Hybrid Control Synthesis for Turing Instability and Hopf Bifurcation of Marine Planktonic Ecosystems With Diffusion

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ABSTRACT Great progress has been made in bifurcation control of systems described by ordinary differential equations. However, the control of Hopf bifurcation and Turing patterns is seldom reported in reaction-diffusion systems, which is formed by partial differential equations. In this paper, a hybrid control synthesis combining state feedback is firstly devised in the reaction-diffusion marine planktonic ecosystem. The Turing instability condition and Hopf bifurcation criterion are derived through carrying out the eigenvalue analysis of the controlled system. The numerical simulations show that the hybrid control strategy can not only suppress the formation of Turing patterns, but also delay or advance the Hopf bifurcation point. Therefore, the desired spatial dynamics behaviors can be generated by manipulate the control gain parameters, so as to achieve the purpose of maintaining the marine ecological balance.

INDEX TERMS Diffusive plankton systems, Hopf bifurcation, turing instability, turing patterns, hybrid control.

I. INTRODUCTION

In recent years, some ecosystem models have been developed as important analytical methods in order to better understand marine ecological energy cycles [1], [2]. Plankton plays a significant role in both marine and lake ecosystems. Plankton which has become the basis for the reproduction of other organisms in the water body and a major source of food for the world in the future is widely distributed and highly capable of reproduction. As the bottom of the food chain, the size of the catch is basically determined by how much plankton is produced [3]. At the same time, if the cyanobacteria, dinoflagellates and other plankton content in the water is too high, they may cause blooms or red tides, resulting in the death of economic aquatic animals which will be harmful to human survival [4]. For decades, many experts and scholars have done a great deal of work on ecological competition networks, taking into account factors such as natural enemies,

age structure, delay in gestation, shelter, infectious diseases, and so on [5]–[8].

The species and quantities of plankton vary with time (mainly seasonal distribution) and space (horizontal distribution and vertical distribution). Based on the reaction-diffusion equation, the interaction between prey and predator can be described more accurately [9]–[11]. Nevertheless, there have been few literatures that introduce species migration called diffusion into dynamic modeling of marine planktonic ecosystems currently. In the reaction-diffusion system proposed by Turing [12], [13], the spatial heterogeneity caused by the internal reaction-diffusion characteristics of the system resulted in the loss of system symmetry and made the system self-organize to produce some spatial patterns. The process of patterns formation is called Turing instability (Turing bifurcation). The symmetry of the system was broken, leading to the formation of Turing patterns. Therefore, we call this phenomenon Turing instability caused by diffusive reaction [14]. In reality, the limit cycles caused by Hopf bifurcation and the patterns caused by Turing instability may exist simultaneously in systems under the

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influence of reaction-diffusion terms [2], [15]–[17]. In the past few years, the focuses of research on some Turing-Hopf bifurcations have been shifted from purely spatial dimensions or time scales to spatio-temporal dynamics. Wang *et al.* [15] investigated the spatiotemporal dynamics of a class of marine planktonic ecosystems containing toxins. Through linearization analysis, the existence conditions of Turing-Hopf bifurcation were determined, and the formation criteria of some specific patterns were derived by the coupling amplitudes of the system. Shi *et al.* [16] considered a predator-prey model with proportional dependence. The two-parameter selection method was utilized to analyze the Turing-Hopf bifurcation issue of the reaction-diffusion equation under Neumann boundary conditions. The stable region, unstable region and Turing-Hopf bifurcation point on the two-parameter plane were acquired, severally. As important dynamic behaviors, Turing instability and Hopf bifurcation frequently appear in actual marine planktonic ecosystems. Reasonable control algorithms adopted for different marine planktonic ecosystems to suppress blooms or red tides and protect humans survival have great and far-reaching significance to our homeland.

Control synthesis is a common method to improve dynamic behaviors of systems [18]–[22]. Based on the previous results [23]–[25], the research on controllers of complex systems has received extensive attention from scholars. At present, various bifurcation control methods such as state feedback method, parameter adjustment method, time delay feedback method and PID control method have been proposed [26]–[29] based on traditional ordinary differential equation (ODE) theorems, but there are few studies on the control of reaction-diffusion systems described by partial differential equations (PDE). Beck *et al.* [30] studied the diffusion equation on the real line of the control parameter. It was shown that travel front solutions for which the rest state behind the front undergo a supercritical Turing or Hopf bifurcation as the increase of control parameter. Kumar and Gangopadhyay [31] studied the dynamic behavior of the equilibrium point of an open nonlinear system other than the traditional Turing pattern with cross-diffusion, and found that the Turing instability condition can be changed by a critical control parameter including self-diffusion. The above-mentioned documents have provided the possibility for scholars to control the dynamic evolution of diffusive reaction systems. However, most of the bifurcation control strategies for diffusion systems are limited to the time-delay feedback control [32], [33]. In addition, a class of time-delay feedback controllers have disadvantages such as difficulties in design and single control mode.

Based on the characteristics of microorganisms that are liable to multiply and combinations with traditional biological methods to control red tides, we utilize artificial introduction of microorganisms to control plankton for the first time. With reference to the design methods of state feedback control and parameter adjustment, we propose a new hybrid bifurcation control strategy that simulates human

intervention to eliminate or delay the adverse effects caused by the occurrence or formation of water blooms and red tides in marine ecological systems.

The main innovations of this paper can be listed as follows:

(1) Although there have been some results on the dynamical behaviors of stability for equilibrium and bifurcation for delayed marine planktonic ecosystems, they did not consider the inevitable diffusion of plankton in the spatiotemporal domain. In order to better reflect real marine phytoplankton ecosystems, this paper introduces the reaction diffusion terms into both phytoplankton and zooplankton.

(2) The necessary and sufficient conditions for Turing instability and Hopf bifurcation dynamics have been determined in the meaning time, by analyzing the characteristic equation of the corresponding system. Observe the formation of Turing patterns and the appearance of Hopf bifurcation periodic solution in the presence of reaction-diffusion phenomenon.

