -\Phi_1\bar{\mathbf{P}}_g^{-1}(\theta^+)\Phi_1^T\end{array}\right]. \quad (26)$$

Moreover, pre- and post-multiplying (26) by $\mathbf{diag}(\Phi_1,I,I,\Phi_2)$ and its transpose leads to

$$0>\left[\begin{array}{cc|c|c}-\bar{P}_g(\theta) & (*) & (*) & (*)\\ -\mathcal{S}^T\bar{\mathbf{G}}_g(\theta,\widetilde{\theta}) & \beta I-\mathcal{R} & 0 & (*)\\ \hline \mathcal{Q}_1\bar{\mathbf{G}}_g(\theta,\widetilde{\theta}) & 0 & -I & 0\\ \hline \bar{\mathbf{A}}_g(\theta,\widetilde{\theta}) & \bar{\mathbf{E}}_g(\theta,\widetilde{\theta}) & 0 & -\Phi_2\Phi_1\bar{\mathbf{P}}_g^{-1}(\theta^+)\Phi_1^T\Phi_2^T\end{array}\right]. \quad (27)\ (28)$$

Therefore, since it follows from Lemma 4 that

$$-\Phi_2\Phi_1\bar{\mathbf{P}}_g^{-1}(\theta^+)\Phi_1^T\Phi_2^T\leq-\mathbf{He}\{\Phi_2\Phi_1\}+\bar{\mathbf{P}}_g(\theta^+),$$

condition (28) is guaranteed by (18). ■

In what follows, by partitioning $\bar{P}_g(\theta)\in\mathbb{R}^{2n_x\times 2n_x}$ and $\bar{\mathbf{P}}_g(\theta^+)\in\mathbb{R}^{2n_x\times 2n_x}$ into block matrices as follows:

$$\bar{P}_g(\theta)=\begin{bmatrix}\bar{P}_g^{(1)}(\theta) & (*)\\ \bar{P}_g^{(2)}(\theta) & \bar{P}_g^{(3)}(\theta)\end{bmatrix}$$
$$\bar{\mathbf{P}}_g(\theta^+)=\begin{bmatrix}\bar{\mathbf{P}}_g^{(1)}(\theta^+) & (*)\\ \bar{\mathbf{P}}_g^{(2)}(\theta^+) & \bar{\mathbf{P}}_g^{(3)}(\theta^+)\end{bmatrix}$$

condition (18) can be rearranged as follows:

$$0>\left[\begin{array}{ccc|c|cc}-\bar{P}_g^{(1)}(\theta) & (*) & (*) & (*) & (*) & (*)\\ -\bar{P}_g^{(2)}(\theta) & -\bar{P}_g^{(3)}(\theta) & (*) & (*) & (*) & (*)\\ \mathbf{(3,1)} & -\mathcal{S}^TG_g(\theta) & \beta I-\mathcal{R} & 0 & (*) & (*)\\ \hline \mathbf{(4,1)} & \mathcal{Q}_1G_g(\theta) & 0 & -I & 0 & 0\\ \hline \mathbf{(5,1)} & \mathbf{(5,2)} & \mathbf{(5,3)} & 0 & \begin{pmatrix}-\mathbf{He}\{Z_{1g}\}\\+\bar{\mathbf{P}}_g^{(1)}(\theta^+)\end{pmatrix} & (*)\\ \mathbf{(6,1)} & \mathbf{(6,2)} & \mathbf{(6,3)} & 0 & \bar{\mathbf{P}}_g^{(2)}(\theta^+) & \begin{pmatrix}-\mathbf{He}\{Z_{2g}\}\\+\bar{\mathbf{P}}_g^{(3)}(\theta^+)\end{pmatrix}\end{array}\right]. \quad (29)$$

where

$$\mathbf{(3,1)}=-\mathcal{S}^TG_g(\theta)Z_{1g}-\mathcal{S}^TH_g(\theta)\bar{F}_g(\widetilde{\theta})$$

$$\begin{aligned}
(\mathbf{4},\mathbf{1}) &= \mathcal{Q}_1\, G_g(\theta)Z_{1g} + \mathcal{Q}_1\, H_g(\theta)\bar{F}_g(\widetilde{\theta})\\
(\mathbf{5},\mathbf{1}) &= A_g(\widetilde{\theta})Z_{1g} + B_g(\widetilde{\theta})\bar{F}_g(\widetilde{\theta}) + L_g(\widetilde{\theta})C_g(\nu)Z_{1g}\\
(\mathbf{5},\mathbf{2}) &= L_g(\widetilde{\theta})C_g(\theta),\ (\mathbf{5},\mathbf{3}) = L_g(\widetilde{\theta})D_g(\theta)\\
(\mathbf{6},\mathbf{1}) &= Z_{2g}A_g(\nu)Z_{1g} + Z_{2g}B_g(\nu)\bar{F}_g(\widetilde{\theta})\\
&\quad -\bar{L}_g(\widetilde{\theta})C_g(\nu)Z_{1g}\\
(\mathbf{6},\mathbf{2}) &= Z_{2g}A_g(\theta) - \bar{L}_g(\widetilde{\theta})C_g(\theta)\\
(\mathbf{6},\mathbf{3}) &= Z_{2g}E_g(\theta) - \bar{L}_g(\widetilde{\theta})D_g(\theta).
\end{aligned}$$

However, condition (29) is still formulated in terms of non-convex matrix inequalities with θ, $\widetilde{\theta}$ and θ^+. Thus, this paper will present a two-step approach to address the non-convex problem based on the fact that (29) implies

$$0 > \left[\begin{array}{cc|c|c} -\bar{P}_g^{(3)}(\theta) & (*) & (*) & (*)\\ -\mathcal{S}^T G_g(\theta) & \beta I - \mathcal{R} & 0 & (*)\\ \hline \mathcal{Q}_1 G_g(\theta) & 0 & -I & 0\\ \hline \begin{pmatrix} Z_{2g}A_g(\theta)\\ -\bar{L}_g(\widetilde{\theta})C_g(\theta)\end{pmatrix} & \begin{pmatrix} Z_{2g}E_g(\theta)\\ -\bar{L}_g(\widetilde{\theta})D_g(\theta)\end{pmatrix} & 0 & -\mathbf{He}\{Z_{2g}\} + \bar{\mathbf{P}}_g^{(3)}(\theta^+) \end{array}\right]. \quad (30)$$

In other words, using (29) and (30), the observer-based control (2) will be designed according to the following steps:

(S1) For a prescribed $\beta = \beta_0 > 0$, solve (30) and obtain $\bar{L}_g(\widetilde{\theta})$ and Z_{2g}. Then, reconstruct the fuzzy observer gain $L_g(\widetilde{\theta}) = Z_{2g}^{-1}\bar{L}_g(\widetilde{\theta})$.

