IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received July 13, 2021, accepted July 19, 2021, date of publication July 30, 2021, date of current version August 9, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3101211

Multi-Objective Optimal Control for Uncertain Singularly Perturbed Systems

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This work was supported by the Youth Foundation of Beijing Nature Science under Grant 4154068, the Youth Talent Cultivation Program of Beijing, the National Natural Science Foundation of China under Grant 61473002 and Grant 61573024, the Fundamental Research Funds for Beijing Universities under Grant 110052971921/030, and the North China University of Technology Yuyou Talent Support Program.

ABSTRACT This paper studies the passive multi-objective optimal control of uncertain singularly perturbed systems. First, the uncertainty is described in a norm-bounded way, and then the state space model of the system is determined. Further time, the problem of passivity can also be transformed into a linear quadratic performance index problem. According to the Lyapunov stability theory, the system robust stability criterion is derived, based on the above conclusion, the corresponding parameterized passive multi-objective controller is designed using state feedback, and the weight matrix and the minimum performance index are solved based on the optimal control theory. Finally, a numerical example is given to illustrate the correctness and feasibility of the control method.

INDEX TERMS Singularly perturbed system, robust control, multi-objective, optimization, LQ-control.

I. INTRODUCTION

Singular perturbed systems widely exist in all areas of human production and life [1]. Due to the particularity of the system itself, the system contains both slow and fast variables, so there are higher requirements for the control accuracy of the system. Since the need of scientific and technological progress, and meanwhile, the singular perturbed system has a wide range of theoretical research and practical value, therefore, the research on singular perturbed theory has become the trend of modern scientific research. In the literature [2]–[5], the predecessors have already made many important achievements in this field. By using the *Riccati* equation, they have obtained relevant conclusions about the H_∞ control and quadratic stability of the singularly perturbed system. Nowadays, scholars advance the research to a whole new level. In [6], data-driven control strategies are used to combine them with fault-tolerant control, and parameter is designed through a new internal model structure. In [7], aiming at the stabilization problem of the multi-parameter quasi-linear singularly perturbed system, the design method of the corresponding controller is given.

The associate editor coordinating the review of this manuscript and approving it for publication was Zhiguang Feng.

However, in actual production and life, it is usually difficult to obtain an accurate mathematical model of the system or the controlled object. At the same time, with the continuous advancement of technology, systems or equipment are gradually becoming more complex and sophisticated. Since the different control methods selected, we tend to simplify and approximate them when we analyze, so the system will inevitably contain uncertainty. These include parameter uncertainty, the influence of external environment or interference on it, and many internal system changes caused by equipment aging. Based on the above factors, the control method we design must enable the system to have strong anti-disturbance ability to meet the performance index requirements we expect. Because of this, the controller design based on uncertainty is more practical.

Robust control is a better way to solve such problems. It is mostly used to solve the control difficulties of the system under the conditions of changing working conditions, external interference and parameter errors. Its control idea is to design a fixed controller to meet the control quality required by uncertain objects.

With the gradual deepening of research, in [8], for discrete uncertain control problems, a control algorithm based on hidden Markov model is designed on the basis of linear matrix inequality and H_∞ control. Because stability is a prerequisite for the system to work normally, some scholars now combine stability with other performances, such as passivity and positive reality and so on, this also makes multi-objective optimization control has more practical application value [9], [10]. In the [11], a multi-objective optimization control algorithm based on *LMI* is given so that the system has better robustness and ability to adapt to external disturbances. In [12], for fuzzy switched systems, in order to reduce the impact of actuators on system performance, a synthesized fault-tolerant output feedback controller is designed for multiple performances. In the [13] and [14], the multi-objective optimization control and passivity are combined and applied to the singularly perturbed system, and with the help of the understanding of the passivity, the design method of system's passive multi-objective optimization controller is finally obtained. In particular, in [14], the adaptive integral sliding mode control is applied to the anti-disturbance observer, the design idea of the observer, the method of controller gain design and ε-bound estimation are given. In [15], using known theories, the research direction is deepened into the cooperative control of multi-agents, and the cooperative problem is equivalently transformed into a dynamic output feedback robust stabilization problem of a singularly perturbed system with linear uncertain cost. Finally, the design scheme of the dynamic controller is given based on the *LMI* method.

