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# A Multi-Service Model of Resources With the Neighboring Choice of Allocation Units

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**ABSTRACT** This article discusses a discretized model of resources to which a mixture of multi-service traffic streams is offered. In the model the connections of individual traffic classes that require a large number of allocation units are always executed using adjacent allocation units. The necessity of occupying the adjacent allocation units is typical, for example, for elastic optical networks, in which these units are called frequency slots. The article proposes a new occupancy distribution specifically designed for these systems. The new occupancy distribution is based on an approximation of the service process by a reversible Markov process. Particular attention is given to the method for calculating the so-called conditional transition probability of the transitions between neighboring states in the Markov process. The analytical results of the modeling are compared with the results of simulations of corresponding network systems.

**INDEX TERMS** Blocking probability, elastic optical network, frequency slot unit, Markov process, multi-service resources.

## I. INTRODUCTION AND RELATED WORK

Present-day optical transport networks make use of DWDM (dense wavelength division multiplexing) transmission with a throughput of 10, 40, 100, or 400 Gbps per wavelength [1]–[7]. Elastic optical networks (EONs) provide the possibility of effective use of the throughput available in optics. Thanks to the application of the elastic DWDM grid (flexible grid), specified in the ITU-T G.694 standard [8], it is possible to allocate an adequate size-dependent optical spectrum to particular connections depending on the required transmission speed. This spectrum is expressed in so-called frequency slot units (FSUs) [3], [9], [10]. The actual size of the allocated spectrum, as well as the number of FSUs, is influenced not only by the demanded transmission speed, but also by the distance, the path quality, and the scheme for the applied modulation [3], [11]–[15].

Demands in EONs can be divided into several classes on the basis of the requirements related to the spectrum. In the EON architecture, each class demands an appropriate number of neighboring (adjacent) FSUs according to the required size

of the optical spectrum/frequency slot for a given connection requesting a specific bitrate. For example, assuming a slot width of 12.5 GHz, the demand for 40 Gbps (100 Gbps, 400 Gbps) requires an optical spectrum that consists of 2 FSUs (4 FSUs, 16 FSUs) using the quadrature phase shift keying (QPSK) modulation [16]. When the modulation format is changed into 16-QAM, then the requirements for the number of FSUs will be reduced by half for all the three demands.

To use the resources of EONs effectively, it is necessary to make use of optical switching networks that are capable of converting and transmitting optical signals in different lengths of light waves. One of the most common switching networks extensively employed in EONs is the three-stage W-S-W (wavelength-space-wavelength) switching network [9], [17]–[20]. The switches of the first and the third stage allow both the frequency slot (wavelength) and the output of a switch (optical fiber) to be changed. The switches of the middle stage, in turn, allow a change in the output link only. In the case of a switch of the middle stage, a change in the frequency of the channel is not possible. The principal limitation of the EONs and nodes under discussion that has a direct influence on the efficiency and performance of the

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system is the number of neighboring slots required for the execution of a given connection. This is one of the constraints defined in the routing and spectrum assignment (RSA) problem [21], [22].

This requirement also significantly influences any analytical methods for modeling such systems. The multi-service mathematical models of telecommunications resources (links) that have been used until now do not take into consideration the necessity of allocating neighboring transmission units. These models take into account only the total number of occupied/free slots in the resources, e.g., [23], [24]. The first attempt to take into consideration the limitations in the occupation of frequency slots was undertaken in [25] and [20]. The aforementioned studies apply the concept of virtual resources that define the possibility of setting up connections in a number of adjacent FSUs of the connection path in an EON. Such a model, however, cannot be applied to the analysis of the occupancy distribution of a single link. This article proposes a model of resources in which connections are always executed in the adjacent allocation units. According to our knowledge, it is the first analytical model that allows the occupancy distribution in a single link of EONs to be determined. Then, on the basis of the occupancy distribution, it is possible to determine the blocking probability for calls of particular traffic classes. The proposed analytical model of a single link will also be used in the future to increase the accuracy of existing methods for determining the blocking probability in EON nodes [20].