(S2) Substitute Z_{2g} and $L_g(\widetilde{\theta})$ (or $\bar{L}_g(\widetilde{\theta})$) into (29). Then, solve (29) and obtain $\bar{F}_g(\widetilde{\theta})$ and Z_{1g}. After that, reconstruct the fuzzy control gain $F_g(\widetilde{\theta}) = \bar{F}_g(\widetilde{\theta})Z_{1g}^{-1}$.

Remark 4: Indeed, various approaches have been used to deal with the noncovex terms that occur in observer-based control design conditions. However, the reason for choosing the two-step approach in this paper is to avoid the complicated iteration process and solve the stabilization conditions as quickly as possible. Furthermore, as the number of variables used to obtain less conservative stabilization conditions increases, it is more advantageous to use this approach because other iteration approaches can take significant time.

IV. RELAXED STABILIZATION CONDITIONS

Before dealing with the relaxation problem of (29) and (30), let us first consider the following condition:

$$0 > \sum_{i=1}^{r}\sum_{j=1}^{r}\widetilde{\theta}_i\widetilde{\theta}_j\Gamma_{ij} + \sum_{i=1}^{r}\theta_i\mathbf{He}\left\{\Gamma_i^{(1)}\Omega\right\} \quad (31)$$

$$+\sum_{i=1}^{r}\sum_{j=1}^{r}\widetilde{\theta}_i\theta_j\mathbf{He}\left\{\Gamma_{ij}^{(2)}\Omega\right\} + \sum_{i=1}^{r}\sum_{\ell=1}^{r}\widetilde{\theta}_i\nu_\ell\mathbf{He}\left\{\Gamma_{i\ell}^{(3)}\Omega\right\} \quad (32)$$

subject to the following error constraints:

$$|\nu_\ell| \le \alpha_\ell < 1,\ \forall \ell \in \mathbb{N}_\theta \quad (33)$$

where $\Gamma_i = \Gamma_i^T \in \mathbb{R}^{p\times p}$, $\Gamma_i^{(1)}$, $\Gamma_{ij}^{(2)}$, $\Gamma_{i\ell}^{(3)} \in \mathbb{R}^{p\times m}$, and $\Omega \in \mathbb{R}^{m\times p}$ $(m < p)$ with full rank.

The following lemma proposes a relaxed condition of (32) subject to (33).

Lemma 7: Condition (32) subject to (33) holds if it is satisfied that

$$0 > \mathbf{M}_{ii},\ \forall i \in \mathbb{N}_\theta \quad (34)$$

$$0 > \frac{1}{r-1}\mathbf{M}_{ij} + \frac{1}{2}(\mathbf{M}_{ij} + \mathbf{M}_{ji}),\ \forall i, j(\neq i) \in \mathbb{N}_\theta \quad (35)$$

where $U_{ij} = U_{ij}^T \in \mathbb{R}^{m\times m}$, $W_i \in \mathbb{R}^{p\times m}$, and

$$\mathbf{M}_{ij} = \left[\begin{array}{c|c} [U_{i\ell}]^{\mathbf{d}}_{\ell\in\mathbb{N}_\theta} & (*)\\ \hline [\Gamma_\ell^{(1)} + \Gamma_{i\ell}^{(2)} + \Gamma_{i\ell}^{(3)} + W_i]_{\ell\in\mathbb{R}_\theta} & \left(\begin{array}{l}\Gamma_{ij} + \mathbf{He}\left\{\Gamma_i^{(1)}\Omega + \Gamma_{ij}^{(2)}\Omega\right\}\\ \quad + \Omega^T\left(-\sum_{\ell=1}^{r}\alpha_\ell^2 U_{i\ell}\right)\Omega\end{array}\right)\end{array}\right].$$

Proof: Based on $\theta_i = \widetilde{\theta}_i + \nu_i$ (see Remark 2), condition (32) can be rearranged as follows:

$$0 > \sum_{i=1}^{r}\sum_{j=1}^{r}\widetilde{\theta}_i\widetilde{\theta}_j\left(\Gamma_{ij} + \mathbf{He}\left\{\Gamma_i^{(1)}\Omega + \Gamma_{ij}^{(2)}\Omega\right\}\right) + \sum_{\ell=1}^{r}\nu_\ell\mathbf{He}\{\mathbf{T}_\ell\Omega\} \quad (36)$$

where

$$\mathbf{T}_\ell = \Gamma_\ell^{(1)} + \sum_{i=1}^{r}\widetilde{\theta}_i\Gamma_{i\ell}^{(2)} + \sum_{i=1}^{r}\widetilde{\theta}_i\Gamma_{i\ell}^{(3)} \in \mathbb{R}^{p\times m}.$$

