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# Adaptive Event-Based Non-Fragile Output Pinning Synchronization Control for Complex Networks With Random Saturations and Cyber-Attacks

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**ABSTRACT** This paper addresses the adaptive event-triggered non-fragile output pinning synchronization control for complex networks subject to random saturations and cyber-attacks. An adaptive event-triggered scheme (AETS) based on the output synchronization error is proposed to save network bandwidth, and a pinning control strategy is employed to reduce the input of control signal. Considering the effect of AETS, randomly occurring saturations and cyber-attacks on the drive and response systems, we investigate a novel output security synchronization error model, and then design an event-triggered non-fragile controller such that the asymptotic stability of the error system can be guaranteed. Meanwhile, we obtain the controller gains and event-triggered matrices in terms of solving the linear matrix inequalities (LMIs). Finally, a simulation example is provided to verify the design method.

**INDEX TERMS** Complex networks, pinning synchronization control, adaptive event-triggered scheme, output saturations, cyber-attacks.

## I. INTRODUCTION

Complex networks generally represent a class of large-scale systems with multiple nodes, in which each node refers to different individuals with specific characteristics. Recently, complex networks have been deeply studied and widely applied in logistics, cloud manufacturing, power grids and so on [1]–[7].

Synchronization, which is an important feature of complex networks, has important implications to the real-world systems. Generally speaking, the nodes in complex networks cannot automatically tend to be synchronized, which will exert an influence on the performance of the system. Therefore, the study on synchronization of complex networks is quite promising. In practice, however, each node in complex networks has different autonomous behavior, thus it is difficult for them to achieve synchronization without external

intervention. This leads to the emergence of some synchronization control strategies, such as global and group synchronization [8]–[11], finite-time synchronization [12], [13], output synchronization [14], [15] and lock synchronization [16], [17]. In fact, it is too expensive to design controllers for all nodes in a large-scale network. To reduce the number of the controlled nodes, some local feedback injections are employed to a small part of network nodes, that is the pinning control [18], [19]. The main objective of such control scheme is to design controllers on a small part of the nodes in complex networks such that the final synchronization of the whole network can be achieved. In current study, we utilize pinning synchronization control strategy based on output error to discuss the synchronization problem for complex networks under network attacks.

Currently, information technology is developing rapidly, which means the structure of complex networks is becoming larger and more and more complex. Therefore, more signals need to be transmitted and an increasing number of data

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will be generated in the process. In practice, however, network bandwidth is limited and low transmission efficiency will be possible to happen in the process of network transmission. To avoid these problems, an effective transmission strategy, i.e., event-triggered scheme (ETS), has been developed for networked systems [20]–[25]. For example, in [22], the distributed event-triggered strategy has been studied for internally coupled second-order nonlinear systems with time variation. In [23], an event-based synchronization control problem has been studied for a class of complex networks with stochastic switching topologies. However, the ETS used in the above literature is static, i.e., the trigger threshold is a fixed constant, which means it cannot reply the impact of network emergencies flexibly. In recent years, an AETS was developed based on the ETS, in which the trigger threshold is a variable function related to the measurement errors. Up to now, there have been some control results based on the AETS (see, e.g., [26]–[32]), however, such communication scheme has not been studied in the design of pinning control for complex networks. Therefore, the event-triggered pinning synchronization control for complex networks is worthy of further investigation.

In the real environment, due to the limitation of physics and technology, the components of the systems cannot receive or send signals indefinitely, which is called the saturation phenomenon. Up to now, some important results have been achieved in responding to the issues of saturation problems [33]–[35]. Output saturation, which is a common nonlinear phenomenon in the real environment, will affect the system performance and even induce instability. Moreover, some random factors such as channel noise often lead to the output saturation, which brings some additional challenges to the stability of the systems [36]–[38]. Based on this, the random saturations are considered in this paper.

In addition, the wireless channel in the data interaction layer is open and the deployment of sensor nodes in the sensing layer is random, which means that the signal transmitted in the network communication channel could be stolen, modified, or discarded maliciously. Among these threats, the most common and destructive one is cyber-attacks which will result a huge security risk to the system. Thus, it is necessary to study effective emergency defense measures against network security problems caused by cyber-attacks. Cyber-attacks cause the internal components of the systems to fail to operate normally or even make the system unstable. Recently, network security is becoming a hot topic in the field of networked control systems and some interesting results have been published [39]–[42]. In [40], the decentralized event-triggered control was considered for the neural networks under the threat of cyber-attacks. As we know, there is little relevant literature on the output security pinning synchronization control of complex networks subject to time-varying delay and random saturations. This situation promotes the further study of this paper.

According to the above discussions, we aim to study the adaptive event-triggered pinning security synchronization

control problem of complex networks with random saturations and time-varying delay. The main contributions are organized as:

- (1) To reduce the stress of network bandwidth greatly, an adaptive event-triggered scheme based on the output synchronization error is proposed. Considering the influence of stochastic cyber-attacks and saturations, a new output pinning synchronization error model is established.
- (2) The proposed event-based synchronization non-fragile control results are more applicable for complex networked systems as the two typical issues are considered in sensor networks, i.e., the communication resource limitation and the controller gain variation are investigated in a unified framework instead of analyzing separately in some existing works.
- (3) A new sufficient criterion for the stability of synchronization error systems is proposed, and the controller gains and AETS are co-designed simultaneously.

The remainder of this paper is organized as: Section 2 is the problem statement. Section 3 gives the main results including stability analysis and controller design. Section 4 provides a numerical example and this paper is concluded in Section 5.

*Notation:* The  $n$ -dimensional Eculidean space is denoted by  $\mathbb{R}^n$ . The set of  $n \times m$  real matrices is represented by  $\mathbb{R}^{n \times m}$ . We use the symbol  $0_m$  to denote a row vector with  $m$  zero elements, and let  $I$  represent an identity matrix with appropriate dimensions.  $\|\cdot\|$  is the Euclidean vector norm.  $diag_n\{X\}$  denotes a  $n$ -dimensional diagonal matrix whose diagonal elements are all  $X$ .  $diag_n\{X_i\}$  represents a  $n$ -dimensional diagonal matrix whose diagonal elements are  $X_1, X_2, \dots, X_n$ .  $col_n\{X\}$  and  $col_n\{X_i\}$  stand for column vectors similar to the definitions of the above diagonal matrices.  $E\{X\}$  is the expectation of the stochastic variable  $X$ . The symbol  $\otimes$  represents the Kronecker product of matrices. The symbol  $*$  in a symmetric matrix stands for the implicit entries of some symmetry terms.

## II. PROBLEM FORMULATION AND SYSTEM MODELING

In this paper, we consider a class of drive-response complex networks with  $N$  coupled nodes. The considered drive system model is as follows:

$$\left\{ \begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i f(x_i(t)) \\ &\quad + c_{1i} \sum_{j=1}^N m_{ij} \Gamma_1 X_j(t) \\ &\quad + c_{2i} \sum_{j=1}^N w_{ij} \Gamma_2 X_j(t - \tau_i(t)), \\ X_i(t) &= E_i x_i(t), \quad i = 1, \dots, N, \end{aligned} \right. \quad (1)$$

where  $x_i(t) \in \mathbb{R}^n$  represents the state of the node  $i$ ,  $X_i(t)$  denotes the output of the node  $i$ ,  $A_i$  and  $B_i$  are the system parameter matrices with appropriate dimensions,  $f(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a continuous nonlinear vector function,  $c_{1i} > 0$ ,  $c_{2i} > 0$  are the coupling strength of the network,

$M = [m_{ij}]_{N \times N}$ ,  $W = [w_{ij}]_{N \times N}$  are the external coupling matrices of the network with  $m_{ij} > 0$  and  $w_{ij} > 0 (i \neq j)$ , but not all zero. Generally, the matrices  $M$  and  $W$  are symmetric and satisfy  $m_{ii} = -\sum_{j=1, j \neq i}^N m_{ij}$ ,  $w_{ii} = -\sum_{j=1, j \neq i}^N w_{ij} (i = 1, 2, \dots, N)$ .  $\Gamma_1$  and  $\Gamma_2$  are inner-coupling matrices between connected nodes,  $\tau_i(t) \in (0, \tau_i]$  is the time-varying delay.  $E_i$  is a given matrix with proper dimensions.

According to (1), we can get the output dynamic model as follows:

$$\begin{aligned} \dot{X}_i(t) = & \bar{A}_i X_i(t) + \bar{B}_i \bar{f}(X_i(t)) + c_{1i} \sum_{j=1}^N m_{ij} \bar{\Gamma}_1 X_j(t) \\ & + c_{2i} \sum_{j=1}^N w_{ij} \bar{\Gamma}_2 X_j(t - \tau_i(t)), \end{aligned} \quad (2)$$

where  $\bar{A}_i = E_i A_i E_i^{-1}$ ,  $\bar{B}_i = E_i B_i$ ,  $\bar{f}(x(t)) = f(E_i^{-1} x(t))$ ,  $\bar{\Gamma}_1 = E_i \Gamma_1$ ,  $\bar{\Gamma}_2 = E_i \Gamma_2$ .

