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Submarine Cable Path Planning Based on Weight Selection of Design Considerations

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ABSTRACT Submarine cables are indispensable in today's international data transmission. In the process of submarine cable path planning, any factors that may potentially affect reliability and cost should be considered. Still, the degree of importance of these factors is difficult to assess accurately. Currently, cable path planning is done manually, meter by meter, over thousands of kilometers by experts that rely on their experience and expertise. This paper provides a submarine cable path planning algorithm based on simulated annealing (SA) and the fast marching method (FMM). Accordingly, we name the algorithm FMM/SA. FMM/SA can be used as a guide and benchmark for cable path planners and also enhances understanding of the multiple considerations and their corresponding weights aiming to further improve the end-results beyond what is obtained currently by experts. In FMM/SA, SA is used to optimize weights of design considerations to minimize the Fréchet distance between existing cable paths and paths with minimized total life-cycle cost obtained by FMM. FMM/SA is demonstrated to be superior to two other algorithms based on random-restart hill-climbing and Monte Carlo using real-life cable paths.

INDEX TERMS Submarine cable path planning, combinatorial optimization, simulated annealing, fast marching method, design considerations, Fréchet distance.

I. INTRODUCTION

With recent technology push and demand pull mainly linked to the introduction of 5G technology and the COVID-19 outbreak, we have seen continuous growth in global data and network traffic. For example, in Europe, the total traffic in internet exchange points has increased by 15-20% [1]. In the foreseeable future, bandwidth demand is expected to double almost every two years [1]. This demand provides opportunities for the submarine optical fiber industry, as the submarine cables carry 99% of international data exchange [2].

With global demand increasing rapidly, sustaining capacity growth of submarine cables will be challenging, potentially causing demand to exceed supply. During the period of 2016-2020, the submarine cable design capacity of major routes increased at a compound annual growth rate

of 26.4% [2]. As of early 2021, there are over 1.3 million kilometers of submarine cables in service worldwide [3]. The industry has been able to keep up with the demand, but it will be necessary to continue focusing on increasing capacity to continue to meet the growing demand. It is estimated that the global production capacity of submarine cables will increase 100% by the end of 2023 [2].

However, the laying and maintenance of submarine cables are costly. According to [4], over 8.9 billion U.S. dollars has been spent on submarine cables from 2016 to 2020. It is estimated that there are already plans for 8.8 billion U.S. dollars to be invested in the laying of new submarine cables in the next three years [4]. Meanwhile, these vast submarine cable systems are prone to faults. Specifically, there are more than 100 faults each year [5]. The Hengchun earthquake in 2006 caused several submarine cables to break [6], resulting in the blockage of the internet and international telephone services in East Asia. In 2013, three divers cut the

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main cable connecting Egypt and Europe, reducing Egypt's internet bandwidth by 60% [7]. Regardless of the cause, the failure of the submarine cable may lead to serious social consequences. For example, in time-sensitive financial operations, a communication interruption of minutes or hours can be very costly. Even a loss of a fraction of a second can cause millions of dollars in losses for financial transactions.

To design a cost-effective and resilient submarine cable network, we need to consider various factors that may cause the submarine cable to break, including natural and anthropological activities [8]. Dinmohammadi *et al.* [9] proposed a modeling method that can predict the wear of submarine cables based on the various submarine environmental conditions and can further evaluate the expected life of the submarine cables. In reality, a resilient and cost-effective submarine cable's path design is achievable by considering a range of factors. Industry experts conduct multiple routes and engineering surveys and constantly modify the cable path meter by meter manually to balance the various considerations. This process is highly time-consuming and expensive. It is conceivable that the more comprehensive the considerations are, the more economical, stable, and reliable the submarine cable will be. However, various considerations are usually not coordinated and may conflict with each other [10].

In addition to basic construction cost consideration (including cable length) as well as considerations for cable resilience (including geological hazards like earthquakes and volcano eruptions, anthropological hazards like fishing and anchoring activities), there are other cable design considerations that are taken into account in cable path planning. Such considerations include but not limited to restricted areas, existing cables/pipelines, seabed slope, water depth, shield level for cables. For more information on cable design considerations, see [11]. See also Section V-B where we list the considerations that we apply to our realistic cable path applications.

In this paper, we first use a real-life existing submarine cable that has a history of resilience, and as they were designed by experts, they are also cost effective. Using this cable path, we derive design considerations' weights. We use discrete Fréchet distance [12] to measure the similarity between the actual submarine cable path and the submarine cable paths designed using the fast marching method (FMM) [13], [14] algorithm. Then, we demonstrate on a second real-life existing cable, or cable segment, that our cable path planning algorithm based on the design considerations and their corresponding weights produces a cable path that is close to that of the second cable that we consider. Using our cable path algorithm, we can design a submarine cable path automatically that satisfies all the considerations with realistic priorities, which will benefit future submarine cable path planning. Path planners can then use the cable path obtained by our algorithm as a guide and benchmark for their cable path planning. Compared with all the existing work, the contributions of this paper are as follows.

- 1) We use the ArcGIS software [15] to combine detailed information on existing submarine cable paths' data with information on the important design considerations described in [8] that include among others the basic construction cost, geological hazards, water depth, seabed slope, anthropological hazards, to consider their impact on the new submarine cable path planning. Moreover, we add a new constraint of altering course angle for FMM to account for the stability of a burying plough carried out by a remotely operated vehicle, further enhancing the result of [8].
- 2) For the first time, we propose a new algorithm that obtains weight values for the design considerations, and use these values for submarine cable path planning. Our new algorithm, which is based on simulated annealing (SA) [16], is named in this paper FMM/SA as it is an application of SA to the FMM algorithm. In FMM/SA, FMM is used to obtain the optimal submarine cable path with the lowest life-cycle cost, and SA is used to continuously adjust the weight of each design consideration with the aim to automatically provide a cable path that will be as close as possible to that of a real-life cable path.
- 3) We demonstrate our FMM/SA algorithm using the data of two real-life existing submarine cables designed by experts with a history of resilience and cost-effectiveness. The first cable path is used to obtain the weights of design considerations. Then the derived weights are put to use to design a cable path close to the second cable path. We compare FMM/SA with the other two algorithms based on the random-restart hill-climbing (RRHC) [17] and Monte Carlo (MC) [18] algorithms, called FMM/RRHC and FMM/MC, and demonstrate the superiority of FMM/SA, as evidenced by significantly lower results for the Fréchet distance for FMM/SA relative FMM/RRHC and FMM/MC, through comparisons using the second real-life cable path.

The remainder of this paper is organized as follows. In Section II, we review related research on cable path planning. In Section III, we formulate the problem of minimizing the Fréchet distance between the real world submarine cable path and the minimal total life-cycle cost paths derived by the FMM algorithm. In Section IV, we provide the detailed information of FMM/SA to solve the problem described in Section III. In Section V, we perform simulations on two real-life existing submarine cable paths to evaluate the performance of FMM/SA, FMM/RRHC, and FMM/MC. Finally, conclusions are drawn in Section VI.

II. BACKGROUND AND RELATED WORK

Currently, submarine cable path planning is done manually by experienced surveyors and designers, which is highly time-consuming and labor-intensive. In [19], Burnett *et al.* guided submarine cable routing in industry by providing

the crucial principles for path selection to lay and maintain submarine cables. An initial route is designed by considering the existing area's relevant data, such as earthquakes, volcanoes, water depths, seabed slopes, and the like. Then the experts will repeatedly continue to survey and investigate the neighboring sea area of this path to further improve the path. The Makai software [20] is also widely used to assist experts in the initial stages of cable route planning by providing all available data required for submarine cable path planners to conduct a desktop study on the preliminary selection of submarine cables.