(3) Combined with the design methods of parameter adjustment and state feedback, the hybrid bifurcation control strategy is applied to the marine planktonic ecosystem with diffusion for the first time. The results demonstrate that the hybrid controller can suppress the occurrence of Turing pattern, change the position of Hopf bifurcation critical value, and enhance the stability and controllability of the system, so as to obtain the expected dynamic behaviors.

The paper is arranged as follows: In Section II, A diffusive marine planktonic ecosystem with a hybrid bifurcation controller is put forward. In Section III, the conditions for local stability of equilibrium point, Turing instability of equilibrium without time delay, and Hopf bifurcation derived by the sum of time delays under the influence of hybrid controller synthesis are given, separately. The theoretical results are verified by two calculation examples and numerical simulations are illustrated in Section IV. Finally, Section V summarizes the paper and indicates the future research.

II. MODEL DESCRIPTION

Motivated by previous work by Jiang *et al.* [34], we put forward a novel marine planktonic ecosystem with diffusion as follows:

$$
\begin{cases}
\dfrac{\partial P(t,x)}{\partial t} = d_1 \Delta P(t,x) + r_1 P(t,x)(1 - \dfrac{P(t,x)}{\delta}) \\
\qquad\qquad - mf(P(t,x))Z(t-\tau_1,x), \\
\dfrac{\partial Z(t,x)}{\partial t} = d_2 \Delta Z(t,x) + r_2 Z(t,x)(1 - \dfrac{Z(t,x)}{\gamma P(t-\tau_2,x)}), \\
t > 0, x \in \Omega,
\end{cases}
\tag{1}
$$

under Neumann boundary conditions

$$
\frac{\partial P(t,x)}{\partial \upsilon} = \frac{\partial Z(t,x)}{\partial \upsilon} = 0, \quad t > 0,\ x \in \partial\Omega,
$$

TABLE 1. The definitions of parameters in system (1).

Symbol	Parameter definition
r_1	Phytoplankton intrinsic growth rate
r_2	Zooplankton intrinsic growth rate
δ	Enviroment carrying capacity for phytoplankton
γ	Quality of phytoplankton as food for zooplankton
γP	Phytoplankton carrying capacity for zooplankton
$mf(P)$	Functional response function
τ_1	Maturation delay of zooplankton
τ_2	Time required to digest phytoplankton

with initial conditions

$$\begin{cases} P(t, x) = \phi_1(t, x) \geq 0, \\ Z(t, x) = \phi_2(t, x) \geq 0, \\ (t, x) \in [-\tau, 0] \times \bar{\Omega}, \end{cases}$$

where $(\phi_1, \phi_2)^T \in C \triangleq C([-\tau, 0], X)$, and X is determined by

$$X = \{(P, Z) : P, Z \in W^{2,2}(\Omega), \frac{\partial P(t, x)}{\partial \upsilon} = \frac{\partial Z(t, x)}{\partial \upsilon} = 0, \\ x \in \partial\Omega\}$$

with the inner product $\langle \cdot, \cdot \rangle$.

$P(t, x)$ and $Z(t, x)$ stand for the population density of phytoplankton and zooplankton at time t and location x, severally. Δ denotes the Laplacian operator in R^n. d_1 and d_2 represent the diffusion coefficients of phytoplankton and zooplankton, receptively. Assume $\Omega = (0, l\pi) (l > 0)$ is a bounded domain with smooth boundary $\partial\Omega$. υ is the outward unit normal vector on $\partial\Omega$. The homogeneous Neumann boundary conditions mean that system (1) is self-contained with zero population flux across the boundary. The remaining parameters of system (1) are shown in Table 1.

Remark 1: If there is no diffusions ($d_1 = d_2 = 0$) in the marine ecological system, system (1) will degenerate into the form of ordinary differential equation cited from [34].

To consider the positive effects of control on the dynamics of plankton, an interaction system between phytoplankton and zooplankton has been investigated in the presence of a hybrid controller:

$$\begin{cases} \frac{\partial P(t, x)}{\partial t} = d_1 \Delta P(t, x) + k_1[r_1 P(t, x)(1 - \frac{P(t,x)}{\delta}) \\ \quad - mf(P(t, x))Z(t - \tau_1, x)] \\ \quad + k_2(P(t, x) - p^*), \\ \frac{\partial Z(t, x)}{\partial t} = d_2 \Delta Z(t, x) + r_2 Z(t, x)(1 - \frac{Z(t,x)}{\gamma P(t-\tau_2,x)}), \\ t > 0, x \in \Omega, \end{cases} \quad (2)$$

where the parameter k_1 and k_2 treated as human-control parameters stand for regulation parameter and state feedback parameter, respectively. p^* is on behalf of the steady-state value of phytoplankton population density.

Remark 2: Here k_2 is called the feedback gain, $k_2 > 0$ is considered as breeding rate which means that the number of phytoplankton will be artificially increased. On the contrary, $k_2 < 0$ is regarded as the capturing rate resulting from

zooplankton which means that the number of phytoplankton will be artificially reduced. k_1 corresponding to k_2 controls the system by changing parameter. Furthermore, $k_1 = 1$ and $k_2 = 0$ mean that there is no artificial control.

Remark 3: The significance of the control is that by artificially increasing or reducing the account of phytoplankton to maintain the marine ecological balance in a local area. Improved by the thought in [35]–[37]. A hybrid control strategy combined with parameter adjustment method and state feedback transmission is designed to control the Turing patterns and Hopf bifurcation generation of system (2) in order to obtain the desired dynamical behaviors.

Remark 4: The single-parameter hybrid control method has a small adjustable parameter area, and it is difficult to achieve control tasks with a high precision range or anti-interference ability. The dual-parameter hybrid control method has two adjustable control gain parameters, which not only enlarges the parameter adjustable area, but also further considers the control effect of parameter disturbance. Compared with the traditional single-parameter hybrid control method, it has better control effects [35], [38].

