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Multiservice Loss Models for Cloud Radio Access Networks

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ABSTRACT In this paper, a cloud radio access network (C-RAN) is considered where the remote radio heads (RRHs) are separated from the baseband units which form a common pool of computational resource units. Depending on their capacity, the RRHs may form one or more clusters. Each RRH accommodates multiservice traffic, i.e., calls from different service-classes with different radio and computational resource requirements. Arriving calls follow a Poisson process and simultaneously require radio and computational resource units in order to be accepted in the serving RRH. If their resource requirements cannot be met then calls are blocked and lost. Otherwise, calls remain in the serving RRH for a generally distributed service time. Assuming the single-cluster C-RAN, we model it as a multiservice loss system, prove that a product form solution exists for the steady-state probabilities and determine call blocking probabilities via an efficient convolution algorithm whose accuracy is validated via simulation. Furthermore, we generalize the previous multiservice loss model by considering the more complex multi-cluster case where RRHs of the same capacity are grouped in different clusters.

INDEX TERMS Cloud-radio access, cluster, call blocking, poisson process, product form, convolution.

I. INTRODUCTION

The cloud radio access network (C-RAN) architecture is considered to be a promising and cost-effective fifth generation (5G) RAN solution which is anticipated to cope with the constantly increasing wireless traffic and the soaring demand for bandwidth-hungry applications, decreased latency and enhanced data rate [1], [2].

According to the C-RAN architecture, a base station is separated into two parts: a) the remote radio head (RRH) which consists of the antenna and the radio frequency components and is responsible for the transmission/reception of the signal and b) the baseband unit (BBU) which is responsible for the baseband signal processing. Quality of service (QoS) can be guaranteed, to the mobile users (MUs), via the C-RAN architecture by deploying a number of RRHs, each of capacity C radio resource units (RUs), and forming a common pool of BBUs. Such a BBU pooling reduces not only the necessary processors for baseband processing but also the operators' capital/operational expenditures as well as the power consumption compared to the traditional RAN architectures [3], [4]. To further benefit from network function virtualization [5], we consider virtualized BBU computational resources (V-BBU) which are connected to the RRHs with a low-latency, high-capacity fronthaul, via the common public radio interface (CPRI) [6].

During the last years, various significant aspects of the 5G C-RAN architecture have been extensively studied, including: a) functional splits and capacity demands of the fronthaul network [7]–[9], b) privacy and security challenges [10], [11] and c) energy and cost saving issues [12]–[15]. On the other hand, only a few papers exist in the literature that study call admission control (CAC) of MUs in a 5G C-RAN and propose efficient formulas, via Markov-based loss/queueing models, for the call blocking probabilities (CBP) computation [16]–[21]. Such formulas are always desirable to have as they reduce the computational complexity of the CBP determination and therefore can be adopted by telecom engineers in network dimensioning/planning procedures.

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In [16], a single cluster of RRHs is considered, where all RRHs have the same capacity in terms of radio RUs. Calls arrive in the RRHs according to a Poisson process and form a single service-class according to their resource requirements. More specifically, a new call requests two RUs: a radio one from the serving RRH and a computational one from the V-BBU. If these RUs are available then the call remains in the serving RRH for a generally distributed service time. Otherwise, call blocking occurs, i.e, there are no available queues to accommodate blocked calls. This single-class-single-cluster (SC-SC) model has a product form solution (PFS) for the steady-state probabilities which is essential for the accurate computation of CBP. The SC-SC model has been extended in [17], [18] to include the notion of overlapping cells and also in [19] to include the case of grouping the RRHs (in terms of their capacity) in more than one clusters. We name the model of [19], single-class-multi-cluster (SC-MC) model. The SC-SC and the SC-MC models have been extended in [20] and in [21], respectively, to include the case of RRHs serving random (Poisson), quasi-random and batched Poisson traffic. Quasi-random traffic is generated by a finite number of MUs while in the case of batched Poisson traffic, calls arrive in the C-RAN in the form of batches which follow a Poisson process [22], [23].

The common characteristic of [16]–[21] is that all RRHs serve single-service calls which require a single radio RU and a single computational RU. However, in contemporary networks it is essential to consider a multi-service environment where calls may have different resource requirements. The call-level analysis of such networks (wired, wireless or even satellite) is always challenging due to the complexity of the underlying multidimensional Markov chains but also essential in network dimensioning and planning [24]–[34].

In this paper, we generalize the SC-SC and SC-MC models by assuming that RRHs can serve many service-classes whose calls have different traffic and resource requirements. A possible application scenario is the enhanced mobile broadband scenario which focuses not only in voice services but also on different and high resource requirements' service-classes such as online 4K video and virtual reality [35]. To the best of our knowledge, this is the first paper that considers the multi-service case in a 5G C-RAN and provides an efficient CBP computation via convolution algorithms. Such algorithms have the advantage that they can incorporate complicated resource sharing policies including the threshold-based policies and the bandwidth reservation policy [36]–[41]. On the other hand, the CBP algorithms proposed in [16] and [19] are difficult to be extended to other resource sharing policies. The corresponding proposed models are named herein multi-class-single-cluster (MC-SC) and multi-class-multi-cluster (MC-MC), respectively, while our contribution can be summarized as follows: 1) we propose the MC-SC model and show that a PFS holds for the steady-state probability distribution, 2) we provide a brute force method as well as a convolution algorithm for the computation of CBP in the MC-SC model, 3) we compare the CBP results of the MC-SC model with those obtained via simulation and the SC-SC model of [16], 4) we propose the MC-MC model and show via a multidimensional Markov chain that a PFS holds for the steady-state probability distribution, 5) we provide a brute force method and a convolution algorithm for the CBP determination in the MC-MC model and 6) we compare the CBP results of the MC-MC model with those obtained via the MC-SC model in order to show the impact of forming different clusters of RRHs on CBP.

The remainder of this paper is the following: In Section II, we propose the MC-SC model. In Section II-A, we determine the steady-state probabilities of the MC-SC model via a PFS while in Sections II-B and II-C, we propose a brute force method as well as a convolution algorithm for the CBP computation, respectively. In Section III, we provide analytical and simulation CBP results for the proposed MC-SC model and analytical CBP results of the SC-SC model of [16]. In Section IV, we propose the MC-MC model. In Section IV-A, we determine the steady-state probabilities of the MC-MC model via a PFS while in Sections IV-B and IV-C, we propose a brute force method and a convolution algorithm for the CBP computation, respectively and in Sections IV-D we compare the MC-MC model against the MC-SC one. We conclude in Section V. In Appendix A, we provide the pseudocode for the software implementation for both the brute force method and the convolution algorithm in the case of the proposed MC-SC model. In Appendix B, we present a tutorial MC-SC example that shows all intermediate CBP calculations for both the brute force method and the convolution algorithm. For the reader's convenience, we include in Table 1 the list of abbreviations adopted in this paper.

TABLE 1. List of abbreviations.

BBU	Baseband units
CAC	Call admission control
CBP	Call blocking probabilities
CPRI	Common public radio interface
C-RAN	Cloud radio access network
MUs	Mobile users
MC-MC	Multi-class-multi-cluster
MC-SC	Multi-class-single-cluster
PFS	Product form solution
QoS	Quality of service
RRH	Remote radio head
RUs	Resource units
SC-MC	Single-class-multi-cluster
SC-SC	Single-class-single-cluster
V-BBU	Virtualized BBU

II. THE MC-SC MODEL

A. THE ANALYTICAL MODEL

Consider the C-RAN of Fig. 1 where the RRHs are separated from the centralized V-BBU. Let M be the total number of RRHs and assume that each RRH has a capacity of C radio RUs which can be allocated to the accepted calls. Similarly, let T be the computational RUs of the V-BBU which can also be allocated to the accepted calls.

The m-th RRH ($m = 1, \ldots, M$) accommodates calls from K_m different service-classes. A call of service-class k ($k = 1, \ldots, K_m$) arrives to the m-th RRH according to a Poisson process with a mean arrival rate of $\lambda_{m,k}$ and requires $b^{\mathrm{r}}_{m,k}$ radio RUs and $b^{\mathrm{c}}_{m,k}$ computational RUs. In what follows, we assume that $b^{\mathrm{r}}_{m,k} = b^{\mathrm{c}}_{m,k}$. The call is accepted in the m-th RRH for a generally distributed service time with mean $\mu^{-1}_{m,k}$ if the required RUs are available at the time of its arrival, i.e., if the occupied radio RUs in the m-th RRH do not exceed the value of $C - b^{\mathrm{r}}_{m,k}$ and the occupied computational RUs do not exceed the value of $T - b^{\mathrm{c}}_{m,k}$. Otherwise, the call is blocked and lost without further affecting the system, i.e., the blocked call does not have the option to enter a queue or retry.

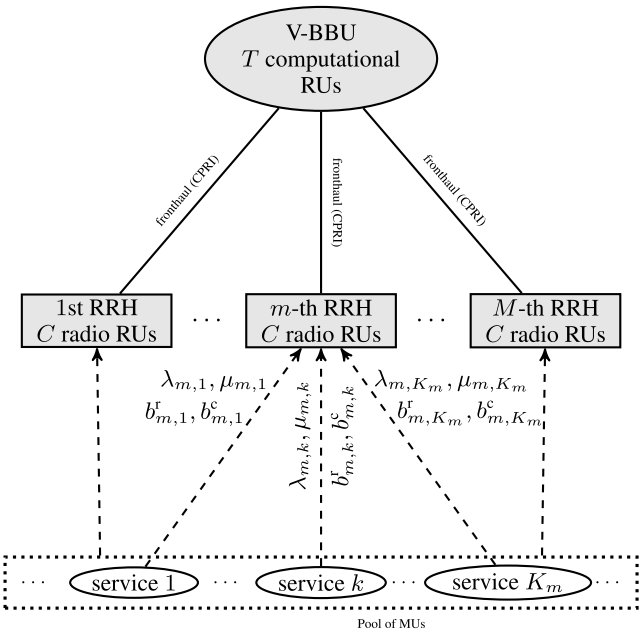

FIGURE 1. The MC-SC model.

To proceed with the analysis of the proposed MC-SC model let $\alpha_{m,k} = \lambda_{m,k}/\mu_{m,k}$ be the offered traffic-load (in erl) of service-class k calls in the m-th RRH and denote as $n_{m,k}$ the number of in-service calls of service-class k ($k = 1, \ldots, K_m$), where $n_{m,k} \geq 0$. Then, the steady-state vector $\boldsymbol{n} = (n_{1,1}, \ldots, n_{1,K_1}, \ldots, n_{m,1}, \ldots, n_{m,k}, \ldots, n_{m,K_m}, \ldots, n_{M,1}, \ldots, n_{M,K_M})$ expresses the number of all in-service calls of all service-classes in all RRHs. By further denoting as $\boldsymbol{\Omega}$ the system's state space, we can express the set of all possible states, via:

$$\boldsymbol{\Omega} = \left\{ \begin{array}{l} \boldsymbol{n} : n_{m,k} \geq 0, \sum_{k=1}^{K_m} n_{m,k} b^{\mathrm{r}}_{m,k} \leq C, \\ \sum_{m=1}^{M} \sum_{k=1}^{K_m} n_{m,k} b^{\mathrm{c}}_{m,k} \leq T \end{array} \right\}. \quad (1)$$

Our first target is to determine the steady-state probability distribution $P(\boldsymbol{n})$. To this end, we denote the additional steady-state vectors $\boldsymbol{n}^{-}_{m,k} = (n_{1,1}, \ldots, n_{1,K_1}, \ldots, n_{m,1}, \ldots, n_{m,k} - 1, \ldots, n_{m,K_m}, \ldots, n_{M,1}, \ldots, n_{M,K_M})$, $\boldsymbol{n}^{+}_{m,k} = (n_{1,1}, \ldots, n_{1,K_1}, \ldots, n_{m,1}, \ldots, n_{m,k} + 1, \ldots, n_{m,K_m}, \ldots, n_{M,1}, \ldots, n_{M,K_M})$ and let $P(\boldsymbol{n}^{-}_{m,k}), P(\boldsymbol{n}^{+}_{m,k})$ be the corresponding steady-state probability distributions. Assuming that the states $\boldsymbol{n}^{-}_{m,k}, \boldsymbol{n}, \boldsymbol{n}^{+}_{m,k}$ belong to the system's state space $\boldsymbol{\Omega}$, we show in Fig. 2 the state transition diagram for service-class k calls in the m-th RRH.

