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Dynamic Output Feedback Control of Networked Systems With Medium Access Constraints

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ABSTRACT This paper is concerned with the dynamic output feedback control issue for a class of networked control systems with medium access constraints, network-induced delays and packet dropouts. It is assumed that the medium access constraints at both sides of the controller obey the Markov distribution, the upper bound of the network-induced delays is less than one sampling period, and the packet dropouts of networked channels can be described by i.i.d Bernoulli distribution processes. Then, the networked control systems are modelled as a class of asynchronous dynamic systems. According to Lyapunov stability theory and switching system theory, the dynamic output feedback controller is designed to guarantee the exponential stability of networked systems with the given decay rate. Finally, an illustrative example is provided to demonstrate the effectiveness of the proposed method.

INDEX TERMS Networked control systems, medium access constraints, network-induced delays, packet dropouts, dynamic output feedback.

I. INTRODUCTION

Networked control systems (NCSs) are closed-loop feedback control systems in which the controllers exchange information with the sensors and actuators over shared communication networks instead of the conventional point-to-point communication [1]–[3]. The utilization of communication networks provides many advantages, such as simple structure, easy installation and maintenance, and remote operation [4], [5]. Therefore, NCSs have been successfully applied to telemedicine, industrial automation, intelligent robots, and other control fields. However, the presence of network in control systems has also given rise to multiple communication constraints for modeling, analysis, and control of NCSs, such as, medium access constraints, network-induced delays, packet dropouts, quantization effects, etc. These communication constraints are potential sources of poor performance and instability, see, e.g., [6]–[9] and references therein. Medium access constraints, packet dropouts and time delays are the most common network-induced problems and they often co-exist in networked systems, furthermore, it may result incomplete measurement of system state in some cases. Thus, the purpose of this paper is to investigate the dynamic output feedback control problem for the networked systems by considering medium access constraints, network-induced delays and packet dropouts simultaneously.

Due to the limitations of network bandwidth and irregular changes of network loads, network-induced delays occur unavoidably during information exchange of the controllers with the sensors and the actuators, respectively. Recently, the control problem of NCSs with network delays has attracted considerable research interests. Generally speaking, network-induced delays can be roughly classified as deterministic delays and time-varying delays according to their characteristics. The deterministic delay is usually described as a fixed constant, see e.g., [10]–[12]. To mention some, an enhanced Smith predictor is used to compensate the delays of the networked systems subject to random delays and uncertainties in [10]. The authors of [12] propose a novel delay scheduled impulsive control method, which utilizes both the

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plant state and integral quadratic constraint dynamic state, as well as the real-time network-induced delay information for gain scheduling feedback control. For the time-varying delays, there are usually two methods for analyzing its effect on the system: the first method is that the system with time-varying delays is modelled as a class of uncertain systems, see e.g., [13]–[16]; the second method is that the time-varying delay is considered as a stochastic process, see e.g., [17]–[21]. For example, to name a few, the networked system with time-varying delays is modelled as a class of time-varying uncertain discrete systems in [13]. The optimal guaranteed cost control problem of NCSs with network delays is investigated in [15]. The authors of [16] investigate the reliable adaptive observer-based output tracking control problem for a class of networked control systems subject to actuator faults, time-varying delays and external disturbances. Based on Markov jump system model, the authors of [17]–[19] solve the stability analysis problem for the NCSs with time-varying delays. When the random delays of NCSs are bounded and the upper bound of the delays is longer than a sampling period, the authors of [20] present the dynamic output feedback controllers by discretizing the model of continuous-time system to build a discrete-time jump system model governed by finite-state Markov chains.

The packet dropouts are another important communication constraint to be considered in the NCSs. Due to network congestion, data packets would often be lost in a random manner via a communication network. Up to now, the phenomenon of packet dropout is usually modeled as random processes, such as the Bernoulli distribution [22]–[24] and Markov distribution [25], [26]. For example, to name a few, the mode-dependent controller design problem is studied for the networked systems with group random access protocols and packet dropouts, which are modeled as independent Bernoulli processes in [23]. The authors of [24] propose an active resilient control strategy for singular networked control systems with packet dropouts and external disturbances based on sampled-data schemes. In [26], the authors discuss the sliding mode control problem for a class of networked systems with Markov packet dropouts. At the same time, many scholars deal with the packet dropout by model-based approaches, such as a switching system method and an asynchronous dynamic system method, see e.g., [27]–[29] and references therein. To mention some, the stabilization of NCSs with network-induced delays and packet dropouts is investigated by adopting a switching systems method in [27]. Based on the asynchronous dynamic system theory, the authors of [29] establish the networked systems model and present the sufficient conditions of the exponential stability for the NCSs with network-induced delays and packet dropouts.

Aside from network-induced delays and packet dropouts, another fundamental communication limitation is the so-called medium access constraints, i.e., the limitations on the number of sensors and actuators that can obtain channel access right to exchange information with the controller simultaneously. In such a scenario, in order to improve the utilization of the limited network bandwidth and the control quality of networked systems, it is necessary to design not only stabilizing controllers but also communication scheduling strategies. Recently, the control problem of NCSs with medium access constraints has attracted increasing research interests, see, e.g., [30]–[33] and references therein. For example, to name a few, the stability of the networked systems under different static scheduling strategies is addressed by utilizing an optimal pointer configuration method in [30]. The authors of [31] investigate the ultimate bounded control problem of NCSs under the influence of TOD dynamic scheduling protocol and uniform quantization. For a class of networked nonlinear control systems, the communication scheduling problem is studied, and a predictive control framework for controller/scheduler co-design is proposed in [33]. In the practice of networked control, it is more common to have a variety of communication constraints simultaneously such as network-induced delays, packet dropout and medium access constraints, see e.g., [34]–[38]. Various communication constraints cause the highly time-varying behavior of system communication, bringing the risk to deteriorate the performance of the system and even cause system's instability. For example, the authors of [35] discuss the modeling and control problem for NCSs with access constraints and packet dropouts, and design an optimal controller to guarantee the systems exponential stability while minimizing the quadratic cost. The NCSs with medium access constraints and random packet dropouts are modeled as discrete-time switched systems, and the stabilizing controller is designed in [36]. The authors of [37] propose the integrated design of controller and communication sequences for NCSs with dynamic access scheduling. The scheme of stability analysis and controller synthesis is proposed for the networked systems by taking into account the Markov access constraints and network delays of the control signal in [38]. Although the above literature have made some progress in the networked systems with multiple communication constraints, there are still some interesting problems that deserve further research. So far, much attention has been focused on how to deal with only the controller-to-actuator communication constraints, but not sensors-to-controller communication constraints. Furthermore, most of the available results on networked systems are confined to the state-based feedback control problems. To the best of the authors' knowledge, very few results have tackled the stabilization of the NCSs with medium access constraints via dynamic output-feedback control, which motivates us to fill this gap.

In response to the aforementioned discussion, this paper aims to explore the dynamic output feedback control problem of NCSs whose sensors and actuators access the medium subject to Markov processes. More specifically, the objective of this paper is to provide a systematic framework of dynamic output feedback control design for the NCSs with multiple communication constraints so as to guarantee the exponential stable performance. The main contributions of this paper can be summarized as follows.

(1) The comprehensive integration of the dynamic output feedback controller design is investigated for the NCSs subject to medium access constraints, network-induced delays and packet dropouts. (2) Multiple communication constraints have been allowed to occur, simultaneously, in both the backward and the forward network channels in the framework of controller design. (3) The calculation procedure of dynamic output feedback gains is presented to guarantee the exponential stability of the NCSs with the given decay rate.

The rest of this paper is organized as follows. Section 2 is the problem description and preliminaries. In Section 3, the design procedure of dynamic output feedback controller is proposed. An illustrative example is provided in Section 4 to demonstrate the effectiveness of the proposed results. Finally, Section 5 concludes the paper and discusses future research directions.

Notation: The notation used throughout the paper is fairly standard. R^n denotes the n-dimensional Euclidean space and $P > 0\,(P \geq 0)$ means that it is real symmetric and positive definite (semi-definite). The superscripts 'T' and '-1' stand for matrix transposition and matrix inverse, respectively. C_a^b is the combinatorial number that b elements are selected from a total of a elements. $diag\{\rho_1, \cdots, \rho_n\}$ stands for a diagonal matrix with the indicated elements on the diagonal and zeros elsewhere. $\Pr\{\xi\}$ means the occurrence probability of the event ξ.

