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Design and Analysis of LT Codes With a Reverse Coding Framework

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ABSTRACT In this paper, an improved LT code with a reverse coding framework is designed to reduce the error floor caused by low-degree information nodes. For the proposed coding scheme, a well-designed threshold is used to mark the information nodes whose degrees are less than the threshold, and these nodes will be coded reversely to connect to enough candidate check nodes. To design the optimal threshold, firstly, the information degree distribution and the check degree distribution of the improved LT code are deduced. Then, the parameter extrinsic information gain-loss-ratio (GLR) is designed to evaluate the convergence behavior of the improved LT code. Finally, the 'slow increase region' of the GLR is set, and the boundary value of this region is used to deduce the optimal threshold which matches with the channel state information (CSI) and decoding overhead. To make the proposed LT code not limited to a fixed code rate, we further modify the proposed scheme. The segment coding method is used to generate a redundant generator matrix, and the check nodes corresponding to this matrix can be transmitted independently and are not limited to a fixed number. Furthermore, the connection relationship between information nodes and check nodes can be easily recorded, which improves the decoding efficiency. The advantages of the improved LT code are that the degree distributions can be formulated, the convergence behavior can be predicted, and the lowest information degree can be adjusted. Simulation results show that the improved LT code can reduce the error floor by up to 4 orders of magnitude. Besides, the designed LT code outperforms the existing LT codes in literature in terms of bit error rate (BER) performance.

INDEX TERMS Luby transform codes, threshold, degree distribution, convergence.

I. INTRODUCTION

Fountain codes [1] were initially designed for the binary erasure channel (BEC), aiming to provide an ideal method for massive data distribution and reliable broadcast [2], [3]. LT codes [4] were the first practical fountain codes. The number of check nodes of LT codes used for decoding at the receiving end can be adaptively changed with the channel state information (CSI), which makes LT codes very suitable for video and audio broadcasting and reliable data transmission [5]–[7]. Although fountain codes were originally designed for BEC, it also has potential value in additive white Gaussian noise (AWGN) channels [8]–[10]. Unfortunately, simulation results in [11] and [12] show that LT codes still encounter an obvious error floor in a binary symmetric channel (BSC) and AWGN channels.

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LT codes are mainly designed by optimizing the check degree distribution and improving the encoding scheme [13]. As for the check degree distribution, several optimization design methods are proposed in [13]–[17]. However, in binary input AWGN (BIAWGN) channels, optimizing the check distribution has a limited effect on lowering the error floor. There are two main reasons: i) There is no causal relationship between the cause of the error floor and the check degree distribution. ii) When the linear programming method is used to design the check distribution, infinite code length and high number of iterations are often considered. Besides, an universal optimal degree distribution does not exist for AWGN channels [18].

The error floor is mainly caused by unreliable low-degree information nodes [19]–[22]. Namely, for the conventional encoding scheme, there is always a small part of information nodes that cannot be connected to enough check node and cannot be restored correctly. To solve this problem, an improved coding scheme is proposed in [19], in which check nodes are forced to preferentially select the information nodes whose degree is small. The improved coding scheme leads almost all information nodes having the same degree, so we call it the equal degree (ED) LT code. The authors in [20] applied the ED coding scheme to the design of unequal erasure protection (UEP) LT codes. At the expense of slightly increased decoding overhead, the error floor of most important bits (MIB) is reduced by nearly 3 orders of magnitude without reducing the bit error rate (BER) performance of least important bits (LIB). In [21], once a check node needs to select d information nodes, it randomly chooses T information nodes and then decides the one which has the lowest degree for connection. The BER performance of the improved LT codes in [21] depends on the parameter T, but the authors do not give a precise design method for T. More parameters are designed for the improved coding scheme in [22], such as the parameter d^* which determines how each check node connects to the information nodes.

Although the above improved coding schemes can reduce the error floor in BEC or AWGN channels, there are still the following problems to be solved. i) The average degree of information nodes is increased blindly, and the coding result is not adjustable. ii) The mathematical expression of information degree distribution is not deduced. iii) In these schemes, the parameters that play an important role in BER performance are not analyzed and designed quantitatively. iv) The lowest degree of information nodes cannot be predicted.

In this paper, an improved LT code with a reverse coding framework is designed to reduce the error floor. Firstly, the threshold T_v is designed to screen out the low-degree information nodes, which will reselect check nodes to connect. Secondly, the information degree distribution and the check degree distribution are deduced, which is important for the extrinsic information transfer (EXIT) chart used to predict the convergence behavior of the improved coding scheme. Then, the parameter the extrinsic information gain-loss-ratio (GLR) is designed to evaluate the optimal threshold. The proposed LT code has the following characteristics. i) The information distribution can be optimized by using the reverse coding framework, and enough check nodes will be accurately connected to each low-degree information node to ensure the high probability of decoding success. ii) The threshold can be adjusted adaptively, and the coding results can match with CSI and decoding overhead, which means that the improved scheme is more targeted. iii) The information distribution and check distribution of the improved scheme can be formulated, which means that the convergence behavior and the BER performance of LT codes can be easily estimated. Besides, simulation results show that the improved coding scheme can obtain better BER performance than the conventional LT scheme. Furthermore, the improved scheme outperforms the coding schemes in [19] and [21] when the optimal threshold is used.

II. PROPOSED LT CODES WITH REVERSE CODING FRAMEWORK

A. THE CONVENTIONAL LT CODES

K information nodes $\mathbf{v} = \{v_1, v_2, \ldots, v_K\}$ are coded at the sender to generate N check nodes $\mathbf{c} = \{c_1, c_2, \ldots, c_N\}$, and these N check nodes are connected to encode nodes one by one. The Tanner graph of the LT code is shown in Fig. 1.

We define the check degree distribution as $\Omega(x) = \sum_{j=1}^{d_c} \Omega_j x^j$, where Ω_j represents the probability of a check node with degree j, and d_c represents the maximum check node degree. We define the information degree distribution as $\Lambda(x) = \sum_{i=0}^{d_v} \Lambda_i x^i$, where Λ_i represents the probability of an information node with degree i, and d_v represents the maximum information node degree. Although the number of check nodes is arbitrary, we still define its instantaneous rate as $R = K/N$ [23]. We also introduce the coding scheme of the conventional LT code here, as shown in Algorithm 1.

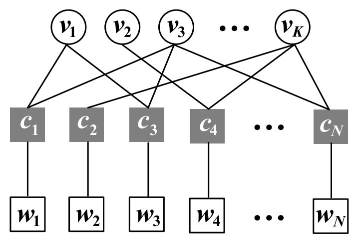

FIGURE 1. The Tanner graph of the LT code.

Algorithm 1 The Conventional LT Code

Step 1: Select a degree d according to the check degree distribution $\Omega(x)$.

Step 2: Randomly select d nodes from all K information nodes.

Step 3: Calculate the XOR value of the selected d information nodes as the current check node.

According to the above definition, the average degree of check nodes is $\beta = \sum_{j=1}^{d_c} \Omega_j j$, and the average degree of information nodes is $\alpha = N\beta/K$. Besides, the fraction of information nodes of degree i can be computed as

$$\Lambda_i = \binom{N}{i} \left(\frac{\beta}{K}\right)^i \left(1 - \frac{\beta}{K}\right)^{N-i} \tag{1}$$

The approximate expression of (1) is calculated as [11]

$$\Lambda_i \approx \frac{\alpha^i}{i!} e^{-\alpha} \tag{2}$$

The edge-degree distribution of check nodes and information nodes are defined as $\rho(x) = \sum_{j=1}^{d_c} \rho_j x^{j-1}$ and $\lambda(x) = \sum_{i=1}^{d_v} \lambda_i x^{i-1}$, where λ_i represents the probability of information-node edges emanating from a degree i information node and ρ_j represents the probability of check-node edges emanating from a degree j check node. The coefficients λ_i and ρ_j can be calculated as [24]

$$\lambda_i = \frac{i\Lambda_i K}{K\sum_{i=1}^{d_v} i\Lambda_i} = \frac{i\Lambda_i}{\sum_{i=1}^{d_v} i\Lambda_i} \tag{3}$$

$$\rho_j = \frac{j\Omega_j N}{N\sum_{j=1}^{d_c} j\Omega_j} = \frac{j\Omega_j}{\sum_{j=1}^{d_c} j\Omega_j} \tag{4}$$

B. THE PROPOSED CODING SCHEME

The error floor of the conventional scheme is inevitable, which can be seen from equation (1). For example, no matter how large the code length N is, the proportion of information nodes with a degree of 0 is not zero. This means that low-degree information nodes are inevitable, that is, the error floor are inevitable.

