Adaptive Dynamic Surface Control for a Class of Dead-Zone Nonlinear Systems via Output Feedback

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ABSTRACT The paper aims to address output feedback problem for a class of nonlinear systems subjected to unknown dead-zone. High-gain K-filters is firstly designed such that unavailable states of system can be reconstructed. Then, with the help of dynamic surface control (DSC) method, an adaptive output feedback controller design is established, which can alleviate undesired influence of the dead-zone and guarantee the semi-global stability of the system. Furthermore, the $\mathcal{L}_\infty$ performance of tracking error is achieved by means of initialization technique. Ultimately, numerical and practical examples clarify the efficiency of the proposed solution.

INDEX TERMS Dynamic surface control, adaptive control, high-gain observer, dead-zone, nonlinear systems.

I. INTRODUCTION

Motivated by the widely application of the industrial motion control systems such as electric servo system and hydraulic servo system, dead-zone characteristic existed in a large number of actuators of the industrial control systems has been extensively discussed.

The pioneering work on control for nonlinear systems subjected to dead-zone could be dated back to [1], where adaptive dead-zone inverse compensation proposal was presented for a linear system with known bound of dead-zone parameters. The proposal ensured global stability and asymptotical tracking for a full state available system. Similarly, adaptive dead-zone inverse was constructed in [2], then linear model reference controller was presented. More recently, the research has been extended to strict-feedback nonlinear systems. In [3] smooth dead-zone inverse was delivered in order that control performance of adaptive backstepping design can be improved.

It is quite difficult to construct the exactly inverse of various dead-zone models from another point of view. Thus robust adaptive thought has been extensively applied to mitigate dead-zone. Instead of constructing dead-zone inverse,

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separating thought is accepted based on the dead-zone characteristic: one is linear part and the other is "disturbance-like" part with unknown bound. In [4], a robust adaptive dynamic surface control (DSC) was provided in order to address the influence from unknown non-symmetric dead-zone in a MIMO nonlinear systems, and initialization technique was offered so as to obtain the $\mathcal{L}_\infty$ performance. When only input and output signals are available, [5] investigated an output feedback modified DSC based on tracking differentiator for a dead-zone nonlinear system. Both steady state and transient performance are achieved under assumption that all control functions are positive as well as bounded. Furthermore, with the aid of the fuzzy logic systems and neural networks techniques, lots of semi-global adaptive robust strategies were published for nonlinear systems disturbed by unknown dead-zone [6]–[9]. Other control schemes concerning the dead-zone can be refer to [6]–[9] and the references cited therein.

Based on the aforementioned results with respect to the dead-zone, we note that most existing control methods require the measurement of all the system states. Thus adaptive output feedback control is still challenging for nonlinear systems with dead-zone. Moreover, [10] proposed an adaptive output feedback control strategy for system subjected to dead-zone input, but the transient performance

could not be guaranteed, which may lead to some undesired behaviors, such as bursting. While in [11], although the transient performance is achieved, a complex dynamic surface design has to be needed due to tracking error transformation. Until now, the difficulties of output feedback control on system subjected to the dead-zone are both states observer construction and relatively simple control strategy in order that the undesirable catastrophic caused by dead-zone can be counteracted and closed-loop stability is obtained. Especially, steady state and transient performance could be guaranteed.

This paper devotes attention to propose an adaptive DSC control via output feedback for uncertain nonlinear systems preceded by dead-zone input. The motivation for the scheme is the high-precision control for some mechanical plans subjected to dead-zone. The study makes these contributions:

- Unlike the common K-filters employed in existing output feedback approach [11], unmeasurable states are reconstructed by high-gain K-filters, which pave the way for improvement of transient performance without tracking error transformation.
- An initialization technique is incorporated into the DSC for purpose of guaranteeing the $\mathcal{L}_\infty$ tracking performance. That is, just by adjusting the value of some design parameters, steady state error as well as maximum overshoot of tracking error could be made arbitrarily small.

The structure of the paper: Section II: Problem statement and some preliminary knowledge are described. Section III: An adaptive output feedback DSC design procedure by means of high-gain K-filters is presented. Section IV: Stability and $\mathcal{L}_\infty$ tracking performance analysis are given. Section V: Two examples are provided to demonstrate effectiveness of this design.

II. PROBLEM STATEMENT AND PRELIMINARIES
In this paper, uncertain nonlinear SISO system is considered:

$$\dot{x} = Ax + f_0(y) + \sum_{i=1}^{r} \theta_i f_i(y) + bG(y)u + d(t),$$
$$y = e_1^T x, \tag{1}$$

where

$$A = \begin{bmatrix} 0 & & \\ \vdots & I_{n-1} & \\ 0 & \cdots & 0 \end{bmatrix}, \quad f_i(y) = \begin{bmatrix} f_{i,1}(y) \\ \vdots \\ f_{i,n}(y) \end{bmatrix}, \quad 0 \le i \le r,$$

$$G(y) = \begin{bmatrix} 0_{(\rho-1)\times 1} \\ g_m(y) \\ \vdots \\ g_0(y) \end{bmatrix}, \quad d(t) = \begin{bmatrix} d_1(t) \\ \vdots \\ d_n(t) \end{bmatrix}, \quad e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$
$$\tag{2}$$

where $x := [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ and $y \in \mathbb{R}$ are state vector and output signal, respectively; $\theta_i, \ i = 1, \ldots, r$ and

b are unknown parameters; the nonlinear smooth functions $f_{i,j}(y), \ i = 1, \ldots, r, \ j = 1, \ldots, n,$ and $g_i(y), \ i = 0, \ldots, m, \ (g_m(y) \neq 0)$ are known; $\rho \, (\ge 1) = $ n-m stands for the system relative order, and $n, \ m$ are known constants; $d \in \mathbb{R}^n$ is the external disturbance; u is unknown dead-zone's output formulated as [12], [13]

$$u(t) = \mathcal{D}(v(t)) = mv(t) + h(t) \tag{3}$$

with

$$h(t) = \begin{cases} -mh_r, & v(t) \ge h_r, \\ -mv(t), & -h_l < v(t) < h_r, \\ mh_l, & v(t) \le -h_l. \end{cases} \tag{4}$$

where $m, \ h_r, \ h_l$ are unknown bounded positive constants, and $h(t)$ satisfies

$$|h(t)| \le \bar{h}, \ \bar{h} = \max\{mh_l, \ mh_r\}. \tag{5}$$

Invoking (1) and (3) yields

$$\dot{x} = Ax + f_0(y) + \sum_{i=1}^{r} \theta_i f_i(y) + b_m G(y) v$$
$$+ bG(y) h(t) + d(t),$$
$$y = e_1^T x, \tag{6}$$

with $b_m = bm$.