The remaining part of the article is structured as follows. Section II proposes a general model of a single optical link with the introduced requirement concerning the necessity of the execution of a connection with the use of adjacent frequency slots. It also presents the structures of offered traffic. Section III presents the most important element of the proposed new method for determining the occupancy distribution, i.e., the method for determining the conditional transition probabilities for the transitions between the neighboring states of the process that describes the service process in the system under consideration. The results of the analytical calculations are then compared with simulation data in Section IV. Section V sums up the article.

## II. OCCUPANCY DISTRIBUTION

Let us designate the resources in which multi-slot connections are always executed by using adjacent FSUs by  $R_{SEQ}$ . The assumption is that the required resource set of neighboring FSUs necessary for a given connection to be executed is randomly chosen from all possible combinations of allocations of this set in the resources  $R_{SEQ}$ . Our further assumption is that the resources  $R_{SEQ}$ , with their capacity also expressed in FSUs, are offered a finite number of Poisson call classes. Each call of a given class demands an appropriate integer number of FSUs to execute a call. The service time for a call in  $R_{SEQ}$  has an exponential pattern. The traffic classes offered to the system under consideration are characterized by the following parameters:

- $M$  – the number of offered call classes in the system under consideration,
- $A_i$  – the intensity of traffic of class  $i$ :

$$A_i = \frac{\lambda_i}{\mu_i}, \quad (1)$$

- $\lambda_i$  – the intensity of a call stream of class  $i$  ( $1 \leq i \leq M$ ),
- $\mu_i$  – the intensity of call service for calls of class  $i$ ,
- $t_i$  – the demanded number of FSUs necessary for a connection of class  $i$  to be executed,
- $V$  – the resource capacity  $R_{SEQ}$  expressed in FSUs.

To determine the occupancy distribution in  $R_{SEQ}$ , in which connections are always executed in adjacent FSUs, the general model of resources with a state-dependent service process will be applied [26]–[30]:

$$n [P(n)]_V = \sum_{i=1}^M A_i t_i \sigma_i(n - t_i) [P(n - t_i)]_V, \quad (2)$$

where:

- $[P(n)]_V$  – occupancy probability of  $n$  FSUs in  $R_{SEQ}$ ,
- $\sigma_i(n)$  – the so-called conditional transition probability, which determines the influence of different kinds of limitations (in this case, the necessity to provide the required neighborhood of occupied slots) on the call stream for particular occupancy states  $n$ ; the systems for which  $\exists_{0 \leq n \leq V} \sigma_i(n) \neq 1$  are called state-dependent systems.

The distribution (2) results from the approximation of the service process in a state-dependent system by a Markovian reversible process. The model is characterized by high accuracy for a large number of systems and mechanisms used in practice, e.g., systems with multi-rate traffic [26], [27], non-full-availability systems [25], [31], limited-availability systems [29], [32], systems with different types of traffic streams [30], [33], [34], systems with threshold mechanisms [28], [35]–[37], and overflow systems [38]–[41]. The parameter  $\sigma_i(n)$  expresses the influence of state dependence on the intensity of call streams between the neighboring states in the Markov process; i.e., it defines which fraction of the total call stream the Markov process can transfer from a given younger state to a neighboring older state. Therefore, the probability  $1 - \sigma_i(n)$  is the conditional blocking probability of the resources for a call stream of class  $i$  in state  $n$ . The total blocking probability for calls of class  $i$  in the resources  $R_{SEQ}$  can then be determined by the following equation:

$$E_i = \sum_{n=1}^V [1 - \sigma_i(n)] [P(n)]_V. \quad (3)$$

To determine the occupancy distribution in  $R_{SEQ}$ , it is necessary to know the probability  $\sigma_i(n)$ , which will be determined in a combinatorial way in the next section.