Let $p(t, x) = P(t - \tau_2, x), z(t, x) = Z(t, x)$ and $\tau = \tau_1 + \tau_2$, then system (2) becomes:

$$\begin{cases} \frac{\partial p(t, x)}{\partial t} = d_1 \Delta p(t, x) + k_1[r_1 p(t, x)(1 - \frac{p(t,x)}{\delta}) \\ \quad - mf(p(t, x))z(t - \tau, x)] \\ \quad + k_2(p(t, x) - p^*), \\ \frac{\partial z(t, x)}{\partial t} = d_2 \Delta z(t, x) + r_2 z(t, x)(1 - \frac{z(t,x)}{\gamma p(t,x)}), \\ t > 0, x \in \Omega. \end{cases} \quad (3)$$

We make the following assumption:

$(H1) f(0) = 0, f'(x) > 0 \text{ for } x \geq 0 \text{ and } \lim_{x \to \infty} f(x) = 1.$

Remark 5: In most of cases, $f(x)$ is represented by Holling I-IV types and Ivlev type [34]. In this paper, we suppose that $f(x)$ as an abstract function which covers most case of the above.

Theorem 1: If $(H1)$ holds, system (3) has only one positive equilibrium $E^* (p^*, z^*)$.

Proof: The positive equilibrium $E^* (p^*, z^*)$ of system (3) satisfies:

$$z^* = \gamma p^*, r_1(\delta - p^*) = \delta m \gamma f(p^*).$$

Let $G(p) = \delta m \gamma f(p) - r_1(\delta - p)$. It is clear that $G(0) = -r_1\delta < 0$ and $G'(p) = \delta m \gamma f'(p) + r_1 > 0$ when $(H1)$ is satisfied. By the zero point theorem, $G(p)$ has only one intersection with the positive half horizontal axis described in Cartesian coordinates, which means that system (3) always exists a unique positive equilibrium $E^* (p^*, z^*)$. ∎

Remark 6: System (3) always has an predator free equilibrium $(\delta, 0)$ which is an unstable saddle point. This implies that the zooplankton population is on the verge of extinction. That is obviously not realistic. The main goal of this paper is to investigate the dynamic behaviors near the positive equilibrium $E^* (p^*, z^*)$.

Remark 7: The following analyses in this paper are based on the assumption $H(1)$ that system (3) has a unique positive equilibrium point $E^*(p^*, z^*)$.

Next, we study the characteristic equation of controlled system (3), and then analyze the effects of the controller on the Turing patterns and Hopf bifurcation.

III. LINEAR STABILITY ANALYSIS

Let $\bar{p} = p - p^*$, $\bar{z} = z - z^*$ and delete '−', and the linearized system of (3) is

$$\begin{pmatrix} \dfrac{\partial p(t, x)}{\partial t} \\ \dfrac{\partial z(t, x)}{\partial t} \end{pmatrix} = D\Delta \begin{pmatrix} p(t, x) \\ z(t, x) \end{pmatrix} + L_1 \begin{pmatrix} p(t, x) \\ z(t, x) \end{pmatrix}$$
$$+ L_2 \begin{pmatrix} p(t - \tau, x) \\ z(t - \tau, x) \end{pmatrix}, \quad (4)$$

where

$$D = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}, \quad L_1 = \begin{pmatrix} k_1 A_{11} + k_2 & 0 \\ A_{21} & A_{22} \end{pmatrix},$$
$$L_2 = \begin{pmatrix} 0 & -k_1 B_{12} \\ 0 & 0 \end{pmatrix},$$

and

$$A_{11} = r_1 - \frac{2r_1 p^*}{\delta} - m\gamma p^* f'(p^*), \quad A_{21} = r_2 \gamma,$$
$$A_{22} = -r_2, \quad B_{12} = mf(p^*).$$

It is well known that the eigenvalues of Δ on X are $-k^2$ ($k \in N_0 \overset{\Delta}{=} \{0, 1, 2, \ldots\}$), and the characteristic equation of (3) is

$$\det(\lambda I - M_k - L_1 - L_2 e^{-\lambda\tau}) = 0,$$

where I is 2×2 identity matrix and $M_k = -k^2 D$. Then we have

$$\lambda^2 + \Lambda(k)\lambda + \Theta(k) + k_1 A_{21} B_{12} e^{-\lambda\tau} = 0, \quad (5)$$

where

$$\Lambda(k) = (d_1 + d_2)k^2 - k_1 A_{11} - k_2 - A_{22},$$
$$\Theta(k) = d_1 d_2 k^4 - [d_1 A_{22} + d_2(k_1 A_{11} + k_2)]k^2$$
$$+ (k_1 A_{11} + k_2)A_{22}.$$

Remark 8: In ecological competition networks, it is too difficult to construct the Lyapunov function precisely. This is almost impossible to investigate the global bifurcation dynamical behaviors of ecological competition networks through nonlinear methods. Hence, a lot of scholars on complex systems use linearization methods to analyze the local bifurcation dynamics of equilibrium point [1], [3], [6], [7], [14], [15]. These previous works illustrate that the linearization method is resultful and effortless for analyzing the local dynamics of ecological competition networks. Therefore, this paper also uses the method of linearization to discuss the local dynamics of marine planktonic ecosystems.

In the following two subsections, we analyze the Turing instability conditions without time delay ($\tau = 0$) of positive

equilibrium $E^*(p^*, z^*)$, and the existence of Hopf bifurcation with time delay ($\tau > 0$) under the influence of hybrid controller.

A. TURING INSTABILITY WITHOUT TIME DELAY ($\tau = 0$)

In this subsection, we investigate the Turing instability of system (3) based on the pattern dynamics theory proposed by Turing [12].

When $\tau = 0$, (5) is reduced to

$$\lambda^2 + \Lambda(k)\lambda + \Theta(k) + k_1 A_{21} B_{12} = 0. \quad (6)$$

We make the following assumptions:

(H2) $A_{11} + A_{22} < 0$, $A_{11}A_{22} + A_{21}B_{12} > 0$;

(H3) $\dfrac{d_1 A_{22} + d_2 A_{11}}{2d_1 d_2} > 0$,

$(d_1 A_{22} + d_2 A_{11})^2 - 4d_1 d_2 (A_{11}A_{22} + A_{21}B_{12}) > 0$;

(H4) $\Lambda(k) > 0$, $\Theta(k) + k_1 A_{21} B_{12} > 0$.

