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Distributed Model Predictive Control for Linear Constrained Systems Based on Time-Varying Terminal Sets

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ABSTRACT A novel distributed model predictive control (DMPC) strategy with time-varying terminal set for linear constrained systems is presented in this paper. To decrease the load of computation of DMPC while ensuring the global optimization, the nominal system is introduced by treating the influence of neighboring subsystems as a bounded disturbance. Then, under the distributed control structure, a distributed predictive control optimization problem containing the nominal state and input can be designed for each subsystem. Furthermore, different from most DMPC approaches, a novel approach to design a terminal constraint set that can be updated in every update time based on the predicted state of the system is proposed. Additionally, the analysis of feasibility and the stability of the proposed DMPC algorithm are described under kinds of the system constraints. Finally, experimental simulation is shown to prove validity by the control scheme in this paper.

INDEX TERMS Distributed model predictive control, time-varying terminal constraint set, linear constrained system.

I. INTRODUCTION

In recent years, model predictive control (MPC) is one of the advanced control technology in complex industrial fields [1]. Some constraints in the complex industrial system [2]–[4] can be built in optimization problems through this control method, and further processed online depend on the MPC algorithm. Therefore, the appearance of MPC has brought huge economic benefits to the industrial control field [5]–[7]. Centralized predictive control is one of the model predictive control methods, which has been widely used in many conventional industrial control systems [8], [9]. The optimization problem of the entire system needs to be solved by this control scheme, therefore, the optimal controller and the optimal system performance can be obtained. If the controller fails, however, the entire system will not work properly [10], [11]. In addition, the load of online computing is increased by the huge scale of the system, and the performance of real-time is more difficult to guarantee [12], [13]. Later, the decentralized predictive control have been proposed by some research scholars. In [14], a decentralized model predictive control scheme with coordinated control is designed, in which the system structure is consisted by a local MPC controller and a game-theoretic supervisory controller. A reasonable method to solve the frequency problem between asynchronous systems is to adopt decentralized predictive control to simplify the system model [15]. Recently, Nikou *et al.* [16] offered a decentralized model predictive control method that can deal with the problem of robust navigation of the system to the working area state when only use local information was created. In another recent work, for dealing with the coupling effects between subsystems, a decentralized predictive control method was presented by Ahandani *et al.* [17]. In the decentralized predictive control mentioned above, the calculation load of the online optimization is reduced. However, the controllers related to the subsystem will not exchange information with each other to improve the system performance. In this case, distributed predictive control is proposed

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by some researchers [18], [19], which allows communication information sharing between subsystem controllers, so that the coupling between subsystems can be fully considered to make the system performance better. To the best of our knowledge, the online calculation load is still large. Subsequently, in the DMPC strategy proposed by Rawlings [20], only the direct coupling effect of the upstream subsystem is considered, while the indirect coupling effect of other subsystems is neglected. In another research method, Zhang *et al.* [21]. have studied a random DMPC by extending deterministic DMPC. Although the load of calculation is reduced, these two methods have a certain degree of requirements on the structure of the system. Additionally, in all the DMPC methods described above, the local terminal controller respects the state and control input constraints when operating only in the terminal constraint set. Therefore, to make the system exist a better performance, it is essential for a suitable terminal set to be constructed. Zheng at al. [22], [23] came up with a DMPC strategy, where the terminal constraint set is a fixed static ellipsoid set, this is a conservative choice. In [24], [25], Dunbar *et al.* have emphasised on a polytopic invariant sets as terminal constraint set in the process of the DMPC design, compared with the ellipsoid set, only a little of degree of conservative is reduced.

Therefore, to reduce the load of online calculations and the conservative brought by the fixed terminal constraint set in DMPC, a novel distributed model predictive control strategy is studied in this paper. Unlike the conventional approaches [19], each subsystem regards the coupling effect of neighboring subsystems as its own disturbance, to directly introduce the nominal system of the corresponding system, thereby reducing the complexity of the DMPC calculation process caused by the disturbance. Subsequently, due to the time-varying terminal constraint set is more practical than constant terminal constraint set, measures to design such terminal constraint set are provided, together with feasibility and stability of the closed-loop system. Based on the time-varying terminal constraint set, the designed novel DMPC algorithm makes the system have a better performance.

Notations: Real sets are expressed as $\Re^n$ and $\Re^m$, $n \times n$ -dimensional matrix is signed as $\Re^{n\times n}$, $I_{1:M}$ is the set of integers from 1 to M. Given an appropriate positive definite matrix Q, $\|x\|_Q^2 = x^TQx$. For arbitrary $g \subseteq \Re^n$, $j \subseteq \Re^n$, the Minkowski sum is represented by $G \oplus J = \{g+j|g \in G, j \in J\}$, and $G \ominus J = \{e \in \Re^n | e + J \subseteq G\}$ is denoted as Pontryagin difference.

