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Development of an Exploring System: An Integrated Approach to Children's Math Education in China

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ABSTRACT In terms of the leap-forward development for education experiment, both the Chinese and English educational experiment have been very successful, while the progress of the children's mathematics education experiment in China has been relatively slow. It is believed that the main reason for this is the lack of an efficient cognitive tool or system that supports students' study and exploration and correspondingly covers the whole curriculum of children's math education. In recent years, basic mathematics education cognitive tools have undergone rapid development, whereas the lack of a common normalization method for continued in-depth inquiry of the relationships among Objects, Algebra, and Geometry (OAG) has led to the fragmentation of students' mathematical knowledge. Thus, it is difficult for students to develop a systematic understanding of mathematics. Therefore this project attempts to construct a Children's Mathematics Exploring System (CMES), which integrates with cognition theory, method, and contents of basic mathematics education to query the essence of the relationships among OAG. Specifically, the functions of CMES include the following: (1) providing cognitive tools with typical characteristics for the relationships among OAG; (2) offering a general normalized open system on which sustainable in-depth research can be conducted and a conversion tool through which the complete textbook unit of the basic mathematics education can be converted to reveal the relationships among OAG; and (3) presenting a set of cognitive theories, methods, and a system of basic mathematics education to further explore the relationships among OAG. This project will accelerate the development of cognitive technology in basic mathematics education, widen its practical applications, reduce the difficulty of mathematics cognition, and finally improve the quality and efficiency of basic mathematics education.

INDEX TERMS Children's math education, exploring system, educational technology.

I. INTRODUCTION


According to developmental psychology, childhood can be divided into early childhood (preschool age), middle childhood (school age), and adolescence (puberty to legal adulthood). The Children mentioned in this research mainly refer to those at preschool age and school age.

Mathematics serve as the language of natural science, and cognitive education in mathematics aims to cultivate basic mathematics ability. With the advent of an information society and the development of Internet and Computing Technologies (ICTs), talent with mathematical ability is

increasingly required. Moreover, countries all over the world pay greater attention to the important and basic role of mathematics cognitive education [1]–[3], and even propose to win the “mathematics war” [4].

Due to the significance and fundamental strategic role, the relevant research on mathematical cognitive education has become a hot spot in various countries [5]–[19]. Indeed, the level of mathematical cognitive technology is endless [21]–[22]. A lot of researchers have paid attention to the practice [20] and investigated the impact of different mathematical technologies on cognitive effects [23].

Davis, the pioneer of cognitive mathematics, has paid great attention to the problem of psychological representation when analyzing the latest views of mathematics education,

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calling for the study of human personality. He has made it clear: “mathematics is not a sign written on paper. Mathematics is a way of thinking, which includes the psychological representation of problem situations and related knowledge, including the analysis of these psychological representations, including the use of annotation” [53]. At present, the cognitive theories involved in mathematical cognitive education mainly include cognitive situation theory [24], cognitive structure theory [25], cognitive diagnosis theory [26], cognitive load theory [27], and cognitive elasticity theory [28].

This research aims to provide the reader with a holistic solution to children's math problems at all levels by developing an exploring system with an integrated approach.

With the deepening of China's technological reform of elementary education and the frequent application of technology in education, the exploration into the leap-forward development of China's elementary education has been solidly placed on the country's agenda [29]. Such development refers to the rapidly enhanced application of ICTs with the aim of observing phenomena to find regularities. Grounded in the theory of language sense [30], great achievements have been made in language learning [31], [32]. However, technology-driven math education in China is not fully investigated. Over the years, both teachers and researchers have come to realize that educational tools can be used to deliver math content across the spectrum for all grades and promote the substantial developments.

To date, technology-supported math software have been utilized to address one or several aspects in the knowledge system of children's math education. What is needed, moreover, is a complete system that can narrow the gap by supporting the teaching of math across elementary and primary grades. The system proposed in this paper is an exploring system based on three types of relationships—(i.e. object and algebra, object and geometry, and algebra and geometry)—, all of which can assist students from all grades in solving math-related problems. The benefit of “uploading” all mathematical knowledge into the framework of “object + algebra + geometry” is that students can grasp the whole picture of mathematics, as well as the relationship between various mathematical elements systematically, and finally gain a unified understanding. In addition, students do not need to learn all kinds of mathematical tools corresponding to different knowledge points. Hence, this system effectively reduces the extra cost in the process of their mathematics learning. Additionally, the relationship between objects, algebra, and geometry is a helpful framework for children's math education because it follows a natural understanding of the world. Unfortunately, it is not conducive to establishing children's understanding of the world, whose unified relationship has already been split by their existing knowledge concept.