The control objective: besides the boundedness of all signals of the closed-loop system, the tracking performance can be guaranteed under the proposed adaptive DSC control scheme.

Assumption 1: Disturbances $d_i(t), \ i = 1, \ldots, n$ satisfy

$$|d_i(t)| \le \bar{d}_i \tag{7}$$

with $\bar{d}_i (\ge 0)$ being unknown constants.

Assumption 2: The sign of b is assumed to be positive for simplicity.

Assumption 3: The reference trajectory y_d is smooth with $y_d(0)$ at designer's disposal, and there exists a known compact set such that $[y_d, \ \dot{y}_d, \ \ddot{y}_d]^T$ belongs to it for all $t \ge 0$.

Assumption 4: The system

$$\dot{x}_i = x_{i+1} + f_{0,i}(y) + \sum_{j=1}^{r} \theta_j f_{j,i}(y) + \frac{g_{n-i}(y)}{g_{n-\rho}(y)} y_0$$
$$- \frac{g_{n-i}(y)}{g_{n-\rho}(y)} x_{\rho+1}, \quad \rho+1 \le i \le n, \tag{8}$$

are bounded-input to bounded-output (BIBO) stable, where $y_0, y, x_2, \ldots, x_\rho$ are inputs and $x_{\rho+1}$ is output.

Remark 1: In contrast to the most related work [14]–[17], system (1) takes $g_i(y)$ into account and the term $bG(y) h(t)$ caused by dead-zone cannot be directly seen as the bounded disturbance, which will increase the difficulty of our control design.

III. HIGH-GAIN OBSERVER BASED ADAPTIVE DSC DESIGN

A. HIGH-GAIN K-FILTER OBSERVER

Firstly, design a proper vector $k = [k_1, k_2, \ldots, k_n]^T$ to make

$$A_0 = A - ke_1^T = \begin{bmatrix} -k_1 & 1 & & \\ -k_2 & & \ddots & \\ \vdots & & & 1 \\ -k_n & 0 & \cdots & 0 \end{bmatrix} \quad (9)$$

become Hurwitz matrix. Now, (6) can be rewritten as:

$$\dot{x} = A_0 x + ky + f_0(y) + \sum_{i=1}^{r} \theta_i f_i(y) + b_m G(y) v$$
$$+ bG(y) h(t) + d(t),$$
$$y = e_1^T x. \quad (10)$$

Owing to the only available output signal, the high-gain K-filter observer should be designed to compensate effect of dead-zone as well as to reconstruct unavailable states:

$$\dot{\zeta} = qA_0\zeta + \Psi^{-1}G(y) v, \quad (11)$$
$$\dot{\omega} = qA_0\omega + qky + \Psi^{-1}f_0(y), \quad (12)$$
$$\dot{\xi}_i = qA_0\xi_i + \Psi^{-1}f_i(y), \ 1 \le i \le r, \quad (13)$$

where $\Psi = diag\{1, q, \ldots, q^{n-1}\}$ and $q(\ge 1)$ is observer gain chosen as a positive parameter. Then estimation of states is given as

$$\hat{x} = \Psi\omega + \sum_{i=1}^{r} \theta_i \Psi \xi_i + b_m \Psi \zeta. \quad (14)$$

To proceed, the estimation error is

$$\epsilon := x - \hat{x}. \quad (15)$$

By means of (10)-(13), and (14), one can conclude

$$\dot{\epsilon} = A\epsilon - q\Psi k\epsilon_1 + bG(y) h(t) + d(t), \quad (16)$$

where ϵ_1 is the first entry of ϵ.

Lemma 1: Let

$$V_\epsilon := \epsilon^T P\epsilon, \quad (17)$$

where a symmetric matrix $P = (\Psi^{-1})^T \bar{P} \Psi^{-1}$, and $\bar{P} = \bar{P}^T > 0$, and $\bar{P}$ satisfies the equation

$$A_0^T \bar{P} + \bar{P} A_0 = -2I, \quad (18)$$

with I being the unit matrix. Then the following inequality holds

$$\dot{V}_\epsilon \le -\varsigma_\epsilon V_\epsilon + D_\epsilon + q^{1-2\rho} \vartheta_\epsilon \varrho(y), \quad q \ge 1 \quad (19)$$

with nonnegative smooth function $\varrho(y)$, and

$$\varsigma_\epsilon := \frac{q}{\lambda_{\max}(\bar{P})}, \quad (20)$$

$$D_\epsilon := q\left(\frac{\|\bar{P}\| \|d\|_{\max}}{q^\rho}\right)^2, \quad (21)$$

$$\vartheta_\epsilon := (\|\bar{P}\| b\bar{h})^2, \quad (22)$$

where $\lambda_{\max}(\bullet)$ denotes the largest eigenvalue of $\bullet$, $\|d\|_{\max}$ presents the maximum value of $\|d(t)\|$.

Proof: We can get the relationship $\epsilon^T P\epsilon = \bar{\epsilon}^T \bar{P} \bar{\epsilon}$ by using the coordinate transformation $\bar{\epsilon} = \Psi^{-1}\epsilon$. And $\bar{\epsilon}$ could result in

$$\dot{\bar{\epsilon}} = qA_0\bar{\epsilon} + \Psi^{-1}(bG(y) h(t) + d(t)). \quad (23)$$

Since A_0 is Hurwitz, (18) holds for positive definite matrix $\bar{P}$. Define

$$V_{\bar{\epsilon}} := \bar{\epsilon}^T \bar{P} \bar{\epsilon}, \quad (24)$$

and time derivative of $V_{\bar{\epsilon}}$ along (23) is

$$\dot{V}_{\bar{\epsilon}} = -2q\bar{\epsilon}^T \bar{\epsilon} + 2\bar{\epsilon}^T \bar{P} \Psi^{-1}(bG(y) h(t) + d(t))$$
$$\le -2q\|\bar{\epsilon}\|^2 + \frac{2\|\bar{\epsilon}\|}{q^{\rho-1}}(\|\bar{P}\| b\bar{h} \|G(y)\|$$
$$+ \|\bar{P}\| \|d\|_{\max}). \quad (25)$$