To overcome the shortcomings of the coding schemes in [19] and [21], the improved scheme with reverse coding framework is proposed in Algorithm 2.

Algorithm 2 The Proposed LT Code

Step 1: Algorithm 1 is adopted to generate N check nodes.

Step 2: Update the degrees of all information nodes. Then screen out the information nodes whose degree is less than the threshold and denote these information nodes as set S. The number of elements in the set S is M. Denote $m = 1$.

Step 3: Mark the degree of the m-th information node in the set S as d_m, where $1 \leq m \leq M$.

Step 4: Randomly select $T_v - d_m$ check nodes whose degrees are greater than 2.

Step 5: If the selected $T_v - d_m$ check nodes have no intersection with the existing d_m check nodes, then connect them to the m-th information node. If the selected $T_v - d_m$ check nodes have an intersection with the existing d_m check nodes, then go to **Step** 4.

Step 6: If $m = M$, go to **Step** 7. If $m < M$, set $m = m+1$ and go to **Step** 3.

Step 7: Update the Tanner graph and recalculate the values of all check nodes. Then output $c_1, c_2, \ldots, c_N$.

We define the number of check nodes as N, and the threshold is T_v. In the first coding phase, the N check nodes will be generated in the conventional way. In the second phase, the information nodes that need to be reversely coded will be screened out and reconnected to enough check nodes. Therefore, the low-degree information nodes will be gradually reduced until eliminated.

Besides, In Algorithm 2, not all check nodes are selected as candidates for selection. The candidate check nodes should meet the following conditions: i) There is no connection relationship with the information node currently to be processed. ii) The degree is not equal to 1 and 2. The reason for meeting the first condition is to avoid repeated edge connections in Tanner graph. The reason for meeting the second condition is that the belief propagation (BP) algorithm will fail due to the lack of check nodes with degrees 1 and 2 [25].

An example is shown in Fig. 2, which contains six information nodes and eight check nodes. We set the threshold as 4, then after the first coding phase is completed, the information nodes that need to be reversely coded are v_1, v_2, v_3, and v_4. According to the design principle of Algorithm 2, the check nodes with degrees greater than 2 can be selected as candidates in the reverse coding process, namely c_1, c_3, c_5, and c_8. In the second phase, information nodes whose degrees are less than the threshold will actively select check nodes to be connected, instead of passively connecting to check nodes like the conventional coding scheme. For information nodes with a degree greater than or equal to the threshold, the existing connection relationship remains unchanged.

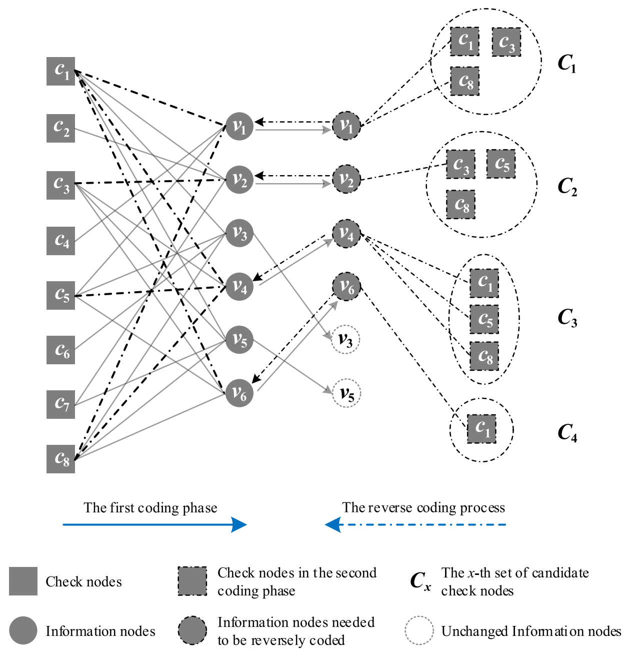

FIGURE 2. An example Tanner graph of the proposed coding scheme for $T_v = 4$.

C. CODING COMPLEXITY ANALYSIS

Compared with the conventional scheme [4], the proposed scheme in this paper increases the operation of sorting information nodes, which inevitably increases the coding complexity. We will analyze it in detail.

The scheme with a reverse coding framework changes the second step of the conventional scheme, and the operation of selecting the check degree and calculate the XOR value is the same as the conventional scheme. Therefore, we will analyze the impact of the way of selecting information nodes on the coding complexity.

In Algorithm 2, the new operations are: i) Sort the information nodes according to their degrees. ii) Compare the degrees of all information nodes with the threshold T_v. iii) Randomly select check nodes in reverse. Similarly, the new operations of the ED scheme in [19] are: i) Sort the information nodes according to their degrees. ii) Compare d with the number of information nodes of each degree. The new operations of the connection choice (CC) scheme in [21] are: i) Randomly selecting T information nodes. ii) Sort the T nodes according to their degrees. iii) Compare the sequence numbers of the T nodes and the currently selected information node. Table 1 shows the new operations and their times of the above schemes when generating N check nodes.

TABLE 1. The new operations and their times of the three schemes.

Coding schemes	Sort	Numeric comparison	Random selection
Conventional scheme	0	0	N
Proposed scheme	1	K	$<N+K$
ED scheme	N-1	N-1	N
CC scheme	$N\beta$	$N(\beta-1)$	$N\beta$

The three improved schemes need to sort the information nodes according to their degrees, which will inevitably increase the coding complexity. If the merge sort algorithm [26] is adopted, the time complexity of the ED scheme is $O(K\log_2 K)$. For the CC scheme, the time complexity is $O(T\log_2 T)$, where T is the parameter in this scheme and $T < K$. However, the number of sorting, numerical comparison, and random selection in this scheme are greater than the scheme in this paper and the ED scheme, so its coding complexity may be relatively high.

Besides, the number of sorting and numerical comparison of the scheme in this paper is less than that of the ED scheme, and the number of random selection is more than that of the scheme. Therefore, the coding complexity of the proposed scheme may be lower than the ED scheme, but both are higher than the conventional algorithm.

TABLE 2. Run time of the three improved schemes.

Coding schemes	Run time (sec)
Conventional scheme	109.5537
Proposed scheme	172.4409
ED scheme	251.4731
CC scheme	616.7659

Table 2 shows the coding running time of these three improved schemes, which are obtained through 2,000 Monte Carlo simulations. We set $K = 1024$, $N = 2048$. The check degree distribution is $\Omega_1(x)$ which is designed for moderate and short length code [27]. Besides, we set the threshold T_v as 11 in the proposed scheme, and $T = 3$ in the CC scheme.

Where

$$\Omega_1(x) = 0.025x + 0.495x^2 + 0.167x^3 + 0.082x^4 + 0.071x^5 + 0.05x^8 + 0.044x^9 + 0.043x^{19} + 0.023x^{66} \quad (5)$$

It can be seen that, compared with the conventional scheme, the coding run time of the three improved schemes will increase. However, the run time of the proposed scheme is relatively short, which is consistent with the above analysis results and also proves the advantages of the scheme in this paper.

