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Detectability of Delayed Boolean Control Networks Based on Full-Order Observer

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ABSTRACT This paper investigates the detectability of delayed Boolean control networks (DBCNs) via the Boolean semi-tensor product (BSTP) method. Firstly, three concepts of weak detectability, strong detectability, and detectability for DBCNs are proposed and the dynamics of DBCN are converted into an equivalent algebraic form. Secondly, a full-order observer of DBCNs is constructed to judge the detectabilities of DBCNs by using BSTP. Thirdly, based on the observer, some necessary and sufficient conditions of weak detectability, strong detectability, and detectability for DBCNs are presented. Finally, two examples are illustrated to verify the obtained results.

INDEX TERMS Boolean control network, time delay, detectability, full-order observer, Boolean semi-tensor product.

I. INTRODUCTION

In control theory, detectability is an important character to investigate the performance of nonlinear systems [1]–[3]. The main focus of detectability is how to determine a unique current state by measuring the input sequences and output sequences, regardless of the initial state. When the current states are uniquely determined, the subsequent states are also determined uniquely. In [3], four kinds of detectability were proposed for discrete event systems, and some effective criteria were obtained based on graph-method. Detectability has many applications, such as fault detection and fault tolerance [4]. During the past twenty years, the study of detectability has received lots of scholars' attention in areas of biology, engineering, computer technology, and so on.

Boolean control network (BCN) is a kind of finite-value discrete-time dynamical systems [5], whose state and input take value from a finite set. The dynamics of BCNs are described by a Boolean difference equation. BCN is a classic model for the investigation of gene regulatory networks [6]. In addition, BCNs are applied to many other areas [7]–[9], such as information security, social networks, digital circuits, and so on.

Recently, semi-tensor of product (STP) method has been introduced to explore BCNs [10]. Several fundamental issues of BCNs are well addressed by resorting to STP, such as controllability and observability [11]–[15], stability [16]–[18], stabilization [19]–[21], and so on. In addition, STP is also applied to game theory [22], [23], finite automata [24], [25], and so on.

Detectability is an important problem for the reconstruction of gene states. Using the STP method, many scholars focus on the detectability problem of BCNs [26]–[31]. Fornasini and Valcher [26] presented the reconstructibility which determines the unique current state and reconstructed an unobservable Boolean network or BCN. Zhang *et al.* [27] proposed a new reconstructibility of BCNs based on a weighted pair graph, which is a generalization of [26]. Wang *et al.* [30] defined three types of detectabilities of BCNs, and proposed some criteria and algorithms for detectability of Boolean networks and BCNs based on a new data model. Recursive methods and a termination condition were presented for the reconstructibility of BCNs in [28]. Furthermore, there exist several new results on the detectability of probabilistic Boolean networks [32]–[34], singular Boolean control networks [35], delayed Boolean control networks (DBCNs) [36] and so on.

As we all know, a state observer is an effective tool to estimate the internal state of nonlinear systems by observing input and output. To estimate the state of BCNs, there exist several forms of a state observer for BCNs [26], [28], [31], [37], [38]. A full-order observer of BCNs was proposed

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in [26]. A Luenberger-like observer of BCNs was presented in [37], which can be observed by all possible input sequences and output sequences. To reduce the computational complexity, a reduced-order observer of BCNs was explored [38].

The delay phenomenon in gene regulatory network is caused by slow biochemical reactions such as gene transcription and translation [42], [43]. It is worth noting that time delay can lead to the poor performance of systems [39]–[41]. Due to the influence of time delay, the obtainment of input and output information may be lagged, which makes the estimation of the current state more complex. Therefore, the detectability of DBCNs is more difficult than BCNs considered in [26], [27], [31]. Sun *et al.* [36] proposed an algorithm to check the reconstructibility of DBCNs based on the weighted pair graph, constructed forest, and finite automata. In this paper, we further propose three types of detectabilities, that is, weak detectability, strong detectability, and detectability. Besides, we present some criteria to verify these three kinds of detectabilities of DBCNs based on a full-order observer. This paper provides a new perspective to study the detectability of DBCNs. As an application of DBCNs, the apoptosis network with time delay [43] is used in Example 2 to verify the obtained results.

The rest of this paper is organized as below. Section II recalls some preliminaries on STP, DBCNs, and three kinds of detectabilities. In Section III, we propose a full-order observer of DBCNs to analyze the three types of detectabilities. In Section IV, we give some necessary and sufficient conditions for weak detectability, strong detectability, and detectability. Two illustrative examples are provided to verify our main results in Section V. In Section VI, a short conclusion is given.

II. PRELIMINARIES

The following notations will be used in the sequel.