There has also significant fundamental research done on submarine cable path planning, considering earthquakes as a factor that causes the submarine cable to break. Msongaleli *et al.* [21] used integer linear programming to optimize the submarine cable path, by minimizing cost subject to constraints that limit cables access to earthquake prone areas. In [22], a dynamic programming method was used to minimize the link breakage probability in the submarine cable network. Zhao *et al.* [23] considered the trade-off between the cost and robustness of the submarine cable path and used the Dijkstra's algorithm [24] to solve cable laying path selection based on seismic data under arbitrary terrain. In [25], curvature constraints are considered since the bending stiffness of the submarine cable or pipeline, and the maneuverability of the ploughs are non-negligible during the path planning process. In [26]–[28], geometric methods were used to evaluate the worst locations that may be most affected by disasters in a given area to avoid being selected in submarine cable path planning. The work in [29]–[31] used a disk failure model on a two-dimensional plane to assume the disaster's scope, and based on this, proposed a path planning program with the least cable cost under the robustness constraint, and finally evaluated the network performance indicators. Oostenbrink and Kuipers [32] defined a pair of nodes and a link between these nodes as a link triple and proposed algorithms to find the cost-effective and disaster-aware link triples that form cable paths based on historical disaster data.

In [8], [33]–[36], Wang *et al.* established a more realistic and accurate model, using triangular two-dimensional manifolds in three-dimensional space to represent the earth's surface. The work in [34] proposed two goals - laying cost and the total number of cable repairs, and formulated the problem as a multi-objective shortest path problem. They used the Pareto frontier to minimize the cost at a given risk level, or the risk at a given cost level. In [35], a shield was added to a submarine cable section in higher risk areas. Wang *et al.* [37] studied the path planning of a submarine cable system with trunk and branch tree topology on the earth's surface. On this basis, the work in [36] considered the latency constraints of different node pairs in the submarine cable topology network and the work in [38] considered the cost of branching units and cable landing stations.

In [33], FMM was used to optimize the submarine cable's path under the premise of only considering the earthquake disaster. FMM is similar to Dijkstra's algorithm and finds an

optimal path by solving the Eikonal equation on a discretized spatial grid [39]. Firstly, a point on the spatial grid network is selected as the start point, and velocity functions defined on each spatial grid point are used to iteratively calculate the arrival time from the start point to each grid point by point. Unlike the Dijkstra's algorithm, FMM moves from the start point towards the end point using a wavefront representing the set of points with equal speed. This process stops when the end point is reached. Secondly, we trace back from the end point to the start point using the information of the shortest arrival time collected in the first step to generate the back-haul path, which is the shortest path.

Most of the existing work on path planning of submarine cables focuses on path optimization under a specific factor. In addition to the earthquake factor that is usually considered, there are many other factors that may affect the cost and reliability of submarine cables that should be considered. More extensive design considerations other than earthquakes were taken into account to solve the challenging submarine cable network problem in [8], where the weight values of different design considerations were derived based on industrial experts' opinions. Wang *et al.* [8] presented the submarine cable path with the lowest life-cycle cost considering all the factors contributing to the cost with specific cost functions. To date, no method can optimize the weight value of each consideration factor for the submarine cable path planning process, which is the main contribution of this paper.

For the combinatorial optimization problem of multiple variables in an extensive solution space, researchers have proposed various algorithms to obtain an acceptable solution within an acceptable time, including randomized and heuristic algorithms. The earliest idea of SA was proposed in [40]. In 1983, Kirkpatrick *et al.* [16] introduced SA into the field of combinatorial optimization as a randomized heuristic algorithm. Then SA was widely used in many cases, providing high quality solutions in an extensive solution space [41], [42]. In [41], a simulated-annealing-based bees algorithm was proved to achieve superior performance over several other baseline schedule algorithms in terms of energy saving for a single-objective constrained optimization problem. Varvarigos and Christodoulopoulos [42] proposed an SA meta-heuristic method to improve the solution of optical network planning problems.

SA is a greedy algorithm built on the hill-climbing (HC) algorithm [17] to select an improved solution from the solution space adjacent to the current solution as the new solution each time until a local optimal solution is reached. It can overcome the HC's weakness that HC is easily stopped in a local optimal solution by accepting the worse solution in the HC process with a certain probability. Specifically, if a moving direction can improve the objective function in SA, then this direction is chosen; otherwise, the selection of this direction, which makes the objective function worse, will be accepted with a certain probability. This probability is a function of the amount of change of the objective function's deterioration at this step. The greater the degree of deterioration, the smaller

the probability of accepting this direction. See Section IV for further information on SA and on the way it is implemented in this paper.

In this paper, we apply the FMM/SA algorithm to derive the weight value of each design consideration to automatically produce cost-effective and reliable real-life submarine cable paths.

III. MODELING AND PROBLEM FORMULATION

A. EARTH'S SURFACE MODELING

We model the earth's surface as a triangulated piecewise-linear 2-dimensional manifold \mathbb{M} in \mathbb{R}^3 to represent the target region \mathbb{T} . We use a 3-dimensional coordinate (x, y, z) to represent the point on \mathbb{M} , where $z = \xi(x, y)$ is the elevation corresponding to the geographic location (x, y) . See [8] for more details on the earth's surface modelling.

B. COST MODELS FOR DESIGN CONSIDERATIONS

We assume that there are K design considerations in the process of a submarine cable path planning and let $[K]$ represent the set $\{1, 2, \dots, K\}$. Then, each of the considerations is indexed by k for $k \in [K]$. Let $\mathcal{W} = \{w_1, w_2, \dots, w_k\}$, where the non-negative w_k is the weight of design consideration k and $\sum_{k=1}^K w_k = 1$. Let $C(X) = \sum_{k=1}^K w_k c_k(X)$ be the life-cycle cost per unit length of the cable passing through location X , where $c_k(X)$ represents the cost function of design consideration k at location X . By implementing the natural parametrization [43], the point X on the cable path λ (Lipschitz continuous [44]) can be formulated as $X = X(t)$, where t is the length of a very small arc segment. Then the total life-cycle cost of the cable path λ is:

$$\mathbb{C}(\lambda) = \int_0^{l(\lambda)} C(X(t)) dt, \quad (1)$$

where $l(\lambda)$ is the total length of cable path λ . Note that in this paper, a cost-effective cable path refers to the path that achieves the minimal total life-cycle cost \mathbb{C} which is based on a weighted average of costs associated with key design considerations over our triangulated manifold model of the earth surface.

C. MEASURE OF SIMILARITY BETWEEN CABLE PATHS

Fréchet proposed the well-known Fréchet distance in [45] to measure the similarity among curves by considering their spatial distances and the ordering of the points along the curves. Discrete Fréchet distance [12] is used in this paper to measure the similarity between the real-world cable path and the cable path generated under a given set of weight values for the design considerations.

To this end, we will need to define a mathematical structure that enables us to compare between cables generated by FMM with various alternative sets of weight values of the design considerations. We will start by recalling that every cable path generated by FMM is formed by a sequence of points. To compare between two cable paths formed by FMM we

consider two such paths defined by the curves U and V where U is the sequence of points $\{u_1, u_2, \dots, u_p\}$ and V is the sequence $\{v_1, v_2, \dots, v_q\}$. The number of points in the two sequences does not have to be the same, and we will define rules on multiple legal ways that we can pair points (u_i, v_k) where u_i is a point in U , and v_k is a point in V , and these pairs will be used in the comparison between U and V . If $p = q$, then the sequence of pairs s will be in the form as follows.

$$s = \{(u_1, v_1), (u_2, v_2), (u_3, v_3), \dots, (u_p, v_p)\}.$$

Now, assume, without loss of generality, that the number of points in U is larger than the number of points in V , namely, $p > q$. Then we have

$$s = \{(u_1, v_{a_1}), (u_2, v_{a_2}), (u_3, v_{a_3}), \dots, (u_p, v_{a_p})\}.$$

We will use the following two rules to generate the sequences of pairs:

- For any two points u_i and u_j in U , if $i < j$, then $a_i \leq a_j$.
- Every point v_j in V should be used to form a pair.