Case 1: Without control ($k_1 = 1$, $k_2 = 0$) and diffusion ($d_1 = d_2 = 0$).

Theorem 2: Assume that $k_1 = 1, k_2 = 0, d_1 = d_2 = 0$ and $\tau = 0$. If (H2) holds, then $E^*(p^*, z^*)$ of system (3) is locally asymptotically stable.

Proof: When $k_1 = 1$, $k_2 = 0$ and $d_1 = d_2 = 0$, (6) becomes

$$\lambda^2 - (A_{11} + A_{22})\lambda + (A_{11}A_{22} + A_{21}B_{12}) = 0. \quad (7)$$

It is obvious by Routh-Hurwitz criterion that all roots of (7) will have negative real parts if (H2) is arrived. Hence, $E^*(p^*, z^*)$ of system (3) is locally asymptotically stable. ∎

Case 2: Without control ($k_1 = 1$, $k_2 = 0$) but with diffusion ($d_1 > 0$, $d_2 > 0$).

Theorem 3: Assume that $k_1 = 1$, $k_2 = 0$, $d_1 > 0$, $d_2 > 0$ and $\tau = 0$. If (H2) and $d_1 A_{22} + d_2 A_{11} < 0$ hold, then $E^*(p^*, z^*)$ of system (3) is locally asymptotically stable.

Proof: When $k_1 = 1$, $k_2 = 0$, $d_1 > 0$ and $d_2 > 0$, (6) turns into

$$\lambda^2 + \phi(k)\lambda + \varphi(k) + A_{21}B_{12} = 0, \quad (8)$$

where

$$\phi(k) = (d_1 + d_2)k^2 - A_{11} - A_{22},$$
$$\varphi(k) = d_1 d_2 k^4 - (d_1 A_{22} + d_2 A_{11})k^2 + A_{11}A_{22}.$$

If (H2) and $d_1 A_{22} + d_2 A_{11} < 0$ hold, we have $\phi(k) \geq -A_{11} - A_{22} > 0$ and $\varphi(k) + A_{21}B_{12} \geq A_{11}A_{22} + A_{21}B_{12} > 0$. This implies that all eigenvalues of (8) have negative real parts for $k \in N_0$, thus $E^*(p^*, z^*)$ is locally asymptotically stable. ∎

Theorem 4: Assume that $k_1 = 1$, $k_2 = 0$, $d_1 > 0$, $d_2 > 0$ and $\tau = 0$. If (H2) and (H3) hold, then $E^*(p^*, z^*)$ of system (3) will undergoes $k - mode$ Turing bifurcation at $k \in N^+ \overset{\Delta}{=} \{1, 2, \ldots\}$.

Proof: Denote

$$g(z) = d_1 d_2 z^2 - (d_1 A_{22} + d_2 A_{11})z + A_{11}A_{22} + A_{21}B_{12}.$$

Note that $g(0) = A_{11}A_{22} + A_{21}B_{12} > 0$ under $(H2)$. If $(H3)$ holds, it is easy to know that $g(z)$ and the positive semi-axis in Cartesian coordinate system have two different intersection points. This implies that $g(z)$ has two positive zero points. Therefore, there exists $k \in N^+$ such that $g(k^2) = \varphi(k) + A_{21}B_{12} < 0$. This ensures that at least one root of (8) will have positive real parts. Then, $E^*(p^*, z^*)$ of system (3) will undergoes $k - mode$ Turing bifurcation. ∎

Remark 9: The Turing instability of system (3) caused by reaction diffusion coefficients d_1, d_2 will initiate the formation of Turing patterns.

Case 3: With control $(k_1 \neq 0, k_2 \neq 0)$ and diffusion $(d_1 > 0, d_2 > 0)$.

Theorem 5: Assume that $k_1 \neq 0, k_2 \neq 0, d_1 > 0, d_2 > 0$ and $\tau = 0$. If $(H4)$ holds, then $E^*(p^*, z^*)$ of system (3) is locally asymptotically stable.

Proof: It is judged by Routh-Hurwitz criterion that when $(H4)$ holds, all roots of (6) have negative real parts. Hence, $E^*(p^*, z^*)$ of system (3) is locally asymptotically stable. ∎

Remark 10: By choosing appropriate k_1 and k_2 of the hybrid controller, we can manipulate the distribution of all roots of (6), so as to get command of the appearance and disappearance of Turing patterns.

B. HOPF BIFURCATION WITH TIME DELAY ($\tau > 0$)

In this subsection, we consider the effects of time delay, diffusion terms and hybrid controller on the dynamics of system (3).

Suppose that $i\omega(\omega > 0)$ is a root of (5), then ω satisfies

$$-\omega^2 + (i\omega)[(d_1 + d_2)k^2 - k_1 A_{11} - k_2 - A_{22}] + d_1 d_2 k^4$$
$$-[d_1 A_{22} + d_2(k_1 A_{11} + k_2)]k^2 + (k_1 A_{11} + k_2)A_{22}$$
$$+k_1 A_{21} B_{12}(\cos \omega\tau - i \sin \omega\tau) = 0. \quad (9)$$

By separating real and imaginary parts of (9), we obtain

$$k_1 A_{21} B_{12} \cos(\omega\tau) = \omega^2 - d_1 d_2 k^4 - (k_1 A_{11} + k_2)A_{22}$$
$$+[d_1 A_{22} + d_2(k_1 A_{11} + k_2)]k^2,$$
$$k_1 A_{21} B_{12} \sin(\omega\tau) = \omega[(d_1 + d_2)k^2 - k_1 A_{11} - k_2 - A_{22}]. \quad (10)$$

