

Received July 17, 2021, accepted August 17, 2021, date of publication August 20, 2021, date of current version August 30, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3106269

A Hybrid Sparrow Search Algorithm Based on Constructing Similarity

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ABSTRACT Sparrow search algorithm (SSA) is easy to fall into local convergence and convergence stagnation. In order to solve these problems, this paper introduced Circle chaos map into the original SSA to improve its global search ability at the beginning of iteration. Meanwhile, it introduced T-distribution variation to affect the sparrow population position update rules in different iteration periods. Finally, we constructed the “similarity function” to measure the “dispersion” of the sparrow population, and formulated the search rules of the sparrow population under different “dispersion”. In order to test the specific optimization performance of the proposed algorithm, the test results of 54 test functions are compared with those of 9 other algorithms which are widely used, and then the test results are analyzed using non-parametric tests in statistics. At the same time, this paper introduces this algorithm into three concrete engineering test problems for testing. The results of these tests all prove that the proposed algorithm has stronger global optimization ability and higher convergence precision compared with other algorithms.

INDEX TERMS Sparrow search algorithm, T-distribution variation, circle chaotic map, intrusion detection, similarity function, adaptive algorithms.

I. INTRODUCTION

In recent years, with the wide application of swarm intelligence optimization algorithms in machine learning, a large number of bionic intelligent algorithms have been proposed and studied [1], [2]. For example, ant colony optimization (ACO) [3], manta ray foraging optimization (MRFO) [4], glowworm swarm optimization (GSO) [5], gray wolf optimization (GWO) [6], artificial ecosystem-based optimization (AEO) [7], shark smell optimization (SSO) [8], flamingo search algorithm (FSA) [9], and sparrow search algorithm (SSA). Among them, SSA was proposed in literature [10] in 2020. This is a new swarm intelligence optimization algorithm. SSA solves specific optimization problems by simulating the behavior of sparrows when foraging in nature. Compared with other intelligent optimization algorithms, SSA has the characteristics of high search accuracy, fast convergence speed, good stability and strong robustness [11]. However, SSA is extremely prone to local convergence and convergence stagnation in the later period of convergence [12]. These problems will directly affect the optimization effect of SSA, resulting in the failure to find the global optimal solution.

The associate editor coordinating the review of this manuscript and approving it for publication was Xujie Li¹.

The problem of SSA is also the concern of swarm intelligence optimization technology. Researchers try to influence the decision in the process of swarm optimization through mathematical techniques. Literature [13] proposed an enhanced whale optimization algorithm (EWOA), which embedding fractional-order chaotic mapping in the search process of EWOA to improve the search accuracy of EWOA. Literature [14] proposed a sinusoidal chaotic gravity search algorithm (SCGSA) as a further step for GSA to get rid of its locally optimal stagnation. Literature [15] proposed an evolutionary programming (EP) using mutations based on the T-probability distribution (TEP). T-probability distributions can be related to Gaussian and Cauchy probability distributions. Its variance can be changed by adjusting n degrees of freedom. Literature [16] proposed an adaptive inverse inertial non-particle swarm optimization algorithm based on simulated annealing to solve the problem that the inverse particle swarm optimization algorithm is easy to fall into local optimum.

On the improvement of the sparrow search algorithm, some of the latest studies include: Literature [17] proposes a lens-learning sparrow search algorithm (LLSSA) to improve the new sparrow search algorithm random and easy to fall into local optimal defects. In the finder stage, the algorithm introduces the reverse learning strategy based on

the lens principle to improve the search range of individual sparrows, and then proposes the variable spiral search strategy to make the follower's search more detailed and flexible. Finally, the simulated annealing algorithm is used to determine the optimal solution. Literature [12] proposes an improved sparrow search algorithm (ISSA) sparrows, this method USES the center of gravity reverse learning mechanism to initialization of population, the population has better spatial distribution, and in the position of the discoverer update part introduces learning coefficient, improve the global search ability of the algorithm. At the same time, mutation operator is used to improve the sparrow position update formula, so as to avoid the local convergence problem.

Inspired by the literature mentioned above, this paper proposes an adaptive sparrow search algorithm (CSSA) based on improved Circle chaos mapping, T-distribution variation and similarity to solve the key problem of local convergence of group optimization algorithm. Currently the sparrow research mainly is to solve the problem of local convergence algorithm, the solution of these problems lies in on the sparrow population of all the sparrow diffusion treatment, but no specific judgment on whether you need a sparrow diffusion, in order to solve this problem and combine the characteristics of the sparrow algorithm itself, this paper constructs a function to measure the clustering and scattered between individuals in a population. That is, the "Similarity function" and then select the optimal strategy through the similarity value. The over-aggregated sparrows were treated with diffusion. The improved Circle chaos map proposed in this paper was used to deal with the position of the sparrows with the general aggregation degree, and the T-distribution variation was used to deal with the position of the remaining sparrows, while the original position remained unchanged. This method is mainly used to solve the problem that the sparrow algorithm is easy to fall into local convergence, which greatly improves the convergence speed of the algorithm and effectively avoids the problem of convergence stagnation.

The main structure of this paper is as follows: In Section II, we describe the basic principles of the original SSA algorithm. In Section III, an improved Circle chaos map and T-distribution variation are proposed, and how these two technologies are applied to the SSA algorithm is deduced and constructed. Finally, the 'similarity function' is constructed to judge the "clustering and dispersion" of individual sparrows. The pseudocode, algorithm flow and computational complexity of CSSA are also discussed. In Section IV, CSSA and four optimization algorithms are tested and compared. Nine benchmark functions with different characteristics are selected for the test function, and the results are discussed. In Section V, we compared CSSA with the ten latest optimization algorithms, and CEC-2015 and CEC-2017 were selected as the test functions. Finally, Wilcoxon Sign-Rank test was conducted on the test results. In Section VI and Section VII, CSSA is introduced into two specific engineering problems

for testing, and the test results of other algorithms are compared. Finally, in Section VIII, we combine CSSA algorithm with SVM model to construct a CSSA-SVM system, which is used as a network intrusion detection system. Compared with other detection systems, this detection system has better detection accuracy. The above test results all show that CSSA and SSA can better solve the local convergence problem, so as to find the global optimal solution.

II. SPARROW SEARCH ALGORITHM

SSA can be simply abstracted into a finder-address-early-warning model. In a D -dimensional search space, if there are N sparrows, the position of the first sparrow in the D -dimensional search space is $X_i = [x_{i1}, \dots, x_{id}, \dots, x_{iD}]$, where $i = 1, 2, \dots, n$, x_{id} represents the position of the i th sparrow in the d th dimension. Finders generally account for 10%-30% of the population. The location update formula is shown in (1).

$$x_{id}^{t+1} = \begin{cases} x_{id}^t \times \exp\left(\frac{-i}{\alpha \times T}\right), & R_2 < ST \\ x_{id}^t + Q \times L, & R_2 \geq ST \end{cases} \quad (1)$$

where, t represents the current iteration number; T is the maximum number of iterations; Is is the uniform random number between 0 and 1; Q is a random number following the standard normal distribution; L represents a matrix of size and elements of 1; $R_2 \in [0,1]$ and $ST \in [0.5,1]$ represent the warning value and the safety value respectively. When $R_2 < ST$, the finder can be widely searched; When $R_2 < ST$, the warning sparrow spots a predator, the finder quickly moves toward safety.

Equation (2), as shown at the bottom of the next page, is the update strategy of entrants. All sparrows in this population that are not finders are entrants. Where: xw_d^t represents the sparrow's worst position in the d dimension when the population is iterated in the t th iteration; xb_d^{t+1} represents the optimal position of sparrow in d dimension in the $(t + 1)$ th iteration of the population.

$$x_{id}^{t+1} = \begin{cases} xb_d^t + \beta(x_{id}^t - xb_d^t), & f_i \neq f_g \\ x_{id}^t + K \left(\frac{x_{id}^t - xw_d^t}{|f_i - f_w| + e} \right), & f_i = f_g \end{cases} \quad (3)$$

Equation (3) is the updating formula of the position of the early warning. In a sparrow population, 20% of the sparrows are generally the early warning sparrows, β representing the step size control parameters and obeying the normal distribution random numbers with a mean of 0 and a variance of 1. K is a random number between -1 to 1, and it is the step size control parameter; e is a minimal constant; f_i is the fitness value of the i th sparrow, f_g and f_w are the optimal and the worst fitness values of the current sparrow population, respectively. When $f_i \neq f_g$, indicates that the sparrow is at the edge of the population, vulnerable to predator attack. When $f_i = f_g$, indicates that the sparrow will get close to other sparrows in time.

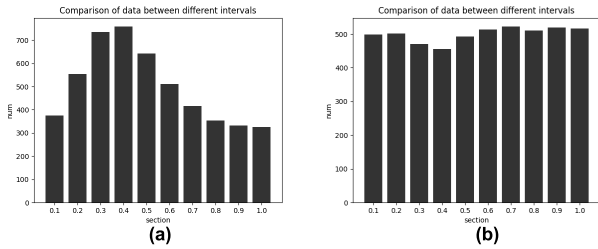


FIGURE 1. The distribution interval of the circle map before(a) and after(b) improvement.

III. ADAPTIVE HYBRID SPARROW ALGORITHM BASED ON THE CONSTRUCTION OF "CONVERGENCE AND DIVERGENCE" SIMILARITY

A. IMPROVED CIRCLE CHAOS MAPPING

At present, there have been a large number of studies on the application of chaotic mapping to the optimization of swarm intelligence algorithms [18]. Circle map is an excellent mapping function with good order, existence and uniqueness [19]. In this paper, the problems existing in the chaos mapping of Circle are mainly improved, and a new improved Circle mapping is constructed to enhance its randomness, so that it can overcome the mapping inequality problems existing in the chaos mapping of Circle. The original formula of chaos mapping of Circle is:

$$C_{i+1} = \text{mod}(C_i + a - (\frac{b}{2\pi}) \sin(2\pi C_i), 1), \quad (4)$$

where, a and b are both mapping parameters, generally $a = 0.2$ and $b = 0.5$ respectively. The distribution diagrams of sequences generated in 5000 iterations in different intervals are shown in Fig. 1a:

It can be clearly seen from Fig. 1a that the values formed by the original Circle map mostly concentrate between 0.2 and 0.6, and their probabilities vary at different values. In order to enhance its randomness, the original chaotic mapping equation of Circle is improved in this paper, and a new mapping function is constructed as follows:

$$C_{i+1} = \text{mod}(C_i + a - (\frac{b}{4\pi}) \sin(2\pi C_i) - (\frac{b}{4\pi}) \cos(2\pi C_i) + \text{random}(0, 1), 1) \quad (5)$$

where, a and b are parameters, generally $a = 0.2$, $b = 0.5$; $\text{random}(0,1)$ is a random number between 0 and 1. The interval distribution diagram of the improved Circle chaotic map is shown in Fig. 1b. Compared with the original Circle chaotic map, the distribution is more uniform and the value is more random. It can be clearly seen from the scatter diagram

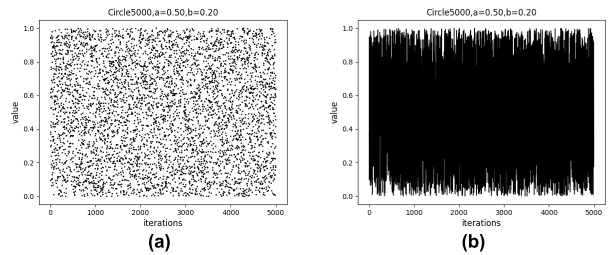


FIGURE 2. Distribution scatter diagram(a) and distribution sequence diagram(b).