- $\mathbb{R}$, $\mathbb{Z}$ and $\mathbb{Z}_+$ denote the set of real numbers, the set of integers, and the set of positive integers, respectively.
- $\mathcal{D} := \{1, 0\}$, and $\mathcal{D}^k := \underbrace{\mathcal{D} \times \cdots \times \mathcal{D}}_{k}$.
- $[1, n]_{\mathbb{Z}_+} := \{1, \cdots, n\}$.
- $\mathbf{1}_k := \Big[\underbrace{1\,1\,\cdots\,1}_{k}\Big]^\top$.
- Given $A \in \mathbb{R}^{m\times m}$, $A^\top$ denotes the transposition of A. $Row_j(A)$ and $Col_i(A)$ represent the j-th row of A and the i-th column of A, respectively. $Col(A)$ represents the set of all columns of A.
- I_k denotes the k dimensional identity matrix, and δ_k^i denotes the i-th column of I_k. $\Delta_k := \{\delta_k^i : i = 1, \cdots, k\}$, $\Delta := \Delta_2$.
- $\delta_k^0 := \Big[\underbrace{0\,0\,\cdots\,0}_{k}\Big]^\top$, and δ_k^0 can be abbreviated as $\delta_k[0]$.
- Set $A = [\delta_n^{i_1}\ \delta_n^{i_2}\ \cdots\ \delta_n^{i_t}]$. We abbreviate A as $A = \delta_n[i_1\ i_2\ \cdots\ i_t]$. If $i_j \in \{1, \cdots, n\}$, $\forall j = 1, \cdots, t$, then A is said to be a logical matrix; if $i_j \in \{0, 1, \cdots, n\}$, $\forall j = 1, \cdots, t$, then A is said to be the generalized logical matrix.
- $\mathcal{L}_{m\times n}$ denotes the set of $m \times n$ dimensional logical matrices. $\mathcal{M}_{m\times n}$ denotes the set of $m \times n$ dimensional generalized logical matrices.
- $W_{[m,n]}$ represents the $mn \times mn$ dimensional swap matrix [10].
- Consider matrix $B = (b_{i,j})_{m\times n} \in \mathbb{R}^{m\times n}$. If $b_{i,j} \in \{0, 1\}$, then B is said to be a Boolean matrix. $\mathcal{B}_{m\times n}$ denotes the set of $m \times n$ dimensional Boolean matrices.
- $\overline{\Delta}_k := \Delta_k \bigcup\{\delta_k^0\}$.
- Consider $a, b \in \{0, 1\}$. The Boolean addition of a and b is defined as $a +_{\mathcal{B}} b = \max\{a, b\}$. The Boolean multiplication of a and b is defined as $a \times_{\mathcal{B}} b = \min\{a, b\}$.
- Consider two Boolean matrices $A \in \mathcal{B}_{m\times n}$ and $B \in \mathcal{B}_{n\times q}$. The Boolean product of A and B is denoted as $A \times_{\mathcal{B}} B$.

Next, we give some necessary preliminaries on STP and Boolean semi-tensor product (BSTP). For details, please refer to [10].

Definition 1 [10]: The semi-tensor product of two matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ is

$$A \ltimes B = (A \otimes I_{\frac{\alpha}{n}}) \times (B \otimes I_{\frac{\alpha}{p}}), \tag{1}$$

where $\alpha = lcm(n, p)$ is the least common multiple of n and p, and $\otimes$ is the Kronecker product.

Definition 2 [10]: The Boolean semi-tensor product of two matrices $A \in \mathcal{B}_{m\times n}$ and $B \in \mathcal{B}_{p\times q}$ is

$$A \ltimes_{\mathcal{B}} B = (A \otimes I_{\frac{\alpha}{n}}) \times_{\mathcal{B}} (B \otimes I_{\frac{\alpha}{p}}), \tag{2}$$

where $\alpha = lcm(n, p)$.

Lemma 1 [10]: Consider the logic vector $x \in \Delta_{k^n}$. It holds that

$$x \ltimes_{\mathcal{B}} x = \Phi_n \ltimes_{\mathcal{B}} x,$$

where $\Phi_n = diag\{\delta_{k^n}^1, \cdots, \delta_{k^n}^{k^n}\}$.

Remark 1: Consider the logic vector $u_i \in \Delta_n$, $i = 1, \cdots, s$. One has

$$^{\mathcal{B}}\prod_{i=1}^{s} u_i = \ltimes_{i=1}^{s} u_i,$$

where $^{\mathcal{B}}\prod_{i=1}^{s} u_i = u_1 \ltimes_{\mathcal{B}} \cdots \ltimes_{\mathcal{B}} u_s$, and $\ltimes_{i=1}^{s} u_i = u_1 \ltimes \cdots \ltimes u_s$.

Identifying $1 \sim \delta_2^1, 0 \sim \delta_2^2$, we have $\Delta \sim \mathcal{D}$. We recall the algebraic form of logical functions.