For example, if $U = \{u_1, u_2, u_3, u_4\}$ and $V = \{v_1, v_2, v_3\}$, then all the satisfied sequences of pairs are:

$$\begin{aligned} &\{(u_1, v_1), (u_2, v_1), (u_3, v_2), (u_4, v_3)\}, \\ &\{(u_1, v_1), (u_2, v_2), (u_3, v_2), (u_4, v_3)\}, \\ &\{(u_1, v_1), (u_2, v_2), (u_3, v_3), (u_4, v_3)\}. \end{aligned}$$

Let S be the set of all possible sequences of pairs as defined above.

For the case $p > q$, we have the discrete Fréchet distance between curves U and V as:

$$\delta_{dF}(U, V) = \min_{s \in S} \left(\max_{i=1, \dots, p} d(u_i, v_{a_i}) \right), \quad (2)$$

where $d(u_i, v_{a_i})$ is the geodesic distance of two points u_{a_i} and v_{b_i} in three-dimensional space.

D. PROBLEM FORMULATION

Let U denote the sequence of points that represents discretized version (on the triangulated manifold model) of a real path curve and V denote the path curve with the minimal total life-cycle cost generated by FMM, which is the solution of the following optimization problem:

$$\min_V \mathbb{C}(V) = \min_V \int_0^{l(V)} C(X(t)) dt. \quad (3)$$

Still, without loss of generality, we assume that the number of points in U is larger than the number of points in V . Then the optimization problem is as follows:

$$\begin{aligned} &\min_{\mathcal{W} \in \mathbb{R}_+^K} \delta_{dF}(U, V). \\ &\text{subject to: } \sum_{k=1}^K w_k = 1. \end{aligned} \quad (4)$$

IV. METHODOLOGIES

In this section, we describe our FMM/SA algorithm, which is based on FMM and SA to solve the problem described in Section III. Subsequently, we present the FMM/SA algorithm's implementation process in detail. Since SA allows worse solutions in each iteration to be accepted with a certain

probability in each iteration, FMM/SA has a solid ability to avoid converging to a local optimum. In order to improve the probability of this algorithm to derive the global optimal solution and speed up the cooling process, we adopt the method of [46] called “very fast simulated annealing algorithm”.

We use FMM in the target region \mathbb{T} as modeled in Section III-A to find the path that achieves a minimal total life-cycle submarine cable cost among all cable path alternatives with given start and end points as those of a real-life existing cable path. An initial solution \mathcal{W}_0 is put to use as the current solution for the first time. Subsequently, we calculate the Fréchet distance between the generated path and the real-life existing cable path using 2. Then we randomly perturb the current solution to generate a new solution and continuously repeat the processes above to converge to a global optimal solution with the execution of SA until no significant improvement can be achieved or SA stops because it reaches the termination temperature.

Detailed implementation process of FMM/SA includes the following steps:

- 1) Set the cooling schedule of SA, which consists of the following set of initial parameters:
 - The initial temperature of cooling T_0 . A sufficiently high initial temperature should be selected to avoid falling into the local optimum;
 - The termination temperature T_f . In order to avoid poor accuracy, this temperature should be kept low;
 - The length L_M^T of the Markov chain, namely the maximal number of iterations at each temperature T .
 - In order to obtain the global optimal solution with probability close to 1, the cooling schedule should have a sufficiently high initial temperature, a sufficiently slow cooling rate, a sufficiently low termination temperature T_f , and sufficient disturbance at each temperature. All of these slow down the convergence speed. In response to this problem, here, we use the following so-called “very fast annealing algorithm” of Ingber [46] that is based on performing perturbations based on the Cauchy-like distribution to generate a new solution, thereby obtaining a function of temperature attenuation $T(r)$.

$$T(r) = T_0 \varphi^{r^{1/D}}, \quad (5)$$

where r is the number of temperature attenuation, D is the dimension of the state space and φ is a non-negative real number. In practical applications, we usually set $D = 1$ or 2 , $0.7 \leq \varphi \leq 1$ [46].

- 2) Specify the feasible solution space and objective function, and generate an initial solution.
 - **Feasible solution space** is \mathbb{R}_+^K since the weights of design considerations should be positive.
 - **Objective function** $\delta_{\text{dF}}(U, V)$ – FMM is here used to calculate the minimal total life-cycle cost cable

path V . Then, the discrete Fréchet distance is calculated as 2. In this case, the smaller the objective function value $\delta_{\text{dF}}(U, V)$ is, the better solution we get.

- **Initial solution** \mathcal{W}_0 – The Robustness of SA is relatively good, that is, the final solution is not too dependent on the selection of the initial solution, so the initial solution is usually selected randomly. We therewith obtain the corresponding initial objective function value $\delta_{\text{dF}}^{\mathcal{W}_0}(U, V)$.
- 3) Generate and accept the new solutions, and store the optimal solutions. Randomly perturb the current solution \mathcal{W} to generate a new solution \mathcal{W}_* . Then calculate the corresponding objective function value $\delta_{\text{dF}}^{\mathcal{W}_*}(U, V)$, and obtain $\Delta\delta_{\text{dF}} = \delta_{\text{dF}}^{\mathcal{W}_*}(U, V) - \delta_{\text{dF}}^{\mathcal{W}_0}(U, V)$. In the classic SA algorithm, the new solution is accepted according to the Metropolis criterion [47]. It is embodied in the algorithm that if $\Delta\delta_{\text{dF}} < 0$, the new solution \mathcal{W}_* is accepted; otherwise, the new solution \mathcal{W}_* is accepted with probability $P = -e^{-\Delta\delta_{\text{dF}}/T}$. Due to the possibility of degradation, the historical optimal solution needs to be recorded as $\mathcal{W}_{\text{best}}$, which corresponds to the historical optimal objective function value $\delta_{\text{dF}}^{\mathcal{W}_{\text{best}}}(U, V)$.
 - 4) At each temperature T , repeat **Step (3)** with the Markov chain length L_M^T , and then lower the temperature according to the setting of the cooling schedule until the termination temperature T_f is reached.

The SA algorithm above accepts the degraded solution with the probability of $P = -e^{-\Delta\delta_{\text{dF}}/T}$ in accordance with the Metropolis criterion. A higher temperature T means a higher probability P . That is, at a higher temperature, the algorithm is likely to accept a worse solution so that it can jump out of the local optimal solution, which significantly reduces the algorithm’s dependence on the initial solution. By slowly lowering the temperature, the worse solution is no longer accepted when the temperature approaches zero, and the algorithm can approach the globally optimal solution. It is clear that, during the execution of FMM/SA, as the temperature parameter T decreases, the probability of the algorithm returning to a global optimal solution monotonically increases, and the probability of returning to a certain non-optimal solution monotonically decreases. Moreover, with enough disturbances and iteration times, the solution generated by SA asymptotically converges to the approximate optimal solution set in a polynomial time.

Algorithm 1 shows the detailed implementation of our FMM/SA algorithm.

V. APPLICATION AND NUMERICAL RESULTS

In this section, we consider two real-life existing submarine cables with histories of resilience and cost-effectiveness that experts designed. We use our FMM/SA algorithm to obtain each design consideration’s weight value using the first real-life existing cable and demonstrate the derived weights

Algorithm 1 FMM/SA Algorithm**Input:**

An existing cable path U , life-cycle cost $c_k(X)$, $k \in [K]$ for k design considerations, initial weights \mathcal{W}_0 of design considerations, initial temperature T_0 , termination temperature T_f , maximal number of iterations L_M^T at temperature T , and temperature attenuation function $T(r)$ with the iteration time r .