(see Fig. 2a) and time sequence diagram (see Fig. 2b) of the map distribution.

The Circle chaotic map constructed above is introduced into the sparrow algorithm to ensure the uniformity and randomness of the sparrow population distribution. Then, the initial value of the i th dimension of the sparrow is shown in (6):

$$x_{ij} = lb + (ub - lb) \times C_i \quad (6)$$

In (6), lb is the lower limit of the search space and ub is the upper limit of the search space. Then the position of the sparrow can be obtained by taking the sequence of length D (D is the dimension of the goal problem).

B. T-DISTRIBUTION VARIATION

Here, this paper introduces the T-distribution in statistics [20]. T-distribution is a special distribution function, which contains parameter degree of freedom n . The smaller n is, the flatter the curve is. Its equation is as follows.

$$f(x, n) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi} \Gamma(\frac{n}{2})} (1 + \frac{x^2}{n})^{-\frac{n+1}{2}}, -\infty < x < +\infty \quad (7)$$

At that time, the T-distribution curve is the Cauchy distribution curve, that is $t(n = 1) = C(0,1)$, $C(0,1)$ is the Cauchy distribution as shown in (8):

$$f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < +\infty \quad (8)$$

The larger n is, the closer the curve is to the normal distribution curve. In the case of $n \rightarrow \infty$, the T-distribution curve is approximately a gaussian distribution curve, this is $t(n \rightarrow \infty) \rightarrow N(0,1)$, where $N(0,1)$ is a Gaussian distribution, as shown in (9):

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2}), -\infty < x < +\infty \quad (9)$$

$$x_{id}^{t+1} = \begin{cases} Q \times \exp(\frac{xw_d^t - x_{id}^t}{i^2}), & i > \frac{n}{2} \\ xb_d^{t+1} + \frac{1}{D} \sum_{d=1}^D (\text{rand}\{-1, 1\} \times |x_{id}^t - xb_d^{t+1}|), & i \leq \frac{n}{2} \end{cases} \quad (2)$$

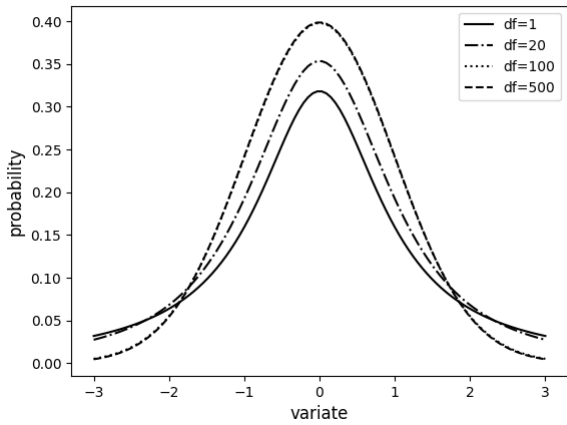


FIGURE 3. Variation of t distribution with different values of n .

In other words, with the change of n value, the distribution curve of T-distribution will gradually approach to Cauchy distribution or Gaussian distribution, as shown in Fig. 3. It can be seen that the Cauchy mutation is more likely than the Gaussian mutation to produce the next generation points far away from the parent. According to the unique properties of T-distribution, this paper proposes the following adaptive sparrow update strategy based on the number of iterations and the fitness value of each sparrow:

In this paper, the T-distribution idea is integrated into the finder update strategy of the sparrow algorithm, as shown in the following equation.

$$x_{id}^{t+1} = \begin{cases} x_{id}^t \cdot \exp(\frac{-i}{\alpha \times T}), & R_2 < ST \\ x_{id}^t + t(t) \times L, & R_2 \geq ST \end{cases} \quad (10)$$

In the reconstructed finder update (10), denotation obeys the T-distribution with T of freedom. When the warning value is greater than the safety value, the population is more inclined to search in the form of Cauchy distribution probability at the early stage of the search, so as to ensure that the finder, as the sparrow with the better position in the whole population, increases the local search ability at the later stage of the iteration, and increases the global search ability at the early stage of the iteration.

At the same time, this paper improves the renewal strategy of participants. As shown in (11), if the sparrow's fitness f_i is greater than the average fitness f_{avg} , it means that the sparrow needs to fly to other locations to search for energy. If the sparrow's fitness f_i is less than or equal to the average fitness f_{avg} , then the sparrow will randomly search for a position near the current optimal position for foraging. At the same time, this paper introduces the T-distribution and the improved Circle mapping, namely C_{i+1} in (11), so as to increase the randomness of its value, where, $rand \{-1,1\}$ represents the random value -1 or 1, and $t(t)$ represents the T-distribution with the degree of freedom of t .

After a round of position updating of the whole sparrow population, T- distribution variation was carried out for some

sparrows with better fitness based on sparrow $x_{i,j}^{old}$ with better fitness. The variation formula was as follows: (12).

$$x_{i,j}^{new} = x_{i,j}^{old} + t(n) \times x_{i,j}^{old} = x_{i,j}^{old} (1 + t(n)) \quad (12)$$

where, n is the current number of iterations, and $t(n)$ represents a T-distribution with n degrees of freedom. Therefore, in the initial stage of optimizing the algorithm, the above formula is also inclined to Cauchy distribution variation, and its variation is more likely to produce the next generation far away from the parent. When the algorithm enters the later stage of optimization, the formula will be inclined to Gaussian distribution variation, and the algorithm will have good local variation performance.

C. SIMILARITY FUNCTION

This paper constructs a new similarity function based on cosine similarity and Euclidean similarity. In this paper, a certain point in the search space is selected as the reference point to calculate the similarity. That is, under the condition that the upper limit of the search space is ub and the lower limit is lb , the coordinate of the reference point determined is $M (ub, ub, \dots, ub)$ and the coordinate of the edge point is $B (lb, lb, \dots, lb)$.

Euclidean similarity is derived from the distance formula between two points in Euclidean space, which measures the absolute distance between each point in multi-dimensional space [21]. Given that the position coordinate of the sparrow is X_i , the Euclidean similarity of this point is as shown in (13).

$$D_i(X_i, M) = \frac{\|X_i - X_j\|}{\|B - M\|} \quad (13)$$

Cosine similarity uses the cosine value of the angle between two vectors in the vector space as the size of the difference between two individuals [22], which pays more attention to the difference between two vectors in direction rather than in distance or length. The cosine similarity after normalization in the sparrow algorithm is shown in (14).

$$C_i(X_i, M) = \frac{1}{2} (1 - \frac{\vec{X}_i \cdot \vec{M}}{\|X_i\| \cdot \|M\|}) \quad (14)$$

Then, the overall similarity calculation method of the i th sparrow is shown in (15) and (16).

$$\rho(x_i) = \sum_{j=1}^M \begin{cases} 1, & M_l \leq \frac{D_i}{D_j} \leq M_u \text{ and } M_l \leq \frac{C_i}{C_j} \leq M_u \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

$$d(x_i) = \frac{\rho(x_i)}{M}, i = 1, 2, 3, \dots, M \quad (16)$$

In (15), M_l and M_u are the parameters, and the values of 0.85 and 1.15 are the optimal after testing. $\rho(x_i)$ is how many sparrows in the population are in a position similar to the i th sparrow. When the sparrow and a sparrow are similar in all dimensions, it is judged that they are similar, and the value is 1; Otherwise, it's 0. $d(x_i)$ is the similarity of the i th sparrow.

D. ALGORITHM FLOW

Aiming at the problem that the SSA is easy to fall into local convergence, this paper introduces the improved Circle chaos map and the improved T-distribution variation, and adds the similarity function constructed for the SSA to form the CSSA, which has a strong global optimization ability. The specific algorithm process is as follows:

Step1: Introduce initial parameters: population size N , finder ratio FR , early-warning ratio WR , dimension M of objective function, maximum number of iterations T , upper bound ub and lower bound lb of search range. The improved Circle chaotic mapping (5) is used to generate N m-dimensional vectors.

Step2: Initialize sparrow population through (6).

Step3: f_i of each sparrow and f_{avg} of the average fitness of the whole population were calculated. Sparrows with the top FR in the fitness ranking were regarded as finders, and the rest were considered as participants. Their distribution was updated according to (10) and (11), as shown at the bottom of the page, and $FR \diamond N$ sparrows were randomly selected to update the positions of early warning sparrows according to (3).

Step4: If $f_i > f_{avg}$, then calculate the similarity value of each sparrow according to (13)-(16), and calculate the average similarity value of the whole sparrow population d_{avg} ; if $d_i > d_{avg}$, introduce the chaotic mapping of improved Circle to diffuse it. If the similarity value of the i th sparrow is less than the average similarity value ($d_i \leq d_{avg}$), then the t-distribution variation is carried out for it according to (12).

Step5: If $f_i \leq f_{avg}$, keep the sparrows in this part to enter the next iteration.

Step6: Boundary detection.

Step7: Whether the ending condition is met. No: go to Step3; Yes: output the optimal sparrow position and the optimal solution.

The algorithm flow chart is shown in Fig. 4.

E. ALGORITHM PSEUDO-CODE

This sub-section shows pseudo-code of CSSA.

F. COMPLEXITY ANALYSIS OF CSSA

This subsection analyzes the complexity of the CSSA. The time complexity and space complexity of CSSA are described separately below.

1) TIME COMPLEXITY

Initializing the population takes $O(n \diamond d)$ time, where n is the population size and d is the dimension size. The

$O(Iter_{Max} \diamond n \diamond d)$ time required to find the fitness of each sparrow, where $Iter_{Max}$ is the maximum number of iterations.

2) SPACE COMPLEXITY

The spatial complexity of the CSSA is $O(n \diamond d)$, where n is the population size and d is the dimension size, that is, the maximum amount of space occupied when initializing the population.

IV. BENCHMARK FUNCTION TESTING

A. SELECTION OF BENCHMARK FUNCTIONS

In order to test the optimization ability of the improved algorithm in different functions, as well as the feasibility of the algorithm and the efficiency of optimization, this paper selects 9 different types of benchmark functions and carries out simulation on their selection of different dimensions to verify the optimization ability of the algorithm in low and high dimensional space, as shown in Table 1.