II. PROBLEM FORMATION

Consider a linear system [9] that combines all the M subsystems

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

where $x(k) \in \Re^n$, $u(k) \in \Re^m$ represent state and control input. System (1) can be divided into M non-overlapping subsystems, hence the state can be indicated $x(k) = (x_1(k), \cdots, x_M(k))$, $n = \sum_{i=1}^M n_i$, where $x_i(k) \in \mathbb{X}_i \subseteq \Re^{n_i}$ is expressed as the state of the i-th subsystem. Under this system framework, the control input $u(k)$ is also divided into M non-overlapping sub-vectors, shown by $u(k) = (u_1(k), \cdots, u_M(k))$. Then, the coefficient matrix of M subsystems are signed as $A_{11} \in \Re^{n_1\times n_1}, \cdots, A_{MM} \in \Re^{n_M\times n_M}, B_{11} \in \Re^{n_1\times n_1}, \cdots, B_{MM} \in \Re^{n_M\times n_M}$, which respectively refer to the diagonal block matrix of A and B matrix. The off-diagonal block matrices A_{ij} and B_{ij} are respectively defined as the coupling terms between subsystems and the influence of the control input $u_j(\cdot)$ upon the state $x_i(\cdot)$.

As for the communication between subsystems, the network topology of all subsystems is indicated as an undirected graph $a = (b, c)$, where $b = \{1, \ldots, M\}$ is signed as a set of subsystems, $c \subseteq \{(i, j) \in b \times b | i \neq j\}$ is edge set. Suppose that set of neighboring of subsystems i is denoted as $\mathbb{N}_i$, which contains i itself, then $|\mathbb{N}_i|$ represents the number of elements in $\mathbb{N}_i$. If the neighbor of subsystem i is subsystems j, information can be transmitted between them. Hence, the i-th subsystem [26] is of the form

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + \sum_{j\in\mathbb{N}_i}(A_{ij}x_j(k) + B_{ij}u_j(k)) \tag{2}$$

where $x_i(k)$, $x_j(k) \in \mathbb{X}_i \subseteq \Re^{n_i}$ and $u_i(k)$, $u_j(k) \in \mathbb{U}_i \subseteq \Re^{m_i}$. $\mathbb{U}_i$ and $\mathbb{X}_i$ represent the constraint sets that contain the origin respectively. Let $\mathcal{W}_i(k) = \sum_{j\in\mathbb{N}_i}(A_{ij}x_j(k) + B_{ij}u_j(k))$, $\mathbb{U} = \prod_{i=1}^M \mathbb{U}_i \subseteq \Re^m$ and $\mathbb{X} = \prod_{i=1}^M \mathbb{X}_i \subseteq \Re^n$, hence the above subsystem model (2) can be rewritten as

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + \mathcal{W}_i(k) \tag{3}$$

and the predictive state of the uncertain system related to (3) at time k is established that

$$x_i(k+t+1|k) = A_{ii}x_i(k+t|k) + B_{ii}u_i(k+t|k) + \mathcal{W}_i(k+t|k) \tag{4}$$

where $x_i(k+t|k) \in \mathbb{X}_i$, $u_i(k+t|k) \in \mathbb{U}_i$. Let set $\mathbb{W}_i(k+t|k) := Co\{\sum_{j\in\mathbb{N}_i} A_{ij}x_j(k+t|k) + B_{ij}u_j(k+t|k) | x_j(k+t|k) \in \mathbb{X}_j, u_j(k+t|k) \in \mathbb{U}_j\}$, from which it can be known that $\mathcal{W}_i(k+t|k) \in \mathbb{W}_i(k+t|k)$. Without considering the influence of the uncertainty of system (3), the nominal model of i-th subsystem associated with equation (3) is signed as

$$z_i(k+t+1) = A_{ii}z_i(k+t) + B_{ii}v_i(k+t) \tag{5}$$

where the nominal state and nominal control input are represented as $z_i(k+t)$ and $v_i(k+t)$.

By leading into nominal system (5), a better DMPC algorithm will be presented in this article, where a time-varying terminal set will be constructed to effectively improve system performance.The detailed description will be presented below.

III. CENTRALIZED MODEL PREDICTIVE CONTROL

Before the DMPC strategy, a centralized model predictive problem is given to facilitate the selection of some parameters in the DMPC design scheme. Firstly, the predictive model for the system (5) is expressed as

$$z_i(k+t+1|k) = A_{ii}z_i(k+t|k) + B_{ii}v_i(k+t|k) \tag{6}$$

where $z_i(k+t|k)$ and $v_i(k+t|k)$ are predictive state and input of nominal system (6). Additionally, $v_i(k+t|k)$ is related to $u_i(k+t|k)$ in (4), the relationship is given by

$$u_i(k+t|k) = v_i(k+t|k) + K_i(x_i(k+t|k) - z_i(k+t|k)) \tag{7}$$

where equation (7) is considered as a feedback strategy to punish the deviation between $x_i(k+t|k)$ and $z_i(k+t|k)$, and control gain K_i to guarantee the asymptotic stability is indicated. Based on the description of (4) and (6), let $\mathbb{L}_i(k+t|k) = x_i(k+t|k) - z_i(k+t|k)$, the following equation holds

$$\mathbb{L}_i(k+t+1|k) = \Gamma\mathbb{L}_i(k+t|k) + \mathcal{W}_i(k+t|k) \tag{8}$$

where $\Gamma = A_{ii}+B_{ii}K_i$ and $\mathbb{L}_i(k+t|k) \in E_i(k+t|k)$ which is a set containing the origin. Furthermore, the set $E_i(k+t+1|k)$ satisfies $E_i(k+t+1|k) := \mathbb{W}_i(k+t|k) \oplus \Gamma E_i(k+t|k)$.In addition, as needed, the following assumption is introduced

Assumption 1([27], [28]): A synchronous update strategy is adopted in this paper, hence, the assumed state is transmitted between neighboring subsystems, the assumed control input is expressed as

$$\hat{v}_i(k+t|k+1) = \begin{cases} v_i^*(k+t|k), & t = 1, \ldots, N-1 \\ K_i z_i^*(k+t|k), & t = N \end{cases}$$