## III. CONDITIONAL TRANSITION PROBABILITY

Let us now consider a state  $n$  of the service process in the resources  $R_{SEQ}$  with a capacity of  $V$  FSUs. In a given

occupancy state of the resources, there are  $w = V - n$  free FSUs. The probability  $\sigma_i(n)$  can be defined as the probability of an event that in the given resources there are  $t_i$  free, located next to one another, adjacent FSUs to be found. The problem is therefore reduced to determining all possible arrangements in which  $t_i$  free FSUs (from among all the  $w$  free FSUs in the resources) happen to be next to one another. Note that the number of all possible arrangements of free  $w$  FSUs in the resources with a capacity of  $V$  FSUs is equal to

$$R_{\text{tot}} = \binom{V}{w}. \tag{4}$$

To make the further discussion easier, we will assign the binary value “1” to a free FSU, whereas to a busy (occupied) FSU we will assign the binary value “0”. In this way, we will reduce our considerations to an analysis of binary sequences of length  $V$ . Now, by analyzing the occupancy of the resources in a given state  $n$ , we will be considering  $k$  sets (sequences) of binary ones separated by  $k - 1$  sets of binary zeros. Further on in the article, the sets of “0s” will be called separating sets, assuming that each separating set includes at least one “0”, whereas the sets of “1s” will be called access sets. In our considerations, we are interested in access sets with a length of at least  $t_i$ , which is tantamount to the possibility of setting up a connection of class  $i$  in  $R_{\text{SEQ}}$ .

**A. THE NUMBER OF ACCESS SETS**

By analyzing all possible arrangements of free FSUs for class  $i$ , our assumption is that the maximum length of an access set is  $t_i$  elements. The adoption of this assumption at this stage makes further combinatorial considerations significantly easier.

The largest number of possible access sets occurs when they are composed of only one element, i.e., one “1”:

$$k_{\text{max}} = w. \tag{5}$$

The smallest number of possible access sets occurs when they are composed of only  $t_i$  elements:

$$k_{\text{min}} = \left\lceil \frac{w}{t_i} \right\rceil. \tag{6}$$

In (6), the supremum is assumed to be on the boundary. If the quotient  $w/t_i$  is a real number, then there are  $\lfloor w/t_i \rfloor$  access sets with a length  $t_i$  and one additional access set with a length  $w - \lfloor w/t_i \rfloor$ .

**B. ARRANGEMENTS IN SEPARATING SETS**

If there are  $k$  access sets, there should be  $k - 1$  separating sets, with each of them including at least one “0” element. Thus, the binary sequence has  $V - w$  “0s”. Therefore, the remaining “0s”, of which there are:

$$p = (V - w) - (k - 1) = V - w - k + 1, \tag{7}$$

can be arranged in

$$u = (k - 1) + 2 = k + 1 \tag{8}$$

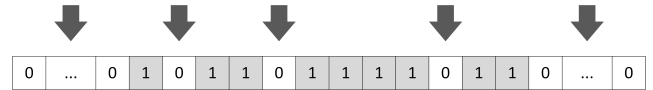


FIGURE 1. Sets in which zeros can be distributed.

sets. The number 2 in (8) indicates the potential occurrence of sets of “0s” at the beginning and end of the binary sequence. These sets are not separating sets, even though they can include zeros. To illustrate the problem under discussion, Fig. 1 shows a binary sequence with four access sets and three separating sets, each of them including one zero. The arrows indicate three separating sets and two additional sets of zeros (at the beginning and end of the binary sequence) that can be completed with zeros. By applying the general equation for the number of arrangements of  $p$  elements in  $u$  sets,

$$R_u(p) = \binom{u + p - 1}{p}, \tag{9}$$

we can determine the number of arrangements  $p = V - w - k + 1$  (7) of “0s” in  $u = k + 1$  (8) sets:

$$R^{“0”, k+1}(V - w - k + 1) = \binom{V - w + 1}{k}. \tag{10}$$

**C. ARRANGEMENTS IN ACCESS SETS**

The number of arrangements of  $w$  elements in  $k$  sets with a maximum capacity of  $T$ , such that in each set there exists at least  $z$  elements, is [29]:

$$F(w, k, T, z) = \sum_{j=0}^{\lfloor \frac{w-kz}{T-z+1} \rfloor} (-1)^j \binom{k}{j} \binom{w - k(z-1) - 1 - j(T - z + 1)}{k-1}. \tag{11}$$

