IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received July 12, 2021, accepted August 23, 2021, date of publication August 24, 2021, date of current version September 2, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3107604

Characteristic Analysis of Judgment Debtors Based on Hesitant Fuzzy Linguistic Clustering Method

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This work was supported by the National Social Science Foundation of China under Grant 18BJY108.

ABSTRACT In most law enforcement cases, judgment debtors have behaviors of evading execution in China, which seriously affects the authority of legal judgments and the judiciary's credibility. Characteristics analysis of judgment debtors plays a vital role in finding out the concealed property and improving efficiency in handling law enforcement cases. Considering the advantages of hesitant fuzzy linguistic term sets (HFLTSs) representing the judgment debtors' attributes and keeping all the original evaluation information on judgment debtors, we develop a hesitant fuzzy linguistic agglomerative hierarchical clustering (HFL-AHC) method to cluster judgment debtors and analyze the main characteristic of judgment debtors with concealing property. In some situations, the existing HFLTS distance cannot divide the judgment debtors. Therefore, we propose some new distance measures to classify the judgment debtors. The clustering results show that the judgment debtors who hide property have a poor evaluation of trading behavior, work, credibility, and consumption behavior.

INDEX TERMS Hesitant fuzzy linguistic term sets (HFLTSs), agglomerative hierarchical clustering (AHC) method, distance measure, judgment debtors, law enforcement.

I. INTRODUCTION

In China, due to the influence of society's low legal consciousness, the lack of social credit system, the imperfect property supervision system, and other factors, the legal documents' automatic performance rate is not high after they come into force. Most of the valid legal documents must be enforced by the court. In recent years, the number of difficult law enforcement cases has increased dramatically. According to the Chinese Supreme People's Court's statistics in 2018, more than 80% of the law enforcement cases, judgment debtors have behaviors of evading or even violently resisting execution, and about 15% of the cases passively waiting for enforcement. Many law enforcement cases cannot be executed smoothly, which seriously affects the authority of legal judgments and the judiciary's credibility.

Judgment debtors act as the subject of law enforcement cases, and the analysis of their characteristics play a vital role in improving efficiency in handling law enforcement cases. Most of the current law enforcement cases only have textual data, and expert judges can only use the law enforcement case's textual information to represent the judgment debtor's characteristics. However, due to the limited understanding and knowledge of law enforcement cases, expert judges are more likely to use fuzzy language to qualitatively assess the attributes of the judgment debtor's information based on their experience in handling cases. For example, expert judges express the risk preference of the judgment debtor's trading behavior. It is difficult to use simple binary logic of "good" or "bad" to describe such information and usually cannot give a precise score, which cannot accurately express the evaluation information's uncertainty and vagueness. Therefore, to get closer to people's perceptions of things, people often use fuzzy linguistic variables to describe them. To better express the uncertainty, Zedeh used fuzzy set theory to depict linguistic information and proposed the concept of fuzzy linguistic variables [19]. Using fuzzy linguistic can more accurately represent the vagueness and uncertainty of the judgment debtors' attributes and make the judgment debtors' attributes have stronger interpretability. Fuzzy linguistic is very important in accurately representing the assessment information of the judgment debtors' attributes [23].

In the characteristic analysis of judgment debtors, because the expert judges may have different opinions in evaluating

The associate editor coordinating the review of this manuscript and approving it for publication was Khalid Aamir.

the judgment debtors' attributes, there is a certain hesitation in the evaluation information. It is usually difficult to use a single linguistic term to evaluate a certain attribute or a variable. A hesitant fuzzy linguistic term set (HFLTS) allows multiple linguistic terms to be used to evaluate the judgment debtors' attribute for keeping all the expert judges' evaluation information. The attribute values represented by HFLTS has some advantages in integrating expert judges' assessment information. The concept of HFLTS was firstly proposed by Rodriguez *et al.* [1]. The current research on HFLTS mainly focuses on multi-attribute decision-making methods [24]–[26], aggregation functions [22], correlation coefficient [7], and distance and similarity measures [2], [3], etc. Little research has been done on clustering methods under a hesitant fuzzy linguistic environment, which cannot meet the requirements of judgment debtor characteristic analysis.

The clustering method is an unsupervised learning method that the judgment debtors are classified into different clusters by some specific criteria, such as distance or similarity. The judgment debtors in the same cluster have a high degree of similarity while ensuring that judgment debtors in different clusters have large differences [9]. Based on the clustering results, we analyze which characteristics of the judgment debtors are more likely to conceal property. The characteristic analysis of the judgment debtors is very important for finding the hidden property in the enforcement cases. Therefore, we introduce a hesitant fuzzy linguistic agglomerative hierarchical clustering (HFL-AHC) method to analyze judgment debtors' characteristics.

The main idea of clustering analysis of the judgment debtors is that HFLTS is used to represent the integrating evaluation information on the judgment debtors' attributes given by expert judges. Then the HFL-AHC method is used to cluster the judgment debtors, which classifies the judgment debtors into those who are concealed property. The characteristics of judgment debtors with concealing property is analyzed by the clustering results. The analysis process is shown in Fig. 1.

The contributions of our work are as follows. Firstly, considering the advantages of HFLTSs representing the judgment debtors' attributes and keeping all the expert judges' evaluation information on judgment debtors, we develop a new clustering method to cluster judgment debtors and analyze the main characteristic of judgment debtors. Secondly, considering the existing HFLTS distance cannot distinguish the judgment debtors in some situations, we propose several new distance measures of HFLTSs to effectively determine the judgment debtors and discuss their properties. Thirdly, a new HFL-AHC method for clustering judgment debtors is developed based on new distance measures. Fourthly, compared with the existing clustering methods, it is more reasonable and suitable for clustering judgment debtors.

The structure of our work is as follows. Section 2 provides some existing researches on judgment debtors and fuzzy clustering methods. Section 3 introduces some concepts of HFLTSs and the agglomerative hierarchical clustering method. Section 4 analyzes the drawbacks of the existing distance measure for classifying the judgment debtors and develops some new distance measures of HFLTSs. Section 5 proposed a new hesitant fuzzy linguistic agglomerative hierarchical clustering method based on new distance measures for clustering the judgment debtors. Section 6 gives an example to illustrate the effectiveness of the developed clustering method and makes a comparative analysis with some other clustering methods. Section 7 presents some conclusions.

II. LITERATURE REVIEW

Clustering the judgment debtors aims to investigate what characteristics make a judgment debtor more likely to conceal property. The existing researches of judgment debtors have focused on law system improvement. Bell [27] analyzed and summarized cases of concealment or transfer of property in the United States and used them in criminal prosecution cases in England to help judges define whether the defendant's actions in similar cases are necessarily intentional concealment and whether the results of the actions constitute a crime. Kupelyants [28] found out it was needlessly onerous that the judgment debtor enforced a foreign judgment under English law and analogized to other legal regimes. Fernandez-Bertier [29] discussed the confiscation system's mechanisms and the effective models of recovery the criminal property. Some scholars have also applied the fuzzy set method to evaluate the possibility of hidden property and debt repayment by judgment debtors in recent years. Zhang *et al.* [30] concluded the factors affecting law enforcement by judgment debtors and developed a hybrid TODIM method for assessing the possibility of enforcing legal instruments. The results showed that the TODIM based method help to improve the efficiency of handling law enforcement cases. Wu *et al.* [31], [32], He *et al.* [33] and Zhang *et al.* [34] evaluated which executor is more likely to conceal property. However, the research findings cannot analyze the characteristics of the judgment debtors with hiding property.