Furthermore, the optimization problem of centralized model predictive control for the system given by (6) is obtained by the following form

$$\begin{aligned} J^*(k) = \min_{v(k+t|k)} & \sum_{t=0}^{N-1} \|z(k+t|k)\|_Q^2 + \|v(k+t|k)\|_R^2 \\ & + \|z(k+N|k)\|_P^2 \\ s.t: & \ z(k+t|k) \in \mathbb{X} \ominus E(k+t|k) \\ & \ v(k+t|k) \in \mathbb{U} \ominus KE(k+t|k) \\ & \ z(k+N|k) \in \mathbb{X}^f \ominus E(k+N|k) \end{aligned} \tag{9}$$

where $\mathbf{z} = [z_1^T, \ldots, z_M^T]^T$ and $\mathbf{v} = [v_1^T, \ldots, v_M^T]^T$ are state and input of the whole nominal system. The terminal constraint set is selected as $\mathbb{X}^f := \{x \in \Re^n | \Omega x \leqslant \mathbf{1}\}$, where the state trajectory starting from $\mathbb{X}^f$ will always keep in $\mathbb{X}^f$ and gradually approach the origin [29]. Furthermore, $\mathbb{X}^f$ is a maximum allowable invariant set, which means that if there exist a control input $v = Kz \in \mathbb{U} \ominus KE$ and state $z \in \mathbb{X}^f \ominus E$, then $(A+BK)z \in \mathbb{X}^f \ominus E$ [30], [31]. Additionally, $\mathbb{X}$ and $\mathbb{U}$ are expressed as $\mathbb{X} := \mathbb{X}_1 \times \mathbb{X}_2 \times \ldots \times \mathbb{X}_M$ and $\mathbb{U} := \mathbb{U}_1 \times \mathbb{U}_2 \times \ldots \times \mathbb{U}_M$. For the positive definite matrix Q, R, P and the control law K, if there exist $z \in \mathbb{X}^f \ominus E$, such that the following inequality is satisfied

$$(A+BK)^T P(A+BK) - P \leqslant -Q - K^T RK \tag{10}$$

Let $X = P^{-1} > 0, \Upsilon = KX$, the equation (10) can be converted as

$$\begin{bmatrix} X & * & * & * \\ AX+B\Upsilon & X & * & * \\ Q^{1/2}X & 0 & I & * \\ R^{1/2}\Upsilon & 0 & 0 & I \end{bmatrix} \geqslant 0 \tag{11}$$

It is worth noting that by solving the linear matrix inequality (11), K and P are obtained.

In this section, based on the nominal system model (6), the centralized model predictive control optimization problem (9) is given. However, in centralized model predictive control, the fixed polyhedron set is considered as the terminal constraint set, which can bring a certain degree of conservative. Therefore, a new method of designing terminal constraint set will be given in the DMPC algorithm.

IV. DISTRIBUTED MODEL PREDICTIVE CONTROL

A novel DMPC algorithm is designed in this section, in which a new time-varying terminal set is constructed. Moreover, the feasibility of the designed algorithm and the stability of the closed-loop system are described as follows.

A. A NOVEL DMPC DESIGN

In this paper, the states of all subsystems are updated synchronously, the assumed state $\hat{z}_j(\cdot)$ and control input $\hat{v}_j(\cdot)$ instead of the actual state and control input are transmitted by neighbor subsystem. Therefore, based on centralized model predictive algorithm, the cost function of DMPC with the predictive horizon N can be described by

$$\begin{aligned} \bar{J}(k) = & \sum\nolimits_{t=0}^{N-1} \|z_i(k+t|k)\|_{Q_i}^2 + \|v_i(k+t|k)\|_{R_i}^2 \\ & + \|z_i(k+N|k)\|_{P_i}^2 \\ & + \sum_{j \in \mathbb{N}_i \setminus i} \sum\nolimits_{t=0}^{N-1} \left\|\hat{z}_j(k+t|k)\right\|_{Q_i}^2 + \left\|\hat{v}_j(k+t|k)\right\|_{R_i}^2 \\ & + \left\|\hat{z}_j(k+N|k)\right\|_{P_i}^2 \end{aligned} \tag{12}$$

and then the optimization problem of DMPC is presented as

$$\begin{aligned} \bar{J}^*(k) = \min_{v_i(k+t|k)} & \ \bar{J}(k) \\ s.t: & \ i \in \mathbb{N}_i \\ & \ z_i(k+t|k) \in \mathbb{X}_i \ominus E_i(k+t|k) \\ & \ v_i(k+t|k)_i \in \mathbb{U}_i \ominus K_i E_i(k+t|k) \\ & \ z_i(k+N|k) \in \mathbb{X}_i^f \ominus E_i(k+N|k) \end{aligned} \tag{13}$$

where $X_i \in \{0,1\}^{n_i \times n}$, $X_{\mathbb{N}_i} \in \{0,1\}^{n_i \times n}$ and $U_i = \{0,1\}^{m_i \times m}$ are the appropriate selection matrices [32], therefore, the parameters Q_i, R_i, P_i and K_i are calculated by $Q = \sum_{i=1}^{M} X_{\mathbb{N}_i}^T Q_i X_{\mathbb{N}_i}$, $R = \sum_{i=1}^{M} U_i R_i U_i$, $P = \sum_{i=1}^{M} X_{\mathbb{N}_i}^T P_i X_{\mathbb{N}_i}$, $K = \sum_{i=1}^{M} U_i^T K_i X_{\mathbb{N}_i}$ variables $z_i = X_i z$, $z_{\mathbb{N}_i} = X_{\mathbb{N}_i} z$ and $v_i = U_i v$ [33].