Adding up the squares of (10) yields

$$\omega^4 + C_1\omega^2 + C_2 = 0, \quad (11)$$

where

$$C_1 = [(d_1 + d_2)k^2 - k_1 A_{11} - k_2 - A_{22}]^2 - 2d_1 d_2 k^4$$
$$+2[d_1 A_{22} + d_2(k_1 A_{11} + k_2)]k^2 - 2(k_1 A_{11} + k_2)A_{22},$$
$$C_2 = \{d_1 d_2 k^4 - [d_1 A_{22} + d_2(k_1 A_{11} + k_2)]k^2$$
$$+(k_1 A_{11} + k_2)A_{22}\}^2 - (k_1 A_{21} B_{12})^2.$$

We propose the following hypotheses:

$(H5)$ $C_1 > 0, C_2 > 0, C_1^2 - 4C_2 \geq 0$ or $C_1^2 - 4C_2 < 0$;
$(H6)$ $C_2 < 0$ or $C_1 < 0, C_1^2 - 4C_2 = 0$;
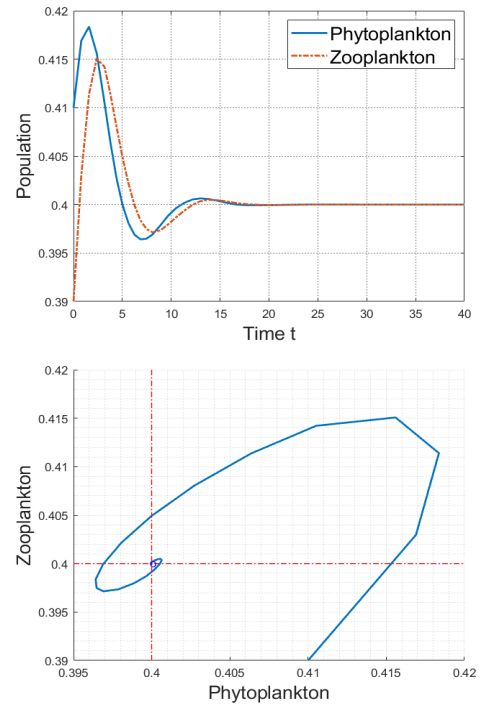$(H7)$ $C_1 < 0, C_2 > 0, C_1^2 - 4C_2 > 0$.

FIGURE 1. Waveform plots and phase portraits of plankton. The numerical results are acquired with $\tau_1 = \tau_2 = 0, d_1 = d_2 = 0, k_1 = 1, k_2 = 0$, and initial condition (0.41, 0.39).

Lemma 1: For (11), we have the following results.
1) If $(H5)$ holds, then (11) has no positive root.
2) If $(H6)$ holds, then (11) has only one positive root.
3) If $(H7)$ holds, then (11) has two positive roots.

The proof of Lemma 1 is easy to prove, so we omit it.

Suppose that (11) has positive roots. Without loss of generality, we assume that it has two positive roots $\omega_j > 0 (j = 1, 2)$. On account of (10), it is obvious that

$$\cos(\omega_j\tau) = \frac{\iota}{\kappa},$$

where

$$\iota = \omega_j^2 - d_1 d_2 k^4 + [d_1 A_{22} + d_2(k_1 A_{11} + k_2)]k^2$$
$$-(k_1 A_{11} + k_2)A_{22},$$
$$\kappa = k_1 A_{21} B_{12}.$$

Therefore

$$\tau_j^{(\gamma)} = \frac{1}{\omega_j}\{\arccos(\frac{\iota}{\kappa}) + 2\gamma\pi\}, \quad (12)$$

where $j = 1, 2$ and $\gamma = 0, 1, 2, \ldots$. Then $\pm i\omega_j$ is a pair of purely imaginary roots of (5) when $\tau = \tau_j^{(\gamma)}$. We define

$$\tau_0 = \tau_{j_0}^{(0)} = \min\{\tau_j^{(0)}, j = 1, 2\} \quad \omega_0 = \omega_{j_0}. \quad (13)$$

Let $\lambda(\tau) = \mu(\tau) + i\omega(\tau)$ is the root of (5) for $\tau = \tau_0$ satisfying $\omega(\tau_0) = \omega_0$ and $\mu(\tau_0) = 0$. Then, differentiating (5) with respect to τ, we have

$$\left(\frac{d\lambda}{d\tau}\right)^{-1} = \frac{[2\lambda + (d_1 + d_2)k^2 - k_1 A_{11} - k_2 - A_{22}]e^{\lambda\tau}}{\lambda k_1 A_{21} B_{12}} - \frac{\tau}{\lambda}.$$
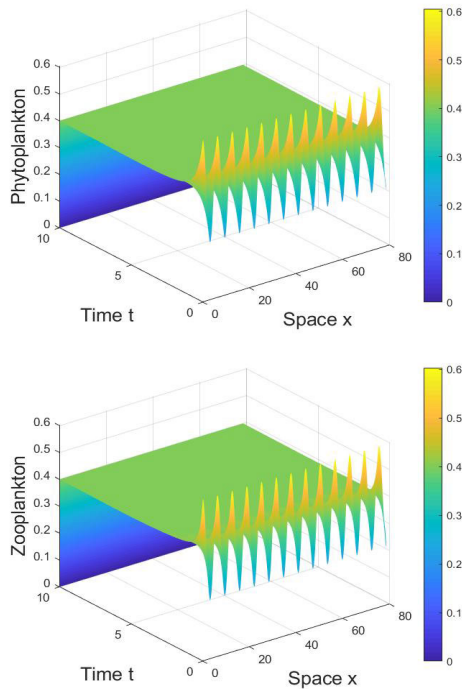
FIGURE 2. Behaviors of plankton for Turing stable conditions. The numerical results are acquired with $\tau_1 = \tau_2 = 0$, $d_1 = 2.5$, $d_2 = 5$, $k_1 = 1$, $k_2 = 0$, and initial condition $(0.41 + 0.2\cos x, 0.41 + 0.2\cos x)$.
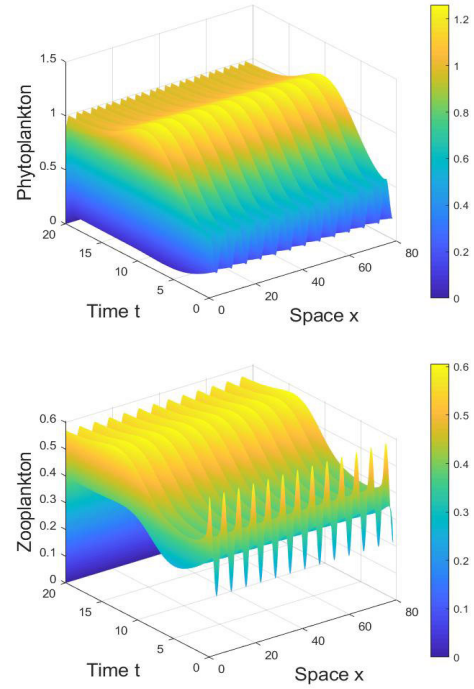
FIGURE 3. Behaviors of plankton for Turing unstable conditions. The numerical results are acquired with $\tau_1 = \tau_2 = 0$, $d_1 = 0.05$, $d_2 = 5$, $k_1 = 1$, $k_2 = 0$, and initial condition $(0.41 + 0.2\cos x, 0.41 + 0.2\cos x)$.