The clustering method provides a useful tool to analyze the characteristics of judgment debtors is analyzed by the clustering result. As an unsupervised learning method, the clustering method mainly divides a group of judgment debtors into several categories according to the principle of large intra-class similarity and small out-of-class similarity [9]. It is widely used in the fields of fuzzy control, medical diagnosis, information retrieval [10]–[12]. According to the clustering technology properties, the commonly used clustering methods mainly include hierarchical clustering, partition-based clustering, and density-based clustering methods [13]. As an essential clustering method, the hierarchical clustering method can be either agglomerative or split. To cluster at different levels, it includes a series of iterative steps. Each layer is made up of merging and cutting techniques. The hierarchical clustering method forms some tree structure of data, which is a widely used clustering technology. It can be divided into two categories [8]: (1) The agglomerative clustering method, in which all objects are regarded as a unique

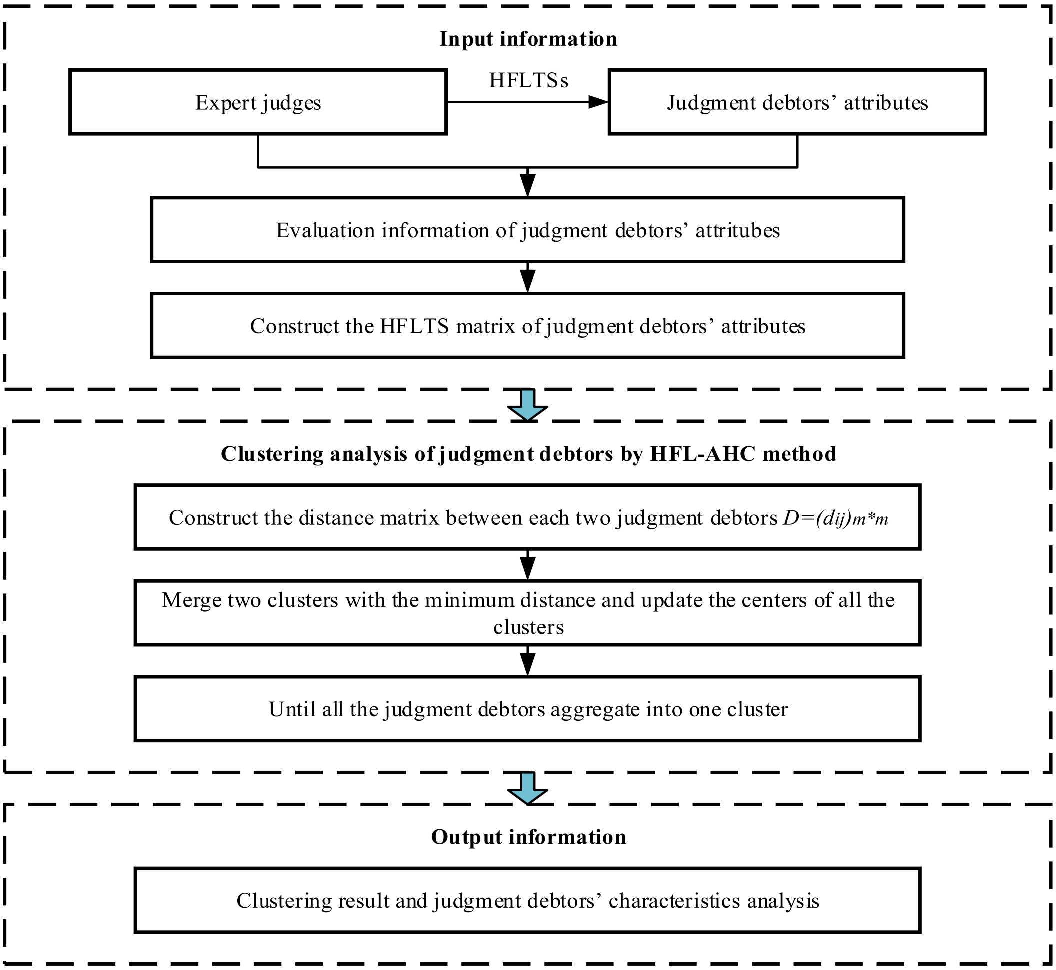

FIGURE 1. The flowchart of judgment debtor characteristics analysis based on the HFL-AHC method.

cluster, gathers into one cluster by the similarity. (2) The divisive clustering method, in which all objects are considered one cluster, and finally, each object is separate from the one cluster and becomes a unique cluster. Moreover, the agglomerative hierarchical clustering is more widely used, and the properties have been paid more attention [14]–[21]. However, the existing AHC methods have been used for a real number [14]–[15], fuzzy information [15], [16], intuitionistic fuzzy information [20], Pythagorean fuzzy information [18], interval type-2 fuzzy information [17], and hesitant fuzzy information [21], and are not suitable to hesitant fuzzy linguistic information.

In real life, the evaluation information of attributes given by expert judges is imprecise, uncertain, or ambiguous. To address the characteristics of evaluation information, scholars have extended clustering methods to fuzzy environments [35], intuitionistic fuzzy environments [36], [37], type-2 fuzzy environments [38]–[40], etc. However, in the judgment debtor characteristic analysis, different expert judges may have different opinions on the judgment debtors' evaluation information. The above fuzzy clustering algorithms do not consider the differences in the opinions of different expert judges. It is usually difficult to use a single linguistic term to evaluate an attribute of judgment debtors. Several possible linguistic terms are retained when processing expert judges' evaluation information [1]. Therefore, the HFLTS has some advantages in expressing the original preference information and expert judges' evaluation information's hesitancy.

HFLTS was introduced by Rodriguez *et al.* [1] to evaluate an attribute by several linguistic terms. For instance, the expert judges provide the attribute values of judgment debtors by linguistic terms, we assume that the linguistic term set (LTS) are as follows: $S = \{s_0$ = Very Poor (VP), s_1 = Poor (P), s_2 = Moderately Poor (MP), s_3 = Medium (M), s_4 = Moderately Good (MG), s_5 = Good (G), s_6 = Very Good (VG)$\}$. Some expert judges think the judgment debtor's credulity is very good, while others think it is moderately good or good, and these judgment debtors cannot persuade each other. Thus, to obtain a reasonable decision result, the evaluating value of the judgment debtors' attributes should be represented by a hesitant fuzzy linguistic element (HFLE) $\{s_4$ = Moderately Good (MG), s_5 = Good (G), s_6 = Very Good (VG)$\}$. It is noted that the HFLE $\{s_4$,

$s_5, s_6\}$ can be described by three linguistic terms and is more objective than that by a single linguistic term. To solve such a problem, Rodriguez *et al.* [1] put forward the concepts of HFLTSs, which provided a more effective tool to describe the expert judges' preference when evaluating the attributes by several linguistic terms. They also discussed some basic operation laws, properties, context-free grammar, and transforming linguistic expression into HFLTS. Subsequently, Liao and Xu [2], Liao *et al.* [3] measured the distance and similarity between two HFLEs or HFLTSs by Hamming distance, Euclidean distance, generalized distance, Hausdorff distance, and cosine distance. The existing distances had no consideration of the hesitance of HFLEs, Zhang *et al.* [5] developed some hesitance-based distance and similarity measures on HFLEs. Besides, Zhang and Wu [22] proposed some hesitant fuzzy linguistic aggregation operators and hesitant fuzzy uncertain linguistic aggregation operators. Inspired by the likelihood-based comparison relation between intervals, Lee and Chen [6] investigated the likelihood-based comparison of HFLTS. They developed some aggregation operators of HFLTS, such as HFLWA, HFLWG, HFLOWA, HFLOWG operators. On the other hand, motivated by traditional fuzzy sets, Liao *et al.* [7] proposed a series of correlation coefficients on HFLTS and applied them to the Chinese medical diagnosis process.

The research on hesitant fuzzy linguistic clustering methods mainly focuses on the transitive closure clustering method based on distance, similarity, and correlation measures of HFLTSs [8], [41]. The transitive closure clustering methods lose a lot of original information while transforming into the hesitant fuzzy linguistic equivalent matrix that cannot meet the need to solve complex qualitative clustering problems such as judgment debtor characteristic analysis. To overcome these disadvantages, we develop a new HFL-AHC method for clustering the judgment debtors.

III. PRELIMINARIES

A. BASIC CONCEPTS ON HFLTSs

Definition 1: [4] Let $S = \{s_0, s_1, \ldots, s_g\}$ be a LTS, $x_i \in X$, and $i = 1, 2, \ldots, N$, then the mathematical form of an HFLTS on X is

$$H_S = \{< x_i, h_S(x_i) > | x_i \in X\}.$$

where $h_S(x_i) : X \to S$ is the possible linguistic terms of the element $x_i \in X$, an HFLE $h_S(x_i) = \{s_{\delta_l}(x_i) \,|\, s_{\delta_l}(x_i) \in S, l = 1, 2, \ldots, L(x_i)\}$ is expressed as with $\delta_l \in \{0, 1, \ldots, g\}$ is the subscript of the linguistic term $s_{\delta_l}(x_i)$, $L(x_i)$ is the number of linguistic terms in $h_S(x_i)$. For convenience, $s_{\delta_l}(x_i)$, $L(x_i)$ can be abbreviated as $s^i_{\delta_l}$, L_i.

