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Capacity Design and Pareto Improvement of Highway Toll Plaza in a Competitive Transport System

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ABSTRACT As the traffic infrastructure for collecting vehicle tolls, the capacity of the toll plaza determines the service level of the entire highway. The capacity of a toll plaza is highly correlated with its operating costs, especially in peak periods. In this paper, it is assumed that in a competitive transportation system, the residential area and the workplace are connected by a highway with a toll plaza which forms a bottleneck, parallel to a mass transit line; commuters can choose to travel by car or by public transport. By establishing an equilibrium model and two mathematical programming models, the capacity designs of the toll plaza were studied during the traffic service period to achieve three objectives, namely, the toll plaza breaking even, profit maximization, and total social cost minimization of the transportation system. The travel modal splits were analyzed under travel equilibrium in three situations, respectively. In addition, a bi-objective optimization model was developed to optimize total profit and total social cost, and a Pareto optimization scheme was analyzed. Finally, the theoretical analyses were also verified by numerical examples.

INDEX TERMS Bi-modal traffic, bi-objective optimization model, capacity design, highway toll plaza.

I. INTRODUCTION

In recent decades, with the acceleration of economic growth and urbanization, rapid expansions and complex changes in developing cities around the world have emerged. Urban expansion results in commuters living further away from workplaces, which, in turn, dramatically increases the demand for motorized vehicles. Simultaneously, the rapid development of highways and mass transit systems in these cities provides major travel services for commuters. Highway transportation plays an important role in the modern economy and social life, although highway construction and maintenance costs are high, therefore requiring a large amount of capital investment. Toll plazas are thus set up on highways, and the fees collected from highway users are the main source of funds for highway construction and daily operation. However, the presence of a toll plaza obstructs the traffic flow. The capacity of toll plaza seriously affects the fluency of highway traffic flow, which is determined by the capacity of toll plazas and the time of service delivery, all being related to operating costs. Without enlarging the toll plaza, the capacity of toll plazas can be improved to a certain extent through the technical training of toll personnel or the upgrading of toll systems. Furthermore, due to land resources and financial constraints, it is unrealistic to enhance traffic capacity at a large scale; therefore, the increasing traffic demand makes the congestion at the toll plaza increasingly serious. For this purpose, transport departments often build mass transit systems (for example, a railway) to provide passengers with an alternative mode of transport.

Once the highway is put into use, the operation department of the highway usually carries out daily management and collects tolls at the toll plaza. However, highway operators do not necessarily have the pricing power. As labor costs continue to rise, the cost of operating toll plazas also increases, making it necessary for operators to reduce operating costs to avoid losses. This is the motivation for this paper to investigate several strategies with consideration of the operating costs

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of toll plazas in a competitive transportation system with highways and mass transit systems.

The main contribution of this paper is to discuss three toll plaza capacity setting schemes and Pareto improvement scheme in a competitive transportation system with highway and public transport, taking into account the operating costs of toll plazas. The analytical and numerical results could help urban transportation system operators determine the appropriate highway toll and the service capacity of toll plaza.

The remainder of this paper is organized as follows. Section 2 categorizes and reviews the related literature. Section 3 provides basic settings of the bi-modal traffic model and the equilibrium of the modal split. Section 4 introduces the operating costs of the toll plaza and investigates the capacity design under different strategies. Section 5 illustrates the feasibility of the proposed models through numerical examples. Finally, the conclusions are given in Section 6.

II. LITERATURE REVIEW

Literature related to three topics, namely, bottleneck models, bi-modal transport systems, and toll plaza capacity designs, are reviewed in this section.

A. BOTTLENECK MODELS

The traffic bottleneck model was first proposed by Vickrey [1], who described that all travelers attempt to minimize their individual travel costs by choosing their departure time in their journey from origin to destination by making trade-offs between travel costs and schedule delay costs. The travel cost can be formulated as queuing delay at a bottleneck, and the schedule delay cost can be formulated as early/late arrival penalties incurred when travelers do not arrive at their destination at the desired time. Thereafter, the issue of heterogeneity is studied from various perspectives in the literature, such as the value of time, schedule delay penalty, desired arrival time, etc [2]–[4]. Many studies have further investigated road congestion pricing, tradable credit and their impact [5]–[7]. Uncertain and variable capacity traffic bottleneck model have also received attention in the literature [8]–[11].

B. BI-MODAL TRANSPORT SYSTEM

With the development of highly integrated urban traffic systems, researchers began to study bi-modal traffic problems in integrated multimodal transport systems. Tabuchi [12] was one of the first scholars to research modal split behavior in a transport system with a physically separated mass transit system parallel to a bottleneck road. Bottleneck roads and co-existing mass transit bi-modal systems have been studied by many scholars from different perspectives. Huang [13] extended Tabuchi's study by considering travelers' crowding cost in carriages with two groups of commuters and four different pricing schemes for comparison, and analyzed how charging policies affected model choice behaviors. Danielis and Marcucci [14] examined the efficiency of different road pricing regimes in reducing the total travel cost in a competitive bi-modal system of highways and transit systems. Huang *et al*. [15] investigated the traffic modal split and commuting patterns of private cars/buses in a bottleneck-constrained highway, and they defined the bus travel cost when considering bus waiting time at bottleneck, schedule delay cost, crowding cost and bus fare, and analyzed the equilibrium with both modal choice and departure time choice. Wu and Huang [16] explored the departure patterns of commuters through analyzing the equilibrium under three road-use pricing strategies. Each strategy was the combination of a time-varying toll and a flat toll, and it was found that the flat toll for minimizing total social cost was negative and financial subsidy was needed to encourage some commuters to leave home earlier or later. Wang and Ding [17] studied modal split in daily travel when there is a railroad parallel to a bottleneck constrained road between home and a workplace, and examined the optimality and efficiency of different railroad fare and road toll schemes with the boundedly rational mode choice behavior of travelers. Under such behavior, commuters did not necessarily choose the mode of which the travel cost was absolutely lower than the mode of the other. Wang *et al*. [18] explored a bi-modal equilibrium network to find optimal parking lots by considering the transit travel cost and the auto travel cost under three strategies: drivers with a tradable parking permit, drivers with a nontradable parking permit, and drivers without a parking permit. Zhang and Guan [19] established an evolutionary game model based on the indifference threshold to analyze the travelers mode choice behavior. The model supposed that the travelers' behavioral adjustment of decision-making of travelers followed the principle of random utility maximization only when the perceived difference in utility between modes was greater than the indifference threshold; otherwise, travelers chose randomly. Liu *et al*. [20] investigated modal split and departure time choices of heterogeneous travelers and two capacity design problems, i.e., total travel cost minimization with budget constraints and total system cost minimization with budget and equity constraints in a bi-modal traffic corridor with a highway and a transit line.