In the case of the binary sequence under consideration, the assumption that it includes  $k$  access sets means that in each of these sets there is at least one element “1”, whereas the maximum number of binary ones in this set is  $t_i$ . Then, the problem boils down to arranging  $w$  ones in  $k$  access sets, each with a maximum capacity of  $t_i$  and with the assumption that in each set there is always at least one “1” element. Assuming then that  $z = 1$  and  $T = t_i$ , on the basis of (11) we get:

$$F(w, k, t_i, 1) = \sum_{j=0}^{\lfloor \frac{w-k}{t_i} \rfloor} (-1)^j \binom{k}{j} \binom{w - jt_i - 1}{k-1}. \tag{12}$$

Equation (12) determines the arrangements in which there can be  $t_i$  “1s” in one access set at the maximum. Such arrangements also include arrangements that do not have  $t_i$  elements in any access set. The number of arrangements in which there always are, at least in one set,  $t_i$  “1s” will be determined by the following equation:

$$R^{“1”, k}(w, t_i) = F(w, k, t_i, 1) - F(w, k, t_i - 1, 1), \tag{13}$$

where  $F(w, k, t_i - 1, 1)$  determines the number of arrangements in  $k$  access sets such that in each set there can be  $t_i - 1$  ones at the maximum:

$$F(w, k, t_i - 1, 1) = \sum_{j=0}^{\lfloor \frac{w-k}{t_i-1} \rfloor} (-1)^j \binom{k}{j} \binom{w-j(t_i-1)-1}{k-1}. \quad (14)$$

**D. THE TOTAL NUMBER OF ARRANGEMENTS**

For the required number of binary ones  $w$  and the required number of access sets  $k$ , the total number of arrangements, calculated under the assumption that the maximum length of the access set is equal to  $t_i$  elements, is:

$$R_k(w, t_i) = R_{\cdot 1 \dots k}(w, t_i) \times R_{\cdot 0 \dots k+1}(V - w - k + 1). \quad (15)$$

Taking (10) and (13) into consideration, as well as the limits on the variation in parameter  $k$  ((5) and (6)), we get

$$R(w, t_i) = \sum_{k=\lceil \frac{w}{t_i} \rceil}^w \binom{V-w+1}{k} F_{\text{sub},1}(w, k, t_i, 1), \quad (16)$$

where

$$F_{\text{sub},1}(w, k, t_i, 1) = F(w, k, t_i, 1) - F(w, k, t_i - 1, 1). \quad (17)$$

**E. DETERMINATION OF THE CONDITIONAL TRANSITION PROBABILITY**

On the basis of (4) and (16), the probability of the event that in a given occupancy state  $n = V - w$  FSUs  $t_i$  free FSUs, arranged next to one another, can be found is

$$h(n, t_i) = \frac{R(V - n, t_i)}{\binom{V-n}{V-n}} = \frac{\sum_{k=\lceil \frac{V-n}{t_i} \rceil}^{V-n} \binom{n+1}{k} F_{\text{sub},2}(V - n, k, t_i, 1)}{\binom{V-n}{V-n}}, \quad (18)$$

where

$$F_{\text{sub},2}(V - n, k, t_i, 1) = F(V - n, k, t_i, 1) - F(V - n, k, t_i - 1, 1). \quad (19)$$