Thus, $Re\left(\frac{d\lambda}{d\tau}\right)^{-1}|_{\tau=\tau_0}$, as shown at the bottom of the page.

If $Re\left(\frac{d\lambda}{d\tau}\right)^{-1}|_{\tau=\tau_0} \neq 0$, then system (3) will meet the traversal condition. According to [39], the Hopf bifurcation occurs when $Im(\lambda_k) \neq 0$, $Re(\lambda_k) = 0$ at $k = 0$, then we have the following theorem.

Theorem 6: Supposing that $(H2)$ holds, the following results hold for system (3). 1)

1) If $(H5)$ holds, then $E^*(p^*, z^*)$ of system (3) is locally asymptotically stable for all $\tau \geq 0$.

2) If either $(H6)$ or $(H7)$ holds, then $E^*(p^*, z^*)$ of system (3) is locally asymptotically stable for $\tau \in [0, \tau_0)$ and unstable for $\tau > \tau_0$.

Remark 11: When $k_1 = 1$, $k_2 = 0$, τ_0 defined in (13) is reduced to the Hopf bifurcation point of system (3) without control.

IV. NUMERICAL SIMULATIONS

In this section, the theoretical results acquired in Section III will be supported by means of two numerical simulations to substantiate the veracity and feasibility.

Example 1: Consider system (2) with $r_1 = 1$, $r_2 = 1$, $\delta = 2$, $\gamma = 1$, $m = 1$, $l = 25$, and $\tau_1 = \tau_2 = 0$. From Theorem 1, we can confirm that system (2) has a unique coexisting equilibrium $E^*(0.4, 0.4)$ by choosing Holling

response function as $f(x) = x/(0.1 + x)$. Then we select different diffusion coefficients to verify the conditions of Turing pattern occurrences, and adopt appropriate control parameters to certify the effects of hybrid controller on Turing patters.

We set $d_1 = d_2 = 0$, $k_1 = 1$, and $k_2 = 0$. It is easy to verify that $(H2)$ is satisfied. It follows from Theorem 2 that $E^*(0.4, 0.4)$ is locally asymptotically stable (see Fig. 1).

We take $d_1 = 2.5$, $d_2 = 5$, $k_1 = 1$, and $k_2 = 0$. From Theorem 3, $E^*(0.4, 0.4)$ of system (2) is still locally asymptotically stable as shown in Fig. 2.

We choose $d_1 = 0.05$, $d_2 = 5$, $k_1 = 1$, and $k_2 = 0$. By Theorem 4, $E^*(0.4, 0.4)$ is Turing unstable (see Fig. 3) and Turing patterns appear (see Fig. 4).

To handle with the appearance of Turing instability and the occurrence of patterns at $E^*(0.4, 0.4)$, we can choose the hybrid controller with $k_1 = -0.3$ and $k_2 = -0.5$. By Theorem 5, $E^*(0.4, 0.4)$ is locally asymptotically stable. It indicates that the Turing instability can be effectively suppressed through manipulating the control gain parameters k_1 and k_2 of the hybrid controller, which are illustrated in Figs. 5 and 6. Therefore, our hybrid control strategy is efficient in eliminating Turing instability and Turing patterns.

Example 2: We study the Hopf bifurcation of system (2) and select $d_1 = 0.05$, $d_2 = 5$, $r_1 = 1$, $r_2 = 1$, $\delta = 2$,

$$Re\left(\frac{d\lambda}{d\tau}\right)^{-1}|_{\tau=\tau_0} = \frac{2\omega_0 \cos\omega_0\tau_0 + \left[(d_1 + d_2)k^2 - k_1 A_{11} - k_2 - A_{22}\right]\sin\omega_0\tau_0}{\omega_0 k_1 A_{21} B_{12}}.$$
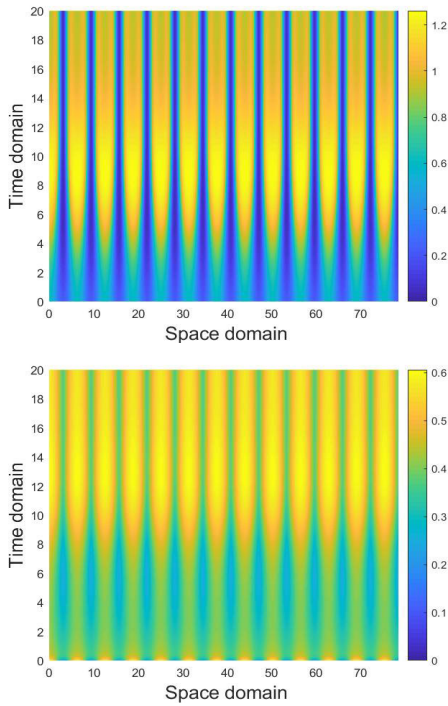
FIGURE 4. Pattern appearance of plankton for Turing unstable conditions. The numerical results are acquired with $\tau_1 = \tau_2 = 0$, $d_1 = 0.05$, $d_2 = 5$, $k_1 = 1$, $k_2 = 0$, and initial condition $(0.41 + 0.2\cos x, 0.41 + 0.2\cos x)$.
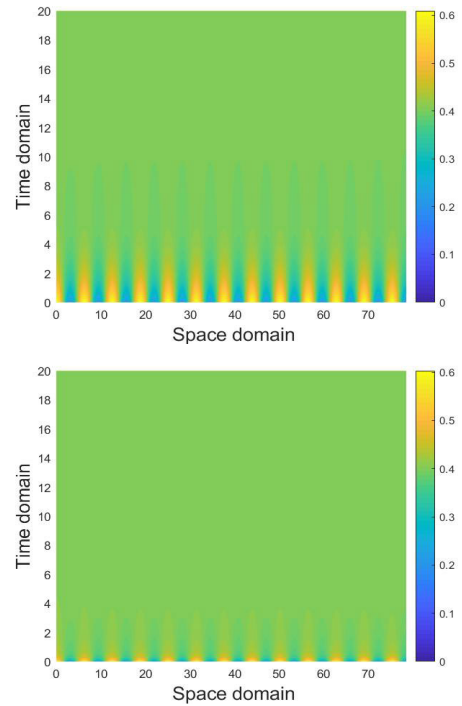