As can be seen from the foregoing DMPC optimization problem(13), the predictive model of the system (6) is known, however, the terminal constraint set $\mathbb{X}_i^f$ is unknown. Since $\mathbb{X}_i^f$ will lead to a large attraction field, hence, it is essential to design a suitable $\mathbb{X}_i^f$ to improve system performance. To give full play to the advantages of distributed computing, $\mathbb{X}_i^f(k)$ needs to be met (14) while changing the size and position

$$\mathbb{X}_1^f(k) \times \mathbb{X}_2^f(k) \times \cdots \times \mathbb{X}_M^f(k) \subset \mathbb{X}^f \tag{14}$$

where at any time $k \geqslant 0$, $\mathbb{X}_i^f(k)$ is defined as

$$\mathbb{X}_i^f(k) := \mathbb{X}_i^f(\alpha_i(k), \beta_i(k)) = \{x \| \|x - \alpha_i(k)\|_\infty \leqslant \beta_i(k)\} \tag{15}$$

hence, the design of $\alpha_i(k)$ and $\beta_i(k)$ are given by the following theorem.

Theorem 1: Given $\alpha=(\alpha_1, \ldots, \alpha_M)$ and $\beta = (\beta_1, \ldots, \beta_M)$. If there are α_i, β_i such that the conditions (16-17) hold, then $\mathbb{X}_i^f(k)$ is a set that can be changed as the update time.

$$\beta_1 = \ldots = \beta_M = \sigma(\alpha) = \min(1 - \Omega_l \alpha)/\|\Omega_l\|_1 \tag{16}$$

$$\alpha_+ = (\alpha_{1+}, \alpha_{2+}, \ldots, \alpha_{M+}) \in \mathbb{X}^f \tag{17}$$

Proof: Based on [33], suppose that $\alpha \in \mathbb{X}^f$, the following set can be defined

$$\{x \in \Re^n : \|x - \alpha\|_\infty \leqslant \vartheta\}.$$

With $v_i \in \{-1, 1\}$, its vertices are represented as

$$\{\alpha \pm \vartheta v_i : i = 1, 2, \ldots, M\}.$$

Therefore, if and only if $\Omega(\alpha \pm \vartheta v_i) \leqslant \mathbf{1}$, the set $\{x \in \Re^n : \|x - \alpha\|_\infty \leqslant \vartheta\}$ lies insides $\mathbb{X}^f$. By letting Ω_l is the l-th row of Ω, there exist $\Omega_l v_i = \|\Omega_l\|_1$ for v_i. Thus, it can be obtained that

$$\vartheta \leqslant (1 - \Omega_l \alpha)/\|\Omega_l\|_1.$$

Then, the maximum value of ϑ is described

$$\sigma(\alpha) = \min(1 - \Omega_l \alpha)/\|\Omega_l\|_1.$$

Furthermore, it has

$$\{x \in \Re^n : \|x - \alpha\|_\infty \leqslant \vartheta\} = \mathbb{X}_1^f(\alpha_1, \beta_1) \times \mathbb{X}_2^f(\alpha_2, \beta_2) \times \ldots \times \mathbb{X}_M^f(\alpha_M, \beta_M).$$

It implies

$$\beta_1 = \beta_2 = \ldots = \beta_M = \sigma(\alpha).$$

where $\sigma(\alpha)$ is a concave function about α, thus, (16) is obtained.

Next, assume that there are α, β, and $\eta = (\eta_1, \eta_2, \ldots, \eta_M)$ with $\eta_i \in \mathbb{X}_i^f(\alpha_i, \beta_i)$ are given, since $\mathbb{X}^f$ is a invariant set, hence, let the following equation hold

$$\alpha_{i+} = \sum\nolimits_{j \in \mathbb{N}_i} (A_{ij} + B_{ij} K_j)\eta_j.$$

Then, it gives

$$\alpha_+ = (\alpha_{1+}, \alpha_{2+}, \ldots, \alpha_{M+}) \in \mathbb{X}^f.$$

The proof is complete.

For each subsystem i, let $\eta_j(k) = x_j(k + N|k)$, the value of $\alpha_i(k + 1)$ is obtained. Then, $\beta_i(k + 1) = \sigma(\alpha(k + 1))$ is given by (16) in Theorem1. Therefore, the terminal constraint set $\mathbb{X}_i^f$ completed the update at time $k + 1$.

Based on terminal constraint set $\mathbb{X}_i^f(k)$, problem (13) can be solved. Then, a detailed description about DMPC algorithm is given as below.

Algorithm

Require: K_i and P_i.

Initialization $z_i(0)$ and $k = 0$.

Repeat

(1) : $\hat{v}_i(k + t|k)$ and $\hat{z}_i(k + t|k)$ are calculated for $t \in [0, N - 1]$ and $i \in \mathbb{N}_i$.

(2) : $\hat{z}_i(k+t|k)$ is sent to the subsystem j, then $\hat{z}_j(k+t|k)$ is received from subsystem j and send to subsystem i.

(3) : $\mathbb{X}_i^f(k)$ is calculated according to (16) and (17).