Meanwhile, there is a nonnegative smooth function $\varrho(y)$ such that

$$\|G(y)\|^2 \le \varrho(y). \quad (26)$$

Together with Young's inequality and (5), one have

$$\dot{V}_{\bar{\epsilon}} \le -q\|\bar{\epsilon}\|^2 + q\left[\left(\frac{\|\bar{P}\| \|d\|_{\max}}{q^\rho}\right)^2 + \left(\frac{\|\bar{P}\| b\bar{h}}{q^\rho}\right)^2 \varrho(y)\right]$$
$$\le -\varsigma_\epsilon V_\epsilon + D_\epsilon + q^{1-2\rho} \vartheta_\epsilon \varrho(y). \quad (27)$$

Hence we arrive at (19). This completes the proof. ∎

B. ADAPTIVE DSC DESIGN

On the basis of above observer, the adaptive DSC design can be proceed smoothly.

Step 1. The first surface error is

$$S_1 = y - y_d. \quad (28)$$

Then

$$\dot{S}_1 = q\omega_2 + \sum_{i=1}^{r} \theta_i(q\xi_{i,2} + f_{i,1}(y)) + b_m q\zeta_2 + f_{0,1}(y)$$
$$+ d_1(t) - \dot{y}_d + \epsilon_2$$
$$= q\omega_2 + \Theta^T \varpi + b_m q\zeta_2 + f_{0,1}(y) + d_1(t)$$
$$- \dot{y}_d + \epsilon_2, \quad (29)$$

with vectors

$$\Theta = [b_m, \theta_1, \ldots, \theta_r]^T,$$
$$\varpi = [0, q\xi_{1,2} + f_{1,1}(y), \ldots, q\xi_{r,2} + f_{r,1}(y)]^T, \quad (30)$$

where $\omega_j, \xi_{i,j}, \zeta_j, \epsilon_j$ denote the *j*th entry of vector $\omega, \xi_i, \zeta, \epsilon$, respectively.

The first virtual control law is

$$\bar{\zeta}_2 = \hat{p}\bar{\zeta}_{2p} = \frac{\hat{p}}{q}\left[-c_1 S_1 - q\omega_2 - f_{0,1}(y) + \dot{y}_d - \hat{\vartheta}\psi S_1\right], \quad (31)$$

with positive design parameter c_1. In the above, $\hat{p}$ denotes estimation of $p\left(=b_m^{-1}\right)$, and $\hat{\vartheta}$ denotes estimation of ϑ, $\vartheta :=$ $\max\{\vartheta_\epsilon, \vartheta_\Theta\}$, ϑ_ϵ is defined in (22), $\vartheta_\Theta := \|\Theta\|^2$, and $\psi = \frac{\mu}{2}\|\varpi\|^2 + q^{1-2\rho}\bar{\varrho}(y)$, $\bar{\varrho}(y)$ is constructed as

$$\bar{\varrho}(y) = \frac{2}{S_1^2 + \delta}\varrho(y) \tag{32}$$

with δ being a small positive constant. Subsequently, the update laws are given by

$$\dot{\hat{\vartheta}} = \gamma_\vartheta \psi S_1^2 - \gamma_\vartheta \eta_\vartheta \hat{\vartheta}, \tag{33}$$

$$\dot{\hat{p}} = -\gamma_p q \bar{\zeta}_{2p} S_1 - \gamma_p \eta_p \hat{p}, \tag{34}$$

with adaptive gains γ_ϑ, γ_p and positive design parameters η_ϑ, η_p.

Let $\bar{\zeta}_2$ pass through filter with constant τ_2:

$$\tau_2 \dot{z}_2 + z_2 = \bar{\zeta}_2, \quad z_2(0) = \bar{\zeta}_2(0). \tag{35}$$

Step i. ($2 \le i \le \rho - 1$) The i-th surface error is

$$S_i = \zeta_i - z_i. \tag{36}$$

Then

$$\dot{S}_i = -qk_i \zeta_1 + q\zeta_{i+1} - \dot{z}_i. \tag{37}$$

The virtual control $\bar{\zeta}_{i+1}$ is

$$\bar{\zeta}_{i+1} = \frac{1}{q}\left(-c_i S_i + qk_i \zeta_1 + \dot{z}_i\right), \tag{38}$$

with positive design parameter c_i.

Let $\bar{\zeta}_{i+1}$ pass through filters with constant τ_{i+1}:

$$\tau_{i+1}\dot{z}_{i+1} + z_{i+1} = \bar{\zeta}_{i+1}, \quad z_{i+1}(0) = \bar{\zeta}_{i+1}(0). \tag{39}$$

Step ρ. The ρ-th surface error is

$$S_\rho = \zeta_\rho - z_\rho, \tag{40}$$

then

$$\dot{S}_\rho = -qk_\rho \zeta_1 + q\zeta_{\rho+1} + q^{1-\rho}g_m(y)v - \dot{z}_\rho. \tag{41}$$

Finally, the actual control v is

$$v = \frac{1}{q^{1-\rho}g_m(y)}\left(-c_\rho S_\rho + qk_\rho \zeta_1 - q\zeta_{\rho+1} + \dot{z}_\rho\right), \tag{42}$$

with positive design parameter c_ρ.

Remark 2: As shown in (33), the estimated parameter ϑ is a scalar instead of a vector with $n + r + 1 - \rho$ parameters, which not only can greatly reduce the computational burden, but also make the controller easy to implement. Moreover, the adaptive laws with dimension one are only used at first step.

IV. STABILITY AND $\mathcal{L}_\infty$ TRACKING PERFORMANCE ANALYSIS

Define

$$y_i = z_i - \bar{\zeta}_i, \quad 2 \le i \le \rho. \tag{43}$$

Same as that in previous DSC methods [4], [18]–[20], from (11)-(13), (28)-(42), there are continuous nonnegative functions B_{i+1}, $i = 1, \ldots, \rho - 1$ such that the following inequations hold:

$$\left|\dot{y}_2 + \frac{y_2}{\tau_2}\right| \le B_2\left(S_1, S_2, y_2, \tilde{\vartheta}, \tilde{p}, \epsilon, y_d, \dot{y}_d, \ddot{y}_d\right), \tag{44}$$

$$\left|\dot{y}_{i+1} + \frac{y_{i+1}}{\tau_{i+1}}\right| \le B_{i+1}(S_1, \ldots, S_{i+1}, y_2, \ldots, y_{i+1},$$

$$\tilde{\vartheta}, \tilde{p}, \epsilon, y_d, \dot{y}_d, \ddot{y}_d), \tag{45}$$

where $\tilde{\vartheta} := \vartheta - \hat{\vartheta}$, $\tilde{p} := p - \hat{p}$.