II. RELATED WORKS

A detailed list of works relating to children's math and CMES is provided below.

A. ANALYSIS OF MATHEMATICS TOOLS FOR CHILDREN

With the in-depth integration of Information Technology and mathematics curricula, many research institutions at home and abroad have developed tools and software for mathematics learning. In particular, the most commonly mentioned tools in the literature included: the “Geometer's Sketchpad” [33]; the “Z+Z Intelligent Education Platform” [34]; and “Jinhua Branch of Mathematical Platform” [35]. There also included the “Primary Mathematics Companion” [36]; the graphing calculator [37]; “Logo language” [38]; “geometric reasoning” [39]; the “Mathematica package” [40]; the “MP_Lab” [41]; the number sense game (NSG) [42]; the “DM_Lab” [43]; the “Children's Enlightenment Master” [44]; the “Child Star of the Enlightenment of Gray Ducks” [45]; the “King of Oral Calculation” [46]; the “Haifeng Learning Courseware” [47]; and other educational software programs and math games, as well as programs such as “Bingo” [48]. These learning tools focus on middle-school mathematics more than primary-school mathematics. Concerned with all the content of primary mathematics, we deemed the most comprehensive software program to be “Primary Mathematics Companion” [36], while others could only support several aspects of primary mathematics content. According to our research, we surmised that the sum of all the software in the inquiry learning researched could not meet the entire contents of primary-school mathematics. The encounter problem—which models the encounter between two vehicles and is a common feature in curricula—was one of the pieces unaccounted for. Although the “Primary Mathematics Companion” [36] manage to cover the full contents, it is far from enough to help children explore, inquire and obtain knowledge since it is primarily used to evaluate and practice what they have learned. Therefore, there still remains room for developing and improving inquiry-based learning for students, so as to best support them in their learning endeavours.

Some of the mathematical learning tools in the game are designed to attract students, but students have to master the relevant mathematical knowledge first in order to win the game. In addition, games are always used to test what has been learned and are not for acquiring new knowledge. With this in mind, supporting students in learning mathematics remains inadequate. A detailed analysis can be seen in the author's PhD thesis [49]. The analysis table follows.

After applying a comparative analysis, it was clear to see that the computer-aided mathematics teaching technology is quite mature both in number and shape. Nevertheless, there is still a long way to modify these software programs in many aspects, such as overall content, renewal of test questions, network supporting, as well as relationships among OAG. These software cannot support real-time evaluation on the basis of supporting mathematics teaching and independent inquiry-based learning; Besides, in terms of knowledge integrity, they fail to explore the relationships among OAG simultaneously. In terms of openness and systematization, they are also unable to perceive the whole content

TABLE 1. Analysis table of children’s mathematics tools.

No	R1	A	R2	G	R3	AS	EU	OP	NS
1				√					
2		√				√			
3		√				√			
4		√							
5		√		√					
6		√				√			
7		√				√			
8				√					
9				√					
10				√					
11				√					
12						√	√		√
13	√								
14		√				√			
15		√							
16		√							
17		√		√					
18		√		√					
19		√							
20	√					√			

R1 Relation between object and algebra
A Algebra
R2 Relation between object and geometry
G Geometry
R3 Relation between algebra and geometry
AS Assessment supporting
EU Exercise problems can be updated
OP The overall perspective of children’s mathematical content
NS Network supporting
1 Geometer’s Sketchpad
2 King of Oral Calculation
3 Child Star of the Enlightenment of Gray Ducks
4 Children’s Enlightenment Master
5 Z+Z Intelligent Education Platform
6 Primary Mathematics Companion
7 Haifeng Learning Courseware
8 DM_LAB
9 PG_Lab
10 MP_Lab
11 Jinhua Branch of Mathematical Platform
12 Interactive teaching system “Happy Instruction”
13 Mathematical software “Zhuhai Coastline” for primary school
14 A complete collection of mathematical puzzle games for primary school students
15 Matlab
16 MathCAD
17 Mathematica package
18 Graphing calculator
19 Maple
20 Bingo

of mathematical cognitive education comprehensively. The software being a closed system results in a lack of necessary interface to adapt to a student’s needs.