Further, using

$$\sum_{\ell=1}^{r}\nu_\ell\mathbf{He}\{\mathbf{T}_\ell\Omega\} = \mathbf{He}\left\{[\mathbf{T}_\ell]_{\ell\in\mathbb{N}_\theta}\cdot(\nu\otimes\Omega)\right\}$$

condition (36) can be converted into

$$0 > \left[\begin{array}{c}\nu\otimes\Omega\\ \hline I\end{array}\right]^T\left[\begin{array}{c|c} 0 & (*)\\ \hline [\mathbf{T}_\ell]_{\ell\in\mathbb{N}_\theta} & (\mathbf{2},\mathbf{2})\end{array}\right]\left[\begin{array}{c}\nu\otimes\Omega\\ \hline I\end{array}\right] \quad (37)$$

where

$$(\mathbf{2},\mathbf{2}) = \sum_{i=1}^{r}\sum_{j=1}^{r}\widetilde{\theta}_i\widetilde{\theta}_j\left(\Gamma_{ij} + \mathbf{He}\left\{\Gamma_i^{(1)}\Omega + \Gamma_{ij}^{(2)}\Omega\right\}\right).$$

Meanwhile, from $\sum_{\ell=1}^{r}\nu_\ell = 0$, it follows that

$$0 = \sum_{\ell=1}^{r}\nu_\ell\mathbf{He}\{\mathbf{W}\Omega\} = \mathbf{He}\left\{[\mathbf{W}]_{\ell\in\mathbb{N}_\theta}\cdot(\nu\otimes\Omega)\right\} \quad (38)$$

where $\mathbf{W} = \sum_{i=1}^{r}\widetilde{\theta}_iW_i \in \mathbb{R}^{p\times m}$ and $W_i \in \mathbb{R}^{p\times m}$. In addition, since (34) implies $0 > U_{i\ell} = U_{i\ell}^T \in \mathbb{R}^{m\times m}$, it follows from (33) that

$$0 \leq \sum_{\ell=1}^{r}(\nu_\ell^2-\alpha_\ell^2)\Omega^T\mathbf{U}_\ell\Omega$$
$$= \begin{bmatrix} \nu\otimes\Omega \\ \hline I \end{bmatrix}^T \left[\begin{array}{c|c} [\mathbf{U}_\ell]^{\mathbf{d}}_{\ell\in\mathbb{N}_\theta} & 0 \\ \hline 0 & \Omega^T\left(-\sum_{\ell=1}^{r}\alpha_\ell^2\mathbf{U}_\ell\right)\Omega \end{array}\right] \times \begin{bmatrix} \nu\otimes\Omega \\ \hline I \end{bmatrix} \tag{39}$$

where $\mathbf{U}_\ell = \sum_{i=1}^{r}\widetilde{\theta}_i U_{i\ell} \in \mathbb{R}^{m\times m}$. As a result, according to the S-procedure, combining (37) with (38) and (39) produces

$$0 > \begin{bmatrix} \nu\otimes\Omega \\ \hline I \end{bmatrix}^T \left[\begin{array}{c|c} [\mathbf{U}_\ell]^{\mathbf{d}}_{\ell\in\mathbb{N}_\theta} & (*) \\ \hline [\mathbf{T}_\ell+\mathbf{W}]_{\ell\in\mathbb{N}_\theta} & (\mathbf{2},\mathbf{2}) \end{array}\right] \begin{bmatrix} \nu\otimes\Omega \\ \hline I \end{bmatrix} \tag{40}$$

where

$$(\mathbf{2},\mathbf{2}) = \sum_{i=1}^{r}\sum_{j=1}^{r}\widetilde{\theta}_i\widetilde{\theta}_j\Big(\Gamma_{ij}+\mathbf{He}\left\{\Gamma_i^{(1)}\Omega+\Gamma_{ij}^{(2)}\Omega\right\}\Big) + \Omega^T\left(-\sum_{\ell=1}^{r}\alpha_\ell^2\mathbf{U}_\ell\right)\Omega.$$

Therefore, condition (40) is guaranteed by

$$0 > \sum_{i=1}^{r}\sum_{j=1}^{r}\widetilde{\theta}_i\widetilde{\theta}_j\mathbf{M}_{ij} \tag{41}$$

and the relaxed condition of (41) is given as (34) and (35) according to Lemma 3. ■

To obtain a finite number of LMIs for (29) and (30) in **(S1)** and **(S2)**, this paper establishes the fuzzy-basis-dependent variables as follows:

$$\bar{P}_g^{(1)}(\theta) = \sum_{i=1}^{r}\theta_i\bar{P}_{gi}^{(1)},\ \bar{P}_g^{(2)}(\theta) = \sum_{i=1}^{r}\theta_i\bar{P}_{gi}^{(2)}$$
$$\bar{P}_g^{(3)}(\theta) = \sum_{i=1}^{r}\theta_i\bar{P}_{gi}^{(3)} \tag{42}$$
$$\bar{F}_g(\widetilde{\theta}) = \sum_{i=1}^{r}\widetilde{\theta}_i\bar{F}_{gi},\ \bar{L}_g(\widetilde{\theta}) = \sum_{i=1}^{r}\widetilde{\theta}_i\bar{L}_{gi}. \tag{43}$$

The following theorem provides the relaxed condition of (30), formulated in terms of LMIs.

Theorem 8: For a prescribed $\beta_0 > 0$, suppose that there exist $0 < \bar{P}_{gi}^{(3)} = \bar{P}_{gi}^{(3)T} \in \mathbb{R}^{n_x\times n_x}$, $Z_{2g} \in \mathbb{R}^{n_x\times n_x}$, $\bar{L}_{gi} \in \mathbb{R}^{n_x\times n_y}$, $W_i^{(1)} \in \mathbb{R}^{n_x\times n_x}$, $U_{g\ell i}^{(1)} = U_{g\ell i}^{(1)T} \in \mathbb{R}^{n_x\times n_x}$, $U_{g\ell i}^{(2)} \in \mathbb{R}^{n_w\times n_x}$ and $U_{g\ell i}^{(3)} = U_{g\ell i}^{(3)T} \in \mathbb{R}^{n_w\times n_w}$ such that the following conditions hold for $g \in \mathbb{N}_\phi$ and $s \in \mathbb{N}_\theta$:

$$0 > \mathbf{M}_{giis},\ \forall i \in \mathbb{N}_\theta \tag{44}$$
$$0 > \frac{1}{r-1}\mathbf{M}_{giis} + \frac{1}{2}\Big(\mathbf{M}_{gijs}+\mathbf{M}_{gjis}\Big),\ \forall i,j(\neq i) \in \mathbb{N}_\theta \tag{45}$$

where

$$\mathbf{M}_{gijs} = \left[\begin{array}{c|c} [U_{i\ell}]^{\mathbf{d}}_{\ell\in\mathbb{N}_\theta} & (*) \\ \hline [\Gamma_{g\ell}^{(1)}+\Gamma_{gi\ell}^{(2)}+W_i]_{\ell\in\mathbb{R}_\theta} & \begin{pmatrix} \Gamma_{gs}+\mathbf{He}\left\{\Gamma_{gi}^{(1)}\Omega+\Gamma_{gij}^{(2)}\Omega\right\} \\ +\Omega^T\left(-\sum_{\ell=1}^{r}\alpha_\ell^2 U_{i\ell}\right)\Omega \end{pmatrix} \end{array}\right]$$

in which

$$\Gamma_{gs} = \left[\begin{array}{cc|c|c} 0 & 0 & 0 & 0 \\ 0 & \beta_0 I-\mathcal{R} & 0 & 0 \\ \hline 0 & 0 & -I & 0 \\ \hline 0 & 0 & 0 & \begin{pmatrix} -\mathbf{He}\{Z_{2g}\} \\ +\sum_{h=1}^{n_\phi}\pi_{gh}\bar{P}_{hs}^{(3)} \end{pmatrix} \end{array}\right]$$

$$W_i = \begin{bmatrix} W_i^{(1)} & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ \hline 0 & 0 \end{bmatrix},\quad \Gamma_{gi}^{(1)} = \begin{bmatrix} -\frac{1}{2}\bar{P}_{gi}^{(3)} & 0 \\ -\mathcal{S}^T G_{gi} & 0 \\ \hline \mathcal{Q}_1 G_{gi} & 0 \\ \hline Z_{2g}A_{gi} & Z_{2g}E_{gi} \end{bmatrix}$$

$$\Gamma_{gij}^{(2)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ \hline -\bar{L}_{gi}C_{gj} & -\bar{L}_{gi}D_{gj} \end{bmatrix}$$

$$\Omega = \left[\begin{array}{cc|c|c} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{array}\right],\quad U_{i\ell} = \begin{bmatrix} U_{i\ell}^{(1)} & U_{i\ell}^{(2)T} \\ U_{i\ell}^{(2)} & U_{i\ell}^{(3)} \end{bmatrix}.$$

Then, for $g \in \mathbb{N}_\phi$, the fuzzy observer gain is given as $L_g(\widetilde{\theta}) = \sum_{i=1}^{r}\widetilde{\theta}_i L_{gi}$, where $L_{gi} = Z_{2g}^{-1}\bar{L}_{gi}$.

Proof: Using (42)–(43) and noting

$$\bar{\mathbf{P}}_g^{(3)}(\theta^+) = \sum_{s=1}^{r}\theta_s^+\sum_{h=1}^{n_\phi}\pi_{gh}\bar{P}_{hs}^{(3)}$$

condition (30) can be represented as

$$0 > \sum_{s=1}^{r}\widetilde{\theta}_s^+\Gamma_{gs} + \sum_{i=1}^{r}\theta_i\mathbf{He}\left\{\Gamma_{gi}^{(1)}\Omega\right\} + \sum_{i=1}^{r}\sum_{j=1}^{r}\widetilde{\theta}_i\theta_j\mathbf{He}\left\{\Gamma_{gij}^{(2)}\Omega\right\}. \tag{46}$$

Therefore, by Lemma 7, the relaxed condition of (46) is given as (44) and (45). ■

Based on Theorem 8, the following theorem provides the relaxed condition of (29), formulated in terms of LMIs.

Theorem 9: For given Z_{2g}, and L_{gi}, suppose that there exist

$$0 < \begin{bmatrix} \bar{P}_{gi}^{(1)} & (*) \\ \bar{P}_{gi}^{(2)} & \bar{P}_{gi}^{(3)} \end{bmatrix} = \begin{bmatrix} \bar{P}_{gi}^{(1)} & (*) \\ \bar{P}_{gi}^{(2)} & \bar{P}_{gi}^{(3)} \end{bmatrix}^T \in \mathbb{R}^{2n_x\times 2n_x}$$

$Z_{1gi} \in \mathbb{R}^{n_x\times n_x}$, $\bar{F}_{gi} \in \mathbb{R}^{n_u\times n_x}$, $W_i^{(1)} \in \mathbb{R}^{n_x\times n_x}$, $W_i^{(2)} \in \mathbb{R}^{n_x\times n_x}$, $W_i^{(3)} \in \mathbb{R}^{n_x\times n_x}$, $U_{i\ell}^{(1)} = U_{i\ell}^{(1)T} \in \mathbb{R}^{n_x\times n_x}$, $U_{i\ell}^{(2)} \in$

$\mathbb{R}^{n_x \times n_x}$, $U_{i\ell}^{(3)} = U_{i\ell}^{(3)T} \in \mathbb{R}^{n_x \times n_x}$, $U_{i\ell}^{(4)} \in \mathbb{R}^{n_w \times n_x}$, $U_{i\ell}^{(5)} \in \mathbb{R}^{n_w \times n_x}$, $U_{i\ell}^{(6)} = U_{i\ell}^{(6)T} \in \mathbb{R}^{n_w \times n_w}$, and $\beta > 0$ such that the following conditions hold for $g \in \mathbb{N}_\phi$ and $s \in \mathbb{N}_\theta$:

$$0 > \mathbf{M}_{giis}, \quad \forall i \in \mathbb{N}_\theta \tag{47}$$

$$0 > \frac{1}{r-1}\mathbf{M}_{giis} + \frac{1}{2}\big(\mathbf{M}_{gijs} + \mathbf{M}_{gjis}\big), \quad \forall i, j (\neq i) \in \mathbb{N}_\theta \tag{48}$$