Definition 2: [3] Let $H^1_S = \left\{\left\langle x_i, \left\{s_{\delta^1_j}(x_i) \,\middle|\, s_{\delta^1_j}(x_i) \in S\right\}\right\rangle | x_i \in X\right\}$ and $H^2_S = \left\{\left\langle x_i, \left\{s_{\delta^2_j}(x_i) \,\middle|\, s_{\delta^2_j}(x_i) \in S\right\}\right\rangle | x_i \in X\right\}$ be two HFLTSs on $X = (x_1, x_2, \cdots, x_n)$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector for $x_i \in X$, and satisfy $\omega_i \geq 0$, $\sum_{i=1}^{n} \omega_i = 1$. And the distance between H^1_S and H^2_S satisfies the following properties.

(1) $0 \leq d\left(H^1_S, H^2_S\right) \leq 1$,

(2) $d\left(H^1_S, H^2_S\right) = 0$, if and only if $H^1_S = H^2_S$,

(3) $d\left(H^1_S, H^2_S\right) = d\left(H^2_S, H^1_S\right)$.

The weighted Hamming distance, Euclidean distance, and generalized distance of HFLTSs can be defined as

$$d_{\omega h}\left(H^1_S, H^2_S\right) = \sum_{i=1}^{n} \omega_i \left[\frac{1}{L_i} \sum_{j=1}^{L_i} \frac{\left|\delta^1_j(x_i) - \delta^2_j(x_i)\right|}{g}\right] \quad (1)$$

$$d_{\omega e}\left(H^1_S, H^2_S\right) = \left[\sum_{i=1}^{n} \omega_i \left[\frac{1}{L_i} \sum_{j=1}^{L_i} \left|\frac{\delta^1_j(x_i) - \delta^2_j(x_i)}{g}\right|^2\right]\right]^{1/2} \quad (2)$$

$$d_{\omega g}\left(H^1_S, H^2_S\right) = \left[\sum_{i=1}^{n} \omega_i \left[\frac{1}{L_i} \sum_{j=1}^{L_i} \left|\frac{\delta^1_j(x_i) - \delta^2_j(x_i)}{g}\right|^\lambda\right]\right]^{1/\lambda} \quad (3)$$

In particular, when $\omega = (1/n, 1/n, \cdots, 1/n)^T$, the weighted Hamming distance, Euclidean distance, and generalized distance of HFLTSs degenerate into the Hamming distance, Euclidean distance, and generalized distance of HFLTSs, can be defined as

$$d_h\left(H^1_S, H^2_S\right) = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{1}{L_i} \sum_{j=1}^{L_i} \frac{\left|\delta^1_j(x_i) - \delta^2_j(x_i)\right|}{g}\right] \quad (4)$$

$$d_e\left(H^1_S, H^2_S\right) = \left[\frac{1}{n} \sum_{i=1}^{n} \left[\frac{1}{L_i} \sum_{j=1}^{L_i} \left|\frac{\delta^1_j(x_i) - \delta^2_j(x_i)}{g}\right|^2\right]\right]^{1/2} \quad (5)$$

$$d_g\left(H^1_S, H^2_S\right) = \left[\frac{1}{n} \sum_{i=1}^{n} \left[\frac{1}{L_i} \sum_{j=1}^{L_i} \left|\frac{\delta^1_j(x_i) - \delta^2_j(x_i)}{g}\right|^\lambda\right]\right]^{1/\lambda} \quad (6)$$

where $\delta^1_j(x_i)$ and $\delta^2_j(x_i)$ are the subscripts of j-th linguistic term in i-th attribute on H^1_S and H^2_S, L_{1_i} and L_{2_i} are the numbers of i-th attribute, $L_i = \max\left(L_{1_i}, L_{2_i}\right)$. When $L_{1_i} \neq L_{2_i}$, the shorter one should be extended by adding some linguistic terms.

Definition 3: [8]. Let $H^1_S, H^2_S, \cdots, H^n_S$ be a set of n HFLTSs, the hesitant fuzzy linguistic generalized operator is defined as

$$HFLG\left(H^1_S, H^2_S, \cdots, H^n_S\right) = \bigoplus_{i=1}^{n} \left(\frac{1}{n} H^i_S\right) = \cup_{s_{\alpha_1} \in H^1_S, s_{\alpha_2} \in H^2_S, \cdots, s_{\alpha_n} \in H^n_S} \left\{s_{\sum_{i=1}^{n} \alpha_i / n}\right\} \quad (7)$$

Definition 4: [42], [43]. Let H^1_S and H^2_S be two HFLTSs, the ordering methods of HFLTSs can be defined as

(1) Partial ordering "$\leq$":

$H^1_S \leq H^2_S$, iff $h^1_S(x_i) \leq h^2_S(x_i)$, iff $s_{\delta^1_j}(x_i) \leq s_{\delta^2_j}(x_i)$, iff $\delta^1_j \leq \delta^2_j$, for all $1 \leq i \leq N$.

(2) Complete ordering "$\leq$":

$H^1_S \leq H^2_S$, iff $s\left(H^1_S\right) \leq s\left(H^2_S\right)$.

where $s\left(H_S^1\right) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{L_i}\sum_{j=1}^{L_i}\frac{\delta_j^1(x_i)}{g}$ and $s\left(H_S^2\right) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{L_i}\sum_{j=1}^{L_i}\frac{\delta_j^2(x_i)}{g}$ are the scores of H_S^1 and H_S^2, respectively.

B. AGGLOMERATIVE HIERARCHICAL CLUSTERING METHOD

Agglomerative hierarchical clustering (AHC) method, the main idea is to regard each sample point as a cluster firstly and then repeatedly merge the two clusters with the closest distance into one cluster until the iteration termination condition is met [21]. The steps of the AHC method are as follows.

Step 1: Assume that there have n objects, and take each object $H_j\,(j = 1, 2, \ldots, n)$ as a unique cluster $\{H_1\}, \{H_2\}, \ldots, \{H_n\}$.

Step 2: Calculate the distance $d\left(H_i, H_j\right)$ between cluster H_i and H_j, and find the smallest distance $d\left(H_i, H_j\right) = \min_{1\le p,q\le n, p\ne q} d\left(H_p, H_q\right)$, then merge the cluster H_i and H_j into a new cluster H_{ij}.

Step 3: Calculate the new center of the cluster H_{ij}, and update the distance matrix until all the clusters are assembled into one cluster.

IV. HESITANCE DEGREE-BASED DISTANCE AND SIMILARITY MEASURES ON HFLTSs

In the clustering process, as an important measure of a similar degree between two judgment debtors, distance measures greatly impact the clustering results. However, the existing distance measures [3] in some cases, do not allow for the classification of judgment debtors based on their evaluation information given by expert judges.

Example 1: Assume that $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$ is an LTS used to represent the evaluation information of judgment debtors, $H_S^1 = \{x, \langle s_3\rangle\}$ and $H_S^2 = \{x, \langle s_2, s_3, s_6\rangle\}$ are attribute values of judgment debtors given by judgment debtors. Now, there has a new judgment debtor with attribute value $H_S = \{x, \langle s_2, s_3, s_4\rangle\}$ to be clustered. The result calculated by the existing distance measures is $d_h\left(H_S^1, H_S\right) = 0.0833$, $d_h\left(H_S^2, H_S\right) = 0.0833$, $d_e\left(H_S^1, H_S\right) = 0.1021$ and $d_e\left(H_S^2, H_S\right) = 0.1443$. In the existing Hamming distance, the new judgment debtor cannot be recognized by any cluster. But in the existing Euclidean distance, the new judgment debtor is grouped into the judgment debtor H_S^1. Through analysis, the existing distance measures cannot classify the judgment debtor.

Therefore, to classify the judgment debtor effectively, we need to develop new distance measures. Considering the hesitance in the process of expert judges evaluating the judgment debtors, the concept of hesitance degree should be introduced.

