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Parametric and Coordinate Control of Oscillating Systems: Physics-Based Oscillation Feedback Design

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ABSTRACT This paper presents and compares two practical implementation schemes based on parametric and coordinate control. The parametric feedback control idea employs a parametric resonance phenomenon. An approximate frequency domain stationarization approach yields the setting for a very simple controller scheme. The application and numerical analysis of the results are given for the pendulum control example, which length varies in a parametric control case. The coordinate and parametric control demonstrate similar damping properties. The study also provides an inventive idea for passive parametric control combined with coordinate control that shows better damping.

INDEX TERMS Parameter-varying systems, time-varying systems, output feedback control, sinusoidal signals, settling time.

I. INTRODUCTION

Modern control theory provides a number of standardized solutions when an engineer is challenged by a situation that falls into the classical framework of oscillation control. Having been an unquestionable advantage, the out-of-shelf feedback control schemes often prevent the practitioners from investigating the inventive ways to reach the desired, and sometimes even better result. The latter requires the understanding of the physics of oscillations: the mathematical model that describes the object and in turn advises the way to control it. In contrast, the mathematical control community tends to stay within a certain family of models. A certain physical system is never given a chance to be observed (and controlled) from different perspectives, unlike being considered through different mathematical models. Thus, if the output of the oscillatory system has to be stabilized, the linear time-invariant model would be the ground for most engineering solutions, out of which classical proportional-integrative (PI) feedback controller will dominate. But what other concepts can be invented if the model is different, for example, when parameters are seen as time-varying?

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Most examples of time-varying parameter control can be found in mechanical systems such as inverted pendulum with vibrating suspension point [1], multi-DOF pendulum [2], and its practical applications: ship roll stabilization [3], control of flexible actuators [4]. For some autonomous technical devices, such as quadcopters or space navigation systems, parametric control is often the only option. The series of articles starting from [5] shows applications of parametric control in spacecraft studies where it is used along with classical coordinate feedback control. One of the newest applications with embedded time-varying parameters are energy harvesting devices [6]. The article [7] discusses semiactive control of vibrations and energy absorption system with periodic time-varying dampers. In [8] time-varying damping is implemented with the usage of electrorheological liquids. A series of articles starting from [9] discuss stabilizing properties of pendulum systems with moving mass. Antisway crane control has the biggest chances to be met among the applications of this set-up, see [10] as an exemplary publication. Physical experimental research was made for the stabilization of gondola oscillations [11].

Thus, the idea of parametric variation produces a wide plurality of new models, regulators, or even new devices. Eventually, the engineer typically needs to know how different control concepts can influence the stabilization of the object to compare them or even better, to systematically generate new out-of-box ideas based on a physical understanding of oscillation. This practice-based approach does not find its way to academically rigorous literature where a specific mathematical model is the departure point of the research. The common idea of the suggested approach is the attempt to find some different ways of control and compare them to the classical control solutions.

The objective of the study is an oscillating system, where classical feedback approach, parametric dampening approach, new parametric feedback approach followed by passive parametric damping, and a combination of them are applied. All these approaches solve the same engineering problem of stabilizing a pendulum while each one is based on a different mathematical model of it.

The main contribution of the work is the demonstration of how different mathematical models of the same problem of oscillation quietening can support the heuristic stage of the conceptual design of stabilizing systems. The article compares mathematically and numerically different approaches to sway stabilization that can be used by field engineers in their creative design of oscillation and vibration dampers in cranes, machines, and possibly in systems of other physical nature. A new method of stabilization based on parametric feedback is presented.

The paper consists of the following parts. In Section II, the case plant which is represented by an ordinary pendulum is presented and described mathematically. Different control paradigms applied to the plant are introduced in corresponding subchapters of Section II. Section III is devoted to numerical modeling of the previously introduced control schemes and contains the modeling results. Further sections contain a discussion of observed results and summarize the study.

II. THEORETICAL PART

Any control system designed with analytical tools of control theory depends on the initial mathematical modeling of the controlled object. Usually, the same physical object can be defined by different mathematical models. For example, the dynamics of physical parameters is often neglected or averaged, but in some cases, it can have a valuable influence on the control system and even can help to stabilize the oscillations. We suggest a new control system design approach based on building time-varying and time-delay parametric feedback and compare different control paradigms using the pendulum-like systems as examples.

In the current paper, we consider a stable pendulum as the control object with the linearized model

$$ml^2\ddot{\alpha} + \zeta_\alpha\dot{\alpha} + mgl\alpha = 0, \tag{1}$$

where α, is angular position, m, is mass of the pendulum, l, is the length of the suspension, ζ_α is the damping factor, and g is the gravity constant.