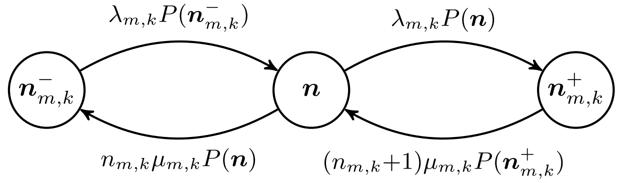

FIGURE 2. State transition diagram for service-class k calls in the m-th RRH of the MC-SC model.

Based on Fig. 2 and since the corresponding Markov chain for the m-th RRH is reversible, we have the following local balance equations (rate-up = rate-down) for the adjacent states: a) $\boldsymbol{n}^{-}_{m,k}$ and $\boldsymbol{n}$ (see (2)) and b) $\boldsymbol{n}$ and $\boldsymbol{n}^{+}_{m,k}$ (see (3)):

$$\lambda_{m,k} P(\boldsymbol{n}^{-}_{m,k}) = n_{m,k}\mu_{m,k} P(\boldsymbol{n}), \quad (2)$$

$$\lambda_{m,k} P(\boldsymbol{n}) = (n_{m,k} + 1)\mu_{m,k} P(\boldsymbol{n}^{+}_{m,k}). \quad (3)$$

In addition, we have the following global balance equation (rate-in = rate-out) for state $\boldsymbol{n}$:

$$\lambda_{m,k} P(\boldsymbol{n}^{-}_{m,k}) + (n_{m,k} + 1)\mu_{m,k} P(\boldsymbol{n}^{+}_{m,k}) = \lambda_{m,k} P(\boldsymbol{n}) + n_{m,k}\mu_{m,k} P(\boldsymbol{n}). \quad (4)$$

The system of (2), (3) and (4) is satisfied by the following PFS for $\boldsymbol{n} \in \boldsymbol{\Omega}$ and $m = 1, \ldots, M$:

$$P(\boldsymbol{n}) = \frac{1}{G} \prod_{m=1}^{M} \prod_{k=1}^{K_m} \frac{\alpha_{m,k}^{n_{m,k}}}{n_{m,k}!}, \quad (5)$$

where: $G \equiv G(\boldsymbol{\Omega}) = \sum_{\boldsymbol{n} \in \boldsymbol{\Omega}} \prod_{m=1}^{M} \prod_{k=1}^{K_m} \alpha_{m,k}^{n_{m,k}}/n_{m,k}!$ refers to the normalization constant and $\boldsymbol{\Omega}$ is the system's state space expressed via (1).

Having calculated $P(\boldsymbol{n})$ via (5) we can determine the CBP of service-class k calls in the m-th RRH. Call blocking occurs either due to insufficient radio RUs in the (serving) m-th RRH or due to insufficient computational RUs in the V-BBU. We distinguish these blocking events by denoting as $B_{\mathrm{r},m,k}$ the CBP of service-class k calls in the m-th RRH due to lack of radio RUs and as $B_{\mathrm{c},m,k}$ the CBP of service-class k calls in the m-th RRH due to lack of computational RUs. Then, the total CBP, $B_{\mathrm{tot},m,k}$, is expressed as follows:

$$B_{\mathrm{tot},m,k} = B_{\mathrm{r},m,k} + B_{\mathrm{c},m,k}. \quad (6)$$

The values of $B_{\mathrm{tot},m,k}$ can be calculated either via a brute force method or via a convolution algorithm (presented in Subsections II-B and II-C, respectively).

B. CBP BASED ON A BRUTE FORCE METHOD

The values of $B_{\mathrm{r},m,k}$ can be computed via (5) as follows:

$$B_{\mathrm{r},m,k} = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}^{C,<T}_{m,k}} P(\boldsymbol{n}), \tag{7}$$

where $\boldsymbol{\Omega}^{C,<T}_{m,k} = \{\boldsymbol{\Omega}^{C}_{m,k} \cap \boldsymbol{\Omega}^{<T}_{m,k}\}$, $\boldsymbol{\Omega}^{C}_{m,k} = \{\boldsymbol{n} : C - b^{\mathrm{r}}_{m,k} < \sum_{y=1}^{K_m} n_{m,y} b^{\mathrm{r}}_{m,y} \leq C\}$, $\boldsymbol{\Omega}^{<T}_{m,k} = \{\boldsymbol{n} : \sum_{x=1}^{M}\sum_{y=1}^{K_x} n_{x,y} b^{\mathrm{c}}_{x,y} \leq T - b^{\mathrm{c}}_{m,k}\}$.

The set $\boldsymbol{\Omega}^{C,<T}_{m,k}$ includes all those blocking states that refer to the unavailability of radio RUs and excludes those blocking states that refer to the lack of computational RUs.

Similarly, by denoting as $\boldsymbol{\Omega}^{T}_{m,k} = \{\boldsymbol{n} : T - b^{\mathrm{c}}_{m,k} < \sum_{x=1}^{M}\sum_{y=1}^{K_x} n_{x,y} b^{\mathrm{c}}_{x,y} \leq T\}$, the CBP of service-class k calls in the m-th RRH due to the lack of computational RUs can be computed via:

$$B_{\mathrm{c},m,k} = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}^{T}_{m,k}} P(\boldsymbol{n}). \tag{8}$$

The set $\boldsymbol{\Omega}^{T}_{m,k}$ includes all those blocking states that refer to the unavailability of computational RUs as well as blocking states that refer to both insufficient radio and computational RUs.

Computing $B_{\mathrm{r},m,k}$ and $B_{\mathrm{c},m,k}$ (and consequently $B_{\mathrm{tot},m,k}$) via (7) and (8) respectively, is accurate (compared to simulation) but complex especially for a system with many service-classes and RRHs of large capacities. This is because it is necessary to enumerate/process $\boldsymbol{\Omega}$ in order to obtain all blocking states (see also Appendix B for a tutorial example). Due to this, the brute force method can be useful for small (tutorial) examples. To circumvent this problem, a convolution algorithm is proposed in the next subsection that leads to the efficient CBP determination.

C. CBP BASED ON A CONVOLUTION ALGORITHM

The proposed convolution algorithm exploits the fact that the MC-SC model has a PFS and consists of the following three steps:

STEP 1

In this step, we determine the occupancy distribution of each RRH. To this end, we initially compute the occupancy distribution for each service-class k of the m-th RRH ($k = 1, \ldots, K_m$), $q_{m,k}(j)$, assuming that only calls of service-class k exist in the m-th RRH:

$$q_{m,k}(j) = \begin{cases} \frac{\alpha^{i}_{m,k}}{i!} q_{m,k}(0), & \text{for } 1 \leq i \leq \lfloor \frac{C}{b^{\mathrm{r}}_{m,k}} \rfloor, j = i b^{\mathrm{r}}_{m,k} \\ 1, & \text{for } j = 0 \\ 0, & \text{otherwise,} \end{cases} \tag{9}$$

where i expresses the number of in-service calls of service-class k in the m-th RRH and j the corresponding occupied radio RUs.

The values of $q_{m,k}(j)$ should be normalized via the constant $G_{m,k} = \sum_{j=0}^{C} q_{m,k}(j)$ and are denoted via $q'_{m,k}(j) = q_{m,k}(j)/G_{m,k}$.

Having determined the values of $q'_{m,k}(j)$, we calculate the aggregated occupancy distribution of the m-th RRH excluding calls of the first service-class, $Q^{m}_{(-1)}$:

$$Q^{m}_{(-1)} = q'_{m,2} * \ldots * q'_{m,k} * \ldots * q'_{m,K_m}, \tag{10}$$

where the convolution operation between two normalized distributions $q'_{m,v}$ and $q'_{m,w}$ is given by:

$$q'_{m,v} * q'_{m,w} = \left\{ \begin{array}{l} q'_{m,v}(0)\cdot q'_{m,w}(0), \sum_{x=0}^{1} q'_{m,v}(x)\cdot q'_{m,w}(1-x), \\ \ldots, \sum_{x=0}^{C} q'_{m,v}(x)\cdot q'_{m,w}(C-x) \end{array} \right\} \tag{11}$$

Finally, to determine the normalized occupancy distribution of the m-th RRH, q'_m, we proceed with the convolution operation $q_m = Q^{m}_{(-1)} * q'_{m,1}$ and the normalization $q'_m(j) = q_m(j)/G_m$ where $G_m = \sum_{j=0}^{C} q_m(j)$.

STEP 2

In this step, we determine the aggregated occupancy distribution of all RRHs apart from the m-th one, via the formula:

$$Q_{(-m)} = q'_1 * \ldots * q'_{m-1} * q'_{m+1} * \ldots * q'_M. \tag{12}$$

The convolution operation between two occupancy distributions q'_v and q'_w is given by:

$$q'_v * q'_w = \left\{ \begin{array}{l} q'_v(0)\cdot q'_w(0), \sum_{x=0}^{1} q'_v(x)\cdot q'_w(1-x), \\ \ldots, \sum_{x=0}^{T} q'_v(x)\cdot q'_w(T-x) \end{array} \right\}. \tag{13}$$

The normalized values of $Q_{(-m)}(j)$, denoted as $Q'_{(-m)}(j)$, can be obtained via the formula $Q'_{(-m)}(j) = Q_{(-m)}(j)/G_{(-m)}$ where $G_{(-m)} = \sum_{j=0}^{T} Q_{(-m)}(j)$.

STEP 3

In this step, we initially determine the convolution operation $Q'_{(-m)} * q'_m$. This operation results to the unnormalized values of $Q_m(j)$ which can be normalized via the constant $G^{*}_m = \sum_{j=0}^{T} Q_m(j)$, resulting in:

$$Q'(j) = \frac{Q_m(j)}{G^{*}_m}. \tag{14}$$

To obtain the values of $Q'(j)$, we may adopt any of the M RRHs since all of them have the same capacity and the same occupancy distribution.

Having determined the computational occupancy distribution, we can calculate the CBP due to lack of computational RUs and radio RUs via (15) and (16), respectively:

$$B_{\mathrm{c},m,k} = \sum_{j=T-b^{\mathrm{c}}_{m,k}+1}^{T} Q'(j), \tag{15}$$

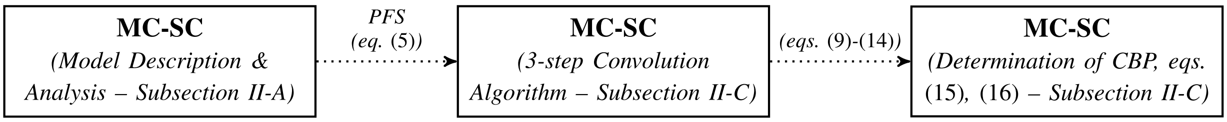

FIGURE 3. **The proposed methodology in the MC-SC model.**

$$B_{\mathrm{r},m,k} = \frac{1}{G^*_m} \sum_{x=C-b^{\mathrm{r}}_{m,k}+1}^{C} q'_m(x) \sum_{y=x}^{T-b^{\mathrm{r}}_{m,k}} Q'_{(-m)}(T-b^{\mathrm{r}}_{m,k}-y). \tag{16}$$

Figure 3 summarizes the proposed methodology which is essential for the CBP determination in the MC-SC model. Initially, it is important to determine the steady-state probabilities via a PFS (note that in non-PFS models, the application of convolution algorithms can become quite complex [36], [38], [41]). Since the proposed model can be analyzed with the aid of a reversible Markov chain, we obtain the PFS of (5). Based on (5), a convolution algorithm is proposed for the exact CBP determination.