The control goal of this paper is: for uncertain singularly perturbed systems, it can still meet the requirements of stability and passivity when there are external disturbances in the system, and then realize the multi-objective optimal control of the given system by setting controller. For the multi-objective optimization of the system, it is generally through the understanding of multiple performance indicators to find the common description form in the performance indicators, and then merge them through rigorous mathematical derivation, so as to obtain the performance indicators that the final system needs to achieve. Based on the research ideas of previous scholars, the study of the multi-objective optimization control algorithm based on passivity for the uncertain singularly perturbed system, through the calculation method of *Lyapunov* theory and *LMI*, has obtained the corresponding stability criterion. Finally, the design method of the parameterized controller is given, and the feasibility and correctness of the control method are verified through the analysis of numerical examples.

II. PROBLEM DESCRIPTION

Consider the specific system:

$$\begin{cases} E_\varepsilon \dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) + D_1\omega(t) \\ z(t) = Cx(t) + B_z u(t) + D_2\omega(t) \end{cases} \tag{1}$$

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad E_\varepsilon = \begin{bmatrix} I_{n1} & 0 \\ 0 & \varepsilon I_{n2} \end{bmatrix} \tag{2}$$

ε is the perturbation parameter, $x(t) \in \mathbb{R}^n$ is the state variable of the system, $u(t) \in \mathbb{R}^m$ is the control input variable of the system, $y(t) \in \mathbb{R}^p$ is the observable output variable of the system, $\omega(t) \in \mathbb{R}^l$ is the disturbance and satisfies $L_2[0, \infty)$. ΔA and ΔB are unknown uncertain matrices, which have the form described by equation (3). A, B, B_z, C, D_1 and D_2 are constant matrices with appropriate dimensions. Assuming that the system is completely controllable.

$$[\Delta A \ \Delta B] = dF(t)\,[E_a \ E_b] \tag{3}$$

where D, E_a, E_b are known constant matrices, describing the structural information of uncertain parameters, $F(t) \in \mathbb{R}^{i\times j}$ are time-varying unknown matrices, satisfying the norm Bounded conditions:

$$F(t)F^T(t) \le I \tag{4}$$

The task of this chapter is to design a state feedback controller

$$u(t) = Kx(t) \tag{5}$$

Make the system (1) meet the gradual stability and meet the index requirements.

The closed-loop system state equation can be obtained by the system (1) and the state feedback controller (5)

$$\begin{cases} E_\varepsilon \dot{x}(t) = A^* x(t) + D_1\omega(t) \\ z(t) = C^* x(t) + D_2\omega(t) \end{cases} \tag{6}$$

then $A^* = A + BK + DF(t)(E_a + E_b K)$, $C^* = C + B_z K$

The definition of passivity is given below.

Definition: In the zero initial state, if the system (1) satisfies the following matrix inequality (7), then the system (1) for non-zero disturbances $\omega(t) \in L_2[0, \infty)$ is passive.

$$\int_0^\infty \omega^T(t)z(t)dt \ge \int_0^\infty \omega^T(t)\omega(t)dt \tag{7}$$

The selection of performance indicators is generally closely related to the mathematical model of the system and the expected system performance. In this chapter, the performance indices that need to be optimized are

$$J = \frac{1}{2}\int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt \tag{8}$$

where Q is a non-negative symmetric weight matrix, and R is a positive definite weight matrix. Bring the controller (5) into the quadratic performance index (8), the linear quadratic performance index can be obtained as

$$J = \frac{1}{2}\int_0^\infty x^T(t)\bar{Q}x(t)dt \tag{9}$$

where, $\bar{Q} = Q + K^T RK$, Q, R_1 and R_2 are waiting matrices related to passivity. Although the manifestation of this performance index is a general linear quadratic performance index, the parameters in it contain the relevant performance requirements necessary for the system.

In order to obtain the main results of this paper, the following three lemmas are given.