B. EXPERIMENTAL ENVIRONMENT

The experiments in this paper are both carried out on a computer configured as Intel(R) Core (TM) i7-10750H CPU @ 2.60GHz 2.59GHz, with a memory size of 16GB and an operating system of Window10. The code part of the experiment was written, run and tested by Python. The version of Python was 3.8.3, the development tool PyCharm version was 2020.3.3, and the mapping tool of the benchmark function was MATLAB, the version was 2018a.

C. IMPROVE THE SSA ALGORITHM TO TEST THE BENCHMARK FUNCTION

In order to verify the specific optimization performance of the CSSA constructed above, nine benchmark functions with different properties were selected in this section to test them in different dimensions (as shown in Table 1). At the same time, the popular PSO, GWO, WOA, and SSA before improvement were selected to optimize the nine benchmark functions in Table 1. The number of fixed iterations is 200, and the population is 50. In order to avoid the contingency of single optimization results, this paper carried out a total of ten tests, and took the optimal values of these test results, and calculated the average value(see AVG), standard deviation(see STD), and average running time(see Runtime). Parameter settings are shown in Table 2.

The test results are shown in Table 3: Through comparison, it can be found that the CSSA has better optimal value than the original SSA, GWO, PSO and WOA in terms of algorithm optimization performance, no matter in low dimensions (F1, F2, F4, F5, F8, and F9) or in high dimensions (F3, F6, and F7).

$$x_{id}^{t+1} = \begin{cases} t(t) \times \exp\left(\frac{xw_d^t - x_{id}^t}{t^2}\right), & f_i > f_{avg} \\ xb_d^{t+1} + \frac{1}{D} \sum_{d=1}^D (rand\{-1, 1\} \times C_{i+1} \times |x_{id}^t - xb_d^{t+1}|), & f_i \leq f_{avg} \end{cases} \quad (11)$$

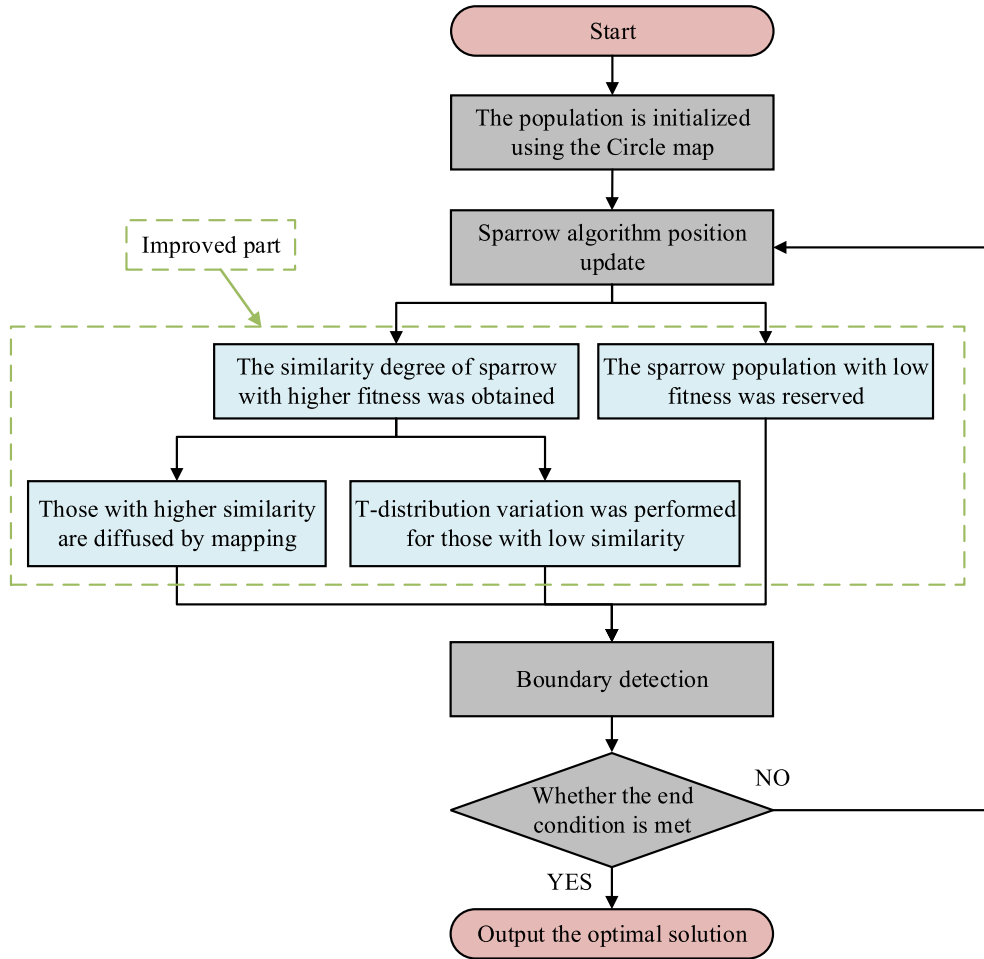


FIGURE 4. CSSA flow chart.

TABLE 1. Selection of benchmark functions.

FUNC	Benchmark function	Function formula	The search space	Dimension	Best	Features
F1	Sphere	$f(x) = \sum_{i=1}^D x_i^2$	[-100,100]	20	0	Unimodal
F2	Rosenbrock	$f(x) = \sum_{i=1}^{D-1} [100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2]$	[-2.48,2.48]	2	0	Unimodal
F3	Ackley	$f(x) = -20 \exp(-0.2 \sqrt{1/D \times \sum_{i=1}^D x_i^2}) - \exp(1/D \times \sum_{i=1}^D \cos(2\pi x_i)^2) + e$	[-32,32]	20	0	Multimodal
F4	Schwefel	$f(x) = 418.4829D - \sum_{i=1}^D x_i \sin \sqrt{ x_i }$	[-500,500]	2	0	Multimodal
F5	Schaffer	$f(x) = 0.5 + ((\sin \sqrt{\sum_{i=1}^D x_i^2})^2 - 0.5) / [1 + 0.001(\sum_{i=1}^D x_i^2)^2]$	[-10,10]	5	0	Multimodal
F6	Rastrigin	$f(x) = \sum_{i=1}^D [x_i^2 - 10 \cos(2\pi x_i) + 10]$	[-5.12,5.12]	20	0	Multimodal
F7	Griewank	$f(x) = 1 + 1/4000 \sum_{i=1}^D x_i^2 - \prod_{i=1}^D \cos(x_i / \sqrt{i})$	[-600,600]	20	0	Multimodal
F8	Three-Hump	$f(x) = 2x_1^2 - 1.05x_1^4 + x_1^6/6 + x_1x_2 + x_2^2$	[-5,5]	2	0	Fixed dimension
F9	Cross-in-Tray	$f(x) = -0.0001 \left(\left \sin(x_1) \sin(x_2) \exp \left(\left 100 - \sqrt{x_1^2 + x_2^2} / \pi \right \right) \right + 1 \right)^{0.1}$	[-10,10]	2	-2.06261	Fixed dimension

It performs well in both single-modal functions (F1 and F2), multi-modal functions (F3, F4, F5, F6, and F7), and fixed-dimensional functions (F8 and F9), among which the

CSSA for F2, F3, and F4 functions is 2-3 orders of magnitude better than the original algorithm. On average, F1, F5, F6, and F7 will reach the optimal value 50-80 iterations ahead

of the original algorithm, which greatly improves the convergence speed and accuracy of the algorithm. Fig. 5 shows a line chart comparing the change trend of the optimal value of the five algorithms in the process of searching for the optimal function from F1 to F9 by randomly selecting one test result out of ten tests conducted by the five algorithms.

V. TEST OF CEC-2015 AND CEC-2017 TEST FUNCTIONS

In this section, I will test and compare CSSA with ten popular swarm intelligence algorithms at the present stage. The test function set I choose is the CEC-2015 data set and the CEC-2017 data set. These ten algorithms are as follows: Particle swarm Optimization (PSO) [23], Whale Optimization Algorithm (WOA) [24], Grey Wolf Optimizer (GWO), Tunicate Swarm Algorithm (TSA) [25], Butterfly Optimization Algorithm (BOA) [26], Satin bowerbird Optimization (SBO) [27], Pigeon Inspired Optimization (PIO) [28], Improved Sparrow Search Algorithm (ISSA), Lens Learning Sparrow Search Algorithm (LLSSA), and Sparrow search algorithm (SSA). We conducted non-parametric Test on the Test results, and the Wilcoxon Sign-Rank Test was selected in this paper. The results are presented in subsection C.

A. EVALUATION OF IEEE CEC-2015 TEST FUNCTIONS

Table 4 is the CEC-2015 benchmark function set. Parameter selection of each algorithm in our test is shown in Table 2, the maximum number of iterations is 1000, the dimension size is shown in Table 4, and the population number is set to 50. The test results are shown in Table 6. As can be seen from Table 6, when testing 9 benchmark functions such as CEC-3, CEC-5, CEC-6, CEC-8, CEC-9, CEC-11, CEC-12, CEC-13, and CEC-15, the test results of CSSA are better than those of the other algorithms. PSO optimizes best when testing functions CEC-3, CEC-4, CEC-7, and CEC-10. Combined with these test results, CSSA is undoubtedly the best optimization algorithm.

B. EVALUATION OF IEEE CEC-2017 TEST FUNCTIONS

Table 5 shows the benchmark functions for CEC-2017. Parameter selection of each algorithm in our test is shown in Table 2, the maximum number of iterations is 1000, the dimension size is shown in Table 5, and the population number is set to 50. The test results are shown in Tables 7 and 8. We can see that the test results of 15 of the 30 test functions of CSSA are better than those of the other four optimization algorithms. These functions are as follows: CEC-1, CEC-2, CEC-5, CEC-6, CEC-7, CEC-9, CEC-12, CEC-13, CEC-15, CEC-18, CEC-19, CEC-24, CEC-25, CEC-27, and CEC-30. PSO has 10 test functions that perform better than the other algorithms, these functions are: CEC-10, CEC-14, CEC-16, CEC-17, CEC-20, CEC-21, CEC-23, CEC-26, CEC-27, and CEC-29. Combined with these test results, CSSA is undoubtedly the best optimization algorithm.

TABLE 2. Parameter settings.