The considered response system model is described as

$$\begin{cases} \dot{y}_i(t) = A_i y_i(t) + B_i f(y_i(t)) + c_{1i} \sum_{j=1}^N m_{ij} \Gamma_1 Y_j(t) \\ \quad + c_{2i} \sum_{j=1}^N w_{ij} \Gamma_2 Y_j(t - \tau_i(t)) + g_i u_i(t), \\ Y_i(t) = E_i y_i(t), \quad i = 1, \dots, N, \end{cases} \quad (3)$$

where  $u_i(t)$  is the control input of the node  $i$ . If the node  $i$  is pinned, then  $g_i > 0$ , otherwise  $g_i = 0$ . Based on (3), the following equation can be obtained

$$\begin{aligned} \dot{Y}_i(t) = & \bar{A}_i Y_i(t) + \bar{B}_i \bar{f}(Y_i(t)) + c_{1i} \sum_{j=1}^N m_{ij} \bar{\Gamma}_1 Y_j(t) \\ & + c_{2i} \sum_{j=1}^N w_{ij} \bar{\Gamma}_2 Y_j(t - \tau_i(t)) + g_i E_i u_i(t), \end{aligned} \quad (4)$$

Let  $\eta_i(t) = Y_i(t) - X_i(t)$  as the output synchronization error. Then, based on (2) and (4), we obtain that

$$\begin{aligned} \dot{\eta}_i(t) = & \bar{A}_i \eta_i(t) + \bar{B}_i \bar{f}(\eta_i(t)) + c_{1i} \sum_{j=1}^N m_{ij} \bar{\Gamma}_1 \eta_j(t) \\ & + c_{2i} \sum_{j=1}^N w_{ij} \bar{\Gamma}_2 \eta_j(t - \tau_i(t)) + g_i E_i u_i(t). \end{aligned} \quad (5)$$

where  $\bar{f}(\eta_i(t)) = \bar{f}(Y_i(t)) - \bar{f}(X_i(t))$ .

The framework of the drive-response control system is shown in Fig. 1. In practice, the bandwidth capacity of the network transmission channel is limited, so it is necessary to save network resources in the process of information transmission. In this paper, an AETS based on output error is proposed. We first provide the following standard assumption of the AETS.

*Assumption 1:* The sampling period is denoted as  $h > 0$  and the triggering time is assumed as  $t_k^i h (k = 0, 1, 2, \dots)$ ,

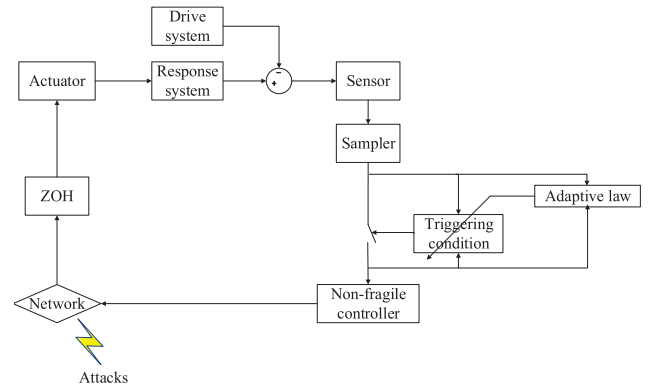


FIGURE 1. Framework of pinning control for complex networks with AETS.

where  $t_k^i$  is a nonnegative integer. In addition, the initial triggering time is set to  $t_0^i h = 0$ .

For the node  $i$ , we define the triggering protocol in the following form.

$$\varrho_i^T(t) \Upsilon_i \varrho_i(t) \leq \xi_i(t) \eta_i^T((t_k^i + j)h) \Upsilon_i \eta_i((t_k^i + j)h), \quad (6)$$

where  $\varrho_i(t) = \eta_i(t_k^i h) - \eta_i((t_k^i + j)h)$ ,  $\Upsilon_i > 0$  is the triggering matrix of the AETS to be designed and  $\xi_i(t)$  is the threshold parameter that satisfies the following condition

$$\dot{\xi}_i(t) = \frac{\varepsilon_i}{\xi_i(t)} \left( \frac{1}{\xi_i(t)} - \lambda_i \right) \varrho_i^T(t) \Upsilon_i \varrho_i(t) \quad (7)$$

where  $0 < \xi_i(t) < 1$ ,  $\varepsilon_i > 0$ ,  $\lambda_i > 0$ .

On the basis of the protocol (6) and the last released instant  $t_k^i h$ , one can define the next released instant  $t_{k+1}^i h$  as

$$\begin{aligned} t_{k+1}^i h = & t_k^i h + \min\{jh | \varrho_i^T(t) \Upsilon_i \varrho_i(t) \\ & > \xi_i(t) \eta_i^T((t_k^i + j)h) \Upsilon_i \eta_i((t_k^i + j)h)\}. \end{aligned} \quad (8)$$

On the other hand, the delay induced by the network is unavoidable when the information is transmitted through network transmission channel. Therefore, the transmission delay is also considered in this paper, which is coincidence with the engineering practice. Here, let  $d_k^i$  as the communication delay of the node  $i$ .

Define  $[t_k^i h + d_k^i, t_{k+1}^i h + d_{k+1}^i) = \bigcup_{w=1}^{r_i} \mu_i^w$ , where

$$\begin{aligned} \mu_i^w = & [t_k^i h + wh - h + d_k^{i_{w-1}}, t_k^i h + wh + d_k^{i_w}), \\ d_k^{i_w} = & \begin{cases} d_k^i, & w \leq r_i - 1, \\ d_{k+1}^i, & w = r_i. \end{cases} \end{aligned}$$

Let  $d_i(t) = t - (t_k^i + j)h$ ,  $t \in \mu_i^w$ , then under the action of zero-order holder (ZOH), the actual sensor measurement error can be represented as below,

$$\bar{\eta}_i(t) = \eta_i(t_k^i h) = \eta_i(t - d_i(t)) + \varrho_i(t) \quad (9)$$

where  $0 \leq d_i(t) \leq h + \max\{d_k^i\} = d_M^i$ .

*Remark 1:* From (7), it can be seen that the error  $\varrho_i(t)$  also approaches to zero if the system is reaching a stable. In addition, the threshold will converge to a constant if there is no disturbance destabilizes this system.

*Remark 2:* If we set  $\varepsilon_i = 0$  in (7), then the triggering condition in (6) becomes to the traditional one as follows

$$\varrho_i^T(t)\Upsilon_i\varrho_i(t) \leq \bar{\xi}_i\eta_i^T((t_k^i + j)h)\Upsilon_i\eta_i((t_k^i + j)h) \quad (10)$$

where  $0 < \bar{\xi}_i \leq 1$  is a given value. Particularly, the above event-triggered scheme becomes the time-triggered scheme if  $\bar{\xi}_i = 0$ . From (6), it can be found that the threshold variable  $\xi_i(t)$  has a significant influence on the transmitted number of packets for a certain time period. In (7),  $\xi_i(t)$  is a sequence of invariable parameter by flexible adjustment of the adaptive law (7).

*Remark 3:* Based on the condition (6), one can know that the sampling time is the discrete instants  $t_k^i$  ( $k = 0, 1, 2, \dots$ ). The minimum triggering interval is  $h$  for the AETS, therefore, Zeno behavior can be avoided under the AETS (6).

The error  $\bar{\eta}_i(t)$  under the randomly occurring saturation nonlinearities can be rewritten as follows:

$$\dot{\bar{\eta}}_i(t) = (1 - \alpha_i(t))\bar{\eta}_i(t) + \alpha_i(t)\rho(\bar{\eta}_i(t)), \quad (11)$$

where  $\rho(\bar{\eta}_i) = [\rho_1(\bar{\eta}_{i1}) \ \rho_2(\bar{\eta}_{i2}) \ \dots \ \rho_\nu(\bar{\eta}_{i\nu})]^T$  is the saturation function, and  $\rho_j(\bar{\eta}_{ij})(j = 1, 2, \dots, \nu)$  satisfies

$$\rho_j(\bar{\eta}_{ij}) = \begin{cases} \sigma_j, & \bar{\eta}_{ij} \geq \sigma_j \\ \bar{\eta}_{ij}, & -\sigma_j < \bar{\eta}_{ij} < \sigma_j \\ -\sigma_j, & -\bar{\eta}_{ij} \leq -\sigma_j \end{cases} \quad j = 1, 2, \dots, \nu. \quad (12)$$

and  $\alpha_i(t) \in \{0, 1\}$  is a random variable subject to Bernoulli distribution and one assumes that  $E\{\alpha_i(t)\} = \alpha_i$ ,  $E\{\alpha_i(t) - \alpha_i\}^2 = \theta_{\alpha_i}^2$ .

According to the results in [43], the saturation signal  $\rho(\bar{\eta}_i(t))$  can be decomposed as below,

$$\rho(\bar{\eta}_i(t)) = \varphi_i(t) + \bar{\eta}_i(t), \quad (13)$$

where  $\varphi_i(t)$  is a nonlinear function satisfying the following condition with  $0 < \varepsilon < 1$ ,

$$\varphi_i^T(t)\varphi_i(t) \leq \varepsilon\bar{\eta}_i^T(t)\bar{\eta}_i(t). \quad (14)$$

Combining (11) and (13), one gets

$$\begin{aligned} \dot{\bar{\eta}}_i(t) &= (1 - \alpha_i(t))\bar{\eta}_i(t) + \alpha_i(t)(\varphi_i(t) + \bar{\eta}_i(t)) \\ &= \bar{\eta}_i(t) + \alpha_i(t)\varphi_i(t). \end{aligned} \quad (15)$$

In this paper, based on the considerations of the possible external disturbances and the fluctuations of the controller gains, the non-fragile controller will be designed, i.e.,  $u_i(t) = (K_i + \Delta K_i)\eta_i(t)$ . Then, combining (9) and (15), the actual input is taken as follows,

$$\begin{aligned} u_i(t) &= (K_i + \Delta K_i)\bar{\eta}_i(t) \\ &= (K_i + \Delta K_i)(\eta(t - d_i(t)) + \varrho_i(t) + \alpha_i(t)\varphi_i(t)), \end{aligned} \quad (16)$$

where  $K_i$  is the gain to be determined and  $\Delta K_i$  is unknown but the norm is bounded. In addition, let  $\Delta K_i = T_i F(t) Z_i$ , where  $T_i, Z_i$  are known constant matrices, and  $F(t)$  is a nonlinear matrix function subject to the condition  $F^T(t)F(t) \leq I$ .