C. TOLL PLAZA CAPACITY DESIGN

A toll plaza is a place where drivers stop to pay bills before entering or leaving a highway. They are therefore bottlenecks on the road. When the number of vehicles arriving is higher than the service capacity of the toll plaza, a queue is formed, resulting in traffic congestion. The length of the queue is directly affected by the traffic demand and the service capacity of the toll plaza. Edie [21] was one of the first scholars who utilized empirical data to investigate the relationship among the flows, the number of toll booths, and the level of service, and proposed a method for determining the number of toll booths required and recommended toll collectors' schedules based on an analysis of data from the Lincoln Tunnel. The operation of toll plazas is also closely related to the charging problem. By altering the pricing strategy, the length of the queue can be controlled and congestion at the toll booth can be alleviated [22]–[26]. Siamak and Francisco [27] have collected and synthesized a large amount of scattered information on highway user costs and incorporated it into an economic analysis of toll plaza operations, examining the operating costs and revenues of conventional toll plazas. Boronico and Siegel [28] used M/M/1 queuing systems to compute the upper bounds of the mean queue lengths and the mean wait times at a toll plaza, and developed a capacity-planning model subject to reliability constraints; subsequently, they have presented a workforce policy through the utilization of a mathematical program. Levinson and Chang [29] tried to monetize the social benefit caused from the implementation of electronic toll collection (ETC) lanes to an existing toll station and found the best combination of ETC lanes and total discount for ETC users so that the monetized social welfare was maximum. Sadoun [30] adopted a micro-simulation model to evaluate the performance of toll plaza systems through delay, the number of toll stations, and service types, i.e., cash or electronic payment, and it was shown in the results that the performance of toll plazas improves as the number of tollbooths increased because less time was spent in queue. Kim [31] built a non-linear integer programming model to study the toll plaza optimization problem, in which the waiting time cost of the vehicles, as determined from the steady-state solution of the queueing model, was minimized. Gu *et al.* [32] presented a model for estimating the vehicle-processing capacities at checkpoints with tandem, staggered, and branch configurations, the models indicated that tandem designs tended to produce the highest capacities among the three alternatives. Kim *et al.* [33] presented an analytical method for dynamically adjusting toll plaza capacity to deal with a sudden shift in demand, which used a proxy measure developed from the discharge rate observed at toll plazas and segments of travel time measured by probe vehicles. Cao *et al.* [34] proposed a framework for finding the optimal profit of toll highways over 5 years of the operating period. Toll rates were adjusted using the updated safety conditions of highway bridges as constraints on the optimization task. Lu and Meng [35] developed a two-stage stochastic programming model to analyze the optimal build-operate-transfer highway capacity under traffic demand uncertainty. Jin *et al.* [36] established a prediction model based on historical multi-source traffic flow data. Based on the prediction results, they proposed an improved human resource planning strategy for toll plazas, so as to reasonably arrange the working times and improve the operational efficiency of the highway in peak periods. The comparison of our contributions to those in reference can be shown in Table 1.

TABLE 1. Comparison of our contributions to those in reference.

Key references	Bi-modal system	Congestion / Crowding effect	Capacity design	Pareto analysis
Tabuchi (1993); Danielis and Marcucci (2002)	Yes	Yes / No	No	No
Huang (2002); Huang et al. (2007); Wu & Huang (2014)	Yes	Yes / Yes	No	No
Wang and Ding (2018); Wang et al. (2019);	Yes	Yes / No	No	No
Zhang and Guan (2019)	Yes	Yes / Yes	No	No
Liu et al. (2020)	Yes	Yes / Yes	Yes	No
Siamak and Francisco (1991)	No	No / No	No	No
Boronico and Siegel (1998)	No	Yes / No	Yes	No
Zhang et al. (2010)	Yes	Yes / Yes	No	No
Sadoun (2005); Gu et al. (2012)	No	Yes / No	Yes	No
Kim (2009); Cao (2020)	No	Yes / No	Yes	No
Kim (2016)	No	Yes / No	Yes	No
Guo and Yang (2009); Tan et al. (2010); Lu and Meng (2018)	No	Yes / No	No	Yes
This paper	Yes	Yes / Yes	Yes	Yes

III. BASIC MODELS

Here, we consider a simplified corridor network that contains two modes to provide transportation services between a residential area and a place of work, as illustrated in Figure 1. Mode A represents a highway with a toll plaza which is a bottleneck located at the leaving point of the highway and has a maximum capacity of s commuters per unit of time. The toll plaza may not be able to operate at maximum capacity due to operating costs including labor or other costs. Therefore, the actual service capacity of the toll plaza is represented as θs. Every automobile commuter pays a highway toll u when they pass through the toll plaza. Mode R represents a mass transit system (for example, a railway) with an assumed infinite capacity. Every morning, The commuters either travel by car on the highway or by train on the railway from the residential area to the workplace, and $N = N_A + N_R$. The notations in this paper are listed in table 2.

In accordance with the empirical evidence [37], it was assumed that $\gamma > \alpha > \beta > 0$.

In this section, we investigate the costs of two travel modes and mode choice at an equilibrium state. To facilitate discussion later, further assumptions are listed below:

Assumption 1: All individuals traveling by car (one person per car) have exactly the same preferred arrival time as train commuters.

Assumption 2: The number of commuters N is large enough that both modes would be simultaneously used by commuters.

TABLE 2. Model parameters and decision variables.