Lemma 2 [10]: Let $f : \mathcal{D}^n \to \mathcal{D}$ be a logical function. Then, there exists a unique matrix $M_f \in \mathcal{L}_{2\times 2^n}$, called the structural matrix of f, satisfying

$$f(x_1, x_2, \cdots, x_n) = M_f \ltimes_{i=1}^{n} x_i,\ x_i \in \Delta. \tag{3}$$

DBCN with n state variables, m input variables and p output variables can be described in the following:

$$\begin{cases} X(t+1) = f(X(t-\tau+1), \cdots, X(t), U(t)), \\ Y(t) = h(X(t)), \end{cases} \tag{4}$$

where $\tau \in \mathbb{Z}_+$ denotes time delay, $X(t) = (x_1(t), \cdots, x_n(t)) \in \mathcal{D}^n$ denotes the state variables at time t, $U(t) = (u_1(t), \cdots, u_m(t)) \in \mathcal{D}^m$ and $Y(t) = (y_1(t), \cdots, y_p(t)) \in \mathcal{D}^p$ represent the input variables and the output variables at time t, respectively, $f = (f_1, \cdots, f_n) : \mathcal{D}^{n\tau+m} \to \mathcal{D}^n$ and $h = (h_1, \cdots, h_p) : \mathcal{D}^n \to \mathcal{D}^p$ are logical functions. $Z(0) = (X(-\tau+1), \cdots, X(-1), X(0)) \in \mathcal{D}^{n\tau}$ denotes an initial state trajectory of DBCN (4). When $\tau = 1$, DBCNs degenerate to BCNs. Therefore, the results in this paper are also applicable to BCNs.

We define the weak detectability, detectability, and strong detectability of DBCN (4).

Definition 3: DBCN (4) is said to be weakly detectable, if there exists $s \in \mathbb{Z}_+$, for any integer $t \geq s$, it holds that $X(t)$ can be determined uniquely by an input sequence $\{U(0), \cdots, U(t-1)\}$ and an output sequence $\{Y(0), \cdots, Y(t)\}$ corresponding to the input sequence.

Definition 4: DBCN (4) is said to be detectable, if there exists $s \in \mathbb{Z}_+$, for any integer $t \geq s$, it holds that $X(t)$ can be determined uniquely by an input sequence $\{U(0), \cdots, U(t-1)\}$ and any output sequence $\{Y(0), \cdots, Y(t)\}$ corresponding to the input sequence.

Definition 5: DBCN (4) is said to be strongly detectable, if there exists $s \in \mathbb{Z}_+$, for any integer $t \geq s$, it holds that $X(t)$ can be determined uniquely by any input sequence $\{U(0), \cdots, U(t-1)\}$ and any output sequence $\{Y(0), \cdots, Y(t)\}$ corresponding to the input sequence.

To simplify the study, we give another three concepts based on the state trajectory of DBCNs, which are equivalent to Definitions 3, 4 and 5.

Definition 6: DBCN (4) is said to be weakly detectable, if there exist a positive integer $s \geq \tau$, an input sequence $\{U(0), \cdots, U(s-1)\}$ and an output sequence $\{Y(0), \cdots, Y(s)\}$ corresponding to the input sequence, such that state trajectory $Z(s) = (X(s-\tau+1), \cdots, X(s))$ can be determined uniquely.

Definition 7: DBCN (4) is said to be detectable, if there exist a positive integer $s \geq \tau$ and an input sequence $\{U(0), \cdots, U(s-1)\}$, such that state trajectory $Z(s) = (X(s-\tau+1), \cdots, X(s))$ can be determined uniquely for any output sequence $\{Y(0), \cdots, Y(s)\}$ corresponding to the input sequence.

Definition 8: DBCN (4) is said to be strongly detectable, if there exists a positive integer $s \geq \tau$, such that for any input sequence $\{U(0), \cdots, U(s-1)\}$ and any output sequence $\{Y(0), \cdots, Y(s)\}$ corresponding to the input sequence, state trajectory $Z(s) = (X(s-\tau+1), \cdots, X(s))$ can be determined uniquely.

Remark 2: If DBCN (4) is strongly detectable, then it must be detectable. Moreover, if DBCN (4) is detectable, then it must be weakly detectable. However, the converse is not true.

Remark 3: When studying the detectability of DBCNs, the estimation of state $X(i)$ cannot be determined by the input sequence and the output sequence, where $-\tau+1 \leq i < 0$ ($i \in \mathbb{Z}$). Thus, the state $X(i)$ can take any values in $\mathcal{D}^n$, where $-\tau+1 \leq i < 0$ ($i \in \mathbb{Z}$). The estimation of state $X(0)$ can be determined by the output sequence $Y(0)$.

Using STP, according to [10], the algebraic form of DBCN (4) can be expressed as follows:

$$\begin{cases} x(t+1) = F \ltimes z(t) \ltimes u(t), \\ y(t) = H \ltimes x(t), \end{cases} \tag{5}$$

where $x(t) = \ltimes_{i=1}^n x_i(t) \in \Delta_{2^n}$, $z(t) = \ltimes_{i=t-\tau+1}^t x(i) \in \Delta_{2^{n\tau}}$, $u(t) = \ltimes_{i=1}^m u_i(t) \in \Delta_{2^m}$, $y(t) = \ltimes_{i=1}^p y_i(t) \in \Delta_{2^p}$, $F \in \mathcal{L}_{2^n \times 2^{n\tau+m}}$, and $H \in \mathcal{L}_{2^p \times 2^n}$.

III. FULL-ORDER OBSERVER OF DBCNs

In this section, based on the system (5), we construct a full-order observer, which will be used to study the detectability of DBCNs.