III. EVALUATION

In this section, we present a C-RAN application example and provide not only analytical but also simulation CBP results in the case of the proposed MC-SC model as well as analytical results for the existing SC-SC model of [16]. Simulation results, which are mean values of seven runs, are obtained via the simulation tool of SIMSCRIPT III [42]. In every run, we allow the generation of two hundred million calls. The initial 5% of these calls is not considered in the CBP results in order to have a warm-up period [43], [44]. The main points of the evaluation section that are examined are the following: a) how close the analytical CBP results are (depicted in line-format in all figures) with the corresponding simulation CBP results (depicted in dot or triangle format in all figures), b) how the increase in offered traffic-load affects CBP and c) how close are the CBP results of the existing SC-SC model with those of the proposed MC-SC model.

In the C-RAN application example, we consider $M = 6$ RRHs of capacity $C = 10$ radio RUs. As far as the computational RUs is concerned, we assume that $T = 30$ RUs. The m-th RRH ($m = 1, \ldots, 6$) serves calls from K_m different service-classes and let $b_{m,k} = b^{\mathrm{c}}_{m,k} = b^{\mathrm{r}}_{m,k}$ be the amount of RUs required by a service-class k call in the m-th RRH. More specifically, the first RRH serves calls from $K_1 = 3$ service-classes. Calls of the first service-class require $b_{1,1} = 1$ RU, calls of the second service-class require $b_{1,2} = 2$ RUs while calls of the third service-class require $b_{1,3} = 3$ RUs. Similarly, the second RRH serves calls from $K_2 = 2$ service-classes, with $b_{2,1} = 2$ RUs and $b_{2,2} = 3$ RUs. On the same hand, the third RRH serves calls from $K_3 = 2$ service-classes, with $b_{3,1} = 1$ RU and $b_{3,2} = 3$ RUs, the fourth RRH serves calls from $K_4 = 2$ service-classes, with $b_{4,1} = 1$ RU and $b_{4,2} = 2$ RUs, the fifth RRH serves calls from only a single service-class with $b_{5,1} = 2$ RUs and finally the sixth RRH serves also calls from a single service-class with $b_{6,1} = 1$ RU. Regarding the offered traffic-load, we initially assume that $\alpha_{m,k} = 1$ erl for all calls in all RRHs (point 1 in the x-axis of Figs. 4-10). Table 2 summarizes the traffic parameters used for the simulation and analytical CBP results of this example.

TABLE 2. **Traffic description parameters of the C-RAN application example.**

$M = 6$ RRHs; $C = 10$ radio RUs; $T = 30$ computational RUs			
	service-class k	band Requirement (RUs)	initial offered traffic load (erl)
1st RRH (K_1=3)	1	$b_{1,1} = 1$	$\alpha_{1,1} = 1$
	2	$b_{1,2} = 2$	$\alpha_{1,2} = 1$
	3	$b_{1,3} = 3$	$\alpha_{1,3} = 1$
2nd RRH (K_2=2)	1	$b_{2,1} = 2$	$\alpha_{2,1} = 1$
	2	$b_{2,2} = 3$	$\alpha_{2,2} = 1$
3rd RRH (K_3=2)	1	$b_{3,1} = 1$	$\alpha_{3,1} = 1$
	2	$b_{3,2} = 3$	$\alpha_{3,2} = 1$
4th RRH (K_4=2)	1	$b_{4,1} = 1$	$\alpha_{4,1} = 1$
	2	$b_{4,2} = 2$	$\alpha_{4,2} = 1$
5th RRH (K_5=1)	1	$b_{5,1} = 2$	$\alpha_{5,1} = 1$
6th RRH (K_6=1)	1	$b_{6,1} = 1$	$\alpha_{6,1} = 1$

We evaluate the C-RAN, by calculating the corresponding CBP for 31 steps, where in each step the offered traffic-load increases by 0.2 erl. Thus, in the last step, we have $\alpha_{m,k} = 7$ erl for all calls in all RRHs (point 31 in the x-axis of Figs 4-10).

For comparison, we consider the SC-SC model of [16] where all calls in all RRHs require a single RU. Since the SC-SC model does not support the case of many service-classes whose calls require a different number of RUs, it is necessary for our comparison to determine the offered traffic-load α_m for every RRH of the SC-SC model. To this end, we consider a load factor l_m ($m = 1, \ldots, 6$), given by $l_m = \sum_{k=1}^{K_m} b_{m,k}$ where K_m and $b_{m,k}$ are the corresponding values of the MC-SC model. The initial values of the offered traffic-load α_m are equal to the corresponding value of l_m, i.e.,: $(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) = (6, 5, 4, 3, 2, 1)$. Similarly to the MC-SC model, we consider 31 steps where in each step the values of α_m are increased by $0.2 l_m$. Thus in the last step, we have $(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) = (42, 35, 28, 21, 14, 7)$.

In Fig. 4, we present the analytical and simulation CBP results ($B_{\mathrm{c},m,k}$), due to lack of computational RUs, of the MC-SC model and the corresponding analytical CBP results (B_{c}) of the SC-SC model. Based on Fig. 4, we observe that: a) analytical and simulation results are almost identical, b) the increase of offered traffic-load results in an increase of the corresponding CBP and c) the SC-SC model cannot capture the behavior of the MC-SC model since the latter refers to a multiservice loss model.

In Figs. 5-7, we consider the first, the second and the third RRH, respectively. More specifically, in Fig. 5, we consider the first RRH and present the analytical and simulation CBP

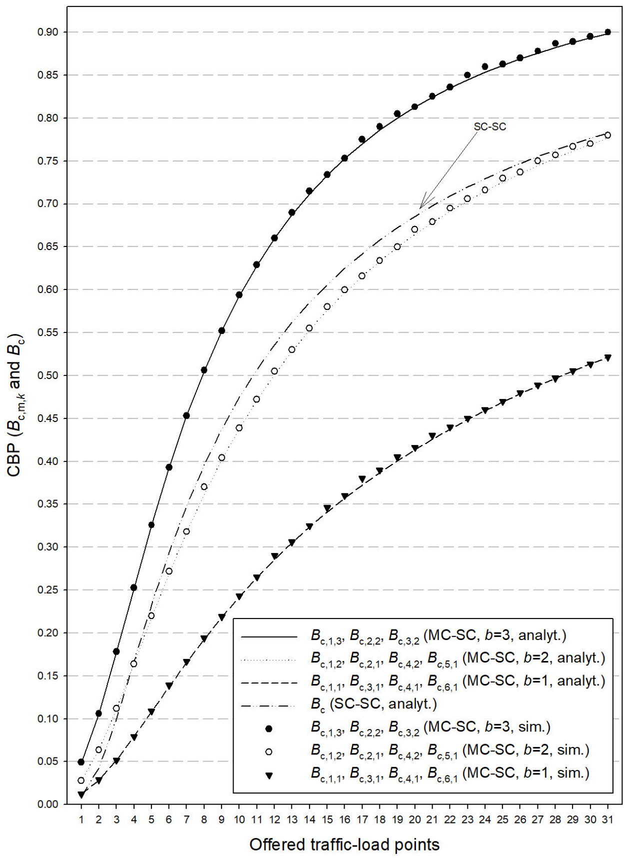

FIGURE 4. CBP due to lack of computational RUs.

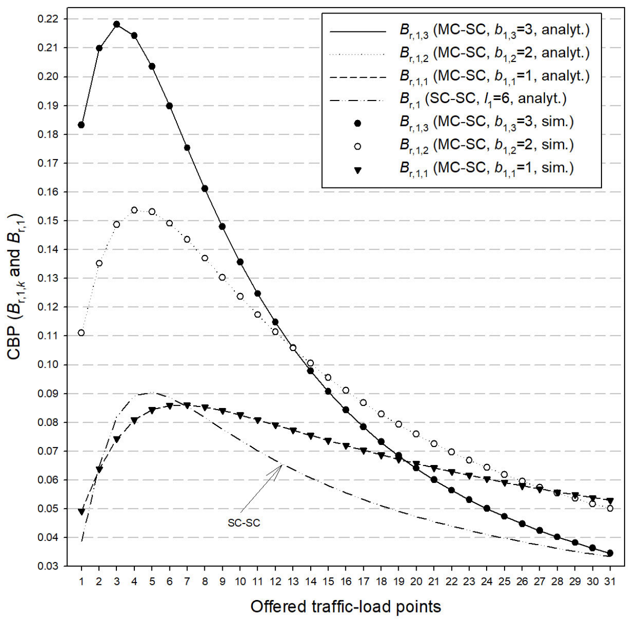

FIGURE 5. CBP due to lack of radio RUs in the 1st RRH.

results of $B_{r,1,k}$ for the three service-classes of the MC-SC model and the corresponding analytical CBP results for the SC-SC model. In the case of the SC-SC model, point 1 in the x-axis of Fig. 5 refers to $\alpha_1 = 6$ erl while point 31

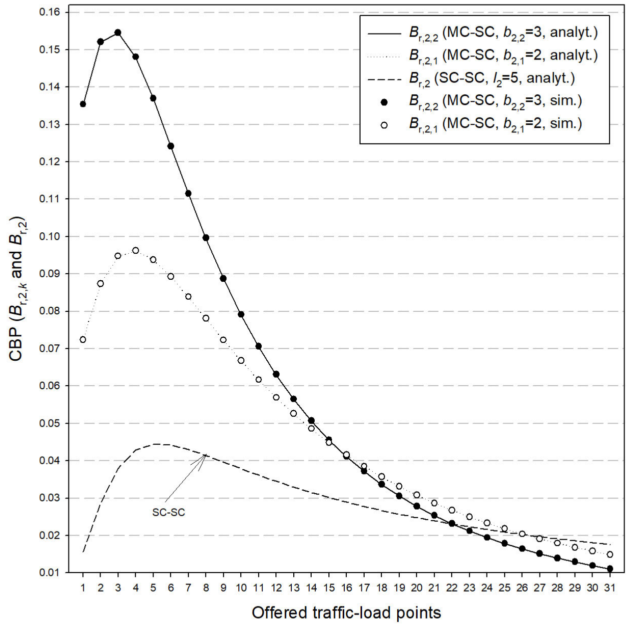

FIGURE 6. CBP due to lack of radio RUs in the 2nd RRH.

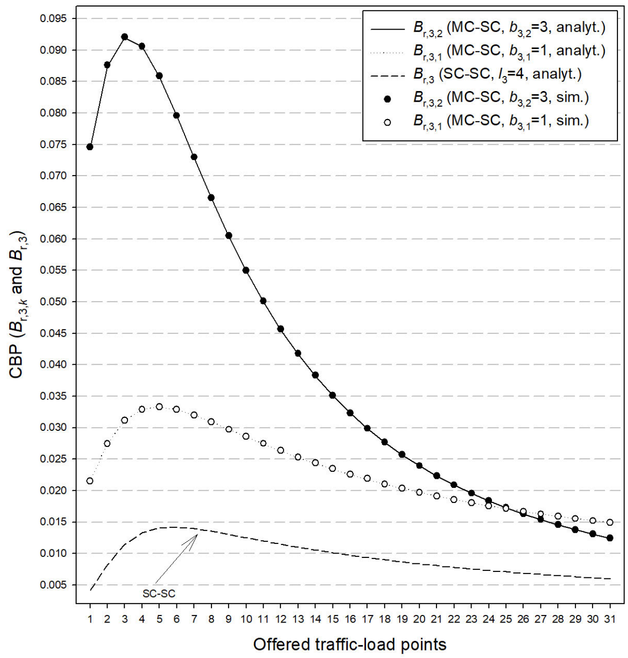

FIGURE 7. CBP due to lack of radio RUs in the 3rd RRH.

refers to $\alpha_1 = 42$ erl. Similarly, in Fig. 6, we consider the second RRH and present the analytical and simulation CBP results of $B_{r,2,k}$ for the two service-classes of the MC-SC model and the corresponding analytical CBP results for the SC-SC model. In the case of the SC-SC model, point 1 in the x-axis of Fig. 6 refers to $\alpha_2 = 5$ erl while point 31 refers to $\alpha_2 = 35$ erl. Finally, in Fig. 7, we consider the third RRH and present the analytical and simulation CBP results of $B_{r,3,k}$ for the two service-classes of the MC-SC model and the corresponding analytical CBP results for the SC-SC model.