θ	The proportion of the toll plaza capacity used, $\theta \in (0,1]$
N	The number of commuters
N_A	The numbers of automobile commuters
N_R	The numbers of train commuters
T_0	The moving/travel time in the absence of a queue
$T(t)$	The waiting time at the bottleneck
$r(t)$	The departure rate at time t
$D(t)$	The queue length with respect to departure time t
t^*	The desired arrival times
t_e	The times when the first commuters depart from home
t_l	The times when the last commuters depart from home
$\tilde{t}$	The departure time with respect to arriving at the destination at the desired time
α	The unit cost of travel time
β	The penalty costs of arriving early at the workplace with regard to the unit time
γ	The penalty costs of arriving late at the workplace with regard to the unit time
u	The toll of the highway
p	The train fare
ω	The unit cost of crowding or discomfort while using the transit system
$g(N_R)$	The discomfort degree generated by body congestion in carriages
F	The fixed cost per unit capacity
κ	The operating cost of unit capacity per unit time
Δt	The operating time
λ	The degree of inclination of the two optimal schemes, $\lambda \in [0,1]$

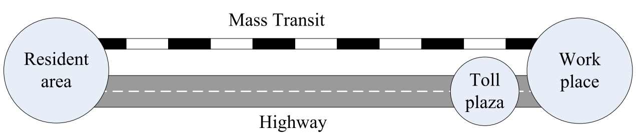

FIGURE 1. Bi-modal traffic system.

Assumption 3: The mass transit can make commuters arrive at work on time, and there is a linear relationship between the discomfort for a commuter caused by body crowding and the number of transit commuters in the carriage.

A. AUTO MODAL

During the morning peak period, N_A homogeneous travelers commute from the residential district to the workplace via the highway with identical desired arrival times t^* at the destination. When a commuter arrives at their workplace, their schedule delay is the difference between the actual and desired arrival times. When traffic flows exceed the actual service capacity of toll plaza θs, a queue forms, and consequently, commuters experience queuing delays. Therefore, the travel time on the highway is given as $T_0 + T(t)$. Without a loss of generality, T_0 is assumed to be zero, and thus $T(t)$ hereafter represents an individual's travel time via the highway with respect to the departure from their home at time t.

Notably, it was assumed that parking spaces were considered to be sufficient and parking fees were ignored. Additionally, it was assumed that there was a one-to-one correspondence between commuters and private vehicles, which means that car-sharing was not taken into consideration. Thus, the individual travel cost $C_A(t)$, which consists of the travel time costs, highway tolls, and schedule delay costs, can be formulated as

$$C_A(t) = \begin{cases} \alpha T(t) + \beta \left(t^* - t - T(t)\right) + u, t \in \left[t_e, \tilde{t}\right), \\ \alpha T(t) + \gamma \left(t + T(t) - t^*\right) + u, t \in \left(\tilde{t}, t_l\right]. \end{cases} \quad (1)$$

The waiting time at the bottleneck $T(t)$ is equal to $D(t)/\theta s$. Mathematically, $D(t)$ can be formulated as

$$D(t) = \max \left\{ \int_{t_e}^{t} r(t)dt - \theta s \left(t - t_e\right), 0 \right\}. \quad (2)$$

Each commuter independently chooses a departure time to minimize their individual travel cost. Through day-to-day travels and learning, all commuters gain a complete understanding of their journey, and the traffic system gradually converges to a stable state. In this state, all commuters will have an identical individual travel cost, and no commuter has an incentive to unilaterally alter their departure time choices, i.e., dynamic user equilibrium. Mathematically, the equilibrium condition can be expressed as

$$\frac{dC_A(t)}{dt} = 0 \quad \text{if } r(t) > 0. \quad (3)$$

According to Equation (3), as given by Arnott *et al.* [38], the equilibrium departure rate can be expressed as the following piecewise linear equation:

$$r(t) = \begin{cases} \dfrac{\alpha}{\alpha - \beta}\theta s, t \in \left[t_e, \tilde{t}\right], \\ \dfrac{\alpha}{\alpha + \gamma}\theta s, t \in \left(\tilde{t}, t_l\right]. \end{cases} \quad (4)$$

The three critical departure times, i.e., t_e, $\tilde{t}$ and t_l, can be expressed as

$$t_e = t^* - \frac{\delta N_A}{\beta \theta s}, \quad \tilde{t} = t^* - \frac{\delta N_A}{\alpha \theta s}, \quad t_l = t^* + \frac{\delta N_A}{\gamma \theta s} \quad (5)$$

Respectively, $\delta = \frac{\beta\gamma}{\beta+\gamma}$. Furthermore, the equilibrium of individual travel costs, denoted as C_A, is given by

$$C_A = \frac{\delta N_A}{\theta s} + u. \quad (6)$$

According to Equation (6), the total travel cost for all automobile commuters, denoted as TC_A, can be expressed by

$$TC_A = \frac{\delta (N_A)^2}{\theta s} + N_A u. \quad (7)$$

The total travel time cost for all commuters, denoted as TTC_A, is expressed by

$$TTC_A = \frac{\delta (N_A)^2}{2\theta s}. \quad (8)$$

The variables t_e, t_l, TC_A and TTC_A are independent of the unit cost of travel time α and are related to the proportion of the toll plaza capacity used, θ.

B. TRAIN MODE

Huang [13] considered body crowding in train carriages and proposed a cost of individual travel for train commuters. Below, we draw lessons from Huang's method to estimate travel costs for train commuters. Let N_R be the number of train commuters, and therefore the individual travel cost of train commuters can be defined as

$$C_R = p + \omega g(N_R), \tag{9}$$

Marginal cost pricing is widely used in the pricing of transportation products. The marginal cost of train travel mainly comprises the expenses on labor, fuel, electricity, and routine materials by the train operators. In reality, most of these expenses are independent of the number of passengers carried. It is assumed that the train fare p is equal to its marginal cost in this paper. The increase in the number of commuters in the carriage will cause commuter's discomfort, to simplify, the discomfort degree $g(N_R)$ is expressed as a linear function of the number of commuters in the carriage, i.e $g(N_R) = N_R$.

C. THE EQUILIBRIUM OF MODAL SPLIT

Next, we investigated the equilibrium state with two modes. We limited our attention to a situation in which both mode A and mode R were used, which means that the equilibrium occurred at an interior point of the highway/transit system. Therefore, the Wardrop equilibrium is defined as when the total cost per commuter on a highway is equal to that using a transit system, i.e., $C_A = C_R$. The equilibrium state of individual travel costs can be expressed by

$$p + \omega N_R = \delta \frac{N_A}{\theta s} + u. \tag{10}$$

According to $N = N_A + N_R$, we can obtain the number of automobile commuters

$$N_A = \frac{\theta s(\omega N + p - u)}{\omega \theta s + \delta}. \tag{11}$$

Therefore, the total travel cost of the highway/transit system from Equations (10) and (11) is

$$TCM = \left(\frac{\delta(\omega N + p - u)}{\omega \theta s + \delta} + u \right) N. \tag{12}$$