(4) : The controller solve optimization problem (13) by receiving measured state $z_i(k)$, then implement $v_i(k) = v_i^*(k|k)$.

(5) : $k = k + 1$.

Remark 1: A key difference between a linear system and a nonlinear system is that the stability of a nonlinear system is not only related to the structure and parameters of the system, but also related to the initial conditions of the system. In addition, nonlinear systems may have multiple equilibrium states. Therefore, to reduce the complexity of system stability analysis, a new DMPC algorithm suitable for linear systems is studied in this paper. In future work, applying the proposed control method to nonlinear systems will be the focus of our research work.

Remark 2: It is worth noting that each subsystem regards the interdependence of its neighboring subsystems as a bounded disturbance of its own dynamics, and its realization needs to increase the range of information obtained by each subsystem. Although a little bit of flexibility of the subsystem is sacrificed by this approach, the corresponding nominal system can be introduced so that a lot of calculation work is completed offline, which reduces the computational burden in the process of DMPC optimization.

B. ANALYSIS OF THE FEASIBILITY AND THE STABILITY

The design method of distributed predictive control is given in the previous subsection. Then the feasibility of the algorithm and the stability of the closed-loop system will be explained in this section and the following theorems can be obtained.

Theorem 2: If the initial state $z_i(0)$ is feasible for optimization problem (13), then it remains feasible throughout the system evolution.

Proof: Suppose $v_i^*(k+t|k)$, $z_i^*(k+t|k)$ are existed for $t \in [0, N]$, and then it can be obtained together with (4) that

$$x_i(k+t+1|k+1) = A_{ii}x_i(k+t|k+1) + \mathcal{W}_i(k+t|k+1) + B_{ii}u_i(k+t|k+1) \tag{18}$$

Then, the control input can be held as

$$u_i(k+t|k+1) = v_i^*(k+t|k) + K_i(x_i(k+t|k+1) - z_i^*(k+t|k)) \tag{19}$$

Furthermore, the following equation is obtained

$$x_i(k+t+1|k+1) = z_i^*(k+t+1|k) + \mathcal{W}_i(k+t|k) + (x_i(k+t|k+1) - z_i^*(k+t|k)) \times (A_{ii} + B_{ii}K_i) \tag{20}$$

Notice the definition of set $E_i(k+t|k+1)$, it can be known after derivation that

$$x_i(k+t+1|k+1) - z_i^*(k+t+1|k) \in E_i(k+t+1|k).$$

Additionally, the result of control input can be shown as

$$u_i(k+t|k+1) - v_i^*(k+t|k) \in K_iE_i(k+t|k).$$

Hence, it inferred that $u_i(k+t|k+1) \in \mathbb{U}_i$ and $x_i(k+t+1|k+1) \in \mathbb{X}_i$. Now, consider the state of the nominal system that

$$z_i(k+t+1|k+1) = A_{ii}z_i(k+t|k+1) + B_{ii}v_i(k+t|k+1) \tag{21}$$

Let the input of the nominal system be structured as

$$v_i(k+t|k+1) = v_i^*(k+t|k) + K_i(z_i(k+t|k+1) - z_i^*(k+t|k)) \tag{22}$$

After derivation with (21) and (22), it has

$$x_i(k+t+1|k+1) = z_i(k+t+1|k+1) + \mathcal{W}_i(k+t|k+1) + (A_{ii} + B_{ii}K_i)(x_i(k+t|k+1) - z_i(k+t|k)) \tag{23}$$

Therefore, the following relationship is gotten that

$$x_i(k+t+1|k+1) - z_i(k+t+1|k+1) \in E_i(k+t+1|k+1).$$

Since $x_i(k+t+1|k+1) \in \mathbb{X}_i$, thus, $z_i(k+t+1|k+1) \in \mathbb{X}_i \ominus E_i(k+t+1|k+1)$. Furthermore, due to

$$z_i^*(k+t|k) \in \mathbb{X}_i \ominus E_i(k+t|k)$$

and

$$z_i(k+t|k+1) \in \mathbb{X}_i \ominus E_i(k+t|k+1).$$

We have

$$z_i(k+t|k+1) - z_i^*(k+t|k) \in E_i(k+t|k) \ominus E_i(k+t|k+1)$$

Since $v_i^*(k+t|k) \in \mathbb{U}_i \ominus K_iE_i(k+t|k)$, hence, it shows that

$$v_i(k+t|k+1) \in \mathbb{U}_i \ominus K_iE_i(k+t|k+1).$$

Next, at time $t = N-1$, the same as the above derivation, it has

$$x_i(k+N|k+1) = z_i^*(k+N|k) + \mathcal{W}_i(k+N-1|k) + (A_{ii} + B_{ii}K_i) \times (x_i(k+N-1|k+1) - z_i^*(k+N|k)) \tag{24}$$