The control schemes to be discussed are shown in Fig. 1. All the pendulums have the same mass and initial length, but the way they are controlled is different. The simplest pendulum with fixed length and lumped mass has a well-known free oscillation profile, that is used as a benchmark for several concepts of control presented further.

Fig. 1a shows the pendulum with a weight with an integrated rotor, capable of producing the reactive torque. When the rotor acceleration is applied properly, the plant oscillating behavior can be controlled with a reactive torque.

The example of a pendulum with variable length is shown in Fig. 1b. The design and properties of these time-varying systems were discussed in [9], [20], and [21]. If the length of the suspension can vary (for example by rotating the drum), we have a chance to control the oscillations, too.

The spring gravitational pendulum (Fig. 1c) shows similar stabilizing characteristics as the previously introduced system and can be implemented with a special design of suspension or suspension point. The main difference of the plant from Fig. 1c from the one from Fig. 1b is the absence of an external rope adjustment device – here the rope length varies ''naturally'' due to centrifugal forces and we can arrive at the same result - shortening of the decay time.

Finally, Figures 1d and 1e present two combined control layouts, where two previous control paradigms are used simultaneously. The plant shown in Fig. 1d is the combination of the systems controlled by the reactive torque and adjusting the length (Fig. 1a and Fig. 1b respectively). The plant shown in Fig. 1e is the combination of the systems controlled by the reactive torque and spring-made adjusting of the length (Fig. 1a and Fig. 1c respectively).

A. COORDINATE CONTROLLER

Let us consider a pendulum system with rigid suspension, which is shown in Fig. 1a. The mass of this system is the electric motor with a free massive rotor. The switching of the rotor's rotation direction generates the reactive torque Te that gradually disappears. The mathematical model of such a system is as follows:

$$\begin{aligned} ml^2\ddot{\alpha} + \zeta_\alpha\dot{\alpha} + mgl\alpha &= T_e \\ T_e &= c_M i = J\ddot{\varphi} \\ u &= Ri + c_e\dot{\varphi} \\ u &= sign\,(\alpha)\,, \end{aligned} \tag{2}$$

where J is rotor moment of inertia, φ is the angle of rotor, $\dot{\varphi}$ and $\ddot{\varphi}$ are corresponding derivatives; T_e is stator reactive rotation moment; u and i are motor's voltage and current respectively; c_M and c_e are the electromagnetic and electric motor's coefficients, respectively.

The control theory formalism describes the input-output (current-rotational rate) relationship of the above system by the transfer function

$$W\,(s) = \frac{M\,(s)}{u\,(s)} = \frac{Ks}{T_m s + 1}, \tag{3}$$

where $K = J/c_e$ is transfer coefficient of stator reactive torque, T_m is the electromechanical time constant of the

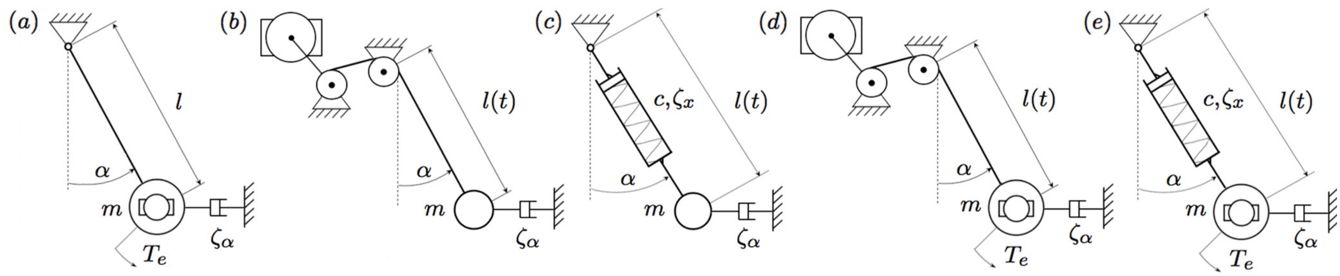

FIGURE 1. The implementations of control schemes. (a) Reactive motor coordinate control system. (b) Variable length pendulum control system. (c) Spring gravitational pendulum. (d, e) Combined control schemes.

motor. As can be seen in this application the motor plays the role of a differentiating element. It can perform the correction function of controlling processes by the control of the reactive torque.