Lemma 1 [16]: Set the matrices $Z(\varepsilon)$ and E_ε be as follows

$$Z(\varepsilon) = \begin{bmatrix} Z_1 + \varepsilon Z_3 & \varepsilon Z_5^T \\ Z_5 & Z_2 + \varepsilon Z_4 \end{bmatrix}$$

$$E_\varepsilon = \begin{bmatrix} I_1 & 0 \\ 0 & \varepsilon I_2 \end{bmatrix} \tag{10}$$

If there is a matrix $Z_i (i = 1, 2, 3, 4)$ with $Z_i = Z_i^T (i = 1, 2, 3, 4)$, which satisfies the following linear matrix inequality

$$\begin{aligned} Z_1 &> 0 \\ \begin{bmatrix} Z_1 + \varepsilon^* Z_3 & \varepsilon^* {Z_5}^T \\ Z_5 & Z_2 + \varepsilon^* Z_4 \end{bmatrix} &> 0 \\ \begin{bmatrix} Z_1 + \varepsilon^* Z_3 & \varepsilon^* {Z_5}^T \\ \varepsilon^* Z_5 & \varepsilon^* Z_2 + \varepsilon^{*2} Z_4 \end{bmatrix} &> 0 \end{aligned} \tag{11}$$

then

$$E_\varepsilon Z(\varepsilon) = Z^T(\varepsilon) E_\varepsilon > 0, \quad \forall \varepsilon \in (0, \varepsilon^*] \tag{12}$$

Lemma 2 [17]: Suppose $x \in R^p$, $x \in R^p$, D and E are constant matrices of appropriate dimensions, then for any appropriate dimension matrix F satisfying $FF^T \le I$, there is

$$2x^T DFEy \le \varepsilon x^T DD^T x + \varepsilon^{-1} y^T E^T Ey \tag{13}$$

Lemma 3 [18]: For a given symmetric matrix (42), where S_{11} is r dimensional, the three conditions in (42) are equivalent.

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{14}$$

$$\begin{aligned} &1)\ S < 0 \\ &2)\ S_{11} < 0, \ \ S_{22} - {S_{12}}^T {S_{11}}^{-1} S_{12} < 0 \\ &3)\ S_{22} < 0, \ \ S_{11} - S_{12} {S_{22}}^{-1} {S_{12}}^T < 0 \end{aligned} \tag{15}$$

III. DESIGN STATE FEEDBACK CONTROLLER

Theorem 1: If there are constant $\psi > 0$, matrix $P_i (i = 1, 2, \cdots, 5)$, satisfying $P_i = P_i^T (i = 1, 2, 3, 4)$ and the following set of matrix inequalities, the closed-loop system (6) for $\forall \varepsilon \in (0, \varepsilon^*]$ is asymptotically stable and satisfies the requirement of passivity.

$$\begin{aligned} P_1 &> 0 \\ \begin{bmatrix} P_1 + \varepsilon^* P_3 & \varepsilon^* {P_5}^T \\ P_5 & P_2 + \varepsilon^* P_4 \end{bmatrix} &> 0 \\ \begin{bmatrix} P_1 + \varepsilon^* P_3 & \varepsilon^* {P_5}^T \\ \varepsilon^* P_5 & \varepsilon P_2 + \varepsilon^{*2} P_4 \end{bmatrix} &> 0 \\ \varphi_1 &< 0 \\ \varphi_1 + \varepsilon^* \varphi_2 &< 0 \end{aligned} \tag{16}$$

where set

$$P_0 = \begin{bmatrix} P_1 & 0 \\ P_5 & P_2 \end{bmatrix} \tag{17}$$

$$\hat{P}_0 = \begin{bmatrix} P_3 & P_5^T \\ 0 & P_4 \end{bmatrix} \tag{18}$$

$$\varphi_1 = \begin{bmatrix} P_0^T \bar{A} + \bar{A}^T P_0 & M & P_0 \bar{N}^T & P_0^T D_1 - \bar{C} \\ M^T & -\psi^{-1} I & 0 & 0 \\ \bar{N} P_0^T & 0 & -\psi I & 0 \\ * & 0 & 0 & 2I - D_2 - D_2^T \end{bmatrix} \tag{19}$$

$$\varphi_1 = \begin{bmatrix} \hat{P}_0^T \bar{A} + \bar{A}^T \hat{P}_0 & 0 & \hat{P}_0^T N & \hat{P}_0^T D_1 \\ 0 & 0 & 0 & 0 \\ N^T \hat{P}_0^T & 0 & 0 & 0 \\ * & 0 & 0 & 0 \end{bmatrix} \tag{20}$$

note: $\bar{A} = A + BK$

Proof: First prove the asymptotic stability of the closed loop system (6). Assume $P_\varepsilon = P_0 + \varepsilon \hat{P}_0$,

$$E_\varepsilon^T P_\varepsilon = \begin{bmatrix} P_1 + \varepsilon P_3 & \varepsilon P_5^T \\ \varepsilon P_5 & \varepsilon P_2 + \varepsilon^2 P_4 \end{bmatrix} \tag{21}$$