II. PROBLEM FORMULATION

In this section, an NCS subject to medium access constraints, network-induced delays and packet dropouts is considered, whose structure is depicted in Figure 1. The linear continuous-time plant is described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t - \tau_k) \\ y(t) = Cx(t) \end{cases}, \tag{1}$$

where $x(t) \in R^n$ is the plant state, $u(t) \in R^m$ is the control input actually executed by the actuators, and $y(t) \in R^l$ is the plant's outputs measured by the sensors. A, B and C are known constant matrices with appropriate dimensions. Due to the introduction of the communication network, there are network delays in the transmission process of the output and control signals. τ_k^{sc} and τ_k^{ac}, respectively, represent the network delay between the sensors-controller, and the controller-actuator in the k-th sampling period. In order to simplify the analysis, the following assumptions are given:

1) The sensors are time-driven, and the controller and the actuators are event-driven;

2) In the k-th sampling period, the network-induced delay τ_k satisfies $\tau_k = \tau_k^{sc} + \tau_k^{ac}$;

3) The controller and the actuators are equipped with data buffers. When packet dropouts occur in the process of network transmission, the data packet at the previous sampling period will be utilized as current control and measured signals.

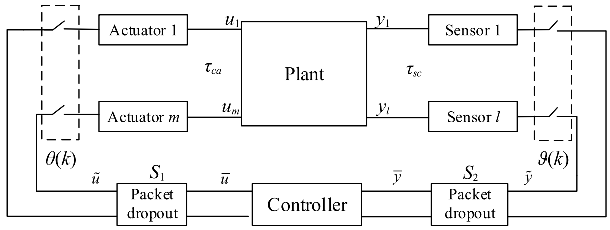

FIGURE 1. The basic configuration of NCSs.

According to the above assumptions, the control input $u(t)$ of the actuators can be expressed as

$$u(t) = \begin{cases} u(k-1), & t_k < t \leq t_k + \tau_k \\ u(k), & t_k + \tau_k < t \leq t_k + T_p \end{cases}, \tag{2}$$

where T_p is the sampling period. Then the discretization model of the controlled plant is

$$\begin{cases} x(k+1) = A_m x(k) + B_{m0}(\tau_k)\, u(k) + B_{m1}(\tau_k)\, u(k-1) \\ y(k) = Cx(k) \end{cases}, \tag{3}$$

where $A_m = e^{AT_p}$, $B_{m0}(\tau_k) = \int_0^{T_p - \tau_k} e^{At} dtB$, $B_{m1}(\tau_k) = \int_{T_1 - \tau_k}^{T_p} e^{At} dtB$. Without loss of generality, it is assumed that the network-induced delay τ_k is a constant, and then $B_{m0}(\tau_k) = B_{m0}$, $B_{m1}(\tau_k) = B_{m1}$.

A. MEDIUM ACCESS CONSTRAINTS

As shown in Figure 1, there are m actuators and l sensors in the NCSs. Due to the bandwidth limitation, the communication medium imposes an upper bound, respectively, $1 \leq h < m$ on the number of actuators and $1 \leq s < l$ on the number of sensors, which may communicate simultaneously with the controller at any time instant k. Therefore, there are total $\nu = C_m^h$ possible medium -access status for the actuators, and $\upsilon = C_l^s$ possible medium-access status for the sensors. In order to describe the medium access state of the h-th actuator in k-th sampling period, the following function is defined:

$$\theta_h(k) = \begin{cases} 1, & u_h(k) = \tilde{u}_h(k) \\ 0, & u_h(k) = 0 \end{cases},$$

where $h \in \{1, 2, \cdots, m\}$. If $\theta_h(k) = 1$, then $\tilde{u}_h(k)$ obtains the access right of communication channels, i.e., $u_h(k) = \tilde{u}_h(k)$. If $\theta_h(k) = 0$, then $\tilde{u}_h(k)$ does not obtain the access right of communication channels, i.e., $u_h(k) = 0$. Let $\theta(k) = [\theta_1(k)\,\theta_2(k)\cdots\theta_m(k)]^T$ denote the channel access status of actuators in the k-th sampling period. Define

$$L(k) = diag\{\theta(k)\},$$

then one has

$$u(k) = L(k)\,\tilde{u}(k). \tag{4}$$

Considering the random feature of medium access constraints, it is assumed that $L(k)$ can be modelled by a

Markov process taking the matrix values in a finite set $L = \{L^1, \cdots, L^v\}$ with the following conditional probability:

$$\Pr\{L(k) = L_i\} = \eta_i(k), \Pr\{L(k) = L_i | L(k-1) = L_j\} = \eta_{ji},$$

where $\eta_{ji} > 0$ is the transition probability of $L(k)$ from the mode j to the mode i, satisfying $\sum_{i=1}^{v} \eta_{ji} = 1$, $j, i \in \{1, 2, \cdots, v\}$. $\eta_i(0)$ is the initial probability of mode i, while $\eta_i(k)$ is the probability of mode i at time k, and $\eta(k) = [\eta_1(k) \cdots \eta_v(k)]^T$.

Similarly, the medium-access status of the sensor s is denoted by another binary valued function

$$\vartheta_s(k) = \begin{cases} 1, & \tilde{y}_s(k) = y_s(k) \\ 0, & \tilde{y}_s(k) = 0 \end{cases},$$

where, $s \in \{1, 2, \cdots, l\}$. $\vartheta_s(k) = 1$ indicates that the sensor s is accessing the communication medium, and the output information is updated, i.e., $\tilde{y}_s(k) = y_s(k)$. Otherwise, $\vartheta_s(k) = 0$ means that the current s-th measured signal is not transmitted and the controller will use the zero value as the measured signal, i.e., $\tilde{y}_s(k) = 0$. Let $\vartheta(k) = [\vartheta_1(k)\, \vartheta_2(k) \cdots \vartheta_l(k)]^T$ represent the medium access status of the sensors at the k-th sampling period. Define

$$M(k) = diag\{\vartheta(k)\},$$

then

$$\tilde{y}(k) = M(k)\, y(k). \tag{5}$$

It is also assumed that $M(k)$ is described by another Markov process that takes the matrix values in finite set $M = \{M^1, \cdots, M^\upsilon\}$ with the following conditional probability:

$$\Pr\{M(k){=}M_p\} = \pi_p(k), \Pr\{M(k){=}M_p | M(k-1) = M_q\} = \pi_{qp},$$

where $\pi_{qp} > 0$, satisfying $\sum_{p=1}^{\upsilon} \pi_{qp} = 1$, $q, p \in \{1, 2, \cdots, \upsilon\}$.

Remark 1: In this paper, it is assumed that the current modes of the Markov process $L(k)$ and $M(k)$ are available at each time instant k. This assumption is critical, since it allows us to avoid the more difficult and generally unsolved 'dual control' problem.

B. PACKET DROPOUT

For the packet dropout issue, this paper utilizes a pair of switches S_1 between the controller and actuators, and S_2 between the sensors and controller to denote the packet-loss phenomenon of the channels. The closed or open status of the switches S_1 and S_2 indicate whether packet dropout occurs during data packet transmission, respectively. It is assumed that, at the k-th sampling period, the closed and open statuses of switches S_1 and S_2 are governed by two i.i.d. Bernoulli processes $\alpha(k)$ and $\beta(k)$ with the following probability distribution:

$$\begin{aligned} &\mathrm{P}\{\alpha(k) = 0\} = \bar{\alpha},\ \mathrm{P}\{\alpha(k) = 1\} = 1 - \bar{\alpha} \\ &\mathrm{P}\{\beta(k) = 0\} = \bar{\beta},\quad \mathrm{P}\{\beta(k) = 1\} = 1 - \bar{\beta}, \end{aligned} \tag{6}$$

where $\alpha(k) = 0$ means that the network switch S_1 is open, and $\alpha(k) = 1$ means that the switch S_1 is closed. Similarly, $\beta(k) = 0$ indicates that the network switch S_2 is open, while $\beta(k) = 1$ indicates that the switch S_2 is closed. From (6), It is easy to know that $\bar{\alpha}$ and $\bar{\beta}$, respectively, are the open probabilities of the switch S_1 and S_2, which are the packet dropout rates; and $1 - \bar{\alpha}$, $1 - \bar{\beta}$, respectively, are the closed probabilities of the switch S_1 and S_2, which are successful transmission rates of the data packet.