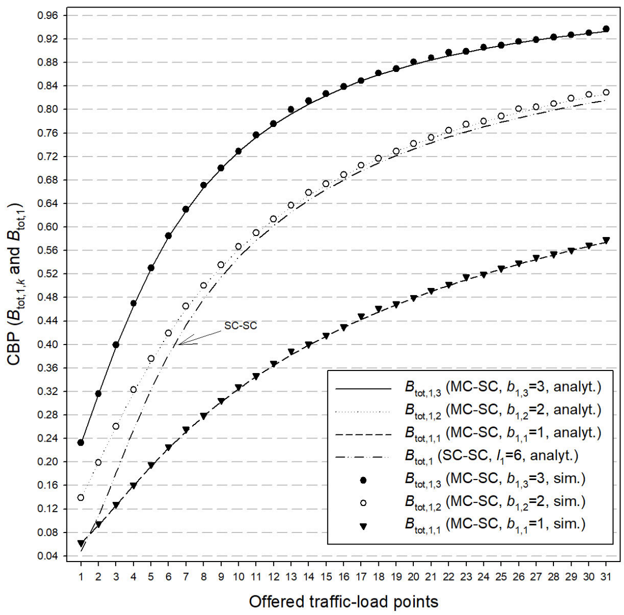

FIGURE 8. Total CBP (1st RRH – all service-classes).

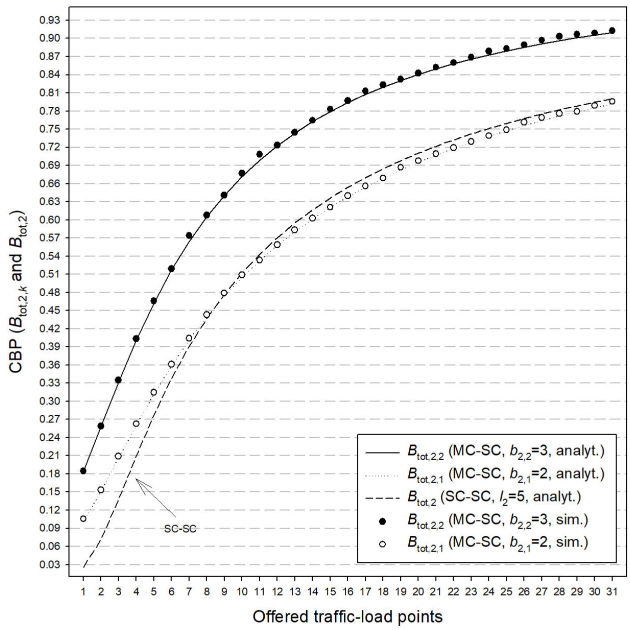

FIGURE 9. Total CBP (2nd RRH – all service-classes).

In the case of the SC-SC model, point 1 in the x-axis of Fig. 7 refers to $\alpha_3 = 4$ erl while point 31 refers to $\alpha_3 = 28$ erl. According to Figs. 5-7, we observe that: a) analytical and simulation CBP results (due to insufficient radio RUs) are almost identical in the case of the MC-SC model which is anticipated since the MC-SC model has a PFS for the steady-state probabilities, b) the CBP results obtained via the SC-SC model cannot be adopted in order to capture the behavior of the MC-SC model and c) the CBP results (due to lack of radio RUs) increase as the offered traffic-load increases but after a point they start to decrease. This behavior can be justified by the fact that the CBP results due to lack of computational RUs, already presented in Fig. 4, increase as the offered traffic-load increases and therefore more radio RUs become available in the corresponding RRHs. Similar conclusions have been drawn for the fourth, fifth and sixth RRH, and therefore the corresponding CBP results are not presented herein

In Figs. 8-10, we consider again the first, the second and the third RRH, respectively, and present the corresponding values of the total CBP (determined via (6) with the aid of (15), (16)). Based on Figs. 8-10, we observe that: a) analytical and simulation results are almost identical, b) the increase of offered traffic-load results in an increase of the corresponding total CBP and c) the SC-SC model cannot capture the behavior of the MC-SC model.

As a final comment we point out that the accuracy of the analytical CBP results (compared to simulation) has been tested for various traffic description parameters and not only those presented in Table 2. In all cases, analytical and simulation results are almost identical a fact that could be anticipated since the MC-SC model has a PFS for the steady state probabilities.

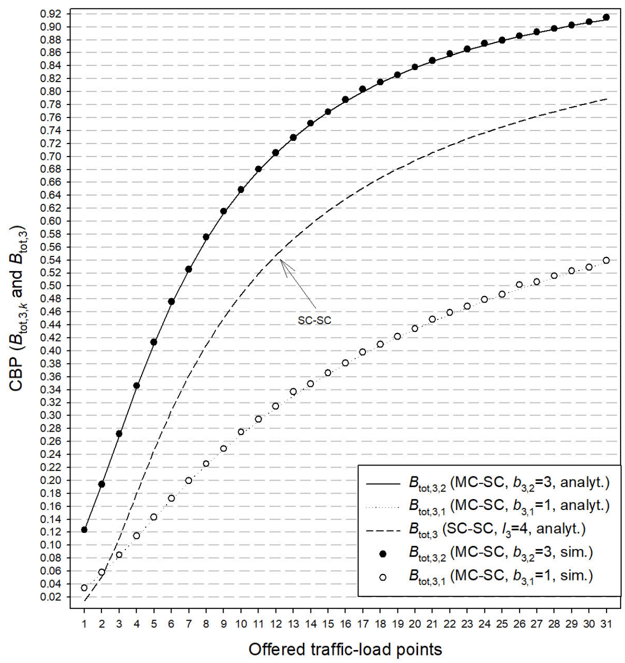

FIGURE 10. Total CBP (3rd RRH – all service-classes).

IV. GENERALIZATION - THE MC-MC MODEL

In this section, we propose the MC-MC model which generalizes the proposed MC-SC model by assuming that RRHs can be grouped in clusters according to their capacity in radio RUs.

A. THE ANALYTICAL MODEL

Consider the C-RAN of Fig. 11 where the RRHs are separated from the centralized V-BBU (of capacity T computational

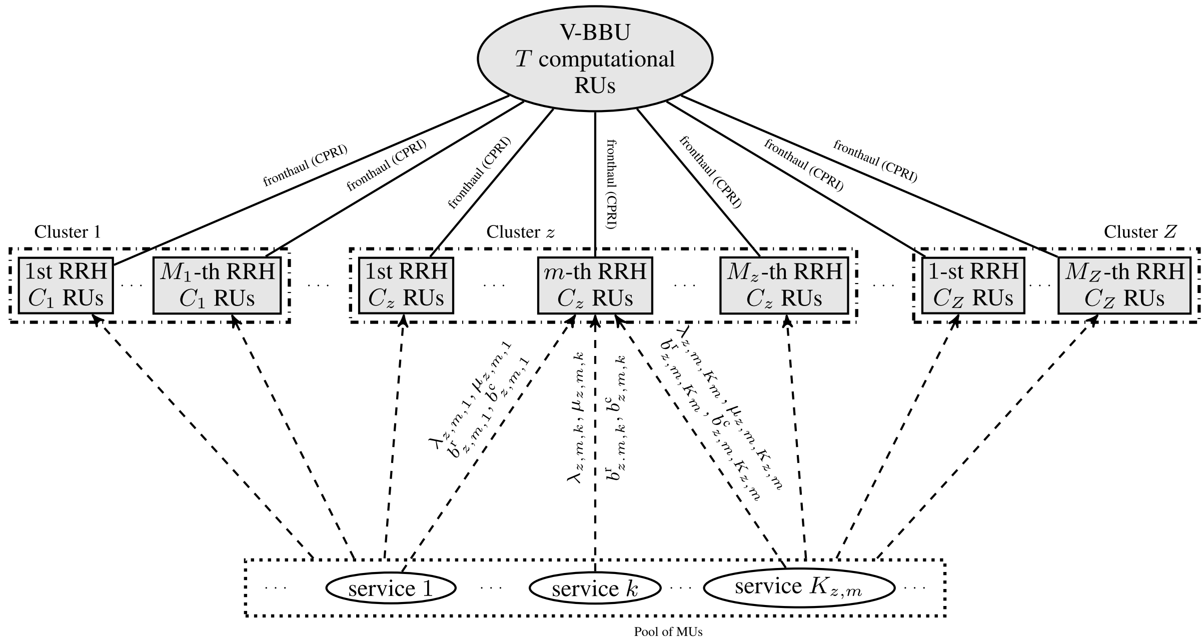

FIGURE 11. The MC-MC model.

RUs) and are grouped into Z different clusters. Cluster z ($z = 1, \ldots, Z$) includes M_z RRHs whose capacity is C_z radio RUs.

The m-th RRH of cluster z ($m = 1, \ldots, M_z$) accommodates Poisson arriving calls from $K_{z,m}$ different service-classes. A call of service-class k ($k = 1, \ldots, K_{z,m}$) arrives to the z, m-th RRH according to a Poisson process with a mean arrival rate of $\lambda_{z,m,k}$ and requires $b^{\mathrm{r}}_{z,m,k}$ radio RUs and $b^{\mathrm{c}}_{z,m,k}$ computational RUs, with $b^{\mathrm{r}}_{z,m,k} = b^{\mathrm{c}}_{z,m,k}$. An accepted call remains in the z, m-th RRH for a generally distributed service time with mean $\mu^{-1}_{z,m,k}$ if the required RUs are available at the time of the call's arrival, i.e., if the occupied radio RUs in the z, m-th RRH do not exceed the value of $C_z - b^{\mathrm{r}}_{z,m,k}$ and the occupied computational RUs do not exceed the value of $T - b^{\mathrm{c}}_{z,m,k}$. Otherwise, the call is blocked and lost.

Let $n_{z,m,k} \geq 0$ be the number of in-service calls of service-class k ($k = 1, \ldots, K_{z,m}$) in the z, m-th RRH. Then, the number of all in-service calls of all service-classes in all RRHs can be expressed by the steady-state vector $\boldsymbol{n} = (n_{1,1,1}, \ldots, n_{1,1,K_{1,1}}, \ldots, n_{z,m,1}, \ldots, n_{z,m,k}, \ldots, n_{z,m,K_{z,m}}, \ldots, n_{Z,M_Z,1}, \ldots, n_{Z,M_Z,K_{Z,M_Z}})$ and the system's state space $\boldsymbol{\Omega}$ can be expressed via:

$$\boldsymbol{\Omega} = \left\{ \begin{array}{l} \boldsymbol{n} : n_{z,m,k} \geq 0, \sum_{k=1}^{K_{z,m}} n_{z,m,k} b^{\mathrm{r}}_{z,m,k} \leq C_z, \\ \sum_{z=1}^{Z} \sum_{m=1}^{M_z} \sum_{k=1}^{K_{z,m}} n_{z,m,k} b^{\mathrm{c}}_{z,m,k} \leq T \end{array} \right\}. \tag{17}$$

To determine the steady-state probability distribution $P(\boldsymbol{n})$, the following additional steady-state vectors are required: $\boldsymbol{n}^-_{z,m,k} = (n_{1,1,1}, \ldots, n_{1,1,K_{1,1}}, \ldots, n_{z,m,1}, \ldots, n_{z,m,k} - 1, \ldots, n_{z,m,K_{z,m}}, \ldots, n_{Z,M_Z,1}, \ldots, n_{Z,M_Z,K_{Z,M_Z}})$,

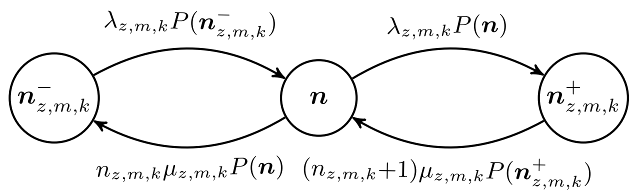

FIGURE 12. State transition diagram for service-class k calls in the z, m-th RRH (MC-MC model).