Algorithm	Parameters	Values
PSO	Inertia coefficient w	0.9
	Parameter c_1	2
	Parameter c_2	2
WOA	Control parameter a_1	[2, 0]
	Control parameter a_2	[-1, -2]
GWO	Control parameter a	[2, 0]
TSA	Parameter P_{min}	1
	Parameter P_{max}	4
BOA	Conversion probability p	0.8
	The power α	[0.1, 0.3]
	Feeling factor c	0.01
SBO	Greatest step size α	0.94
	Mutation probability p	0.04
	Scaling factor z	0.02
PIO	Landmark operator N_{c1}	Round ($Max_{iter} \times 0.7$)
	Compass operator N_{c2}	$Max_{iter} - N_{c1}$
ISSA	The number of the producers	70%
	The sparrow is aware of the danger of gravity	20%
	The early warning value	0.6
LLSSA	The number of the producers	70%
	The sparrow is aware of the danger of gravity	20%
	The early warning value	0.6
	L	Random [-1,1]
SSA	Coefficient of change K	5
	The number of the producers	70%
	The sparrow is aware of the danger of gravity	20%
CSSA	The early warning value	0.6
	The number of the producers	70%
	The sparrow is aware of the danger of gravity	20%
	The early warning value	0.6
	M_l	0.85
	M_u	0.15

C. ANALYSIS OF STATISTICAL SIGNIFICANCE

Wilcoxon Signed-Rank Test (WSRT), as a nonparametric test, can effectively assess statistical significance difference between two optimizers. The statistical results of WSRT on 45 benchmark functions in 30 runs are presented in Tables 9 to 12, where T+ and T- are calculated and their p-values can be obtained. “=” indicates the case in which there is no significance difference between CSSA and its competitor, “+” indicates that CSSA performs worse than the comparison algorithm at the 95% significance level ($\alpha = 0.05$), and “-” indicates that CSSA performs better than the comparison algorithm. We can clearly see from the table that the optimization effect of CSSA is obviously better than other

TABLE 3. Comparison of optimization results of SSA, CSSA, GWO, PSO, and WOA.

FUNC	Attribute	SSA	CSSA	GWO	PSO	WOA
F1	AVG	5.62E-79	0.00E+00	1.01E-15	3.00E+00	1.07E-09
	STD	9.35E-78	0.00E+00	3.82E-14	8.75E+00	5.24E-09
	Runtime	0.493s	1.370s	1.588s	1.024 s	0.970 s
F2	AVG	3.60E-03	2.28E-05	3.85E-05	9.34E-03	2.69E-03
	STD	9.01E-03	6.14E-04	6.01E-05	3.25E-02	4.39E-03
	Runtime	0.296 s	0.927s	0.390 s	0.236s	0.171 s
F3	AVG	1.32E-08	7.69E-11	5.29E-08	3.17E+00	5.78E-05
	STD	2.81E-07	3.24E-11	8.93E-07	2.85E+00	3.73E-05
	Runtime	0.425s	1.483s	2.491 s	1.272 s	1.462 s
F4	AVG	4.06E-02	3.78E-04	1.32E-02	1.45E-03	9.36E-02
	STD	8.34E-02	7.18E-04	4.96E-02	5.76E-03	1.01E-01
	Runtime	0.166 s	0.447s	0.208 s	0.195 s	0.142 s
F5	AVG	0.00E+00	0.00E+00	2.12E-03	2.21E-03	2.31E-03
	STD	0.00E+00	0.00E+00	2.92E-03	2.77E-03	2.84E-03
	Runtime	0.277 s	0.823s	0.512 s	0.362s	0.312 s
F6	AVG	0.00E+00	0.00E+00	2.47E-08	1.17E+02	3.74E-03
	STD	0.00E+00	0.00E+00	7.20E-08	1.21E+02	8.45E-03
	Runtime	0.322 s	1.135s	1.627 s	0.853 s	1.001 s
F7	AVG	0.00E+00	0.00E+00	1.04E-14	6.22E-01	7.24E-10
	STD	0.00E+00	0.00E+00	5.68E-13	8.29E-01	1.53E-09
	Runtime	0.882 s	2.128s	2.474 s	1.154s	1.351s
F8	AVG	2.14E-238	1.06E-301	6.91E-111	1.90E-06	1.04E-83
	STD	0.00E+00	0.00E+00	5.41E-89	8.51E-06	6.00E-75
	Runtime	0.167s	0.660s	0.202 s	0.173 s	0.125 s
F9	AVG	-2.06E+00	-2.06E+00	-2.06E+00	-2.06E+00	-2.06E+00
	STD	9.36E-11	1.26E-07	5.67E-07	7.48E-06	4.91E-03
	Runtime	0.156 s	0.608s	0.201 s	0.176 s	0.131 s

TABLE 4. IEEE CEC-2015 benchmark test functions.

No.	Functions	Related basic functions	Dim	f_{min}
CEC-1	Rotated Bent Cigar Function	Bent Cigar Function	30	100
CEC-2	Rotated Discus Function	Discus Function	30	200
CEC-3	Shifted and Rotated Weierstrass Function	Weierstrass Function	30	300
CEC-4	Shifted and Rotated Schwefel's Function	Schwefel's Function	30	400
CEC-5	Shifted and Rotated Katsuura Function	Katsuura Function	30	500
CEC-6	Shifted and Rotated HappyCat Function	HappyCat Function	30	600
CEC-7	Shifted and Rotated HGBat Function	HGBat Function	30	700
CEC-8	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	Griewank's Function; Rosenbrock's Function	30	800
CEC-9	Shifted and Rotated Expanded Scaffer's F6 Function	Expanded Scaffer's F6 Function	30	900
CEC-10	Hybrid Function 1 ($N = 3$)	Schwefel's Function; Rastrigin's Function; High Conditioned Elliptic Function	30	1000
CEC-11	Hybrid Function 2 ($N = 4$)	Griewank's Function; Weierstrass Function; Rosenbrock's Function; Scaffer's F6 Function	30	1100
CEC-12	Hybrid Function 3 ($N = 5$)	Katsuura Function; HappyCat Function; Schwefel's Function; Expanded Griewank's plus Rosenbrock's Function; Ackley's Function	30	1200
CEC-13	Composition Function 1 ($N = 5$)	Rosenbrock's Function; Bent Cigar Function; Discus Function; High Conditioned Elliptic Function; High Conditioned Elliptic Function	30	1300
CEC-14	Composition Function 2 ($N = 3$)	Schwefel's Function; Rastrigin's Function; High Conditioned Elliptic Function	30	1400
CEC-15	Composition Function 3 ($N = 5$)	HGBat Function; Rastrigin's Function; Schwefel's Function; Weierstrass Function; High Conditioned Elliptic Function	30	1500

algorithms in the whole test function set. Compared with SSA, ISSA, and LLSSA, the optimization result of CSSA is obviously a global optimization algorithm. These results also prove that CSSA can better solve the local convergence problem.

VI. SPEED REDUCER DESIGN PROBLEM

The main purpose of this engineering design problem is to minimize the weight of the reducer [29], as shown in Fig. 6.

When dealing with this problem, the following requirements need to be met [30]: bending stress of the gear teeth, surface stress, transverse deflections of the shafts, stresses in the shafts.

The design problem of this project has seven design variables: face width (b), module of teeth (m), number of teeth in the pinion (p), length of the first shaft between bearings (l_1), length of the second shaft between bearings (l_2), diameter of first (d_1) shafts, and diameter of second shafts (d_2). The

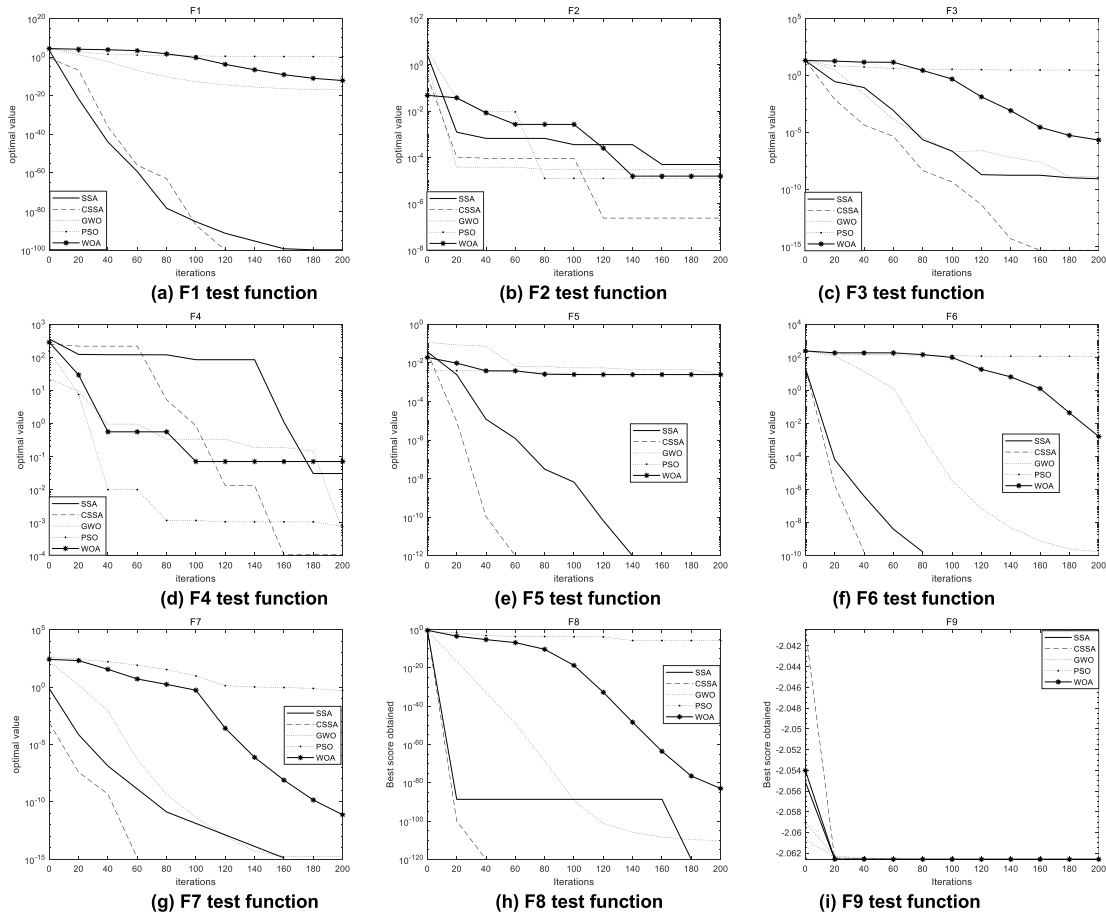


FIGURE 5. Convergence plot (F1-F9).

specific mathematical formula for this engineering design problem is as follows:

Consider:

$$\vec{z} = [z_1 \ z_2 \ z_3 \ z_4 \ z_5 \ z_6 \ z_7]$$

$$= [b \ m \ p \ l_1 \ l_2 \ d_1 \ d_2],$$

Minimize:

$$f(\vec{z}) = 0.7854z_1z_2^2(3.3333z_3^2 + 14.9334z_3 - 43.0934) - 1.508z_1(z_6^2 + z_7^2) + 7.4777(z_6^3 + z_7^3) + 0.7854(z_4z_6^2 + z_5z_7^2)$$