B. PARAMETRIC CONTROLLER

The problem of stabilization via parametric controller may be formalized differently as Brockett stabilization problem [12], parametric resonance and antiresonance approach [13], [14], vibrational stabilization [15]. In most cases, the control of the time-varying parameter is used in a form of feedforward control when the output response is not measured. This approach is not universal and usually works for the stabilization of some unstable systems or for parametric excitation of stable systems [16]. At the same time, stable systems after the introduction of time-varying parameter often meet the unstable frequency of the output coordinate oscillations [9]. This causes a phase shift variation and beating or resonance phenomena as a result. In such cases, the synchronization feedback to maintain the phase shift is needed. The simplest realization of such feedback is shown in [9] where the shape of the periodic function governing the parameter is adjusted. The results of [9] are rather experimental and do not have generalization to higher-order systems. The series of articles starting from [17] show several extensions for higher-order systems. The parametrically controlled system with periodical time-varying parameters can be analyzed as a linear time-invariant system after some trigonometric approximations. The embodiment of this result is described in the patent for a new type of parametrically controlled system [18]. The characteristics of this system are discussed in [19]. Similar results for the second-order systems can be found in the articles [20], [21] and were received independently from [16]–[18].

The common scheme of a parametrically controlled system was recently discussed in [22]. The parametric control feedback links the output (controlled) coordinate $x(t)$ with one of the parameters of the system. In the simplest case, the parameter is the coefficient, which is multiplied by the output coordinate $x(t)$. This multiplication is an additive component of the equation, and it can be treated as control input. We denote this additive component as $y(t)$ and assume the following parametric control feedback

$$y(t) = x(t)\, k \operatorname{sign}\left(\frac{d}{dt}\left[x(t-\tau)\right]^2\right), \tag{4}$$

where k is a gain, τ is a time-delay.

The physical meaning of this mathematical expression is the following. The square of the output signal doubles the frequency of parametric oscillations. The differentiation eliminates the constant component. The sign-function removes the influence of the magnitude of coordinate oscillations on the magnitude of parametric variation. Finally, the variable gain value k controls the magnitude of parametric variations and the delay τ delivers the phase shift between the periodic variation of the parameter and the output coordinate oscillations. This phase shift is the critical parameter we will use to control the output oscillations.

In [14] it is shown that in the case of periodical variation of parameters the parametrically controlled system can be analyzed as a linear time-invariant system after some trigonometric approximations. The development of the result [14] became a patent for a new type of parametrically controlled system [15]. Some scientific applications of approximation methods used for parametrically controlled systems are given in [16].

We are going to stay in the framework of mono-frequency approximation in our analysis, so let us assume that

$$x(t) = A \sin \omega t.$$

where A is the maximum coordinate value and ω is the frequency of oscillations. Then (4) takes the form of

$$\begin{aligned} y(t) &= (A \sin \omega t)\, k \operatorname{sign}\left[A^2 \omega \sin 2\omega (t-\tau)\right] \\ &\cong \frac{2kA}{\pi} \cos(\omega t - \varphi), \end{aligned} \tag{5}$$

where $\varphi = 2\omega t$. In (5) we substitute the square periodic variations by the first harmonic component of its Fourier series, whereas the third and higher harmonics are neglected. According to (5) the transfer function of the parameter-driven by parametric feedback can be defined as

$$W(j\varphi) = \frac{2k}{\pi} e^{-j\varphi}. \tag{6}$$

If the rest of the system is described with transfer function formalism, then the Nyquist criterion can be used to estimate the stability, as well as magnitude and phase stability margins can be calculated. The larger the stability margins are the faster the system is stabilized.

1) VARIABLE-LENGTH PENDULUM

As an example, let us consider the application of the given approach for the variable-length pendulum, shown in Fig. 1b. Since the specific method for implementing the adjustment of the length of the pendulum is not specified, this example can be considered an illustration of the general case of applying the described theory. In this case, the realization of the parametric control would mean the controllable variation of the pendulum length $l = l(t)$ as the function of the sway angle α (Fig. 1b). Our goal is not just to avoid parametric resonance. We can vary the length of the pendulum to damp the oscillation faster than if they were free. In other words, we plan to find "parametric anti-resonance" conditions.

While the physical parameters are the same (the mass is chosen to be equal to the mass of the motor from the coordinate control scheme), the mathematical model (1) changes to

$$ml(t)^2\ddot{\alpha} + 2ml(t)\dot{l}(t)\dot{\alpha} + \zeta_\alpha\dot{\alpha} + mgl(t)\alpha = 0 \tag{7}$$

assuming $l(t) = l_0 + \Delta l(t)$, where $\Delta l(t) \ll l_0$. Let us divide (7) by $ml(t)^2$ and expand the resulting functions $l(t)$ in Taylor series near l_0. Having retained the linear terms only we obtain

$$\ddot{\alpha} + 2\dot{l}(t)\left[\frac{1}{l_0} - \frac{1}{l_0^2}\Delta l(t)\right]\dot{\alpha} + \left[\frac{1}{ml_0^2} - \frac{2}{ml_0^3}\Delta l(t)\right]\zeta_\alpha\dot{\alpha} + \left[\frac{1}{l_0} - \frac{1}{l_0^2}\Delta l(t)\right]g\alpha = 0. \tag{8}$$