III. DEGREE DISTRIBUTIONS OF THE PROPOSED LT CODE

Algorithm 2 designed in this paper is to predict and analyze the performance changes of LT codes, and to obtain the lowest error floor with minimal overhead. The EXIT chart can be used to analyze the convergence behavior of LT codes. The key of the EXIT chart is to obtain the information degree distribution and the check degree distribution [28], [29]. Therefore, the degree distributions of Algorithm 2 need to be formulated to analyze the convergence behavior.

A. ANALYSIS OF THE INFORMATION DEGREE DISTRIBUTION

The information nodes whose degrees are less than the threshold are forced to connect to check nodes in the reverse coding phase in Algorithm 2. Therefore, on the one hand, the information nodes with degrees less than the threshold are eliminated, and all $\Lambda_i\ (0 \le i \le T_v - 1)$ values are accumulated to Λ_{T_v}. On the other hand, the proportion of all information nodes whose degrees are greater than T_v are not changed, that is, the values of $\Lambda_i\ (T_v < i \le d_v)$ are the same as the conventional information degree distribution.

We denote the information degree distribution of the proposed scheme as $\Lambda_n(x) = \sum_{i=0}^{d_v} \Lambda_i^{(n)} x^i$. The proportion of information nodes with degree T_v can be calculated as

$$\Lambda_{T_v}^{(n)} = \sum_{i=0}^{T_v} \Lambda_i \quad (6)$$

Then, the proposed information degree distribution is

$$\Lambda_n(x) = \sum_{i=0}^{T_v} \Lambda_i x^{T_v} + \sum_{i=T_v+1}^{d_v} \Lambda_i x^i \quad (7)$$

where $\Lambda_i = \binom{N}{i}\left(\frac{\beta}{K}\right)^i\left(1-\frac{\beta}{K}\right)^{N-i}$

B. ANALYSIS OF THE CHECK DEGREE DISTRIBUTION

We denote the check degree distribution after the first coding phase as $\Omega(x) = \sum_{j=1}^{d_c} \Omega_j x^j$. The proportion of check nodes with degrees 1 and 2 remains unchanged after the reverse coding phase, that is, the coefficients Ω_1 and Ω_2 remain unchanged while all other coefficients have changed.

We denote the check degree distribution after the reverse coding phase as

$$\Omega_n(x)=\sum_{j=1}^{d_{c(n)}}\Omega_j^{(n)}x^j \tag{8}$$

where $d_{c(n)}$ represents the maximum check node degree of the proposed scheme.

We denote the average number of information nodes that need to be reversely coded is

$$K_a=K\sum_{i=0}^{T_c-1}\Lambda_i \tag{9}$$

The K_a information nodes contain T_v degrees, which are $0, 1, \ldots, T_v-1$ respectively. We denote the average number of check nodes that need to be reversely coded is

$$N_a=N\left(1-\Omega_1-\Omega_2\right) \tag{10}$$

In Algorithm 2, the information nodes whose degree is less than the threshold will randomly select check nodes to connect. Therefore, the Poisson distribution can be used to approximate the distribution of check nodes. It should be noted that when deducing the $\Omega_n(x)$, the number of check nodes that should be eliminated is not subtracted, but all N_a check nodes are considered to be selected. The reasons for the approximate calculation are as follows. i) N_a is much larger than the number of check nodes that need to be eliminated when the code length is long. Therefore, N_a can be used to approximate the true value. ii) When N_a is a constant value, the computational complexity of deducing the $\Omega_n(x)$ is lower.

The Poisson distribution can be used to approximate the true information degree distribution because the conventional coding scheme meets the following two conditions. i) The coefficients of the check degree distribution are already known. ii) The random selection method is used by the check nodes to select neighbor information nodes.

In fact, in the reverse coding phase, the roles of information nodes and check nodes are exchanged. The degrees contained in these K_a information nodes are $0, 1, \ldots, T_v-1$. The average number of information nodes of each degree is $K\Lambda_0$, $K\Lambda_1, \ldots, K\Lambda_{T_v-1}$ respectively. Then, the number of check nodes that need to be connected to information nodes of each degree is $T_v, T_v-1, \ldots, 1$.

Therefore, the degree distribution of the K_a information nodes that need to be coded reversely can be denoted as the reverse information degree distribution:

$$\Omega_\Lambda(x)=\sum_{j=1}^{T_v}\Omega_j^{(\Lambda)}x^j \tag{11}$$

where $\Omega_j^{(\Lambda)}=K\Lambda_{j-1}\big/K_a$.

After the reverse coding phase, the average degree of the K_a information nodes increases by

$$\beta_\Lambda=\sum_{j=1}^{T_v}j\Omega_j^{(\Lambda)} \tag{12}$$

After the reverse coding phase, the average degree of the N_a check nodes increases by

$$\alpha_\Omega=\beta_\Lambda\frac{K_a}{N_a} \tag{13}$$

If the degrees of the N_a check nodes are initialized to 0 before the reverse coding phase, then their degree distribution can be replaced by a Poisson distribution with a mean value of α_Ω. We denote the reverse check degree distribution as

$$\Lambda_\Omega(x)=\sum_{i=0}^{d_{c(\Omega)}}\Lambda_i^{(\Omega)}x^i \tag{14}$$

where $d_{c(\Omega)}=[K_a]$, and $[\cdot]$ represents the ceiling function.

Then the $d_{c(n)}$ can be calculated as

$$d_{c(n)}=d_c+[K_a] \tag{15}$$

The $\Lambda_i^{(\Omega)}$ can be calculated as

$$\Lambda_i^{(\Omega)}=\frac{\alpha_\Omega^i}{i!}e^{-\alpha_\Omega} \tag{16}$$

In fact, $\Lambda_i^{(\Omega)}$ represents the probability that the degrees of these N_a check nodes increase i in the reverse coding phase. However, the problem is to calculate the sum of the degrees obtained by each check node in the first and second phases, that is, the final check degree distribution needs to be solved. Therefore, the degree distribution $\Lambda_\Omega(x)$ obeyed by the N_a check nodes needs to be superposed on $\Omega(x)$.

Since the check nodes with degrees 1 and 2 do not need to be coded reversely, their degree distribution coefficients remain unchanged, namely $\Omega_1^{(n)}=\Omega_1$ and $\Omega_2^{(n)}=\Omega_2$. For a check node with degree m, the probability of its degree becoming $m+i$ is $\Omega_m\Lambda_i^{(\Omega)}$ after the reverse coding phase, where $0\le i\le d_{c(\Omega)}$ and $3\le m\le d_c$. This is for a certain check node. For all check nodes, the following matrix can be used to illustrate.

$$\mathbf{P}=\begin{bmatrix}\Omega_3\Lambda_0^{(\Omega)} & \Omega_4\Lambda_0^{(\Omega)} & \cdots & \Omega_{d_c}\Lambda_0^{(\Omega)}\\ \Omega_3\Lambda_1^{(\Omega)} & \Omega_4\Lambda_1^{(\Omega)} & \cdots & \Omega_{d_c}\Lambda_1^{(\Omega)}\\ \vdots & \vdots & \ddots & \vdots\\ \Omega_3\Lambda_{d_{c(\Omega)}}^{(\Omega)} & \Omega_4\Lambda_{d_{c(\Omega)}}^{(\Omega)} & \cdots & \Omega_{d_c}\Lambda_{d_{c(\Omega)}}^{(\Omega)}\end{bmatrix} \tag{17}$$

It can be seen that when the reverse coding phase is completed, the maximum degree of the check node will become $d_c+d_{c(\Omega)}$, and the minimum degree will still be 3. Then, for any check node whose degree is $j\left(3\le j\le d_c+d_{c(\Omega)}\right)$, it could be generated in the following two ways. i) The check node whose degree is originally j is not connected to any information node during the reverse coding phase. ii) The check node whose degree is originally not j reconnected to the information nodes, which increases its degree to j.