From which, we have

$$x_i(k+N|k+1) - z_i^*(k+N|k) \in E_i(k+N|k).$$

Due to $z_i^*(k+N|k) \in \mathbb{X}_i^f \ominus E_i(k+N|k)$, it holds that

$$x_i(k+N|k+1) \in \mathbb{X}_i^f \subset \mathbb{X}_i$$

and

$$u_i(k+N-1|k+1) \in \mathbb{U}_i.$$

For nominal system, it is obvious that

$$v_i(k+N-1|k+1) \in \mathbb{U}_i \ominus K_iE_i(k+N-1|k+1).$$

By reasons of $x_i(k+N|k+1) \in \mathbb{X}_i^f$ and $x_i(k+N|k+1) - z_i(k+N|k+1) \in E_i(k+N|k+1)$, it renders that

$$z_i(k+N|k+1) \in \mathbb{X}_i^f \ominus E_i(k+N|k+1).$$

Finally, at time $t = N$, owing to $x_i(k+N|k+1) \in \mathbb{X}_i^f$, thus, $u_i(k+N|k+1)$ renders $x_i(k+N+1|k+1) \in \mathbb{X}_i^f$. The same as above that $z_i(k+N+1|k+1) \in \mathbb{X}_i^f$. It is relevant that

$$x_i(k+N+1|k+1) - z_i(k+N+1|K+1) \in E_i(k+N+1|K+1).$$

Hence, it shows that

$$z_i(k+N+1|k+1) \in \mathbb{X}_i^f \ominus E_i(k+N+1|k+1)$$

and

$$v_i(k+N|k+1) \in \mathbb{U}_i \ominus K_iE_i(k+N|k+1).$$

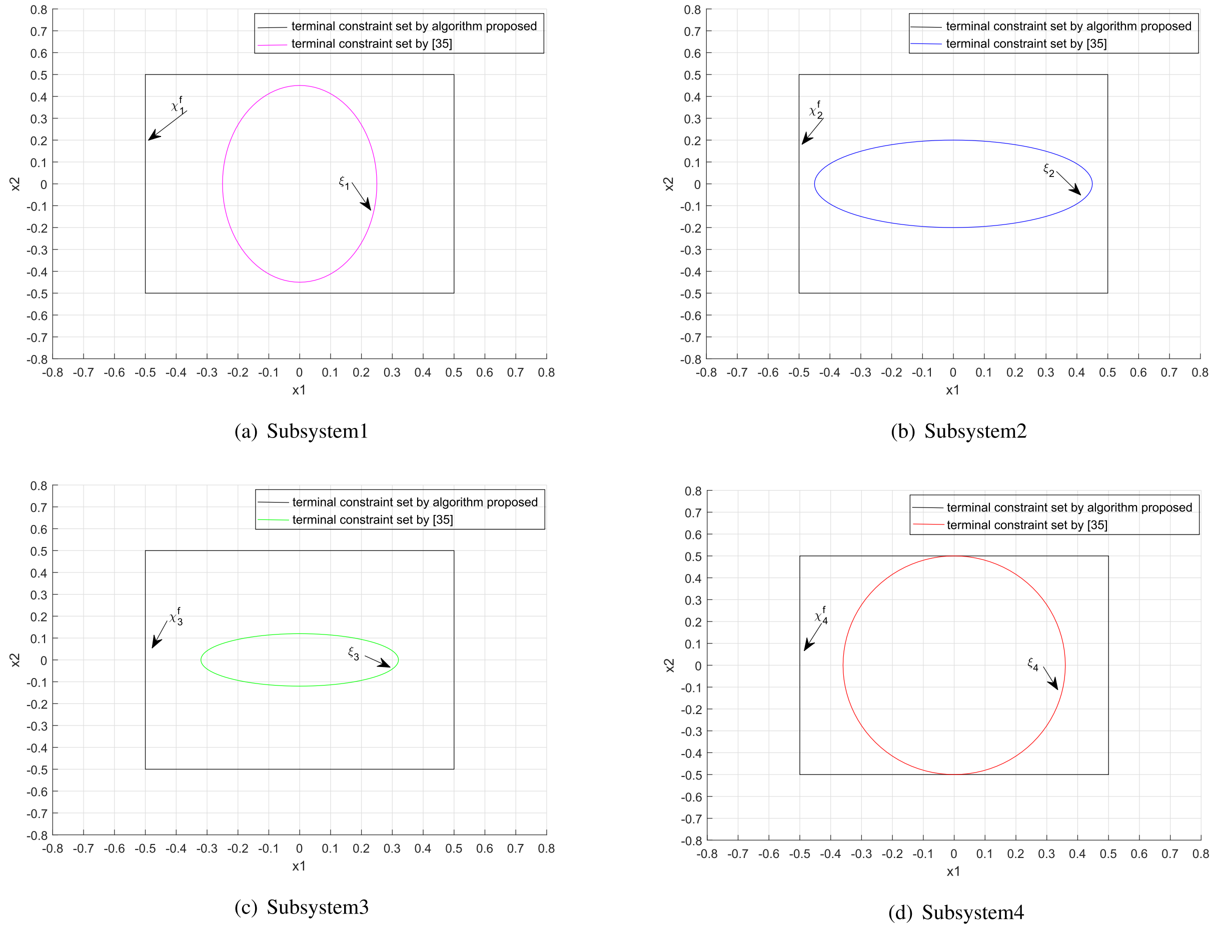
The proof is complete.

In the next theorem, the stability of the system will be proved.

Theorem 3: Suppose that optimization problem (13) is feasible by implementing $v_i^*(k|k)$, after that, whole system is asymptotically stable.

Proof: For all $t \in [1, N]$ at time $k+1$, let the control input $\tilde{v}_i(k+t|k+1) = \hat{v}_i(k+t|k+1)$ is feasible solution of (13). Furthermore, the obtained cost by feasible solution and optimal solution are signed as $\tilde{J}(k+1)$ and $\bar{J}^*(k+1)$. Therefore,it has

$$\bar{J}^*(k+1) - \bar{J}^*(k) \leqslant \tilde{J}(k+1) - \bar{J}^*(k)$$



(a) Subsystem1

(b) Subsystem2

(c) Subsystem3

(d) Subsystem4

FIGURE 1. The trajectories of four terminal constraint sets under two different algorithms.