From the inequality group (16) and the lemma 4 can be obtained

$$E_\varepsilon^T P_\varepsilon = P_\varepsilon^T E_\varepsilon > 0 \tag{22}$$

Choose the Lyapinov function as

$$V(t, \varepsilon) = x^T(t) E_\varepsilon^T P_\varepsilon x(t) > 0 \tag{23}$$

then the function of *Lyapunov* can be derived from t:

$$\begin{aligned} \dot{V}(t, \varepsilon) &= 2x^T(t) E_\varepsilon^T P_\varepsilon \dot{x}(t) \\ &= 2x^T(t) E_\varepsilon^T (P_\varepsilon \dot{x}(t)) \\ &= x^T (P_\varepsilon^T A^* + A^{*T} P_\varepsilon) x(t) \\ &\quad + 2x^T(t) P_\varepsilon^T D_1 \omega(t) \end{aligned} \tag{24}$$

When $\omega(t) = 0$, there is a set of matrix inequalities (16) to get $\dot{V}(t, \varepsilon) < 0$, so the closed-loop system (6) is in $\varepsilon \in (0, \varepsilon^*]$ is asymptotically stable.

The passivity of the closed-loop system (6) is discussed below.

Rewrite the passive performance index (7) into the following form:

$$\begin{aligned} J_{zw} &= \int_0^\infty 2[\omega^T(t)\omega(t) - \omega^T(t) z(t)] dt \\ &= \int_0^\infty 2[\omega^T(t)\omega(t) - \omega^T(t) z(t) + \dot{V}(t, \varepsilon)] dt \\ &\quad + V(t, \varepsilon)|_{t=0} - V(t, \varepsilon)|_{t=\infty} \end{aligned} \tag{25}$$

In the zero initial state, there are $V(t, \varepsilon)|_{t=0} = 0$ and $V(t, \varepsilon)|_{t=\infty} \ge 0$, so by the formula (25) is available

$$J_{zw} \le \int_0^\infty 2[\omega^T(t)\omega(t) - \omega^T(t) z(t) + \dot{V}(t, \varepsilon)] dt \tag{26}$$

where

$$\xi(t) = \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}, \tag{27}$$

$$\Psi = \begin{bmatrix} P_\varepsilon^T A^* + A^{*T} P_\varepsilon & P_\varepsilon^T D_1 - \bar{C} \\ * & 2I - D_2 - D_2^T \end{bmatrix} \tag{28}$$

From the set of inequalities (16), we can get $\Psi < 0$, that is, $J_{zw} < 0$

The following is a relevant proof of the feasibility of robust variance control

The certificate is complete.

The parameterized representation of the state feedback controller is given below.

Theorem 2: If there are constant $\tau > 0$, matrix $X_i(i = 1, 2, \cdots, 5)$ and Y satisfy $X_i = X_i^T(i = 1, 2, 3, 4)$ and the following matrix inequality (28), then the closed-loop system (6) for $\forall\varepsilon \in (0, \varepsilon^*]$ is asymptotically stable and satisfies passivity The gain of the state feedback controller is $K(\omega) = Y(X_0 + \varepsilon\hat{X}_0)^{-1}$.

$$\begin{aligned} X_1 &> 0 \\ \begin{bmatrix} X_1 + \varepsilon^* X_3 & \varepsilon^* {X_5}^T \\ \varepsilon^* X_5 & X_2 + \varepsilon^* X_4 \end{bmatrix} &> 0 \\ \begin{bmatrix} X_1 + \varepsilon^* X_3 & \varepsilon^* {X_5}^T \\ \varepsilon^* X_5 & \varepsilon^* X_2 + \varepsilon^{*2} X_4 \end{bmatrix} &> 0 \\ \Pi_1 &< 0 \\ \Pi_1 + \varepsilon^* \Pi_2 &< 0 \end{aligned} \tag{29}$$