When the network switch S_1 and S_2 are closed, the packet dropout does not occur, that is,

$$\bar{y}(k) = \tilde{y}(k), \tilde{u}(k) = \bar{u}(k). \tag{7}$$

When the network switch S_1 and S_2 are open, the packet dropout occurs during the data transmission, and the data packet at the previous sampling period will be utilized as current control and measured signals, that is,

$$\bar{y}(k) = \bar{y}(k-1), \tilde{u}(k) = \tilde{u}(k-1). \tag{8}$$

By considering (4), (5), (7) and (8), the control vector actuated by the actuators, and the measured output received by the controller can be expressed as follows:

$$u(k) = \alpha(k) L_i \bar{u}(k) + (1 - \alpha(k)) L_j \bar{u}(k-1), \tag{9}$$

$$\bar{y}(k) = \beta(k) M_p y(k) + (1 - \beta(k)) M_q y(k-1), \tag{10}$$

where $L_i = L(k)$, $L_j = L(k-1)$, $M_p = M(k)$, $M_q = M(k-1)$.

C. SYSTEM MODELING

By combining (3), (9) and (10), the NCSs model with network-induced delays, access constraints, and packet dropout is obtained

$$\begin{cases} x(k+1) = A_m x(k) + B_{m0} u(k) + B_{m1} u(k-1) \\ u(k) = \alpha(k) L_i \bar{u}(k) + (1 - \alpha(k)) L_j \bar{u}(k-1) \\ \bar{y}(k) = \beta(k) M_p C x(k) + (1 - \beta(k)) M_q C x(k-1) \end{cases} \tag{11}$$

The following form of dynamic output feedback controller is presented

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c \bar{y}(k) \\ \bar{u}(k) = C_c x_c(k) + D_c \bar{y}(k) \end{cases}, \tag{12}$$

where $x_c(k)$ is the controller state, and $\bar{u}(k)$ is the output of the dynamic output feedback controller. A_c, B_c, C_c and D_c are controller parameters with appropriate dimensions, which are to be determined. Define

$$z(k) = \begin{bmatrix} x^T(k) & x_c^T(k) & \tilde{u}^T(k-1) & \bar{y}^T(k) \end{bmatrix}^T,$$

then the closed-loop model of NCSs with dynamic output feedback can be expressed as

$$z(k+1) = \Phi_r z(k), \tag{13}$$

where $r = 1, \cdots, 4$. When $\alpha(k) = 0$ and $\beta(k) = 0$,

$$\Phi_1 = \begin{bmatrix} A_m & 0 & B_{m0} L_i + B_{m1} L_j & 0 \\ 0 & A_c & 0 & B_c \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}.$$

When $\alpha(k) = 1$ and $\beta(k) = 1$,

$$\Phi_2 = \begin{bmatrix} A_m & B_{m0}L_iC_c & B_{m1}L_j & B_{m0}L_iD_c \\ 0 & A_c & 0 & B_c \\ 0 & C_c & 0 & D_c \\ M_PCA_m & M_PCB_{m0}L_iC_c & M_PCB_{m1}L_j & M_PCB_{m0}L_iD_c \end{bmatrix}.$$

When $\alpha(k) = 0$ and $\beta(k) = 1$,

$$\Phi_3 = \begin{bmatrix} A_m & 0 & B_{m0}L_i + B_{m1}L_j & 0 \\ 0 & A_c & 0 & B_c \\ 0 & 0 & I & 0 \\ M_pCA_m & 0 & M_pC(B_{m0}L_i + B_{m1}L_j) & 0 \end{bmatrix}.$$

When $\alpha(k) = 1$ and $\beta(k) = 0$,

$$\Phi_4 = \begin{bmatrix} A_m & B_{m0}L_iC_c & B_{m1}L_j & B_{m0}L_iD_c \\ 0 & A_c & 0 & B_c \\ 0 & C_c & 0 & D_c \\ 0 & 0 & 0 & I \end{bmatrix}.$$

Obviously, if the values of $\alpha(k)$ and $\beta(k)$ are different, the value of Φ_r is also different. The activated subsystem of the closed-loop networked systems is determined by the status of the network switch S_1 and S_2.

*Remark2:*This paper mainly focus on the design problem of dynamic output feedback control for a class of networked control systems with medium access constraints, network-induced delays and packet dropouts. In order to facilitate the analysis and design, the network-induced delay is described as a deterministic delay, which partly reduces the complexity of control synthesis. Furthermore, in the design of proposed dynamic output feedback controller, the switching probability of Markov access constraints is assumed to be known. For the medium access constraints with unknown or partially known switching probability, how to design a dynamic output feedback controller is truly an interesting issue that needs to be further study.

Remark 3: Note that, under the proposed dynamic output feedback controller, the networked system with multiple communication constraints can be modelled as a class of asynchronous dynamic systems with random switching characteristics, which allows us to utilize well developed asynchronous dynamic theory to implement controller design to guarantee the closed-loop networked systems exponential mean square stability.

III. MAIN RESULTS

In what follows, we will provide a systematic framework of the dynamic output feedback controller design for the presented closed-loop networked systems. To state the subsequent results, some definitions and lemmas are introduced as follows.

Definition 1[39]*:* Defining ρ_r as the NCSs' structure event rate described by the matrix Φ_r, it is clear that the ρ_r must satisfy the condition below:

$$\sum\nolimits_{r=1}^{4} \rho_r = 1, r = 1, \cdots, 4. \tag{14}$$

Therefore, the closed-loop networked systems with multiple communication constraints can be regarded as an asynchronous dynamic system with the structural event rate ρ_r, which is determined by the packet dropout occurrence probability $\bar{\alpha}$ and $\bar{\beta}$.

Definition 2[40]*:* For an asynchronous dynamic system, if its trajectories satisfy

$$\lim_{k\to\infty} \lambda^k \|x(k)\| = 0, \tag{15}$$

for some $\lambda > 1$, it is said that the system is exponentially stable, and the largest λ is referred to as the decay rate of the system.

Lemma 1[39]*:* For an asynchronous dynamic system constrained by the structural events rate ρ_r:

$$x(k+1) = g_r(x(k)),$$

where $r = 1, \cdots, n$, and n is the number of events presented in the system. If there exists a Lyapunov function $V(x(k)) : R^n \to R^+$ that satisfies

$$c\|x(k)\|^2 \le V(x(k)) \le d\|x(k)\|^2,$$

where c, $d > 0$, and for scalar $\lambda > 0$ and $\lambda_r > 0$, $V(x(k))$ satisfies the following conditions:

$$V(x(k+1)) - V(x(k)) \le \left(\lambda_r^{-2} - 1\right) V(x(k)), \tag{16}$$

$$\prod_{r=1}^{n} \lambda_r^{\rho_r} > \lambda > 1, \tag{17}$$

it is said that the asynchronous dynamic system is exponentially stable in the sense of Definition 2.

Lemma 2[41]*:* Given the symmetric matrices H_1, H_2 and H_3, where $H_1 = H_1^T$ and $H_2 = H_2^T$, then $H_1 - H_3^T H_2^{-1} H_3 < 0$ holds if and only if

$$\begin{bmatrix} H_1 & H_3^T \\ H_3 & H_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} H_2 & H_3 \\ H_3^T & H_1 \end{bmatrix} < 0.$$

First of all, we will present the relationship between the structural event rate ρ_r and packet dropout occurrence probabilities $\bar{\alpha}$ and $\bar{\beta}$.