In the following, the algebraic form of full-order observer is given, which is similar to the observer of [37]:

$$\begin{cases} \hat{x}(t) = \Phi_n^\top \ltimes_{\mathcal{B}} F \ltimes_{\mathcal{B}} \hat{z}(t-1) \ltimes_{\mathcal{B}} u(t-1) \\ \qquad\quad \ltimes_{\mathcal{B}} H^\top \ltimes_{\mathcal{B}} y(t), \quad t \in \mathbb{Z}_+ \\ \hat{x}(0) = H^\top \ltimes_{\mathcal{B}} y(0), \\ \hat{x}(i) = \mathbf{1}_{2^n}, \quad i = -\tau+1, \cdots, -1, \end{cases} \tag{6}$$

where $\hat{x}(t) \in \mathcal{B}_{2^n \times 1}$, $\hat{z}(t) = \ltimes_{i=t-\tau+1}^t \hat{x}(i) \in \mathcal{B}_{2^{n\tau} \times 1}$, and $y(t) \in \Delta_{2^p}$ represent the estimation of state, the estimation of state trajectory and the output at time t, respectively, and $u(t-1) \in \Delta_{2^m}$ represents the input at time $t-1$.

Remark 4: In the sequel, the matrix product is BSTP, denoted by $\ltimes_{\mathcal{B}}$. We often omit the symbol "$\ltimes_{\mathcal{B}}$". Based on Remark 1, for $u(i) \in \Delta_{2^m}$ and $y(i) \in \Delta_{2^p}$, $i = 1, \cdots, s$, it is easy to see that ${}^{\mathcal{B}}\prod_{i=1}^s u(i) = \ltimes_{i=1}^s u(i)$, and ${}^{\mathcal{B}}\prod_{i=1}^s y(i) = \ltimes_{i=1}^s y(i)$.

Based on (6), we give the equivalent algebraic form of the observer as follows:

$$\begin{cases} \hat{x}(t) = M\hat{z}(t-1)u(t-1)y(t), \quad t \in \mathbb{Z}_+ \\ \hat{x}(0) = H^\top y(0), \\ \hat{x}(i) = \mathbf{1}_{2^n}, \quad i = -\tau+1, \cdots, -1, \end{cases} \tag{7}$$

where $M = \Phi_n^\top (I_{2^n} \otimes H^\top) F \in \mathcal{M}_{2^n \times 2^{n\tau+m+p}}$.

Proof: Based on (6), we have

$$\begin{aligned} \hat{x}(t) &= \Phi_n^\top F \hat{z}(t-1)u(t-1)H^\top y(t) \\ &= \Phi_n^\top (I_{2^n} \otimes H^\top) F \hat{z}(t-1)u(t-1)y(t) \\ &:= M\hat{z}(t-1)u(t-1)y(t). \end{aligned}$$

Therefore, (7) is equivalent to (6). □

According to (7), we conclude

$$\begin{aligned} \hat{z}(t) &= \ltimes_{i=t-\tau+1}^t \hat{x}(i) = \ltimes_{i=t-\tau+1}^{t-1} \hat{x}(i)\hat{x}(t) \\ &= \ltimes_{i=t-\tau+1}^{t-1} \hat{x}(i) M \hat{z}(t-1)u(t-1)y(t) \\ &= \ltimes_{i=t-\tau+1}^{t-1} \hat{x}(i) M \ltimes_{i=t-\tau}^{t-1} \hat{x}(i)u(t-1)y(t) \\ &= \ltimes_{i=t-\tau+1}^{t-1} \hat{x}(i) M \hat{x}(t-\tau) \end{aligned}$$

$$\begin{aligned}
&\ltimes_{i=t-\tau+1}^{t-1}\hat{x}(i)u(t-1)y(t)\\
&= (I_{2^{n(\tau-1)}}\otimes M)\ltimes_{i=t-\tau+1}^{t-1}\hat{x}(i)\hat{x}(t-\tau)\\
&\quad \ltimes_{i=t-\tau+1}^{t-1}\hat{x}(i)u(t-1)y(t)\\
&= (I_{2^{n(\tau-1)}}\otimes M)W_{[2^n,2^{n(\tau-1)}]}\hat{x}(t-\tau)\\
&\quad [\ltimes_{i=t-\tau+1}^{t-1}\hat{x}(i)]^2u(t-1)y(t)\\
&= (I_{2^{n(\tau-1)}}\otimes M)W_{[2^n,2^{n(\tau-1)}]}(I_{2^n}\otimes\Phi_{n(\tau-1)})\\
&\quad \ltimes_{i=t-\tau}^{t-1}\hat{x}(i)u(t-1)y(t)\\
&:= P\ltimes_{i=t-\tau}^{t-1}\hat{x}(i)u(t-1)y(t)\\
&:= P\hat{z}(t-1)u(t-1)y(t), \qquad (8)
\end{aligned}$$

where $t\in\mathbb{Z}_+$, $P=(I_{2^{n(\tau-1)}}\otimes M)W_{[2^n,2^{n(\tau-1)}]}(I_{2^n}\otimes\Phi_{n(\tau-1)})\in\mathcal{M}_{2^{n\tau}\times 2^{n\tau+m+p}}$. Note that pseudo-commutative law comes from [10].