Due to the open principle of network communication, the transmitted information is vulnerable to malicious attacks, which may damage the stability of the systems. Here, we consider a case that the non-fragile controller is attacked by a class of random deception attacks, and we assume that the attack signal is modeled as nonlinear matrix function  $h(u_i(t))$  based on the input  $u_i(t)$ . Then, after considering the deception attack, the actual input in (16) is rewritten as follows,

$$\bar{u}_i(t) = u_i(t) + \beta_i(t)h(u_i(t)), \quad (17)$$

where  $\beta_i(t) \in \{0, 1\}$  is a Bernoulli random variable and is independent of  $\alpha_i(t)$ , and we assume that  $E\{\beta_i(t)\} = \beta_i$ ,  $E\{(\beta_i(t) - \beta_i)^2\} = \beta_i(1 - \beta_i) = \theta_{\beta_i}^2$ .

*Remark 4:* According to (17), one can easily know that if  $\beta_i(t) = 0$ , then  $\bar{u}_i(t) = u_i(t)$ , which means that the controller is not threatened by cyber-attacks. Otherwise,  $\bar{u}_i(t) = u_i(t) + h(u_i(t))$ , which means that the controller is suffered from cyber-attacks.

Combining (5) and (17), we can obtain the output synchronization error system as follows

$$\begin{aligned} \dot{\eta}_i(t) &= \bar{A}_i\eta_i(t) + \bar{B}_i\bar{f}(\eta_i(t)) + c_{1i} \sum_{j=1}^N m_{ij}\bar{\Gamma}_1\eta_j(t) \\ &\quad + c_{2i} \sum_{j=1}^N w_{ij}\bar{\Gamma}_2\eta_j(t - \tau_i(t)) \\ &\quad + g_i E_i((K_i + \Delta K_i)(\eta_i(t - d_i(t)) + \varrho_i(t) \\ &\quad + \alpha_i(t)\varphi_i(t) + \beta_i(t)h(u_i(t))), \end{aligned} \quad (18)$$

for  $i = 1, 2, \dots, N$ .

According to (18) and the Kronecker product of the matrix, one has the following augmented error model,

$$\begin{aligned} \dot{\eta}(t) &= \bar{A}\eta(t) + \bar{B}\bar{f}(\eta(t)) + C_1 M \otimes \bar{\Gamma}_1 \eta(t) \\ &\quad + C_2 W \otimes \bar{\Gamma}_2 \eta(t - \tau(t)) \\ &\quad + GE[(K + \Delta K)(\eta(t - d(t)) \\ &\quad + \varrho(t) + \alpha(t)\varphi(t) + \beta(t)h(u(t))), \end{aligned} \quad (19)$$

where

$$\begin{aligned} \bar{A} &= \text{diag}_N\{\bar{A}_i\}, \bar{f}(\eta(t)) = \text{col}_N\{\bar{f}(\eta_i(t))\}, \\ C_1 &= \text{diag}_N\{c_{1i}\}, h(u(t)) = \text{col}_N\{h(u_i(t))\}, \\ G &= \text{diag}_N\{g_i\} \otimes I, \beta(t) = \text{diag}_N\{\beta_i(t)\} \otimes I, \\ K &= \text{diag}_N\{K_i\}, \Delta K = \text{diag}_N\{\Delta K_i\}, \\ \eta(t) &= \text{col}_N\{\eta_i(t)\}, \bar{B} = \text{diag}_N\{\bar{B}_i\}, \\ \varrho(t) &= \text{col}_N\{\varrho_i(t)\}, \varphi(t) = \text{col}_N\{\varphi_i(t)\}, \\ \alpha(t) &= \text{diag}_N\{\alpha_i(t)\} \otimes I, C_2 = \text{diag}_N\{c_{2i}\}, \\ E &= \text{diag}_N\{E_i\}. \end{aligned}$$

This paper aims to investigate the event-based output pinning synchronization control for the drive-response complex networks subject to random saturations and cyber-attacks. Before giving the main results, we first introduce the following necessary lemmas and assumptions.

*Lemma 1:* [8] For given  $d > 0$ , if the function  $d(t)$  satisfies  $d(t) \in (0, d_M]$ , then for  $\hat{e}(t) : (0, d_M] \rightarrow \mathbb{R}^n$ , there exists  $U > 0$  such that

$$-d \int_{t-d}^t \dot{\eta}^T(s)U\dot{\eta}(s)ds \leq \mu^T(t)\Sigma\mu(t) \quad (20)$$

where

$$\mu(t) = [\eta^T(t) \quad \eta^T(t-d(t)) \quad \eta^T(t-d)]^T$$

$$\Sigma = \begin{bmatrix} -U & * & * \\ U & -2U & * \\ 0 & U & -U \end{bmatrix}.$$

*Lemma 2:* [21] Assume that  $L_1$  and  $L_2$  are real matrices with appropriate dimensions, and  $F(t)$  satisfies  $F^T(t)F(t) \leq I$ . Then, for any scalar  $\iota > 0$ , one has

$$L_2^T F(t)L_1^T + L_1 F(t)^T L_2 \leq \iota L_1 L_1^T + \iota^{-1} L_2^T L_2 \quad (21)$$

*Assumption 2:* The functions  $h(\cdot)$  and  $f(\cdot)$  describing the attack signal and nonlinear dynamics of the system are assumed to satisfy the following Lipschitz conditions, respectively.

$$\|h(x) - h(y)\| \leq \|\Omega_1(x - y)\|, \quad (22)$$

$$\|f(x) - f(y)\| \leq \|\Omega_2(x - y)\|, \quad (23)$$

where  $\Omega_v, (v = 1, 2)$  are two upper bound matrices.

### III. MAIN RESULTS

In this part, two theorems are given. Theorem 1 provides a sufficient condition for the asymptotic stability of the system (19). Theorem 2 gives a co-design method of the controller and AETS based on the LMI approach.

*Theorem 1:* For given positive parameters  $\alpha_i, \beta_i (i = 1, \dots, N), v_j > 0 (j = 1, 2), c_{1i}, c_{2i}, (i = 1, \dots, N), \varsigma_j (j = 1, 2), d_M = \max_N \{d_M^i\}, \tau_M = \max_N \{\tau_i\}, 0 < \epsilon < 1$  and matrices  $\bar{\Gamma}_k > 0 (k = 1, 2), \bar{A} > 0, \bar{B} > 0, G = \text{diag}_N \{g_i\}, \bar{E} > 0, \Omega_i (i = 1, 2), \epsilon, \lambda$ , the augmented error system (19) is asymptotically stable under the proposed AETS and random cyber-attacks, if there exist  $P = \text{diag}_N \{P_i\} > 0, R_i (i = 1, 2) > 0, U_i (i = 1, 2) > 0$  and matrices  $\Upsilon, \hat{K}$ , such that the following matrix inequality holds

$$\Phi = \begin{bmatrix} \Phi_{11} & * & * & * & * & * & * & * \\ \Phi_{21} & -\tilde{U} & * & * & * & * & * & * \\ \Phi_{31} & 0 & -\tilde{U} & * & * & * & * & * \\ \Phi_{41} & 0 & 0 & -\tilde{U} & * & * & * & * \\ \Phi_{51} & 0 & 0 & 0 & -\varsigma_2 I & * & * & * \\ \Phi_{61} & 0 & 0 & 0 & 0 & -\varsigma_2 I & * & * \\ \Phi_{71} & 0 & 0 & 0 & 0 & 0 & -I & * \\ \Phi_{81} & 0 & 0 & 0 & 0 & 0 & 0 & -\varsigma_1 I \end{bmatrix} < 0, \quad (24)$$

where

$$\Phi_{11} = \begin{bmatrix} \Lambda_{11} & * & * & * \\ \Lambda_{21} & \Lambda_{22} & * & * \\ 0 & \Lambda_{32} & -U_2 - R_2 & * \\ 0 & \Lambda_{42} & 0 & -U_1 - R_1 \end{bmatrix},$$

$$\Lambda_{11} = P(\bar{A} + C_1 M \otimes \bar{\Gamma}_1) + (\bar{A} + C_1 M \otimes \bar{\Gamma}_1)^T P + R_1 + R_2 - U_1 - U_2, \hat{K} = K + \Delta K,$$

$$\Lambda_{21} = \text{col}\{B^T P, (C_2 W \otimes \bar{\Gamma}_2)^T P + U_2, \hat{K}^T E^T G^T P + U_1, \hat{K}^T E^T G^T P, \hat{K}^T \alpha^T E^T G^T P, \beta^T E^T G^T P\},$$