Theorem 1: For the closed-loop networked systems (13), the corresponding structural event rates are ρ_r, $r = 1, \cdots, 4$. The following relationship holds for the structural event rate ρ_r and packet dropout rate $\bar{\alpha}$ and $\bar{\beta}$:

$$\begin{cases} \rho_1 = \bar{\alpha}\bar{\beta} \\ \rho_2 = (1-\bar{\alpha})\left(1-\bar{\beta}\right) \\ \rho_3 = \bar{\alpha}\left(1-\bar{\beta}\right) \\ \rho_4 = (1-\bar{\alpha})\,\bar{\beta} \end{cases}. \tag{18}$$

Proof: The closed and open statuses of the network switch S_1 are represented by events M_1 and $\bar{M}_1$, respectively. The closed and open statuses of the network switch S_2 are represented by events M_2 and $\bar{M}_2$, respectively. According to the definition of the structural event rate ρ_r, there is

$$\rho_1 = Pr\left\{\overline{M}_1\overline{M}_2\right\}.$$

Because the switches S_1 and S_2 are independent of each other, the events M_1 and M_2, M_1 and $\bar{M}_2$, $\bar{M}_1$ and M_2, $\bar{M}_1$ and $\bar{M}_2$ are also independent. From the probability property of independent events, one has

$$\rho_1 = Pr\left\{\overline{M}_1\overline{M}_2\right\} = \Pr\left\{\overline{M}_1\right\}\cdot\Pr\left\{\overline{M}_2\right\} = \bar{\alpha}\bar{\beta}.$$

Similarly, it is obtained

$$\rho_2 = Pr\left\{M_1M_2\right\} = \Pr\left\{M_1\right\}\cdot\Pr\left\{M_2\right\} = (1-\bar{\alpha})\left(1-\bar{\beta}\right),$$
$$\rho_3 = Pr\left\{\overline{M}_1M_2\right\} = \Pr\left\{\overline{M}_1\right\}\cdot\Pr\left\{M_2\right\} = \bar{\alpha}\left(1-\bar{\beta}\right),$$
$$\rho_4 = Pr\left\{M_1\overline{M}_2\right\} = \Pr\left\{M_1\right\}\cdot\Pr\left\{\overline{M}_2\right\} = (1-\bar{\alpha})\,\bar{\beta}.$$

This completes the proof.

With the definition of structural event rate ρ_r, the exponential stability conditions for the closed-loop networked systems with multiple communication constraints are derived.

Theorem 2: Consider the closed-loop networked systems (13), with network-induced delays $0 \le \tau_k < T$, medium access constraints $L(k)$ and $M(k)$, and packet dropout rate $\bar{\alpha}$ and $\bar{\beta}$. If there are scalars $\lambda_r > 0$, $r = 1, \cdots, 4$, and positive definite matrices R, F, Z, and G, which satisfy the following conditions:

$$\lambda_1^{\bar{\alpha}\bar{\beta}}\lambda_2^{(1-\bar{\alpha})(1-\bar{\beta})}\lambda_3^{\bar{\alpha}(1-\bar{\beta})}\lambda_4^{(1-\bar{\alpha})\bar{\beta}} > \lambda > 1, \tag{19}$$

$$\begin{bmatrix} -\lambda_1^{-2}Y & 0 & 0 & 0 & YA_m^T & 0 \\ * & -\lambda_1^{-2}S & 0 & 0 & 0 & SA_c^T \\ * & * & (1-\lambda_1^{-2})X & 0 & \Delta_{35} & 0 \\ * & * & * & (1-\lambda_1^{-2})J & 0 & JB_c^T \\ * & * & * & * & -Y & 0 \\ * & * & * & * & * & -S \end{bmatrix} < 0, \tag{20}$$

where $\Delta_{35} = X(B_{m0}L_i + B_{m1}L_j)^T$,

$$\begin{bmatrix} -\lambda_2^{-2}Y & 0 & 0 & 0 & YA_m^T & \Phi_{16} & 0 & 0 \\ * & -\lambda_2^{-2}S & 0 & 0 & \Phi_{25} & \Phi_{26} & SA_c^T & SC_c^T \\ * & * & -\lambda_2^{-2}X & 0 & \Phi_{35} & \Phi_{36} & 0 & 0 \\ * & * & * & -\lambda_2^{-2}J & \Phi_{45} & \Phi_{46} & JB_c^T & JD_c^T \\ * & * & * & * & -Y & 0 & 0 & 0 \\ * & * & * & * & * & -J & 0 & 0 \\ * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & -X \end{bmatrix} < 0, \tag{21}$$

where $\Phi_{16} = YA_m^T(M_pC)^T$, $\Phi_{25} = SC_c^T(B_{m0}L_i)^T$, $\Phi_{26} = SC_c^T(B_{m0}L_i)^T(M_pC)^T$, $\Phi_{35} = X(B_{m1}L_j)^T$, $\Phi_{36} = X(B_{m1}L_j)^T(M_pC)^T$, $\Phi_{45} = JD_c^T(B_{m0}L_i)^T$, $\Phi_{46} = JD_c^T(B_{m0}L_i)^T(M_pC)^T$,

$$\begin{bmatrix} -\lambda_3^{-2}Y & 0 & 0 & 0 & YA_m^T & \Gamma_{16} & 0 \\ * & -\lambda_3^{-2}S & 0 & 0 & 0 & 0 & SA_c^T \\ * & * & (1-\lambda_3^{-2})X & 0 & \Gamma_{35} & \Gamma_{36} & 0 \\ * & * & * & -\lambda_3^{-2}J & 0 & 0 & JB_c^T \\ * & * & * & * & -Y & 0 & 0 \\ * & * & * & * & * & -J & 0 \\ * & * & * & * & * & * & -S \end{bmatrix} < 0, \tag{22}$$

where $\Gamma_{16} = YA_m^T(M_pC)^T$, $\Gamma_{35} = X(B_{m0}L_i + B_{m1}L_j)^T$, $\Gamma_{36} = X(B_{m0}L_i + B_{m1}L_j)^T(M_pC)^T$,

$$\begin{bmatrix} -\lambda_4^{-2}Y & 0 & 0 & 0 & YA_m^T & 0 & 0 \\ * & -\lambda_4^{-2}S & 0 & 0 & \text{H}_{25} & SA_c^T & SC_c^T \\ * & * & -\lambda_4^{-2}X & 0 & \text{H}_{35} & 0 & 0 \\ * & * & * & \text{H}_{44} & \text{H}_{45} & JB_c^T & JD_c^T \\ * & * & * & * & -Y & 0 & 0 \\ * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & -X \end{bmatrix} < 0, \tag{23}$$

where, $\text{H}_{25} = SC_c^T(B_{m0}L_i)^T$, $\text{H}_{35} = X(B_{m1}L_j)^T$, $\text{H}_{44} = (1-\lambda_4^{-2})J$, $\text{H}_{45} = JD_c^T(B_{m0}L_i)^T$,
it is said that the closed-loop systems (13) are exponentially stable, and the decay rate is $\lambda_1^{\bar{\alpha}\bar{\beta}}\lambda_2^{(1-\bar{\alpha})(1-\bar{\beta})}\lambda_3^{\bar{\alpha}(1-\bar{\beta})}\lambda_4^{(1-\bar{\alpha})\bar{\beta}}$.

Proof: It is clear that the closed-loop systems (13) have four subsystems. Form (17), then we can obtain

$$\lambda_1^{\bar{\alpha}\bar{\beta}}\lambda_2^{(1-\bar{\alpha})(1-\bar{\beta})}\lambda_3^{\bar{\alpha}(1-\bar{\beta})}\lambda_4^{(1-\bar{\alpha})\bar{\beta}} > \lambda > 1. \tag{24}$$

Suppose that a Lyapunov function $V(x(k))$: $R^n \to R^+$ satisfies $c\|x(k)\|^2 \le V(x(k)) \le d\|x(k)\|^2$. Choose the symmetric and positive definite matrix P, Q, Z and F, and define the Lyapunov function $V(x(k))$ as

$$V(x(k)) = x^T(k)Px(k) + x_c^T(k)Qx_c(k) + \tilde{u}^T(k-1)Z\tilde{u}(k-1) + \bar{y}^T(k)F\bar{y}(k), \tag{25}$$

According to Lemma 1, if

$$\begin{aligned} &V(x(k+1)) - \lambda_r^{-2}V(x(k)) \\ &= x^T(k+1)Px(k+1) + x_c^T(k+1)Qx_c(k+1) \\ &\quad + \tilde{u}^T(k)Z\tilde{u}(k) \\ &\quad + \bar{y}^T(k+1)F\bar{y}(k+1) - \lambda_r^{-2}x^T(k)Px(k) \\ &\quad - \lambda_r^{-2}x_c^T(k)Qx_c(k) \\ &\quad - \lambda_r^{-2}\tilde{u}^T(k-1)Z\tilde{u}(k-1) - \lambda_r^{-2}\bar{y}^T(k)F\bar{y}(k) < 0, \end{aligned} \tag{26}$$

holds, then the closed-loop networked systems (13) are exponentially stable.