The solutions of (8) for fixed strictly periodical variation of the parameter $\Delta l(t)$ are well-studied by Floquet theory and are represented as several periodical functions multiplied by corresponding exponents. The results of [9], [20], [21] show experimental algorithms to control the period of the parameter $\Delta l(t)$. In contrast to these methods, the suggested feedback (5) is supported by an analytical approach for its adjustment. This feedback filters oscillating movements of the output, automatically controls the period of parametric variation, guarantees preset amplitude of the parameter, and maintains needed time delay and phase shift.

We use trigonometric approximation to demonstrate the effect. The last equation can be seen as one with periodically time-varying coefficients generated by $\Delta l(t)$ and $\dot{l}(t)$. Let us split the equation into time-varying and time-invariant parts:

$$\ddot{\alpha} + \frac{\zeta_\alpha}{ml_0^2}\dot{\alpha} + \frac{g}{l_0}\alpha = -\frac{2}{l_0}\dot{l}(t)\dot{\alpha} + \frac{2}{l_0^2}\dot{l}(t)\Delta l(t)\dot{\alpha} + \frac{2\zeta_\alpha}{ml_0^3}\Delta l(t)\dot{\alpha} + \frac{g}{l_0^2}\Delta l(t)\alpha. \tag{9}$$

The equation can be seen as a regular pendulum (left part) with feedback forces given by certain time-varying gains (right part). To investigate the oscillating character of solutions and to find the value of k, which is critical for stability, we assume the mono-frequency approximation of all the time-varying terms in the equation (9). We use the same feedback and approach for analysis as in (4) and (5) using the following substitutions:

$$\begin{aligned}\alpha(t) &= A\sin\omega t,\\ \Delta l(t) &= k\,\mathrm{sign}\left(\frac{d}{dt}[\alpha(t-\tau)]^2\right)\\ &\cong \frac{4k}{\pi}sin(2\omega t + \varphi).\end{aligned} \tag{10}$$

We can derive the terms $\dot{l}(t)$ and $\dot{l}(t)\Delta l(t)$ as harmonic functions and leave in the harmonic approximations of time-varying terms $\Delta l(t)\alpha$, $\Delta l(t)\dot{\alpha}$, $\dot{l}(t)\Delta l(t)\dot{\alpha}$ and $\dot{l}(t)\dot{\alpha}$ only oscillations of frequency ω. After Laplace transform and substitution of the Laplace operator $s = j\omega$ with j as an imaginary unit we obtain the condition of the frequency ω stability loss

$$-\omega^2 + \frac{\zeta_\alpha}{ml_0}j\omega + \frac{g}{l_0^2} = -\frac{2}{l_0}\frac{4k\omega^2}{\pi}e^{j0.5\pi}e^{-j\varphi} + \frac{2\zeta_\alpha}{ml_0^3}\frac{2k\omega}{\pi}e^{-j\varphi} + \frac{g}{l_0^2}\frac{2k}{\pi}e^{j0.5\pi}e^{-j\varphi}. \tag{11}$$

It is important to point out that the phase shift here is one of the design parameters: we are interested in that φ that provides the maximal stability margin. Another adjusting coefficient is the value of the parametric feedback gain k. We can gather the adjusting coefficients k and φ in the right part of the equation as follows

$$\left[-\omega^2 + \frac{\zeta_\alpha}{ml_0}j\omega + \frac{g}{l_0^2}\right]\left[\frac{-4\omega^2}{l_0}j - \frac{2\zeta_\alpha s}{ml_0^3}j + \frac{g}{l_0^2}j\right]^{-1} = \frac{2k}{\pi}e^{-j\varphi}. \tag{12}$$

The last equation defines the stability border in the space of k and φ. The calculation can be done numerically, graphically, or analytically for some cases.

2) SPRING GRAVITATIONAL PENDULUM

Let us consider a spring pendulum in the gravitation field, which also has sway oscillations (Fig. 1c) and technically may be considered as a special case of a variable-length pendulum. This system can be seen from an interesting perspective. Indeed, there is "natural" (passive) parametric stabilizing control. The centrifugal force reaches its maximum as the pendulum sway crosses the vertical axis and extends the length of the pendulum since there is a spring. Thus, the length of the pendulum depends on the angular position like in parametric control. The equation of the motion and its frequency analysis is given in [17]. As one can see, the spring gravitational pendulum may carry out the parametric control

TABLE 1. The parameters of the gravitational pendulum.