where

$$\mathbf{M}_{gijs} = \left[\begin{array}{c|c} [U_{i\ell}]^{\mathbf{d}}_{\ell \in \mathbb{N}_\theta} & (*) \\ \hline [\Gamma_{g\ell}^{(1)} + \Gamma_{gi\ell}^{(2)} + \Gamma_{gi\ell}^{(3)} + W_i]_{\ell \in \mathbb{R}_\theta} & q \; \left(\begin{array}{c} \Gamma_{gijs} + \mathbf{He}\left\{\Gamma_{gi}^{(1)}\Omega + \Gamma_{gij}^{(2)}\Omega\right\} \\ + \Omega^T \left(-\sum_{\ell=1}^{r} \alpha_\ell^2 U_{i\ell}\right)\Omega \end{array}\right) \end{array}\right]$$

in which

$$\Gamma_{gijs} = \left[\begin{array}{ccc|c|cc} 0 & 0 & 0 & 0 & (*) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta I - \mathcal{R} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -I & 0 & 0 \\ \hline \begin{pmatrix} A_{gi}Z_{1g} \\ +B_{gi}\bar{F}_{gj} \end{pmatrix} & 0 & 0 & 0 & \begin{pmatrix} -\mathbf{He}\{Z_{1g}\} \\ +\sum_{h=1}^{n_\phi} \pi_{gh}\bar{P}_{hs}^{(1)} \end{pmatrix} & (*) \\ 0 & 0 & 0 & 0 & \sum_{h=1}^{n_\phi} \pi_{gh}\bar{P}_{hs}^{(2)} & \begin{pmatrix} -\mathbf{He}\{Z_{2g}\} \\ +\sum_{h=1}^{n_\phi} \pi_{gh}\bar{P}_{hs}^{(3)} \end{pmatrix} \end{array}\right]$$

$$\Gamma_{gi}^{(1)} = \left[\begin{array}{ccc} -\frac{1}{2}\bar{P}_{gi}^{(1)} & 0 & 0 \\ -\bar{P}_{gi}^{(2)} & -\frac{1}{2}\bar{P}_{gi}^{(3)} & 0 \\ -\mathcal{S}^T G_{gi}Z_{1g} & -\mathcal{S}^T G_{gi} & 0 \\ \hline \mathcal{Q}_1 G_{gi}Z_{1g} & \mathcal{Q}_1 G_{gi} & 0 \\ \hline 0 & 0 & 0 \\ 0 & Z_{2g}A_{gi} & Z_{2g}E_{gi} \end{array}\right]$$

$$U_{i\ell} = \begin{bmatrix} U_{i\ell}^{(1)} & (*) & (*) \\ U_{i\ell}^{(2)} & U_{i\ell}^{(3)} & (*) \\ U_{i\ell}^{(4)} & U_{i\ell}^{(5)} & U_{i\ell}^{(6)} \end{bmatrix}$$

$$\Gamma_{gij}^{(2)} = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\mathcal{S}^T H_{gj}\bar{F}_{gi} & 0 & 0 \\ \hline \mathcal{Q}_1 H_{gj}\bar{F}_{gi} & q0 & 0 \\ \hline 0 & L_{gi}C_{gj} & L_{gi}D_{gj} \\ 0 & -\bar{L}_{gi}C_{gj} & -\bar{L}_{gi}D_{gj} \end{array}\right]$$

$$W_i = \left[\begin{array}{ccc} W_i^{(1)} & W_i^{(2)} & 0 \\ W_i^{(3)} & W_i^{(4)} & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\Gamma_{gi\ell}^{(3)} = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline L_{gi}C_{g\ell}Z_{1g} & 0 & 0 \\ Z_{2g}A_{g\ell}Z_{1g} + Z_{2g}B_{g\ell}\bar{F}_{gi} - \bar{L}_{gi}C_{g\ell}Z_{1g} & 0 & 0 \end{array}\right]$$

$$\Omega = \left[\begin{array}{ccc|c|cc} I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \end{array}\right].$$

Then, closed-loop system (3) is stochastically stable and strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-$\beta$-dissipative, and the fuzzy control gain is given as $F_g(\tilde{\theta}) = \sum_{i=1}^{r} \tilde{\theta}_i \bar{F}_{gi} Z_{1g}^{-1}$.

Proof: Noting

$$\begin{bmatrix} \bar{\mathbf{P}}_g^{(1)}(\theta^+) & (*) \\ \bar{\mathbf{P}}_g^{(2)}(\theta^+) & \bar{\mathbf{P}}_g^{(3)}(\theta^+) \end{bmatrix} = \sum_{s=1}^{r} \theta_s^+ \sum_{h=1}^{n_\phi} \pi_{gh} \begin{bmatrix} \bar{P}_{hs}^{(1)} & (*) \\ \bar{P}_{hs}^{(1)} & \bar{P}_{hs}^{(2)} \end{bmatrix}$$

we can represent condition (29) as follows:

$$0 > \sum_{s=1}^{r}\sum_{i=1}^{r}\sum_{j=1}^{r} \widetilde{\theta}_s^+ \widetilde{\theta}_i \widetilde{\theta}_j \Gamma_{gijs} + \sum_{i=1}^{r} \theta_i \mathbf{He}\left\{\Gamma_{gi}^{(1)}\Omega\right\} + \sum_{i=1}^{r}\sum_{j=1}^{r} \widetilde{\theta}_i \theta_j \mathbf{He}\left\{\Gamma_{gij}^{(2)}\Omega\right\} + \sum_{i=1}^{r}\sum_{\ell=1}^{r} \widetilde{\theta}_i \nu_\ell \mathbf{He}\left\{\Gamma_{gi\ell}^{(3)}\Omega\right\}. \tag{49}$$

Therefore, by Lemma 7, the relaxed condition of (49) is given as (47) and (48). ■

Remark 5: In order to reduce the amount of computation generated from the relaxation variables used in Theorem 4.2, it is possible to exclude $U_{i\ell}^{(2)} \in \mathbb{R}^{n_x \times n_x}$, $U_{i\ell}^{(4)} \in \mathbb{R}^{n_w \times n_x}$ and $U_{i\ell}^{(5)} \in \mathbb{R}^{n_w \times n_x}$ from (47) and (48). Then, the number of scalar variables is reduced by $r^2(n_x^2 + n_x n_w)$, but the performance can decrease as the value of α_i in (33) increases (see Table 1).