$\boldsymbol{n}^+_{z,m,k} = (n_{1,1,1}, \ldots, n_{1,1,K_{1,1}}, \ldots, n_{z,m,1}, \ldots, n_{z,m,k} + 1, \ldots, n_{z,m,K_{z,m}}, \ldots, n_{Z,M_Z,1}, \ldots, n_{Z,M_Z,K_{Z,M_Z}})$. In addition, let $P(\boldsymbol{n}^-_{z,m,k})$, $P(\boldsymbol{n}^+_{z,m,k})$ be the corresponding steady-state probability distributions. By further assuming that $\boldsymbol{n}^-_{z,m,k}, \boldsymbol{n}, \boldsymbol{n}^+_{z,m,k}$ belong to the system's state space $\boldsymbol{\Omega}$, Fig. 12 presents the state transition diagram for service-class k calls in the z, m-th RRH.

Based on Fig. 12, we have the following global balance equation (rate-in = rate-out) for state $\boldsymbol{n}$ (see (18)) as well as the following local balance equations (rate-up = rate-down) for the adjacent states: a) $\boldsymbol{n}^-_{z,m,k}$ and $\boldsymbol{n}$ (see (19)) and b) $\boldsymbol{n}$ and $\boldsymbol{n}^+_{z,m,k}$ (see (20)):

$$\begin{aligned} \lambda_{z,m,k} P(\boldsymbol{n}^-_{z,m,k}) + (n_{z,m,k} + 1)\mu_{z,m,k} P(\boldsymbol{n}^+_{z,m,k}) \\ = \lambda_{z,m,k} P(\boldsymbol{n}) + n_{z,m,k}\mu_{z,m,k} P(\boldsymbol{n}), \end{aligned} \tag{18}$$

$$\lambda_{z,m,k} P(\boldsymbol{n}^-_{z,m,k}) = n_{z,m,k}\mu_{z,m,k} P(\boldsymbol{n}), \tag{19}$$

$$\lambda_{z,m,k} P(\boldsymbol{n}) = (n_{z,m,k} + 1)\mu_{z,m,k} P(\boldsymbol{n}^+_{z,m,k}). \tag{20}$$

The system of (18), (19) and (20) is satisfied by the PFS of (21) for $\boldsymbol{n} \in \boldsymbol{\Omega}$, $z = 1, \ldots, Z$ and $m = 1, \ldots, M_z$:

$$P(\boldsymbol{n}) = \frac{1}{G} \prod_{z=1}^{Z} \prod_{m=1}^{M_z} \prod_{k=1}^{K_{z,m}} \frac{\alpha^{n_{z,m,k}}_{z,m,k}}{n_{z,m,k}!}, \tag{21}$$

where: $\alpha_{z,m,k} = \lambda_{z,m,k}/\mu_{z,m,k}$ is the offered traffic-load (in erl) for calls of the service-class k in the z, m-th RRH, $G \equiv G(\mathbf{\Omega}) = \sum_{\boldsymbol{n}\in\mathbf{\Omega}} \prod_{z=1}^{Z} \prod_{m=1}^{M} \prod_{k=1}^{K_{z,m}} \alpha_{z,m,k}^{n_{z,m,k}}/n_{z,m,k}!$ refers to the normalization constant.

Having calculated the values of $P(\boldsymbol{n})$ via (21), we can determine the CBP of service-class k calls in the z, m-th RRH: a) due to lack of radio RUs, $B_{\mathrm{r},z,m,k}$ and b) due to lack of computational RUs, $B_{\mathrm{c},z,m,k}$. In addition, we can compute the total CBP, $B_{\mathrm{tot},z,m,k}$, as follows:

$$B_{\mathrm{tot},z,m,k} = B_{\mathrm{r},z,m,k} + B_{\mathrm{c},z,m,k}. \tag{22}$$

The values of $B_{\mathrm{tot},z,m,k}$ can be determined either via a brute force method (see Subsection IV-B) or via a convolution algorithm (see Subsection IV-C).

B. THE BRUTE FORCE EVALUATION METHOD

The values of $B_{\mathrm{r},z,m,k}$ can be computed via (23):

$$B_{\mathrm{r},z,m,k} = \sum_{\boldsymbol{n}\in\mathbf{\Omega}_{z,m,k}^{C_z,<T}} P(\boldsymbol{n}), \tag{23}$$

where $\mathbf{\Omega}_{z,m,k}^{C_z,<T} = \{\mathbf{\Omega}_{z,m,k}^{C_z} \cap \mathbf{\Omega}_{z,m,k}^{<T}\}$, $\mathbf{\Omega}_{z,m,k}^{C_z} = \{\boldsymbol{n} : C_z - b_{z,m,k}^{\mathrm{r}} < \sum_{w=1}^{K_{z,m}} n_{z,m,w} b_{z,m,w}^{\mathrm{r}} \leq C_z\}$, $\mathbf{\Omega}_{z,m,k}^{<T} = \{\boldsymbol{n} : \sum_{x=1}^{Z} \sum_{y=1}^{M_x} \sum_{w=1}^{K_{x,y}} n_{x,y,w} b_{x,y,w}^{\mathrm{c}} \leq T - b_{z,m,k}^{\mathrm{c}}\}$. Note that the set $\mathbf{\Omega}_{z,m,k}^{C_z,<T}$ includes the blocking states that refer to the lack of radio RUs and excludes the blocking states that refer to the lack of computational RUs.

Similarly, by denoting $\mathbf{\Omega}_{z,m,k}^{T} = \{\boldsymbol{n} : T - b_{z,m,k}^{\mathrm{c}} < \sum_{x=1}^{Z} \sum_{y=1}^{M_x} \sum_{w=1}^{K_{x,y}} n_{x,y,w} b_{x,y,w}^{\mathrm{c}} \leq T\}$, the set that includes all those blocking states that refer to the unavailability of computational RUs as well as the blocking states that refer to both insufficient radio and computational RUs, we can compute the CBP of service-class k calls due to lack of computational RUs via:

$$B_{\mathrm{c},z,m,k} = \sum_{\boldsymbol{n}\in\mathbf{\Omega}_{z,m,k}^{T}} P(\boldsymbol{n}). \tag{24}$$

To circumvent the enumeration/processing of $\mathbf{\Omega}$ which is essential for the implementation of the brute force method we propose a convolution algorithm in the next subsection that leads to the efficient CBP determination.

C. CBP BASED ON A CONVOLUTION ALGORITHM

The proposed convolution algorithm is based on the PFS of the MC-MC model and consists of the following three steps:

STEP 1

In this step, the occupancy distribution of each RRH is computed. To this end, we initially compute the occupancy distribution for each service-class k of the z, m-th RRH ($k = 1, \ldots, K_{z,m}$, $m = 1, \ldots, M_z$, $z = 1, \ldots, Z$), $q_{z,m,k}(j)$, assuming that only calls of service-class k exist in the z, m-th RRH:

$$q_{z,m,k}(j) = \begin{cases} \frac{\alpha_{z,m,k}^{i}}{i!} q_{z,m,k}(0), & \text{for } 1 \leq i \leq \lfloor \frac{C_z}{b_{z,m,k}^{\mathrm{r}}} \rfloor, \\ & j = i b_{z,m,k}^{\mathrm{r}} \\ 1, & \text{for } j = 0 \\ 0, & \text{otherwise,} \end{cases} \tag{25}$$

where i expresses the number of in-service calls of service-class k in the z, m-th RRH and j the corresponding occupied radio RUs. The values of $q_{z,m,k}(j)$ should be normalized via $G_{z,m,k} = \sum_{j}^{C_z} q_{z,m,k}(j)$ and are denoted via $q'_{z,m,k}(j) = q_{z,m,k}(j)/G_{z,m,k}$.

Having computed the values of $q'_{z,m,k}(j)$, we proceed with the determination of the aggregated occupancy distribution of the z, m-th RRH excluding the first service-class calls, $Q_{(-1)}^{z,m}$:

$$Q_{(-1)}^{z,m} = q'_{z,m,2} * \ldots * q'_{z,m,k} * \ldots * q'_{z,m,K_{z,m}}, \tag{26}$$

where the convolution operation between $q'_{z,m,v} \equiv q'_a$ and $q_{z,m,w} \equiv q'_b$ is computed via:

$$q'_a * q'_b = \left\{ \begin{array}{l} q'_a(0)\cdot q'_b(0), \sum_{x=0}^{1} q'_a(x)\cdot q'_b(1-x), \\ \ldots, \sum_{x=0}^{C_z} q'_a(x)\cdot q'_b(C_z - x) \end{array} \right\}. \tag{27}$$

Finally, the computation of the normalized occupancy distribution of the z, m-th RRH, $q'_{z,m}$, is based on the convolution operation $q_{z,m} = Q_{(-1)}^{z,m} * q'_{z,m,1}$ and the normalization $q'_{z,m}(j) = q_{z,m}(j)/G_{z,m}$ where $G_{z,m} = \sum_{j=0}^{C_z} q_{z,m}(j)$.

STEP 2

In this step, we proceed with the determination of the aggregated occupancy distribution of all RRHs apart from the z, m-th one, via:

$$Q_{(-(z,m))} = q'_{1,1} * \ldots * q'_{z,m-1} * q'_{z,m+1} * \ldots * q'_{Z,M_Z}. \tag{28}$$

The convolution operation between two occupancy distributions q'_v and q'_w is given by (13), while the normalized values of $Q_{(-(z,m))}(j)$, denoted as $Q'_{(-(z,m))}(j)$, can be obtained via $Q'_{(-(z,m))}(j) = Q_{(-(z,m))}(j)/G_{(-(z,m))}$ where $G_{(-(z,m))} = \sum_{j=0}^{T} Q_{(-(z,m))}(j)$.

STEP 3

In this step, we initially determine the convolution operation $Q'_{(-(z,m))} * q'_{z,m}$. This operation leads to the unnormalized values of $Q_{z,m}(j)$ which can be normalized via $G_{z,m}^{*} = \sum_{j=0}^{T} Q_{z,m}(j)$, resulting in:

$$Q'(j) = \frac{Q_{z,m}(j)}{G_{z,m}^{*}}. \tag{29}$$

To determine the values of $Q'(j)$, we may consider any of the z, m RRHs since all of them have the same capacity (C_z) and the same occupancy distribution.

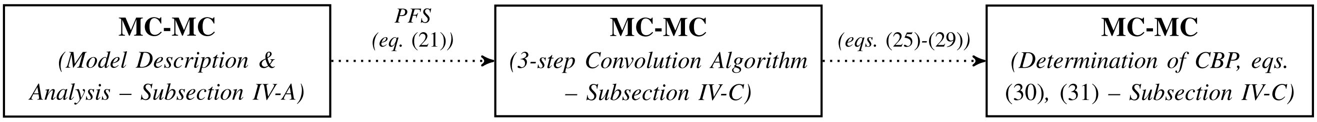

FIGURE 13. The proposed methodology in the MC-MC model.

TABLE 3. CBP comparison between the MC-SC and the MC-MC models.