Definition 5: Let H_S be an HFLTS on $X = (x_1, x_2, \cdots, x_n)$, $H_S = \{< x, h_S(x) > |x \in X\}$, $h_S(x) = \left\{s_{\delta_j}(x)\,\middle|\, j = 1, 2, \cdots, L_i, s_{\delta_j}(x) \in S\right\}$. The hesitance degree of $h_S(x_i)$ and H_S are defined as follows.

$$\hbar\left(h_S(x_i)\right) = \begin{cases}\sqrt{\frac{1}{\binom{L_i}{2}}\sum_{k>j=1}^{L_i}\left(\frac{\delta_k(x_i)-\delta_j(x_i)}{g}\right)^2}, & L_i > 1\\ 0, & L_i = 1\end{cases},$$

$$\hbar\left(H_S\right) = \frac{1}{n}\sum_{i=1}^{n}\hbar\left(h_S(x_i)\right).$$

where, $\binom{L_i}{2} = \frac{1}{2}L_i\left(L_i - 1\right)$, L_i is the number of linguistic terms in $h_S(x_i)$.

Example 2: Let $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$ be a set of linguistic terms, $H_S^1 = \{\langle x_i, \{s_2, s_5\}\rangle | x_i \in X\}$ and $H_S^2 = \{\langle x_i, \{s_1, s_4, s_6\}\rangle | x_i \in X\}$ be two HFLTSs. Thus, the hesitance degree $\hbar\left(H_S^1\right) = \sqrt{\frac{1}{1}\left(\frac{5-2}{8}\right)^2} = 0.3750$ and $\hbar\left(H_S^2\right) = \sqrt{\frac{1}{3}\left(\left(\frac{6-1}{8}\right)^2 + \left(\frac{6-4}{8}\right)^2 + \left(\frac{4-1}{8}\right)^2\right)} = 0.4449$. The hesitance degree of H_S^2 is greater than that of H_S^1.

Next, we define the new ordering methods considering hesitance degree of HFLTSs:

Definition 6: Let H_S^1 and H_S^2 be two HFLTSs on $X = (x_1, x_2, \cdots, x_n)$, then

The strict component-wise ordering of HFLTSs:

$H_S^1 \le H_S^2$, iff $h_S^1(x_i) \le h_S^2(x_i)$ and $\hbar\left(h_S^1(x_i)\right) \ge \hbar\left(h_S^2(x_i)\right)$, $1 \le i \le n$, $1 \le j \le L_i$.

Complete ordering "$\le$":

$H_S^1 \le H_S^2$, iff $s\left(H_S^1\right) \le s\left(H_S^2\right)$ and $\hbar\left(H_S^1\right) \ge \hbar\left(H_S^2\right)$.

where $\hbar\left(H_S^1\right)$, $\hbar\left(H_S^2\right)$ are the hesitance degree of H_S^1 and H_S^2, respectively.

Based on the above hesitance degree of HFLTSs, some novel distances considering hesitance degree are developed as follows.

Definition 7: Let H_S^1 and H_S^2 be two HFLTSs on $X = (x_1, x_2, \cdots, x_n)$, the Hamming distance, Euclidean distance, and generalized distance including hesitance degree between H_S^1 and H_S^2 are defined as follows.

$$d_{hh}\left(H_S^1, H_S^2\right) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{L_i+1}\left(\left|\hbar\left(h_S^1(x_i)\right) - \hbar\left(h_S^2(x_i)\right)\right| + \sum_{j=1}^{L_i}\left|\frac{\delta_j^1(x_i) - \delta_j^2(x_i)}{g}\right|\right)\right) \tag{8}$$

$$d_{he}\left(H_S^1, H_S^2\right) = \left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{L_i+1}\left(\left|\hbar\left(h_S^1(x_i)\right) - \hbar\left(h_S^2(x_i)\right)\right|^2 + \sum_{j=1}^{L_i}\left|\frac{\delta_j^1(x_i) - \delta_j^2(x_i)}{g}\right|^2\right)\right)\right)^{1/2} \tag{9}$$

$$d_{hg}\left(H_S^1, H_S^2\right) = \left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{L_i+1}\left(\left|\hbar\left(h_S^1(x_i)\right) - \hbar\left(h_S^2(x_i)\right)\right|^{\lambda} + \sum_{j=1}^{L_i}\left|\frac{\delta_j^1(x_i) - \delta_j^2(x_i)}{g}\right|^{\lambda}\right)\right)\right)^{1/\lambda} \tag{10}$$

where $\lambda > 0$, $\delta_j^1(x_i)$ and $\delta_j^2(x_i)$ are the subscripts of j-th linguistic term in i-th attribute on H_S^1 and H_S^2, and $L_i = \max\left\{l\left(h_S^1(x_i)\right), l\left(h_S^2(x_i)\right)\right\}$.

In some cases, the distance measures of HFLTSs should consider the weight of the element $x \in X$. Here, the weighted Hamming distance, Euclidean distance, and generalized distance for HFLTSs are defined.

$$d_{whh}\left(H_S^1, H_S^2\right) = \sum_{i=1}^{n}\omega_i\left(\frac{1}{L_i+1}\left(\left|\hbar_S^1(x_i) - \hbar_S^2(x_i)\right| + \sum_{j=1}^{L_i}\left|\frac{\delta_j^1(x_i) - \delta_j^2(x_i)}{g}\right|\right)\right) \tag{11}$$

$$d_{whe}\left(H_S^1, H_S^2\right) = \left(\sum_{i=1}^{n}\omega_i\left(\frac{1}{L_i+1}\left(\left|\hbar_S^1(x_i) - \hbar_S^2(x_i)\right|^2 + \sum_{j=1}^{L_i}\left|\frac{\delta_j^1(x_i) - \delta_j^2(x_i)}{g}\right|^2\right)\right)\right)^{1/2} \tag{12}$$

$$d_{whg}\left(H_S^1, H_S^2\right) = \left(\sum_{i=1}^{n}\omega_i\left(\frac{1}{L_i+1}\left(\left|\hbar_S^1(x_i) - \hbar_S^2(x_i)\right|^{\lambda} + \sum_{j=1}^{L_i}\left|\frac{\delta_j^1(x_i) - \delta_j^2(x_i)}{g}\right|^{\lambda}\right)\right)\right)^{1/\lambda} \tag{13}$$

where ω_i is the weight of the element x_i, and satisfy $0 \le \omega_i \le 1$, $\sum_{i=1}^{n}\omega_i = 1$, $\lambda > 0$.

Theorem 1: Let H_S^1, H_S^2 and H_S^3 be three HFLTSs on $X = (x_1, x_2, \cdots, x_n)$, the distance measures d_{hh}, d_{he}, d_{hg} and d_{whh}, d_{whe}, d_{whg} satisfy the following properties.

(1) $0 \le d\left(H_S^1, H_S^2\right) \le 1$,

(2) $d\left(H_S^1, H_S^2\right) = 0$, iff $H_S^1 = H_S^2$,

(3) $d\left(H_S^1, H_S^2\right) = d\left(H_S^2, H_S^1\right)$,

(4) If $H_S^1 \le H_S^2 \le H_S^3$, then $d\left(h_S^1(x), h_S^2(x)\right) \le d\left(h_S^1(x), h_S^3(x)\right)$ and $d\left(h_S^2(x), h_S^3(x)\right) \le d\left(h_S^1(x), h_S^3(x)\right)$.

Proof:

(1) It is obvious.

(2) If $d\left(H_S^1, H_S^2\right) = 0$, namely, $\sum_{j=1}^{L_i}\left|\frac{\delta_j^1(x_i) - \delta_j^2(x_i)}{g}\right|^{\lambda} = 0$, $\left|\hbar\left(h_S^1(x_i)\right) - \hbar\left(h_S^2(x_i)\right)\right|^{\lambda} = 0$, then $\delta_j^1(x_i) = \delta_j^2(x_i)$, thus $H_S^1 = H_S^2$. If $H_S^1 = H_S^2$, then $\delta_j^1(x_i) = \delta_j^2(x_i)$, thus $d\left(h_S^1, h_S^2\right) = 0$. From the above analysis, $d\left(H_S^1, H_S^2\right) = 0$, iff $H_S^1 = H_S^2$.