Output:

$\mathcal{W}_{\text{best}}$ that can provide a minimal life-cycle cost path V with a minimal value of $\delta_{\text{dF}}^{\mathcal{W}}(U, V)$.

```

1:  $r=1$ ;
2:  $V = \min_V \int_0^{l(V)} \sum_{k=1}^K w_k c_k(X)$ ;
3:  $\delta_{\text{current}} = \delta_{\text{dF}}^{\mathcal{W}_0}(U, V)$ ;
4: while  $T(r) > T_f$  do
5:   for  $i = 1, \dots, L_M^T$  do
6:     for  $k = 1, \dots, K$  do
7:        $\sigma \sim \mathcal{U}(-1, 1)$ ;
8:        $w_k = w_k + \sigma$ ;
9:     end for
10:     $\mathcal{W}_{\text{new}} = \{w_1, w_2, \dots, w_k\}$ ;
11:     $V = \min_V \int_0^{l(V)} \sum_{k=1}^K w_k c_k(X)$ ;
12:    if  $k_j > 0, \forall k \in K$  then
13:       $\delta_{\text{new}} = \delta_{\text{dF}}^{\mathcal{W}_{\text{new}}}(U, V)$ ;
14:      if  $\delta_{\text{new}} < \delta_{\text{current}}$  then
15:         $\mathcal{W}_{\text{current}} = \mathcal{W}_{\text{new}}$ ;
16:         $\delta_{\text{current}} = \delta_{\text{new}}$ ;
17:        if  $\delta_{\text{new}} < \delta_{\text{best}}$  then
18:           $\mathcal{W}_{\text{best}} = \mathcal{W}_{\text{new}}$ ;
19:           $\delta_{\text{best}} = \delta_{\text{new}}$ ;
20:        end if
21:      else
22:         $\sigma \sim \mathcal{U}(0, 1)$ ;
23:        if  $\sigma < e^{-(\delta_{\text{new}} - \delta_{\text{current}})/T(r)}$  then
24:           $\mathcal{W}_{\text{current}} = \mathcal{W}_{\text{new}}$ ;
25:           $\delta_{\text{current}} = \delta_{\text{new}}$ ;
26:        else
27:           $\mathcal{W}_{\text{new}} = \mathcal{W}_{\text{current}}$ ;
28:        end if
29:      end if
30:    end if
31:  end for
32:   $r = r + 1$ ;
33: end while
34: return  $\mathcal{W}_{\text{best}}$ .

```

on the second real-life existing cable. Based on the derived weights of design considerations from the first real-life existing cable, we demonstrate that we can design, using FMM/SA, a cost-effective and resilient submarine cable path that achieves consistency with a real-life path planning of the second cable by experts. Also, we compare FMM/MC and

FMM/RRHC with FMM/SA to demonstrate the superiority of FMM/SA. In addition to the illustrations of the various differences between the various resulted cable path using the resulted curve shapes, we also provide specific numerical results in terms of running time and detailed Fréchet distance.

A. GENERAL INFORMATION OF TWO REAL-LIFE EXISTING SUBMARINE CABLES

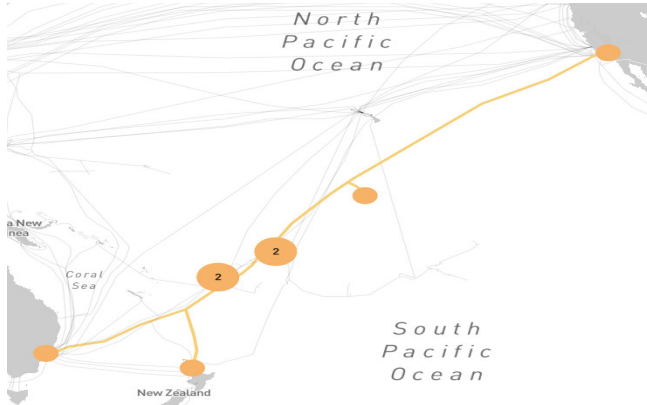
Southern Cross NEXT [48] is located in the Pacific Ocean comprising a Trans-Pacific trunk route linking Coogee Beach, Australia with Hermosa Beach, California, USA, and branches to Takapuna Beach, New Zealand, to Suva, to Savusavu, to Apia, to Tokelau, and also a link to Kiribati. Fig. 1(a) illustrates the topology of this submarine cable network. We select the longest segment of this submarine cable system (denoted by **Cable SX**) as the first real-life existing cable path and implement our FMM/SA algorithm on it. Our aim is to design a cable path that has a minimal Fréchet distance with the realistic cable path from the start point (34.053389° N, 118.245335° W) on Hermosa Beach (USA) represented by the red dot and the end point (5.062067° N, 160.200084° W) represented by the black dot in Fig. 1(b).

South America-1 (SAM-1) cable network [49] is located in Latin America, connecting the United States, Puerto Rico, Brazil, Argentina, Chile, Peru and Guatemala. Fig. 2(a) shows the topology of this cable network. We select the cable segment that connects San Juan (Puerto Rico) and Boca Raton (USA) as the second real-life existing cable path (denoted by **Cable SAM**), see in Fig. 2(b). We generate cable paths using FMM algorithm under the derived weight values of design considerations obtained from FMM/SA, FMM/RRHC, and FMM/MC algorithms, respectively, to evaluate the performances of these algorithms, namely the similarity between the generated cable paths and the second real-life existing cable path.

It should be noted that in reality, there is no standard to judge whether a submarine cable must be the most cost-effective. Therefore, though we set Cable SX as the first real-life existing cable for our FMM/SA method to obtain weights, we cannot directly call it the only standard cable path. It can only be said that the submarine cable path we chose is the optimal path that designed by experts who spend significant amount of time designing, investigating, and weighing a variety of design considerations based on existing data and technology.

B. DESIGN CONSIDERATIONS

In this case, for any point $X(x, y, z) \in \mathbb{M}$, we take into account the following design considerations that contribute to the total life-cycle cost of a submarine cable path. All the parameter settings are derived from our previous work [8]. Besides, all lengths and distances are measured in kilometers. And all the cost is measured in dollars. Notice that the units “dollars” representing the total life-cycle cost should not be taken as the actual prediction for the cable cost, because they are a measure obtained as a summary cost function

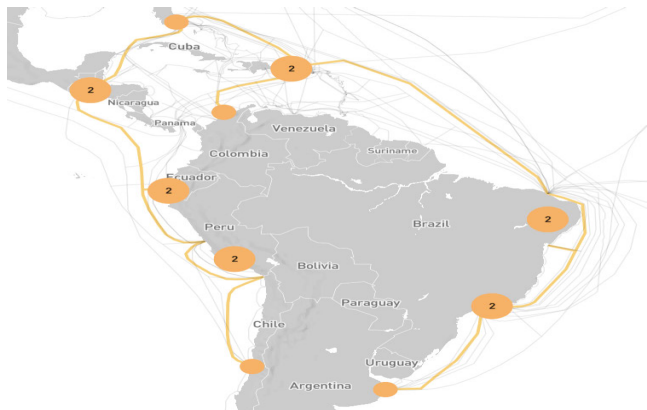


(a) Southern Cross NEXT cable network topology. Source: [50].