Then, the probability of check nodes with degree j after the reverse coding phase can be calculated as

$$\Omega_j^{(n)}=\sum_{m=3}^{j}\Omega_m\Lambda_{j-m+1}^{(\Omega)} \tag{18}$$

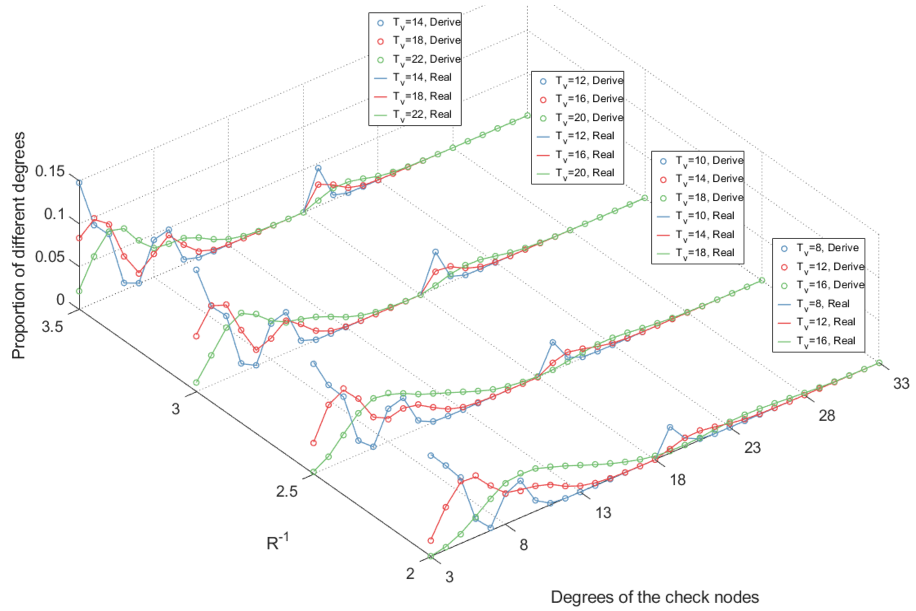

FIGURE 3. **Comparison of simulation results and derived results of the check degree distribution.**

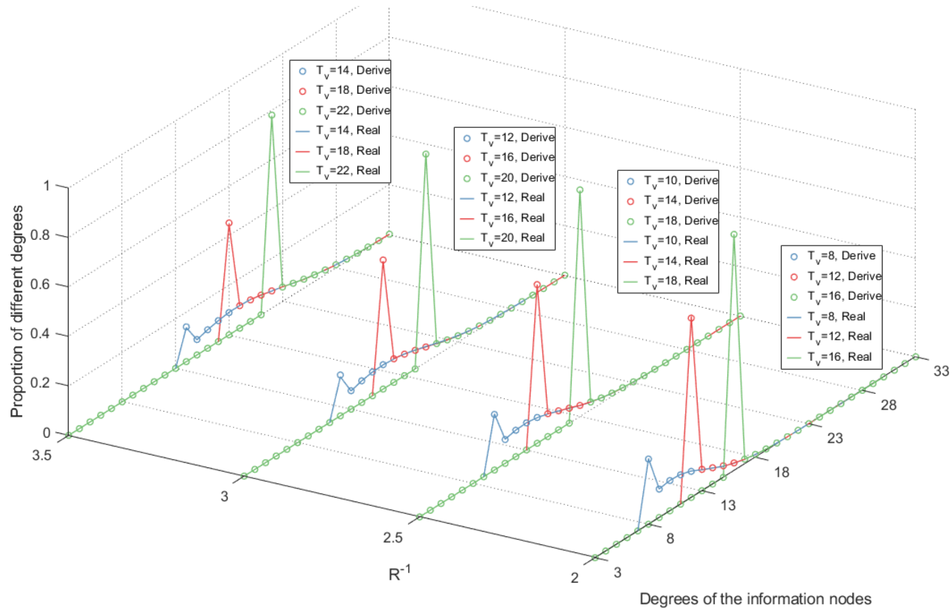

FIGURE 4. **Comparison of simulation results and derived results of the information degree distribution.**

Therefore, the final check degree distribution is

$$\Omega_n(x) = \Omega_1 x + \Omega_2 x^2 + \sum_{j=3}^{d_{c(n)}} \left(\sum_{m=3}^{j} \Omega_m \Lambda_{j-m+1}^{(\Omega)} \right) x^j \quad (19)$$

C. PROOF OF THE DERIVED DEGREE DISTRIBUTIONS

To verify whether the conclusions in Section III-A and III-B are correct, some simulation results are given, as shown in Fig. 3 and Fig. 4.

In the above two figures, the two different check degree distributions $\Omega_1(x)$ and $\Omega_2(x)$ are used [27]. Besides, to facilitate analysis, we set the abscissa axis as the reciprocal of the code rate.

Where

$$\Omega_2(x) = 0.015x + 0.495x^2 + 0.167x^3 + 0.082x^4 + 0.071x^5 + 0.049x^8 + 0.048x^9 + 0.05x^{19} + 0.023x^{66} \quad (20)$$

For simulations, we consider the following schemes:

i) We set $R^{-1} = 2$, and the thresholds are 8, 12, and 16, respectively. The check degree distribution $\Omega_1(x)$ is used.

ii) We set $R^{-1} = 2.5$, and the thresholds are 10, 14, and 18, respectively. The check degree distribution $\Omega_1(x)$ is used.

iii) We set $R^{-1} = 3$, and the thresholds are 12, 16, and 20, respectively. The check degree distribution $\Omega_2(x)$ is used.

iv) We set $R^{-1} = 3.5$, and the thresholds are 14, 18, and 22, respectively. The check degree distribution $\Omega_2(x)$ is used.

All results are obtained through 200 Monte Carlo simulations. In addition, we also give the derived results for comparison.

It can be seen from the two figures that the simulation results are entirely consistent with the derived results, which proves the correctness of the degree distribution mathematical expressions we derived in Section III-A and III-B.

IV. DESIGN AND ANALYSIS OF THE PROPOSED LT CODE

A. CONVERGENCE ANALYSIS OF THE PROPOSED SCHEME

The EXIT chart is used to analyze the convergence behavior of LT codes. Denote the monotonically increasing function $J(\theta)$ as [30]

$$J(\theta) = 1 - \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{+\infty} e^{-\frac{(\xi-\theta^2/2)^2}{2\theta^2}} \log_2\left(1+e^{-\xi}\right) d\xi \tag{21}$$

Its unique inverse function is $\theta = J^{-1}(I)$. The approximated calculation method of $J(\cdot)$ and $J^{-1}(\cdot)$ is given in [31]. We divide the LT decoder into the check node decoder (CND) and the information node decoder (IND), and the EXIT function of the conventional CND is

$$I_{E,C} = 1 - \sum_{j=1}^{d_c} \rho_j J\left(\sqrt{(j-1)\sigma_C^2 + \sigma_{ch}^2}\right) \tag{22}$$

where

$$\sigma_{ch}^2 = \left[J^{-1}\left(1 - J\left(\sqrt{4/\sigma_n^2}\right)\right)\right]^2 \tag{23}$$

$$\sigma_C^2 = \left[J^{-1}\left(1 - I_{A,C}\right)\right]^2 \tag{24}$$

The σ_n^2 is the noise variance of BIAWGN channels. $I_{A,C}$ represents the priori mutual information of the CND.

The EXIT function of the conventional IND is

$$I_{E,I} = \sum_{i=1}^{d_{v(\max)}} \lambda_i J\left(\sqrt{(i-1)\sigma_I^2}\right) \tag{25}$$

where $\sigma_I = J^{-1}(I_{A,I})$ and $I_{A,I}$ represents the priori mutual information of the IND.