Based on (8), we can obtain that

$$\begin{aligned}
\hat{z}(t) &= P\hat{z}(t-1)u(t-1)y(t)\\
&= P^2\hat{z}(t-2)\ltimes_{i=t-2}^{t-1}u(i)y(i+1)\\
&= \cdots\\
&= P^t\hat{z}(0)\ltimes_{i=0}^{t-1}u(i)y(i+1)\\
&= P^t\ltimes_{i=-\tau+1}^{0}\hat{x}(i)\ltimes_{i=0}^{t-1}u(i)y(i+1)\\
&= P^t\mathbf{1}_{2^{n(\tau-1)}}H^\top y(0)\ltimes_{i=0}^{t-1}u(i)y(i+1), \qquad (9)
\end{aligned}$$

where $t\in\mathbb{Z}_+$, $P^t=\underbrace{P\ltimes_{\mathcal{B}}\cdots\ltimes_{\mathcal{B}}P}_{t}$.

IV. VERIFICATION OF DETECTABILITIES FOR DBCNs

In this section, using the full-order observer, we present some necessary and sufficient conditions for the strong detectability, weak detectability, and detectability of DBCNs.

Theorem 1: DBCN (4) is weakly detectable, if and only if there exist $l\geq\tau$ ($l\in\mathbb{Z}_+$) and $q\in\{1,\cdots,2^{ml+p(l+1)}\}$ such that

$$Col_q\left(P^l\mathbf{1}_{2^{n(\tau-1)}}H^\top\right)\in\Delta_{2^{n\tau}}. \qquad (10)$$

Proof: (Necessity) Consider the algebraic form (5). According to Definition 6, the state trajectory $Z(s)=(X(s-\tau+1),\cdots,X(s))$ can be estimated by using an input sequence and an output sequence $\{Y(0),\cdots,Y(s)\}$ corresponding to the input sequence.

Based on (7) and (9), we have

$$\begin{aligned}
\hat{z}(s) &= P\hat{z}(s-1)u(s-1)y(s)=\cdots\\
&= P^{s-1}\hat{z}(1)\ltimes_{i=1}^{s-1}u(i)y(i+1)\\
&= P^s\hat{z}(0)\ltimes_{i=0}^{s-1}u(i)y(i+1)\\
&= P^s\mathbf{1}_{2^{n(\tau-1)}}H^\top y(0)\ltimes_{j=0}^{s-1}u(j)y(j+1). \qquad (11)
\end{aligned}$$

Then, there exist $s\geq\tau$ ($s\in\mathbb{Z}_+$) and $y(0)\ltimes_{j=0}^{s-1}u(j)y(j+1)\in\Delta_{2^{ms+p(s+1)}}$, such that $\hat{z}(s)\in\Delta_{2^{n\tau}}$ holds. Set $l=s$. Let $y(0)\ltimes_{j=0}^{l-1}u(j)y(j+1)=\delta^q_{2^{ml+p(l+1)}}$. Therefore, there exist a positive integer $l\geq\tau$ and $q\in\{1,\cdots,2^{ml+p(l+1)}\}$ such that (10) holds.

(Sufficiency) Assume that (10) holds for $l\geq\tau$ ($l\in\mathbb{Z}_+$) and $q\in\mathbb{Z}_+$. Then, there exist $l\in\mathbb{Z}_+$ and $y(0)\ltimes_{j=0}^{l-1}u(j)y(j+1)=\delta^q_{2^{ml+p(l+1)}}$ satisfying (11).

Based on (11), one can obtain that

$$\hat{z}(l)=P^l\mathbf{1}_{2^{n(\tau-1)}}H^\top y(0)\ltimes_{j=0}^{l-1}u(j)y(j+1). \qquad (12)$$

Thus, we have $\hat{z}(l)\in\Delta_{2^{n\tau}}$. Then, there exist $l\geq\tau$ ($l\in\mathbb{Z}_+$), an input sequence $\{U(0),\cdots,U(l-1)\}$ and an output sequence $\{Y(0),\cdots,Y(l)\}$ corresponding to the input sequence, such that $\hat{z}(l)\in\Delta_{2^{n\tau}}$ holds. Therefore, by Definition 6, DBCN (4) is weakly detectable. □

Theorem 2: DBCN (4) is strongly detectable, if and only if there exists $l\geq\tau$ ($l\in\mathbb{Z}_+$) such that

$$Col\left(P^l\mathbf{1}_{2^{n(\tau-1)}}H^\top\right)\subseteq\overline{\Delta}_{2^{n\tau}}. \qquad (13)$$

Proof: (Necessity) Based on (7) and (9), one can obtain (11). According to Definition 8, there exists $s\geq\tau$ ($s\in\mathbb{Z}_+$), for any input sequence and any output sequence $\{Y(0),\cdots,Y(s)\}$ corresponding to the input sequence, it holds that $\hat{z}(s)\in\Delta_{2^{n\tau}}$. In addition, for any input sequence and any output sequence independent of the input sequence, we have $\hat{z}(t)=\delta^0_{2^{n\tau}}$. Set $l=s$. Therefore, there exists $l\geq\tau$ ($l\in\mathbb{Z}_+$) such that (13) holds.