Taking the derivative of N_A and TCM with respect to θ for Equations (10) and (11), we have

$$\frac{dN_A}{d\theta} = \frac{\omega s \delta (p - u + \omega N)}{(\omega \theta s + \delta)^2}, \tag{13}$$

$$\frac{dTCM}{d\theta} = -\frac{\delta \omega N s(\omega N + p - u)}{(\omega \theta s + \delta)^2}. \tag{14}$$

Due to the assumption that both modes are used, the number of automobile commuters N_A is non-negative, thus $\omega N + p - u > 0$ from Equation (11). Therefore, from Equations (13) and (14), we can derive

$$\frac{dN_A}{d\theta} > 0 \quad \text{and} \quad \frac{dTCM}{d\theta} < 0. \tag{15}$$

In the above, the derivative of the number of automobile commuters N_A with respect to the parameter θ is greater than 0, and the derivative of the bi-modal system cost, TCM with respect to θ is less than 0, indicating that with the increase in θ, i.e., an increase in the capacity of the toll plaza, the number of automobile commuters increases and the bi-modal system cost decreases.

IV. CAPACITY DESIGN OF HIGHWAY TOLL PLAZA

A. THE OPERATING COST OF TOLL PLAZA

Operation costs of the toll plaza are the necessary expenses to maintain normal operation of the toll plaza, which can be divided into fixed costs and variable costs. The fixed cost consists of facility costs and fixed operating costs, mainly related to tollgate capacity. The variable costs include labor, power, and maintenance costs related to tollgate capacity and operating time. Let $OC = F + \kappa \Delta t$ be the toll plaza operating cost per unit capacity, and the operating time Δt is expressed as the time period between the first and last car to pass through the toll plaza during the peak hour. Thus, the total operating cost of capacity θs is

$$TOC = (F + \kappa \Delta t)\theta s. \tag{16}$$

The travel time is assumed to be 0, according to the earliest departure time and the latest departure time in Equation (5); therefore, the operating time is obtained as

$$\Delta t = t_l - t_e = \frac{N_A}{\theta s}. \tag{17}$$

Then, TOC can be rewritten as

$$TOC = F\theta s + \kappa N_A. \tag{18}$$

The total profit TP is

$$TP = (u - \kappa) N_A - F\theta s. \tag{19}$$

From Equation (18), we know that because F, s and κ are constant, the total operating cost of toll plaza is positively correlated with the parameter θ and number of automobile commuters N_A.

B. CAPACITY DESIGN SCHEMES UNDER DIFFERENT STRATEGIES

In reality, toll plazas can control and optimize the traffic flow distribution of highways and transit systems by changing the capacity. In this section, under the premise of a fixed transit fare p and highway toll u, three capacity design schemes are studied, which aim to make the toll plaza break-even, maximize the profits, or minimize the total social cost of the bi-modal system.

1) THE CAPACITY DESIGN SCHEME WHICH MAKES THE TOLL PLAZA BREAK EVEN

In states of equilibrium, N_A commuters pass through the toll plaza and the total amount paid is uN_A. The toll plaza breaks even when the total revenue uN_A is equal to the total operating cost TOC, which is

$$N_A u = \theta F s + \kappa N_A. \tag{20}$$

Combining this with Equation (10), the proportion of the toll plaza capacity used θ_1 under this situation can be obtained:

$$\theta_1 = \frac{(u-\kappa)(\omega N + p - u) - \delta F}{\omega F s}. \tag{21}$$

The number of automobile commuters is

$$N_A^1 = N - \frac{\delta F - (u-k)(p-u)}{\omega(u-\kappa)}. \tag{22}$$

2) THE CAPACITY DESIGN SCHEME WHICH MAXIMIZES THE PROFITS

Next, we considered the proportion of capacity needed to maximize the profit of the toll plaza, i.e., to maximize the difference between total tolls and total operating costs. The mathematical programming form of the problem is

$$\begin{aligned} &\text{Maximize } TP(\theta, N_A, N_R) = (u-\kappa)N_A - F\theta s \\ &\text{Subject to } \omega N_R - \frac{\delta}{\theta s} N_A = u - p, \\ &\qquad N_R + N_A = N. \end{aligned} \tag{23}$$

Solving the maximization problem (23) (See appendix A), we derive the proportion of the toll plaza capacity used θ_2, as

$$\theta_2 = \frac{\sqrt{\delta F (u-\kappa)(\omega N + p - u)} - \delta F}{\omega F s}. \tag{24}$$

In this situation, the result is

$$N_A^2 = \frac{1}{\omega}(\omega N + p - u)\left(1 - \sqrt{\frac{\delta F}{(u-\kappa)(\omega N + p - u)}}\right). \tag{25}$$

3) THE CAPACITY DESIGN SCHEME WHICH MINIMIZES THE TOTAL SOCIAL COST

In this section, we discuss a capacity design that minimizes the total social cost in a competing system consisting of highways and public transport. The total social cost includes three parts, namely, the total cost of automobile commuters, the total cost of transit commuters, and the total operating cost of toll plazas. Then, a minimization model can be formulated as follows

$$\begin{aligned} &\text{Minimize } TSC(\theta, N_A, N_R) = \frac{\delta}{\theta s}(N_A)^2 + \omega (N_R)^2 \\ &\qquad + \kappa N_A + p N_R + \theta F s \\ &\text{subject to } \omega N_R - \frac{\delta}{\theta s} N_A = u - p, \\ &\qquad N_R + N_A = N. \end{aligned} \tag{26}$$

Using the Lagrange multiplier method, the proportion of the toll plaza capacity used θ_3 and the number of automobile commuters can be obtained as (See appendix B)

$$\theta_3 = \frac{\sqrt{\delta F (\omega N + u - \kappa)(\omega N + p - u)} - \delta F}{\omega F s}, \tag{27}$$

and

$$N_A^3 = \frac{1}{\omega}(\omega N + p - u)\left(1 - \sqrt{\frac{\delta F}{(\omega N + u - \kappa)(\omega N + p - u)}}\right). \tag{28}$$

Due to $TP(\theta_2) \geq TP(\theta_1) = 0$, we can derive that $\theta_1 \geq \theta_2$. In addition, the difference between θ_2 and θ_3 is

$$\theta_2 - \theta_3 = -\frac{N\sqrt{\delta F(\omega N + p - u)}}{Fs\left(\sqrt{u-\kappa} + \sqrt{(\omega N + u - \kappa)}\right)} < 0. \tag{29}$$

According to Equations (25) and (28), this can be rewritten as

$$N_A^2 - N_A^3 = \frac{\sqrt{\delta F(\omega N + p - u)}}{\omega} \times \left(\frac{1}{\sqrt{\omega N + u - \kappa}} - \frac{1}{\sqrt{u-\kappa}}\right) < 0. \tag{30}$$

From the above two expressions, we know that the capacity of a toll plaza with a minimum social cost is greater than the one with maximum profit. In other words, the profit of a toll plaza is not necessarily the largest when a bi-modal transportation system reaches social optimal. The reason is that adding operating costs of toll plazas to increase traffic capacity can reduce the individual travel cost of automobile commuters, thus reducing the total social cost.