Subject to:

$$g_1(\vec{z}) = \frac{27}{z_1z_2^2z_3} - 1 \leq 0, \quad g_2(\vec{z}) = \frac{397.5}{z_1z_2^2z_3} - 1 \leq 0,$$

$$g_3(\vec{z}) = \frac{1.93z_4^3}{z_2z_6^2z_3} - 1 \leq 0, \quad g_4(\vec{z}) = \frac{1.93z_5^3}{z_2z_7^2z_3} - 1 \leq 0,$$

$$g_5(\vec{z}) = \frac{[(745(z_4/z_2z_3))^2 + 16.9 \times 10^6]^{1/2}}{110z_6^3} - 1 \leq 0,$$

$$g_6(\vec{z}) = \frac{[(745(z_5/z_2z_3))^2 + 157.5 \times 10^6]^{1/2}}{85z_7^3} - 1 \leq 0,$$

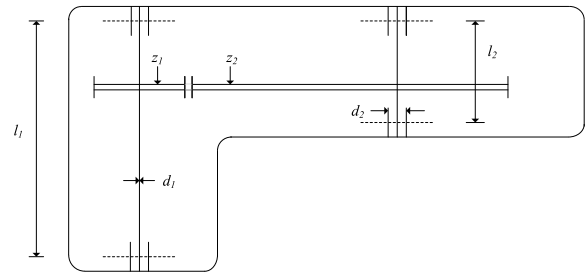


FIGURE 6. Reducer design problem diagram.

$$g_7(\vec{z}) = \frac{z_2z_3}{40} - 1 \leq 0, \quad g_8(\vec{z}) = \frac{5z_2}{z_1} - 1 \leq 0,$$

$$g_9(\vec{z}) = \frac{z_1}{12z_2} - 1 \leq 0, \quad g_{10}(\vec{z}) = \frac{1.5z_6 + 1.9}{z_4} - 1 \leq 0,$$

$$g_{11}(\vec{z}) = \frac{1.1z_7 + 1.9}{z_5} - 1 \leq 0,$$

where, $2.6 \leq z_1 \leq 3.6$, $0.7 \leq z_2 \leq 0.8$, $17 \leq z_3 \leq 28$, $7.3 \leq z_4 \leq 8.3$, $7.3 \leq z_5 \leq 8.3$, $2.9 \leq z_6 \leq 3.9$, $5.0 \leq z_7 \leq 5.5$.

After testing, the test results are shown in Table 13 and Fig. 7. The optimal solution of SSA is [3.5459843, 0.70607238, 17.05650762, 7.51461454, 8.18743412, 3.48360511,

TABLE 5. IEEE CEC-2017 benchmark test functions.

No.	Functions	Dim	f_{min}
CEC-1	Shifted and Rotated Bent Cigar Function	30	100
CEC-2	Shifted and Rotated Sum of Different Power Function	30	200
CEC-3	Shifted and Rotated Zakharov Function	30	300
CEC-4	Shifted and Rotated Rosenbrock's Function	30	400
CEC-5	Shifted and Rotated Rastrigin's Function	30	500
CEC-6	Shifted and Rotated Expanded Scaffer's Function	30	600
CEC-7	Shifted and Rotated Lunacek Bi_Rastrigin Function	30	700
CEC-8	Shifted and Rotated Non-Continuous Rastrigin's Function	30	800
CEC-9	Shifted and Rotated Levy Function	30	900
CEC-10	Shifted and Rotated Schwefel's Function	30	1000
CEC-11	Hybrid Function 1 ($N = 3$)	30	1100
CEC-12	Hybrid Function 2 ($N = 3$)	30	1200
CEC-13	Hybrid Function 3 ($N = 3$)	30	1300
CEC-14	Hybrid Function 4 ($N = 4$)	30	1400
CEC-15	Hybrid Function 5 ($N = 4$)	30	1500
CEC-16	Hybrid Function 6 ($N = 4$)	30	1600
CEC-17	Hybrid Function 6 ($N = 5$)	30	1700
CEC-18	Hybrid Function 6 ($N = 5$)	30	1800
CEC-19	Hybrid Function 6 ($N = 5$)	30	1900
CEC-20	Hybrid Function 6 ($N = 6$)	30	2000
CEC-21	Composition Function 1 ($N = 3$)	30	2100
CEC-22	Composition Function 2 ($N = 3$)	30	2200
CEC-23	Composition Function 3 ($N = 4$)	30	2300
CEC-24	Composition Function 4 ($N = 4$)	30	2400
CEC-25	Composition Function 5 ($N = 5$)	30	2500
CEC-26	Composition Function 6 ($N = 5$)	30	2600
CEC-27	Composition Function 7 ($N = 6$)	30	2700
CEC-28	Composition Function 8 ($N = 6$)	30	2800
CEC-29	Composition Function 9 ($N = 3$)	30	2900
CEC-30	Composition Function 10 ($N = 3$)	30	3000

TABLE 6. AVG and STD deviation of best optimal solution for 30 independent runs on CEC-2015 benchmark test functions.

FUNC	CEC-1	CEC-2	CEC-3	CEC-4	CEC-5	CEC-6	CEC-7	CEC-8	CEC-9	CEC-10	CEC-11	CEC-12	CEC-13	CEC-14	CEC-15	
PSO	AVG	9.33E+06	2.74E+05	3.21E+02	4.70E+02	4.86E+03	9.82E+05	7.12E+02	2.56E+05	1.03E+03	3.20E+05	1.83E+03	1.32E+03	1.30E+03	3.69E+04	1.60E+03
	STD	7.22E+06	1.44E+06	8.62E-02	2.07E+01	1.25E+03	8.33E+05	2.63E+00	1.41E+05	5.68E+01	3.06E+05	1.19E+02	3.44E+01	2.73E-02	2.34E+03	1.34E-02
WOA	AVG	1.07E+08	6.19E+08	3.21E+02	6.94E+02	5.94E+03	4.39E+06	7.60E+02	8.71E+05	1.08E+03	5.85E+06	2.42E+03	1.33E+03	1.30E+03	3.99E+04	1.69E+03
	STD	4.00E+07	3.51E+08	1.31E-01	5.55E+01	6.05E+02	3.64E+06	3.82E+01	4.56E+05	1.50E+02	2.76E+06	2.97E+02	3.54E+01	1.70E-02	2.71E+03	9.77E+01
GWO	AVG	1.20E+09	7.34E+10	3.21E+02	7.77E+02	7.88E+03	3.86E+07	1.02E+03	1.26E+07	1.38E+03	3.76E+07	2.52E+03	1.41E+03	2.05E+03	1.20E+05	3.22E+04
	STD	4.14E+08	8.99E+09	4.92E-02	2.64E+01	4.17E+02	3.66E+07	9.19E+01	8.71E+06	1.25E+02	3.10E+07	4.31E+02	1.78E+01	4.96E+02	1.87E+04	1.01E+04
TSA	AVG	2.59E+08	2.33E+10	3.21E+02	7.29E+02	7.11E+03	7.36E+06	8.14E+02	2.08E+06	1.13E+03	1.26E+07	2.05E+03	1.36E+03	1.33E+03	6.46E+04	3.49E+03
	STD	1.65E+08	7.59E+09	5.36E-02	3.04E+01	5.96E+02	3.58E+06	4.64E+01	1.88E+06	1.16E+02	1.06E+07	4.01E+02	2.69E+01	3.94E+01	8.41E+03	8.16E+02
BOA	AVG	1.65E+09	7.06E+10	3.21E+02	8.03E+02	7.84E+03	9.99E+07	1.25E+03	1.29E+07	1.21E+03	6.99E+07	2.05E+03	1.39E+03	1.34E+03	9.53E+04	6.95E+04
	STD	7.51E+08	1.11E+10	4.53E-02	1.77E+01	3.13E+02	7.55E+07	1.91E+02	8.52E+06	6.77E+01	4.74E+07	3.80E+02	1.36E+01	9.48E+01	1.86E+04	2.78E+04
SBO	AVG	3.23E+06	6.31E+04	3.21E+02	5.84E+02	4.22E+03	1.83E+06	7.18E+02	4.20E+05	1.05E+03	7.68E+05	1.99E+03	1.32E+03	1.34E+03	2.56E+04	1.60E+03
	STD	1.86E+06	2.23E+04	6.25E-02	3.00E+01	8.35E+02	1.07E+06	1.53E+01	2.25E+05	1.08E+02	3.94E+05	5.14E+02	2.77E+01	2.88E+01	5.28E+03	9.92E+03
PIO	AVG	2.33E+08	1.61E+10	3.21E+02	7.40E+02	7.90E+03	1.08E+07	7.68E+02	2.15E+06	1.09E+03	4.78E+06	2.19E+03	1.34E+03	1.30E+03	4.39E+04	2.08E+03
	STD	4.76E+07	2.78E+09	4.42E-02	2.50E+01	3.34E+02	5.15E+06	1.81E+01	1.09E+06	8.82E+01	2.11E+06	3.41E+02	1.15E+01	2.77E-02	1.47E+03	3.81E+02
ISSA	AVG	9.11E+06	1.23E+06	3.21E+02	6.60E+02	5.13E+03	1.05E+06	8.51E+02	3.86E+05	1.16E+04	3.51E+06	2.15E+03	1.32E+03	1.30E+03	4.58E+04	1.89E+03
	STD	8.00E+06	9.45E+06	2.03E+02	3.91E+02	5.02E+03	8.49E+05	5.13E+01	3.72E+05	1.55E+02	2.84E+05	1.24E+03	1.45E+02	9.69E+00	4.21E+04	1.54E+03
LLSSA	AVG	6.55E+06	8.70E+05	3.21E+02	5.50E+02	4.65E+03	8.43E+05	8.13E+02	2.67E+05	1.16E+04	5.09E+05	1.75E+03	1.28E+03	1.30E+03	4.44E+04	1.79E+03
	STD	5.42E+06	4.61E+06	5.66E+01	1.94E+02	9.75E+02	8.13E+05	5.12E+02	1.96E+05	1.55E+02	4.90E+05	1.36E+03	1.25E+03	6.93E-01	2.14E+04	8.74E+00
SSA	AVG	1.28E+07	3.95E+06	3.21E+02	6.93E+02	5.45E+03	3.27E+06	8.59E+02	4.00E+05	1.17E+03	3.85E+06	2.67E+03	1.34E+03	1.30E+03	4.83E+04	1.95E+03
	STD	5.24E+07	2.35E+06	5.82E-02	5.05E+01	8.53E+02	2.51E+06	6.03E+01	4.04E+05	1.50E+02	2.90E+06	5.23E+02	2.93E+01	3.07E+01	8.62E+03	2.32E+02
CSSA	AVG	7.04E+06	5.22E+05	3.21E+02	5.22E+02	4.01E+03	6.53E+05	7.84E+02	2.51E+05	9.21E+02	3.94E+05	1.51E+03	1.22E+03	1.30E+03	3.72E+04	1.60E+03
	STD	1.11E+06	4.52E+04	2.82E-02	1.61E+01	3.18E+02	2.37E+05	1.26E+01	2.31E+05	1.16E+02	2.95E+05	3.29E+02	2.35E+01	8.29E+00	5.61E+03	8.34E+01

5.2899798], the corresponding optimal value is 3100.355814. At the same time, the optimal solution of CSSA is [3.51058267, 0.70141566, 17.02162146, 7.48680426, 8.11370926, 3.35946343, 5.32393441], the corresponding optimal value is 3045.542 957. It can be clearly seen from the test results that CSSA's optimization results are better than SSA when dealing with reducer design problems.