Taking into consideration that a connection with a demand of  $t_i$  FSUs can be executed if there exists an access set that is composed of  $t_i$  or  $t_i + 1 \dots$  or  $w$  free FSUs, we eventually obtain

$$\sigma_i(n) = \sum_{s=t_i}^{V-n} h(n, s). \quad (20)$$

**IV. NUMERICAL EXAMPLES**

The proposed model of EON resources, which requires the allocation of  $t_i$  adjacent FSUs to service a call of class  $i$ , is an approximate model. To determine the accuracy of the proposed distribution and the possibility of applying the model to model and optimize EONs, the results of the analytical

calculations were compared with the results of simulations for a selected number of resources (Fig. 1). The channel width is equal to the product of the number of requested FSUs and the basic channel width of 12.5 GHz. Two example systems, whose parameters are given below, were chosen for modeling:

**System 1**

- Capacity of EON link:  $V = 32$  FSUs,
- Structure of offered traffic:  $M = 3, t_1 = 1$  FSU,  $\mu_1 = 1, t_2 = 2$  FSUs,  $\mu_2 = 1, t_3 = 3$  FSUs,  $\mu_3 = 1$ .

**System 2**

- Capacity of EON link:  $V = 40$  FSUs,
- Structure of offered traffic:  $M = 4, t_1 = 1$  FSU,  $\mu_1 = 1, t_2 = 2$  FSUs,  $\mu_2 = 1, t_3 = 3$  FSUs,  $\mu_3 = 1, t_4 = 5$  FSUs,  $\mu_4 = 1$ .

**System 3**

- Capacity of EON link:  $V = 320$  FSUs,
- Structure of offered traffic:  $M = 3, t_1 = 1$  FSU,  $\mu_1 = 1, t_2 = 20$  FSUs,  $\mu_2 = 1, t_3 = 30$  FSUs,  $\mu_3 = 1$ .

The results of the simulation experiments are presented in the form of points in Figs. 2 and 5 with confidence intervals (22), which were determined on the basis of the Student's t-distribution (with 95% confidence level) for five series, for two values of traffic  $a$  offered to a single FSU. Traffic  $a$  offered to a single FSU can be determined on the basis of the following equation:

$$a = \frac{\sum_{i=1}^M t_i \lambda_i / \mu_i}{V}. \quad (21)$$

The duration of each of the series was determined on the basis of the time required to generate 10,000,000 calls of the least active class. In each of the cases, the confidence interval did not exceed 5% of the average value of the result of the simulation experiment. The results of the analytical calculations are shown in the graphs in the form of solid lines.

The confidence intervals are determined in the following way:

$$\left( \bar{X} - t_\alpha \frac{\sigma}{\sqrt{r}}; \bar{X} + t_\alpha \frac{\sigma}{\sqrt{r}} \right), \quad (22)$$

where  $\bar{X}$  is the arithmetic mean calculated from  $r$  results (simulation runs) and  $t_\alpha$  is the value of Student's t-distribution for  $r - 1$  degrees of freedom. The parameter  $\sigma$  which determines the standard deviation, is determined using the following equation:

$$\sigma^2 = \frac{1}{r-1} \sum_{s=1}^r x_s^2 - \frac{r}{r-1} \bar{X}^2, \quad (23)$$

where  $x_s$  is the result obtained in the  $s$ -th course of the simulation.

Figs. 2 and 4 show the values of the occupancy distributions for the systems (Systems 1 and 2) under investigation in relation to the  $n$  occupancy state of the system, for  $a = 0.7$  and  $a = 0.9$  erlangs, using a logarithmic scale. Fig. 6 also shows the dependence of the occupancy distribution value

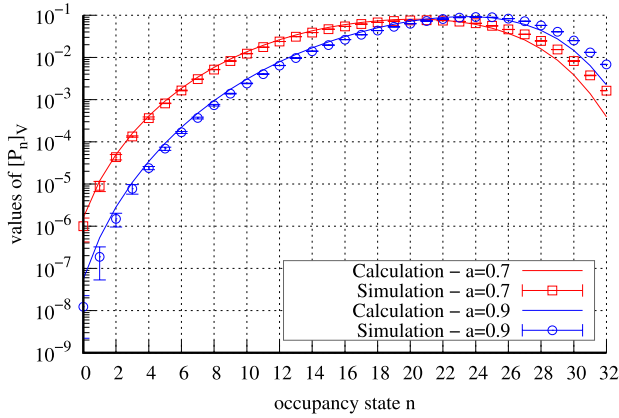


FIGURE 2. Occupancy distribution in System 1.