where

$$X_0 = \begin{bmatrix} X_1 & 0 \\ X_5 & X_2 \end{bmatrix}, \quad \hat{X}_0 = \begin{bmatrix} X_3 & X_5^T \\ 0 & X_4 \end{bmatrix} \tag{30}$$

$$\Pi_1 = \begin{bmatrix} N & D & {E_1}^{*T} & D_1 - X_0^T C + Y^T B_z^T \\ D^T & -\tau^{-1} I & 0 & 0 \\ {E_1}^* & 0 & -\tau I & 0 \\ * & 0 & 0 & 2I - D_2 - D_2^T \end{bmatrix} \tag{31}$$

$$\Pi_2 = \begin{bmatrix} \hat{X}_0^T A^T + A\hat{X}_0 & 0 & {E_2}^{*T} & D_1 \\ 0 & 0 & 0 & 0 \\ {E_2}^* & 0 & 0 & 0 \\ * & 0 & 0 & 0 \end{bmatrix} \tag{32}$$

note: $N = X_0^T A^T + AX_0 + Y^T B + B^T Y$, ${E_1}^* = E_a X_0 + E_b Y$, ${E_2}^* = E_a \hat{X}_0$

Proof: Perform proper matrix transformation on the inequality group (16). Performing contract transformation, that is, multiply $\mathrm{diag}\{P_\varepsilon^{-T}, I\}$ on the left, multiply $\mathrm{diag}\{P_\varepsilon^{-1}, I\}$ on the right, set $X = P_\varepsilon^{-1}$, then the set of inequalities (29) can be obtained.

From this, the calculation formula of the weight matrix R can be obtained:

$$R = -(K(\varepsilon)P_\varepsilon^{-1} B^{-T})^{-1} \tag{33}$$

The weight matrix Q is the solution of the following Riccati equation.

$$A^T P_\varepsilon + P_\varepsilon^T A + Q - P_\varepsilon^T B R^{-1} B^T P_\varepsilon = 0 \tag{34}$$

IV. NUMERICAL EXAMPLE

Consider the uncertain singularly perturbed system (1), in which the system parameters are set as follows:

$$E_\varepsilon = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix}; \quad A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}; \quad B = \begin{bmatrix} 1.2 \\ 0.3 \end{bmatrix};$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}; \quad D = \begin{bmatrix} 0.5 & 0 \\ 0 & -0.5 \end{bmatrix}; \quad D_1 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix};$$

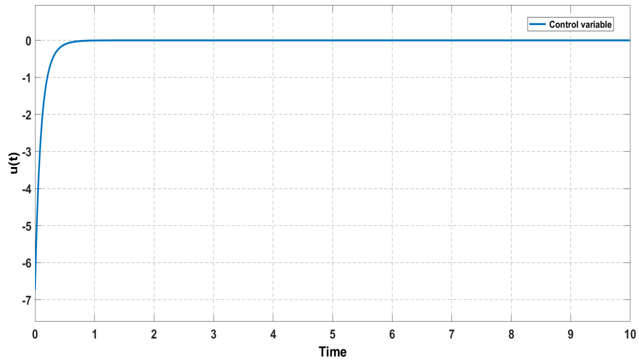

FIGURE 1. Control variable when $\omega(t) = 0$.

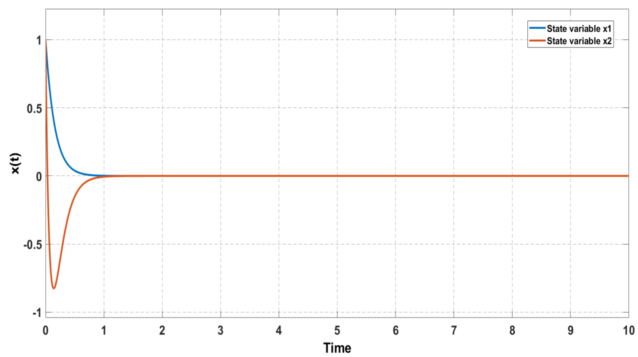

FIGURE 2. State variable when $\omega(t) = 0$.

$$E_a = \begin{bmatrix} 0.4 & 0.2 \\ 0.5 & 0.4 \end{bmatrix}; \quad E_b = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}; \quad x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$
$$F = 0.5; \quad D_2 = 2; \quad B_z = 2$$

The results obtained by simulation calculation are as follows

$$P_\varepsilon = \begin{bmatrix} 0.1611 & -0.0677 \\ -0.6775 & 3.4729 \end{bmatrix}, \quad K = \begin{bmatrix} -5.9510 & -0.7885 \end{bmatrix} \tag{35}$$

The weight matrix is calculated as follows

$$Q = \begin{bmatrix} 1.6723 & 2.2265 \\ 2.2265 & 53.0777 \end{bmatrix}, \quad R = 0.0201 \tag{36}$$

The final calculated performance index is $J = 16.61$.