VII. WELDED BEAM DESIGN PROBLEM

The main purpose of this engineering design problem is to minimize the manufacturing cost of welded beams, as shown

in Fig. 8. When dealing with this problem, the following requirements need to be met [16]: shear stress (τ), bending stress (θ) in the beam, buckling load (P_c) on the bar, end deflection (δ) of the beam.

This project design problem has four design variable: thickness of weld (h), length of the clamped bar (l), height of the bar (t), thickness of the bar (b). The specific mathematical formula for this engineering design problem is as follows:

Consider:

$$\vec{z} = [z_1 \ z_2 \ z_3 \ z_4] = [h \ l \ t \ b],$$

TABLE 7. AVG and STD deviation of best optimal solution for 30 independent runs on CEC-2017 benchmark test functions (CEC1-CEC15).

FUNC	CEC-1	CEC-2	CEC-3	CEC-4	CEC-5	CEC-6	CEC-7	CEC-8	CEC-9	CEC-10	CEC-11	CEC-12	CEC-13	CEC-14	CEC-15	
PSO	AVG	4.05E+03	9.82E+03	2.47E+04	5.44E+02	5.68E+02	6.00E+02	8.31E+02	8.70E+02	1.09E+03	2.02E+03	1.23E+03	5.67E+04	1.58E+04	2.48E+04	1.48E+04
	STD	4.35E+03	7.76E+04	7.70E+03	3.37E+01	1.99E+01	3.12E-01	2.64E+01	2.08E+01	2.32E+02	1.20E+03	6.35E+01	1.07E+05	1.65E+04	2.21E+04	2.09E+04
WOA	AVG	8.14E+07	9.45E+05	2.18E+05	5.95E+02	7.71E+02	6.73E+02	1.25E+03	1.01E+03	9.40E+03	6.52E+03	2.78E+03	9.48E+07	2.30E+05	2.06E+06	1.01E+05
	STD	4.71E+07	2.28E+07	6.54E+04	4.87E+01	6.31E+01	1.35E+01	7.02E+01	6.09E+01	3.26E+03	9.80E+02	1.19E+03	6.71E+07	2.80E+05	1.89E+06	8.02E+04
GWO	AVG	3.83E+10	8.57E+08	8.05E+04	9.04E+03	8.86E+02	6.87E+02	1.32E+03	1.12E+03	9.46E+03	8.54E+03	6.27E+03	9.65E+09	6.29E+09	1.56E+06	1.01E+08
	STD	4.96E+09	1.24E+08	9.02E+03	1.52E+03	1.34E+01	7.15E+00	5.57E+01	1.71E+01	9.91E+02	4.17E+02	1.42E+03	2.26E+09	2.67E+09	8.64E+05	1.38E+08
TSA	AVG	1.23E+10	6.75E+09	7.09E+04	1.69E+03	8.11E+02	6.77E+02	1.25E+03	1.04E+03	8.86E+03	7.47E+03	3.15E+03	1.33E+09	2.59E+08	1.26E+06	4.40E+06
	STD	2.37E+09	1.42E+09	5.25E+03	6.14E+02	2.82E+01	6.24E+00	7.16E+01	2.52E+01	1.01E+03	7.91E+02	1.11E+03	9.69E+08	2.05E+08	6.78E+05	4.10E+06
BOA	AVG	3.40E+10	6.50E+09	7.06E+04	1.48E+04	8.92E+02	6.79E+02	1.35E+03	1.12E+03	9.01E+03	8.44E+03	6.90E+03	6.22E+09	3.37E+09	1.64E+06	1.01E+08
	STD	6.07E+09	9.05E+09	8.95E+03	1.97E+03	1.32E+01	4.87E+00	3.85E+01	1.23E+01	1.27E+03	3.34E+02	1.70E+03	2.35E+09	1.53E+09	1.06E+06	9.32E+07
SBO	AVG	1.09E+04	3.00E+04	2.39E+04	4.78E+02	6.69E+02	6.37E+02	1.11E+03	9.30E+02	4.04E+03	4.67E+03	1.22E+03	1.30E+06	1.60E+04	4.65E+05	5.31E+03
	STD	4.47E+03	7.80E+03	1.20E+04	1.37E+01	2.67E+01	5.22E+00	7.56E+01	2.26E+01	8.66E+02	5.36E+02	2.86E+01	7.83E+05	1.12E+04	3.55E+05	4.34E+03
PIO	AVG	1.73E+10	6.16E+09	7.48E+04	1.61E+03	8.09E+02	6.54E+02	1.43E+03	1.11E+03	9.19E+03	8.34E+03	3.16E+03	1.29E+09	3.80E+08	2.21E+05	6.99E+07
	STD	2.84E+09	2.81E+08	1.31E+04	2.47E+02	2.35E+01	7.25E+00	6.70E+01	2.27E+01	2.15E+03	3.42E+02	5.69E+02	3.40E+08	1.36E+08	1.31E+05	3.09E+07
ISSA	AVG	4.21E+05	3.30E+05	7.02E+04	5.51E+02	6.24E+02	6.51E+02	1.24E+03	9.14E+02	3.12E+03	4.13E+03	1.46E+03	4.59E+05	4.15E+04	8.98E+04	3.16E+04
	STD	4.10E+05	5.40E+05	9.80E+03	1.25E+01	2.14E+01	1.16E+00	1.64E+01	4.12E+01	5.47E+02	7.22E+02	1.48E+05	3.44E+03	4.52E+02	3.22E+04	2.53E+04
LLSSA	AVG	4.52E+04	7.31E+05	7.15E+04	5.42E+02	5.83E+02	6.29E+02	8.12E+02	9.09E+02	2.95E+02	3.75E+02	1.31E+03	4.49E+04	3.18E+03	5.18E+04	7.17E+03
	STD	1.41E+04	3.19E+05	2.34E+04	4.21E+02	1.42E+02	1.47E+01	1.42E+01	8.13E+02	1.47E+02	2.16E+02	4.48E+04	1.37E+03	3.44E+04	4.89E+03	
SSA	AVG	4.37E+07	5.97E+06	7.26E+04	5.79E+02	7.85E+02	6.64E+02	1.32E+03	9.86E+02	5.39E+03	5.73E+03	1.54E+03	3.92E+07	1.14E+05	6.81E+05	4.71E+04
	STD	4.25E+07	3.95E+06	1.38E+04	3.75E+01	3.64E+01	6.71E+00	2.54E+01	2.42E+01	4.10E+02	1.68E+02	2.97E+07	6.65E+04	7.44E+05	3.39E+04	
CSSA	AVG	3.90E+03	3.19E+03	6.97E+04	5.12E+02	5.21E+02	6.00E+02	7.52E+02	8.81E+02	1.05E+03	2.12E+03	1.24E+03	6.25E+03	9.86E+03	2.70E+04	4.96E+03
	STD	3.40E+03	9.88E+02	9.54E+03	9.74E+00	1.81E+01	5.31E+01	9.38E+00	2.53E+01	3.20E+02	5.93E+01	1.56E+02	4.56E+03	9.46E+03	2.36E+04	4.59E+03

TABLE 8. AVG and STD deviation of best optimal solution for 30 independent runs on CEC-2017 benchmark test functions (CEC16-CEC30).

FUNC	CEC-16	CEC-17	CEC-18	CEC-19	CEC-20	CEC-21	CEC-22	CEC-23	CEC-24	CEC-25	CEC-26	CEC-27	CEC-28	CEC-29	CEC-30	
PSO	AVG	2.43E+03	1.96E+03	1.00E+06	1.82E+04	2.25E+03	2.37E+03	4.30E+03	2.72E+03	2.74E+03	2.91E+03	4.24E+03	3.25E+03	3.26E+03	3.21E+03	3.91E+03
	STD	2.87E+02	1.38E+02	1.49E+06	1.41E+04	1.39E+02	3.32E+01	2.14E+03	2.60E+01	4.76E+01	1.53E+01	4.05E+02	1.66E+01	2.73E+01	1.27E+02	4.20E+03
WOA	AVG	3.84E+03	2.63E+03	3.61E+06	5.25E+06	2.80E+03	2.59E+03	6.89E+03	3.05E+03	3.19E+03	3.00E+03	7.55E+03	3.40E+03	3.36E+03	4.87E+03	9.45E+03
	STD	4.44E+02	2.60E+02	3.30E+06	4.90E+06	1.95E+02	5.28E+01	1.99E+03	1.02E+02	9.26E+01	3.26E+01	9.12E+02	8.75E+01	3.63E+01	4.02E+02	6.38E+02
GWO	AVG	4.98E+03	3.18E+03	1.30E+07	1.53E+08	2.89E+03	2.68E+03	8.08E+03	3.56E+03	3.86E+03	4.34E+03	9.77E+03	4.47E+03	3.19E+03	6.56E+03	3.38E+03
	STD	4.99E+02	3.09E+02	1.07E+07	9.93E+07	1.74E+02	3.13E+01	1.14E+03	1.04E+02	1.43E+02	3.36E+02	3.58E+02	2.40E+02	4.39E+02	5.77E+02	8.69E+02
TSA	AVG	3.72E+03	2.54E+03	6.56E+06	6.41E+06	2.76E+03	2.61E+03	8.75E+03	3.21E+03	3.41E+03	3.31E+03	8.30E+03	3.70E+03	4.00E+03	5.32E+03	4.78E+03
	STD	4.46E+02	2.52E+02	3.48E+06	3.02E+06	1.98E+02	3.86E+01	1.31E+03	8.90E+01	8.82E+01	1.72E+02	9.38E+02	1.56E+02	3.53E+02	4.20E+02	2.13E+02
BOA	AVG	6.02E+03	4.10E+03	1.28E+07	9.88E+07	2.79E+03	2.37E+03	3.96E+03	3.48E+03	3.71E+03	4.85E+03	1.03E+04	4.39E+03	6.79E+03	7.17E+03	7.28E+03
	STD	9.40E+02	1.02E+03	1.05E+07	9.42E+07	1.05E+02	6.11E+01	3.35E+02	2.34E+02	3.21E+02	2.77E+02	1.19E+03	3.31E+02	4.43E+02	8.96E+02	9.10E+02
SBO	AVG	2.86E+03	2.44E+03	1.34E+06	8.85E+03	2.62E+03	2.42E+03	5.54E+03	3.13E+03	3.43E+03	2.89E+03	7.03E+03	3.35E+03	3.22E+03	4.05E+03	3.30E+03
	STD	3.02E+02	2.75E+02	1.34E+06	6.37E+03	2.00E+02	2.51E+01	2.22E+03	7.75E+01	8.69E+01	1.79E+01	1.73E+03	3.04E+02	2.26E+01	2.88E+02	7.06E+02
PIO	AVG	3.67E+03	2.64E+03	4.48E+06	1.12E+08	2.78E+03	2.59E+03	4.90E+03	2.95E+03	3.10E+03	4.13E+03	6.39E+03	3.33E+03	3.98E+03	4.66E+03	3.98E+03
	STD	1.94E+02	1.14E+02	1.98E+06	4.84E+07	1.48E+02	1.69E+01	1.94E+03	2.71E+01	2.31E+01	3.19E+02	1.03E+03	3.22E+01	2.46E+02	2.20E+02	1.46E+02
ISSA	AVG	3.17E+03	3.81E+03	9.14E+05	8.15E+04	2.74E+03	3.41E+03	5.32E+03	3.22E+03	3.17E+03	2.92E+03	5.89E+03	3.65E+03	3.21E+03	4.65E+03	3.06E+03
	STD	1.56E+03	3.11E+03	1.31E+05	1.10E+04	2.26E+03	1.74E+02	1.77E+03	3.19E+02	1.21E+03	2.31E+03	5.41E+02	1.22E+03	1.45E+02	1.52E+03	4.52E+02
LLSSA	AVG	2.95E+03	2.79E+03	9.23E+05	1.95E+04	2.68E+03	3.31E+03	5.18E+03	3.18E+03	3.01E+03	2.85E+03	5.61E+03	3.62E+03	3.30E+03	4.41E+03	3.05E+03
	STD	1.46E+03	1.49E+02	4.58E+04	1.50E+04	1.62E+03	3.11E+03	8.68E+02	9.85E+02	1.25E+03	1.65E+03	5.59E+03	1.24E+03	3.14E+03	4.10E+03	5.14E+02
SSA	AVG	3.86E+03	4.36E+03	9.68E+05	9.16E+05	2.89E+03	3.52E+03	7.30E+03	3.30E+03	3.44E+03	2.99E+03	9.09E+03	3.86E+03	3.36E+03	5.81E+03	3.10E+03
	STD	6.06E+02	3.29E+02	1.36E+06	8.91E+05	2.65E+02	4.22E+01	7.73E+02	1.39E+02	1.07E+02	3.11E+01	1.11E+03	3.17E+02	3.02E+01	8.00E+02	5.01E+02
CSSA	AVG	2.97E+03	3.70E+03	9.10E+05	8.10E+03	2.55E+03	3.24E+03	4.53E+03	3.10E+03	2.52E+03	2.74E+03	5.12E+03	3.61E+03	3.20E+03	4.22E+03	3.05E+03
	STD	2.95E+03	9.27E+01	5.07E+04	6.25E+03	2.21E+03	5.13E+02	1.25E+03	1.42E+02	1.21E+01	1.69E+00	4.32E+02	6.93E+00	1.25E+02	4.52E+02	1.25E+02