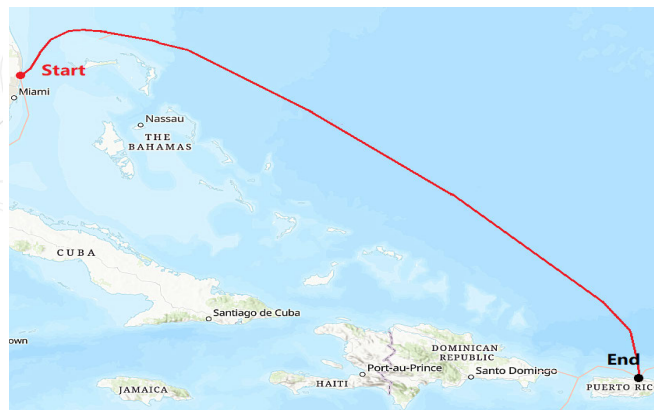


(b) The first real-life existing submarine cable path (Cable SX) drawn using ArcGIS.

FIGURE 1. Southern cross NEXT cable network.



(a) SAM-1 cable network topology. Source: [50].



(b) The second real-life existing submarine cable path (Cable SAM) drawn using ArcGIS.

FIGURE 2. SAM-1 cable network.

which is based on the various costs associated with the design considerations and their weights (that are subjective measures of importance).

- 1) Basic construction cost $c_1(X)$. It involves the laying, maintenance and removal cost of submarine cables. We define $c_1(X)$ as a constant number, that is, $c_1(X) = 27,000$ \$.
- 2) Geological hazards $c_2(X)$, specifically, earthquakes with magnitudes greater than 4.5 and volcanic eruptions. We assume that there are α earthquakes and β volcanic eruptions in total in target region \mathbb{T} , then we have

$$c_2(X) = \sum_{i_e=1}^{\alpha} c_e(X, i_e) + \sum_{i_v=1}^{\beta} c_v(X, i_v) \quad (6)$$

, where $c_e(X, i_e)$ and $c_v(X, i_v)$ are the cost caused by an earthquake i_e and a volcanic eruption i_v . Let

$$c_e(X, i_e) = a_1 e^{1.3 \ln \text{PGV}(X) - 7.21}, \quad (7)$$

and

$$\text{PGV}(X) = 2.04 + 0.422 \times (M_w - 6) - 0.0373 \times (M_w - 6)^2 - \log_{10} d(X, i_e), \quad (8)$$

which represents the peak ground velocity (PGV) at location X , where M_w and $d(X, i_e)$ are the earthquake magnitude of i_e and the distance between point X and earthquake i_e , respectively. And let

$$c_v(X, i_v) = \begin{cases} a_2, & \text{if } d(X, i_v) \leq 3\text{km}, \\ a_2 e^{3-2d(X, i_v)}, & \text{otherwise,} \end{cases} \quad (9)$$

where a_2 is a very large number for avoiding these volcanos and $d(X, i_v)$ is the distance between point X and volcano i_v , respectively.

- 3) Seabed slope $c_3(X)$. We define

$$c_3(X) = \begin{cases} a_3 e^{l_1(X) - 20}, & \text{if } l_1(X) > 20^\circ, \\ \frac{a_3(l_1(X) - 10)}{10}, & \text{if } 20^\circ \geq l_1(X) \geq 10^\circ, \\ 0, & \text{otherwise,} \end{cases} \quad (10)$$

where a_3 is a very large number for avoiding steep areas and $l_1(X)$ is the slope at location X .

4) Water depth $c_4(X)$. Let

$$c_4(X) = \begin{cases} a_4, & \text{if } l_2(X) \leq 0, \\ a_4 e^{-4l_2(X)}, & \text{if } 1\text{km} \geq l_2(X) > 0, \\ a_4 e^{-3-l_2(X)}, & \text{otherwise,} \end{cases} \quad (11)$$

where a_4 is a very large number for avoiding placing cable on the land and $l_2(X)$ is the water depth at location X . Note that $l_2(X) < 0$ means the location X is underwater.

5) Anthropological hazards $c_5(X)$, specifically, fishing and anchoring activities. We define $c_5(X) = c_f(X) + c_a(X)$, where $c_f(X)$ and $c_a(X)$ are the cost caused by fishing and anchoring activities, respectively. Let

$$c_f(X) = \begin{cases} 0, & \text{if } l_2(X) < 0, \\ 0.001725a_5, & \text{if } 0 \leq l_2(X) \leq 0.3, \\ 0.000275a_5, & \text{if } 0.3 < l_2(X) \leq 1, \\ 0.0001a_5, & \text{otherwise,} \end{cases} \quad (12)$$

and

$$c_a(X) = \begin{cases} 0, & \text{if } l_2(X) < 0, \\ 0.000575a_5, & \text{if } 0 \leq l_2(X) \leq 0.3, \\ 0.00005a_5, & \text{otherwise,} \end{cases} \quad (13)$$

where a_5 is a very large number for avoiding the shallow water area.

6) Protected areas $c_6(X)$, specifically, seagrass and coral areas. We define

$$c_6(X) = \begin{cases} a_6, & \text{if } X \text{ is located in the protected area,} \\ 0, & \text{otherwise,} \end{cases} \quad (14)$$

where a_6 is a very large number for avoiding these protected areas.

For this case, let $\mathcal{W} = \{w_1, w_2, w_3, w_4, w_5, w_6\}$, which represents the importance of the design considerations (1)-(6) above. By implementing the weights, we have the life-cycle cost per unit length of the cable passing through location X as $C(X) = \sum_{i=1}^6 w_i c_i(X)$. To implement FMM/SA, we hereby set the initial value \mathcal{W}_0 as $\mathcal{W}_0 = \{0.28, 0.091, 0.35, 0.091, 0.09, 0.098\}$. Also, in our experiments, we set $a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = 3 \times 10^6\$$.

C. DATA SOURCE

We use ArcGIS software and data from [50] to obtain each point’s geological data on the path of Cable SX and Cable SAm, that is, longitude, latitude, and elevation.

- The global terrain data for ocean and land is available in the General Bathymetric Chart of the Oceans (GEBCO, <https://www.gebco.net>) at 15 arc-second intervals. This data can provide us a triangulated manifold model \mathbb{M} with the distance between two adjacent grid points in the range of 350 to 650 meters. Also, we use the GEBCO

data to calculate the seabed slope and water depth for considerations (3) and (4).

- The earthquake data for consideration (2) is provided by United States Geological Survey (USGS, <https://earthquake.usgs.gov/>).
- The information of volcano eruptions for consideration (2) is obtained from National Oceanic and Atmospheric Administration (NOAA, <https://www.ngdc.noaa.gov/>).
- The protected areas for seagrass and corals are derived from World Conservation Monitoring Centre (WCMC, <https://data.unep-wcmc.org/datasets/>), we use it for consideration (6).

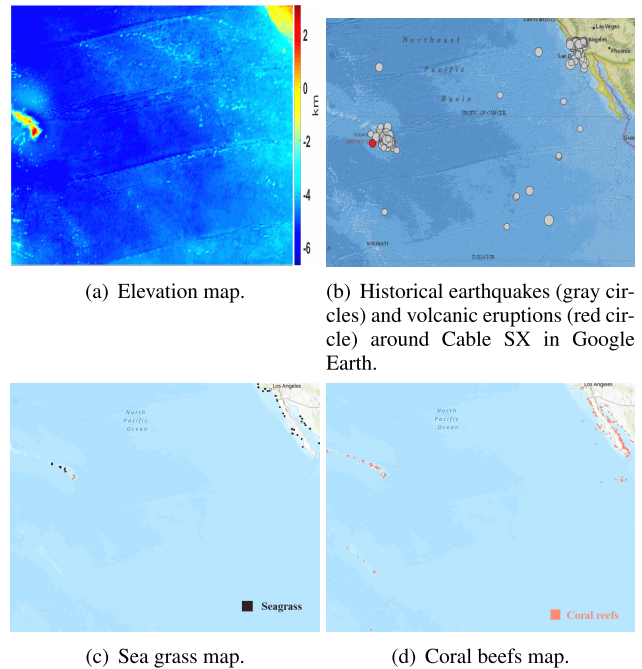


FIGURE 3. Design considerations of cable SX.