Remark 6: Although the proposed method is based on the commonly used LMI approach, congruent transformation technique and relaxation process, this method has more contribution to answering how to use the LMI approach, how to define the congruent transformation matrix, and how to make the relaxation process less conservative. In particular, this method is uniquely developed to be suitable for dealing with the problem of designing observer-based control for a class of discrete-time Markovian jump fuzzy systems under error constrains of mismatched fuzzy basis functions.

V. ILLUSTRATIVE EXAMPLE

Let us consider the following single-link robot arm model with the system mode $g \in \mathbb{N}_\phi = \{1, 2, 3\}$, adopted in [6], [8], [9]:

$$\ddot{\vartheta}(t) = -\frac{M_g}{J_g}\bar{g}\ell\sin(\vartheta(t)) - \frac{R}{J_g}\dot{\vartheta}(t) + \frac{1}{J_g}u(t) + w(t) \tag{50}$$

where $\vartheta(t)$ and $\dot{\vartheta}(t)$ stand for the angular position and angular velocity of the arm, respectively; and $M_g \in \{0.75, 1.5, 2\}$, $J_g \in \{1, 2, 2.5\}$, $\ell = 0.5$, $\bar{g} = 9.81$ and $R = 2$ denote the payload mass, the inertia moment, the arm length, the gravity acceleration and the viscous friction coefficient, respectively.

Further, let us define $x_1(t) = \vartheta(t)$ and $x_2(t) = \dot{\vartheta}(t)$, and assume that only $\vartheta(t)$ is measurable. Then, using the first-order Euler approximation with the sampling time $T_s = 0.5$, the discrete-time state-space representation of (50) is given as

$$\begin{cases} x_1(k+1) = x_1(k) + T_s x_2(k) \\ x_2(k+1) = -\dfrac{T_s M_g}{J_g}\bar{g}\ell\sin(x_1(k)) \\ \qquad\qquad + \left(1 - \dfrac{T_s R}{J_g}\right) + \dfrac{T_s}{J_g}u(k) + T_s w(k). \end{cases}$$

Further, as in [29] and [30], the nonlinear function $\sin(x_1(k))$ can be represented as

$$\sin(x_1(k)) = \theta_1(x_1(k))x_1(k) + \eta\theta_2(x_1(k))x_1(k)$$

where $\eta = 0.01/\pi$, $\theta_1(x_1(k))$, $\theta_2(x_1(k)) \in [0, 1]$, and $\theta_1(x_1(k)) + \theta_2(x_1(k)) = 1$. Thus, the normalized fuzzy basis function is given as

$$\theta_1(x_1(k)) = \begin{cases} \dfrac{\sin(x_1(k)) - \eta x_1(k)}{(1-\eta)x_1(k)}, & x_1(k) \neq 0 \\ 1, & x_1(k) = 0 \end{cases}$$
$$\theta_2(x_1(k)) = 1 - \theta_1(x_1(k)).$$

Accordingly, the discrete-time MJFS model of (50) is described as follows:

Plant rule 1: **IF** $x_1(k)$ is "about 0"
THEN

$$\begin{cases} x(k+1) = A_{g1}x(k) + B_{g1}u(k) + E_{g1}w(k) \\ y(k) = C_{g1}x(k) + D_{g1}w(k) \\ z(k) = G_{g1}x(k) + H_{g1}u(k) \end{cases} \tag{51}$$

Plant rule 2: **IF** $x_1(k)$ is "about π or $-\pi$"
THEN

$$\begin{cases} x(k+1) = A_{g2}x(k) + B_{g2}u(k) + E_{g2}w(k) \\ y(k) = C_{g2}x(k) + D_{g2}w(k) \\ z(k) = G_{g2}x(k) + H_{g2}u(k) \end{cases} \tag{52}$$

TABLE 1. Comparison of dissipativity performance level β according to α_1 and α_2.

(α_1, α_2)	Theorem 4.2	Theorem 4.2 with $U_{i\ell}^{(2)}, U_{i\ell}^{(4)}, U_{i\ell}^{(5)} = 0$
(0,0): matched	4.2972	4.2972
(0.01,0.01)	4.2707	4.2463
(0.05,0.05)	3.6848	3.4175
(0.1, 0.1)	2.8919	1.5756

where $x(k) = \left[x_1^T(k)\; x_2^T(k)\right]^T$, and

$$A_{g1} = \begin{bmatrix} 1 & T_s \\ -\dfrac{T_s M_g}{J_g}\bar{g}\ell & 1 - \dfrac{T_s R}{J_g} \end{bmatrix}$$
$$A_{g2} = \begin{bmatrix} 1 & T_s \\ -\dfrac{\eta T_s M_g}{J_g}\bar{g}\ell & 1 - \dfrac{T_s R}{J_g} \end{bmatrix}$$
$$B_{g1} = B_{g2} = \begin{bmatrix} 0 \\ \dfrac{T_s}{J_g} \end{bmatrix}, \quad E_{g1} = E_{g2} = \begin{bmatrix} 0 \\ T_s \end{bmatrix}$$
$$C_{g1} = C_{g2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_{g,1} = D_{g,2} = 0.1$$
$$G_{g1} = G_{g2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad H_{g,1} = H_{g,2} = 0.1.$$

As a result, using a singleton fuzzifier, product-fuzzy inference, and a center-average defuzzifier, the blended system model of (51) and (52) can be expressed as (1). In addition, the transition probabilities are given as

$$[\pi_{gh}]_{g,h\in\mathbb{N}_\phi} = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}.$$