We can define Lyapunov function as:

$$V = \sum_{i=1}^{\rho} V_i \tag{46}$$

with

$$V_1 = \frac{1}{2}S_1^2 + \frac{1}{2}y_2^2 + \frac{1}{2\gamma_\vartheta}\tilde{\vartheta}^2 + \frac{b_m}{2\gamma_p}\tilde{p}^2 + V_\epsilon, \tag{47}$$

$$V_i = \frac{1}{2}S_i^2 + \frac{1}{2}y_{i+1}^2, \quad 2 \le i \le \rho - 1, \tag{48}$$

$$V_\rho = \frac{1}{2}S_\rho^2. \tag{49}$$

Theorem 1: Consider the system (1) with dead-zone input (3), high-gain K-filters (11)-(13), intermediate controllers (31) and (38), first-order filters (35), (39), update laws and actual control provided by (33), (34) and (42), respectively. Suppose assumptions 1-4 hold. Then for any given positive constants C_1, C_2, if the initial condition of (46) satisfy $V(0) \le C_2$ and $y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le C_1$, there are parameters q, c_i $(i = 1, \ldots, \rho)$, τ_i $(i = 2, \ldots, \rho)$, η_ϑ, η_p, γ_ϑ, γ_p such that all signals are semi-global bounded. Furthermore, steady state error can be made arbitrarily small. Specifically, tracking error satisfies

$$\lim_{t \to \infty}|S_1| \le \lim_{t \to \infty}\sqrt{2V} \le \sqrt{\frac{Q}{r_\epsilon}}, \tag{50}$$

where r_ϵ is a positive constant,

$$Q = \frac{\iota}{2}(\rho - 1) + \frac{\eta_\vartheta}{2}\vartheta^2 + \frac{b_m\eta_p}{2}p^2 + \frac{1}{2\mu}$$

$$+ \frac{1}{2}\|d_1\|^2 + \bar{\vartheta} + D_\epsilon. \tag{51}$$

Positive constants ι, μ and $\bar{\vartheta}$ will be presented later.

Proof: Define the compact sets

$$\Omega_1 = \left\{(y_d, \dot{y}_d, \ddot{y}_d) : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le C_1\right\},$$

$$\Omega_2 = \Bigg\{(S_1, \ldots, S_\rho, y_2, \ldots, y_\rho, \tilde{\vartheta}, \tilde{p}, \epsilon) : \sum_{i=1}^{\rho}S_i^2 + \sum_{i=1}^{\rho-1}y_{i+1}^2$$

$$+ \frac{1}{\gamma_\vartheta}\tilde{\vartheta}^2 + \frac{b_m}{\gamma_p}\tilde{p}^2 + 2\epsilon^T P\epsilon \le 2C_2\Bigg\}. \tag{52}$$

It is easy to have compact set $\Omega_1 \times \Omega_2$. In the compact set $\Omega_1 \times \Omega_2$, there exist M_{i+1}, the maximum value of B_{i+1} in (44), (45). Furthermore, from (44), (45) and the Young's inequality, we get

$$y_{i+1}\dot{y}_{i+1} \leq -\frac{y_{i+1}^2}{\tau_{i+1}} + M_{i+1}|y_{i+1}|,$$

$$M_{i+1}|y_{i+1}| \leq \frac{M_{i+1}^2 y_{i+1}^2}{2\iota} + \frac{\iota}{2}, \quad 1 \leq i \leq \rho - 1, \quad (53)$$

where ι is arbitrary number. Considering Young's inequality and $\bar{\epsilon} = \Psi^{-1}\epsilon$ in Lemma 1, we have

$$S_1 S_2 \leq \frac{1}{2}S_1^2 + \frac{1}{2}S_2^2, \quad S_1 y_2 \leq \frac{1}{2}S_1^2 + \frac{1}{2}y_2^2,$$

$$S_1 \epsilon_2 \leq \frac{q^2}{2}S_1^2 + \frac{1}{2\lambda_{\min}(\bar{P})}V_\epsilon,$$

$$S_1 d_1(t) \leq \frac{1}{2}S_1^2 + \frac{1}{2}\|d_1\|^2,$$

$$S_1 \Theta^T \varpi \leq \frac{\mu S_1^2 \vartheta_\Theta \varpi^T \varpi}{2} + \frac{1}{2\mu},$$

$$\eta_\vartheta \tilde{\vartheta}\hat{\vartheta} \leq \frac{\eta_\vartheta}{2}\vartheta^2 - \frac{\eta_\vartheta}{2}\tilde{\vartheta}^2,$$

$$b_m \eta_p \tilde{p}\hat{p} \leq \frac{b_m \eta_p}{2}p^2 - \frac{b_m \eta_p}{2}\tilde{p}^2. \quad (54)$$

Using (19), (29)-(34), (53), (54), then

$$\dot{V}_1 = S_1 \dot{S}_1 + y_2 \dot{y}_2 - \frac{1}{\gamma_\vartheta}\tilde{\vartheta}\dot{\hat{\vartheta}} - \frac{b_m}{\gamma_p}\tilde{p}\dot{\hat{p}} + \dot{V}_\epsilon$$

$$\leq S_1(-c_1 S_1 + q b_m(S_2 + y_2)) + \frac{1}{2}S_1^2 + \frac{1}{2}\|d_1\|^2$$

$$+ \frac{q^2}{2}S_1^2 + \frac{1}{2\lambda_{\min}(\bar{P})}V_\epsilon + \frac{1}{2\mu} - \frac{q}{\lambda_{\max}(\bar{P})}V_\epsilon$$

$$+ \vartheta q^{1-2\rho}\left(\varrho(y) - \bar{\varrho}(y)S_1^2\right) + \eta_\vartheta \tilde{\vartheta}\hat{\vartheta} + b_m \eta_p \tilde{p}\hat{p}$$

$$- \frac{y_2^2}{\tau_2} + \frac{M_2^2 y_2^2}{2\iota} + \frac{\iota}{2} + D_\epsilon$$