$$\Lambda_{22} = \text{diag}\{-\varsigma_1 I, -2U_2, -2U_1 + \epsilon \Upsilon, -\epsilon \lambda \Upsilon, -\epsilon I, -\varsigma_2 I\},$$

$$\Lambda_{32} = [0 \quad U_2 \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\Lambda_{42} = [0 \quad 0 \quad U_1 \quad 0 \quad 0 \quad 0],$$

$$\Phi_{21} = [\Phi_{211} \quad \Phi_{212}],$$

$$\Phi_{211} = [\tilde{U}(\bar{A} + C_1 M \otimes \bar{\Gamma}_1) \quad \tilde{U}B \quad \tilde{U}C_2 W \otimes \bar{\Gamma}_2],$$

$$\Phi_{212} = [\tilde{U}GE\hat{K} \quad \tilde{U}GE\hat{K} \quad \tilde{U}GE\alpha\hat{K} \quad \tilde{U}GE\beta \quad 0 \quad 0],$$

$$\Phi_{31} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \tilde{U}GE\theta_1\hat{K} \quad 0 \quad 0 \quad 0],$$

$$\Phi_{41} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \tilde{U}GE\theta_2 \quad 0 \quad 0],$$

$$\Phi_{51} = [0_{1 \times 3} \quad \varsigma_2 \Omega_2 \hat{K} \quad \varsigma_2 \Omega_2 \hat{K} \quad \varsigma_2 \Omega_2 \alpha \hat{K} \quad 0_{1 \times 3}],$$

$$\Pi_{61} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \varsigma_2 \Omega_2 \theta_1 \hat{K} \quad 0 \quad 0 \quad 0],$$

$$\Phi_{71} = [0 \quad 0 \quad 0 \quad \epsilon I \quad \epsilon I \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\Phi_{81} = [\varsigma_1 \Omega_1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\tilde{U} = d_M^2 U_1 + \tau_M^2 U_2, \quad \epsilon = \text{diag}_N \{\epsilon_i\} \otimes I,$$

$$\lambda = \text{diag}_N \{\lambda_i\} \otimes I, \quad \Upsilon = \text{diag}_N \{\Upsilon_i\},$$

$$\theta_1 = \text{diag}_N \{\theta_{1i}\} \otimes I, \quad \theta_2 = \text{diag}_N \{\theta_{2i}\} \otimes I,$$

$$\Omega_1 = \text{diag}_N \{\Omega_{1i}\}, \quad \Omega_2 = \text{diag}_N \{\Omega_{2i}\},$$

$$\alpha = \text{diag}_N \{\alpha_i\} \otimes I, \quad \beta = \text{diag}_N \{\beta_i\} \otimes I.$$

*Proof:* For the system (19), the Lyapunov functional is constructed in the following form.

$$V(t) = \sum_{i=1}^4 V_i(t), \quad (25)$$

where

$$V_1(t) = \eta^T(t)P\eta(t),$$

$$V_2(t) = \int_{t-d_M}^t \eta^T(s)R_1\eta(s)ds + \int_{t-\tau_M}^t \eta^T(s)R_2\eta(s)ds,$$

$$V_3(t) = d_M \int_{t-d_M}^t \int_s^t \dot{\eta}^T(v)U_1\dot{\eta}(v)dv ds + \tau_M \int_{t-\tau_M}^t \int_s^t \dot{\eta}^T(v)U_2\dot{\eta}(v)dv ds,$$

$$V_4(t) = \frac{1}{2} \xi^T(t)\xi(t), \quad \xi(t) = \text{col}_N \{\xi_i(t)\},$$

Taking the derivative of  $V(t)$  on  $t$  and taking expectation on it, one obtains

$$E\{\dot{V}(t)\} = \sum_{i=1}^4 E\{\dot{V}_i(t)\}, \quad (26)$$

where

$$E\{\dot{V}_1(t)\} = 2\eta^T(t)P\dot{\eta}(t)$$

$$= 2\eta^T(t)P[(\bar{A} + C_1 M \otimes \bar{\Gamma}_1)\eta(t) + \bar{B}f(\eta(t)) + C_2 W \otimes \bar{\Gamma}_2 \eta(t - \tau(t)) + GE[(K + \Delta K)(\eta(t - d(t))) + \varrho(t) + \alpha(t)\varphi(t)) + \beta(t)h(u(t))],$$

$$\begin{aligned}
 E\{\dot{V}_2(t)\} &= \eta^T(t)(R_1 + R_2)\eta(t) \\
 &\quad - \eta^T(t - d_M)R_1\eta(t - d_M) \\
 &\quad - \eta^T(t - \tau_M)R_2\eta(t - \tau_M), \\
 E\{\dot{V}_3(t)\} &= E\{d_M \int_{t-d_M}^t [\dot{\eta}^T(s)U_1\dot{\eta}(s) - \dot{\eta}^T(t)U_1\dot{\eta}(t)]ds \\
 &\quad + \tau_M \int_{t-\tau_M}^t [\dot{\eta}^T(s)U_2\dot{\eta}(s) - \dot{\eta}^T(t)U_2\dot{\eta}(t)]ds\} \\
 &= E\{d_M^2\dot{\eta}^T(t)U_1\dot{\eta}(t) + \tau_M^2\dot{\eta}^T(t)U_2\dot{\eta}(t) \\
 &\quad - d_M \int_{t-d_M}^t \dot{\eta}^T(s)U_1\dot{\eta}(s)ds \\
 &\quad - \tau_M \int_{t-\tau_M}^t \dot{\eta}^T(s)U_2\dot{\eta}(s)ds\} \\
 &= E\{\dot{\eta}^T(t)\tilde{U}\dot{\eta}(t)\} - E\{d_M \int_{t-d_M}^t \dot{\eta}^T(s)U_1\dot{\eta}(s)ds \\
 &\quad - \tau_M \int_{t-\tau_M}^t \dot{\eta}^T(s)U_2\dot{\eta}(s)ds\}, \\
 E\{\dot{V}_4(t)\} &= \xi^T(t)\dot{\xi}(t) \\
 &= \varepsilon_1\left(\frac{1}{\xi_1(t)} - \lambda_1\right)\varrho_1^T(t)\Upsilon_1\varrho_1(t) \\
 &\quad + \varepsilon_2\left(\frac{1}{\xi_2(t)} - \lambda_2\right)\varrho_2^T(t)\Upsilon_2\varrho_2(t) \\
 &\quad + \dots + \varepsilon_N\left(\frac{1}{\xi_N(t)} - \lambda_N\right)\varrho_N^T(t)\Upsilon_N\varrho_N(t).
 \end{aligned}$$

Combining (6) and  $E\{\dot{V}_4(t)\}$ , one can get the following inequality

$$\begin{aligned}
 E\{\dot{V}_4(t)\} &\leq \varepsilon_1\eta_1^T(t - d_1(t))\Upsilon_1\eta_1(t - d_1(t)) \\
 &\quad + \varepsilon_2\eta_2^T(t - d_2(t))\Upsilon_2\eta_2(t - d_2(t)) + \dots \\
 &\quad + \varepsilon_N\eta_N^T(t - d_N(t))\Upsilon_N\eta_N(t - d_N(t)) \\
 &\quad - \varepsilon_1\lambda_1\varrho_1^T(t)\Upsilon_1\varrho_1(t) \\
 &\quad - \varepsilon_2\lambda_2\varrho_2^T(t)\Upsilon_2\varrho_2(t) \\
 &\quad - \dots - \varepsilon_N\lambda_N\varrho_N^T(t)\Upsilon_N\varrho_N(t) \\
 &= \eta^T(t - d(t))\varepsilon\Upsilon\eta(t - d(t)) \\
 &\quad - \varrho^T(t)\varepsilon\lambda\Upsilon\varrho(t). \tag{27}
 \end{aligned}$$

Notice that  $\dot{\eta}(t) = \mathcal{A} + GE\hat{K}(\alpha(t) - \alpha)\varphi(t) + GE(\beta(t) - \beta)h(u(t))$ , where  $\mathcal{A} = (\bar{A} + C_1M \otimes \bar{\Gamma}_1)\eta(t) + \bar{B}\bar{f}(\eta(t)) + C_2W \otimes \bar{\Gamma}_2\eta(t - \tau(t)) + GE[\hat{K}(\eta(t - d(t)) + \varrho(t) + \alpha\varphi(t)) + \beta h(u(t))]$ .

Thus, one has

$$\begin{aligned}
 E\{\dot{\eta}^T(t)\tilde{U}\dot{\eta}(t)\} &= \mathcal{A}^T\tilde{U}\mathcal{A} \\
 &\quad + \varphi^T(t)\hat{K}^T\theta_1^TE^TG^T\tilde{U}GE\theta_1\hat{K}\varphi(t) \\
 &\quad + h^T(u(t))\theta_2^TE^TG^T\tilde{U}GE\theta_2h(u(t)). \tag{28}
 \end{aligned}$$

Based on Assumption 2, for any scalar  $\varsigma_1 > 0$  and  $\varsigma_2 > 0$ , we can derive the following two inequalities

$$\varsigma_1 f^T(t)f(t) \leq \varsigma_1 \eta^T(t)\Omega_1^T\Omega_1\eta(t), \tag{29}$$

$$\varsigma_2 h^T(u(t))h(u(t)) \leq \varsigma_2 u^T(t)\Omega_2^T\Omega_2u(t). \tag{30}$$

Furthermore, we can rewrite (16) as

$$u(t) = (K_i + \Delta K_i)(\mathcal{B} + (\alpha(t) - \alpha)\varphi(t)), \tag{31}$$

where  $\mathcal{B} = \eta(t - d(t)) + \varrho(t) + \alpha\varphi(t)$ .