Parameter	Value
m	1 kg
l	1 m
ζ_α	0.05 N · s/rad
g	10 m/s^2
c	40 N · s/m
ζ_x	0.1 N/m
$\alpha(0)$	0.1 rad
$\dot{\alpha}(0)$	0 rad/s

task without artificial length changing mechanism and control tuning. The pendulum stabilization occurs only due to the properties of the system. In addition, the numerical value of oscillations suppression in this scheme is quite good.

C. COMBINED CONTROL

When we consider 3 basic paradigms of control (Fig. 1a-c), some of them can be combined. Although the combination is not proved mathematically and nonlinear synergy can have negative as well as a positive influence, nevertheless this combination provides two more control concepts: variable length coordinate control (Fig. 1d) and spring gravitational pendulum coordinate control (Fig. 1e). The basic paradigms together with combined control models will be numerically analyzed in the next chapter.

III. NUMERICAL EXAMPLES

The control schemes shown in Fig. 1 are simulated with MATLAB Simulink environment. The chosen values of the system parameters are shown in Table 1. These values are the same for all the further control settings.

A. UNDRIVEN PENDULUM EXPERIMENT

The undriven pendulum will be the reference case to compare with the suggested control schemes. Using the values from Table 1 we arrive at the pendulum transfer function:

$$W(s) = \frac{\alpha(s)}{T_e(s)} = \frac{1}{s^2 + 0.05s + 10}, \tag{13}$$

where s is the Laplace operator. We will compare the oscillation control results by the settling time evaluation. We define the settling time T_s as the intersection of the envelope curve to the oscillations with a 10% error band. In the case of free pendulum oscillations settling time T_s is 91.5 s (Fig. 2).

B. COORDINATE CONTROL EXPERIMENT

The coordinate control setup by the reactive moment of the motor was built according to (Fig. 1a). The modeled control loop is presented in (Fig. 3). We include the "sign" block in the feedback loop to model the switching between clockwise and counterclockwise direction of the torque T_e. Such relay control is useful for practical reasons. Indeed, the control torque is reactive. So, the switching of motor provides better and is easier to realize comparing to varying its rotational rate. Thus, the "sign" element in the sensor circuit would tell the

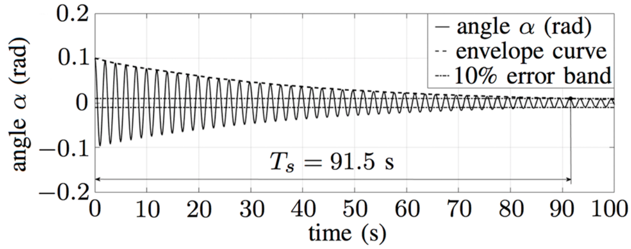

FIGURE 2. The uncontrolled pendulum oscillations.

motor to reverse as the pendulum crosses its vertical position. Although it immediately brings the design into the nonlinear control domain, the frequency of oscillations slightly varies with time, the output still decays, and its settling time can be evaluated. Thus, the oscillations under the coordinate control are stabilized in approximately 14 s (Fig. 4).

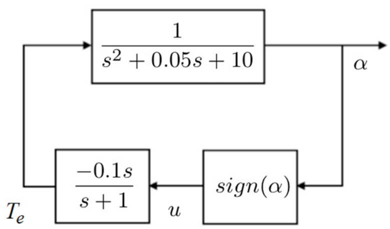

FIGURE 3. The model of coordinate stable pendulum control by the reactive torque of the electric motor's rotor. The model describes the setup of Fig. 1a.

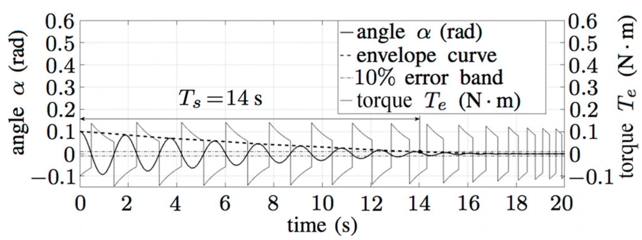

FIGURE 4. The oscillations of the coordinate stable pendulum control by the reactive moment of electric motor's rotor.

C. PARAMETRIC CONTROL EXPERIMENT

We consider equation (7) as the basic equation for modeling and numerical experiments of the system shown in Fig. 1b. The equation (12) is used for the analytical calculation of stability. We can denote the left part of (12) as $W_{LTI}^{-1}(j\omega)$ and the right part as $W_{LTV}(j\varphi)$. The open-loop Nyquist plots $W_{LTI}^{-1}(j\omega)\, W_{LTV}(j\varphi)$ for several $\varphi = 2\omega\tau$ and the same $k = 0.02$ are shown in Fig. 5.