C. CAPACITY DESIGN OF A HIGHWAY TOLL PLAZA WITH BI-OBJECTIVE OPTIMIZATION

As mentioned before, both the total profits of the toll plaza, TP, and the total social cost of traffic systems, TSC, are meaningful measures of system performance from different perspectives. Here, we consider the maximization of TP and the minimization of TSC simultaneously. In order to coordinate the different expectations of toll plaza operators and the traffic management department, the maximization problem (23) is transformed into the minimization problem and concurrently associated with the minimization problem (26). Thus, we have generated the following bi-objective minimization problem,

$$\text{Minimize} \begin{pmatrix} -TP(\theta, N_A, N_R) \\ TSC(\theta, N_A, N_R) \end{pmatrix} = \begin{pmatrix} F\theta s - (u-\kappa)N_A \\ \frac{\delta}{\theta s}(N_A)^2 + \omega(N_R)^2 + \kappa N_A + p N_R + \theta F s \end{pmatrix}, \tag{31}$$

the Pareto optimal solution of the bi-objective problem (31) can be seen as a negotiation outcome between the traffic management department and toll plaza operators. Additionally, it confirms there is no other feasible solution of the unknown design factors in Equation (31) that could improve one of its two objectives without having a detrimental effect on the other. A similar notion of Pareto optimization has been used by Guo and Yang [39], Tan *et al.* [40], and Lu and Meng [41] for different bi-criteria problems. To solve Equation (31), a commonly used method would be to transfer it into a single objective via the weighting method (similarly to Tan *et al.* [40]; Lu and Meng [41]). Theoretically, any Pareto optimal schemes must uniquely solve the following scalar programming model:

$$\begin{aligned} &\text{Minimize } BI(\theta, N_A, N_R) = -\lambda TP(\theta, N_A, N_R) \\ &\qquad + (1-\lambda) TSC(\theta, N_A, N_R) \\ &\text{subject to } \omega N_R - \frac{\delta}{\theta s} N_A = u - p, \\ &\qquad N_R + N_A = N. \end{aligned} \tag{32}$$

In this situation, the polar point $\lambda = 1$ means that the toll plaza operators take full control of the traffic system, whereas $\lambda = 0$ indicates that the traffic management department takes full control of the traffic system. When $0 < \lambda < 1$, neither the traffic management department nor toll plaza operators will be better off without reducing the other's benefit. Notably, the Pareto efficiency is not necessarily the economic efficiency; it is a tradeoff here between the traffic management department and toll plaza operators.

Solving minimization problem (32) (See appendix C), the proportion of the toll plaza capacity used with bi-objective optimization, θ_λ, can be obtained:

$$\theta_\lambda = \frac{\sqrt{\delta F\,[(1-\lambda)\,\omega N + u - \kappa]\,(\omega N + p - u)} - \delta F}{\omega F s}. \tag{33}$$

As discussed above, when $\lambda = 1$, the proportion of the toll plaza capacity used $\theta_\lambda = \theta_2$; when $\lambda = 0$, the proportion $\theta_\lambda = \theta_3$.

The toll plaza must at least make ends meet, i.e., $0 < TP(\theta_\lambda)$, and according to Equations (11), (19), and (33), we can derive

$$\lambda > 1 - \frac{u-\kappa}{\omega F \delta N}\left[(u-\kappa)(\omega N + p - u) - F\delta\right]. \tag{34}$$

Based on the given value range of the parameter λ and the inequality in Equation (34), the value range of the parameter λ, satisfying both the revenue of toll plaza greater than 0 and the bi-objective minimization problem (Equation (31)), can be obtained:

$$max\left\{1 - \frac{u-\kappa}{\omega F \delta N}\left[(u-\kappa)(\omega N + p - u) - F\delta\right], 0\right\} < \lambda < 1. \tag{35}$$

We explored the extent to which the system performance varies when an alternative Pareto optimum scheme is considered. To measure the deviation, we defined the following two ratios, called system performance deviation factors:

$$\rho_\lambda^{TP} = \frac{TP_{max} - TP_\lambda}{TP_{max}} \quad \text{and } \rho_\lambda^{TSC} = \frac{TSC_\lambda - TSC_{min}}{TSC_{min}}. \tag{36}$$

Clearly, $\rho_\lambda^{TP} \le 1$ and $\rho_\lambda^{TSC} \ge 0$ for any feasible θ_λ. Specifically, TP_λ and TSC_λ are the total profits of the toll plaza and the total social cost of Pareto optimization under parameter λ, respectively. Furthermore, let TP_{max} and TSC_{min} denote the maximum total profits of the toll plaza and the minimum total social costs which are realized, respectively. For a given Pareto optimal capacity ratio θ_λ, we are interested in knowing how far the system disabilities TP_λ and TSC_λ could deviate from their optimal values, TP_{max} and TSC_{min}, respectively.

Let ρ_1^{TP} denote the values of the two factors when $\theta = \theta_3$, and ρ_0^{TSC} denote the values of the two factors when $\theta = \theta_2$. Thus,

$$\rho_0^{TP} = 1 - \frac{TP_0}{TP_{\max}} \quad \text{and } \rho_1^{TSC} = \frac{TSC_1}{TSC_{\min}} - 1. \tag{37}$$

ρ_0^{TP} measures the deviation of the total profits from the toll plaza when the total social cost is minimized and the maximum total profits of the toll plaza are achieved; analogously, ρ_1^{TSC} measures the deviation of the total social cost when the total profit of the toll plaza is maximized and the total social cost when it is minimized. Therefore, for any parameter λ, we have $\rho_\lambda^{TP} \le \rho_0^{TP}$ and $\rho_\lambda^{TSC} \le \rho_1^{TSC}$.