When $r = 1$, Φ_1 occurs. Let

$$\bar{z}(k) = \left[x^T(k)\,x_c^T(k)\,\tilde{u}^T(k-1)\;\bar{y}^T(k)\right]^T.$$

From (26), it is yielded that

$$\begin{aligned} &V(x(k+1)) - \lambda_r^{-2}V(x(k)) \\ &= x^T(k+1)Px(k+1) + x_c^T(k+1)Qx_c(k+1) \\ &\quad + \tilde{u}^T(k)Z\tilde{u}(k) \\ &\quad + \bar{y}^T(k+1)F\bar{y}(k+1) - \lambda_r^{-2}x^T(k)Px(k) \\ &\quad - \lambda_r^{-2}x_c^T(k)Qx_c(k) - \lambda_r^{-2}\tilde{u}^T(k-1)Z\tilde{u}(k-1) \\ &\quad - \lambda_r^{-2}\bar{y}^T(k)F\bar{y}(k) < 0, \\ &= \bar{z}^T(k)\Omega_1\bar{z}(k) < 0, \end{aligned}$$

where

$$\Omega_1 = \begin{bmatrix} A_m^T P A_m - \lambda_1^{-2} P & 0 & \mathrm{K}_{13} & 0 \\ * & A_c^T Q A_c - \lambda_1^{-2} Q & 0 & A_c^T Q B_c \\ * & * & \mathrm{K}_{33} & 0 \\ * & * & * & \mathrm{K}_{44} \end{bmatrix}, \tag{27}$$

where, $\mathrm{K}_{13} = A_m^T P(B_{m0}L_i + B_{m1}L_j)$,
$\mathrm{K}_{33} = (B_{m0}L_i + B_{m1}L_j)^T P(B_{m0}L_i + B_{m1}L_j) + Z - \lambda_1^{-2} Z$,
$\mathrm{K}_{44} = B_c^T Q B_c + F - \lambda_1^{-2} F$.

Then (27) can be written as follows (28), as shown at the bottom of the page.

Applying Lemma 2, one has (29), as shown at the bottom of the page, where $\Lambda_{22} = A_c^T Q A_c - \lambda_1^{-2} Q$, $\Lambda_{44} = B_c^T Q B_c + F - \lambda_1^{-2} F$.

Applying Lemma 2 again for (29) yields (30), as shown at the bottom of the next page, where $O_{35} = (B_{m0}L_i + B_{m1}L_j)^T$.

Then, pre-multiplying and post-multiplying the inequality (30) by $diag\{P^{-1}, Q^{-1}, Z^{-1}, Y^{-1}, I, I\}$ and letting $X = Z^{-1}, Y = P^{-1}, S = Q^{-1}, J = F^{-1}$ yields (20).

Along the similar lines of case $r = 1$, the results follow immediately for the case $r = 2$, $r = 3$ and $r = 4$, (31) as shown at the bottom of the next page, where $\Pi_{16} = A_m^T (M_p C)^T$, $\Pi_{25} = C_c^T (B_{m0}L_i)^T$,
$\Pi_{26} = C_c^T (B_{m0}L_i)^T (M_p C)^T$, $\Pi_{35} = (B_{m1}L_j)^T$,
$\Pi_{36} = (B_{m1}L_j)^T (M_p C)^T$, $\Pi_{45} = D_c^T (B_{m0}L_i)^T$,
$\Pi_{46} = D_c^T (B_{m0}L_i)^T (M_p C)^T$. (32) as shown at the bottom of the next page, where $\Theta_{16} = A_m^T (M_P C)^T$, $\Theta_{33} = \left(1 - \lambda_3^{-2}\right) Z$, (33) as shown at the bottom of the next page.

$\Theta_{35} = (B_{m0}L_i + B_{m1}L_j)^T$, $\Theta_{36} = (B_{m0}L_i + B_{m1}L_j)^T (M_P C)^T$, where $\Sigma_{25} = C_c^T (B_{m0}L_i)^T$, $\Sigma_{35} = (B_{m1}L_i)^T$.
$\Sigma_{44} = (1 - \lambda_4^{-2})F$, $\Sigma_{45} = D_c^T (B_{m0}L_i)^T$.

Similarly, pre-multiplying and post-multiplying the inequality (31) by $diag\{P^{-1}, Q^{-1}, S^{-1}, Y^{-1}, I, I, I, I\}$ and letting $X = Z^{-1}$, $Y = P^{-1}$, $S = Q^{-1}$, $J = F^{-1}$ yields (21).

By pre-multiplying and post-multiplying the inequality (32) by $diag\{P^{-1}, Q^{-1}, Z^{-1}, T^{-1}, I, I, I\}$ and letting $X = Z^{-1}$, $Y = P^{-1}$, $S = Q^{-1}$, $J = F^{-1}$, (22) can be obtained.

By pre-multiplying and post-multiplying the inequality (33) by $diag\{P^{-1}, Y^{-1}, S^{-1}, U^{-1}, I, I, I\}$ and letting $X = Z^{-1}$, $Y = P^{-1}$, $S = Q^{-1}$, $J = F^{-1}$, (23) can be obtained.

This completes the proof.

Theorem 3: Consider the closed-loop networked systems (13), with network-induced delays $0 \leq \tau_k < T_p$, medium access constraints $L(k)$ and $M(k)$, and packet dropout rates $\bar{\alpha}$ and $\bar{\beta}$. If there are scalars $\lambda_r > 0$, $r = 1, \cdots, 4$, and positive definite matrices X, Y, S, and J, and matrices H, M, N, and W, which satisfy the following conditions (34) and (35), as shown at the bottom of the next page, where $\Upsilon_{35} = X(B_{m0}L_i + B_{m1}L_j)^T$, (36) as shown at the bottom of the 9th page, where $\Xi_{16} = YA_m^T (M_P C)^T$, $\Xi_{25} = N^T (B_{m0}L_i)^T$, $\Xi_{26} = N^T (B_{m0}L_i)^T (M_p C)^T$, $\Xi_{35} = X(B_{m1}L_j)^T$, $\Xi_{36} = X(B_{m1}L_j)^T (M_p C)^T$, $\Xi_{45} = W^T (B_{m0}L_i)^T$, $\Xi_{46} = W^T (B_{m0}L_i)^T (M_p C)^T$, (37) as shown at the bottom of the 9th page, where $\Psi_{16} = YA_m^T (M_P C)^T$,

$$\begin{aligned} \Omega_1 &= \begin{bmatrix} -\lambda_1^{-2} P & 0 & 0 & 0 \\ * & A_c^T Q A_c - \lambda_1^{-2} Q & 0 & A_c^T Q B_c \\ * & * & \left(1 - \lambda_1^{-2}\right) Z & 0 \\ * & * & * & B_c^T Q B_c + F - \lambda_1^{-2} F \end{bmatrix} \\ &\quad + \begin{bmatrix} A_m^T P A_m & 0 & A_m^T P(B_{m0}L_i + B_{m1}L_j) & 0 \\ * & 0 & 0 & 0 \\ * & * & (B_{m0}L_i + B_{m1}L_j)^T P(B_{m0}L_i + B_{m1}L_j) & 0 \\ * & * & * & 0 \end{bmatrix} \\ &= \begin{bmatrix} -\lambda_1^{-2} P & 0 & 0 & 0 \\ * & A_c^T Q A_c - \lambda_1^{-2} Q & 0 & A_c^T Q B_c \\ * & * & \left(1 - \lambda_1^{-2}\right) Z & 0 \\ * & * & * & B_c^T Q B_c + F - \lambda_1^{-2} F \end{bmatrix} \\ &\quad + \left[A_m \ 0 \ (B_{m0}L_i + B_{m1}L_j) \ 0\right]^T \left(-P^{-1}\right) \left[A_m \ 0 \ (B_{m0}L_i + B_{m1}L_j) \ 0\right] < 0. \end{aligned} \tag{28}$$

$$\Omega_1 = \begin{bmatrix} -\lambda_1^{-2} P & 0 & 0 & 0 & A_m^T \\ * & \Lambda_{22} & 0 & A_c^T Q B_c & 0 \\ * & * & \left(1 - \lambda_1^{-2}\right) Z & 0 & \left(B_{m0}L_i + B_{m1}L_j\right)^T \\ * & * & * & \Lambda_{44} & 0 \\ * & * & * & 0 & -P^{-1} \end{bmatrix} < 0, \tag{29}$$