The Nyquist plot can be used to find the critical value of τ for any k. The comparison of the stability borders obtained with an above-mentioned approximate analytical approach and numerically by means of MATLAB Simulink are shown in Fig. 6. The lower values of k leave the system stable for any τ. The highest values of k lead to stability loss for any value of τ. The chosen control value $k = 0.02$ remains the sensitivity of the stability to the value of τ. The oscillation process for $k = 0.02$ and $\tau = 0$ is shown in Fig. 7.

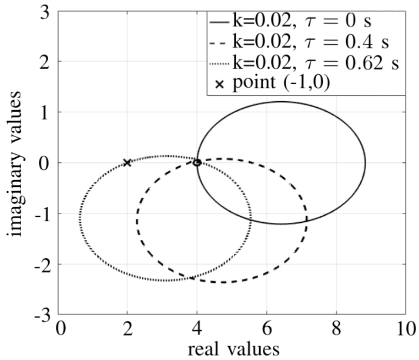

FIGURE 5. The Nyquist plot for the linearized system.

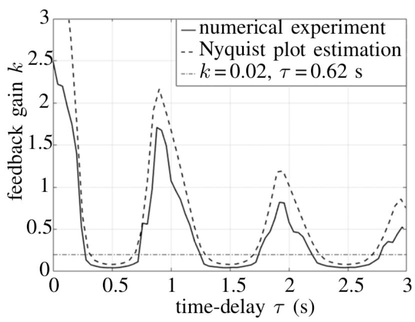

FIGURE 6. Nyquist and numerical estimation of the stability border for the parametric controller.

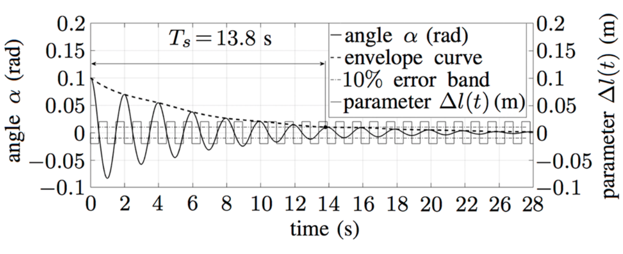

FIGURE 7. Parametrically controlled oscillations and square shape of parametric variations.

D. SPRING GRAVITATIONAL PENDULUM EXPERIMENT

The spring gravitational pendulum model was built as an alternative for a variable-length pendulum control system. The main task here is the choice of the stiffness of the spring. The natural frequency of the spring pendulum from Fig. 1c must be equal to the doubled frequency of the swaying pendulum oscillations. The experimentally obtained oscillations are shown in Fig. 8.

E. COMBINED PARAMETRIC-COORDINATE CONTROL SCHEMES

Here we present the results of two combined control schemes. Coordinate-parametric control system represented by a pendulum with adjustable length (Fig. 1d), and another coordinate-parametric control scheme implemented with springing suspension (Fig. 1e). In such control typologies, we apply parametric and coordinate control methods simultaneously. Results of the adjustable length pendulum numerical modeling are shown in Fig. 9, and results of the spring-suspended pendulum numerical modeling are shown in Fig. 10.

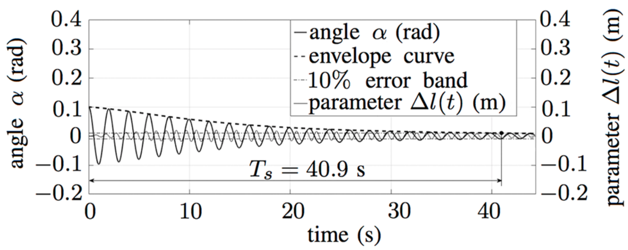

FIGURE 8. The signal output of uncontrolled spring pendulum.

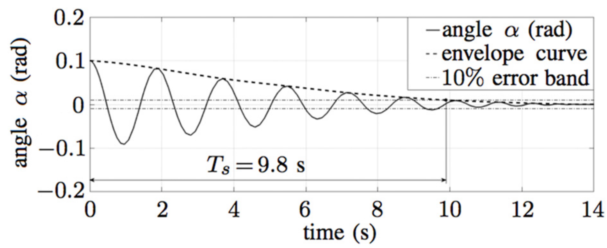

FIGURE 9. The oscillations of the general case parametric-coordinate control system, embodied with an adjustable-length pendulum.