(Sufficiency) Based on (11), one can get (12), which shows that $\hat{z}(l)\in\overline{\Delta}_{2^{n\tau}}$. When $\hat{z}(t)=\delta^0_{2^{n\tau}}$, for any input sequence and any output sequence independent of the input sequence, the state trajectory does not exist. When $\hat{z}(t)\in\Delta_{2^{n\tau}}$, for any input sequence and any output sequence $\{Y(0),\cdots,Y(l)\}$ corresponding to the input sequence, the state trajectory can be uniquely determined. Therefore, by Definition 8, DBCN (4) is strongly detectable. □

Theorem 3: DBCN (4) is detectable, if and only if there exist $l\geq\tau$ ($l\in\mathbb{Z}_+$) and $q\in\{1,\cdots,2^{ml}\}$ such that

$$\begin{cases}P^l\mathbf{1}_{2^{n(\tau-1)}}H^\top\Lambda_l\delta^q_{2^{ml}}\neq\mathbf{1}^\top_{2^{p(l+1)}}\delta^0_{2^{n\tau}},\\ Col\left(P^l\mathbf{1}_{2^{n(\tau-1)}}H^\top\Lambda_l\delta^q_{2^{ml}}\right)\subseteq\overline{\Delta}_{2^{n\tau}},\end{cases} \qquad (14)$$

where $\Lambda_l=W_{[2^m,2^p]}\left[{}^{\mathcal{B}}\prod_{i=2}^{l}(I_{2^{(i-1)m}}\otimes W_{[2^m,2^{ip}]})\right]$.

Proof: (Necessity) According to Definition 7, the state trajectory $Z(s)=(X(s-\tau+1),\cdots,X(s))$ can be estimated by using an input sequence and any output sequence $\{Y(0),\cdots,Y(s)\}$ corresponding to the input sequence. Moreover, for any input sequence and any output sequence independent of the input sequence, the state trajectory does not exist.

Based on (7), (9) and (11), we have

$$\begin{aligned}
&y(0)\ltimes_{i=0}^{s-1}u(i)y(i+1)\\
&\quad= W_{[2^m,2^p]}u(0)\ltimes_{k=0}^{1}y(k)\ltimes_{i=1}^{s-1}u(i)y(i+1)\\
&\quad= W_{[2^m,2^p]}(I_{2^m}\otimes W_{[2^m,2^{2p}]})\ltimes_{j=0}^{1}u(j)\\
&\qquad \ltimes_{k=0}^{2}y(k)\ltimes_{i=2}^{s-1}u(i)y(i+1)\\
&\quad= \cdots
\end{aligned}$$

$$= W_{[2^m,2^p]}\Big[{}^{\mathcal{B}}\prod_{i=2}^{s}(I_{2^{(i-1)m}} \otimes W_{[2^m,2^{ip}]})\Big] \ltimes_{j=0}^{s-1} u(j) \ltimes_{k=0}^{s} y(k) = \Lambda_s \ltimes_{j=0}^{s-1} u(j) \ltimes_{k=0}^{s} y(k), \quad (15)$$

where $\Lambda_s = W_{[2^m,2^p]}\Big[{}^{\mathcal{B}}\prod_{i=2}^{s}(I_{2^{(i-1)m}} \otimes W_{[2^m,2^{ip}]})\Big] \in \mathcal{L}_{2^{ms+ps}\times 2^{ms+ps}}$.

According to (15) and (11), one has

$$\hat{z}(s) = P^s \mathbf{1}_{2^{n(\tau-1)}} H^\top \Lambda_s \ltimes_{j=0}^{s-1} u(j) \ltimes_{i=0}^{s} y(i). \quad (16)$$

Then, there exist $s \geq \tau$ ($s \in \mathbb{Z}_+$) and $\{U(0), \cdots, U(s-1)\}$, for any output sequence $\{Y(0), \cdots, Y(s)\}$ corresponding to the input sequence, it holds that $\hat{z}(s) \in \Delta_{2^{n\tau}}$. Besides, for any input sequence and any output sequence independent of the input sequence, one obtains $\hat{z}(s) = \delta_{2^{n\tau}}^0$.

Thus, there exist $s \geq \tau$ ($s \in \mathbb{Z}_+$) and $\ltimes_{j=0}^{s-1} u(j) = \delta_{2^{ms}}^q$, such that

$$P^s \mathbf{1}_{2^{n(\tau-1)}} H^\top \Lambda_s \delta_{2^{ms}}^q \neq \mathbf{1}_{2^{p(s+1)}}^\top \delta_{2^{n\tau}}^0,$$

and for any $\ltimes_{j=0}^{s} y(j) \in \Delta_{2^{p(s+1)}}$, it holds that $\hat{z}(s) \in \overline{\Delta}_{2^{n\tau}}$. Set $l = s$. Therefore, there exists $\ltimes_{j=0}^{l-1} u(j) = \delta_{2^{ml}}^q$ such that (14) holds for any $\ltimes_{j=0}^{l} y(j) \in \Delta_{2^{p(l+1)}}$.

(Sufficiency) Assume that there exist $l \geq \tau$ ($l \in \mathbb{Z}_+$) and $q \in \mathbb{Z}_+$ such that (14) holds. Then, there exist $l \in \mathbb{Z}_+$ and $\ltimes_{j=0}^{l-1} u(j) = \delta_{2^{ml}}^q$ such that (14) holds.