$\Psi_{35} = X(B_{m0}L_i + B_{m1}L_j)^T$, $\Psi_{36} = X(B_{m0}L_i + B_{m1}L_j)^T(M_PC)^T$,

$$\begin{bmatrix} -\lambda_4^{-2}Y & 0 & 0 & 0 & YA_m^T & 0 & 0 \\ * & -\lambda_4^{-2}S & 0 & 0 & E_{25} & H^T & N^T \\ * & * & -\lambda_4^{-2}X & 0 & E_{35} & 0 & 0 \\ * & * & * & E_{44} & E_{45} & M^T & W^T \\ * & * & * & * & -Y & 0 & 0 \\ * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & -X \end{bmatrix} < 0, \quad (38)$$

where $E_{25} = N^T(B_{m0}L_i)^T$, $E_{35} = X(B_{m1}L_i)^T$, $E_{44} = (1-\lambda_4^{-2})J$, $E_{45} = W^T(B_{m0}L_i)^T$, then there exists dynamic output feedback controller

$$A_c = HS^{-1}, B_c = MJ^{-1}, C_c = NS^{-1}, D_c = WJ^{-1}, \quad (39)$$

that renders the closed-loop networked systems (13) exponentially stable.

Proof: Using the variable substitution method, let $A_cS = H, B_cJ = M, C_cS = N, W = D_cJ$. Substituting them

$$\Omega_1 = \begin{bmatrix} -\lambda_1^{-2}P & 0 & 0 & 0 & A_m^T & 0 \\ * & -\lambda_1^{-2}Q & 0 & 0 & 0 & A_c^T \\ * & * & (1-\lambda_1^{-2})Z & 0 & O_{35} & 0 \\ * & * & * & (1-\lambda_1^{-2})F & 0 & B_c^T \\ * & * & * & * & -P^{-1} & 0 \\ * & * & * & * & * & -Q^{-1} \end{bmatrix} < 0, \quad (30)$$

$$\Omega_2 = \begin{bmatrix} -\lambda_2^{-2}P & 0 & 0 & 0 & A_m^T & \Pi_{16} & 0 & 0 \\ * & -\lambda_2^{-2}Q & 0 & 0 & \Pi_{25} & \Pi_{26} & A_c^T & C_c^T \\ * & * & -\lambda_2^{-2}Z & 0 & \Pi_{35} & \Pi_{36} & 0 & 0 \\ * & * & * & -\lambda_2^{-2}F & \Pi_{45} & \Pi_{46} & B_c^T & D_c^T \\ * & * & * & * & -P^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & -F^{-1} & 0 & 0 \\ * & * & * & * & * & * & -Q^{-1} & 0 \\ * & * & * & * & * & * & * & -Z^{-1} \end{bmatrix} < 0, \quad (31)$$

$$\Omega_3 = \begin{bmatrix} -\lambda_3^{-2}P & 0 & 0 & 0 & A_m^T & \Theta_{16} & 0 \\ * & -\lambda_3^{-2}Q & 0 & 0 & 0 & 0 & A_c^T \\ * & * & \Theta_{33} & 0 & \Theta_{35} & \Theta_{36} & 0 \\ * & * & * & -\lambda_3^{-2}F & 0 & 0 & B_c^T \\ * & * & * & * & -P^{-1} & 0 & 0 \\ * & * & * & * & * & -F^{-1} & 0 \\ * & * & * & * & * & * & -Q^{-1} \end{bmatrix} < 0, \quad (32)$$

$$\Omega_4 = \begin{bmatrix} -\lambda_4^{-2}P & 0 & 0 & 0 & A_m^T & 0 & 0 \\ * & -\lambda_4^{-2}Q & 0 & 0 & \Sigma_{25} & A_c^T & C_c^T \\ * & * & -\lambda_4^{-2}Z & 0 & \Sigma_{35} & 0 & 0 \\ * & * & * & \Sigma_{44} & \Sigma_{45} & B_c^T & D_c^T \\ * & * & * & * & -P^{-1} & 0 & 0 \\ * & * & * & * & * & -Q^{-1} & 0 \\ * & * & * & * & * & * & -Z^{-1} \end{bmatrix} < 0, \quad (33)$$

$$\lambda_1^{\bar{\alpha}\bar{\beta}}\lambda_2^{(1-\bar{\alpha})(1-\bar{\beta})}\lambda_3^{\bar{\alpha}(1-\bar{\beta})}\lambda_4^{(1-\bar{\alpha})\bar{\beta}} > \lambda > 1, \quad (34)$$

$$\begin{bmatrix} -\lambda_1^{-2}Y & 0 & 0 & 0 & YA_m^T & 0 \\ * & -\lambda_1^{-2}S & 0 & 0 & 0 & H^T \\ * & * & (1-\lambda_1^{-2})X & 0 & \Upsilon_{35} & 0 \\ * & * & * & (1-\lambda_1^{-2})J & 0 & M^T \\ * & * & * & * & -Y & 0 \\ * & * & * & * & * & -S \end{bmatrix} < 0, \quad (35)$$

Algorithm 1 The Algorithm of Dynamic Output Feedback Control Design

Step1. Calculate the matrices X, Y, S, and J, and matrices H, M, N, and W by solving (35)-(38) through standard numerical software.

Step2. Derive the controller parameters A_c, B_c, C_c and D_c by solving (39).

into (20), (21), (22) and (23) yields (35), (36), (37) and (38). This completes the proof.

By means of Theorem 3, the dynamic output feedback controller design algorithm is summarized as follows:

Remark 4: It is noteworthy that different from the literature [30]–[38], this paper not only considers the Markov access constraints between the controller and sensors/ actuators, but also Bernoulli packet dropouts and network-induced delays. In such a framework, the synthesis procedure of dynamic output feedback controller is proposed for the networked systems with simultaneously considering multiple communication constraints.

IV. NUMERICAL EXAMPLE

In order to demonstrate the effectiveness of the proposed results, two examples are given.

Example 1: Consider the quadruple-tank process proposed in [42] and the schematic diagram of the process is shown in Figure. 2. The target of the networked controller is to remotely control the levels in Tank 1 and Tank 2 with two pumps. The process inputs are v_1 and v_2 that are input voltages to the pumps, and the outputs are y_1 and y_2 that are voltages from level measurement devices. The linearized state-space equation of the quadruple-tank process is given by

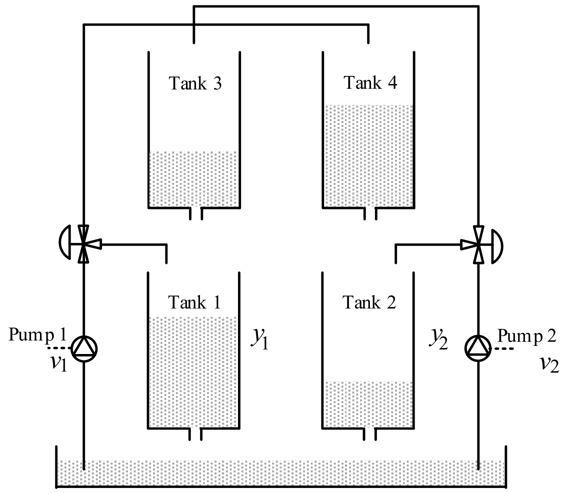

FIGURE 2. Schematic diagram of the quadruple-tank process.