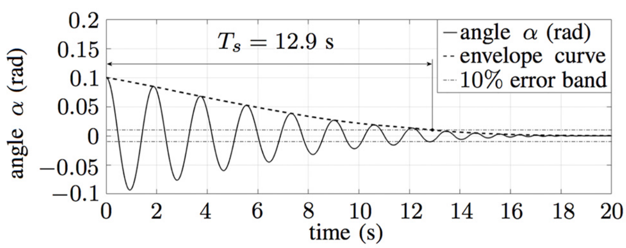

FIGURE 10. The oscillations of the special-case parametric-coordinate controlled system driven by spring pendulum.

IV. DISCUSSION

In the present paper, we have considered different control topologies for the sedation oscillatory systems on the instance of the swinging pendulum case. The key objective of this study was to elaborate on the control technique which allows making oscillating systems to rest in the shortest time and keep the system stable. On the basis of the obtained numerical

TABLE 2. Different control methods comparison.

Control method	Damping time T_s, s
Uncontrolled	91.5
Coordinate controlled	14
Parametrically controlled (general case: adjustable-length pendulum)	13.8
Parametrically controlled (spring gravitational control)	40.9
Combined parametric-coordinate control (general case: adjustable-length pendulum)	9.8
Combined parametric-coordinate control (spring pendulum)	12.9

experiments, the time interval that is required for the damping of oscillations to an amplitude equal to 10% error band was found for each considered case and collected in Table 2.

From Table 2 we can find and compare the pendulum decay time of all the cases being considered in the paper. The largest decay time of 91.5 s was observed when a simple uncontrolled pendulum was modeled. Application of any of the considered control methods reduced the decay time to varying degrees. The application of the coordinate control reduces the pendulum damping time by 84.6 % (from 91.5 s to 14 s). The parametric control has the potential to reduce damping time even more – by 84.9%. It is notable that even the simplest case of the passive parametric control system, realized by using a spring in the suspension of the pendulum, shows the decay time 55.3% less than the decay time of the reference uncontrolled pendulum. The damping time reached in systems, where the parametric and coordinate control was combined, is the shortest among all observed cases. The combination of coordinate control with parametric control generally showed the most effective pendulum decay time reduction: from 91.5 s to 9.8 s, i.e., by 89.3%. The combination of coordinate control with the spring pendulum also reduces the decay time significantly – by 85.9%.

This work has shown that despite the fact that the coordinate and parametric methods of oscillation control are widely known and can reduce the settling time of oscillatory systems, they have only a moderate decay time reduction rate when used alone. At the same time, simultaneous use of coordinate and parametric methods offers a great potential to reduce the oscillating systems' damping time most effectively.

V. CONCLUSION

The study delivers two messages that can be useful for design engineers and researchers in the field of periodic motion control. Firstly, we demonstrate how the conceptual design of the control for the same physical object can be assisted by the ability to observe it from different perspectives, or, in other words, to describe it by different mathematical models. We compare the efficiency of the concepts and their combinations on a basic benchmark problem of pendulum stabilization. Second, we present the method of parametric feedback that is naturally derived from one of the concepts. All the results are supported by numerical simulations. The presented schemes of coordinate, parametric, and passive dynamic compensation control are simple and therefore important for practice schemes for oscillation feedback control. While coordinate control is well known and used by engineers, parametric feedback control seems to need more studies before it becomes standard practice. The numerical results show approximately the same performance of the closed-loop system, but the combination of the methods yields the best damping results. The presented study may be utilized by the manufacturers of lifting and transportation equipment, construction engineers, and other fields' specialists, where it is necessary to quickly and effectively suppress various oscillations. The study is limited by the type of systems in focus: physical objects that demonstrate essentially "one degree of freedom" behavior with a high Q-factor. Having focused on similar physical objects, we compare the outputs of various mathematical models, that is intuitively understandable but very economical in terms of formal mathematical sense. The review of literature should have been given a more structured and exhaustive form. Further research will address other schemes of parametric and nonlinear control that inventively engage the physics of the plant and concurrently combine the controller design with admissible changes in the object.