This chapter considers the control system conditions under three disturbances $\omega(t)$, which are as follows:

(1) $\omega(t) = 0$, the control variable, state variable and output variable curve are shown in Figure 1 to Figure 3 respectively

It can be seen from Figure 1-3, when $\omega(t) = 0$, the system can reach asymptotic stability.

(2) $\omega(t) = 0.1$, the curves of control variables, state variables and output variables are shown in Figure 4 to Figure 6 respectively.

Also it can be concluded from Figure 4-6, when $\omega(t) = 0$, the system has a small error, but it can still be guaranteed that the system in Lyapunov stable.

(3) $\omega(t) = 0.1\sin(t)$, the curves of control variables, state variables and output variables are shown in Figure 7 to Figure 9 respectively.

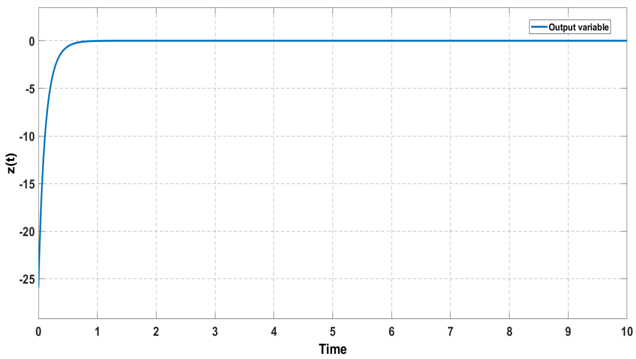

FIGURE 3. Output variable when $\omega(t) = 0$.

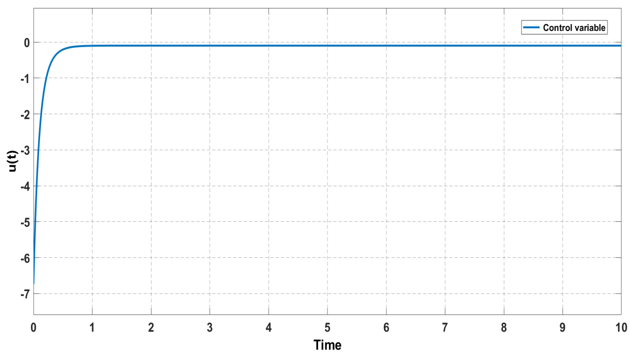

FIGURE 4. Control variable when $\omega(t) = 0.1$.

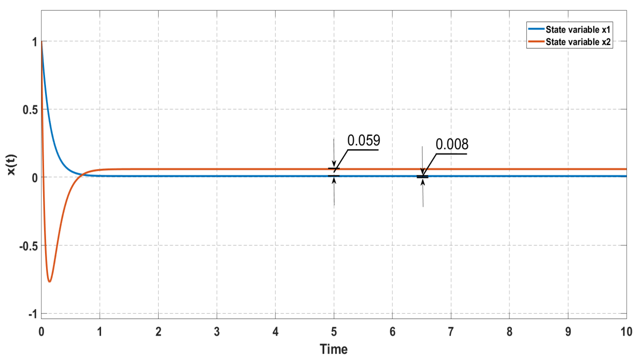

FIGURE 5. State variable when $\omega(t) = 0.1$.

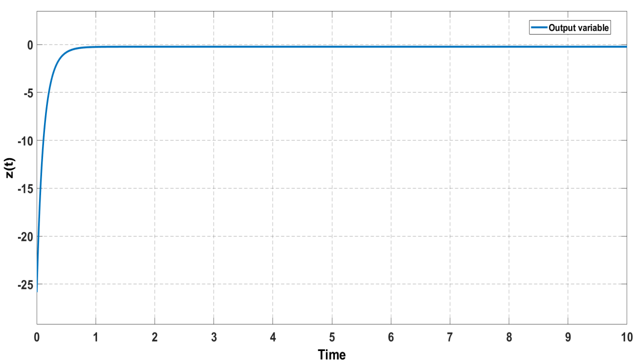

FIGURE 6. Output variable when $\omega(t) = 0.1$.