$$\dot{x} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u,$$

$$y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x.$$

where A_i is the cross-section of Tank i, with the parameter values $A_1 = A_3 = 28cm^2$ and $A_2 = A_4 = 32cm^2$, α_i is the cross-section of outlet hole i, with the parameter values $\alpha_1 = 0.195cm^2, \alpha_2 = 0.167cm^2, \alpha_3 = 0.126cm^2$ and $\alpha_4 = 0.123cm^2$. The voltage applied to Pump i is v_i and the corresponding flow is $k_i v_i$. The parameters γ_1, $\gamma_2 \in (0, 1)$ are determined from the values set prior to an

$$\begin{bmatrix} -\lambda_2^{-2}Y & 0 & 0 & 0 & YA_m^T & \Xi_{16} & 0 & 0 \\ * & -\lambda_2^{-2}S & 0 & 0 & \Xi_{25} & \Xi_{26} & H^T & N^T \\ * & * & -\lambda_2^{-2}X & 0 & \Xi_{35} & \Xi_{35} & 0 & 0 \\ * & * & * & -\lambda_2^{-2}J & \Xi_{45} & \Xi_{46} & M^T & W^T \\ * & * & * & * & -Y & 0 & 0 & 0 \\ * & * & * & * & * & -J & 0 & 0 \\ * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & -X \end{bmatrix} < 0, \quad (36)$$

$$\begin{bmatrix} -\lambda_3^{-2}Y & 0 & 0 & 0 & YA_m^T & \Psi_{16} & 0 \\ * & -\lambda_3^{-2}S & 0 & 0 & 0 & 0 & H^T \\ * & * & (1-\lambda_3^{-2})X & 0 & \Psi_{35} & \Psi_{36} & 0 \\ * & * & * & -\lambda_3^{-2}J & 0 & 0 & M^T \\ * & * & * & * & -Y & 0 & 0 \\ * & * & * & * & * & -J & 0 \\ * & * & * & * & * & * & -S \end{bmatrix} < 0, \quad (37)$$

experiment. The flow to Tank 1 is $\gamma_1 k_1 v_1$ and the flow to Tank 4 is $(1-\gamma_1)k_1 v_1$ and similarly for Tank 2 and Tank 3. The acceleration of gravity is denoted by g, with the parameter values $g = 981cm/s^2$. The time constants are, respectively, $T_1 = 23s^{-1}$, $T_2 = 31s^{-1}$, $T_3 = 22s^{-1}$, $T_4 = 26s^{-1}$. The measured parameter is $k_c = 0.5V/cm$.

The purpose of this paper is to derive a dynamic output feedback controller to render the networked systems exponentially mean-square stability with the given decay rate. The sampling period T_p is taken as 2s and the network-induced delays are $\tau_k = 0.5$s, then the system dynamics after discretization is governed by the following parameters:

$$A = \begin{bmatrix} 0.5134 & 0 & 0.1437 & 0 \\ 0 & 0.7165 & 0 & 0.2388 \\ 0 & 0 & 0.8007 & 0 \\ 0 & 0 & 0 & 0.7165 \end{bmatrix},$$

$$B_{m0} = \begin{bmatrix} 0.0421 & 0.0096 \\ 0.0116 & 0.0278 \\ 0 & 0.1322 \\ 0.0967 & 0 \end{bmatrix}, B_{m1} = \begin{bmatrix} 0.0100 & 0.0063 \\ 0.0079 & 0.0078 \\ 0 & 0.0394 \\ 0.0272 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.5000 & 0 \\ 0 & 0.5000 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T.$$

This paper considers the worst case scenario that $p = 1$ and $q = 1$, i.e., only one actuator and one sensor can have access to channels at any time. Then, the actuators and sensors access sequences set are

$$\left\{M_\rho^1, M_\rho^2\right\} = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right\}, \left\{M_\sigma^1, M_\sigma^2\right\} = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right\}$$

In the simulation, assume the following transition matrices for the Markov chains in (4) and (5), respectively

$$\eta = \begin{bmatrix} 0.6250 & 0.3750 \\ 0.4530 & 0.5470 \end{bmatrix}, \pi = \begin{bmatrix} 0.4550 & 0.5450 \\ 0.3630 & 0.6370 \end{bmatrix}.$$

The stochastic communication sequences generated for the actuators and sensors are shown in Figure 3 and Figure 4, where ‘1’ and ‘2’ in the y-axis denote the actuators communication matrices L_1 or L_2, and the sensors communication matrices M_1 or M_2. Assuming that packet dropout probabilities are $\bar{\alpha} = \bar{\beta} = 0.2$. According to Theorem 1, we obtain the structural event rate ρ_r as follows:

$$\rho_1 = 0.04, \rho_2 = 0.64, \rho_3 = 0.16, \rho_4 = 0.16.$$

Select

$$\lambda_1 = 0.6524, \lambda_2 = 1.2000, \lambda_3 = 0.9288, \lambda_4 = 0.8247.$$

Substituting them into (19) yields

$$\lambda_1^{\rho_1}\lambda_2^{\rho_2}\lambda_3^{\rho_3}\lambda_4^{\rho_4} = 1.0586 > 1.$$

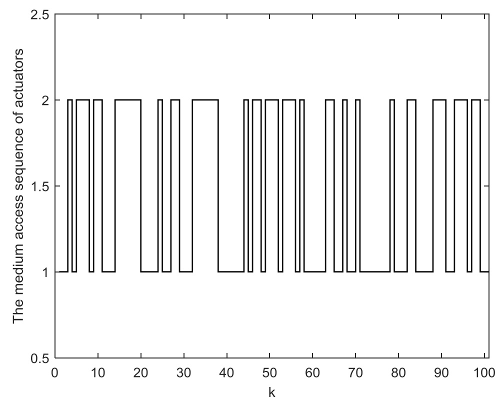

FIGURE 3. Medium access sequence of actuators.

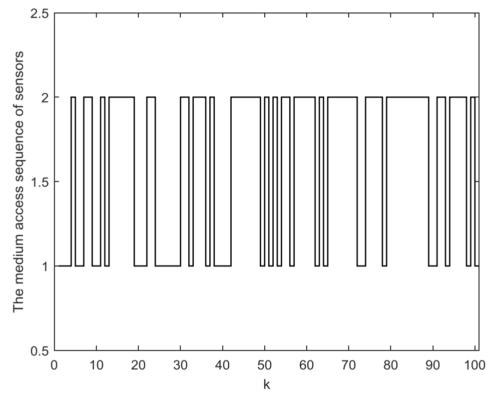

FIGURE 4. Medium access sequence of sensors.

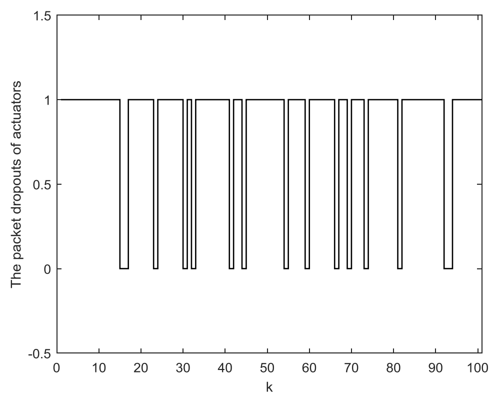

FIGURE 5. Packet dropouts of actuators.

The packet dropout curves of the actuators and sensors are shown in Figure 5 and Figure 6, where ‘1’ and ‘0’ in the y-axis denote the cases of the packet arrival and packet loss, respectively. Figure 7 shows subsystem switching trajectory of the closed-loop networked systems (13), where ‘1’, ‘2’, ‘3’ and ‘4’ in the y-axis indicate the subsystem of the asynchronous dynamic systems. According to Theorem 3, and using Matlab LMI Toolbox, the following dynamic output

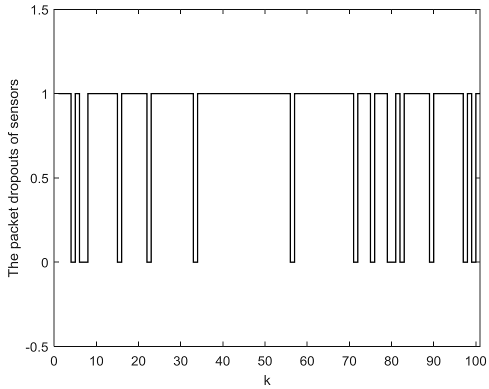

FIGURE 6. Packet dropouts of sensors.