Fig. 3 and Fig. 4 shows the details of elevation map, geological hazards, and protected areas around Cable SX and Cable SAm, respectively. The elevation map is used for calculations of water depth and seabed slope.

D. NUMERICAL RESULTS ON CABLE SX

In practice, the alter-course angles for ploughs should generally not exceed 25 degrees according to [11]. We hereby set the FMM algorithm to march in only the 50-degree fan-shaped range in front of the current direction during the marching process, see Fig. 6. The solution of FMM/SA does not strongly depend on the initial solution so we use a randomly generated initial solution for FMM/SA. Table 1 shows the parameter settings of the cooling schedule. Note that we select a sufficiently high initial temperature ($T_0 = 500$) because we have experimented with the values $T_0 = 100, 200, 300, 400, 500$ and we have observed a diminishing marginal benefit as we approach $T_0 = 500$. See Fig.5 for the relevant minimal Fréchet distance as a function of T_0 values.

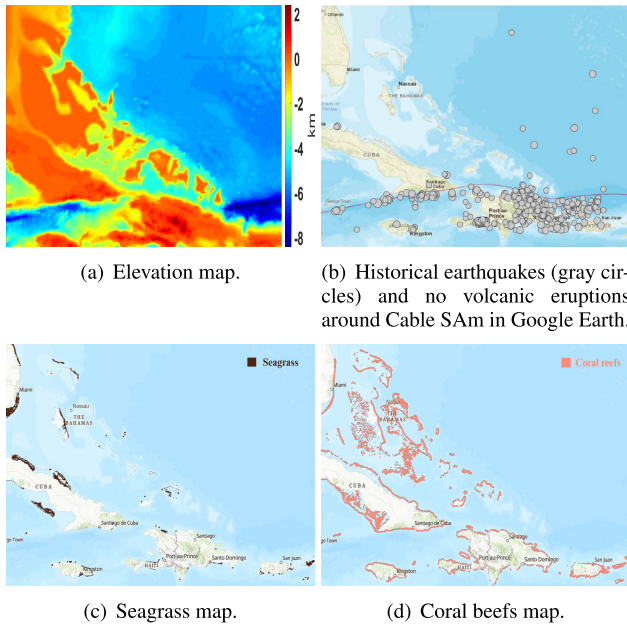


FIGURE 4. Design considerations of cable SAM.

The results are shown in Fig. 7 and Table 2, where Cable SX is the Southern Cross NEXT cable path and the Paths 1-4 are minimal life-cycle cost paths generated by FMM/SA under different design considerations' weights. Path 1 is the optimal cable path obtained using FMM/SA while Paths 2-4 are the intermediate results. All the results are obtained using a Dell G7-7590 laptop (32GB RAM, 2.60 GHz Intel(R) Core(TM) i7-9750H CPU) for running the codes in Matlab R2017b.

We can see in Fig. 7 that as the running time increases, the curve of the submarine cable path that we generated keeps getting closer to the curve of Cable SX, and the Fréchet distance between Cable SX and the curve of generated cable path keeps decreasing. Compared with Paths 2-4, Path 1 has almost overlapped with Cable SX, meaning a relatively good result has been achieved. Table 2 provides the detailed Fréchet distances, running times, total lengths, and total

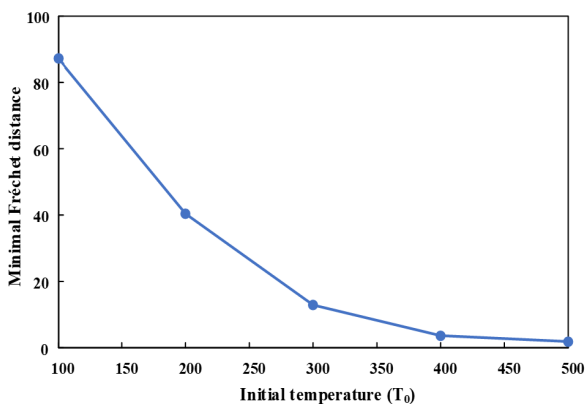


FIGURE 5. Results of minimal Fréchet distances for different initial temperatures T_0 .

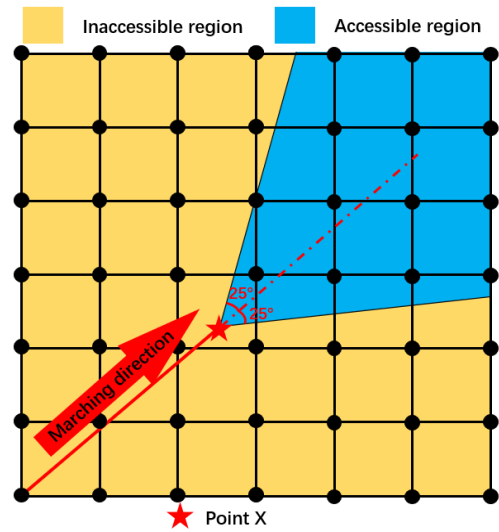


FIGURE 6. Alter-course constraint for FMM in cable path planning application.

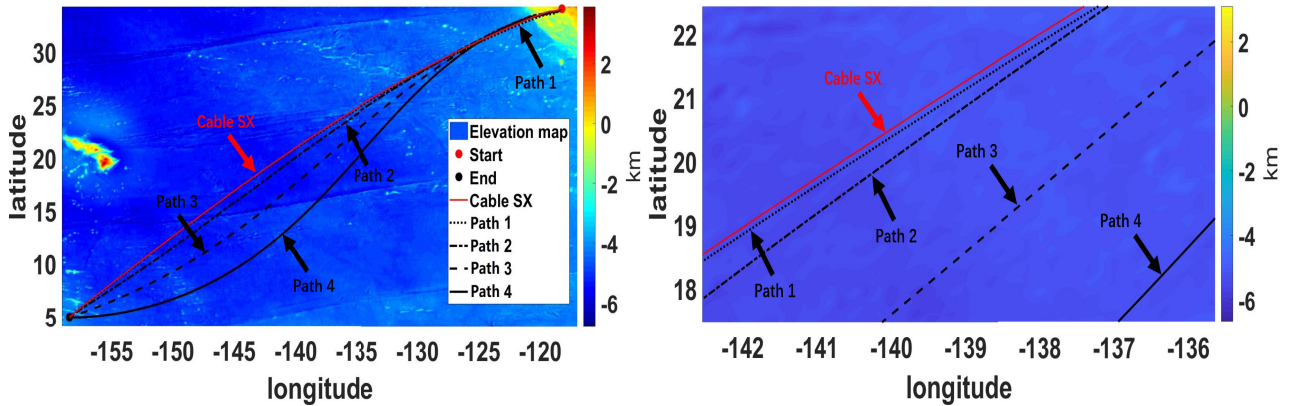
TABLE 1. Cooling schedule of FMM/SA.

Parameter	Value
T_0	500
T_f	5
$T(r)$	$T(r) = T_0 * 0.8^{r^{1/2}}$

life-cycle cost of Paths 1-4. It can be seen from Table 2 that, as time used to assess the weights increases, the closer our cable path is to Cable SX, but the time required to make further improvements in getting closer to Cable SX will increase greatly. A partially enlarged view is given in Fig. 7(b), which shows the difference between the various curves more clearly. Note that although Path 1 and Cable SX are very close to each other and almost overlapped, the Fréchet distance between them is still not 0 (see in Table 2). This is because there are more considerations in the design process of Cable SX. We only consider the considerations with public data here. Better results (more closer to Cable SX than Path 1) will be obtained if data of more design considerations is provided.