FIGURE 6. Pattern disappearance of plankton for Turing stable conditions in the presence of hybrid controller. The numerical results are acquired with $\tau_1 = \tau_2 = 0$, $d_1 = 0.05$, $d_2 = 5$, $k_1 = -0.3$, $k_2 = -0.5$, and initial condition $(0.41 + 0.2\cos x, 0.41 + 0.2\cos x)$.
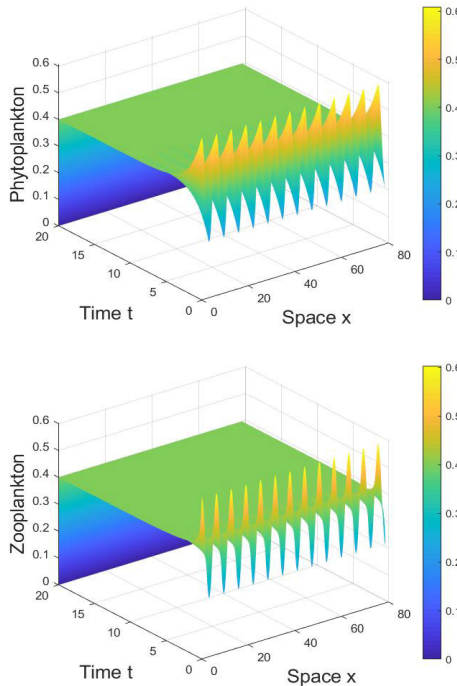

FIGURE 5. Behaviors of plankton for Turing stable conditions. The numerical results are acquired with $\tau_1 = \tau_2 = 0$, $d_1 = 0.05$, $d_2 = 5$, $k_1 = -0.3$, $k_2 = -0.5$, and initial condition $(0.41 + 0.2\cos x, 0.41 + 0.2\cos x)$.
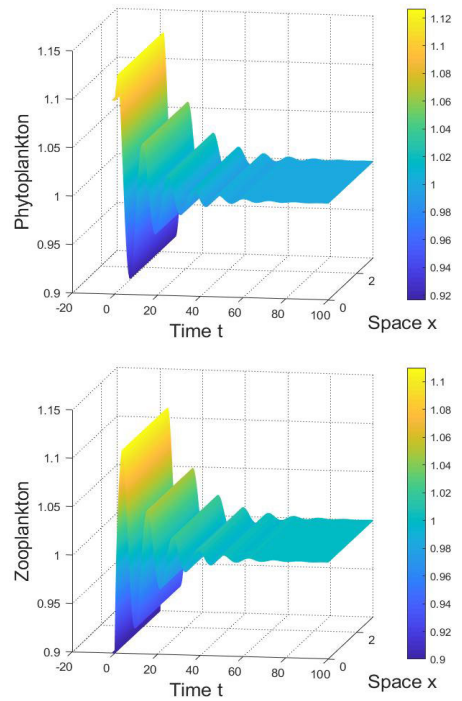

FIGURE 7. Behaviors of plankton for Hopf stable conditions. The numerical results are acquired with $\tau = \tau_1 + \tau_2 = 2.6 < \tau_0 = 2.6727$, $d_1 = 0.05$, $d_2 = 5$, $k_1 = 1$, $k_2 = 0$, and initial condition $(1.1, 0.9)$.

$\gamma = 1$, $m = 1$, $l = 1$, and $f(x) = x/(1 + x)$. Under these parameters, system (2) has a unique coexisting equilibrium $E^*(1, 1)$. By direct computation, we have $\omega_0 = 0.5491$ and $\tau_0 = 2.6727$ while $k_1 = 1$, $k_2 = 0$. It follows