There may be slight deviations in the convergence analysis using the EXIT chart. However, referring to the conclusion in the literature [19], it can be known that such a deviation is acceptable. Further, the EXIT function of the proposed LT code can be computed as

$$I_{E,C}^{(n)} = 1 - \sum_{j=1}^{d_c+d_{c(\Omega)}} \rho_j^{(n)} J\left(\sqrt{(j-1)\sigma_C^2 + \sigma_{ch}^2}\right) \tag{26}$$

$$I_{E,I}^{(n)} = \sum_{i=T_v}^{d_v} \lambda_i^{(n)} J\left(\sqrt{(i-1)\sigma_I^2}\right) \tag{27}$$

where $\lambda_i^{(n)}$ represents the information-node edges distribution of the proposed LT code and $\rho_j^{(n)}$ represents the check-node edges distribution of the proposed LT code.

Some simulation results are given as shown in Fig. 5. We set $R^{-1} = 2$, $E_s/N_0 = 5$dB. The check degree distribution $\Omega_1(x)$ is adopted. For the proposed LT code, not only the degree distribution of information nodes is changed, but also the degree distribution of check nodes. Therefore, both the IND curve and CND curve of the proposed scheme deviate from their original positions, and the greater the threshold, the greater the deviation. Besides, the deviation position and deviation amplitude of the two curves are not the same, which indicates that the decoding convergence trajectory may be narrow in the front and wide in the back. This may increase the number of iterations and increase the decoding complexity. More seriously, when the threshold is too large, the IND curve and the CND curve will even intersect, which will cause decoding failure.

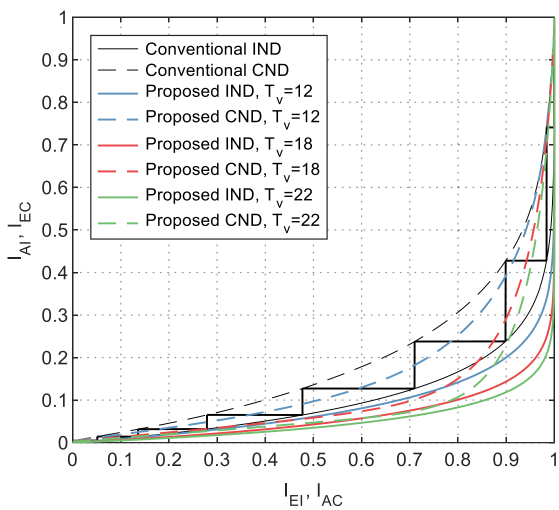

FIGURE 5. Convergence behavior of the proposed LT code under different thresholds.

Therefore, to ensure decoding success, an optimal threshold should be elaborately designed to meet the following requirements. i) The IND should have a large extrinsic information gain, and the CND should have a small extrinsic information loss. ii) The decoding convergence region should keep open, and there should be no partial narrowness.

B. DESIGN PRINCIPLES OF THE THRESHOLD

The proposed scheme will not only move the EXIT curves to the left, but also compress the decoding convergence region, and even cause decoding failure when the threshold is large. Therefore, although the proposed scheme can reduce the error floor of the LT code, the threshold is not the larger the better, and there should be an optimal value.

TABLE 3. The GLR under different SNR and decoding overhead.

GLR	E_s/N_0=0dB			E_s/N_0=1dB			E_s/N_0=2dB		
	R^{-1}=2	R^{-1}=2.1	R^{-1}=2.2	R^{-1}=2	R^{-1}=2.1	R^{-1}=2.2	R^{-1}=2	R^{-1}=2.1	R^{-1}=2.2
T_v=5	0.6462	0.6125	0.5818	0.5889	0.5583	0.5302	0.5469	0.5184	0.4924
T_v=6	0.6733	0.6394	0.6084	0.6136	0.5828	0.5545	0.5698	0.5412	0.5150
T_v=7	0.6913	0.6575	0.6265	0.6301	0.5993	0.5711	0.5851	0.5565	0.5303
T_v=8	0.7034	0.6698	0.6391	0.6412	0.6106	0.5825	0.5955	0.5670	0.5409
T_v=9	0.7115	0.6782	0.6477	0.6486	0.6182	0.5904	0.6024	0.5742	0.5483
T_v=10	0.7166	0.6838	0.6536	0.6533	0.6233	0.5958	0.6068	0.5789	0.5533
T_v=11	0.7196	0.6872	0.6573	0.6561	0.6265	0.5993	0.6095	0.5819	0.5566
T_v=12	0.7212	0.6891	0.6596	0.6576	0.6283	0.6013	0.6109	0.5836	0.5586
T_v=13	0.7219	0.6899	0.6607	0.6583	0.6291	0.6024	0.6116	0.5845	0.5596
T_v=14	0.7219	0.6901	0.6611	0.6584	0.6294	0.6028	0.6118	0.5848	0.5601

To satisfy the above requirements, we consider introducing the extrinsic information gain-loss-ratio (GLR) to design the threshold and other parameters. The loss of the extrinsic information can be computed as

$$L_{E,C} = \sum_{I_{A,C}=0}^{I_{A,C}\to 1} I_{E,C} - I_{E,C}^{(n)} \tag{28}$$

The gain of the extrinsic information can be computed as

$$G_{E,I} = \sum_{I_{A,I}=0}^{I_{A,I}\to 1} I_{E,I}^{(n)} - I_{E,I} \tag{29}$$

We define the GLR as

$$GLR = \frac{G_{E,I}}{L_{E,C}} \tag{30}$$

Maximizing GLR is used as one of the principles for designing the optimal threshold. Table 3 shows the calculation results of GLR under different thresholds. The check degree distribution $\Omega_1(x)$ is adopted.

The following conclusions can be drawn from the results in Table 3.

i) When the signal-to-noise ratio (SNR) and decoding overhead are constant, the GLR will increase as the threshold increases. This shows that although the total loss of the CND is greater than the total gain of the IND, the gap between the two will gradually narrow. Therefore, a larger threshold seems to be better for the proposed scheme, but this is an ideal situation without considering the code length and the number of iterations.

ii) When the SNR and code rate are different, the GLRs at the same T_v are not equal. This means it seems difficult to design the optimal threshold using only a fixed GLR value. It is necessary to analyze the trend of GLR, and other necessary principles need to be considered.

We can also observe in Table 3 that the increase rate of the GLR will slow down when the GLR reaches a certain value. Therefore, to solve the optimal threshold, we set a 'slow increase region' of the GLR to measure whether the decoding convergence trajectory will get narrow.

Denote the increase rate threshold of GLR to analyze the boundary value of the slow increase region. The increase rate of GLR (IR-GLR) corresponding to the i-th ($i \geq 2$) threshold can be computed as

$$IR - GLR = \frac{GLR_i - GLR_{i-1}}{GLR_{i-1}} \times 100\% \tag{31}$$

We denote the increase rate threshold of GLR as T_{IR}. Then when the IR-GLR of the i-th GLR is less than the T_{IR}, it means that the i-th GLR has entered the slow increase region. Then all the GLRs in the region and the thresholds corresponding to these GLRs will be marked as not selectable. Besides, the boundary threshold at which the IND curve and the CND curve exactly intersect is the maximum threshold $T_{v(\max)}$. The design method of the optimal threshold is outlined in Algorithm 3.

Then the optimal threshold under arbitrary parameter constraints can be solved according to Algorithm 3. Table 4 shows the optimal thresholds of the check degree distribution $\Omega_1(x)$ when $T_{IR} = 0.5\%$, $T_{IR} = 0.3\%$, and $T_{IR} = 0.2\%$. It can be seen that changes in decoding overhead are more likely to cause changes in the optimal threshold, while changes in the SNR have a smaller impact on the optimal threshold.