MC-SC, $C=10$					MC-MC, $C_1=12, C_2=11, C_3=4$					
m	k	$B_{\mathrm{r},m,k}$	$B_{\mathrm{c},m,k}$	$B_{\mathrm{tot},m,k}$	z	m	k	$B_{\mathrm{r},z,m,k}$	$B_{\mathrm{c},z,m,k}$	$B_{\mathrm{tot},z,m,k}$
1	1	0.049200208	0.011931635	0.061131843	1	1	1	0.022287523	0.016665128	0.038952651
	2	0.110855337	0.027972131	0.138827468			2	0.051007543	0.037932006	0.088939549
	3	0.183083340	0.048950372	0.232033712			3	0.086377887	0.064447276	0.150825163
2	1	0.072262721	0.027972131	0.100234852	2	1	1	0.049483937	0.037932006	0.087415943
	2	0.135298147	0.048950372	0.184248519			2	0.081710708	0.064447276	0.146157984
3	1	0.021507273	0.011931635	0.033438908		2	1	0.008714734	0.016665128	0.025379862
	2	0.074455583	0.048950372	0.123405955			2	0.048915230	0.064447276	0.113362506
4	1	0.003820514	0.011931635	0.015752149		3	1	0.001495769	0.016665128	0.018160897
	2	0.011160787	0.027972131	0.039132918			2	0.004623877	0.037932006	0.042555883
5	1	0.001936090	0.027972131	0.029908221		4	1	0.001805142	0.037932006	0.039737148
6	1	0.000000064	0.011931635	0.011931699	3	1	1	0.013933326	0.016665128	0.030598454

Based on (29), we can now compute the CBP due to lack of computational RUs and radio RUs via (30) and (31), respectively:

$$B_{\mathrm{c},z,m,k} = \sum_{j=T-b^{\mathrm{c}}_{z,m,k}+1}^{T} Q'(j), \tag{30}$$

$$B_{\mathrm{r},z,m,k}) = \frac{1}{G^{*}_{z,m}} \sum_{x=C_z-b^{\mathrm{r}}_{z,m,k}+1}^{C_z} q'_{z,m}(x) \times \sum_{y=x}^{T-b^{\mathrm{r}}_{z,m,k}} Q'_{(-(z,m))}(T-b^{\mathrm{r}}_{z,m,k}-y). \tag{31}$$

Figure 13 summarizes the proposed methodology which is essential for the CBP determination in the MC-MC model. The method is analogous to the one shown in Figure 3. Initially, it is important to determine the steady-state probabilities via a PFS. Since the proposed model can be analyzed with the aid of a reversible Markov chain, we obtain the PFS of (21). Based on the PFS, a convolution algorithm is proposed for the exact CBP determination.

D. AN APPLICATION EXAMPLE

Contrary to the MC-SC model, in the MC-MC model RRHs can be grouped in clusters according to their capacity in radio RUs. The purpose of the following application example is to show how the existence of different clusters may affect CBP.

In what follows, we consider again the C-RAN example of Section III whose traffic parameters per service-class are summarized in Table 2. In that example, we assume that the six RRHs have the same capacity $C = 10$ radio RUs while $T = 30$ computational RUs. In the case of the MC-MC model, we consider again the same six RRHs (servicing the same service-classes) but we assume that $Z = 3$ clusters exist. The first cluster ($z = 1$) includes only the first RRH whose capacity is now $C_1 = 12$ RUs. The second cluster ($z = 2$) includes the next four RRHs whose capacity is now $C_2 = 11$ RUs. Finally, the third cluster ($z = 3$) includes the sixth RRH whose capacity is now $C_3 = 4$ RUs. The choice of C_1, C_2 and C_3 is based on the following criteria: a) the total amount of radio RUs in both the MC-SC and the MC-MC models remains the same, i.e., 60 radio RUs and b) the capacity of the first RRH has been increased (from 10 to 12 radio RUs) since this RRH accommodates calls from three service-classes, while the capacity of the last RRH has been decreased (from 10 to 4 radio RUs) since it accommodates only calls of a single service-class with $b_{3,1,1} = 1$ RU.

Assuming that the offered traffic-load, $\alpha_{z,m,k} = 1$ erl for all service-classes in all RRHs, Table 3 presents the analytical CBP results for both models and all service-classes in all RRHs. Based on Table 3, we see that: a) the MC-SC model cannot capture the behavior of the MC-MC model, b) even a slight change in the values of C_z can affect the CBP due to lack of radio RUs ($B_{\mathrm{r},z,m,k}$) or computational RUs ($B_{\mathrm{c},z,m,k}$). Besides, we would like to point out that the CBP behavior in the MC-MC model when the offered traffic-load increases is similar to that of the MC-SC model (see Section III) while the simulation CBP results of the MC-MC model are almost identical to the corresponding analytical CBP results and therefore are not presented herein.

V. CONCLUSION

In this paper, we propose two multirate loss models, the MC-SC and the MC-MC models, for the performance evaluation of a C-RAN that accommodates many service-classes whose calls arrive in the RRHs according to a Poisson process and have different resource requirements. New calls are accepted in the serving RRH for a generally distributed service time, if their resource requirements (in terms of radio and computational RUs) are available. Otherwise call blocking occurs. We showed that both models have a PFS for the determination of the steady-state probabilities and provided convolution algorithms for the efficient CBP calculation. The accuracy of these algorithms was verified via simulation. As a future extension of this paper, we will study either single or multi-cluster C-RAN that accommodate many service-classes whose in-service calls may occupy RUs between a minimum and a maximum value, expressing the so called "elastic traffic" [45], [46].

APPENDIX A PSEUDOCODE

In Appendix A, we provide the pseudocode for the software implementation for both the brute force method and the convolution algorithm in the case of the proposed MC-SC model. In the pseudocode, we use the notation of section II (whenever possible). To this end, the main notation is the following:

- M — the number of RRHs;
- C — capacity of each RRH (radio RUs);
- T — capacity of V-BBU (computational RUs);
- K_m — the number of service-classes served by the m-th RRH;
- $\boldsymbol{n}$ — steady-state vector, which contains the number, $n_{m,k}$, of all in-service calls of all service-classes in all RRHs.

A. BRUTE FORCE METHOD

The implementation of the brute force method relies on procedures 1 to 5. More specifically, to describe the brute force method we start with the determination of the system's state space $\boldsymbol{\Omega}$. The latter can be considered as an array of steady-state vectors. Procedure 1 (`spaceBuild`) requires the following parameters:

- $\boldsymbol{n}$ — steady-state vector;
- m — the m-th RRH;
- k — service-class k of the m-th RRH;
- T_{sum} — expresses the occupied computational RUs in the steady-state vector, $\boldsymbol{n}$;
- C_{sum} — expresses the occupied radio RUs in the steady-state vector, $\boldsymbol{n}$, m-th RRH.

To determine the system's state space $\boldsymbol{\Omega}$, we call procedure 1 assuming a zero steady-state vector $\boldsymbol{n}$ (all elements, $n_{m,k}$, are 0) and $m \leftarrow 1$, $k \leftarrow 1$, $T_{\text{sum}} \leftarrow 0$, $C_{\text{sum}} \leftarrow 0$.

Procedures 2 (`productForm`) and 3 (G) implement the PFS of (5). More specifically, by dividing the result of `productForm(`$\boldsymbol{n}$`)` with G, we obtain the probability of steady-state $\boldsymbol{n}$, $P(\boldsymbol{n})$.

Procedures 4 and 5 implement the CBP $B_{\text{r},m,k}$ (7) and $B_{\text{c},m,k}$ (8), respectively.

B. CONVOLUTION ALGORITHM

The implementation of the convolution algorithm relies on procedures 6-19.

Procedure 6 (`convolution`) implements the discrete convolution operation of two distributions q_v and q_w, defined on the set $[0, j] \subset \mathbb{Z}$ and requires the following parameters:

- q_v — variable structure that describes the first distribution;
- q_w — variable structure that describes the second distribution;
- j — the upper bound of the set $[0, j]$.

Procedure 6 is required in procedures 10, 11, 12, 13, 16.

Procedure 1 State Space $\boldsymbol{\Omega}$, `spaceBuild`

Require: $\boldsymbol{n}$, $m \le M$, $k \le K_m$, T_{sum}, C_{sum}
  while $T_{\text{sum}} \le T$ **do**
    if $k+1 \le K_m$ **then**
      `spaceBuild` ($\boldsymbol{n} \leftarrow \boldsymbol{n}, m \leftarrow m, k \leftarrow k+1, T_{\text{sum}} \leftarrow T_{\text{sum}}, C_{\text{sum}} \leftarrow C_{\text{sum}}$) {call `spaceBuild` for next service-class, $k+1$}
    else
      `spaceBuild` ($\boldsymbol{n} \leftarrow \boldsymbol{n}, m \leftarrow m+1, k \leftarrow 1, T_{\text{sum}} \leftarrow T_{\text{sum}}, C_{\text{sum}} \leftarrow 0$) {call `spaceBuild` for next RRH, $m+1$}
    end if
    if $T_{\text{sum}} + b^{\text{c}}_{m,k} \le T$ **and** $C_{\text{sum}} + b^{\text{r}}_{m,k} \le C$ **then**
      Increase the element $n_{m,k}$ of the state $\boldsymbol{n}$ by 1 {add service-class k call to the m-th RRH}
      $T_{\text{sum}} \leftarrow T_{\text{sum}} + b^{\text{c}}_{m,k}$ {occupy computational RUs by the added call}
      $C_{\text{sum}} \leftarrow C_{\text{sum}} + b^{\text{r}}_{m,k}$ {occupy radio RUs by the added call}
      Add state $\boldsymbol{n}$ to the system's state space $\boldsymbol{\Omega}$
    end if
  end while

Procedure 2 Unnormalized Probability of Steady-State $\boldsymbol{n}$, `productForm`

Require: steady-state $\boldsymbol{n}$
Ensure: P
  $P \leftarrow 1$
  for $m \leftarrow 1$ **to** M **do**
    for $k \leftarrow 1$ **to** K_m **do**
      $P \leftarrow P \times \alpha_{m,k}^{n_{m,k}} / n_{m,k}!$
    end for
  end for

Procedure 3 Normalisation Constant, G

Ensure: G
  $G \leftarrow 0$
  for all $\boldsymbol{n} \in \boldsymbol{\Omega}$ **do**
    $G \leftarrow G +$ `productForm(`$\boldsymbol{n}$`)`
  end for

STEP 1

The first step of the proposed convolution algorithm involves procedures 7, 8, 9, 10, 11 and 12.

Procedure 7 represents (9). It accepts as parameters the service-class k calls serviced in the m-th RRH and the amount j of radio RUs that are occupied by those calls.