Figure 2 plots the Pareto optimal frontier in (TP, TSC) space to illustrate the two factors, ρ_λ^{TP} and ρ_λ^{TSC}. The two points $(\rho_0^{TP}, 0)$ and $(0, \rho_1^{TSC})$ represent the optimal total benefits and total social costs, respectively.

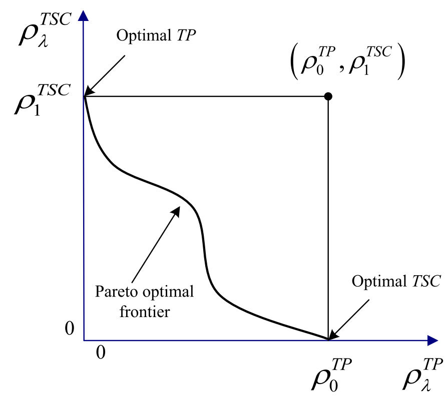

FIGURE 2. Pareto optimal frontier in (TP, TSC) space.

V. NUMERICAL ANALYSIS

In this section, we outline the numerical analysis verifying the analytical analysis of the capacity design schemes of toll plazas with bi-modal transportation systems. The number of commuters, transit fares, highway tolls, and other parameters of the example is shown in Table 3.

Figure 3 shows the modal split of the competitive system against the parameter θ. With the increase in parameter θ, the number of automobile commuters increases, and the number of train commuters decrease. This is because the increased capacity of the toll plaza reduces the individual travel cost for automobile commuters, causing train commuters to transfer to the highway.

TABLE 3. The parameter values of numerical example.

parameter	value	parameter	value
α	1.2 USD/h	s	200 veh/h
β	0.6 USD/h	F	4
γ	3.0 USD/h	N	500 veh
ω	0.01	u	10 USD
p	6 USD	κ	6 USD/h

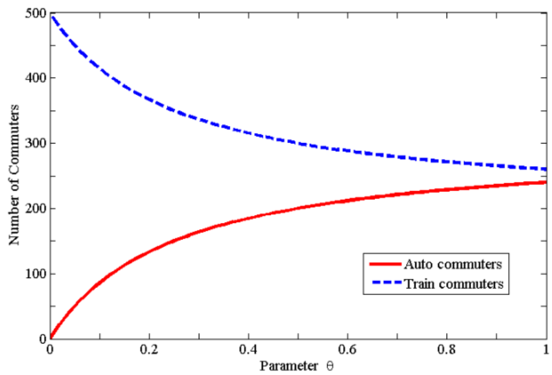

FIGURE 3. Numbers of automobile commuters and train commuters plotted against the parameter θ.

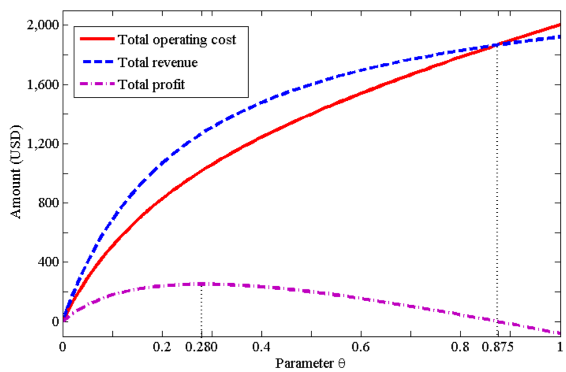

FIGURE 4. The total operating cost, total revenue, and total profit plotted against the parameter θ.

Figure 4 shows the variation trend of the total operating costs, total toll revenues, and total profits of the toll plaza against the parameter θ. The operating cost and the total toll revenue increase with θ, and the total profit increases first and then decreases as θ increases. When $\theta = 0.280$, the total profit of the toll plaza is the greatest. When θ is greater than 0.875, the total profit is less than 0, i.e., the toll plaza is running at a loss. Figure 5 shows the change in the total social cost with respect to θ. It can be seen in the figure that the total social cost function graph is convex, and the minimum value, USD 4285.6, is obtained at $\theta = 0.618$.

The variations of parameter θ with respect to the unit operating cost κ for the three capacity design schemes are shown in Figure 6. On the assumption that $\kappa \in [0, 5]$, from the figure, we can see that as κ increases, θ decreases. Due to the increase in unit operating cost, the individual travel cost to automobile commuters increases accordingly; then, some automobile commuters abandon their cars and choose

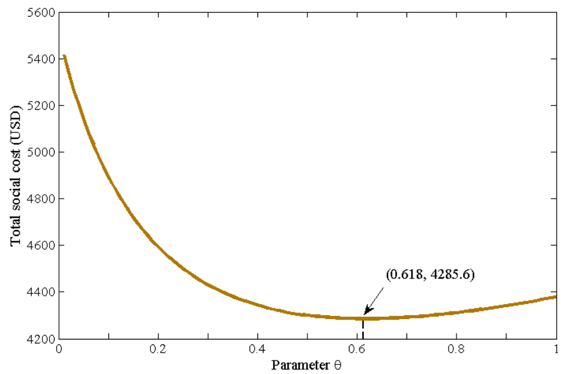

FIGURE 5. Total social cost plotted against the parameter θ.

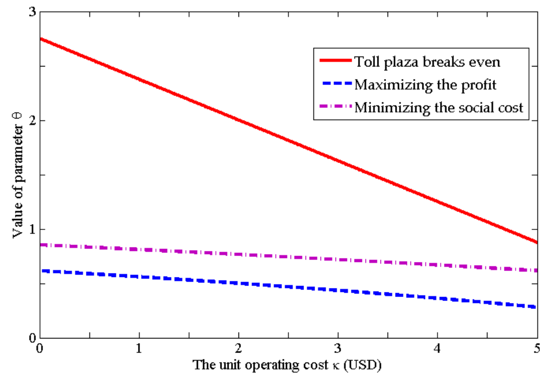

FIGURE 6. The value of parameter θ against the unit operating cost κ.

to travel on the mass transit system. Therefore, toll plaza operators respond by reducing capacity to reduce the total operating costs.

Figure 7 shows the changes in the value of θ under the three strategies when the highway toll value ranges from 0 to 12 and other parameters are fixed. In other words, under different highway toll standards, the toll plaza capacity design can make three different strategic goals achieved. It can be observed in the figure that with the increase in highway toll u, all three curves first increase and then decrease.