In figure 7-9, when $\omega(t) = 0.1\sin(t)$, the system also fluctuates slightly, but it can also ensure that the system in Lyapunov stable.

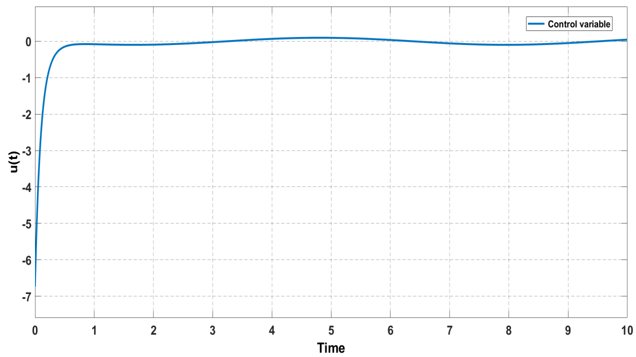

FIGURE 7. Control variable when $\omega(t) = 0.1\sin(t)$.

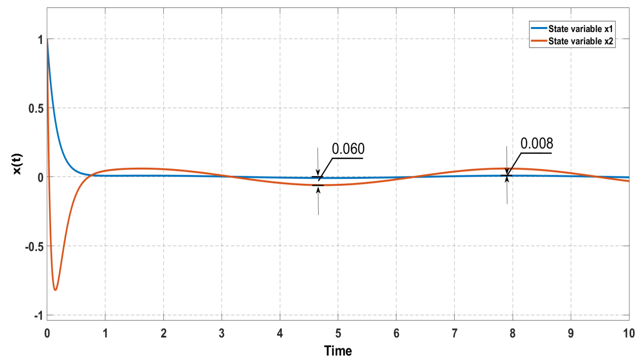

FIGURE 8. State variable when $\omega(t) = 0.1\sin(t)$.

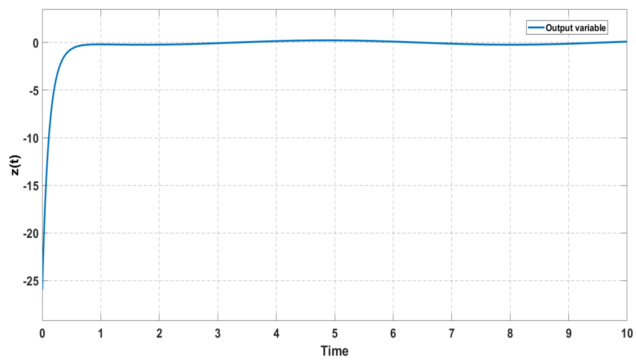

FIGURE 9. Output variable when $\omega(t) = 0.1\sin(t)$.

V. CONCLUSION

This dissertation has done a certain research on the passive multi-objective optimization control problem of uncertain singularly perturbed systems, and transformed it into a standard linear quadratic performance index problem. Using linear matrix inequalities to derive theorems and conditions that the controller should satisfy. Finally, the global optimal solution of the matrix inequality is solved according to the Riccati equation. Through simulation examples, the validity of the theory is verified. The simulation results show that when there is uncertainty in the system, the system can also have better output results, which further proves the correctness of the theory.

At the same time, in the process of research, it was discovered that this topic also contains some deeper research content:

(1)The time-delay phenomenon mostly exists in the actual industrial production process, such as: signal transmission and the controlled object itself has certain subsequent characteristics, etc., especially, for uncertain singular perturbation systems, this phenomenon will be more common, so it is extremely important to add the analysis of time-delay variables into the research content and has considerable practical value.

(2)For nonlinear singular perturbation systems and control problems with more than two targets. Since there is no complete linear system in actual engineering, at the same time, with the continuous development of science and technology, there is an increasing need to optimize the control of the three goals and above of the system. At present, there are still few studies on the above issues.

ACKNOWLEDGMENT
The authors would like to thank the reviewers for their hard work.

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