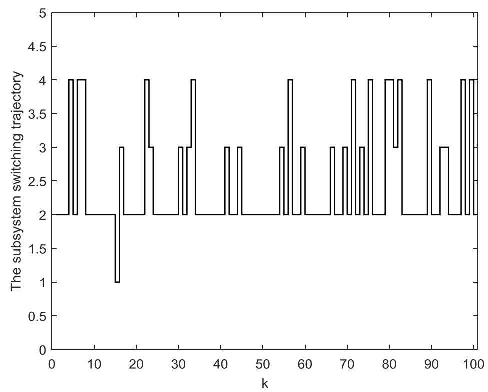

FIGURE 7. The subsystem switching trajectory.

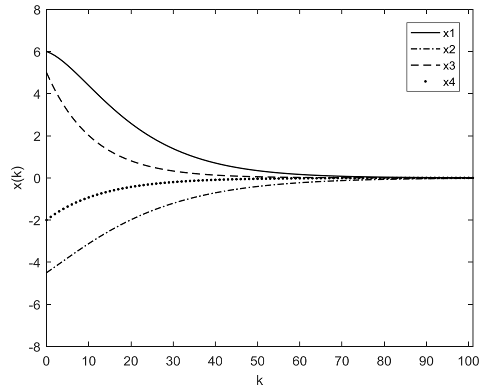

FIGURE 8. State trajectories of example 1.

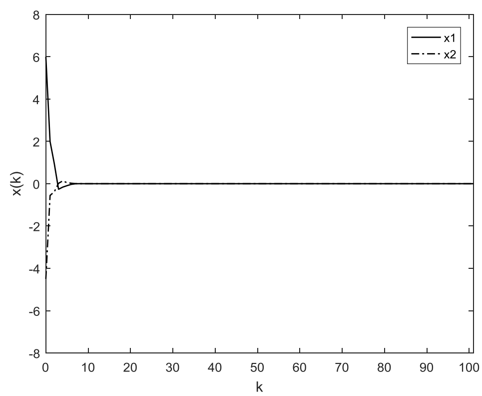

FIGURE 9. State trajectories of example 2.

feedback controller gains are obtained.

$$A_c = \begin{bmatrix} 0.2975 & 0.0052 & 0.0109 & 0.0036 \\ 0.0051 & 0.2954 & -0.0069 & -0.0036 \\ 0.0110 & -0.0070 & 0.3072 & -0.0013 \\ 0.0036 & -0.00360 & -0.0013 & 0.3086 \end{bmatrix},$$

$$B_c = \begin{bmatrix} 0.0425 & -0.0891 \\ 0.0206 & 0.1418 \\ 0.0633 & -0.0094 \\ 0.0420 & 0.0351 \end{bmatrix},$$

$$C_c = \begin{bmatrix} 0.1924 & -0.1314 \\ 0.0871 & 0.1872 \\ -0.1327 & 0.2566 \\ -0.0282 & 0.0633 \end{bmatrix}^T,$$

$$D_c = \begin{bmatrix} 0.2526 & 0.0023 \\ 0.0021 & 0.2618 \end{bmatrix}.$$

Applying Theorem 3, it is known that the closed-loop systems (13) is exponential stability with the decay rate 1.0586. Set the initial state $x(0) = [6 - 4.55 - 2]^T$. The corresponding state trajectories is depicted in Figure 8 under the given initial conditions. The simulation result has confirmed that the proposed dynamic output feedback controller performs very well.

Example 2: Next, consider a numerical example about a continuous-time plant in the form of (1) with the following parameters:

$$A = \begin{bmatrix} 0.0000 & 1.1200 \\ -1.2370 & -2.4000 \end{bmatrix}, B = \begin{bmatrix} 0.0000 & 1.3560 \\ 1.1080 & 0.0000 \end{bmatrix},$$

$$C = \begin{bmatrix} 1.2250 & 0.0000 \\ 0.0000 & 1.1360 \end{bmatrix}.$$

Similarly, the sampling period T_p is also taken as 2s, and the network-induced delays are $\tau_k = 0.5$s, then the system dynamics after discretization is governed by the following parameters:

$$A_m = \begin{bmatrix} 0.3265 & 0.2107 \\ -0.2307 & -0.1249 \end{bmatrix}, B_{m0} = \begin{bmatrix} 0.4665 & 1.5665 \\ 0.2804 & -0.6305 \end{bmatrix},$$

$$B_{m1} = \begin{bmatrix} 0.1368 & 0.2706 \\ -0.0720 & -0.1849 \end{bmatrix}, C = \begin{bmatrix} 1.2250 & 0.0000 \\ 0.0000 & 1.1360 \end{bmatrix}.$$

In the simulation, the parameters of packet arrival probabilities and access sequences are same as those of Example 1 with the initial state $x(0) = [6 - 4.5]^T$. The corresponding state trajectories are depicted in Figure 9 under the given initial conditions. The simulation result has confirmed that the proposed dynamic output feedback controller has excellent performance.

For comparison, the method from [38] is also simulated under the same conditions. The state trajectories are depicted in Figure 8, and Table 1 lists the summary of quadratic

TABLE 1. Comparison of quadratic performance index.

Control scheme	State index	Control index	The quadratic performance index
Our result	8.5834e+002	0.0036e+002	8.5798e+002
Guo [38]	9.1329e+002	0.0333e+002	9.1662e+002

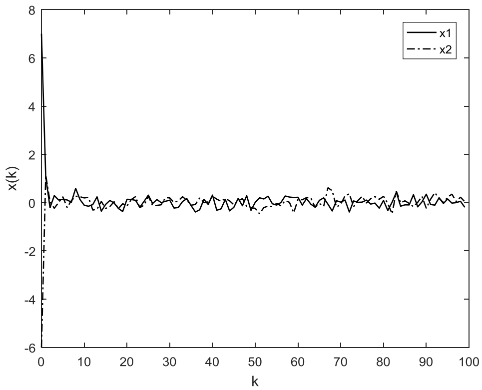

FIGURE 10. State trajectories of [38].

performance index with the state weighting matrix $Q = 10I_4$, and the control weighting matrix $R = I_4$ for the presented scheme in our paper as well as that of [38]. It can be seen from Figure 7, Figure 8 and Table 1 that the proposed dynamic output feedback controller can not only guarantee the networked systems to be exponential stability, but also have the better quadratic performance index than the method of [38]. Nevertheless, from the controller synthesis procedure, it is known that the suggested state feedback controller of [38] is dependent on the modes of the Markov chain and more easily solvable than the proposed control scheme.

V. CONCLUSION

In this paper, the design problem of dynamic output feedback controller has been investigated for a class of networked systems with medium access constraints, network-induced delays and packet dropouts. The case in which the deterministic time delay is less than one sampling period, two independent Markov chains are implemented to assign channel access to the sensors and actuators, and packet dropouts are described as i.i.d Bernoulli processes are taken into account. In such a framework, the closed-loop networked systems are modelled as a class of asynchronous dynamic systems. Based on Lyapunov stability theory and switching system theory, the dynamic output feedback controller is proposed to guarantee the exponential stability of closed-loop networked systems with the given decay rate. Finally, the effectiveness of the proposed method is verified by the numerical examples.

It is assumed in this paper that the network-induced delays are deterministic. Such an assumption might not fit some existing well-designed MAC protocols. Thus, future work will involve the controller synthesis for the NCSs with stochastic access constraints and random delays. Besides, since signal quantization and packet dropouts are common in network-based control systems, it is more interesting to develop the controller design by taking into simultaneous consideration of the effects of medium access constraints, packet dropouts, and signal quantization. This is another research direction in the future.

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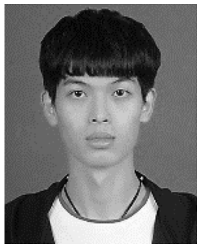

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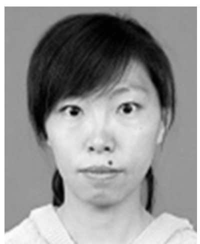

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