(3) It is easily noted that

$$\begin{aligned} d\left(H_S^1, H_S^2\right) &= \left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{L_i+1}\left(\alpha\left|\hbar\left(h_S^1(x_i)\right) - \hbar\left(h_S^2(x_i)\right)\right|^{\lambda} + \beta\sum_{j=1}^{L_i}\left|\frac{\delta_j^1(x_i) - \delta_j^2(x_i)}{g}\right|^{\lambda}\right)\right)\right)^{1/\lambda} \\ &= \left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{L_i+1}\left(\alpha\left|\hbar\left(h_S^1(x_i)\right) - \hbar\left(h_S^2(x_i)\right)\right|^{\lambda} + \beta\sum_{j=1}^{L_i}\left|\frac{\delta_j^2(x_i) - \delta_j^1(x_i)}{g}\right|^{\lambda}\right)\right)\right)^{1/\lambda} \\ &= d\left(H_S^2, H_S^1\right). \end{aligned}$$

Thus, $d\left(H_S^1, H_S^2\right) = d\left(H_S^2, H_S^1\right)$.

(4) If $H_S^1 \le H_S^2 \le H_S^3$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, L_i$, known from Definition 6, $h_S^1(x_i) \le h_S^2(x_i) \le h_S^3(x_i)$ and $\hbar_S^1(x_i) \ge \hbar_S^2(x_i) \ge \hbar_S^3(x_i)$, thus

$$\begin{aligned} \left|\hbar\left(h_S^1(x_i)\right) - \hbar\left(h_S^2(x_i)\right)\right| &\le \left|\hbar\left(h_S^1(x_i)\right) - \hbar\left(h_S^3(x_i)\right)\right|, \\ \left|\hbar\left(h_S^2(x_i)\right) - \hbar\left(h_S^3(x_i)\right)\right| &\le \left|\hbar\left(h_S^1(x_i)\right) - \hbar\left(h_S^3(x_i)\right)\right|, \\ \left|h_S^1(x) - h_S^2(x)\right| &\le \left|h_S^1(x) - h_S^3(x)\right|, \\ \left|h_S^2(x) - h_S^3(x)\right| &\le \left|h_S^1(x) - h_S^3(x)\right|. \end{aligned}$$

Because $\lambda > 0$, then

$$\left(\sum_{i=1}^{n}w_i\left(\frac{1}{L_i+1}\left(\left|\hbar_S^1(x_i) - \hbar_S^2(x_i)\right|^{\lambda} + \sum_{j=1}^{L_i}\left|\frac{\delta_j^1(x_i) - \delta_j^2(x_i)}{g}\right|^{\lambda}\right)\right)\right)^{1/\lambda} \le \left(\sum_{i=1}^{n}w_i\left(\frac{1}{L_i+1}\left(\left|\hbar_S^1(x_i) - \hbar_S^3(x_i)\right|^{\lambda} + \sum_{j=1}^{L_i}\left|\frac{\delta_j^1(x_i) - \delta_j^3(x_i)}{g}\right|^{\lambda}\right)\right)\right)^{1/\lambda},$$

$$\left(\sum_{i=1}^{n}w_i\left(\frac{1}{L_i+1}\left(\alpha\left|\hbar_S^1(x_i) - \hbar_S^2(x_i)\right|^{\lambda} + \beta\sum_{j=1}^{L_i}\left|\frac{\delta_j^2(x_i) - \delta_j^3(x_i)}{g}\right|^{\lambda}\right)\right)\right)^{1/\lambda} \le \left(\sum_{i=1}^{n}w_i\left(\frac{1}{L_i+1}\left(\alpha\left|\hbar_S^1(x_i) - \hbar_S^3(x_i)\right|^{\lambda} + \beta\sum_{j=1}^{L_i}\left|\frac{\delta_j^1(x_i) - \delta_j^3(x_i)}{g}\right|^{\lambda}\right)\right)\right)^{1/\lambda}.$$

Namely, $d\left(h_S^1(x), h_S^2(x)\right) \leq d\left(h_S^1(x), h_S^3(x)\right)$ and $d\left(h_S^2(x), h_S^3(x)\right) \leq d\left(h_S^1(x), h_S^3(x)\right)$.

Next, to verify the effectiveness of the new distance measures of HFLTSs, we give the example.

Example 3: (Continued to **Example 1**). The result calculated by the new distance measures is $d_{hh}\left(H_S^1, H_S\right) = 0.1067$, $d_{hh}\left(H_S^2, H_S\right) = 0.1103$, $d_{he}\left(H_S^1, H_S\right) = 0.1250$ and $d_{he}\left(H_S^2, H_S\right) = 0.1574$. From the result calculated by the new Hamming distance and Euclidean distance, the new judgment debtor H_S divides into the cluster H_S^1. From the analysis, the new distance measures are more reasonable than the existing distance measures.

V. HFL-AHC METHOD FOR CLUSTERING THE JUDGMENT DEBTORS

Based on the above distance measures of HFLTSs, we introduce a new HFL-AHC method for clustering the judgment debtors to analyze their characteristics.

Assume that $X = \{x_1, x_2, \ldots, x_m\}$ is a set of judgment debtors' attributes, $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ is the weight vector of judgment debtors' attributes with $\omega_i \geq 0, i = 1, 2, \ldots, m$, $\sum_{i=1}^{m} \omega_i = 1$, and $A_j\,(j = 1, 2, \ldots, n)$ is a collection of n judgment debtors represented by HFLTSs, which means that have n judgment debtors to be clustered, the mathematical form can be expressed as follows.

$$A_j = \left\{ < x_i, h_S^{A_j}(x_i) > | x_i \in X \right\}, \quad j = 1, 2, \ldots, n.$$

Inspired by the classical AHC method [15], a novel hesitant fuzzy linguistic agglomerative hierarchical clustering (HFL-AHC) method is developed to cluster the judgment debtors. The steps of the HFL-AHC method are as follows.

Step 1: Take each judgment debtor $A_j\,(j = 1, 2, \ldots, n)$ as a unique cluster $\{A_1\}, \{A_2\}, \ldots, \{A_n\}$.

Step 2: Calculate the distance $d\left(A_i, A_j\right)$ between the judgment debtors A_i and A_j by Eqs. (8) - (13), then construct the distance matrix between each two judgment debtors $D = \left(d_{ij}\right)_{n\times n}$. Find the two judgment debtors with the smallest distance, $d\left(A_i, A_j\right) = \min_{1\leq p,q\leq n, p\neq q} d\left(A_p, A_q\right)$ in the distance matrix $D = \left(d_{ij}\right)_{n\times n}$, and merge the judgment debtors A_i and A_j into a new cluster A_{ij}.

Step 3: Calculate the new center of the cluster A_{ij} by Eq. (7). Update the distance matrix by calculating the distance between the new cluster A_{ij} and the other clusters.

Step 4: Repeat Step 2 and Step 3 until all the clusters are grouped into one cluster.

The process of the above HFL-AHC method is shown in Fig. 2.

VI. A NUMERICAL EXAMPLE FOR CLUSTERING THE JUDGMENT DEBTORS

A. NUMERICAL EXAMPLE

Next, we give an example to illustrate the effectiveness of clustering the judgment debtors by the HFL-AHC method.

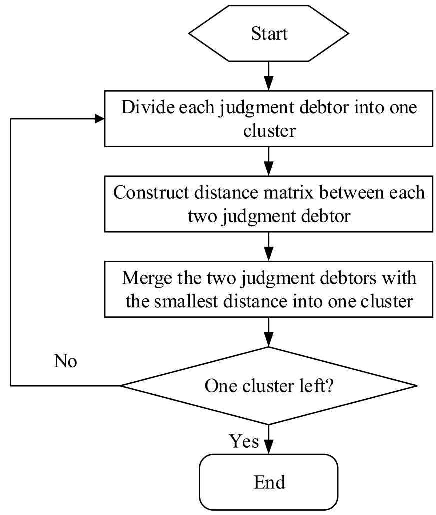

FIGURE 2. Flow chart of HFL-AHC method.

TABLE 1. Attributes of judgment debtors.