To sum, there are many kinds of software tools supporting children’s mathematical cognition. Unfortunately, most of these cognitive tools are oriented to a certain knowledge point or a certain aspect of mathematics. It’s a pity that software programs, which can cover and support the overall content of mathematics cognitive education, are relatively few in each program.

The educational software tools are often based on assessments and games, which are of great help for the consolidation of existing knowledge. However, it is difficult for these tools to provide effective support for the understanding of new knowledge in the classroom (including the learning of new concepts, principles, and methods). Although they are technically mature in the presentation of geometry, they fail to illustrate the relationship of object and algebra and that of object and geometry. Still there’re defects and drawback in the theory and method of children’s mathematical exploration tools which can explore the relationships among OAG.

The main reason is that mathematical cognitive education involves multiple grades, with various contents and difficulties. As a result, it is not easy to provide a unified exploration system for mathematical cognitive education. Therefore, The producers of mathematical educational tools tend to conduct research from a certain aspect, but fail to start from an overall perspective of children’s mathematical cognitive education. Importantly, content for mathematics cognitive education should be merged and classified with all related objects summed up and all kinds of relationships between these objects mined out. From here, all objects and relationships are integrated and developed. Hence, each product designed can only provide teaching and learning support for part of the current mathematics cognitive education (even a very small part of the teaching content). Apart from that, most of the teaching content does not have the same overall cognitive system to support the students’ cognitive inquiry process in each grade. Even for the same topic, it needs to be learned using two different kinds of recognition knowledge tools. Consequently, children will have to make adaption constantly, after learning a variety of new software cognitive tools, which simply results in additional burdens. What teachers and students do hope to use is an unified exploring system which will span the whole learning process from junior to senior, so as to minimize the extra burden.

It is under this background that this research puts forward a system of children’s mathematical cognitive education tools that can further explore the relationships among OAG. In fact, it is simply impossible to develop a flexible system that is convenient to use and can cover most of the content in the Chinese children’s mathematics curriculum. However, it is indeed possible to develop an exploratory system which covers most of the content in the curriculum. For one thing, elementary mathematics is relatively simple, for another, there’s standard syllabus formulated by the Ministry of Education in China.

Also in this research, we have selected lessons from 12 textbooks published by People’s Education Press of China

(PEPC). The workload is very large and the depth of the content differs. In fact, although the focus of content differs, the same exploring scene will be involved again during children's different learning stage due to the unique nature of spiral progression for math learning. Therefore, all the mathematical knowledge in the 12 textbooks can be divided and recombined according to each scene, and the exploring tools for corresponding scenes can be designed. In order to adapt to the different depth of content learners' knowledge, these exploring tools will allow learners to adjust the scene or system parameters. Beginners are allowed to use the default parameters to make the learning of CMES easy. Additionally, the OAG in mathematics has its own internal relations. We follow those internal relations and develop a series of exploring tools of OAG, thus forming a system of children's exploring math. The author hopes that this system can support children's mathematical cognitive education, assist them in exploring mathematical OAG independently, and improve their abilities and the teaching quality of children's mathematical cognitive education significantly. This study addresses and tries to overcome the challenge to provide a unified theory, method, and system of inquiry tools that can explore the entirety of mathematics cognitive education.

According to our project LFDIEEE (Leap Forward Development and Innovation Experiment of Elementary Education) in Primary Mathematics Learning, math teachers from primary school mentioned that the encounter problem, which involves a situation where two vehicles approach each other and students are required to solve for variables such as the speed, time, and distance, was one of the key cruxes that had no suitable tool. However, it's difficult for kids to handle such model of encountering since they seldom practice that in math class. What's worse, the actual situation of computing bidirectional movement is uncontrollable. As a result, what teachers considered ideal was an interactive, multimedia, animated, inquiry-based learning tool, but unfortunately, they generally could not find this ideal tool.