TABLE 9. Statistical comparisons of CSSA vs. PSO, WOA, GWO, TSA, and BOA. (CEC-2015).

FUNC	CSSA vs. PSO			CSSA vs. WOA			CSSA vs. GWO			CSSA vs. TSA			CSSA vs. BOA			
	p-value	T-	T+	Winner	p-value	T-	T+	Winner	p-value	T-	T+	Winner	p-value	T-	T+	Winner
CEC-1	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-2	2.37E-06	435	30	+	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-3	1.00E+00	0	0	=	1.00E+00	0	0	=	1.00E+00	0	0	=	1.00E+00	0	0	=
CEC-4	5.00E-03	368	97	+	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-5	1.73E-06	0	465	-	5.73E-06	13	452	-	1.73E-06	0	465	-	1.92E-06	1	464	-
CEC-6	1.73E-06	0	465	-	6.74E-06	14	451	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-7	1.73E-06	465	0	+	1.73E-06											

TABLE 10. Statistical comparisons of CSSA vs. SBO, TLBO, PIO, and SSA. (CEC-2015).

FUNC	CSSA vs. SBO				CSSA vs. PIO				CSSA vs. ISSA				CSSA vs. LLSSA				CSSA vs. SSA			
	p-value	T-	T+	Winner	p-value	T-	T+	Winner	p-value	T-	T+	Winner	p-value	T-	T+	Winner	p-value	T-	T+	Winner
CEC-1	2.05E-04	413	52	+	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-2	1.73E-06	465	0	+	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-3	1.00E+00	0	0	=	1.00E+00	0	0	=	1.00E+00	0	0	=	1.00E+00	0	0	=	1.00E+00	0	0	=
CEC-4	1.73E-06	465	0	+	1.73E-06	0	465	-	1.73E-06	0	465	-	3.13E-02	33	432	-	1.73E-06	0	465	-
CEC-5	1.73E-06	0	465	-	1.73E-06	0	465	-	1.77E-02	99	366	-	7.19E-02	145	320	=	1.74E-04	50	415	-
CEC-6	1.73E-06	0	465	-	1.73E-06	0	465	-	5.67E-03	98	367	-	7.52E-02	146	319	=	5.00E-06	11	454	-
CEC-7	1.73E-06	465	0	+	1.73E-06	465	0	+	4.11E-06	2	463	-	1.89E-04	51	414	-	1.73E-06	0	465	-
CEC-8	1.38E-03	77	388	-	1.73E-06	0	465	-	1.65E-01	165	300	=	3.49E-01	278	187	=	1.30E-02	111.5	353.5	-
CEC-9	8.00E-03	83	385	-	2.00E-05	28	437	-	1.03E-05	18	447	-	3.11E-05	30	435	-	6.96E-06	14	451	-
CEC-10	5.71E-04	65	400	-	1.73E-06	0	465	-	2.35E-06	3	462	-	2.84E-05	29	436	-	2.35E-06	3	462	-
CEC-11	3.55E-02	101.5	276.5	-	8.10E-05	30	376	-	1.14E-05	19	446	-	1.80E-05	24	441	-	6.96E-06	14	451	-
CEC-12	1.73E-06	0	465	-	1.33E-03	84	381	-	1.60E-04	49	416	-	3.38E-03	90	375	-	7.08E-04	43	422	-
CEC-13	2.37E-06	30	435	-	1.00E+00	0	0	=	1.00E+00	0	0	=	1.00E+00	0	0	=	1.00E+00	0	0	=
CEC-14	1.60E-05	442	23	+	1.73E-06	0	465	-	3.52E-06	7	458	-	3.52E-06	7	458	-	2.35E-06	3	462	-
CEC-15	1.00E+00	0	0	=	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-

TABLE 11. Statistical comparisons of CSSA vs. PSO, WOA, GWO, TSA, and BOA. (CEC-2017).

FUNC	CSSA vs. PSO				CSSA vs. WOA				CSSA vs. GWO				CSSA vs. TSA				CSSA vs. BOA			
	p-value	T-	T+	Winner	p-value	T-	T+	Winner	p-value	T-	T+	Winner	p-value	T-	T+	Winner	p-value	T-	T+	Winner
CEC-1	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-2	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-3	1.73E-06	465	0	+	1.73E-06	0	465	-	1.73E-06	0	465	-	6.00E-02	141	324	=	1.40E-02	113	352	-
CEC-4	1.65E-01	300	165	=	4.07E-05	33	432	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-5	5.29E-04	64	401	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-6	1.00E+00	0	0	=	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-7	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-8	2.89E-01	284	181	=	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-9	8.00E-06	15	450	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-10	1.57E-02	350	115	+	2.88E-06	23	442	-	1.73E-06	0	465	-	2.13E-06	2	465	-	1.73E-06	0	465	-
CEC-11	1.57E-02	115	350	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-12	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-13	3.68E-02	131	334	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-14	3.49E-01	278	187	=	2.60E-05	5	460	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-15	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-16	3.16E-02	337	128	+	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-17	2.45E-01	289	176	=	1.73E-06	465	0	+	1.73E-06	0	465	-	1.92E-06	464	1	=	2.80E-01	180	285	=
CEC-18	1.73E-06	0	465	-	1.06E-04	28	437	-	1.73E-06	0	465	-	1.92E-06	1	464	-	1.73E-06	0	465	-
CEC-19	1.73E-06	0	465	-	1.92E-06	1	464	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-20	1.25E-01	307	158	=	1.73E-06	0	465	-	1.73E-06	0	465	-	1.65E-01	165	300	=	1.73E-06	0	465	-
CEC-21	7.34E-01	216	249	=	1.57E-02	115	350	+	1.73E-06	465	0	+	1.73E-06	465	0	+	1.73E-06	165	0	+
CEC-22	4.78E-01	267	198	=	1.06E-04	44	421	-	2.13E-06	2	465	-	8.00E-06	15	450	-	4.72E-02	329	136	+
CEC-23	9.37E-02	151	314	-	8.00E-06	450	15	+	1.73E-06	0	465	-	4.78E-01	198	267	=	1.73E-06	0	465	-
CEC-24	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-25	1.73E-06	0	465	-	1.92E-06	1	464	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-26	7.19E-01	250	215	=	1.92E-06	1	464	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-27	8.22E-02	317	148	=	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-28	1.73E-06	0	465	-	1.73E-06	0	465	-	2.89E-01	181	284	=	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-29	8.77E-01	240	225	=	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-30	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-

TABLE 12. Statistical comparisons of CSSA vs. SBO, TLBO, PIO, and SSA. (CEC-2017).

FUNC	CSSA vs. SBO				CSSA vs. PIO				CSSA vs. ISSA				CSSA vs. LLSSA				CSSA vs. SSA			
	p-value	T-	T+	Winner	p-value	T-	T+	Winner	p-value	T-	T+	Winner	p-value	T-	T+	Winner	p-value	T-	T+	Winner
CEC-1	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-2	1.73E-06	0	465	-	1.73E-06	0	465	-	1.92E-06	1	464	-	1.92E-06	1	464	-	1.73E-06	0	465	-
CEC-3	8.00E-06	449	16	+	1.40E-02	113	352	-	1.40E-02	113	352	-	7.50E-01	230	235	=	1.73E-06	0	465	-
CEC-4	3.52E-06	458	7	+	1.73E-06	0	465	-	1.60E-04	49	416	-	7.32E-03	71	394	-	1.06E-04	44	421	-
CEC-5	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-6	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.02E-05	18	447	-	1.73E-06	0	465	-
CEC-7	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.60E-04	49	416	-	1.73E-06	0	465	-
CEC-8	1.73E-06	0	465	-	1.73E-06	0	465	-	3.12E-02	33	432	-	3.61E-03	91	374	-	1.73E-06	0	465	-
CEC-9	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-10	7.19E-02	145	320	=	1.73E-06	0	465	-	3.59E-04	59	406	-	5.12E-02	139	326	=	6.30E-05	38	427	-
CEC-11	1.80E-02	348	117	+	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-12	1.00E-03	70	395	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-13	1.02E-01	153	321	-	1.73E-06	0	465	-	1.73E-06	0	465	-	3.52E-06	7	458	-	1.73E-06	0	465	-
CEC-14	2.35E-06	3	462	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-	1.73E-06	0	465	-
CEC-15	1.36E-05	21	444	-	1.73E-06	0	465	-	1.89E-04	51	414	-	9.59E-01	230	235	=	1.00E-05	18	447	-
CEC-16	2.35E-06	3	462	-	1.73E-06	0	465	-	3.11E-05	30	435	-	3.88E-04	60	405	-	1.73E-06	0	465	-
CEC-17	4.00E-06	7	458	-	1.73E-06	465	0	+	1.73E-06	0	465	-	5.75E-06	12	453	-	1.73E-06	0	465	-
CEC-18	1.40E-02	113	352	-	2.35E-06	3	462	-	3.49E-01	187	278	=	7.81E-01	219	246	=	2.71E-01	179	286	=
CEC-19	3.50E-02	130	335	-	1.73E-06	0	465	-	7.10E-02	145	320	=	6.26E-01							

TABLE 13. The comparison between the optimal solution and the optimal value of the reducer problem.