Attributes	Description
Trading Behavior	(1) Whether the judgment debtor has transferred the real estate, land, vehicles, vessels, securities, equity, intellectual property, etc., during the litigation period. (2) In securities trading, the degree of risk preference. The more the act of transferring property, the more risk-averse the securities transaction is, and the worse the evaluation.
Work	Whether the judgment debtors have a steady job and a steady income. The more stable the job, the better the rating
Credibility	(1) Whether the judgment debtor often lied during the judge handling the law enforcement cases. (2) Whether the judgment debtor is regularly late on bank credit cards, online lending platforms, and payment platforms. (3) Whether the judgment debtor has often been dishonest in historical cases, etc. The better the honesty is, the better the rating is.
Consumer Behavior	Whether the judgment debtors often have consumption behavior inconsistent with their consumption ability, travels by plane, high-speed rail, and other high-grade transportation, and consumes in star hotels, restaurants, dance halls, and other high-grade consumption places. The more the judgment debtors consume, the worse the rating

To find the characteristics of judgment debtor concealing property, we regard trading behavior (C_1), work (C_2), credibility (C_3), and consumer behavior (C_4) as the attributes, the description of the four attributes are shown in Table 1. The weight of each attribute is provided by expert judges as $\omega = (0.20, 0.25, 0.30, 0.25)^T$. Assume that the LTS are presented as follows: $S = \{s_0 =$ Very Poor (VP), $s_1 =$ Poor (P), $s_2 =$ Moderately Poor (MP), $s_3 =$ Medium (M), $s_4 =$ Moderately Good (MG), $s_5 =$ Good (G), $s_6 =$ Very

Good (VG)}. The decision matrix of judgment debtors represented by HFLTSs is shown in Table 2. For example, the value of judgment debtor A_1 under attribute C_1 is greater than good, indicating that some expert judges think the trading behavior C_1 of judgment debtor A_1 is good, while others think that it is very good.

TABLE 2. Decision matrix of judgment debtors represented by HFLTSs.

Judgment debtors	C_1	C_2	C_3	C_4
A_1	$\{s_5,s_6\}$	$\{s_2,s_3,s_4\}$	$\{s_5,s_6\}$	$\{s_4,s_5,s_6\}$
A_2	$\{s_2,s_3,s_4\}$	$\{s_4,s_5\}$	$\{s_4,s_5\}$	$\{s_4,s_5,s_6\}$
A_3	$\{s_1,s_2,s_3\}$	$\{s_2,s_3\}$	$\{s_2,s_3,s_4\}$	$\{s_1,s_2,s_3\}$
A_4	$\{s_2,s_3,s_4\}$	$\{s_2,s_3\}$	$\{s_4,s_5\}$	$\{s_2,s_3,s_4\}$
A_5	$\{s_3,s_4\}$	$\{s_2,s_3,s_4\}$	$\{s_3,s_4\}$	$\{s_3,s_4\}$
A_6	$\{s_1,s_2,s_3\}$	$\{s_1,s_2\}$	$\{s_1,s_2,s_3\}$	$\{s_1,s_2\}$
A_7	$\{s_5,s_6\}$	$\{s_4,s_5,s_6\}$	$\{s_5,s_6\}$	$\{s_4,s_5,s_6\}$

The steps of the HFL-AHC method are as follows.

Step 1: Take each judgment debtor $A_j (j = 1, 2, 3, 4, 5, 6, 7)$ as a unique cluster $\{A_1\}, \{A_2\}, \{A_3\}, \{A_4\}, \{A_5\}, \{A_6\}$ and $\{A_7\}$.

Step 2: Calculate the weighted Hamming distance of HFLTSs between each two judgment debtors by Eq.(11). When two HFLEs have different lengths, the shorter one should add the minimum linguistic term, and the distance matrix is shown in Table 3.

TABLE 3. Distance matrix of each judgment debtors in the first iteration.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7
A_1	-	0.1404	0.2903	0.1821	0.1787	0.3814	**0.0625**
A_2		-	0.2332	0.1250	0.1441	0.3244	0.1196
A_3			-	0.1082	0.1148	0.0912	0.3528
A_4				-	0.0816	0.1994	0.2446
A_5					-	0.2028	0.2412
A_6						-	0.4439
A_7							-

Judgment debtor A_1 and A_7 have the smallest distance, and combine them into one cluster. Thus, the seven judgment debtors divide into six clusters: $\{A_1, A_7\}, \{A_2\}, \{A_3\}, \{A_4\}, \{A_5\}$ and $\{A_6\}$.

Step 3: Update the new cluster centers by Eq.(7). The new cluster centers are as follows.

$$\begin{aligned} c\{A_1, A_7\} &= f\{A_1, A_7\} \\ &= \{< x_1, \{s_5, s_5, s_6\} >, < x_2, \{s_3, s_4, s_5\} >, \\ &< x_3, \{s_5, s_5, s_6\} >, < x_4, \{s_4, s_5, s_6\} >\}, \\ c\{A_2\} &= A_2, c\{A_3\} = A_3, c\{A_4\} = A_4, \\ c\{A_5\} &= A_5, c\{A_6\} = A_6. \end{aligned}$$

Calculate the distance between each cluster and the other five clusters, and the result is shown in Table 4.

TABLE 4. Distance matrix of each judgment debtors in the second iteration.

	$C\{A_1,A_7\}$	A_2	A_3	A_4	A_5	A_6
$C\{A_1,A_7\}$	-	0.1092	0.3215	0.2133	0.2099	0.4127
A_2		-	0.2332	0.1250	0.1441	0.3244
A_3			-	0.1082	0.1148	0.0912
A_4				-	**0.0816**	0.1994
A_5					-	0.2028
A_6						-

Then, group judgment debtor A_4 and A_5 with the smallest distance into one cluster. In such a case, the seven judgment debtors group into five clusters: $\{A_1, A_7\}, \{A_2\}, \{A_3\}, \{A_4, A_5\}$ and $\{A_6\}$.

Step 4: update the new cluster centers by Eq.(7). The new cluster centers are as follows.

$$\begin{aligned} c\{A_1, A_7\} &= f\{A_1, A_7\} \\ &= \{< x_1, \{s_5, s_5, s_6\} >, < x_2, \{s_3, s_4, s_5\} >, \\ &< x_3, \{s_5, s_5, s_6\} >, < x_4, \{s_4, s_5, s_6\} >\}, \\ c\{A_4, A_5\} &= f\{A_4, A_5\} \\ &= \{< x_1, \{s_{2.5}, s_3, s_4\} >, < x_2, \{s_2, s_{2.5}, s_{3.5}\} >, \\ &< x_3, \{s_{3.5}, s_{3.5}, s_{4.5}\} >, < x_4, \{s_{2.5}, s_3, s_4\} >\}, \\ c\{A_2\} &= A_2, c\{A_3\} = A_3, c\{A_6\} = A_6. \end{aligned}$$

The distance between every two clusters is calculated and shown in Table 5.

TABLE 5. Distance matrix of each judgment debtors in the third iteration.

	$C\{A_1,A_7\}$	A_2	$C\{A_3,A_6\}$	A_4	A_5
$C\{A_1,A_7\}$	-	0.1092	0.3690	0.2133	0.2099
A_2		-	0.2807	0.1250	0.1441
A_3			-	0.1557	0.1607
$C\{A_4,A_5\}$				-	**0.0816**
A_6					-

Merge judgment debtor A_3 and A_6 into one cluster. Then, the seven judgment debtors aggregate into four clusters: $\{A_1, A_7\}, \{A_2\}, \{A_3, A_6\}$ and $\{A_4, A_5\}$.

Step 5: update the new cluster centers, and the new cluster centers are as follows.

$$\begin{aligned} c\{A_1, A_7\} &= f\{A_1, A_7\} \\ &= \{< x_1, \{s_5, s_5, s_6\} >, < x_2, \{s_3, s_4, s_5\} >, \\ &< x_3, \{s_5, s_5, s_6\} >, < x_4, \{s_4, s_5, s_6\} >\}, \\ c\{A_3, A_6\} &= f\{A_3, A_6\} \\ &= \{< x_1, \{s_1, s_2, s_3\} >, < x_2, \{s_{1.5}, s_{1.5}, s_{2.5}\} >, \\ &< x_3, \{s_{1.5}, s_{2.5}, s_{3.5}\} >, < x_4, \{s_1, s_{1.5}, s_{2.5}\} >\}, \\ c\{A_4, A_5\} &= f\{A_4, A_5\} \\ &= \{< x_1, \{s_{2.5}, s_3, s_4\} >, < x_2, \{s_2, s_{2.5}, s_{3.5}\} >, \\ &< x_3, \{s_{3.5}, s_{3.5}, s_{4.5}\} >, < x_4, \{s_{2.5}, s_3, s_4\} >\}, \\ c\{A_2\} &= A_2. \end{aligned}$$

The update distance matrix is shown in Table 6.