From (30), we can get

$$\omega = \varsigma_2 u^T(t)\Omega_2^T\Omega_2u(t) - \varsigma_2 h^T(u(t))h(u(t)) \geq 0. \tag{32}$$

Taking the expectation of the formula (32), the following inequality can be derived

$$\begin{aligned}
 E\{\omega\} &= \varsigma_2 \mathcal{B}^T\Omega_2^T\Omega_2\mathcal{B} + \varsigma_2 \varphi^T(t)\theta_1^T\Omega_2^T\Omega_2\theta_1\varphi(t) \\
 &\quad - \varsigma_2 h^T(u(t))h(u(t)) \geq 0. \tag{33}
 \end{aligned}$$

Based on Lemma 1 and (26)-(33), one further comes to

$$\begin{aligned}
 E\{\dot{V}(t)\} &\leq 2\eta^T(t)P[(\bar{A} + C_1M \otimes \bar{\Gamma}_1)\eta(t) + \bar{B}\bar{f}(\eta(t)) \\
 &\quad + C_2W \otimes \bar{\Gamma}_2\eta(t - \tau(t)) \\
 &\quad + GE[(K + \Delta K)(\eta(t - d(t)) \\
 &\quad + \varrho(t) + \alpha(t)\varphi(t)) + \beta(t)h(u(t))] \\
 &\quad + \eta^T(t)(R_1 + R_2)\eta(t) - \eta^T(t - d_M)R_1\eta(t - d_M) \\
 &\quad - \eta^T(t - \tau_M)R_2\eta(t - \tau_M) + \mathcal{A}^T\tilde{U}\mathcal{A} \\
 &\quad + \varphi^T(t)\hat{K}^T\theta_1^TE^TG^T\tilde{U}GE\theta_1\hat{K}\varphi(t) \\
 &\quad + h(u(t))^T\theta_2^TE^TG^T\tilde{U}GE\theta_2h(u(t)) \\
 &\quad + \mu_1^T(t)\Sigma_1\mu_1(t) + \mu_2^T(t)\Sigma_2\mu_2(t) \\
 &\quad + \eta^T(t - d(t))\varepsilon\Upsilon\eta(t - d(t)) - \varrho^T(t)\varepsilon\lambda\Upsilon\varrho(t) \\
 &\quad + \varsigma_1\eta^T(t)\Omega_1^T\Omega_1\eta(t) - \varsigma_1\bar{f}^T(\eta(t))\bar{f}(\eta(t)) \\
 &\quad + \varsigma_2\mathcal{B}^T\Omega_2^T\Omega_2\mathcal{B} + \varsigma_2\varphi^T(t)\theta^T\Omega_2^T\Omega_2\theta\varphi(t) \\
 &\quad - \varsigma_2 h^T(u(t))h(u(t)) + \varepsilon^2\bar{\eta}^T(t)\bar{\eta}(t) - \varepsilon\varphi^T(t)\varphi(t) \\
 &= \zeta^T(t)(\bar{\Phi}_{11} + \Psi_5 + \Psi_6)\zeta(t). \tag{34}
 \end{aligned}$$

where

$$\mu_1(t) = [\eta^T(t) \quad \eta^T(t - d(t)) \quad \eta^T(t - d_M)]^T$$

$$\mu_2(t) = [\eta^T(t) \quad \eta^T(t - \tau(t)) \quad \eta^T(t - \tau_M)]^T$$

$$\Sigma_1 = \begin{bmatrix} -U_1 & * & * \\ U_1 & -2U_1 & * \\ 0 & U_1 & -U_1 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} -U_2 & * & * \\ U_2 & -2U_2 & * \\ 0 & U_2 & -U_2 \end{bmatrix}$$

$$\begin{aligned}
 \zeta(t) &= \text{col}\{\eta(t), \bar{f}(\eta(t)), \eta(t - \tau(t)), \eta(t - d(t)), \varrho(t), \\
 &\quad \varphi(t), h(u(t)), \eta(t - \tau_M), \eta(t - d_M)\},
 \end{aligned}$$

$$\bar{\Phi}_{11} = \begin{bmatrix} \bar{\Lambda}_{11} & * & * \\ \bar{\Lambda}_{21} & \bar{\Lambda}_{22} & * \\ \bar{\Lambda}_{31} & \bar{\Lambda}_{32} & \bar{\Lambda}_{33} \end{bmatrix},$$

$$\begin{aligned}
 \bar{\Lambda}_{11} &= P(\bar{A} + C_1M \otimes \bar{\Gamma}_1) + (\bar{A} + C_1M \otimes \bar{\Gamma}_1)^T P + R_1 \\
 &\quad + R_2 - U_1 - U_2 + \varsigma_1\Omega_1^T\Omega_1,
 \end{aligned}$$

$$\bar{\Lambda}_{21} = \text{col}\{B^T P, (C_2W \otimes \bar{\Gamma}_2)^T P + U_2, \hat{K}^T E^T G^T P + U_1\},$$

$$\bar{\Lambda}_{22} = \text{diag}\{-\varsigma_1 I, -2U_2, \Psi_1\},$$

$$\bar{\Lambda}_{31} = \text{col}\{\hat{K}^T E^T G^T P, \hat{K}^T \alpha^T E^T G^T P, \beta^T E^T G^T P, 0, 0\},$$

$$\bar{\Lambda}_{32} = [0_{5 \times 1} \quad \bar{\Lambda}_{322} \quad \bar{\Lambda}_{323}],$$

$$\bar{\Lambda}_{322} = \text{col}\{0, 0, 0, U_2, 0\}, \quad \bar{\Lambda}_{323} = \text{col}\{\varepsilon^2 I, 0, 0, 0, U_1\},$$

$$\bar{\Lambda}_{33} = \text{diag}\{\Psi_2, \Psi_3, \Psi_4, -U_2 - R_2, -U_1 - R_1\},$$

$$\begin{aligned} \Psi_1 &= \epsilon^2 I + \epsilon \Upsilon - 2U_1, & \Psi_2 &= \epsilon^2 I - \epsilon \lambda \Upsilon, \\ \Psi_3 &= \hat{K} \theta_1^T E^T G^T \tilde{U} G E \theta_1 \hat{K} + \varsigma_2 \theta_1^T \Omega_2^T \Omega_2 \theta_1 - \epsilon I, \\ \Psi_4 &= \theta_2^T E^T G^T \tilde{U} G E \theta_2 - \varsigma_2 I, & \Psi_5 &= \vartheta_1 \tilde{U} \vartheta_1^T, \\ \Psi_6 &= \varsigma_2 \vartheta_2 \Omega_2^T \Omega_2 \vartheta_2, \\ \vartheta_1 &= \text{col}\{(\bar{A} + C_1 M \otimes \bar{\Gamma}_1)^T, \bar{B}^T, (C_2 W \otimes \bar{\Gamma}_2)^T, (G E \hat{K})^T, \\ &\quad (G E \hat{K} \alpha)^T, \beta^T, 0, 0\}, \\ \vartheta_2 &= \text{col}\{0, 0, 0, \hat{K}^T, \hat{K}^T, (\hat{K} \alpha)^T, 0, 0, 0\}. \end{aligned}$$

On the basis of Lemma 3, it can be known that  $\tilde{\Phi}_{11} + \Psi_5 + \Psi_6 < 0$  is equivalent to the matrix  $\Phi < 0$ . According to (24),  $\Phi < 0$  holds. Therefore,  $\tilde{\Phi}_{11} + \Psi_5 + \Psi_6 < 0$  holds. Thus, we have  $E\{\dot{V}(t)\} < 0$ , such that the system (19) is asymptotically stable. The end of the proof.

*Remark 5:* In (25), the constructed Lyapunov functional contains both the double integrals terms of time-delays and the threshold parameter states of triggers. Compared with the existing literature such as [44], where the Lyapunov functional only considered basic one-fold integrals in deriving the sufficient condition of systems stability, the obtained results in Theorem 1 are expected to be less conservative.

Theorem 1 only provides a sufficient condition for the stability of the system (19) and does not solve the design of the event-triggered controller. Based on this, Theorem 2 provides a design method of controller gains  $K_i (i = 1, 2, \dots, N)$  and trigger matrices  $\Upsilon_i (i = 1, 2, \dots, N)$ .