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REFERENCES

[1] A. A. Mailybaev and A. P. Seyranian, "Stabilization of statically unstable systems by parametric excitation," *J. Sound Vibrat.*, vol. 323, nos. 3–5, pp. 1016–1031, Jun. 2009.
[2] I. M. Arkhipova and A. Luongo, "Stabilization via parametric excitation of multi-dof statically unstable systems," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 19, no. 10, pp. 3913–3926, Oct. 2014.
[3] V. Calabro and A. Caiti, "Parametric control allocation for vessels equipped with two non-fully rotable thrusters," *IFAC Proc. Volumes*, vol. 45, no. 27, pp. 368–373, 2012.
[4] H. Yabuno and S. Kobayashi, "Motion control of a flexible underactuated manipulator using resonance in a flexible active arm," *Int. J. Mech. Sci.*, vol. 174, May 2020, Art. no. 105432.
[5] V. Yu Rutkovsky and S. D. Zemlyakov, "Coordinate-parametric control system for spacecraft: Functioning algorithms," in *Proc. 9th IFAC/ESA Symp. Autom. Control Space*, 1983, pp. 103–109.
[6] M. Scapolan, M. G. Tehrani, and E. Bonisoli, "Energy harvesting using parametric resonant system due to time-varying damping," *Mech. Syst. Signal Process.*, vol. 79, pp. 149–165, Oct. 2016.
[7] F. Di Monaco, M. Ghandchi Tehrani, S. J. Elliott, E. Bonisoli, and S. Tornincasa, "Energy harvesting using semi-active control," *J. Sound Vibrat.*, vol. 332, no. 23, pp. 6033–6043, Nov. 2013.
[8] Y. Liu, H. Matsuhisa, H. Utsuno, and J. G. Park, "Variable damping and stiffness vibration control with magnetorheological fluid dampers for two degree-of-freedom system," *JSME Int. J. Ser. C*, vol. 49, no. 1, pp. 156–162, 2006.

[9] W. Szyszkowski and D. S. D. Stilling, ''On damping properties of a frictionless physical pendulum with a moving mass,'' *Int. J. Non-Linear Mech.*, vol. 40, no. 5, pp. 669–681, 2005.

[10] H. Chen and N. Sun, ''Nonlinear control of underactuated systems subject to both actuated and unactuated state constraints with experimental verification,'' *IEEE Trans. Ind. Electron.*, vol. 67, no. 9, pp. 7702–7714, Oct. 2020.

[11] H. Matsuhisa, ''Development of vibration absorber of gondola,'' *Trans. Jpn. Soc. Mech. Eng. Ser.*, vol. 78, no. 788, pp. 1026–1030, 2012.

[12] R. W. Brockett, ''A stabilization problem,'' in *Open Problems in Mathematical Systems and Control Theory, Communications and Control Engineering*. Berlin, Germany: Springer, 1998, pp. 75–78.

[13] L. Pešek, P. Šulc, V. Bula, and J. Cibulka, ''Parametric 'Anti-resonance' phenomenon as active damping tool of beam flexural vibration,'' *Procedia Eng.*, vol. 144, pp. 1031–1038, Jan. 2016.

[14] M. Rades, ''Resonance and antiresonance,'' in *Encyclopedia of Vibration*, 1st ed. New York, NY, USA: Academic, 2001, pp. 1046–1055.

[15] X. Cheng, Y. Tan, and I. Mareels, ''On robustness analysis of linear vibrational control systems,'' *Automatica*, vol. 87, pp. 202–209, Jan. 2018.

[16] L. Chechurin and S. Chechurin, *Physical Fundamentals of Oscillations*. Cham, Switzerland: Springer, 2017, p. 264.

[17] S. L. Chechurin and L. S. Chechurin, ''Elements of physical oscillation and control theory,'' in *Proc. IEEE Int. Conf. Phys. Control*, St. Petersburg, Russia, vol. 2, Aug. 2003, pp. 589–594.

[18] S. Chechurin, L. Chechurin, and A. Mandrik, ''Approach to stabilization of oscillating object output,'' RU Patent 2 393 520, Jun. 27, 2010.

[19] A. V. Mandrik, L. S. Chechurin, and S. L. Chechurin, ''Frequency analysis of parametrically controlled oscillating systems,'' in *IFAC-PapersOnLine*, vol. 48, no. 11, pp. 651–655, 2015.

[20] M. Kure, J. Busek, T. Vyhlídal, and S.-I. Niculescu, ''Damping oscillation of suspended payload by up and down motion of the pivot base–time delay algorithms for UAV applications,'' *IFAC-PapersOnLine*, vol. 52, no. 18, pp. 121–126, 2019.

[21] A. Abe, ''Non-linear control technique of a pendulum via cable length manipulation: Application of particle swarm optimization to controller design,'' *FME Trans.*, vol. 41, no. 4, pp. 265–270, 2013.

[22] L. Chechurin, S. Chechurin, and A. Mandrik, ''Parametric control of oscillations,'' in *Cyber-Physical Systems and Control, CPS&C* (Lecture Notes in Networks and Systems), vol. 95, D. Arseniev, L. Overmeyer, H. Kälviäinen, and B. Katalinic, Eds. Cham, Switzerland: Springer, 2020, pp. 98–107.

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