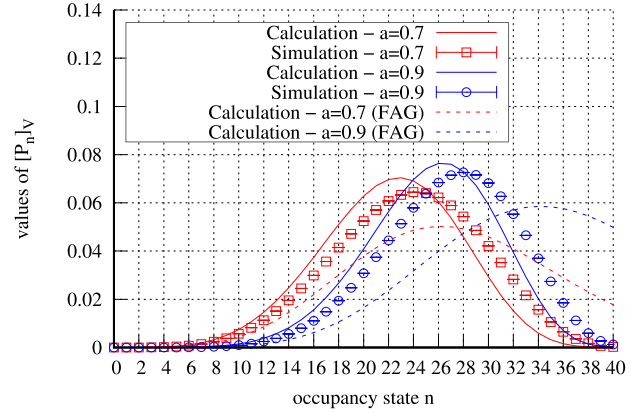


FIGURE 5. Occupancy distribution in System 2 (comparison with FAG).

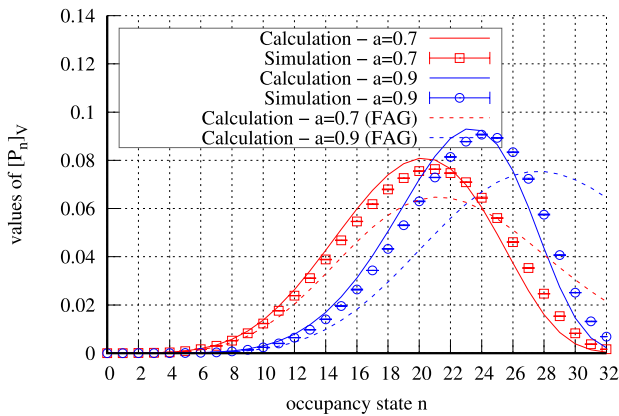


FIGURE 3. Occupancy distribution in System 1 (comparison with FAG).

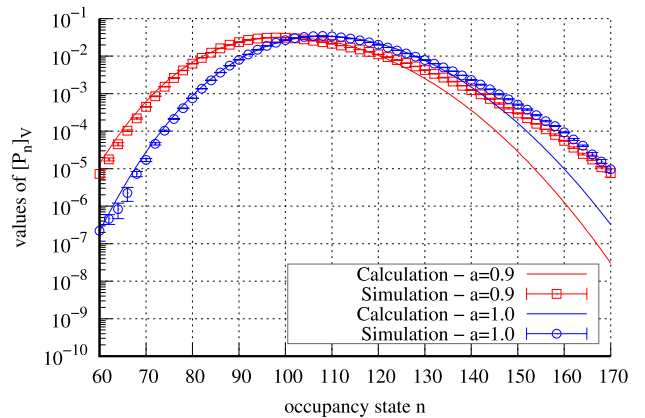


FIGURE 6. Occupancy distribution in System 3.

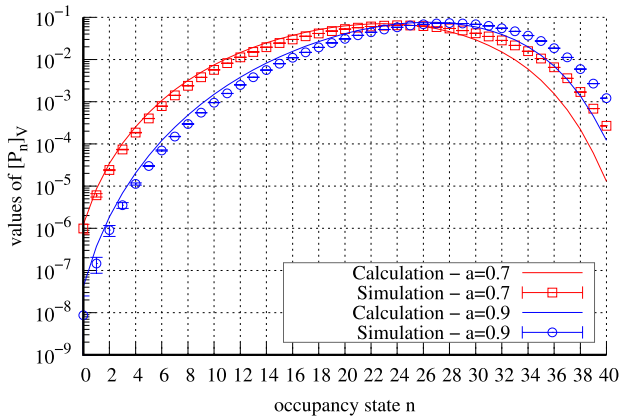


FIGURE 4. Occupancy distribution in System 2.