$$\leq -\left(c_1 - q b_m - \frac{q^2}{2} - \frac{1}{2}\right)S_1^2$$

$$- \left(\frac{1}{\tau_2} - \frac{q b_m}{2} - \frac{M_2^2}{2\iota}\right)y_2^2$$

$$- \left(\frac{q}{\lambda_{\max}(\bar{P})} - \frac{1}{2\lambda_{\min}(\bar{P})}\right)V_\epsilon$$

$$+ \frac{q b_m}{2}S_2^2 + \vartheta q^{1-2\rho}\left(\varrho(y) - \bar{\varrho}(y)S_1^2\right)$$

$$+ \frac{\eta_\vartheta}{2}\vartheta^2 - \frac{\eta_\vartheta}{2}\tilde{\vartheta}^2 + \frac{b_m \eta_p}{2}p^2 - \frac{b_m \eta_p}{2}\tilde{p}^2$$

$$+ \frac{1}{2}\|d_1\|^2 + \frac{1}{2\mu} + \frac{\iota}{2} + D_\epsilon. \quad (55)$$

Similarly, the time derivatives of (48), (49) are

$$\dot{V}_i \leq -c_i S_i^2 + q(S_{i+1} + y_{i+1})S_i - \frac{y_{i+1}^2}{\tau_{i+1}}$$

$$+ \frac{M_{i+1}^2 y_{i+1}^2}{2\iota} + \frac{\iota}{2}$$

$$\leq -(c_i - q)S_i^2 - \left(\frac{1}{\tau_{i+1}} - \frac{M_{i+1}^2}{2\iota} - \frac{q}{2}\right)y_{i+1}^2$$

$$+ \frac{q}{2}S_{i+1}^2 + \frac{\iota}{2}, \quad 2 \leq i \leq \rho - 1, \quad (56)$$

and

$$\dot{V}_\rho = -c_\rho S_\rho^2 = -\left(c_\rho - \frac{q}{2}\right)S_\rho^2 - \frac{q}{2}S_\rho^2. \quad (57)$$

Substituting (55), (56), (57) into the time derivatives of (46), we have

$$\dot{V} \leq -\left(c_1 - q b_m - \frac{q^2}{2} - \frac{1}{2}\right)S_1^2$$

$$- \left(c_2 - \frac{q b_m}{2} - q\right)S_2^2$$

$$- \sum_{i=3}^{\rho-1}\left(c_i - \frac{3q}{2}\right)S_i^2 - \left(c_\rho - \frac{q}{2}\right)S_\rho^2$$

$$- \left(\frac{1}{\tau_2} - \frac{M_2^2}{2\iota} - \frac{q b_m}{2}\right)y_2^2$$

$$- \sum_{i=2}^{\rho-1}\left(\frac{1}{\tau_{i+1}} - \frac{M_{i+1}^2}{2\iota} - \frac{q}{2}\right)y_{i+1}^2$$

$$- \left(\frac{q}{\lambda_{\max}(\bar{P})} - \frac{1}{2\lambda_{\min}(\bar{P})}\right)V_\epsilon$$

$$- \frac{\eta_\vartheta}{2}\tilde{\vartheta}^2 - \frac{b_m \eta_p}{2}\tilde{p}^2 + \vartheta q^{1-2\rho}\left(\varrho(y) - \bar{\varrho}(y)S_1^2\right)$$

$$+ \frac{\iota}{2}(\rho - 1) + \frac{\eta_\vartheta}{2}\vartheta^2 + \frac{b_m \eta_p}{2}p^2 + \frac{1}{2}\|d_1\|^2$$

$$+ \frac{1}{2\mu} + D_\epsilon. \quad (58)$$

To ensure the desired objective, the following inequalities for the corresponding design parameters should be held:

$$q \geq 2\lambda_{\max}(\bar{P})r_\epsilon + \frac{\lambda_{\max}(\bar{P})}{\lambda_{\min}(\bar{P})},$$

$$c_1 \geq q b_m + \frac{q^2}{2} + \frac{1}{2} + r_\epsilon,$$

$$c_2 \geq \frac{q b_m}{2} + q + r_\epsilon,$$

$$c_i \geq \frac{3q}{2} + r_\epsilon, \quad 3 \leq i \leq \rho - 1$$

$$c_\rho \geq \frac{q}{2} + r_\epsilon,$$

$$\frac{1}{\tau_2} \geq \frac{q b_m}{2} + \frac{M_2^2}{2\iota} + r_\epsilon,$$

$$\frac{1}{\tau_{i+1}} \geq \frac{q}{2} + \frac{M_{i+1}^2}{2\iota} + r_\epsilon, \quad 2 \leq i \leq \rho - 1,$$

$$\eta_\vartheta \geq \frac{2r_\epsilon}{\gamma_\vartheta},$$

$$\eta_p \geq \frac{2r_\epsilon}{\gamma_p}. \quad (59)$$

Moreover, from (32), rewrite $\vartheta q^{1-2\rho} \left(\varrho (y) - \bar{\varrho} (y) S_1^2 \right)$ as $\vartheta q^{1-2\rho} \varrho (y) \left(1 - \frac{2}{1 + \frac{\delta}{S_1^2}} \right)$. Obviously, if $S_1 > \sqrt{\delta}$, then $\vartheta q^{1-2\rho} \left(\varrho (y) - \bar{\varrho} (y) S_1^2 \right) < 0$, while on the other hand, if $S_1 \leq \sqrt{\delta}$, the term $\vartheta q^{1-2\rho} \left(\varrho (y) - \bar{\varrho} (y) S_1^2 \right)$ has an upper bound, which we denote as $\bar{\vartheta}$.

Viewing (55)-(59), the time derivative of (46) satisfies

$$\dot{V} \leq -2r_\epsilon V + Q \qquad (60)$$

with Q defined by (51). Let r_ϵ satisfy

$$r_\epsilon \geq \frac{Q}{2C_2}. \qquad (61)$$

Then if $V = C_2$, we have $\dot{V} \leq 0$. Hence, given $V (0) \leq C_2$, one can obtain that $V (t) \leq C_2$, $t \geq 0$. Further, according to (60), we obtain

$$V (t) \leq e^{-2r_\epsilon t} V (0) + \frac{Q}{2r_\epsilon} \left(1 - e^{2r_\epsilon t} \right), \quad t \geq 0. \qquad (62)$$

Hence,

$$\lim_{t \to \infty} V (t) \leq \frac{Q}{2r_\epsilon}, \qquad (63)$$

and

$$\lim_{t \to \infty} |S_1| \leq \lim_{t \to \infty} \sqrt{2V} \leq \sqrt{\frac{Q}{r_\epsilon}}, \qquad (64)$$

from which we note steady state error could be made arbitrarily small under sufficiently large r_ϵ.