To obtain the normalization constant $G^{-1}_{m,k}$, we can use procedure 8. Here, we increase the value of j by $b^{\text{r}}_{m,k}$ in

Procedure 4 CBP Due to Insufficient Radio RUs, $B_{r,m,k}$

Require: $m \leq M, k \leq K_m$
Ensure: $B_{r,m,k}$
  $B_{r,m,k} \leftarrow 0$
  for all $\boldsymbol{n} \in \boldsymbol{\Omega}$ **do**
    $C_{\text{occupied}} \leftarrow$ occupied radio RUs in the m-th RRH by calls that belong to steady-state $\boldsymbol{n}$
    $T_{\text{occupied}} \leftarrow$ occupied computational RUs by calls that belong to steady-state $\boldsymbol{n}$
    if $C - b^{\text{r}}_{m,k} < C_{\text{occupied}}$ **and** $T_{\text{occupied}} \leq T - b^{\text{c}}_{m,k}$ **then**
      $B_{r,m,k} \leftarrow B_{r,m,k} + \texttt{productForm}(\boldsymbol{n})/G$
    end if
  end for

Procedure 5 CBP Due to Insufficient Computational RUs, $B_{c,m,k}$

Require: $m \leq M, k \leq K_m$
Ensure: $B_{c,m,k}$
  $B_{c,m,k} \leftarrow 0$
  for all $\boldsymbol{n} \in \boldsymbol{\Omega}$ **do**
    $T_{\text{occupied}} \leftarrow$ occupied computational RUs in steady-state $\boldsymbol{n}$
    if $T_{\text{occupied}} > T - b^{\text{c}}_{m,k}$ **then**
      $B_{c,m,k} \leftarrow B_{c,m,k} + \texttt{productForm}(\boldsymbol{n})/G$
    end if
  end for

Procedure 6 Convolution Operation, `convolution`

Require: q_v, q_w, j
Ensure: $(q_v * q_w)(j)$
  $q \leftarrow 0$
  for $i \leftarrow 0$ **to** j **do**
    $q \leftarrow q + q_v(i) \times q_w(j-i)$
  end for

Procedure 7 Occupancy Distribution of Service-Class m, k, $q_{m,k}(j)$

Require: $m \leq M, k \leq K_m, j$
Ensure: Unnormalized probability that j radio RUs are occupied by service-class k calls in the m-th RRH.
  if $j = 0$ **then**
    return 1 {second case of (9)}
  end if
  $i \leftarrow \lfloor \frac{j}{b^{\text{r}}_{m,k}} \rfloor$ {the number of in-service calls of service-class k in the m-th RRH}
  if $i \leq 0$ **or** $i > C$ **or** $j \bmod b^{\text{r}}_{m,k} > 0$ **then**
    return 0 {third case of (9)}
  end if
  return $\frac{\alpha^i_{m,k}}{i!}$ {first case of (9)}

each iteration, since the domain of (9) is $j \in \{ib^{\text{r}}_{m,k} : i = 0, \ldots, \lfloor C/b^{\text{r}}_{m,k} \rfloor\}$.

Procedure 9 is a simple division of the previously defined procedures 7 and 8.

Procedure 8 Normalization Constant, $G_{m,k}$

Require: $m \leq M, k \leq K_m$
Ensure: Normalization constant for $q_{m,k}$, $G_{m,k}$
  $G_{m,k} \leftarrow 0$
  for $j \leftarrow 0$ **to** C with step $b^{\text{r}}_{m,k}$ **do**
    $G_{m,k} \leftarrow G_{m,k} + q_{m,k}(j)$
  end for

Procedure 9 Normalized Occupancy Distribution, $q'_{m,k}(j)$

Require: $m \leq M, k \leq K_m, j$
Ensure: Probability that j radio RUs are occupied by service-class k calls in the m-th RRH.
  return $q_{m,k}(j)/G_{m,k}$

Procedure 10 Aggregated Occupancy Distribution of the m-Th RRH Excluding Calls of the First Service-Class, $Q^m_{(-1)}$

Require: $m \leq M$
Ensure: $Q^m_{(-1)}(j)$, for $j \in [0, C]$
  $Q^m_{(-1)} \leftarrow q'_{m,2}$
  for $k \leftarrow 3$ **to** K_m **do**
    InterAgg an empty distribution
    for $j \leftarrow 0$ **to** C **do**
      InterAgg $\leftarrow$ `convolution`$(Q^m_{(-1)}, q'_{m,k}, j)$
    end for
    $Q^m_{(-1)} \leftarrow$ *InterAgg*
  end for

Procedure 11 Normalization Constant, G_m

Require: $m \leq M$
Ensure: Normalization constant for $Q^m_{(-1)} * q'_{m,1}$, G_m
  $G_m \leftarrow 0$
  for $j \leftarrow 0$ **to** C **do**
    $G_m \leftarrow G_m +$ `convolution`$(Q^m_{(-1)}, q'_{m,1}, j)$
  end for

Procedure 12 Occupancy Distribution of the m-Th RRH, $q'_m(j)$

Require: $m \leq M, j$
Ensure: Probability that j radio RUs are occupied in the m-th RRH
  return `convolution`$(Q^m_{(-1)}, q'_{m,1}, j)/G_m$

The process to calculate (10) is shown in procedure 10. The aggregated convolution between several distributions requires the calculation of all intermediate convolutions. In procedure 10, this is accomplished by declaring a temporary distribution variable *InterAgg*.

The first step is completed with procedures 11 and 12. The latter returns the values of the occupancy distribution of the m-th RRH, which are necessary for the next steps of the algorithm.

Procedure 13 Aggregated Occupancy Distribution of All RRHs Apart From the m-Th One, $Q_{(-m)}(j)$

Require: $m \leq M, j \leq T$
Ensure: $Q_{(-m)}(j)$
$Q_{(-m)} \leftarrow$ occupancy distribution of the first RRH that is not the m-th one
for l all other RRHs except the one above and the m-th one **do**
  InterAgg an empty distribution
  for $j \leftarrow 0$ **to** T **do**
    $InterAgg \leftarrow \texttt{convolution}(Q_{(-m)}, q'_l, j)$
  end for
  $Q_{(-m)} \leftarrow InterAgg$
end for

Procedure 14 Normalization Constant, $G_{(-m)}$

Require: $m \leq M$
Ensure: Normalization constant for $Q_{(-m)}$ distribution
$G_{(-m)} \leftarrow 0$
for $j \leftarrow 0$ **to** T **do**
  $G_{(-m)} \leftarrow G_{(-m)} + Q_{(-m)}(j)$
end for

Procedure 15 Normalized Occupancy Distribution of All RRHs Apart From the m-Th One, $Q'_{(-m)}(j)$

Require: $m \leq M, j \leq T$
Ensure: Probability that j computational RUs are occupied by calls in all RRHs except the m-th one
return $Q_{(-m)}(j)/G_{(-m)}$

Procedure 16 Normalization Constant, G^*_m

Require: $m \leq M$
Ensure: Normalization constant for $Q_{(-m)} * q'_m$, G^*_m
$G^*_m \leftarrow 0$
for $j \leftarrow 0$ **to** T **do**
  $G^*_m \leftarrow G^*_m + \texttt{convolution}(Q_{(-m)}, q'_m, j)$
end for

STEP 2
Procedures 13, 14 and 15 form the second step of the algorithm.

Procedure 13 resembles procedure 10, since it also computes the aggregated convolutions, even though different distributions are involved and the domain is enlarged up to T, i.e., $j \in [0, \ldots, T]$.

Procedure 14 calculates the normalization constant $G_{(-m)}$ for the distribution $Q_{(-m)}$.

The result of the second step of the algorithm is the calculation of $Q'_{(-m)}(j)$ and it is produced via procedure 15.

STEP 3
To obtain the normalized values of the occupancy distribution of the computational RUs, procedures 16 and 17 are required.

Procedure 17 Occupancy Distribution of the MC-SC Model, $Q'(j)$

Ensure: Probability that j computational RUs are occupied
return $\texttt{convolution}(Q_{(-1)}, q'_1, j)/G^*_m$

Procedure 18 CBP Due to Insufficient Computational RUs, $B_{c,m,k}$

Require: $m \leq M, k \leq K_m$
Ensure: $B_{c,m,k}$
$B_{c,m,k} \leftarrow 0$
for $j > T - b^c_{m,k}$ **to** T **do**
  $B_{c,m,k} \leftarrow B_{c,m,k} + Q'(j)$
end for

Procedure 19 CBP Due to Insufficient Radio RUs, $B_{r,m,k}$

Require: $m \leq M, k \leq K_m$
Ensure: $B_{r,m,k}$
$B_{r,m,k} \leftarrow 0$
for $x > C - b^r_{m,k}$ **to** C **do**
  $sum \leftarrow 0$
  for $y \leftarrow x$ **to** $T - b^r_{m,k}$ **do**
    $sum \leftarrow sum + Q'_{(-m)}(T - b^r_{m,k} - y)$
  end for
  $B_{r,m,k} \leftarrow B_{r,m,k} + q'_m(x) \times sum$
end for
$B_{r,m,k} \leftarrow B_{r,m,k}/G^*_m$

Finally, to obtain CBP, via (15) and (16), one can apply procedures 18 and 19, respectively.

APPENDIX B
TUTORIAL
We consider a C-RAN as in the setup of the proposed MC-SC model with $M = 2$ RRHs of capacity $C = 3$ radio RUs and $T = 4$ computational RUs. The m-th RRH ($m \in \{1, 2\}$) serves calls from K_m different service-classes and let $b_{m,k} = b^c_{m,k} = b^r_{m,k}$ be the amount of RUs required by a service-class k call in the m-th RRH. The first RRH serves $K_1 = 2$ service-classes. The first service-class calls of the first RRH require $b_{1,1} = 1$ RUs, while the second service-class calls require $b_{1,2} = 2$ RUs. The second RRH serves $K_2 = 1$ service-class and the calls of this service-class require $b_{2,1} = 1$ RUs. The mean arrival rate of all calls is $\lambda_{1,1} = \lambda_{1,2} = \lambda_{2,1} = 1$ and the mean service time is $\mu^{-1}_{1,1} = \mu^{-1}_{1,2} = \mu^{-1}_{2,1} = 1$, thus, the offered traffic load is $\alpha_{1,1} = \alpha_{1,2} = \alpha_{2,1} = 1$ erl.

The cardinality of $\boldsymbol{\Omega}$, as described by (1), for the above C-RAN example, is $|\boldsymbol{\Omega}| = 18$ states. All steady states $\boldsymbol{n} = (n_{1,1}, n_{1,2}, n_{2,1})$ are presented in the first column of Table 4 while the corresponding values of $P(\boldsymbol{n})$ are presented in the second column. Column j presents the occupied computational RUs by the in-service calls of steady-state $\boldsymbol{n}$. Columns j_1 and j_2 present the occupied radio RUs in the first and the second RRH, respectively. Finally, we denote with A_r and A_c the availability of radio and computational

TABLE 4. State space Ω.

$\boldsymbol{n}$	$P(\boldsymbol{n})$	j	j_1	j_2	A_r	A_c
(0,0,0)	12/137	0	0	0	⊙ ⊙ ⊙	⊙ ⊙ ⊙
(0,0,1)	12/137	1	0	1	⊙ ⊙ ⊙	⊙ ⊙ ⊙
(0,0,2)	6/137	2	0	2	⊙ ⊙ ⊙	⊙ ⊙ ⊙
(0,0,3)	2/137	3	0	3	⊙ ⊙ ⊗	⊙ ⊗ ⊙
(0,1,0)	12/137	2	2	0	⊙ ⊗ ⊙	⊙ ⊙ ⊙
(0,1,1)	12/137	3	2	1	⊙ ⊗ ⊙	⊙ ⊗ ⊙
(0,1,2)	6/137	4	2	2	⊙ ⊗ ⊙	⊗ ⊗ ⊗
(1,0,0)	12/137	1	1	0	⊙ ⊙ ⊙	⊙ ⊙ ⊙
(1,0,1)	12/137	2	1	1	⊙ ⊙ ⊙	⊙ ⊙ ⊙
(1,0,2)	6/137	3	1	2	⊙ ⊙ ⊙	⊙ ⊗ ⊙
(1,0,3)	2/137	4	1	3	⊙ ⊙ ⊗	⊗ ⊗ ⊗
(1,1,0)	12/137	3	3	0	⊗ ⊗ ⊙	⊙ ⊗ ⊙
(1,1,1)	12/137	4	3	1	⊗ ⊗ ⊙	⊗ ⊗ ⊗
(2,0,0)	6/137	2	2	0	⊙ ⊗ ⊙	⊙ ⊙ ⊙
(2,0,1)	6/137	3	2	1	⊙ ⊗ ⊙	⊙ ⊗ ⊙
(2,0,2)	3/137	4	2	2	⊗ ⊗ ⊙	⊗ ⊗ ⊗
(3,0,0)	2/137	3	3	0	⊗ ⊗ ⊙	⊙ ⊗ ⊙
(3,0,1)	2/137	4	3	1	⊗ ⊗ ⊙	⊗ ⊗ ⊗

RUs, respectively. We also introduce the following visual encoding: the symbol ⊙ means there are available RUs and the symbol ⊗ means that the RUs are not sufficient for the next call. By placing three such visual encodings in a row, we can describe the availability of RUs for each type of incoming calls. Based on the above, the first icon refers to the first service-class calls in the 1st RRH. The second icon refers to the second service-class calls in the 1st RRH. Finally, the third icon refers to the first service-class calls in the 2nd RRH. Thus, the columns A_r and A_c presents the availability of radio and computational RUs, respectively, for the next incoming call of all service-classes in both RRHs.