*Theorem 2:* For given scalars  $\alpha_i, \beta_i (i = 1, \dots, N)$ ,  $\nu_i > 0 (i = 1, 2)$ ,  $c_{1i}, c_{2i}, (i = 1, \dots, N)$ ,  $\varsigma_i (i = 1, 2)$ ,  $\iota_i (i = 1, 2)$ ,  $\sigma_i (i = 1, 2)$ ,  $d_M = \max_N \{d_M^i\}$ ,  $\tau_M = \max_N \{\tau_i\}$ ,  $0 < \epsilon < 1$  and matrices  $\bar{\Gamma}_i > 0 (i = 1, 2)$ ,  $\bar{A} > 0, \bar{B} > 0$ ,  $G = \text{diag}_N \{g_i\}$ ,  $\bar{E} > 0$ ,  $\Omega_i (i = 1, 2)$ ,  $Z = \text{diag}_N \{Z_i\}$ ,  $T = \text{diag}_N \{T_i\}$ ,  $\epsilon$  and  $\lambda$ , the system (19) is asymptotically stable, if there exist matrices  $Y = \text{diag}_N \{Y_i\}$ ,  $\bar{R}_i (i = 1, 2)$ ,  $\tilde{U}_i (i = 1, 2)$ ,  $X, \tilde{\Upsilon}$ , such that the following LMI holds

$$\Pi = \begin{bmatrix} \tilde{\Phi} & * & * & * & * \\ \Pi_1 & -\iota_1 I & * & * & * \\ \Pi_2 & 0 & -\iota_1 I & * & * \\ \Pi_2 & 0 & 0 & -\iota_2 I & * \\ \Pi_2 & 0 & 0 & 0 & -\iota_2 I \end{bmatrix} < 0, \quad (35)$$

where

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Phi}_{11} & * & * & * & * & * & * & * \\ \tilde{\Phi}_{21} & \hat{U} & * & * & * & * & * & * \\ \tilde{\Phi}_{31} & 0 & \hat{U} & * & * & * & * & * \\ \tilde{\Phi}_{41} & 0 & 0 & \hat{U} & * & * & * & * \\ \tilde{\Phi}_{51} & 0 & 0 & 0 & -\varsigma_2 I & * & * & * \\ \tilde{\Phi}_{61} & 0 & 0 & 0 & 0 & -\varsigma_2 I & * & * \\ \tilde{\Phi}_{71} & 0 & 0 & 0 & 0 & 0 & -I & * \\ \tilde{\Phi}_{81} & 0 & 0 & 0 & 0 & 0 & 0 & -\varsigma_1 I \end{bmatrix},$$

$$\tilde{\Phi}_{11} = \begin{bmatrix} \Phi_{111} & * & * & * & * \\ \tilde{\Phi}_{112} & \Xi_1 & * & * & * \\ \tilde{\Phi}_{113} & 0 & -\varsigma_2 I & * & * \\ \tilde{\Phi}_{114} & 0 & 0 & -\bar{U}_2 - \bar{R}_2 & * \\ \tilde{\Phi}_{115} & 0 & 0 & 0 & -\bar{U}_1 - \bar{R}_1 \end{bmatrix},$$

$$\tilde{\Phi}_{111} = \begin{bmatrix} \tilde{\Lambda} & * & * & * & * \\ B^T & -\varsigma_1 I & * & * & * \\ \Xi_2 & 0 & -2\bar{U}_2 & * & * \\ \Xi_3 & 0 & 0 & -2\bar{U}_1 + \epsilon \tilde{\Upsilon} & * \\ \Xi_4 & 0 & 0 & 0 & -\epsilon \lambda \tilde{\Upsilon} \end{bmatrix},$$

$$\tilde{\Lambda} = (\bar{A} + C_1 M \otimes \bar{\Gamma}_1) Y + Y (\bar{A} + C_1 M \otimes \bar{\Gamma}_1)^T + \bar{R}_1 + \bar{R}_2 - \bar{U}_1 - \bar{U}_2,$$

$$\tilde{\Phi}_{112} = [X^T \alpha^T E^T G^T \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\tilde{\Phi}_{113} = [\beta^T E^T G^T \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\tilde{\Phi}_{114} = [0 \quad 0 \quad \bar{U}_2 \quad 0 \quad 0],$$

$$\tilde{\Phi}_{115} = [0 \quad 0 \quad 0 \quad \bar{U}_1 \quad 0], \quad \Xi_1 = \epsilon(-2\sigma_2 Y + \sigma^2 I),$$

$$\Xi_2 = Y(C_2 W \otimes \bar{\Gamma}_2)^T + \bar{U}_2, \quad \Xi_3 = X^T E^T G^T + \bar{U}_1,$$

$$\Xi_4 = X^T E^T G^T, \quad \tilde{\Phi}_{21} = [\tilde{\Phi}_{211} \quad \tilde{\Phi}_{212}],$$

$$\tilde{\Phi}_{211} = [(\bar{A} + C_1 M \otimes \bar{\Gamma}_1) Y \quad B \quad C_2 W \otimes \bar{\Gamma}_2 Y \quad G E X],$$

$$\tilde{\Phi}_{212} = [G E X \quad G E \alpha X \quad G E \beta \quad 0 \quad 0],$$

$$\tilde{\Phi}_{31} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad G E \theta_1 X \quad 0 \quad 0 \quad 0],$$

$$\tilde{\Phi}_{41} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad G E \theta_2 \quad 0 \quad 0],$$

$$\tilde{\Phi}_{51} = [0 \quad 0 \quad 0 \quad \varsigma_2 \Omega_2 X \quad \varsigma_2 \Omega_2 X \quad \varsigma_2 \Omega_2 \alpha X \quad 0 \quad 0 \quad 0],$$

$$\tilde{\Pi}_{61} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \varsigma_2 \Omega_2 \theta_1 X \quad 0 \quad 0 \quad 0],$$

$$\tilde{\Phi}_{71} = [0 \quad 0 \quad 0 \quad \epsilon Y \quad \epsilon Y \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\tilde{\Phi}_{81} = [\varsigma_1 \Omega_1 Y \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\hat{U} = -2\sigma_2 Y + \sigma_2^2 \tilde{U},$$

$$\Pi_1 = [0 \quad 0 \quad 0 \quad \iota_1 Z Y \quad \iota_1 Z Y \quad 0_9],$$

$$\Pi_2 = [T^T E^T G^T \quad 0_8 \quad T^T E^T G^T \quad 0 \quad 0 \quad T^T \Omega_2^T \quad 0],$$

$$\Pi_3 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \iota_2 Z Y \quad 0_8],$$

$$\Pi_4 = [\Pi_{41} \quad \Pi_{42}].$$

$$\Pi_{41} = [T^T \alpha^T E^T G^T \quad 0_8 \quad T^T \alpha^T E^T G^T].$$

$$\Pi_{42} = [T^T \theta_1^T E^T G^T \quad 0 \quad T^T \alpha^T \Omega_2^T \quad T^T \theta_1^T \Omega_2^T].$$

Furthermore, the gain matrices and trigger matrices are designed by

$$K_i = X_i Y_i^{-1}, \quad \Upsilon_i = Y_i^{-1} \tilde{\Upsilon}_i Y_i^{-1}, \quad i = 1, 2, \dots, N. \quad (36)$$

*Proof:* According to the congruence transformation of the matrix, it can be known that the matrix inequality (24) holds, if and only if the following inequality (37) holds.

$$\begin{bmatrix} \Phi_{11} & * \\ \tilde{\Lambda}_{21} & \tilde{\Lambda}_{22} \end{bmatrix} < 0, \quad (37)$$

where

$$\tilde{\Lambda}_{21} = \text{col}\{\tilde{\Phi}_{21}, \tilde{\Phi}_{31}, \tilde{\Phi}_{41}, \Phi_{51}, \Phi_{61}, \Phi_{71}, \Phi_{81}\},$$

$$\tilde{\Lambda}_{22} = \text{diag}\{\tilde{\Lambda}_{221}, \tilde{\Lambda}_{222}\},$$

$$\tilde{\Lambda}_{221} = \text{diag}\{-P\tilde{U}P, -P\tilde{U}P, -P\tilde{U}P\},$$

$$\tilde{\Lambda}_{222} = \text{diag}\{-\varsigma_2 I, -\varsigma_2 I, -I, -\varsigma_1 I\},$$

$$\tilde{\Phi}_{21} = [\tilde{\Phi}_{211} \quad \tilde{\Phi}_{212}],$$

$$\tilde{\Phi}_{211} = [P(\bar{A} + C_1 M \otimes \bar{\Gamma}_1) \quad PB \quad PC_2 W \otimes \bar{\Gamma}_2],$$

$$\tilde{\Phi}_{212} = [PGE\hat{K} \quad PGE\hat{K} \quad PGE\alpha\hat{K} \quad PGE\beta \quad 0 \quad 0],$$

$$\begin{aligned} \bar{\Phi}_{31} &= [0 \ 0 \ 0 \ 0 \ 0 \ PGE\theta_1\hat{K} \ 0 \ 0 \ 0], \\ \bar{\Phi}_{41} &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ PGE\theta_2 \ 0 \ 0]. \end{aligned}$$

For any positive scalar  $\sigma$ , we have

$$(\tilde{U} - \sigma^{-1}P)\tilde{U}^{-1}(\tilde{U} - \sigma^{-1}P) \geq 0, \quad (38)$$

which yields

$$-P\tilde{U}^{-1}P \leq -2\sigma P + \sigma^2\tilde{U}. \quad (39)$$

Thus, taking the place of  $-P\tilde{U}^{-1}P$  by  $-2\sigma P + \sigma^2\tilde{U}$  in (37), it can be verified that (37) holds if the following matrix inequality holds.