C. THE MODIFIED REVERSE CODING SCHEME

According to the design method in Section IV-B, we can design the optimal threshold for any given code rate and SNR.

TABLE 4. The optimal thresholds for different channel parameters when T_{IR}, $T_{IR} = 0.3\%$, and $T_{IR} = 0.2\%$.

SNR	R^{-1}	Optimal Threshold (T_{IR}=0.5%)	Optimal Threshold (T_{IR}=0.3%)	Optimal Threshold (T_{IR}=0.2%)	SNR	R^{-1}	Optimal Threshold (T_{IR}=0.5%)	Optimal Threshold (T_{IR}=0.3%)	Optimal Threshold (T_{IR}=0.2%)
0.0dB	2.0	10	11	12	0.5dB	2.0	10	11	12
	2.1	10	11	12		2.1	11	11	12
	2.2	11	12	12		2.2	11	12	12
	2.3	11	12	13		2.3	11	12	13
	2.4	11	12	13		2.4	11	12	13
	2.5	12	13	13		2.5	12	13	13
1.0dB	2.0	10	11	12	1.5dB	2.0	10	11	12
	2.1	11	11	12		2.1	11	11	12
	2.2	11	12	12		2.2	11	12	12
	2.3	11	12	13		2.3	11	12	13
	2.4	11	12	13		2.4	11	12	13
	2.5	12	13	13		2.5	12	13	13

Algorithm 3 The Optimal Threshold Scheme

Step 1: Calculate the maximum threshold and set the initial threshold range as $\{1, T_{v(\max)}\}$.
Step 2: Calculate the degree distributions and GLR corresponding to each threshold.
  for $m = 1$: $T_{v(\max)}$
  i) According to the conclusion in Section III, calculate the information degree distribution and check degree distribution of the m-th threshold.
  ii) According to the conclusion in Section IV-B, calculate the GLR of the m-th threshold and record it as GLR_m.
  end
Step 3: Calculate the IR-GLR of each threshold and predict the optimal threshold.
for $m = 2$: $T_{v(\max)}$
  i) Calculate the IR-GLR of the m-th threshold and record it as IR-GLR_m.
  ii) If the IR-GLR_m is greater than T_{IR}, then return to **Step 3**
i). If the IR-GLR_m is less than T_{IR}, then go to **Step 4**.
  end
Step 4: Output the optimal threshold as $T_v = m - 1$.

However, on the one hand, this method seems to make the LT code limited to a fixed bit rate, rather than rateless. We know that the characteristic of fountain codes is that the code rate is not fixed, and any number of check nodes can be continuously generated. Therefore, we need to modify the LT coding scheme proposed in this paper to make it rateless.

On the other hand, the connection graph between the information node and the check node should be easily obtained. The LT code generally uses a generator matrix $\mathbf{G}$ to record the connection relationship [32]. Where $\mathbf{G}$ is the $K \times N$ binary generator matrix. If the element in the i-th row and the j-th column is 1, it means that there is a connecting edge between the i-th information node and the j-th check node. The product of the information nodes and the generator matrix are the check nodes.

$$\mathbf{c} = \mathbf{v}\mathbf{G} \tag{32}$$

Therefore, we intend to use the segment coding method to achieve the rateless characteristic of the proposed scheme, and use the redundant generator matrix to record the connection relationship.

This is because we noticed that the optimal threshold T_v and the number of check nodes also matched segment by segment. For example, when $T_{IR} = 0.2\%$, the optimal thresholds corresponding to $R^{-1} = 2.0, 2.1$, and 2.2 are all 12, and it becomes 13 until $R^{-1} = 2.3$ and 2.4.

We suppose that the maximum number of check nodes that can be tolerated in the receiver decoder is $N_{\max}$, that is, the maximum number of columns of the redundant generation matrix is $N_{\max}$. When the number of received check nodes reaches $N_{\max}$ but the information nodes are still not recovered correctly, we consider that this decoding failure.

We fix SNR, check degree distribution, and K. According to Algorithm 3, we can calculate all the optimal thresholds and their corresponding number of check nodes. Suppose they are $[T_{v(1)}, T_{v(2)}, \ldots, T_{v(\tau)}]$ and $[K + K_{\sigma(1)}, K + K_{\sigma(1)} + K_{\sigma(2)}, \ldots, N_{\max}]$. Where the numerical difference between these optimal thresholds is 1.

According to Algorithm 2, the first $K + K_{\sigma(1)}$ check nodes are generated, and $T_{v(1)}$ is the optimal threshold. Then a basic matrix is obtained, which is used to record the connection relationship of the first coding segment. When the next $K_{\sigma(2)}$ check nodes are generated, the degrees of all information nodes should not be less than $T_{v(2)}$. First, all K information nodes are used as candidate nodes, and $K_{\sigma(2)}$ check nodes are generated according to the conventional scheme [4]. Then, the information nodes whose degrees are less than $T_{v(2)}$ are screened out and coded reversely. However, it should be noted that the check nodes that can be selected at this time are only the nodes with a degree greater than 2 among the $K_{\sigma(2)}$ check nodes, not all check nodes.

Continue to code until $N_{\max}$ check nodes are generated. Then, all information nodes will have degrees not less than $T_{v(\tau)}$. At the same time, we also obtain a redundant generator matrix $\mathbf{G_{max}}$ with K rows and $N_{\max}$ columns. Fig. 6 shows the generator matrix using the segment coding method.

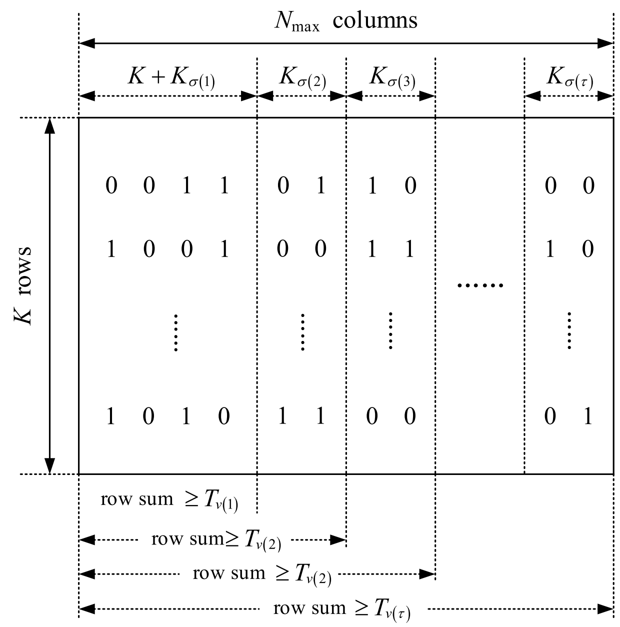

FIGURE 6. **The framework graph of the rateless redundant generator matrix of the modified proposed scheme.**

The advantages of segment coding are as follows:

i) The design goal of the proposed scheme is achieved. That is, when any number of check nodes is generated, the minimum information degree is not less than the optimal threshold corresponding to the number of check nodes.

ii) The proposed scheme can realize the rateless characteristic. Each check node is not related to the previous check nodes and can be sent in sequence and independently.

iii) The sender and receiver can easily record the connection relationship. The same redundant generator matrix $\mathbf{G_{max}}$ can be stored at both the sender and receiver, which is used to record the edges between check nodes and information nodes. When coding and decoding, we only need to extract the column vectors in sequence.

V. SIMULATION RESULTS

In this section, the BER performance of the proposed coding scheme and the other schemes are presented. BPSK modulation is used and all results are obtained through 1,000,000 Monte Carlo simulations. The maximum number of BP decoding iterations is 50. The code length K used in simulations is set to 1024. The check degree distribution $\Omega_1(x)$ is used.