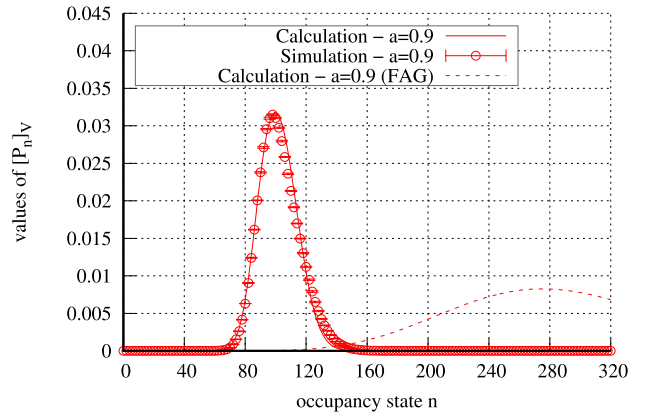


FIGURE 7. Occupancy distribution in System 3 for  $a = 0.9$  Erl (comparison with FAG).

on the occupancy state  $n$ , for  $a = 0.9$  and  $a = 1.0$  erlang, and for a system with a much larger capacity (System 3). The obtained results (Figs. 2, 4 and 6) show high accuracy of the developed method for determining the occupancy distribution with the requirement of the distribution of adjacent frequency slots.

To show the influence of the proposed method for determining the conditional transition probability on the accuracy of assessment of the occupancy distribution in EONs,

Figs. 3, 5, 7, and 8 additionally show a comparison of the obtained results with the results based on a model of resources that does not include the requirement of the arrangement of adjacent resources. The most widely used model of this type is the Kaufman-Roberts model [42], [43], i.e., a model of a single link with a complete sharing policy, also known as the full-availability group (FAG). In this model, whether a call is admitted for service or not is decided by the number of unoccupied FSUs and not by their arrangement relative to

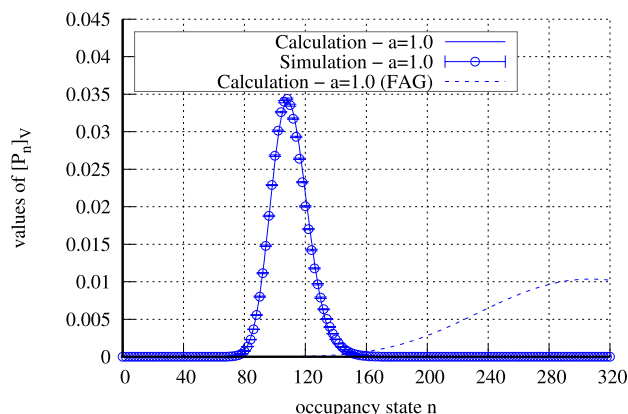


FIGURE 8. Occupancy distribution in System 3 for  $\alpha = 1.0$  Erl (comparison with FAG).

one another. The obtained results, presented using a linear scale (Figs. 3, 5, 7, and 8), unequivocally show the increased accuracy of the proposed method.

## V. CONCLUSION

This article proposes a new occupancy distribution for resources in EONs. The model takes into consideration the necessity of allocating a call in adjacent frequency slots. To the best of our knowledge, this is the first multi-service model of resources that takes into consideration occupancies exclusively in adjacent FSUs. The model is based on the general model of resources with a state-dependent service process, in which the conditional transition probabilities between neighboring states of the service process are determined in a combinatorial way. The present article proposes a method for determining this probability. The comparison of the analytical calculations with the data obtained in the digital simulation shows high accuracy of the proposed solution and, consequently, the possibility of applying the solution in the analyses and optimization of EONs.

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