from Theorem 6 that $E^*(1, 1)$ is locally asymptotically stable when $\tau = \tau_1 + \tau_2 = 2.6 < \tau_0 = 2.6727$ (see Fig. 7),
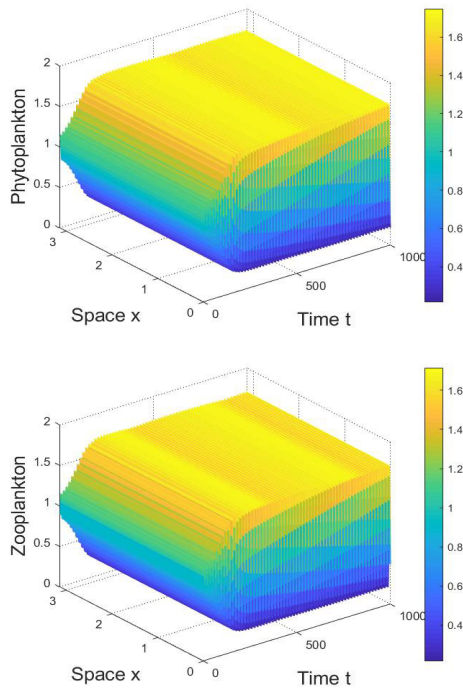
FIGURE 8. Behaviors of plankton for Hopf unstable conditions. The numerical results are acquired with $\tau = \tau_1 + \tau_2 = 2.8 > \tau_0 = 2.6727$, $d_1 = 0.05$, $d_2 = 5$, $k_1 = 1$, $k_2 = 0$, and initial condition $(1.1, 0.9)$.
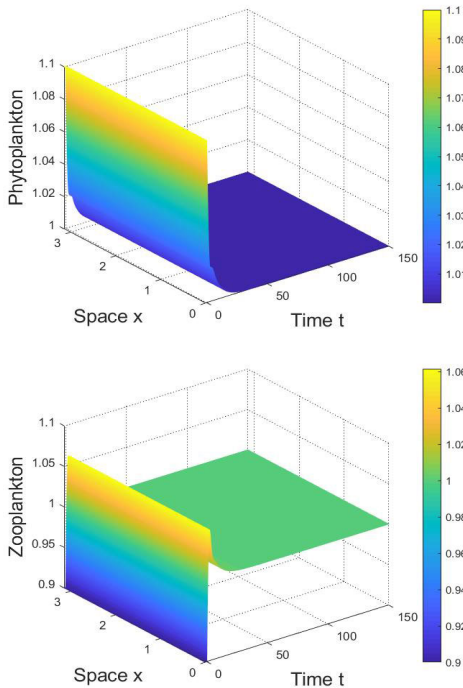

FIGURE 9. Behaviors of plankton in the presence of hybrid controller. The numerical results are acquired with $\tau = \tau_1 + \tau_2 = 2.8 > \tau_0 = 2.6727$, $d_1 = 0.05$, $d_2 = 5$, $k_1 = -0.3$, $k_2 = -0.5$, and initial condition $(1.1, 0.9)$.

while $E^*(1, 1)$ loses its stability and the bifurcation occurs $\tau = \tau_1 + \tau_2 = 2.8 > \tau_0 = 2.6727$ (see Fig. 8).

Next, we adopt the same hybrid bifurcation control strategy above with $k_1 = -0.3$, $k_2 = -0.5$. Unlike the case of
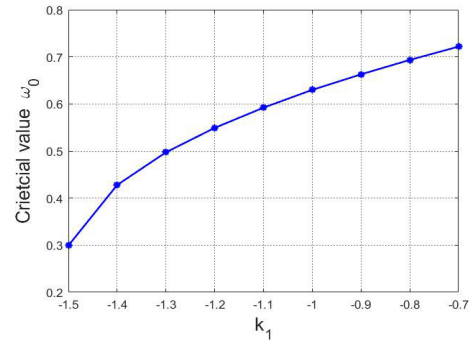

FIGURE 10. The influence of k_1 on the critical value ω_0 for system (2) with $k_2 = -0.5$.
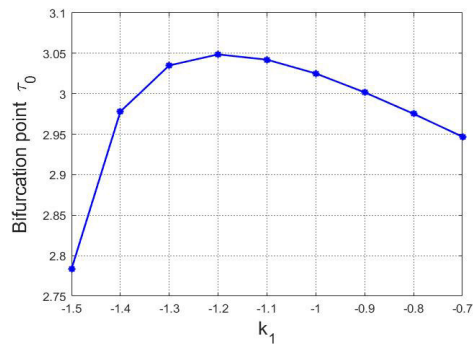

FIGURE 11. The influence of k_1 on the bifurcation point τ_0 for system (2) with $k_2 = -0.5$.

TABLE 2. The influence of regulation parameter k_1 on critical frequency ω_0 and Hopf bifurcation point τ_0 in the case of $k_2 = -0.5$ fixed.

k_1	ω_0	τ_0
-0.7	0.2992	2.7835
-0.8	0.4278	2.9780
-0.9	0.4972	3.0350
-1	0.5491	3.0486
-1.1	0.5920	3.0421
-1.2	0.6293	3.0250
-1.3	0.6628	3.0017
-1.4	0.6934	2.9752
-1.5	0.7217	2.9469

$k_1 = 1$, $k_2 = 0$, $E^*(1, 1)$ becomes locally asymptotically stable for $\tau = \tau_1 + \tau_2 = 2.8$ (see Fig. 9).

Last but not least, for the controlled system (2) with $k_1 \neq 0$, $k_2 \neq 0$, we further make a thorough inquiry of the relationship between the regulation parameter k_1 of the hybrid controller, the critical frequency ω_0 and the Hopf bifurcation point τ_0 in the case of $k_2 = -0.5$ fixed. As k_1 decreases, the critical frequency ω_0 rises, and the Hopf bifurcation point τ_0 lags to a certain extent and then advances (see Figs. 10 and 11). The results are also diagramed in Table 2.

V. CONCLUSION
This paper extends the reaction-diffusion terms into the marine planktonic ecosystem with multiple delays, and takes into account the influence of diffusive reaction and hybrid controller on the dynamic behaviors of system (1). Firstly, we analyze the Turing instability of the non-delay

phytoplankton-zooplankton system and further find that Turing patterns resulted from reaction-diffusion terms can be eliminated via reasonably selecting the parameters of the hybrid controller. Next, by means of adopting the sum of delays as the bifurcation parameter, the sufficient conditions for the stability of controlled system (2) have been given and results indicate that reasonable parameter adjustment can advance or postpone the critical value of bifurcation, so as to achieve the desired spatial dynamics behaviors. Furthermore, the functional line chart of the hybrid control parameters on the onset of Hopf bifurcation has been provided. Finally, two numerical examples are given to show the correctnesses of the theorems. The results demonstrate that the hybrid control synthesis plays an significant role in optimizing the dynamic performance of complex systems.

The spatial position information involved in this paper is limited to one dimension. In future work, we will extend it to two or even three dimensions to further investigate the different formations of Turing patterns through amplitude equations.

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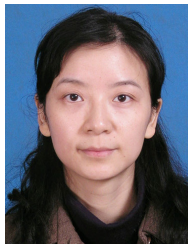
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