Algorithm	SSA	CSSA
b	3.5459843	3.51058267
m	0.70607238	0.70141566
p	17.05650762	17.02162146
l_1	7.51461454	7.48680426
l_2	8.18743412	8.11370926
d_1	3.48360511	3.35946343
d_2	5.2899798	5.32393441
g_1	-0.10455608	-0.08159821
g_2	-0.22710295	-0.20566232
g_3	-0.5382335	-0.46741284
g_4	-0.88768303	-0.89252475
g_5	-0.11034235	-0.00796445
g_6	-0.00181118	-0.02078878
g_7	-0.69892178	-0.70151921
g_8	-0.00440565	-0.00099823
g_9	-0.58148952	-0.58291699
g_{10}	-0.05179332	-0.07314324
g_{11}	-0.0572165	-0.04404661
Optimal value	3100.355814	3045.542957

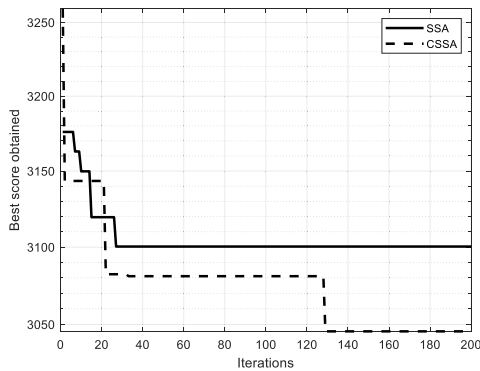


FIGURE 7. Welded beam design problem.

$$\tau'' = \frac{6000(14 + 0.5l)\sqrt{0.25(l^2 + (h + t)^2)}}{2[\sqrt{2}hl(l^2/12 + 0.25(h + t)^2)]}$$

$$Pc(\vec{z}) = \frac{4.013 \times 30 \times 10^6 \sqrt{36 \times z_3^2 z_4^6}}{14 \times 14} \left(1 - \frac{z_3}{28} \sqrt{0.625}\right).$$

After testing, the test results are shown in Table 14 and Fig. 9. The optimal solution of SSA is [0.13561309, 6.77516019, 9.74777439, 0.18708584], the corresponding optimal value is 1.96039428. At the same time, the optimal solution of CSSA is [0.17324276, 4.00413158, 9.793008, 0.18093193], the corresponding optimal value is 1.66751421. It can be clearly seen from the test results that CSSA is better than SSA when dealing with the design problems of welded beams.

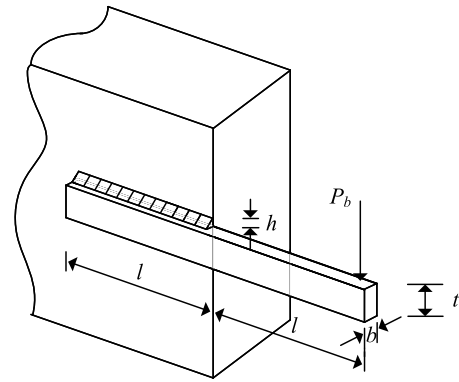


FIGURE 8. Welded beam design problem.

TABLE 14. The comparison between the optimal solution and the optimal value of the welded beam design problem.

Algorithm	SSA	CSSA
h	0.13561309	0.17324276
l	6.77516019	4.00413158
t	9.74777439	9.793008
b	0.18708584	0.18093193
g_1	-1840.089235	-33.15998228
g_2	-1648.328237	-954.216793
g_3	-0.23733175	-0.23708155
g_4	-0.05147276	-0.00768917
g_5	-164497.1839	-148662.4156
g_6	-0.01061309	-0.04824276
g_7	-3.17532837	-3.46210299
Optimal value	1.96039428	1.66751421

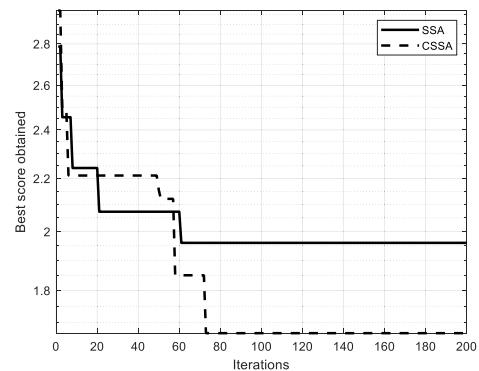


FIGURE 9. Convergence curve of welded beam design problem.

VIII. APPLICATION TEST OF CSSA ALGORITHM IN NETWORK INTRUSION DETECTION

In order to verify the feasibility and improvement of the CSSA proposed in this paper in practical application, the Support Vector Machine (SVM) model commonly used in network intrusion detection [31]–[33] was selected in this paper. The CSSA can be used to solve the parameter optimization problem of the model, that is, to find the optimal solution of

TABLE 15. Comparison of SVM optimization results.

Models	Test set Accuracy	Dos prediction accuracy	Probe prediction accuracy	R2L prediction accuracy	U2R prediction accuracy
GA-SVM	88.21%	86.25%	47.36%	43.75%	94.28%
SSA-SVM	91.37%	93.91%	75.20%	91.48%	89.13%
CSSA-SVM	94.26%	96.12%	81.59%	95.28%	92.76%

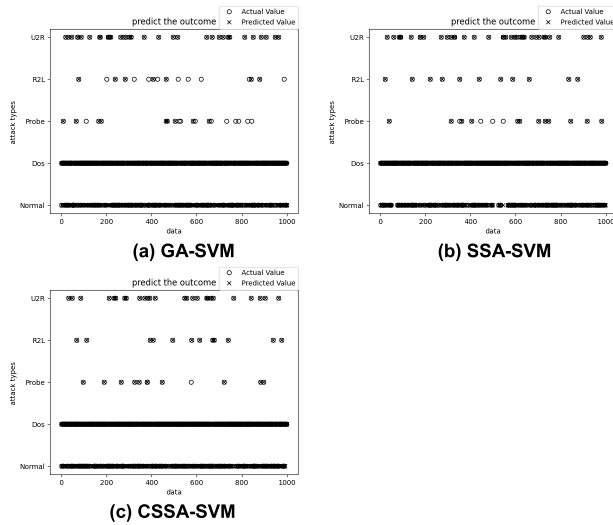


FIGURE 10. Distribution of network intrusion detection prediction results.

the parameter C in the SVM model and the parameter g of the radial basis kernel function. The data source selected in this section is KDD99 data set, in which there are four main network attack means: DoS attack, Probe attack, R2L attack, and U2R attack.

Firstly, the data set was processed. 5000 pieces of data were randomly selected as the experimental data source in this section, and the quantitative and normalized processing was carried out. Then, 20% of the 5000 data sources were randomly selected as a sub-data set. Meanwhile, in order to avoid the influence of other uncontrollable factors on the accuracy of training, this paper adopted ten-fold cross validation and obtained its average value to obtain the results. At the same time, this section uses the same method to optimize the SVM model respectively for sparrow search algorithm (SSA) and genetic algorithm (GA), and finally compares the prediction results of these three models. In this section, the upper bound of the search space of SSA, GA and CSSA is set as 0.001, the lower bound of the search space is set as 200, the maximum number of iterations is 20, the dimension is 2, the crossover rate of GA is 0.8, and the mutation rate is 0.15. The predicted results are shown in Table 15.

As can be seen from Table 15, compared with GA and SSA, the overall prediction accuracy of CSSA in the optimized SVM model is 3% higher than that of SSA and 6% higher than that of GA. In addition, for different attacks, the prediction

accuracy of intrusion detection models built according to this algorithm has increased to different degrees. From the 10 randomly selected experimental results, the scatter diagram of the prediction results of the SVM model and the actual results is shown in Fig. 10.

IX. CONCLUSION

This article mainly aims at SSA easy to fall into local optimal problems, introduces the improved chaotic mapping Circle, as well as the t distribution variation, and puts forward a kind of function to measure similarity of sparrow cluster divergence to improve the original sparrow algorithm. Circle chaotic mapping can maximum limit increase the randomness of the sparrow population distribution, t distribution variation can satisfy different iterations the sparrow population for value to update the size of the size, the similarity function can be used to measure each iteration cycle the sparrow population aggregation and dispersion, thereby minimize CSSA in a local convergence problem. The algorithm is based on different population distribution of adaptive iteration times and adjust the sparrow position. In order to test the performance of the algorithm, this paper mainly carries out the following five experiments: In experiment 1, we mainly test and compare the existing good algorithm with the test function of the original SSA and CSSA in the optimization, the experimental results show that the CSSA can better solve the problem of local convergence of the original algorithm, at the same time greatly increases the convergence speed. In Experiment 2, we tested and compared CSSA with ten other latest optimization algorithms. CEC-2015 and CEC-2017 were selected as Test function sets, and Wilcoxon Sign-Rank Test was used. In experiment 3 and experiment 4, we used CSSA and SSA to optimize problem reducer design problem and welded beam design problem. The experimental results show that the test results of CSSA are better than that of SSA, because CSSA has a strong global optimization capability. In Experiment 5, we applied CSSA to the network intrusion detection model and constructed the CSSA-SVM model. The experimental results show that the CSSA-SVM model has higher prediction accuracy, which verifies the feasibility of the algorithm in practical application. These experimental results show that CSSA has been able to greatly solve the local convergence problem and is an optimization algorithm with local and global optimization ability.

Even though CSSA has a strong performance and performance in the optimization performance, it has a relatively obvious improvement on the original SSA algorithm, but there is also a certain direction of improvement, that is, the running cost of CSSA algorithm is also increased compared with SSA, which will be one of the directions for the next improvement. In addition to that, the application of CSSA in specific practical projects still needs further research and verification, which will also be the main direction of future improvement and development.

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