A. BER PERFORMANCE OF THE PROPOSED SCHEME AND OTHER SCHEMES

Fig. 7 and Fig. 8 show the BER performance of the proposed scheme under different SNRs and R^{-1}. The R^{-1} is set to 2 in Fig. 7 and the SNR is set to 1dB in Fig. 8. The following conclusions can be drawn.

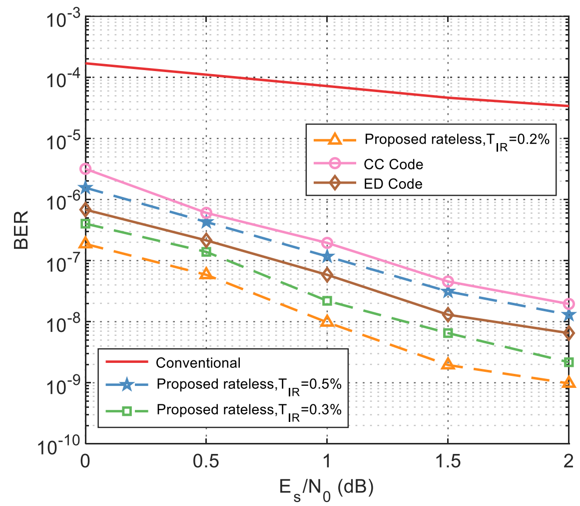

FIGURE 7. **The BER performance of the proposed LT code, the ED LT code, and the CC LT code under different SNR for $R^{-1} = 2$.**

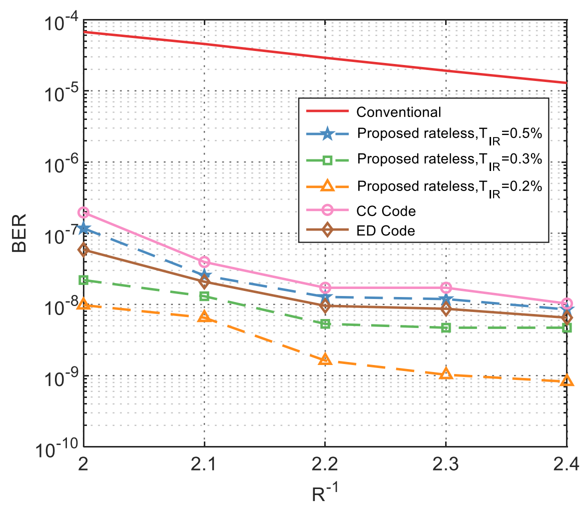

FIGURE 8. **The BER performance of the proposed LT code, the ED LT code, and the CC LT code under different decoding overhead for $E_S/N_0 = 1$dB.**

i) The proposed scheme can significantly reduce the error floor. For example, compared with the conventional LT code, the proposed LT code can reduce the error floor by 4 orders of magnitude for $T_{IR} = 0.2\%$ in Fig. 7. This is because the low-degree information nodes are eliminated, which also proves the correctness of the proposed scheme.

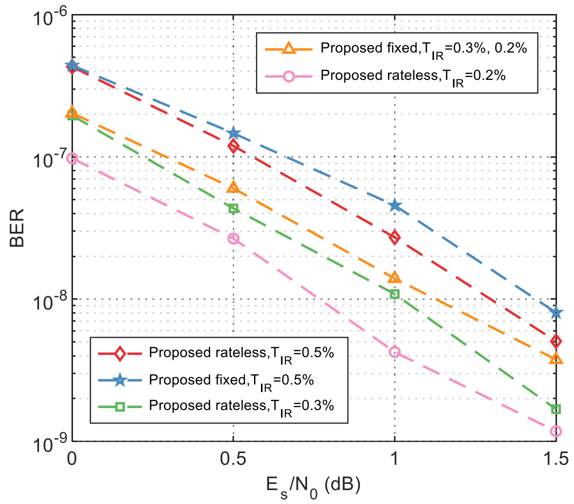

FIGURE 9. **The BER performance of the proposed LT code and the modified LT code under different SNRs for $R^{-1} = 2.2$.**

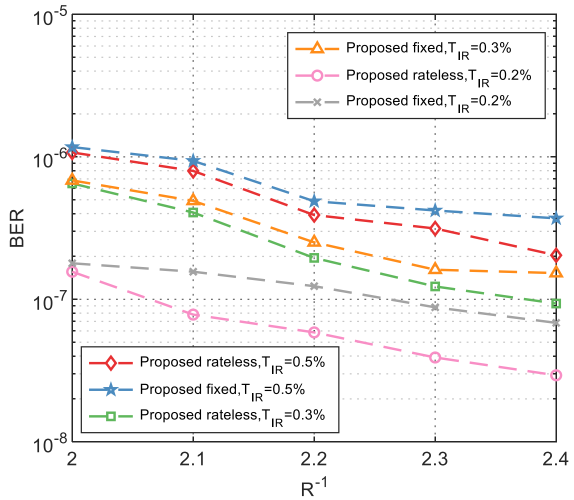

FIGURE 10. **The BER performance of the proposed LT code and the modified LT code under different decoding overhead for $E_S/N_0 = 0$dB.**

ii) The proposed scheme also outperforms the ED scheme and the CC scheme for $T_{IR} = 0.3\%$ and $T_{IR} = 0.2\%$. For example, when $R^{-1} = 2.4$ and $E_s/N_0 = 1dB$, the BER of the proposed scheme is nearly an order of magnitude lower than the two improved schemes. This is because the lowest information degree can be regulated by adjusting the threshold, so the BER lower bound can also be reduced. While the average information degree of the two improved schemes can only be blindly increased without a target.

iii) The design method of the optimal threshold is feasible. In the above two cases, when $T_{IR} = 0.5\%$, the BER performance of the proposed scheme is not better than the other two schemes. However, by reducing the T_{IR}, a lower error floor than these two schemes can be achieved. Besides, the BER can be quickly reduced to below 10^{-8} for $T_{IR} = 0.2\%$ under different SNRs. It means that the error floor can be significantly reduced at the expense of a slight loss of the convergence behavior.

B. BER PERFORMANCE OF THE MODIFIED SCHEME

Fig. 9 and Fig. 10 show the BER performance of the Algorithm 2 and the modified reverse coding scheme under different SNRs and R^{-1}. The R^{-1} is set to 2.2 in Fig. 9 and the SNR is set to 0dB in Fig. 10. The following conclusions can be drawn.

The modified scheme outperforms the original proposed scheme. In Fig. 9, for example, if 10^{-8} is considered as the standard, the required SNR is 0.7dB for the modified scheme when $T_{IR} = 0.2\%$, while the original proposed scheme is 1.1dB. This means that the modified scheme can obtain a coding gain of 0.4dB. In Fig. 10, for example, if 10^{-7} is considered as the standard, the required coding overhead is 2.06 for the modified scheme when $T_{IR} = 0.2\%$, while the original proposed scheme is 2.25. This means that the modified scheme reduces the coding overhead of 0.19. This is because the modified scheme will generate check nodes in segments to eliminate low-degree information nodes, so that the average degrees of the check nodes and information nodes are larger than the original proposed scheme. Therefore, it has a higher elimination efficiency and a lower error floor.

VI. CONCLUSION

The conventional LT codes will encounter a high error floor in the BEC and AWGN channels, which is caused by low-degree information nodes. To solve this problem, an improved coding scheme is proposed in this paper. A well-designed threshold is designed to screen out the low-degree information nodes, which will be coded reversely to ensure that they can obtain degrees not less than the threshold. We studied the connection rules between the information nodes and the check nodes for the proposed scheme and deduced the mathematical expression of their degree distributions. We estimated the convergence behavior of the scheme and designed the parameter GLR to establish the relationship between convergence and threshold. We also set a 'slow increase region' of the GLR to solve the optimal threshold. Besides, we modified the proposed scheme to prevent it from being restricted to a fixed code rate. Therefore, the check nodes can be transmitted independently and are not limited by a fixed number. Numerical results indicate that the error floor of the proposed scheme can be lowered by 4 orders of magnitude. Besides, by adjusting the threshold, the proposed scheme can achieve better BER performance than the ED scheme and the CC scheme, which proves the advantages of the scheme in this paper. As a part of the future work, it will also be interesting to investigate coding scheme with no loss of convergence, under the light of our proposed framework.