Based on Theorem 8 with $\beta_0 = 1$, Table 1 shows the comparison of dissipativity performance level β for several α_1 and α_2, obtained by Theorem 9, where $\mathcal{Q} = -0.01$, $\mathcal{S} = 0.2$ and $\mathcal{R} = 5$ are used for simulation. That is, from Table 1, it can be seen that the lower the degree of mismatch (i.e., the smaller the value of α_i), the higher the performance level we can obtain. In addition, Table 1 reveals that the larger the value of α_i (i.e., the more serious mismatch), the more $U_{i\ell}^{(2)}$, $U_{i\ell}^{(4)}$, and $U_{i\ell}^{(5)}$ play important roles in achieving better performance. Meanwhile, for $(\alpha_1, \alpha_2) = (0.1, 0.1)$, Theorem 8 and Theorem 9 provide the following observer and control gains:

$$L_{11} = \begin{bmatrix} 0.9849 \\ -1.5855 \end{bmatrix}, \quad L_{21} = \begin{bmatrix} 1.2008 \\ -1.3345 \end{bmatrix}$$
$$L_{31} = \begin{bmatrix} 1.1483 \\ -1.3830 \end{bmatrix}, \quad L_{12} = \begin{bmatrix} 1.0525 \\ -0.1889 \end{bmatrix}$$
$$L_{22} = \begin{bmatrix} 1.1589 \\ -0.0554 \end{bmatrix}, \quad L_{32} = \begin{bmatrix} 1.0567 \\ -0.1765 \end{bmatrix}$$
$$F_{11} = \begin{bmatrix} -0.0883 & -1.7360 \end{bmatrix}$$
$$F_{21} = \begin{bmatrix} 1.1171 & -4.0562 \end{bmatrix}$$
$$F_{31} = \begin{bmatrix} 1.5524 & -5.8559 \end{bmatrix}$$

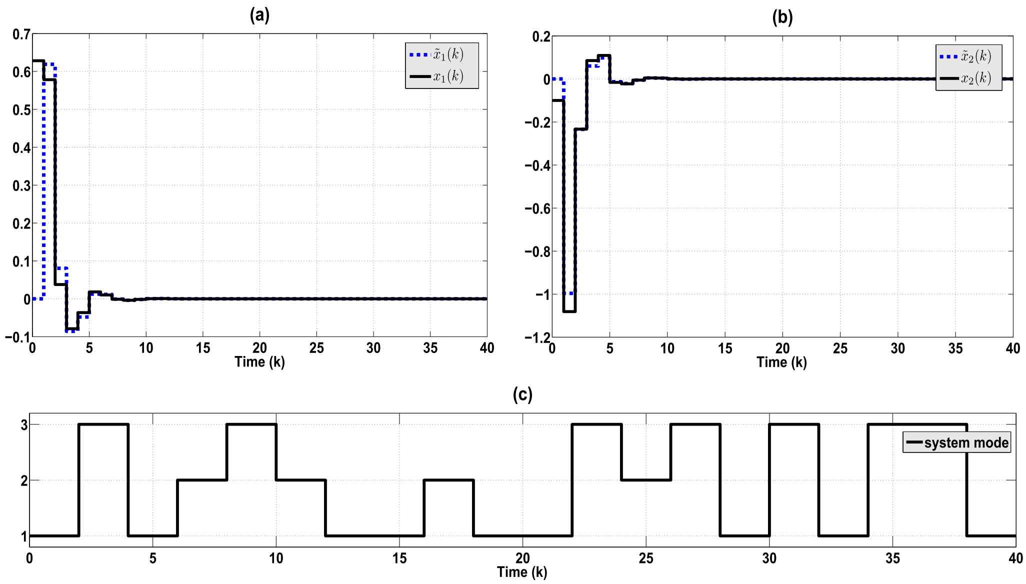

FIGURE 1. State response of the closed-loop system: (a) $x_1(k)$ and $\tilde{x}_1(k)$, (b) $x_2(k)$ and $\tilde{x}_2(k)$, and (c) system operation mode $\phi(k)$.

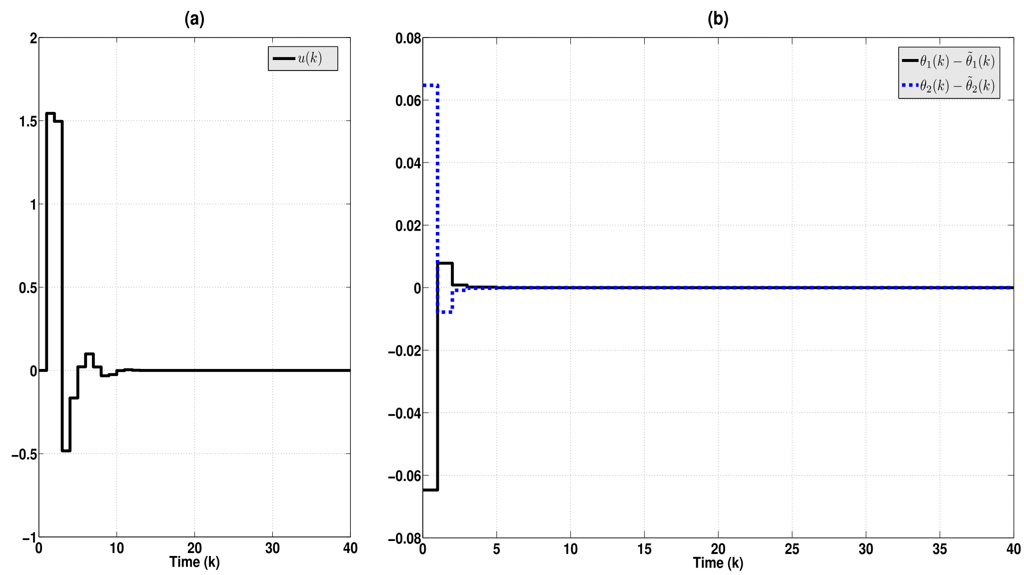

FIGURE 2. Simulation results: (a) control input, and (b) error between $\theta_i(k)$ and $\tilde{\theta}_i(k)$.