The derivatives with respect to u of θ_1, θ_2 and θ_3 of Equations (21), (24), and (27) are

$$\frac{d\theta_1}{du} = \frac{-2u + \kappa + \omega N + p}{\omega F s}, \tag{38}$$

$$\frac{d\theta_2}{du} = \frac{\sqrt{\delta F}\,(-2u + \kappa + \omega N + p)}{\omega F s \sqrt{(u-\kappa)(\omega N + p - u)}}, \tag{39}$$

$$\frac{d\theta_3}{du} = \frac{\sqrt{\delta F}(-2u + \kappa + p)}{\omega F s \sqrt{(\omega N + u - \kappa)(\omega N + p - u)}}. \tag{40}$$

Through simple calculations, when $u < \frac{\kappa + \omega N + p}{2}$, we have $\frac{d\theta_1}{du} < 0$ and $\frac{d\theta_2}{du} < 0$. therefore, θ_1 and θ_2 increase monotonically with respect to u. However, when $u > \frac{\kappa+\omega N+p}{2}$, θ_1 and θ_2 are monotonically decreasing with respect to u. In addition, the monotonicity of θ_3 is different on both sides at $u = \frac{\kappa+p}{2}$. These conclusions are verified in Figure 6.

The change in the value of θ with the three strategies of capacity design with respect to the total number of

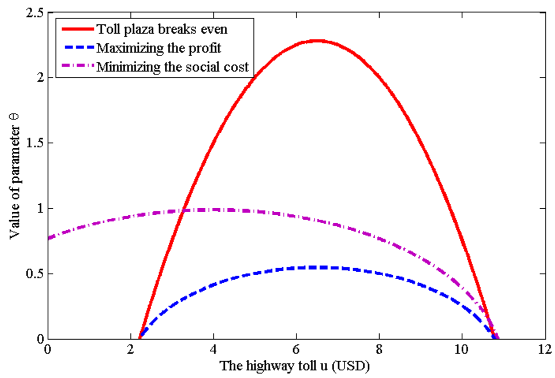

FIGURE 7. The value of parameter θ against the highway toll u.

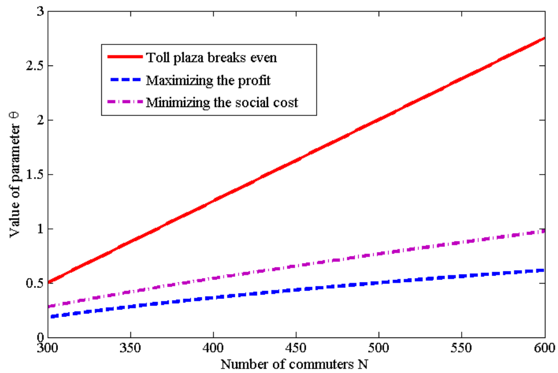

FIGURE 8. The value of θ against N.

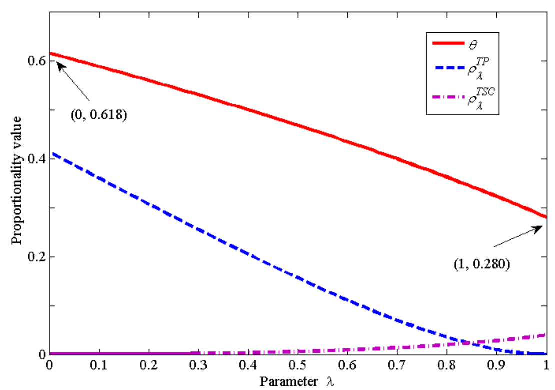

FIGURE 9. The value of the proportion θ, ρ_λ^{TP}, and ρ_λ^{TSC} against λ.

commuters, N, is shown in Figure 8. When $N \in [300, 600]$, the increase in total traffic demand will increase the proportion θ of capacity under the three strategies. Moreover, when the total traffic demand exceeds a certain amount, it may be necessary to expand the toll plaza to increase the maximum capacity, to ensure the balance of revenue and expenditure of the toll plaza, although its upper bound may be determined by geographical restrictions.

Figure 9 shows that the proportion of the toll plaza capacity used, θ, and system performance deviation factors, ρ_λ^{TP} and ρ_λ^{TSC}, change with parameter λ under the bi-objective optimization model. The proportions θ and ρ_λ^{TP} monotonically decrease with respect to λ, and ρ_λ^{TSC} monotonically increases with respect to λ. Specifically, the degree of inclination for maximizing the total profits of the toll plaza will increase

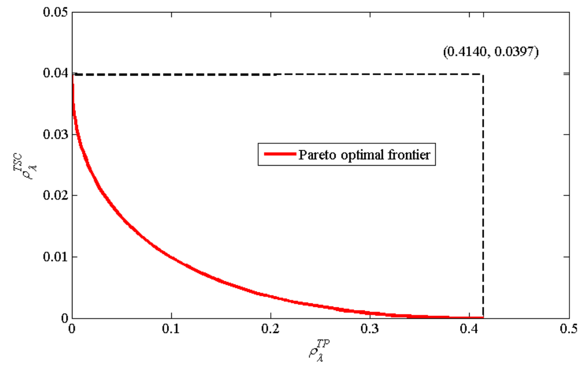

FIGURE 10. Pareto optimal frontier of the example.

if there is any inclination in the bi-objective optimization model. In contrast, the total social cost decreases. In addition, points $(1, 0.280)$ and $(0, 0.618)$ in Figure 8 correspond to the values of $\theta = 0.280$ and $\theta = 0.618$ in Figure 4 and 5, respectively.

The Pareto optimal frontier is shown in Figure 10. When $\lambda \in [0, 1]$, the system performance deviation factors ρ_λ^{TP} and ρ_λ^{TSC} are valued in the range $[0, 0.4140]$ and $[0, 0.0397]$, respectively, and ρ_λ^{TSC} is decreasing with respect to ρ_λ^{TP}.

Overall, it can be observed that the numerical results confirm our analytical results.