From the above analysis, V is proven to be bounded, and then $S_1, \ldots, S_\rho$, $y_2, \ldots, y_\rho$, $\tilde{\vartheta}$, $\tilde{p}$, and ϵ are bounded. Moreover, y_d is bounded due to Assumption 3, which implies y is bounded. What is more, the nonlinear smooth functions $f_{i,j} (y)$, $g_i (y)$, $\varrho (y)$ and $\bar{\varrho} (y)$ are bounded, and then ω, ξ_i and ψ are bounded. Note that

$$x = \Psi \omega + \sum_{i=1}^{r} \theta_i \Psi \xi_i + b_m \Psi \zeta + d (t) + \epsilon, \qquad (65)$$

and Assumption 1, which implies that ζ_1 is bounded. Viewing (31) and (35), we have z_2 is bounded. Then it can be checked from (36) that ζ_2 is bounded. Similarly, we obtain that $z_3, \ldots, z_\rho$, $\zeta_3, \ldots, \zeta_\rho$ are bounded due to (36)-(40). From (65), we can know that $x_2, \ldots, x_\rho$ are bounded. With Assumption 4, $x_{\rho+1}$ is bounded. Furthermore, $\zeta_{\rho+1}$ is bounded. Then, according to (42), we can get bounded v. Also, from (11), we can get bounded ζ. Hence, we have shown that uniformly ultimate boundedness is guaranteed for all signals in the system. ∎

Until now, we only discussed steady state tracking performance. Transient state performance also should be considered. What follows is a conclusion aims to solve the problem.

Theorem 2: Consider the system discussed in Theorem 1. Select initial values of high-gain K-filters (11)-(13), update law (33), (34) to be zero and let $\omega_1 (0) = y (0)$, $y_d (0) = y (0)$. Then $V (t)$ can satisfy

$$V (t) \leq \frac{Q}{2r_\epsilon} + \frac{\lambda_{\max} \left(\bar{P} \right)}{q^2} \| \epsilon (0) \|^2, \qquad (66)$$

and $\mathcal{L}_\infty$ tracking performance satisfies

$$\| S_1 \|_\infty \leq \sqrt{\frac{Q}{r_\epsilon} + \frac{2}{q^2} \lambda_{\max} \left(\bar{P} \right) \| \epsilon (0) \|^2}. \qquad (67)$$

Proof: Let $y_d (0) = y (0)$, and by (28),

$$S_1 (0) = y (0) - y_d (0) = 0. \qquad (68)$$

We set the initial conditions of (33), (34) to ensure $\hat{\vartheta} (0) = 0$, $\hat{p} (0) = 0$. From (31), $\bar{\zeta}_2 (0) = 0$ can be obtained. Hence, $z_2 (0) = 0$, $\dot{z}_2 (0) = 0$ are achieved by considering (35). Similarly, in light of (36)-(40), we can deduce

$$S_i (0) = 0, \quad 2 \leq i \leq \rho. \qquad (69)$$

Moreover, together with (35)-(39) and (43), we have

$$y_{i+1} (0) = 0, \quad 1 \leq i \leq \rho - 1. \qquad (70)$$

Noting (46) and (59), and from (69) and (70), we can get

$$\begin{aligned} V (0) &= \frac{1}{2\gamma_\vartheta} \vartheta^2 + \frac{b_m}{2\gamma_p} p^2 + V_\epsilon (0) \\ &\leq \frac{1}{2r_\epsilon} \left(\frac{\eta_\vartheta}{2} \vartheta^2 + \frac{b_m \eta_p}{2} p^2 \right) + V_\epsilon (0) \\ &\leq \frac{Q}{2r_\epsilon} + V_\epsilon (0). \end{aligned} \qquad (71)$$

Now, substituting (71) into (62) yields

$$\begin{aligned} V (t) &\leq \frac{Q}{2r_\epsilon} + \epsilon^T (0) P \epsilon (0) e^{-2r_\epsilon t} \\ &\leq \frac{Q}{2r_\epsilon} + \epsilon^T (0) P \epsilon (0) \\ &\leq \frac{Q}{2r_\epsilon} + \lambda_{\max} \left(\bar{P} \right) \left\| \Psi^{-1} \epsilon (0) \right\|^2. \end{aligned} \qquad (72)$$

The initial conditions of the K-filters imply that $\xi_i (0) = 0 \, (1 \leq i \leq r)$, $\zeta (0) = 0$. We can deduce that $\hat{x}_1 (0) = \omega_1 (0)$. Considering $\epsilon = x - \hat{x}$ and $\omega_1 (0) = y (0) = x_1 (0)$, it can be obtained that $\epsilon_1 (0) = 0$. Then for $q \geq 1$, we have

$$\left\| \Psi^{-1} \epsilon (0) \right\| \leq \frac{1}{q} \| \epsilon (0) \|. \qquad (73)$$

Thus, we get the upper bound of $V (t)$ as follows

$$V (t) \leq \frac{Q}{2r_\epsilon} + \frac{\lambda_{\max} \left(\bar{P} \right)}{q^2} \| \epsilon (0) \|^2, \qquad (74)$$

which implies that $\mathcal{L}_\infty$ tracking performance

$$\| S_1 \|_\infty \leq \left\| \sqrt{2V} \right\|_\infty \leq \sqrt{\frac{Q}{r_\epsilon} + \frac{2}{q^2} \lambda_{\max} \left(\bar{P} \right) \| \epsilon (0) \|^2}. \qquad (75)$$

Therefore, by tunning the design parameters in (59), we can make the $\mathcal{L}_\infty$ performance of tracking error,

namely $\|S_1\|_\infty$, converge to arbitrarily small region of the equilibrium for all initial condition. ∎

Remark 3: From (75), the high gain q of the high gain K-filters is crucial to guarantee the transient tracking performance. And the common K-filters only ensure the steady state tracking performance, cannot obtain the transient tracking performance.