Based on (16), we have

$$\hat{z}(l) = P^l \mathbf{1}_{2^{n(\tau-1)}} H^\top \Lambda_l \ltimes_{j=0}^{l-1} u(j) \ltimes_{i=0}^{l} y(i).$$

Then $\hat{z}(l) \in \overline{\Delta}_{2^{n\tau}}$. When $\hat{z}(t) = \delta_{2^{n\tau}}^0$, there exists an input sequence $\{U(0), \cdots, U(l-1)\}$, such that for any output sequence independent of the input sequence, the state trajectory does not exist. When $\hat{z}(t) \in \Delta_{2^{n\tau}}$, there exists an input sequence $\{U(0), \cdots, U(l-1)\}$, such that the state trajectory can be uniquely determined for any output sequence $\{Y(0), \cdots, Y(l)\}$ corresponding to the input sequence. By Definition 7, DBCN (4) is detectable. □

Remark 5: For DBCN (4), the upper bound of l in Theorems 1, 2 and 3 is $l_{\max} = 2^{2^{n\tau}}$.

Remark 6: Theorem 2 also provides a sufficient condition for the verification of detectability and weak detectability. In addition, Theorem 3 is also applicable to verifying the weak detectability.

V. ILLUSTRATIVE EXAMPLES

In this section, we give two examples to illustrate the obtained results.

Example 1: Consider the following DBCN:

$$\begin{cases} x_1(t+1) = u(t) \wedge \{\neg x_1(t-1) \vee \neg x_2(t)\}, \\ x_2(t+1) = u(t) \wedge \{\neg x_1(t) \wedge \neg x_2(t)\}, \\ y(t) = x_1(t) \wedge x_2(t), \end{cases} \quad (17)$$

where x_i denotes the state variable, $i = 1, 2$, u denotes the input variable, and y denotes the output variable.

Using the STP method, we give the algebraic form of (17) as same as (5), where

$$F = \delta_4[4\ 4\ 2\ 4\ 4\ 4\ 1\ 4\ 4\ 4\ 2\ 4\ 4\ 4\ 1\ 4\ \ 2\ 4\ 2\ 4\ 2\ 4\ 1\ 4\ 2\ 4\ 2\ 4\ 2\ 4\ 1\ 4],$$
$$H = \delta_2[1\ 2\ 2\ 2]. \quad (18)$$

According to the algebraic form, we present a full-order observer which is the same to (7), where

$$M = \Phi_2^\top (I_4 \otimes H^\top) F. \quad (19)$$

Split M into four equal blocks $M = [M_1, M_2, M_3, M_4]$, where

$$M_1 = \delta_4[0\ 4\ 0\ 4\ 0\ 2\ 0\ 4\ 0\ 4\ 0\ 4\ 1\ 0\ 0\ 4],$$
$$M_2 = \delta_4[0\ 4\ 0\ 4\ 0\ 2\ 0\ 4\ 0\ 4\ 0\ 4\ 1\ 0\ 0\ 4],$$
$$M_3 = \delta_4[0\ 2\ 0\ 4\ 0\ 2\ 0\ 4\ 0\ 2\ 0\ 4\ 1\ 0\ 0\ 4],$$
$$M_4 = \delta_4[0\ 2\ 0\ 4\ 0\ 2\ 0\ 4\ 0\ 2\ 0\ 4\ 1\ 0\ 0\ 4].$$

Based on (8), we have

$$P = (I_4 \otimes M) W_{[4,4]} (I_4 \otimes \Phi_2) = [P_1, P_2, P_3, P_4] \in \mathcal{M}_{16\times 64},$$

and split P into four equal blocks $P = [P_1, P_2, P_3, P_4]$, where

$$P_1 = \delta_{16}[0\ 4\ 0\ 4\ 0\ 6\ 0\ 8\ 0\ 12\ 0\ 12\ 13\ 0\ 0\ 16],$$
$$P_2 = \delta_{16}[0\ 4\ 0\ 4\ 0\ 6\ 0\ 8\ 0\ 12\ 0\ 12\ 13\ 0\ 0\ 16],$$
$$P_3 = \delta_{16}[0\ 2\ 0\ 4\ 0\ 6\ 0\ 8\ 0\ 10\ 0\ 12\ 13\ 0\ 0\ 16],$$
$$P_4 = \delta_{16}[0\ 2\ 0\ 4\ 0\ 6\ 0\ 8\ 0\ 10\ 0\ 12\ 13\ 0\ 0\ 16].$$

According to (9), we get

$$(\mathbf{1}_4 H^\top)^\top = \delta_2[1\ 2\ 2\ 2\ 1\ 2\ 2\ 2\ 1\ 2\ 2\ 2\ 1\ 2\ 2\ 2].$$

Calculate $P^l \mathbf{1}_4 H^\top$, where $l \geq 1$ and $l \in \mathbb{Z}_+$. When $l = 3$, we have

$$Col_q(P^3 \mathbf{1}_4 H^\top) \in \Delta_{16},$$

where

$$q \in \Psi := \{18, 20, 22, 24, 29, 32, 50, 52, 61, 64, 70, 72, 77, 80, 82, 84, 86, 88, 93, 96, 114, 116, 125, 128\}.$$