MC is designed based on the Monte Carlo's idea, which is a random search algorithm, and it is the basis of the RRHC and SA algorithms. MC will randomly determine many solutions and then select the solution that minimizes the objective function. RRHC is similar to SA but continuously restarts the HC process to jump out of a local optimal solution. In the HC process, RRHC always moves towards a direction that can improve the objective function.

The results of FMM/RRHC and FMM/MC are shown in Fig. 8 and Fig. 9, respectively. Table 3 and 4 give numerical results of Path 5 generated by FMM/RRHC and Path 6 generated by FMM/MC. Table 5 shows \mathcal{W}_{best} values which can generate the closest paths with Cable SX for three different methods. Noted that the total life-cycle cost for Cable SX is different in Table 3 and 4 because these two tables use different \mathcal{W} derived from FMM/RRHC and FMM/MC,



(a) Comparison among five cable paths. Cable SX is the Southern cable path in the real world, Path 1 (the path that most resembles Cable SX, represented by black dot line), Path 2 (represented by black dash-dot line), Path 3 (represented by black dash line) and Path 4 (represented by black line) are all generated by FMM/SA.

(b) Partially enlarged view of (a).

FIGURE 7. The results of FMM/SA.

TABLE 2. Results of FMM/SA.

	Fréchet distance with Cable SX (kilometers)	Running time (seconds)	Total Length (kilometers)	Total life-cycle cost (millions of dollars)
Cable SX	0	NA	5351.1	1112.58
Path 1	1.856	15411	5352.8	1113.21
Path 2	5.994	5462	5356.3	1113.84
Path 3	55.275	75	5403.5	1125.75
Path 4	241.370	15	5578.3	1150.06

respectively. Compared with Path 6, Path 5 has a closer shape to Cable SX, a smaller Fréchet distance, and a shorter time consumption.

We draw Path 1, Path 5, Path 6, and Cable SX in one figure for further comparison, see in Fig. 10. We can clearly see that FMM/SA can better solve this problem within a limited time. Given the data of a submarine cable path in the real world and the cost functions of all the design considerations, FMM/SA can continuously approach the actual submarine cable curve (Cable SX) at a faster speed, see Fig. 7 and Table 2. In contrast, FMM/MC takes 50,297 seconds to find the path result (Path 6), which is almost the same as Path 3 by FMM/SA taking only 75 seconds. This is because FMM/MC based on Monte Carlo’s idea exhibits significant variability while finding the solution. It usually performs better for solving problems in a finite solution space. In the objective function 4, each $w_k, \forall k \in [K]$ can be any positive real number, meaning that the solution space is infinite. It is challenging for FMM/MC to find a near-optimal solution in an infinite solution space. In comparison, FMM/RRHC solves the dilemma that the traditional HC algorithm is prone to be trapped in the local optimal through the operation of continuously random restart and obtains a path result closer to FMM/SA (Path 5). However, to look in detail, the Fréchet distance with Cable SX is still greater than that of Path 1

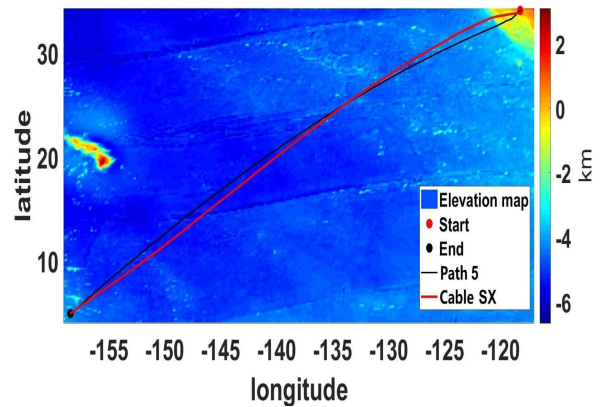


FIGURE 8. Result of FMM/RRHC. Path 5 is the best solution of FMM/RRHC, which has minimal Fréchet distance with cable SX.

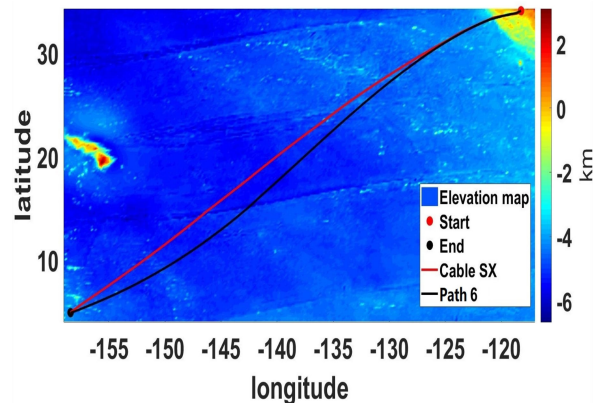
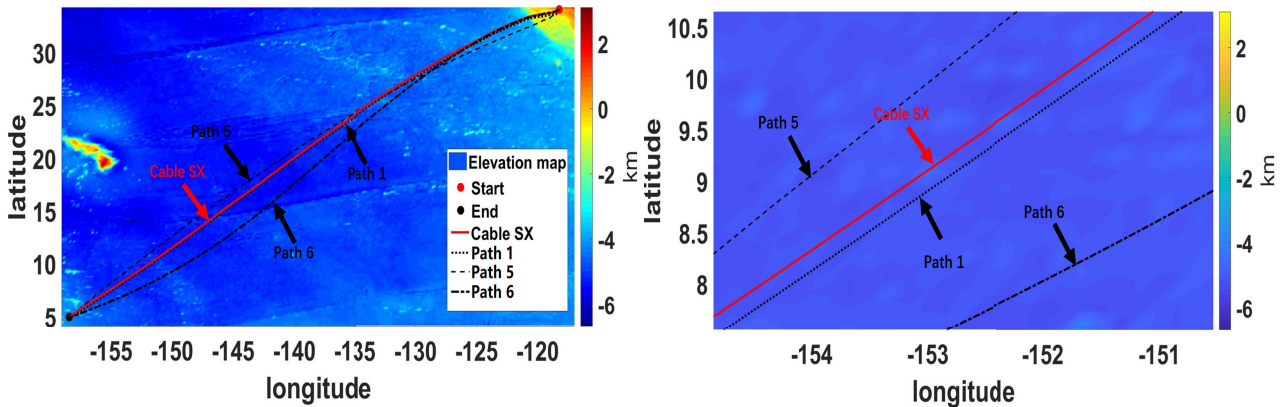


FIGURE 9. Result of FMM/MC. Path 6 is the best solution of FMM/MC, which has minimal Fréchet distance with cable SX.

generated by FMM/SA, and it takes much more time to obtain the results.



(a) Comparison among Cable SX, Path 1, Path 5, and Path 6.

(b) Partially enlarged view of (a).

FIGURE 10. Comparison of the cable paths generated by FMM/SA (Path 1), FMM/RRHC (Path 5), and FMM/MC (Path 6).

TABLE 3. Results of FMM/RRHC.

	Fréchet distance with Cable SX (kilometers)	Running time (seconds)	Total Length (kilometers)	Total life-cycle cost (million of dollars)
Cable SX	0	NA	5351.1	1215.04
Path 5	9.732	31776	5359.6	1217.61

TABLE 4. Results of FMM/MC.

	Fréchet distance with Cable SX (kilometers)	Running time (seconds)	Total Length (kilometers)	Total life-cycle cost (millions of dollars)
Cable SX	0	NA	5351.1	1055.08
Path 6	38.136	50297	5387.5	1063.32

TABLE 5. w_{best} values for design considerations by FMM/SA (Path 1), FMM/RRHC (Path 5), and FMM/MC (Path 6).