TABLE 6. Distance matrix of each judgment debtors in the fourth iteration.

	$C\{A_1,A_7\}$	A_2	$C\{A_3,A_6\}$	$C\{A_4,A_5\}$
$C\{A_1,A_7\}$	-	**0.1092**	0.3690	0.2139
A_2		-	0.2807	0.1368
$C\{A_3,A_6\}$			-	0.1597
$C\{A_4,A_5\}$				-

Judgment debtor A_2 and the cluster $c\{A_1, A_7\}$ have the smallest distance, merge them into one cluster. Thus, the seven judgment debtors divide into three clusters: $\{A_1, A_2, A_7\}, \{A_3, A_6\}$ and $\{A_4, A_5\}$.

Step 6: update the new cluster centers, and the new cluster centers are as follows.

$$\begin{aligned} c\{A_1, A_2, A_7\} &= f\{A_1, A_2, A_7\} \\ &= \{<x_1, \{s_{3.5}, s_4, s_5\}>, <x_2, \{s_{3.5}, s_4, s_5\}>, \\ &\quad <x_3, \{s_{4.5}, s_{4.5}, s_{5.5}\}>, <x_4, \{s_4, s_5, s_6\}>\}, \\ c\{A_3, A_6\} &= f\{A_3, A_6\} \\ &= \{<x_1, \{s_1, s_2, s_3\}>, <x_2, \{s_{1.5}, s_{1.5}, s_{2.5}\}>, \\ &\quad <x_3, \{s_{1.5}, s_{2.5}, s_{3.5}\}>, <x_4, \{s_1, s_{1.5}, s_{2.5}\}>\}, \\ c\{A_4, A_5\} &= f\{A_4, A_5\} \\ &= \{<x_1, \{s_{2.5}, s_3, s_4\}>, <x_2, \{s_2, s_{2.5}, s_{3.5}\}>, \\ &\quad <x_3, \{s_{3.5}, s_{3.5}, s_{4.5}\}>, <x_4, \{s_{2.5}, s_3, s_4\}>\}. \end{aligned}$$

The update distance matrix is shown in Table 7.

TABLE 7. Distance matrix of each judgment debtors in the fifth iteration.

	$C\{A_1,A_2,A_7\}$	$C\{A_3,A_6\}$	$C\{A_4,A_5\}$
$C\{A_1,A_2,A_7\}$	-	0.3271	0.1674
$C\{A_3,A_6\}$		-	**0.1597**
$C\{A_4,A_5\}$			-

The cluster $c\{A_3, A_6\}$ and $c\{A_4, A_5\}$ have the smallest distance, group them into one cluster. Thus, the seven judgment debtors divide into two clusters: $\{A_1, A_2, A_7\}$ and $\{A_3, A_4, A_5, A_6\}$.

Finally, aggregate the two clusters $c\{A_3, A_6\}$ and $c\{A_4, A_5\}$ into one cluster.

The above process of the HFL-AHC method is shown in Fig. 3.

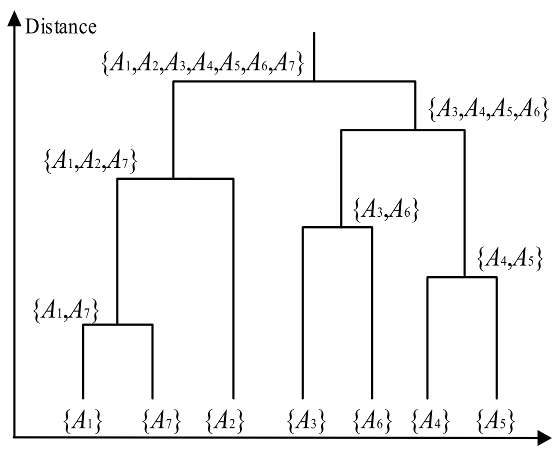

FIGURE 3. Clustering results from the HFL-AHC method.

When all the judgment debtors group into two clusters, the cluster $\{A_3, A_4, A_5, A_6\}$ has significantly lower attribute value than the cluster $\{A_1, A_2, A_7\}$ in terms of trading behavior, work, credibility, and consumption behavior. The former judgment debtors have more or less one or several of the following characteristics: (1) during the litigation period, the judgment debtors maliciously transfer their property from bank accounts, land, vehicles, securities, etc.; the judgment debtors prefer risks in the process of securities trading; (2) the judgment debtor has no work or less steadily work; (3) the judgment debtors often lie in the process of assisting senior judges, and often overdue credit cards, network lending platforms, etc.; (4) the judgment debtors often go out to places that do not match their consumption capacity, etc. Therefore, such judgment debtors are more likely to conceal property.

TABLE 8. Clustering result from the HFL-BMC method.

Confidence level	Clustering result
$0.9375 \le \alpha \le 1$	$\{A_1\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\},\{A_7\}$
$0.9139 < \alpha \le 0.9375$	$\{A_1,A_7\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\}$
$0.8896 < \alpha \le 0.9139$	$\{A_1,A_7\},\{A_2\},\{A_3,A_6\},\{A_4\},\{A_5\}$
$0.8819 < \alpha \le 0.8896$	$\{A_1,A_7\},\{A_2\},\{A_3,A_6\},\{A_4,A_5\}$
$0.8708 < \alpha \le 0.8819$	$\{A_1,A_7\},\{A_2,A_4,A_5\},\{A_3,A_6\}$
$0.8688 < \alpha \le 0.8708$	$\{A_1,A_2, A_4,A_5,A_7\},\{A_3,A_6\}$
$0 < \alpha \le 0.8688$	$\{A_1,A_2,A_3,A_4,A_5,A_6,A_7\}$

TABLE 9. Clustering result from the HFL-CCC method.

Confidence level	Clustering result
$0.9957 \le \alpha \le 1$	$\{A_1\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\},\{A_7\}$
$0.9881 < \alpha \le 0.9957$	$\{A_1\},\{A_2\},\{A_3\},\{A_4\},\{A_5,A_7\},\{A_6\}$
$0.9802 < \alpha \le 0.9881$	$\{A_1,A_5,A_7\},\{A_2\},\{A_3\},\{A_4\},\{A_6\}$
$0.9773 < \alpha \le 0.9802$	$\{A_1,A_5,A_7\},\{A_2\},\{A_3,A_6\},\{A_4\}$
$0.9772 < \alpha \le 0.9773$	$\{A_1,A_2,A_5,A_7\},\{A_3,A_6\},\{A_4\}$
$0.9771 < \alpha \le 0.9772$	$\{A_1,A_2,A_5,A_7\},\{A_3,A_4,A_6\}$
$0 < \alpha \le 0.9771$	$\{A_1,A_2,A_3,A_4,A_5,A_6,A_7\}$

B. COMPARATIVE ANALYSIS

In this section, we will compare the HFL-AHC method with the two other clustering methods.

(1) Hesitant fuzzy linguistic Boole matric clustering (HFL-BMC) method

The main idea of the HFL-BMC method is to construct the hesitant fuzzy linguistic similarity matrix of judgment debtors, transform it into the equivalent similarity matrix and divide all the judgment debtors into different clusters by setting different confidence levels. The similarity measure of HFLTSs is defined as follows [41].

$$\begin{aligned} s_h\left(h_S^1, h_S^2\right) = 1 - \sum_{i=1}^{n} w_i \left(\frac{1}{L_i+1}\left(\left|u\left(h_S^1(x_i)\right)\right.\right.\right. \\ \left.\left.\left. - u\left(h_S^2(x_i)\right)\right| + \left|\frac{\delta_j^1(x_i) - \delta_j^2(x_i)}{g}\right|\right)\right) \end{aligned} \tag{14}$$

The similarity matrix of judgment debtors is derived from Eq. (14) as follows.

$$S = \begin{pmatrix} 1.0000 & 0.8292 & 0.6729 & 0.8083 & 0.8368 & 0.5833 & 0.9375 \\ 0.8292 & 1.0000 & 0.7507 & 0.8819 & 0.8271 & 0.6646 & 0.8708 \\ 0.6729 & 0.7507 & 1.0000 & 0.8688 & 0.8292 & 0.9139 & 0.6104 \\ 0.8083 & 0.8819 & 0.8688 & 1.0000 & 0.8896 & 0.7826 & 0.7458 \\ 0.8368 & 0.8271 & 0.8292 & 0.8896 & 1.0000 & 0.7674 & 0.7743 \\ 0.5833 & 0.6646 & 0.9139 & 0.7826 & 0.7674 & 1.0000 & 0.5208 \\ 0.9375 & 0.8708 & 0.6104 & 0.7458 & 0.7743 & 0.5208 & 1.0000 \end{pmatrix}$$

TABLE 10. The comparison of three different methods.