After derivation, we have

$$\begin{aligned}
&\tilde{J}(k+1) - \bar{J}^*(k) \\
&\quad \leqslant \sum_{i\in\mathbb{N}_i} \|\tilde{z}_i(k+N+1|k+1)\|_{P_i}^2 + \|\tilde{z}_i(k+N|k+1)\|_{Q_i}^2 \\
&\qquad + \|\tilde{v}_i(k+N|k+1)\|_{R_i}^2 - \left\|z_i^*(k+N|k)\right\|_{P_i}^2 \\
&\qquad - \left\|z_i^*(k|k)\right\|_{Q_i}^2 - \left\|v_i^*(k|k)\right\|_{R_i}^2 \\
&\quad \leqslant \sum_{i\in\mathbb{N}_i} - \left\|z_i^*(k|k)\right\|_{Q_i}^2 - \left\|v_i^*(k|k)\right\|_{R_i}^2 \\
&\quad \leqslant 0
\end{aligned}$$

it holds $\bar{J}^*(k+1) - \bar{J}^*(k) \leqslant 0$.The proof is complete.

It can be seen from Theorem 2 that $z_i(k+1)$ and $v_i(k+1)$ satisfy all the constraints (13), thus, problem (13) is feasible by utilizing the algorithm at any time $k > 0$. Theorem 3 shows that the cost of the system is decreased at each update time. If $\bar{J}(0)$ is a bounded value, according to the Lyapunov stability theorem, it can be obtained that the cost of the system is bounded in the entire time domain. Then, the state of the system asymptotically converge to 0. So far, the stability of the global system is proved.

V. NUMERICAL EXAMPLE

The system consisting of four subsystems is considered to confirm the effectiveness of the proposed algorithm, of which the model parameters are given as $A_{ii} = \begin{bmatrix} 1.1 & 1 \\ 0 & 1.3 \end{bmatrix}$, $B_{ii} = B_{ij} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $A_{ij} = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0.2 \end{bmatrix}$. For any subsystem, control input constraint with $u_i(k) \in \mathbb{U}_i = \{u_i(k) \in \Re^m | -2.5 \leqslant u_i(k) \leqslant 2.5\}$. The primary parameters K_i and P_i of the proposed method are obtained by calculating (11), the calculation result are $K_1 = \begin{bmatrix} -0.6939 & -1.056 \end{bmatrix}$,$K_2 = \begin{bmatrix} 0.7452 & 1.113 \end{bmatrix}$ $K_3 = \begin{bmatrix} -0.7312 & -1.084 \end{bmatrix}$, $K_4 = \begin{bmatrix} -0.7312 & -1.084 \end{bmatrix}$ and $P_1 = \begin{bmatrix} 4.294 & 0.3814 \\ 0.3814 & 2.267 \end{bmatrix}$, $P_2 = \begin{bmatrix} 8.062 & 0.8835 \\ 0.8835 & 4.362 \end{bmatrix}$. $P_3 = P_4 = \begin{bmatrix} 6.306 & -0.6004 \\ -0.6004 & 3.340 \end{bmatrix}$.

At the initial time, assume that the corresponding states are given as $x_1(0) = \begin{bmatrix} 2.794 & 2.5 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} 2.08 & 2.2 \end{bmatrix}^T$, $x_3(0) = \begin{bmatrix} 2.194 & -2.305 \end{bmatrix}^T$, $x_4(0) = \begin{bmatrix} -2.8 & 2.5 \end{bmatrix}^T$. For $i = 1, 2, 3, 4$, $\alpha_i(0) = 0$, $\beta_i(0) = 0.5$ are given for the update of terminal constraint set. The weights in the cost are chosen

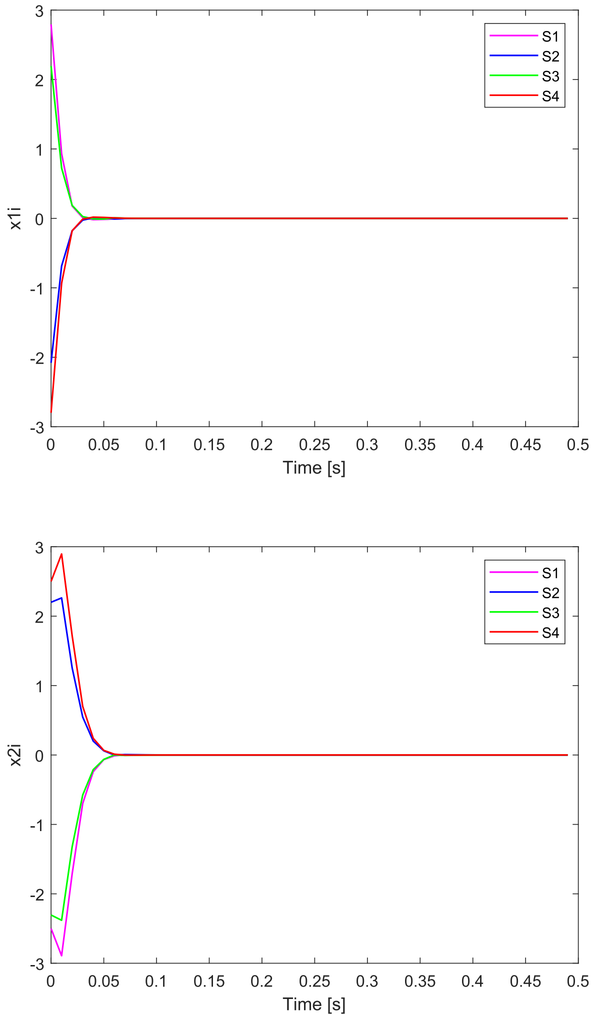


FIGURE 2. The state trajectories of four subsystems.