$$F_{12} = \begin{bmatrix} -3.6646 & -1.8692 \end{bmatrix}$$
$$F_{22} = \begin{bmatrix} -7.2478 & -5.5246 \end{bmatrix}$$
$$F_{32} = \begin{bmatrix} -8.9807 & -7.4305 \end{bmatrix}.$$

Fig. 1-(a) and Fig. 1-(b) show the state response of the closed-loop system with $x(0) = [0.2\pi, \ -0.1]^T$, $\tilde{x}(0) \equiv 0$, and $w(t) \equiv 0$; and Fig. 1-(c) shows the behavior of the system mode $\phi(t)$. As shown in Fig. 1-(a) and Fig. 1-(b), the estimated state $\tilde{x}(t)$ follows $x(t)$ well and the system state $x(t)$ also converges to zero as time increases, which demonstrates the availability of the obtained observer and control gains. In addition, Fig. 2-(a) shows the control input $u(k)$ generated by the mode-dependent observer-based controller; and Fig. 2-(b) shows the error $\nu_i(k) = \theta_i(k) - \tilde{\theta}_i(k)$, for $i = 1, 2$, which arises from the state estimation error as mentioned in Remark 1. From Fig. 2-(b), it can be seen that the value of $\nu_i(k)$ never exceed the value of $\alpha_i = 0.1$. In what follows, Fig. 3-(a) shows the performance output response of the closed-loop system with $x(0) \equiv 0$, $\tilde{x}(0) \equiv 0$ and $w(k) = 0.4\exp^{-0.1(k-15)}\sin(0.3(k-15))$; Fig. 3-(b) shows the behavior of the system mode $\phi(t)$; and Fig. 3-(c) shows the error $\nu_i(k) = \theta_i(k) - \tilde{\theta}_i(k)$ caused by the disturbance $w(k)$ as mentioned in Remark 1, which illustrates that the

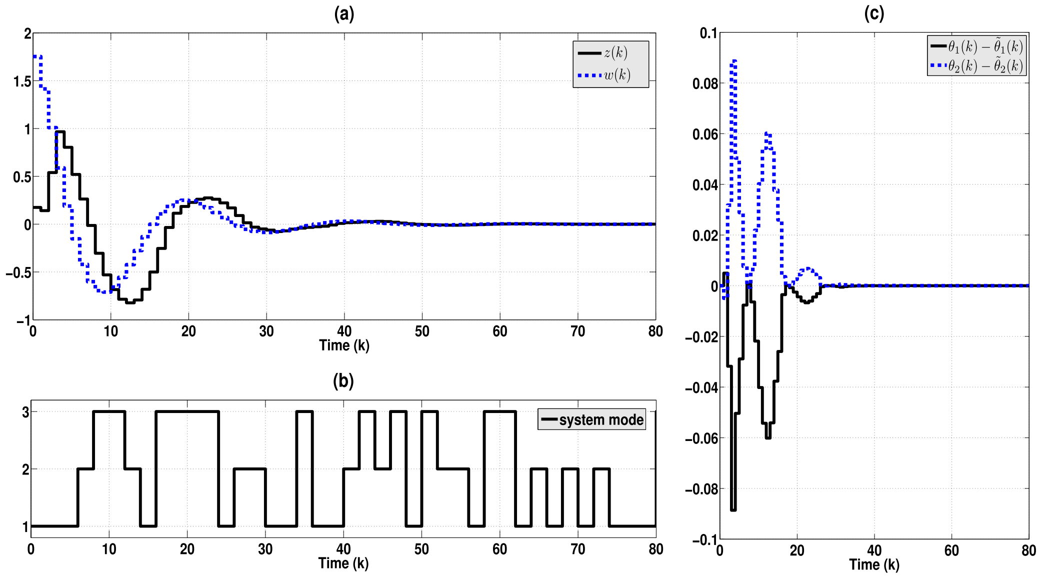

FIGURE 3. **Performance output response: (a) $z(k)$ and $w(k)$, (b) system mode $\phi(k)$, and (c) error between $\theta_i(k)$ and $\tilde{\theta}_i(k)$.**

value of $\nu_i(k)$ never exceed the value of $\alpha_i = 0.1$. Especially, from Fig. 3-(a), it can be found that the dissipativity level obtained from the performance output response $z(k)$ and disturbance $w(k)$ meets the value given in Table 1 as follows: $\sum_{k=0}^{80} \mathcal{W}(z(k), w(k)) / \sum_{k=0}^{80} ||w(k)||^2 = 5.1634 > \beta = 2.8919$.

Remark 7: In the literature, to the best of our knowledge, there are no comparable studies addressing the problem of designing observer-based control for a class of discrete-time Markovian jump fuzzy systems under error constrains of mismatched fuzzy basis functions. Thus, unfortunately, this paper cannot provide comparisons with other papers to highlight the advantage of the proposed method. Instead, our results can be used for comparison when more advanced control design methods are proposed for the same stabilization problem.

Remark 8: In the numerical example, the practicality of the proposed stabilization condition is verified using the single-link robot arm system. However, in the end, it will be necessary to substantiate the proposed method through some experimental results as well as numerical results. Although the experiments are not performed in this paper because other auxiliary processes and techniques related to hardware design must be added, the proposed method will continue to evolve so that it can be extended to experiments that even consider measurement and actuator faults.

VI. CONCLUDING REMARKS

In this paper, we have focused on the problem of designing an observer-based dissipative control for a class of discrete-time Markovian jump fuzzy systems under mismatched fuzzy basis functions. Specifically, to deal with the influence of mismatched fuzzy basis functions in the observer-based stabilization problem of MJFSs, we have proposed a simple method to solve the decoupling problem within two steps and provided a useful relaxation technique based on the error of the mismatched fuzzy basis functions. In the future, we will extend the proposed control design method to be suitable for nonhomogeneous MJFSs and develop a more advanced decoupling technique.

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