Hence, in terms of content adaptation, almost all the current software still cannot satisfy the need of children's math learning under the guidance of standard mathematics syllabus for elementary schools. According to the survey, teachers also said that several typical math issues had no suitable tools in current exploring software system, such as the model of encountering, overall arrangement, coding interpretation, random possibility. Among the relevant pedagogical tools for addressing the gap, one of the key issues is "encounter problem". Taking this need as the starting point, we try to develop the exploring tools. Finally, we design the exploring system for children's math.

B. THINKING OF CMES

Generally speaking, children's mathematics cognitive education involves the knowledge for preschool and grades 1 - 6, and includes various contents and difficulties. This study asks how to make these complicated elements of knowledge covered by the same system? In other words, what is the

relationship among the contents of mathematical cognitive education? Which theory and method can be used to construct a mathematical cognitive education system that can explore the essential relationships among OAG? It has become a key scientific issue in this study. Because the research on the relationships among OAG is the cornerstone of this study, we argue that the best way to realize the exploration of the relationships among OAG is to build the corresponding relationships among them into the exploration system itself. Specifically, the relationships of OAG in mathematical cognitive education includes three categories: the relationship between objects and algebra, between objects and geometry, and that between algebra and geometry. While each category can be divided into several subcategories, too. For example, Geometer's Sketchpad belongs to the category of geometric operation tools under shapes in the relationship of objects and geometry. How to obtain and organize these relationships in OAG in children's math education? Our solution of development is to merge and classify all the content in mathematical cognitive education, summarize all related objects, and excavate all kinds of objects, as well as the algebraic and geometric topics among these objects. Finally, we aim to integrate all objects and relationships among OAG to establish a unified model of a cognitive theoretical method and system that can explore OAG, providing a holistic way for us to solve key scientific problems. This solution may seem different from the aforementioned solution; but, in fact, they are completely consistent.

Below is a discussion of the connection to the solution of development. The development strategy here is holistic. As a whole, there is always a problem about which should be done first and which should be done second. The meeting or encounter problem mentioned before is chosen as a starting point for the overall development. Although the encounter problem is a specific problem, its development still falls within the above-mentioned overall development plan. The development of the encounter problem is only a part of the overall development. The difference lies in that we do this work with a more comprehensive set of components, and we get closer to encompassing the whole set of knowledge elements in the curriculum.

The research related to this topic is mainly conducted based on the following aspects of thinking. Finally, through the design and development of CMES, we realized these aspects of thinking (See Table 2).

III. RESEARCH METHODS AND STEPS

A. RESEARCH METHODS

In the early stages, our team published relevant research results related to the design principles of the overall perspective of math education [50], the construction of mathematical teaching support model and exploration technology in primary school mathematics [51], automation assessment technology of computer skills and educational technology [52], online game addiction mechanism, and more. These results of the recent publications are of great significance to CMES.

TABLE 2. Thinking of CMES and related aspects.

Aspects	Research focus
Essential relationships of OAG	<ul style="list-style-type: none"> ● What are the typical characteristics of the relationships of object and algebra, object and geometry, and algebra and geometry in CMES? ● How to construct the model of CMES?
Objects (the basic elements of CMES)	<ul style="list-style-type: none"> ● How to set up mathematical objects? What identity operations should all objects have? How to construct the differences among objects under the premise of the same operation? ● How are the objects stored? Is it an independent document or an open space document? How to recover the type and state of each object exactly when it is called in again after storage?
Alternation between objects and algebra	<ul style="list-style-type: none"> ● How to present objects and algebra? If we use the physical graph to represent objects, how can we ensure that the number and the figure change automatically, and vice versa? ● If there is a relationship among objects, how to ensure that the related objects change at the same time? ● How to satisfy the students' independent setting of third-party action conditions and automatic occurrence of related actions when the action conditions are met? For example, when two cars meet, they can automatically continue driving in the opposite direction. ● In the encounter problem, how to set the moving speed of two vehicle objects on the computer screen under the premise of ensuring the best visual effect? Is it necessary to balance the speed of visual effect first? How to set the balance value to ensure the best visual animation effect? In order to ensure that the user's preset actions are executed after the meeting time, how to judge and ensure that the two vehicles meet only once and are not interfered by the error range? Since each car has its length and the meeting will continue for a while.
Alternation between object and geometry	<ul style="list-style-type: none"> ● How to extract the corresponding geometry from a real object? ● Should the system use an image recognition method or an object-related graphics method? ● How to obtain a physical drawing similar to a geometric object's graphic features?
Overall arrangement	<ul style="list-style-type: none"> ● How to establish and present the elements or steps of the overall planning object? ● How to explore the overall planning elements and steps? ● How to judge the correctness of students' overall exploration results?
The meaning of code	<ul style="list-style-type: none"> ● How to present a random code? How to let students explore the meaning of each part of the code? ● How to judge whether the students understand the correct meaning of each part of the code?
Sequence and set	<ul style="list-style-type: none"> ● How to find out the meaning of a quantity sequence of multiple physical pictures with different quantities? ● How to enable students to use physical drawings with different quantities to explore the meaning of sequence? ● How to judge the correctness of students' use of real image sequences? ● How to explore the intersection, union, and complement of sets based on graphs? ● How to judge the correctness of a set based on a graph?
Quantity	<ul style="list-style-type: none"> ● How to set the relevant properties of objects in a physical figure?