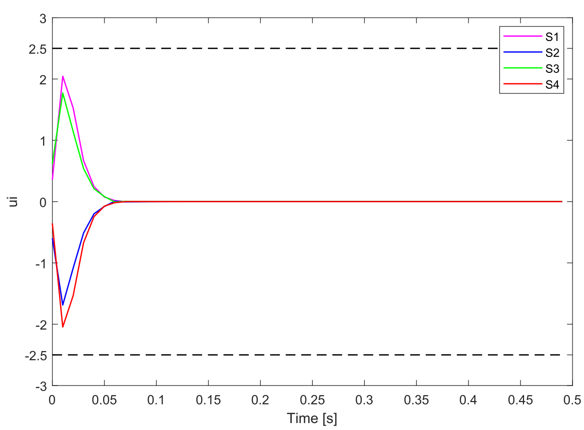


FIGURE 3. The control input trajectories of four subsystems.

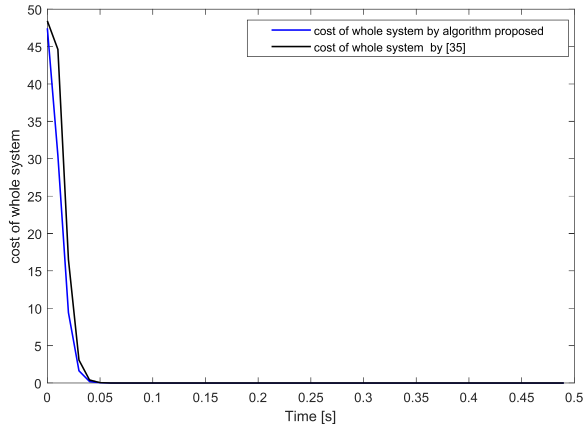


FIGURE 4. The trajectory of cost of whole system under two algorithms.

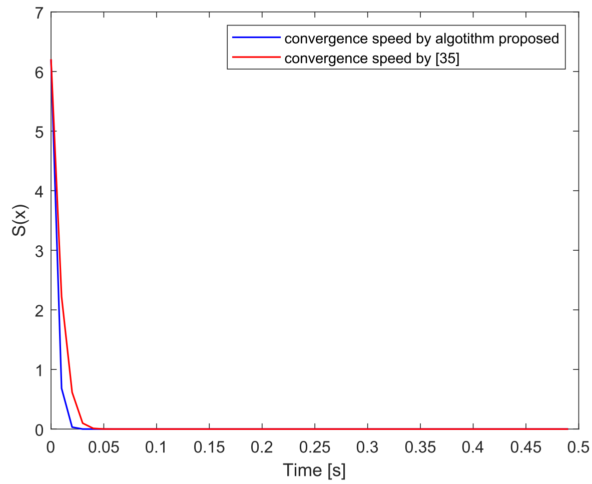


FIGURE 5. The trajectory of convergence speed under two algorithms.

in the same way of [34] as follows: $Q_i = I$ and $R_i = 0.01$. The simulation results are diaplyed in Figs.1-5.

The work in the literature [34] use a block-diagonal of P matrix as terminal constraint set design, for example, the terminal constraint set of i-th subsystem is of the form $\xi_i := \{x_i : \|x_i\|^2_{P_i} \leqslant r_i\}$. Then, by calculating the terminal constraint set under two different algorithms, a larger terminal constraint set is obtained by comparing with the algorithm in [34], which will make the system performance better. The comparison result are appeared in Fig.1.

Figs.2-3 display the control inputs and states of the four subsystems, it conclude that the control inputs which obtained by solving optimization problem (13) content the constraint, and corresponding states of four subsystems from different locations reach to the origin.

By implementing the algorithm proposed, the optimal control inputs and optimal states are obtained after solving the quadratic programming problem (13). Subsequently, the cost of the whole system can be calculated by (12). Additionally, the cost of the entire system is also calculated by adopting algorithm in [34]. Obviously, it can be seen from Fig.4 that the system cost is reduced faster with the algorithm proposed. It also implies that the system performance is better under the algorithm proposed in this paper.

Then, in terms of convergence speed, the offered DMPC algorithm is compared with algorithm by [34]. Like [35], a function $S(x) = \sum \|x_i\|^2 / |\mathbb{N}_i|$ is established to picture convergence speed. From Fig.5, we can see that $S(x)$ is reduced in two algorithms at the same time. While, the $S(x)$ by put forward DMPC algorithm in this paper converges faster.

VI. CONCLUSION

A fancy DMPC algorithm for linear coupled systems is proposed in this article. The objective function related to the nominal system is constructed to reduce load of calculation, which can be achieved by each system treating the influence of its neighboring systems as a disturbance. Furthermore, a new design method is proposed for the terminal constraint set to make the system performance better. Moreover, the DMPC optimization problem is solved. Finally, simulation example prove the validity of the DMPC algorithm.

In future research work, event-triggered model predictive control of nonlinear systems [36], [37] will be the focus of our study.

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