Clusters	The developed HFL-AHC method	The HFL-BMC method [41]	The HFL-CCC method [8]
7	$\{A_1\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\},\{A_7\}$	$\{A_1\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\},\{A_7\}$	$\{A_1\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\},\{A_7\}$
6	$\{A_1,A_7\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\}$	$\{A_1,A_7\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\}$	$\{A_1\},\{A_2\},\{A_3\},\{A_4\},\{A_5,A_7\},\{A_6\}$
5	$\{A_1,A_7\},\{A_2\},\{A_4,A_5\},\{A_3\},\{A_6\}$	$\{A_1,A_7\},\{A_2\},\{A_3,A_6\},\{A_4\},\{A_5\}$	$\{A_1,A_5,A_7\},\{A_2\},\{A_3\},\{A_4\},\{A_6\}$
4	$\{A_1,A_7\},\{A_2\},\{A_3,A_6\},\{A_4,A_5\}$	$\{A_1,A_7\},\{A_2\},\{A_3,A_6\},\{A_4,A_5\}$	$\{A_1,A_5,A_7\},\{A_2\},\{A_3,A_6\},\{A_4\}$
3	$\{A_1,A_2,A_7\},\{A_4,A_5\},\{A_3,A_6\}$	$\{A_1,A_7\},\{A_2,A_4,A_5\},\{A_3,A_6\}$	$\{A_1,A_2,A_5,A_7\},\{A_3,A_6\},\{A_4\}$
2	$\{A_1,A_2,A_7\},\{A_3,A_4,A_5,A_6\}$	$\{A_1,A_2,A_4,A_5,A_7\},\{A_3,A_6\}$	$\{A_1,A_2,A_5,A_7\},\{A_3,A_4,A_6\}$
1	$\{A_1,A_2,A_3,A_4,A_5,A_6,A_7\}$	$\{A_1,A_2,A_3,A_4,A_5,A_6,A_7\}$	$\{A_1,A_2,A_3,A_4,A_5,A_6,A_7\}$

Then, construct the equivalent similarity matrix by setting different confidence levels. The clustering result is shown in Table 8.

(2) Hesitant fuzzy linguistic correlation coefficient based clustering (HFL-CCC) method

The main idea of the HFL-CCC method is to construct the hesitant fuzzy linguistic equivalent correlation coefficient matrix of judgment debtors and divide all the judgment debtors into different clusters by different confidence levels. The correlation coefficient of HFLTSs is defined as follows [8].

$$\rho\left(H_S^1,H_S^2\right)=\frac{C\left(H_S^1,H_S^2\right)}{\left(E\left(H_S^1\right)\cdot E\left(H_S^2\right)\right)^{1/2}}=\frac{\sum_{i=1}^{N}\left(\frac{1}{L_i}\sum_{l=1}^{L_i}\left(\frac{\delta_l^1(x_i)}{g}\cdot\frac{\delta_l^2(x_i)}{g}\right)\right)}{\left(\sum_{i=1}^{N}\left(\frac{1}{L_i}\sum_{l=1}^{L_i}\left(\frac{\delta_l^1(x_i)}{g}\right)^2\right)\cdot\sum_{i=1}^{N}\left(\frac{1}{L_i}\sum_{l=1}^{L_i}\left(\frac{\delta_l^2(x_i)}{g}\right)^2\right)\right)^{1/2}} \tag{15}$$

The correlation coefficient matrix of judgment debtors is obtained by Eq.(15) and shown as:

$$C=\begin{pmatrix}1.0000&0.9494&0.9398&0.9771&0.9881&0.9534&0.9842\\0.9494&1.0000&0.9700&0.9659&0.9765&0.9487&0.9773\\0.9398&0.9700&1.0000&0.9772&0.9602&0.9802&0.9673\\0.9771&0.9659&0.9772&1.0000&0.9732&0.9619&0.9764\\0.9881&0.9765&0.9602&0.9732&1.0000&0.9687&0.9957\\0.9534&0.9487&0.9802&0.9619&0.9687&1.0000&0.9643\\0.9842&0.9773&0.9673&0.9764&0.9957&0.9643&1.0000\end{pmatrix}$$

Then, transform the above matrix into the equivalent correlation coefficient matrix. The equivalent matrix is

$$C^{16}=C^8\circ C^8=\begin{pmatrix}1.0000&0.9773&0.9771&0.9771&0.9881&0.9771&0.9881\\0.9773&1.0000&0.9771&0.9771&0.9773&0.9771&0.9773\\0.9771&0.9771&1.0000&0.9772&0.9771&0.9802&0.9771\\0.9771&0.9771&0.9772&1.0000&0.9771&0.9772&0.9771\\0.9881&0.9773&0.9771&0.9771&1.0000&0.9771&0.9957\\0.9771&0.9771&0.9802&0.9772&0.9771&1.0000&0.9771\\0.9881&0.9773&0.9771&0.9771&0.9957&0.9771&1.0000\end{pmatrix}=C^8$$

Thus, C^8 is an equivalent correlation matrix. The clustering result is shown in Table 9.

(3) Discussion

The clustering results by the HFL-AHC method, the HFL-BMC method, and the HFL-CCC method are shown in Table 10. The clustering result derived from the HFL-AHC method is distinguished from the HFL-BMC method and the HFL-CCC method. The main reasons are as follows.

Firstly, the hesitance degree-based distance measures have some advantages of expressing expert judges' evaluation information's hesitance. For example, judgment debtor A_2 is grouped into the HFL-AHC method cluster while combined $\{A_4, A_5\}$ in the HFL-BMC method. The developed hesitance degree-based distance measures cause the differences in the clustering results. Therefore, the hesitance degree-based distance measures are more reasonable in the judgment debtor characteristic analysis.

Secondly, both the HFL-BMC method and the HFL-CCC method are transitive closure clustering methods. The two transitive closure clustering methods take lots of calculations in the process of constructing the equivalent matrix, while the HFL-AHC method does not need to convert the distance matrix into the equivalent distance matrix. Namely, the HFL-AHC method has fewer calculations than the HFL-BMC method and the HFL-CCC method. Meanwhile, the two transitive closure clustering methods may lose some original evaluation information given by expert judges in the process of constructing the equivalent matrix. Thus, the HFL-AHC method is obviously different from the HFL-CCC method.

VII. CONCLUSION

Considering the advantages of HFLTSs representing the judgment debtors' attributes and keeping all the expert judges' evaluation information on judgment debtors, we develop the HFL-AHC method to cluster judgment debtors and analyze the main characteristic of judgment debtors with concealing property. The conclusions of our work are as follows.

Firstly, considering the advantages of HFLTSs representing the judgment debtors' attributes and keeping all the expert judges' evaluation information on judgment debtors, we develop a new clustering method to cluster judgment debtors and analyze the main characteristic of judgment debtors.

Secondly, considering the existing HFLTS distance cannot distinguish the judgment debtors in some situations, we propose several new distance measures of HFLTSs to effectively determine the judgment debtors. An example is given to illustrate the new hesitance degree-based distance measures are more suitable for clustering the judgment debtors than the existing distance measures.

Thirdly, a new HFL-AHC method for clustering judgment debtors is developed based on new distance measures. The clustering results show that the judgment debtors who hide property have a poor evaluation of trading behavior, work, credibility, and consumption behavior.

Fourthly, compared with the existing clustering methods, the HFL-AHC method takes fewer computations and keep the original evaluation information by expert judges. Thus, it is more reasonable to find the characteristics of judgment debtors than the other two clustering methods.

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