Remark 4: In the proposed controller, design parameters q, $c_i, i = 1, \ldots, \rho$, $\tau_{i+1}, i = 1, \ldots, \rho - 1$, η_ϑ, η_p, and adaptive gains γ_ϑ, γ_p should be adjusted for a good transient tracking performance, then tracking error in (50) and (67) can be reduced. Since there are no explicit guideline to quantify the relationship of these parameters, some suggestions are given for obtaining good tracking performance.

- By (50), increasing the value of r_ϵ can reduce the tracking error. Further, in view of (59), increasing the values of q, $c_i, i = 1, \ldots, \rho$, η_ϑ, η_p and adaptive gains γ_ϑ, γ_p and decreasing the values $\tau_{i+1}, i = 1, \ldots, \rho - 1$ is able to make r_ϵ larger. However, the increase of η_ϑ, η_p will cause the increase of Q in (51). Based on the above observation, we can fix η_ϑ, η_p first and Q is independent of r_ϵ. By this means, r_ϵ can be arbitrarily increased without increasing Q and the maximum overshoot of tracking error is arbitrarily small.

- Theoretically, we can arbitrarily increase q, $c_i, i = 1 \cdots \rho$, , γ_ϑ, γ_p, and reduce $\tau_{i+1}, i = 1 \cdots \rho - 1$ to increase r_ϵ and make the tracking error arbitrarily small. From (11)-(13), (30)-(35) and (39)-(42), however, the tuning process may result in large amplitude of control signal. There is no effective method to relieve these phenomena, which would be an interesting problem in the future.

V. SIMULATION EXAMPLES

Two examples are provided to verify effectiveness of developed design.

Example 1: Consider the system with external disturbance:

$$\dot{x}_1 = x_2 + y + y^2 + 0.1 \cos t, \quad x_1(0) = 0.1,$$
$$\dot{x}_2 = 0.5y + \left(1 + y^2\right) + u + 0.3 \sin t, \quad x_2(0) = 0,$$
$$y = x_1, \tag{76}$$

which implies $n = \rho = 2$, $r = 1$,

$$f_0(y) = \begin{bmatrix} y \\ 0.5y \end{bmatrix}, \quad f_1(y) = \begin{bmatrix} y^2 \\ 1 + y^2 \end{bmatrix},$$

$$G(y) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad d(t) = \begin{bmatrix} 0.1 \cos t \\ 0.3 \sin t \end{bmatrix}, \tag{77}$$

and dead-zone parameters are $m = 2$, $h_l = 0.5$, $h_r = 0.3$. The reference trajectory is $y_d(= \sin(2t) + \cos(t))$, $y_d(0) = 0.1$. According to (11)-(13), the high-gain K-filters characterized by

$$\dot{\zeta} = qA_0\zeta + \Psi^{-1}G(y)v, \quad \zeta(0) = 0, \tag{78}$$
$$\dot{\omega} = qA_0\omega + qky + \Psi^{-1}f_0(y), \quad \omega(0) = 0, \tag{79}$$
$$\dot{\xi}_1 = qA_0\xi_i + \Psi^{-1}f_1(y), \quad \xi_1(0) = 0, \tag{80}$$
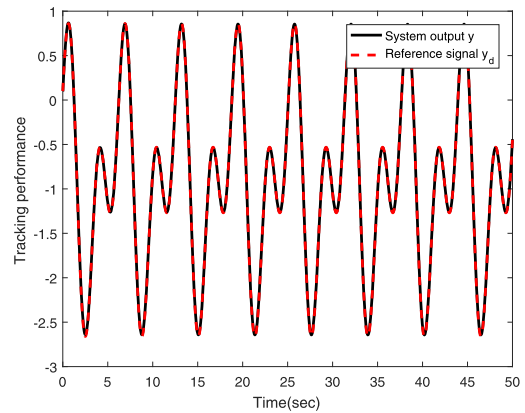

FIGURE 1. Tracking performance for Example 1.
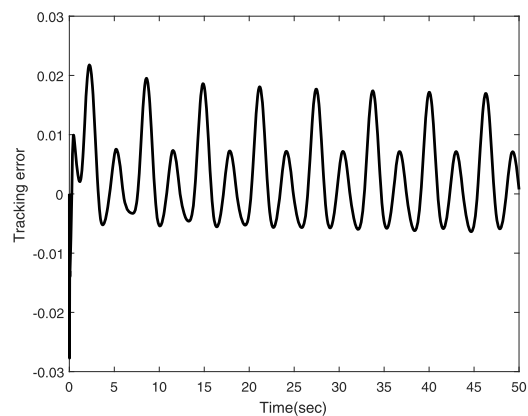

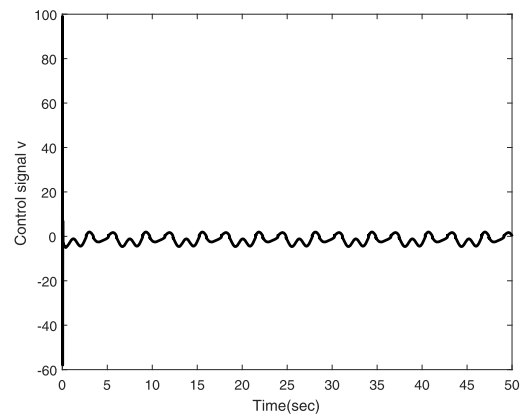**FIGURE 2.** Tracking error for Example 1.

FIGURE 3. Control signal v for Example 1.

with $q = 4$, and

$$k = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad A_0 = \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix}, \quad \Psi^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1/4 \end{bmatrix}. \tag{81}$$