For any $q' \in [1, 128]_{\mathbb{Z}_+} \backslash \Psi$, we obtain $Col_{q'}(P^3 \mathbf{1}_4 H^\top) = \delta_{16}^0$. Based on Theorem 2, DBCN (17) is strongly detectable. Based on Remarks 2 and 6, it naturally holds that DBCN (17) is weakly detectable and detectable. □

Example 2: Consider the following apoptosis network with time delay [43]:

$$\begin{cases} x_1(t+1) = \neg x_2(t-1) \wedge u(t), \\ x_2(t+1) = \neg x_1(t-1) \wedge x_3(t-1), \\ x_3(t+1) = x_2(t-1) \vee u(t), \\ y(t) = x_1(t), \end{cases} \quad (20)$$

where x_1, x_2, x_3 and u represent the inhibitor of apoptosis proteins (*IAP*), the concentration level of the active caspase 3 ($C3\ a$), the concentration level of the active caspase 8 ($C8\ a$), and the concentration level of the tumor necrosis factor

(*TNF*, a stimulus), respectively, and y denotes the output variable.

Using the STP method, we give the algebraic form of (20) as same as (5), where

$$\begin{aligned} F = \delta_8[&7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\\ &7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\\ &3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\\ &3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\\ &5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\\ &7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\\ &1\ 6\ 1\ 6\ 1\ 6\ 1\ 6\ 1\ 6\ 1\ 6\ 1\ 6\ 1\ 6\\ &3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8\ 3\ 8],\\ H = \delta_2[&1\ 1\ 1\ 1\ 2\ 2\ 2\ 2]. \end{aligned} \tag{21}$$

Based on the algebraic form, we give a full-order observer which is the same to (7), where

$$M = \Phi_3^\top (I_8 \otimes H^\top)F.$$

According to (8) and (9), we have

$$P = (I_8 \otimes M)W_{[8,8]}(I_8 \otimes \Phi_3) \in \mathcal{M}_{64\times 256}$$

and

$$\begin{aligned} (\mathbf{1}_8 H^\top)^\top = \delta_2[&1\ 1\ 1\ 1\ 2\ 2\ 2\ 2\ 1\ 1\ 1\ 1\ 2\ 2\ 2\ 2\\ &1\ 1\ 1\ 1\ 2\ 2\ 2\ 2\ 1\ 1\ 1\ 1\ 2\ 2\ 2\ 2\\ &1\ 1\ 1\ 1\ 2\ 2\ 2\ 2\ 1\ 1\ 1\ 1\ 2\ 2\ 2\ 2\\ &1\ 1\ 1\ 1\ 2\ 2\ 2\ 2\ 1\ 1\ 1\ 1\ 2\ 2\ 2\ 2]. \end{aligned}$$

Now, calculate $P^l \mathbf{1}_8 H^\top$, where $l \geq 1$ and $l \in \mathbb{Z}_+$. When $l = 2^{64}$ and $q = 1$, we obtain

$$Col_q(P^l \mathbf{1}_8 H^\top) \in \Delta_{64}.$$

However, when $l = 2^{64}$ and $q' = 4^{(2^{64})}$, we have

$$Col_{q'}(P^3 \mathbf{1}_8 H^\top) \subseteq \{\delta_{64}^{38}, \delta_{64}^{40}, \delta_{64}^{46}, \delta_{64}^{48}, \delta_{64}^{54}, \delta_{64}^{56}, \delta_{64}^{62}, \delta_{64}^{64}\}.$$

Based on Theorems 1 and 2, DBCN (20) is weakly detectable, but not strongly detectable.

When $l = 4$ and $q'' = 1$, we obtain $\Lambda_4 = W_{[2,2]}(I_2 \otimes W_{[2,4]})(I_4 \otimes W_{[2,8]})(I_8 \otimes W_{[2,16]})$ and

$$\begin{cases} P^4 \mathbf{1}_8 H^\top \Lambda_4 \delta_{16}^{q''} \neq \mathbf{1}_{32}^\top \delta_{64}^0, \\ Col(P^4 \mathbf{1}_8 H^\top \Lambda_4 \delta_{16}^{q''}) \subseteq \overline{\Delta}_{64}. \end{cases} \tag{22}$$

By Theorem 3, DBCN (20) is detectable.

Theoretical results reveal that gene states of apoptosis network can be determined by the knowledge of *IAP* and *TNF*. □

VI. CONCLUSION

In this paper, we have studied three kinds of detectabilities for DBCNs based on a full-order observer. We have presented the concepts of weak detectability, strong detectability, and detectability for DBCNs. We have constructed a full-order observer of DBCNs, based on which, we have proposed some criteria for the weak detectability, strong detectability, and detectability of DBCNs.

Future works will focus on studying strong periodic detectability, periodic detectability, and weak periodic detectability of DBCNs. Another interesting topic is to explore the detectability of large-scale BCNs [44] with time delay by using network aggregation and pinning control.

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