$$\bar{\Pi} = \begin{bmatrix} \Phi_{11} & * & * & * & * & * & * & * & * \\ \bar{\Phi}_{21} & \hat{U} & * & * & * & * & * & * & * \\ \bar{\Phi}_{31} & 0 & \hat{U} & * & * & * & * & * & * \\ \bar{\Phi}_{41} & 0 & 0 & \hat{U} & * & * & * & * & * \\ \Phi_{51} & 0 & 0 & 0 & -\varsigma_2 I & * & * & * & * \\ \Phi_{61} & 0 & 0 & 0 & 0 & -\varsigma_2 I & * & * & * \\ \Phi_{71} & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ \Phi_{81} & 0 & 0 & 0 & 0 & 0 & 0 & -\varsigma_1 I & * \end{bmatrix} < 0. \quad (40)$$

Based on the product of the matrix, one can rewrite the matrix  $\bar{\Pi}$  in the following form.

$$\bar{\Pi} = \hat{\Pi} + \hat{\Pi}_1^T F(t) \hat{\Pi}_2 + \hat{\Pi}_2^T F^T(t) \hat{\Pi}_1 + \hat{\Pi}_3^T F(t) \hat{\Pi}_4 + \hat{\Pi}_4^T F^T(t) \hat{\Pi}_3, \quad (41)$$

where

$$\begin{aligned} \hat{\Pi} &= \bar{\Pi}|_{\hat{K}=K}, \quad \hat{\Pi}_1 = [0 \ 0 \ 0 \ Z \ Z \ 0_9], \\ \hat{\Pi}_2 &= [\hat{\Pi}_{21} \ \hat{\Pi}_{22}], \\ \hat{\Pi}_{21} &= [T^T E^T G^T P \ 0_8 \ T^T E^T G^T P], \\ \hat{\Pi}_{22} &= [0 \ 0 \ T^T \Omega_2^T \ 0], \\ \hat{\Pi}_3 &= [0 \ 0 \ 0 \ 0 \ 0 \ Z \ 0_8], \quad \hat{\Pi}_4 = [\hat{\Pi}_{41} \ \hat{\Pi}_{42}], \\ \hat{\Pi}_{41} &= [T^T \alpha^T E^T G^T P \ 0_8 \ T^T \alpha^T E^T G^T P], \\ \hat{\Pi}_{42} &= [T^T \theta_1^T E^T G^T P \ 0 \ T^T \alpha^T \Omega_2^T \ T^T \theta_1^T \Omega_2^T]. \end{aligned}$$

By using Lemma 2, for any  $\iota_i > 0 (i = 1, 2)$ , we know that

$$\bar{\Pi} \leq \hat{\Pi} + \iota_1 \hat{\Pi}_1^T \hat{\Pi}_1 + \iota_1^{-1} \hat{\Pi}_2^T \hat{\Pi}_2 + \iota_2 \hat{\Pi}_3^T \hat{\Pi}_3 + \iota_2^{-1} \hat{\Pi}_4^T \hat{\Pi}_4. \quad (42)$$

On the basis of (42), (40) holds if the following inequality holds

$$\hat{\Pi} + \iota_1 \hat{\Pi}_1^T \hat{\Pi}_1 + \iota_1^{-1} \hat{\Pi}_2^T \hat{\Pi}_2 + \iota_2 \hat{\Pi}_3^T \hat{\Pi}_3 + \iota_2^{-1} \hat{\Pi}_4^T \hat{\Pi}_4 < 0. \quad (43)$$

Then employing the Schur complement to (43), one has

$$\check{\Pi} = \begin{bmatrix} \hat{\Pi} & * & * & * & * \\ \hat{\Pi}_1 & -\iota_1 I & * & * & * \\ \hat{\Pi}_2 & 0 & -\iota_1 I & * & * \\ \hat{\Pi}_3 & 0 & 0 & -\iota_2 I & * \\ \hat{\Pi}_4 & 0 & 0 & 0 & -\iota_2 I \end{bmatrix} < 0. \quad (44)$$

Letting  $P = \text{diag}_N\{P_i\}$ , and defining  $P^{-1} = Y$ ,  $P_i^{-1} = Y_i$ , pre- and post-multiplying the both sides of (44) by

$\text{diag}\{R, I, I, I, I, I, I, I, I\}$  with  $R = \text{diag}\{Y, I, Y, Y, Y, Y, I, Y, Y, Y, Y, Y\}$ , and denoting  $X = KY$ ,  $\Upsilon = \text{diag}_N\{\Upsilon_i\}$ ,  $\bar{R}_i = YR_iY (i = 1, 2)$ ,  $\bar{U}_i = YU_iY (i = 1, 2)$ ,  $\bar{\Upsilon}_i = Y_i\Upsilon_iY_i$ , it yields

$$\check{\Pi} = \begin{bmatrix} \check{\Pi} & * & * & * & * \\ \Pi_1 & -\iota_1 I & * & * & * \\ \Pi_2 & 0 & -\iota_1 I & * & * \\ \Pi_2 & 0 & 0 & -\iota_2 I & * \\ \Pi_2 & 0 & 0 & 0 & -\iota_2 I \end{bmatrix} < 0, \quad (45)$$

where

$$\check{\Pi} = \begin{bmatrix} \check{\Phi}_{11} & * & * & * & * & * & * & * & * \\ \check{\Phi}_{21} & \hat{U} & * & * & * & * & * & * & * \\ \check{\Phi}_{31} & 0 & \hat{U} & * & * & * & * & * & * \\ \check{\Phi}_{41} & 0 & 0 & \hat{U} & * & * & * & * & * \\ \check{\Phi}_{51} & 0 & 0 & 0 & -\varsigma_2 I & * & * & * & * \\ \check{\Phi}_{61} & 0 & 0 & 0 & 0 & -\varsigma_2 I & * & * & * \\ \check{\Phi}_{71} & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ \check{\Phi}_{81} & 0 & 0 & 0 & 0 & 0 & 0 & -\varsigma_1 I & * \end{bmatrix},$$

$$\check{\Phi}_{11} = \begin{bmatrix} \Phi_{111} & * & * & * & * \\ \check{\Phi}_{112} & -\epsilon YY & * & * & * \\ \check{\Phi}_{113} & 0 & -\varsigma_2 I & * & * \\ \check{\Phi}_{114} & 0 & 0 & -\bar{U}_2 - \bar{O}_2 & * \\ \check{\Phi}_{115} & 0 & 0 & 0 & -\bar{U}_1 - \bar{O}_1 \end{bmatrix}.$$

Similar to (39), for any  $\sigma_2 > 0$ , one has

$$-\epsilon YY \leq \epsilon(-2\sigma_2 Y + \sigma_2^2 I). \quad (46)$$

Replacing  $-\epsilon YY$  by  $\epsilon(-2\sigma_2 Y + \sigma_2^2 I)$ , then (45) holds only if (35) holds. To sum up, we can conclude that the LMI (35) is a sufficient condition of the matrix inequality (24), which means that the designed event-based controller can guarantee the asymptotically stable of the system (19). Moreover, we can get  $K = XY^{-1}$  and  $\Upsilon = Y^{-1}\bar{\Upsilon}Y^{-1}$ , that is,  $K_i = X_i Y_i^{-1}$  and  $\Upsilon_i = Y_i^{-1}\bar{\Upsilon}_i Y_i^{-1}$ . The end of the proof.

#### IV. SIMULATION EXAMPLE

Here, one provides an example to verify the effectiveness of above theoretical results. Consider the systems (1) and (3) with four nodes, and the pinning matrix is chosen as  $G = \text{diag}\{2.2, 3.4, 0, 0\}$ . Moreover, we choose the coupled configuration matrices and inner coupled matrices of the two systems as following:

$$M = \begin{bmatrix} -1.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & -1.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -1.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & -1.5 \end{bmatrix},$$

$$W = \begin{bmatrix} -2.4 & 0.8 & 0.8 & 0.8 \\ 0.8 & -2.4 & 0.8 & 0.8 \\ 0.8 & 0.8 & -2.4 & 0.8 \\ 0.8 & 0.8 & 0.8 & -2.4 \end{bmatrix},$$

$$\Gamma_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}.$$



In this example, one chooses the following functions  $f(x_i(t))$  and  $h(u_i(t))$  to describe the nonlinearity and network attack, respectively.

$$f(x_i(t)) = \begin{bmatrix} x_{i1}(t)\sin(0.15x_{i1}(t)) \\ 0.1x_{i2}(t)\sin(0.15x_{i1}(t)) \end{bmatrix},$$

$$h(u_i(t)) = \begin{bmatrix} 0.1\tanh(x_{i1}(t)) \\ 0.12\tanh(x_{i2}(t)) \end{bmatrix}.$$

Then, we can calculate the upper bound of  $f(\cdot)$  and  $g(\cdot)$  as

$$\Omega_1 = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.12 \end{bmatrix},$$

respectively.