The design parameters are $c_1 = 120$, $c_2 = 10$, $\eta_\vartheta = 0.00005$, $\eta_p = 0.00001$, $\tau_2 = 0.01$, $\delta = 0.1$, $\mu = 1$, $\varrho(y) = 1.6y^2$ and the adaptive gains $\gamma_\vartheta = 80$, $\gamma_p = 30$. The simulation results in Figs. 1-3 demonstrate our developed scheme. Fig. 1 and Fig. 2 depict the tracking performance and tracking error. Fig. 3 illustrate control signal v. Then we
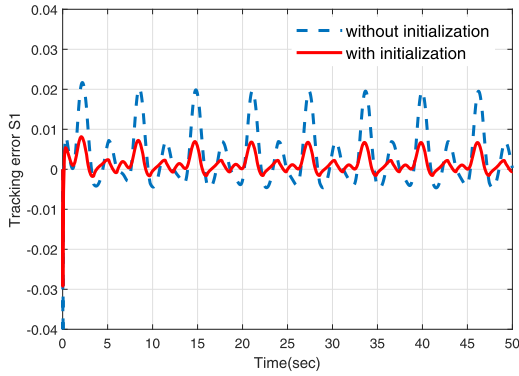
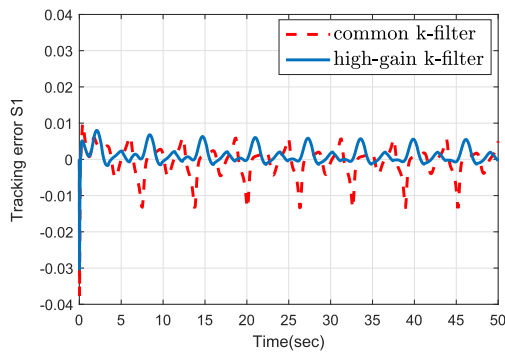**FIGURE 4.** Tracking error obtained by initialization for Example 1.

FIGURE 5. Tracking errors by common and high gain K-filters for Example 1.

increase the values of q, c_1, c_2 to 10, 200, 100 respectively and decrease the value of τ_2 to 0.005. The simulation results for tracking error is given by Fig. 4. It is clear that $\mathcal{L}_\infty$ tracking performance can be guaranteed by the control scheme with initialization and the correctness of the Theorem 2 is verified. Moreover, in contrast to the common K-filters in [11], the high gain K-filters exhibit better tracking performance in Fig. 5.

Example 2: Consider single-link robot motion dynamic model [21]:

$$M\ddot{q}_\theta + 0.5m_0gl\sin(q_\theta) = \tau_q + \eta_0(t),$$
$$y = q_\theta, \qquad (82)$$

some detailed description of the model can be referred to [21]. $\eta_0(t)$ is the unknown time-varying disturbance. Then let $x_1 = q_\theta$, $x_2 = \dot{q}_\theta$ and (82) can be rewritten as:

$$\dot{x}_1 = x_2, \quad x_1(0) = 0.1,$$
$$\dot{x}_2 = -\frac{m_0gl}{2M}\sin(x_1) + \frac{1}{M}u + \eta_M, \quad x_2(0) = 0,$$
$$y = x_1, \qquad (83)$$

which implies $\eta_M = \frac{\eta_0(t)}{M}$, and

$$f_0(y) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad f_1(y) = \begin{bmatrix} 0 \\ -\sin(y) \end{bmatrix},$$
$$G(y) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad d(t) = \begin{bmatrix} 0 \\ \sin(0.5t) + 0.2\cos(t) \end{bmatrix}. \qquad (84)$$

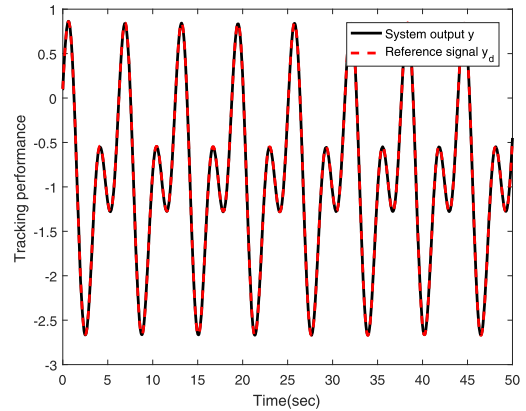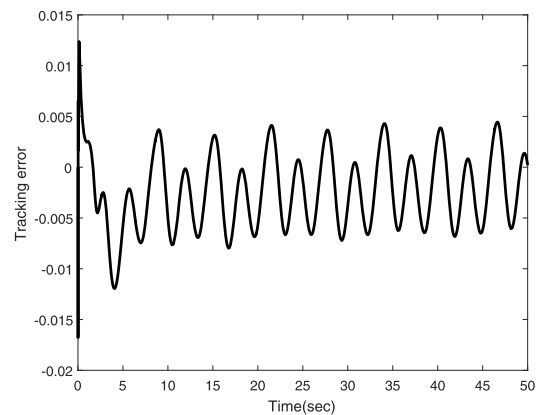**FIGURE 6.** Tracking performance for Example 2.
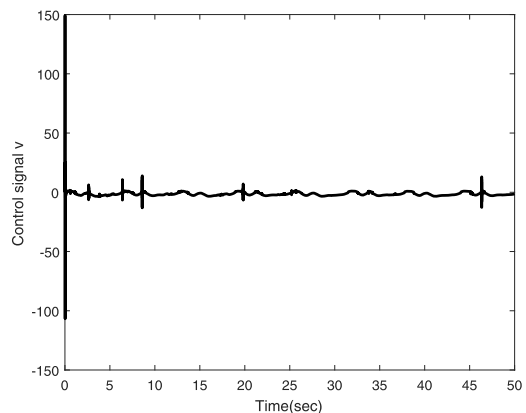

FIGURE 7. Tracking error for Example 2.

FIGURE 8. Control signal v for Example 2.

$M = 0.5$ kg/m^2, $g = 9.8$ m/s^2, $m_0 = 1.0$ kg, $l = 1.0$ m. The parameters of dead-zone, the reference trajectory and the high-gain K-filters parameters are the same as those of example 1. And design parameters are set as $c_1 = 200$, $c_2 = 1$, $\eta_\vartheta = 0.00005$, $\eta_p = 0.00001$, $\tau_2 = 0.005$, $\delta = 0.1$, $\mu = 1$, $\varrho(y) = 1.6y^2$. The adaptive gains $\gamma_\vartheta = 80$, $\gamma_p = 30$. Figs. 6-8 show the validity of the developed scheme and satisfactory control performance is obtained.

VI. CONCLUSION

An adaptive DSC strategy with high-gain K-filters is proposed to cancel undesirable influence from input dead-zone in this paper. Also, it can be shown that design complexity of controller is greatly reduced due to the first order filters and simplified adaptive update laws. Moreover, the explicit function of $\mathcal{L}_\infty$ norm of tracking performance is given. However, our control design only can guarantee stability of closed-loop system in the sense of semi-global stability, and thus more attention should be paid on this problem in the further research.

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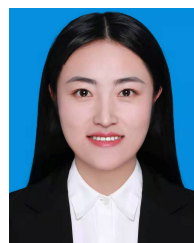
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