REFERENCES

[1] J. W. Byers, M. Luby, and M. Mitzenmacher, "A digital fountain approach to asynchronous reliable multicast," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 8, pp. 1528–1540, Oct. 2002.

[2] J. Huang, Z. Fei, C. Cao, M. Xiao, and X. Xie, "Reliable broadcast based on online fountain codes," *IEEE Commun. Lett.*, vol. 25, no. 2, pp. 369–373, Feb. 2021.

[3] S. S. Borkotoky and M. B. Pursley, "Fountain-coded broadcast distribution in multiple-hop packet radio networks," *IEEE/ACM Trans. Netw.*, vol. 27, no. 1, pp. 29–41, Feb. 2019.

[4] M. Luby, "LT codes," in *Proc. IEEE Symp. Found. Comput. Sci.*, Vancouver, BC, Canada, Nov. 2002, pp. 271–280.

[5] Z. Zhang, H. Zhang, H. Dai, X. Chen, and D. O. Wu, "Fountain-coded file spreading over mobile networks," *IEEE Trans. Wireless Commun.*, vol. 16, no. 10, pp. 6766–6778, Oct. 2017.

[6] W. B. Abbas, P. Casari, and M. Zorzi, "Controlled flooding of fountain codes," *IEEE Trans. Wireless Commun.*, vol. 16, no. 7, pp. 4698–4710, Jul. 2017.

[7] M. Abughalwa and M. O. Hasna, "A secrecy study of UAV based networks with fountain codes and FD jamming," *IEEE Commun. Lett.*, vol. 25, no. 6, pp. 1796–1800, Jun. 2021.

[8] J. Shang, W. Xu, C.-H. Lee, X. Yuan, P. Zhang, and J. Lin, "REF codes: Intermediate performance oriented fountain codes with feedback," *IEEE Trans. Veh. Technol.*, vol. 69, no. 11, pp. 13148–13164, Nov. 2020.

[9] K. Zhang, J. Jiao, Z. Huang, S. Wu, and Q. Zhang, "Finite block-length analog fountain codes for ultra-reliable low latency communications," *IEEE Trans. Commun.*, vol. 68, no. 3, pp. 1391–1404, Mar. 2020.

[10] H. Zhang, M. Yang, W. Bao, X. Li, and J. Wang, "Short-frame fountain code for plasma sheath with 'communication Windows,"' *IEEE Trans. Veh. Technol.*, vol. 69, no. 12, pp. 15569–15579, Dec. 2020.

[11] R. Palanki and J. S. Yedidia, "Rateless codes on noisy channels," in *Proc. IEEE Int. Symp. Inf. Theory*, Chicago, IL, USA, Jun./Jul. 2004, p. 37.

[12] L. Zhang and L. Su, "Goal-oriented design of optimal degree distribution for LT codes," *IET Commun.*, vol. 14, no. 16, pp. 2658–2665, Oct. 2020.

[13] O. Etesami and A. Shokrollahi, "Raptor codes on binary memoryless symmetric channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 2033–2051, May 2006.

[14] M. Shirvanimoghaddam and S. Johnson, "Raptor codes in the low SNR regime," *IEEE Trans. Commun.*, vol. 64, no. 11, pp. 4449–4460, Nov. 2016.

[15] A. Kharel and L. Cao, "Analysis and design of physical layer raptor codes," *IEEE Commun. Lett.*, vol. 22, no. 3, pp. 450–453, Mar. 2018.

[16] S. Xu and D. Xu, "Optimization design and asymptotic analysis of systematic Luby transform codes over BIAWGN channels," *IEEE Trans. Commun.*, vol. 64, no. 8, pp. 3160–3168, Aug. 2016.

[17] H. Fadhel, A. Kharel, and L. Cao, "Performance of raptor codes on the BIAWGN channel in the presence of SNR mismatch," in *Proc. IEEE Wireless Commun. Netw. Conf.*, Seoul, South Korea, May 2020, pp. 1–6.

[18] W. Yu, K. Narayanan, J. Cheng, and J. Wu, "Raptor codes with descending order degrees for AWGN channels," *IEEE Commun. Lett.*, vol. 24, no. 1, pp. 29–33, Jan. 2020.

[19] I. Hussain, M. Xiao, and L. K. Rasmussen, "Error floor analysis of LT codes over the additive white Gaussian noise channel," in *Proc. IEEE Global Telecommun. Conf.*, Houston, TX, USA, Dec. 2011, pp. 1–5.

[20] I. Hussian, M. Xiao, and K. Rasmussen, "Design of LT codes with equal and unequal erasure protection over binary erasure channels," *IEEE Commun. Lett.*, vol. 17, no. 2, pp. 261–264, Feb. 2013.

[21] C. M. Chen and Y. P. Chen, "Connection choice code," *IEICE Trans. Commun.*, vol. E97-B, pp. 1350–1357, Jul. 2014.

[22] L. J. Chang, C. H. Wang, and J. K. Zao, "An error-floor reduction technique for short-length LT codes," in *Proc. Int. Symp. Inform. Theory Appl.*, Melbourne, VIC, Australia, Oct. 2014, pp. 279–283.

[23] H. Tian, D.-F. Zhao, Y.-F. Yang, and R. Xue, "Research of LT code based on key information feedback in deep space communication," *IEEE Access*, vol. 8, pp. 103956–103972, Jun. 2020.

[24] A. Kharel and L. Cao, "Improved fountain codes for BI-AWGN channels," in *Proc. IEEE Wireless Commun. Netw. Conf.*, San Francisco, CA, USA, Mar. 2017, pp. 1–6.

[25] F. Lazaro, G. Liva, G. Bauch, and E. Paolini, "Bounds on the error probability of Raptor codes under maximum likelihood decoding," *IEEE Trans. Inf. Theory*, vol. 67, no. 3, pp. 1537–1558, Mar. 2021.

[26] J. Lobo and S. Kuwelkar, "Performance analysis of merge sort algorithms," in *Proc. IEEE Int. Conf. Electron. Sustain. Commun. Syst.*, Coimbatore, India, Jul. 2020, pp. 110–115.

[27] W. Zhang, S. Hranilovic, and C. Shi, "Soft-switching hybrid FSO/RF links using short-length Raptor codes: Design and implementation," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 9, pp. 1698–1708, Dec. 2009.

[28] F. Lazaro, G. Liva, and G. Bauch, "Inactivation decoding of LT and Raptor codes: Analysis and code design," *IEEE Trans. Commun.*, vol. 65, no. 10, pp. 4114–4127, Oct. 2017.

[29] Y. Zhang, Z. Fan, and L. Meng, "Rateless coded multi-user uplink transmission with distributed fronthaul compression in cloud RAN," *IEEE Access*, vol. 8, pp. 186710–186724, Oct. 2020.

[30] S. T. Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 1727–1737, Oct. 2001.

[31] S. T. Brink, G. Kramer, and A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," *IEEE Trans. Commun.*, vol. 52, no. 4, pp. 670–678, Apr. 2004.

[32] D. Jia, Z. Fei, J. Yuan, S. Tian, and J. Kuang, "A hybrid EF/DF protocol with rateless coded network code for two-way relay channels," *IEEE Trans. Commun.*, vol. 64, no. 8, pp. 3133–3147, Aug. 2016.

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