Other parameter matrices of the systems are given as

$$A_1 = \begin{bmatrix} -0.9 & 0.26 \\ 0.9 & -0.26 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.7 & 0.12 \\ 0.7 & -0.12 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.98 & 0.8 \\ 0.98 & -0.8 \end{bmatrix}, \quad A_4 = \begin{bmatrix} -0.34 & 0.9 \\ 0.34 & -0.9 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1.2 & -0.1 \\ 0 & 0.4 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2.3 & 3.7 \\ 0 & 0.1 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 1.5 & 0 \\ -0.1 & 0.2 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 4.6 & 0.32 \\ -2.1 & 0.45 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.17 & 0 \\ 0 & 0.11 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.12 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 0.11 & 0 \\ 0 & 0.14 \end{bmatrix}.$$

The saturation function is chosen as

$$\rho(\bar{\eta}_i) = \begin{cases} 0.05, & \bar{\eta}_j \geq 0.05 \\ \bar{\eta}_j, & -0.05 < \bar{\eta}_j < 0.05, \\ -0.05, & -\bar{\eta}_j \leq -0.05 \end{cases}$$

and the uncertain parameter matrices are selected as

$$T_1 = \begin{bmatrix} -0.045 & 0.1 \\ 0.23 & -0.05 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0.062 & 0.13 \\ 0.3 & -0.01 \end{bmatrix},$$

$$T_3 = \begin{bmatrix} 0.092 & 0.2 \\ 0.2 & 0.07 \end{bmatrix}, \quad T_4 = \begin{bmatrix} 0.011 & 0.14 \\ 0.1 & 0.035 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 0.24 & 0 \\ 0 & 0.34 \end{bmatrix}, \quad Z_2 = \begin{bmatrix} 0.36 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$Z_3 = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad Z_4 = \begin{bmatrix} 0.24 & 0 \\ 0 & 0.84 \end{bmatrix}.$$

We set the expectations of the probabilities of stochastic saturations and network attacks as  $\alpha_i = 0.32(i = 1, 2, 3, 4)$  and  $\beta_i = 0.85(i = 1, 2, 3, 4)$ , respectively. The coupled strengths are  $c_{11} = 0.4, c_{12} = 0.21, c_{13} = 0.42, c_{14} = 0.97, c_{21} = 0.8, c_{22} = 0.56, c_{23} = 0.69, c_{24} = 0.81$ . Other relevant parameters are  $\varsigma_1 = 8.2, \varsigma_2 = 7.7, \epsilon = 0.9, \iota_1 = 6.31, \iota_2 = 1.53, \sigma_1 = 0.18, \sigma_2 = 0.8$ . For the given bounds of delay  $d_M = 0.02, \tau_M = 0.05$ , and adaptive event-triggered parameter scalar  $\varepsilon_i = 0.3(i = 1, 2, 3, 4), \lambda_i = 60(i = 1, 2, 3, 4)$ , one obtains the gains of the controller and AETS as below by utilizing the MATLAB to solve Theorem 2,

$$K_1 = \begin{bmatrix} -1.1945 & -1.1179 \\ -0.8697 & -1.3348 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.8462 & -0.3243 \\ -0.1966 & -1.2964 \end{bmatrix},$$

TABLE 1. Results of triggers with different cases.

	Node 1	Node 2	Node 3	Node 4
$\varepsilon_i = 0.01$	21	20	45	48
$\varepsilon_i = 0.1$	19	16	43	51
$\varepsilon_i = 0.3$	16	18	44	28
Total trigger times	56	54	132	127

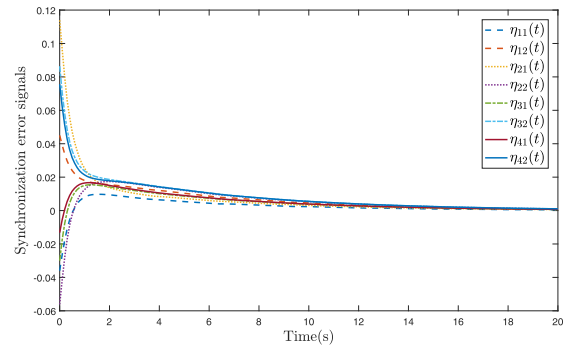


FIGURE 2. Synchronization errors  $\eta_i(t)(i = 1, 2, 3, 4)$  under control.

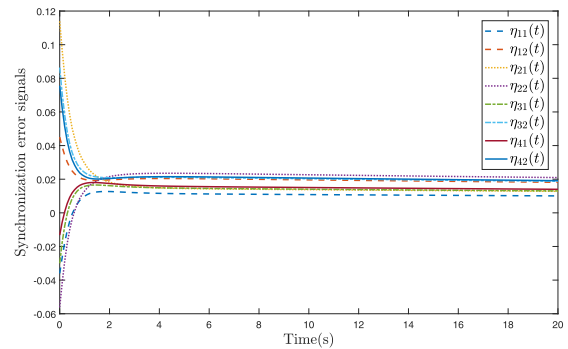


FIGURE 3. Synchronization errors  $\eta_i(t)(i = 1, 2, 3, 4)$  without control.

$$\Upsilon_1 = \begin{bmatrix} 1.1773 & -0.1139 \\ -0.1139 & 0.6507 \end{bmatrix}, \quad \Upsilon_2 = \begin{bmatrix} 1.0623 & -0.2758 \\ -0.2758 & 0.9019 \end{bmatrix},$$

$$\Upsilon_3 = \begin{bmatrix} 0.9411 & -0.4324 \\ -0.4324 & 0.8171 \end{bmatrix}, \quad \Upsilon_4 = \begin{bmatrix} 0.5367 & -0.2436 \\ -0.2436 & 0.9611 \end{bmatrix}.$$

Choose the initial state of the system (1) as  $x_1^T(0) = [0.37 \quad -0.52]^T, x_2^T(0) = [0.01 \quad -0.17]^T, x_3^T(0) = [0.41 \quad -0.88]^T, x_4^T(0) = [0.24 \quad -0.66]^T$ , and the initial state of the system (3) as  $y_1^T(0) = [0.07 \quad -0.07]^T, y_2^T(0) = [0.68 \quad -0.68]^T, y_3^T(0) = [0.16 \quad -0.16]^T, y_4^T(0) = [0.12 \quad -0.12]^T$ . Moreover, the initial values of the trigger threshold are selected as  $\xi_1(0) = 0.401, \xi_2(0) = 0.39, \xi_3(0) = 0.42, \xi_4(0) = 0.426$ . According to the gains obtained above, we can get the following simulation results. Table 1 gives the detailed trigger times for the four nodes in the different parameters of  $\varepsilon_i$ . It is clearly that the trigger times of the controlled node 1 and node 2 are less than the other two uncontrolled nodes, which are reasonable. Figs. 2-3 display the responses of synchronization errors with and without control input, respectively. Based on these two figures, it is easily to find that output synchronization errors converge

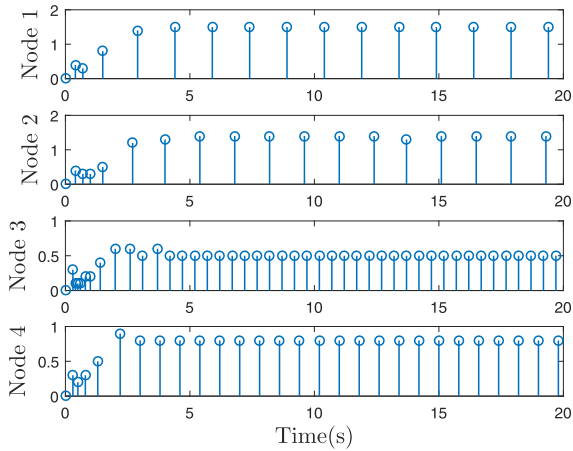


FIGURE 4. The adaptive event-triggered instants and intervals.

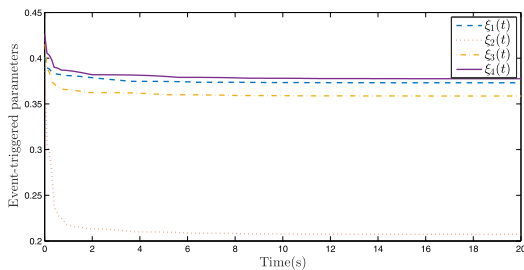


FIGURE 5. Trajectories of event-triggered parameters  $\xi_i(t)(i = 1, 2, 3, 4)$ .

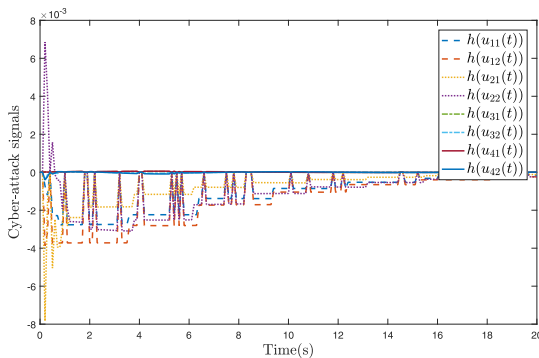


FIGURE 6. Trajectories of attack functions  $h(u_i(t))(i = 1, 2, 3, 4)$ .

to zero in the case of employing the designed control input signals. Fig. 4 depicts the released intervals and instants of the AETS. Fig. 5 gives the trajectories of the adaptive event-triggered threshold parameter. Fig. 6 displays the curves of the attack functions  $h(u_i(t))$ . These results demonstrate that the design method of the non-fragile controllers proposed is effective.

### V. CONCLUSION

In this paper, we have investigated the adaptive output pinning synchronization control for delayed complex networks with random saturations and cyber-attacks. Different from the traditional event triggered scheme, we use the adaptive law to flexibly adjust the threshold of the event triggered condition. The sufficient conditions for the stability of related systems are derived in terms of stochastic analysis technique and

Lyapunov stability method. By using LMI method, the gains of the controller and AETS have been got simultaneously. One key point of this paper is that the AETS-based output pinning synchronization control problem is firstly studied for a class of delayed complex networks with random saturations and cyber-attacks. In our future work, we will focus on the pinning synchronization control for the drive-response complex systems with adaptive coupling strength.

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