	FMM/SA	FMM/RRHC	FMM/MC
w_1 (Basic construction cost)	0.1695	0.1697	0.1321
w_2 (Geological hazards)	0.3852	0.4720	0.4324
w_3 (Seabed slope)	0.1645	0.1756	0.1766
w_4 (Water depth)	0.0215	0.0229	0.0235
w_5 (Anthropological hazards)	0.0739	0.0895	0.0809
w_6 (Protected areas)	0.1851	0.0703	0.1545

E. NUMERICAL RESULTS ON CABLE SAM

We demonstrate the derived weights for the design considerations listed in Section V-B on the second real-life existing cable, namely, Cable SAM. We provide three cable paths, denoted by Path 7, Path 8, and Path 9, which are generated under the design considerations' weights obtained from FMM/SA, FMM/RRHC, and FMM/MC, respectively, with minimal total life-cycle cost.

From the results shown in Fig. 11, we can clearly see that Path 7 generated under the weights obtained from FMM/SA in Section V-D is much closer to the second real-life existing cable path, namely, Cable SAM, where Path 8 and Path 9 use the weights derived from FMM/RRHC and FMM/MC, respectively. FMM/SA is then proved to have superiority among these alternatives. And we can design a cable path that achieves a consistency of all the design considerations with that of the real-life existing cable path (Cable SAM). Table 6 provides the numerical results in terms of total length and Fréchet distance between Cable SAM and Path 7, 8, and 9. Note that in Table 6, we use normalized total life-cycle cost (set the total life-cycle cost for Cable SAM to 1) for easy comparison among Path 7, 8, and 9.

We have demonstrated that learning the design considerations' weights from the 5,351.1 kilometer-long (with over 9,000 data points) Cable SX in one part of the world (Pacific Ocean), and then using these weights for cable path planning between the end-point of cable SAM in a different part of the world (Latin America) can provide a path which we call Path 7 that is very close to the actual real-life path of cable SAM derived based on the traditional approach. Considering the fact that Cable SX and Cable SAM are from very different parts of the world and that the length of Cable SAM is 1,791.2 kilometers with over 3,000 data points and all the data points are counted in the triangulated manifold model, the consistently close matching that has been achieved for the entire length of Cable SAM is somewhat surprising. Fig. 12 the histogram of the geodesic distance between the data points from Cable SAM and Path 7. These results provide a certain indication that design considerations' weights are to a certain extent independent of the location of the cable.

However, as we mentioned in Section I, we must clarify again that our cable path planning based on the learned design considerations' weights should only be used as a guide and reference to assist the planner and not replace the traditional approach completely. Planners can use our design as a suggestion for consideration where human expertise

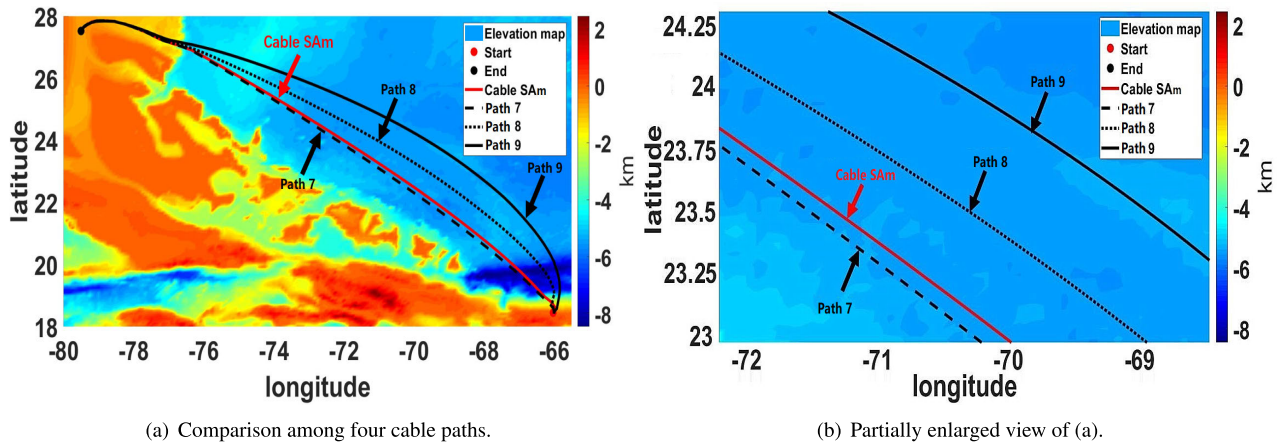


FIGURE 11. Comparison of the cable SAm with the cable paths generated by FMM using the weights derived from FMM/SA (Path 7), FMM/RRHC (Path 8), and FMM/MC (Path 9).

TABLE 6. Results of cable paths generated by FMM using weights derived from FMM/SA (Path 7), FMM/RRHC (Path 8) and FMM/MC (Path 9).

	Fréchet distance with Cable SAm (kilometers)	Total Length (kilometers)	Normalized total life-cycle cost
Cable SAm	0	1791.2	1
Path 7	3.341	1788.8	0.9928
Path 8	40.937	1825.0	1.0304
Path 9	104.682	1877.1	1.0833

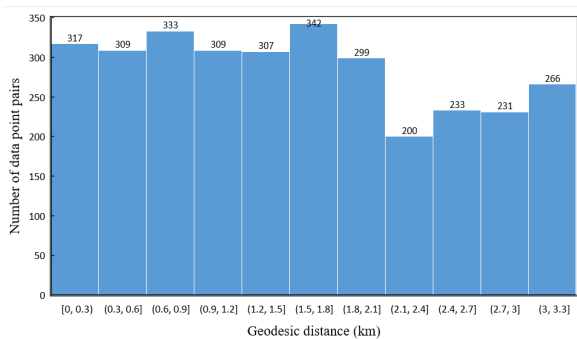


FIGURE 12. The histogram of the geodesic distance between data point u_i (from Cable SAm) and v_i (from Path 7).

and experience are still required. Planners that are fully aware of the design considerations used by the algorithm and their weights, can compare their traditional design with our proposed path planning, and if a discrepancy between the two is found in a certain segment of the cable path, the human planners can decide whether or not the path planning based on the traditional approach needs to be modified. For example, if planners do not consider earthquake hazards (as was the case in the past), and by comparing with the path obtained by the algorithm, they discover a discrepancy in an earthquake-prone area (because the cable does consider earthquake hazards), they may modify their path planning in that area. Unlike the case in autonomous driving systems where the aim is towards fully autonomous self-driving vehicles

that will fully replace human drivers to avoid accidents made by human errors that are often made because of wrong split-second trajectory (path-planning) decisions, in cable path planning, the decisions are not so time-critical, and the human planner should have the final say on the cable path but still benefit from having our automatic path planning as reference.

VI. CONCLUSION

We have proposed an algorithm named FMM/SA for automatic cable path planning in an irregular 2D manifold in a 3D Euclidean space. FMM/SA is based on SA that uses a real-life existing cost-effective and resilient submarine cable path to obtain the weights of various design considerations in practice together with FMM that performs the path planning optimization that relies on the weights obtained using SA. Based on the two real-life existing cable paths, the weights derived from the first real-life existing cable path are demonstrated on the second real-life existing cable path to show that our FMM/SA algorithm can provide a cable path that takes various design considerations into account with realistic priorities and is close to the second cable path that we consider. In addition, we compare FMM/SA with the other two algorithms based on RRHC and MC, which we call FMM/RRHC and FMM/MC, respectively. The simulation results have demonstrated a significant superiority of FMM/SA over these three algorithms. Our proposed FMM/SA algorithm enables us to understand the priorities and preferences of the various design considerations adopted by planners, which helps achieve automation of submarine cable path planning. Given the close matching we have achieved by the cable path generated by FMM/SA, the work in this paper can be a first step towards standardization of a solution for the generation of a cable path that can be used as a guide and benchmark for cable path planners to further improve future cable path planning.

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