As an example, let us examine the fourth row from Table 4 which refers to the steady-state $\boldsymbol{n} = (0, 0, 3)$, i.e., to the state where three calls of service-class 1 are serviced in the second RRH. This steady-state occurs with probability $P(\boldsymbol{n}) = 2/137$. In state $(0, 0, 3)$, in-service calls occupy $j = 3$ computational RUs from the V-BBU, $j_1 = 0$ radio RUs from the first RRH and $j_2 = 3$ radio RUs from the second RRH. The field A_r shows that the second RRH has no available radio RUs for service-class 1 calls and therefore if a call of service-class 1 arrives at the second RRH it will be blocked and lost. The field A_c shows that the V-BBU cannot provide enough computational RUs for a new call of service-class 2 of the first RRH, thus if a call of service-class 2 arrives at the first RRH it will be blocked and lost. On the other hand, if a call of service-class 1 arrives at the first RRH it will be accepted.

Based on Table 4, we can apply the brute force method in order to initially obtain the sets $\boldsymbol{\Omega}_{m,k}^{C,<T}$ and $\boldsymbol{\Omega}_{m,k}^{T}$ and then calculate the values of $B_{r,m,k}$ and $B_{c,m,k}$ as described in (7) and (8), respectively. To decide whether a state belongs to the set $\boldsymbol{\Omega}_{m,k}^{C,<T}$ we need the field A_r to indicate that there are no sufficient RUs (⊗) and the field A_c to indicate that there are sufficient RUs (⊙) for the next call of service-class m, k. To decide whether a state belongs to the set $\boldsymbol{\Omega}_{m,k}^{T}$ we only need the field A_c to show that there are no sufficient RUs (⊗) for the next call of service-class m, k. To this end, we have:

$$\boldsymbol{\Omega}_{1,1}^{C,<T} = \{(1, 1, 0), (3, 0, 0)\} \rightarrow B_{r,1,1} = 14/137$$

$$\boldsymbol{\Omega}_{1,2}^{C,<T} = \{(0, 1, 0), (2, 0, 0)\} \rightarrow B_{r,1,2} = 18/137$$

$$\boldsymbol{\Omega}_{2,1}^{C,<T} = \{(0, 0, 3)\} \rightarrow B_{r,2,1}q = 2/137$$

$$\boldsymbol{\Omega}_{1,1}^{T} = \left\{\begin{matrix}(0, 1, 2), (1, 0, 3), (1, 1, 1),\\ (2, 0, 2), (3, 0, 1)\end{matrix}\right\} \rightarrow B_{c,1,1} = \frac{25}{137}$$

$$\boldsymbol{\Omega}_{1,2}^{T} = \left\{\begin{matrix}(0, 0, 3), (0, 1, 1), (0, 1, 2),\\ (1, 0, 2), (1, 0, 3), (1, 1, 0),\\ (1, 1, 1), (2, 0, 1), (2, 0, 2),\\ (3, 0, 0), (3, 0, 1)\end{matrix}\right\} \rightarrow B_{c,1,2} = \frac{65}{137}$$

$$\boldsymbol{\Omega}_{2,1}^{T} = \left\{\begin{matrix}(0, 1, 2), (1, 0, 3), (1, 1, 1),\\ (2, 0, 2), (3, 0, 1)\end{matrix}\right\} \rightarrow B_{c,2,1} = \frac{25}{137}$$

In what follows, we apply the proposed convolution algorithm for this example.

STEP 1

For each service-class k of each m-th RRH, compute the values of $q_{m,k}(j)$ using (9) ($q_{m,k}(0) = 1$ and $\alpha_{m,k} = 1$).

Service-class 1, first RRH:

$$j = 1 \rightarrow q_{1,1}(1) = (1^1/1!)1 \implies q_{1,1}(1) = 1$$
$$j = 2 \rightarrow q_{1,1}(2) = (1^2/2!)1 \implies q_{1,1}(2) = 1/2$$
$$j = 3 \rightarrow q_{1,1}(3) = (1^3/3!)1 \implies q_{1,1}(3) = 1/6$$

The summation of the above, along with $q_{1,1}(0) = 1$, results in the normalization factor $G_{1,1} = 2\frac{2}{3}$. The corresponding normalized values of $q'_{1,1}(j)$ are the following: $q'_{1,1}(0) = 3/8$; $q'_{1,1}(1) = 3/8$; $q'_{1,1}(2) = 3/16$; $q'_{1,1}(3) = 1/16$.

Service-class 2, first RRH [1]:

$$j = 2 \rightarrow q_{1,2}(2) = (1^1/1!)1 \implies q_{1,2}(1) = 1$$

Since the normalization factor is $G_{1,2} = 2$, the corresponding normalized values of $q'_{1,2}(j)$ are the following: $q'_{1,2}(0) = 1/2$; $q'_{1,2}(2) = 1/2$.

Service-class 1, second RRH:

$$j = 1 \rightarrow q_{2,1}(1) = (1^1/1!)1 \implies q_{2,1}(1) = 1$$
$$j = 2 \rightarrow q_{2,1}(2) = (1^2/2!)1 \implies q_{2,1}(2) = 1/2$$
$$j = 3 \rightarrow q_{2,1}(3) = (1^3/3!)1 \implies q_{2,1}(3) = 1/6$$

Since the normalization factor is $G_{2,1} = 2\frac{2}{3}$, the corresponding normalized values of $q'_{2,1}(j)$ are the following: $q'_{2,1}(0) = 3/8$; $q'_{2,1}(1) = 3/8$; $q'_{2,1}(2) = 3/16$; $q'_{2,1}(3) = 1/16$.

Next, we determine the aggregated occupancy distribution of the m-th RRH excluding calls of the first service-class, $Q^{m}_{(-1)}$. Since the first RRH serves two service-classes, the $Q^{m=1}_{(-1)} \equiv q'_{1,2}$.

[1]Note that service-class 2 calls (in the 1st RRH) require $b_{1,2} = 2$ RUs and therefore for $j = 2$, there is only one call, $i = 1$, while $q_{1,2}(1)$ or $q_{1,2}(3)$ cannot occur, thus $q_{1,2}(1) = q_{1,2}(3) = 0$.

The goal of step 1 is the determination of the normalized occupancy distribution of the m-th RRH, q'_m. To this end, we first calculate $q_m = Q^m_{(-1)} * q'_{m,1}$.

For the first RRH, we have:

$$j = 0 \to q_1(0) = q'_{1,1}(0)q'_{1,2}(0) = 3/16$$

$$j = 1 \to q_1(1) = \sum_{x=0}^{1} q'_{1,1}(x) \cdot q'_{1,2}(1-x) = \frac{3}{8} \cdot 0 + \frac{3}{8} \cdot \frac{1}{2} = 3/16$$

$$j = 2 \to q_1(2) = \sum_{x=0}^{2} q'_{1,1}(x) \cdot q'_{1,2}(2-x) = \frac{3}{8} \cdot \frac{1}{2} + \frac{3}{8} \cdot 0 + \frac{3}{16} \cdot \frac{1}{2} = 9/32$$

$$j = 3 \to q_1(3) = \sum_{x=0}^{3} q'_{1,1}(x) \cdot q'_{1,2}(3-x) = \frac{3}{8} \cdot 0 + \frac{3}{8} \cdot \frac{1}{2} + \frac{3}{16} \cdot 0 + \frac{1}{16} \cdot \frac{1}{2} = 7/32$$

The normalization factor $G_{2,1} = 7/8$. The corresponding normalized values of $q'_1(j)$ are the following: $q'_1(0) = 3/14$; $q'_1(1) = 3/14$; $q'_1(2) = 9/28$; $q'_1(3) = 1/4$.

For the second RRH, we have that $q'_2 \equiv q'_{2,1}$ (since the RRH services only one service-class), i.e., $q'_2(0) = 3/8$; $q'_2(1) = 3/8$; $q'_2(2) = 3/16$; $q'_2(3) = 1/16$.

STEP 2

Determine the values of $Q'_{(-m)}$. Since we have two service classes, the $Q'_{(-1)} \equiv q'_2$ and $Q'_{(-2)} \equiv q'_1$.

STEP 3

Calculate $Q_m(j) = Q'_{(-m)} * q'_m$ for the first RRH.

$$j = 0 \to Q_1(0) = q'_2(0)q'_1(0) = 9/112$$

$$j = 1 \to Q_1(1) = \sum_{x=0}^{1} q'_2(x) \cdot q'_1(1-x) = \frac{3}{8} \cdot \frac{3}{14} + \frac{3}{8} \cdot \frac{3}{14} = 18/112$$

$$j = 2 \to Q_1(2) = \sum_{x=0}^{2} q'_2(x) \cdot q'_1(2-x) = \frac{3}{8} \cdot \frac{9}{28} + \frac{3}{8} \cdot \frac{3}{14} + \frac{3}{16} \cdot \frac{3}{14} = 54/224$$

$$j = 3 \to Q_1(3) = \sum_{x=0}^{3} q'_2(x) \cdot q'_1(3-x) = \frac{3}{8} \cdot \frac{1}{4} + \frac{3}{8} \cdot \frac{9}{28} + \frac{3}{16} \cdot \frac{3}{14} + \frac{1}{16} \cdot \frac{3}{14} = 60/224$$

$$j = 4 \to Q_1(3) = \sum_{x=0}^{4} q'_2(x) \cdot q'_1(4-x) = \frac{3}{8} \cdot 0 + \frac{3}{8} \cdot \frac{1}{4} + \frac{3}{16} \cdot \frac{9}{28} + \frac{1}{16} \cdot \frac{3}{14} + 0 \cdot \frac{3}{14} = 75/448$$

The normalization factor $G^*_1 = 411/448$. The corresponding normalized values of the computational occupancy distribution $Q'(j)$ are the following: $Q'(0) = 36/411$; $Q'(1) = 72/411$; $Q'(2) = 108/411$; $Q'(3) = 120/411$; $Q'(4) = 75/411$.

The same values are also obtained for the second RRH, i.e., $G^*_2 \equiv G^*_1 = 411/448$.

Via (15), we calculate the CBP due to the lack of computational RUs for calls of service-class k in the m-th RRH, as follows:

$$B_{c,1,1} = \sum_{j=4}^{4} Q'(j) = Q'(4) = 75/411$$

$$B_{c,1,2} = \sum_{j=3}^{4} Q'(j) = Q'(3) + Q'(4) = 195/411$$

$$B_{c,2,1} = \sum_{j=4}^{4} Q'(j) = Q'(4) = 75/411$$

Via (16), we calculate the CBP due to the lack of radio RUs for calls of service-class k in the m-th RRH:

$$B_{r,1,1} = \frac{1}{G^*_1} \sum_{x=3}^{3} q'_1(x) \sum_{y=x}^{3} Q'_{(-1)}(3-y) = \frac{448}{411} \cdot \frac{1}{4} \cdot \frac{3}{8} = \frac{42}{411}$$

$$B_{r,1,2} = \frac{1}{G^*_1} \sum_{x=2}^{3} q'_1(x) \sum_{y=x}^{2} Q'_{(-1)}(2-y) = \frac{448}{411} \cdot \frac{9}{28} \cdot \frac{3}{8} = \frac{54}{411}$$

$$B_{r,2,1} = \frac{1}{G^*_2} \sum_{x=3}^{3} q'_2(x) \sum_{y=x}^{3} Q'_{(-2)}(3-y) = \frac{448}{411} \cdot \frac{1}{16} \cdot \frac{3}{14} = \frac{6}{411}$$

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