VI. CONCLUSION

Effective operation of toll plazas is a significant guarantee for highways to provide sustainable transportation services. The revenue of toll plazas is a major source of funding for the repayment of highway construction loans and routine maintenance, renovation, and expansion. In a bi-modal transportation system, commuters usually choose the mode which is most beneficial to them. In this paper, a traffic system in which highways and public transport facilities coexist and compete was taken as the research object, and a bi-modal travel equilibrium model was adopted to study and determine the capacity of highway toll plazas. Considering the operating cost of the toll plaza, we deduced the capacity proportions of the toll plaza required in three situations, i.e., breaking even, maximizing the profit of the toll plaza, and minimizing the total social cost. The numerical analysis verified the analytical analysis. This research has important managerial implications; for example, a highway management department could control traffic flow by adjusting the capacity of the toll plaza so that the traffic system can achieve different aims. From our results, there are several aspects worth pointing out. Firstly, the service hours of toll plazas can be determined by the earliest departure time and the latest departure time of highway commuters. Secondly, under certain parameters, when traffic demands exceed a specified value, toll plazas can be upgraded or expanded to increase maximum capacity, to ensure the balance between income and expenditure. Thirdly, highway tolls priced too high or too low will reduce the capacity of the toll plaza. Finally, the single-objective optimization problem has difficultly meeting the requirements of multiple departments; however, the bi-objective optimization model can be used to obtain a Pareto capacity design scheme, which can be adopted by toll plaza operators and traffic management departments.

The following aspects are limitations of our research and suggested issues for further study. (1) In this paper, to simplify the analysis and focus on the investigation, we made some assumptions that caused the model to differ from actual traffic. Relaxing some assumptions, such as incorporating commuters' heterogeneity, elastic demand, and operational characteristics of highway toll plazas during off-peak hours will be one direction for our future research. (2) Our investigation was qualitative and has not been empirically verified by actual traffic data; therefore, it will be necessary to calibrate our survey results based on actual traffic data in the future. (3) In the field of public transportation, dynamic departure and multiple pricing strategies are not usually considered during peak periods.

APPENDIX

A. DERIVATION OF THE CAPACITY DESIGN SCHEME WHICH MAXIMIZES THE PROFITS

$$\begin{aligned}&\text{Maximize } TP(\theta, N_A, N_R) = (u-\kappa)N_A - F\theta s\\ &\text{Subject to } \omega N_R - \frac{\delta}{\theta s}N_A = u - p,\\ &\qquad N_R + N_A = N.\end{aligned} \tag{A.1}$$

Substituting the constraints into the target function, we obtain

$$TP(\theta) = \frac{\theta s(u-\kappa)(\omega N - u + p)}{\omega\theta s + \delta} - \theta F s. \tag{A.2}$$

The derivative of the function $TP(\theta)$ with respect to θ is

$$\frac{dTP(\theta)}{d\theta} = \frac{\delta s(u-\kappa)(\omega N - u + p)}{(\omega\theta s + \delta)^2} - Fs. \tag{A.3}$$

Letting $\frac{dTP(\theta)}{d\theta} = 0$, we derive

$$\theta = \frac{\sqrt{\delta F(u-\kappa)(p+\omega N)} - \delta F}{\omega F s}. \tag{A.4}$$

B. DERIVATION OF THE CAPACITY DESIGN SCHEME WHICH MINIMIZES THE TOTAL SOCIAL COST

$$\begin{aligned}&\text{Minimize } TSC(\theta, N_A, N_R) = \frac{\delta}{\theta s}(N_A)^2 + \omega(N_R)^2\\ &\qquad + \kappa N_A + pN_R + \theta F s\\ &\text{Subject to } \omega N_R - \frac{\delta}{\theta s}N_A = u - p,\\ &\qquad N_R + N_A = N.\end{aligned} \tag{B.1}$$

Substitute N_R into the target function $TSC(\theta, N_A, N_R)$, and then write the Lagrange function of Equation (B1) as

$$\begin{aligned}L(N_A, \theta, \lambda) = &\frac{\delta}{\theta s}(N_A)^2 + \omega(N_R)^2 + \kappa N_A + pN_R\\ &+ \theta F s + \lambda\left[\omega(N - N_A) - \frac{\delta}{\theta s}N_A - u - p\right].\end{aligned} \tag{B.2}$$

The partial derivative of the function $L(N_A, \theta, \lambda)$ with respect to N_A, θ and λ, results in

$$\begin{cases}\dfrac{\partial L}{\partial N_A} = -2\omega N + k - p + 2(\omega + \dfrac{\delta}{\theta s})N_A - \lambda(\omega + \dfrac{\delta}{\theta s}),\\ \dfrac{\partial L}{\partial \theta} = Fs - \dfrac{\delta(N_A)^2}{\theta^2 s} + \dfrac{\delta N_A}{\theta^2 s}\lambda,\\ \dfrac{\partial L}{\partial \lambda} = \omega N - u + p - (\omega + \dfrac{\delta}{\theta s})N_A.\end{cases} \tag{B.3}$$

Setting the partial derivatives equal to 0, and solving Equation (B3), the parameter θ and the number of automobile commuters to minimize the total social cost is

$$\theta = \frac{\sqrt{\delta F(\omega N + u - \kappa)(\omega N - u + p)} - \delta F}{\omega F s}, \tag{B.4}$$

$$N_A = \frac{1}{\omega}(\omega N - u + p) \times \left(1 - \sqrt{\frac{\delta F}{(\omega N + u - \kappa)(\omega N - u + p)}}\right). \tag{B.5}$$

C. DERIVATION OF THE CAPACITY DESIGN SCHEME WHICH BI-OBJECTIVE OPTIMIZATION

$$\begin{aligned}&\text{Minimize } BI(\theta, N_A, N_R) = -\lambda TP(\theta, N_A, N_R)\\ &\qquad + (1-\lambda)\, TSC(\theta, N_A, N_R)\\ &\text{subject to } \omega N_R - \frac{\delta}{\theta s}N_A = u - p.\\ &\qquad N_R + N_A = N.\end{aligned} \tag{C.1}$$

From the constraints, we can solve for N_R and N_A with respect to θ, and substitute it into the objective function $BI(\theta, N_A, N_R)$, thus

$$\begin{aligned}BI(\theta) = &(1-\lambda)\left[\omega N\frac{(u-p)\,\theta s + \delta N}{\omega\theta s + \delta} + pN\right]\\ &+ (\kappa - u)\left[N - \frac{(u-p)\,\theta s + \delta N}{\omega\theta s + \delta}\right] + \theta F s.\end{aligned} \tag{C.2}$$

Letting $\frac{d\mathrm{BI}(\theta)}{d\theta} = 0$, we derive

$$\theta = \frac{\sqrt{[(1-\lambda)\,\omega N + u - \kappa]\,(\omega N + p - u)\,\delta F} - \delta F}{F\omega s}. \tag{C.3}$$

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