TABLE 2. (Continued.) Thinking of CMES and related aspects.

General and normalized open space	<ul style="list-style-type: none"> ● How to measure these attributes? Quality requires balance tools, length and height require a ruler, and angle requires a protractor. Should all these exploration tools be included in CMES? ● How to present all kinds of mathematical objects and related relationships in open space? ● Is there a corresponding library of OAG, as well as their mutual relationships in this space? ● What general functions should the open space have to ensure students' exploration, draft paper tools, measuring tools or other tools? ● How to make the associated objects change and be present in coordination when the setting of an object is changed in the open space?
Concept exploration	<ul style="list-style-type: none"> ● Can we provide a large number of dynamic, operable, and exploratory imagery materials to students for understanding mathematical concept? ● How to make the objects in the related problems move? (e.g., travel problems.) ● How can mathematical terms and concepts automatically appear when a dynamic inquiry operation is carried out?
Evaluation and openness	<ul style="list-style-type: none"> ● The CMES should have the diversified testing performance. ● The openness of CMES includes the open update of its own content and its own program, rather than open source.
Theory, Method, and System	<ul style="list-style-type: none"> ● Can we put forward a set of theories, methods, and systems for exploring the relationships among OAG? ● Can we summarize all the related objects from the perspective of cognitive education in children's mathematics, dig out all kinds of essential relations among OAG in depth, and then integrate and develop all objects and relations? ● What contribution does the CMES make to the existing theories and methods?

We absorbed the latest achievements of both domestic and foreign researchers in mathematical exploration tools, cognitive effect measurement, cognitive theory. Additionally, we drew on representative thoughts and experiences; used some cognitive tools, methods, and theories in the field of mathematical cognitive education both at home and abroad, and actively discussed with these distinguished scholars as well. Finally we integrated various results into our exploring system.

Four research methods had been used in this study, including literature research, system science, mathematics, and scientific experiment. The details are as follows.

1) LITERATURE METHOD

This method was used for researching mathematical exploring tools and related theories or methods. In particular, this study carried out a full literature review on the mathematical cognitive ability related to OAG cognition, surveyed the supporting tools of mathematical cognitive ability, and the measuring tools or scales of its promoting role. The review also included a survey of the educational psychology basis and learning science basis of relevant scientific experiments,

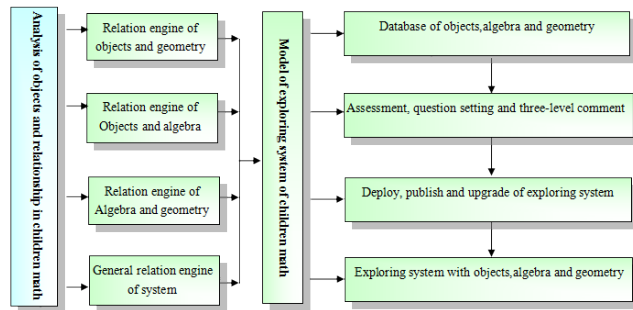


FIGURE 1. Overall research technology route of exploring system.

and investigated the promotion or training of relevant basic cognitive ability by corresponding tools.

2) SYSTEM SCIENCE METHOD

This method was used to study objects, algebra, geometry, and their essential relations. Meanwhile, it was used to study the mathematical cognitive education tools and the mathematical model, structure, and design method of the system. In terms of specific development technology and methods, we used this research method to carry out abstract object analysis, constructed all mathematical objects and related objects in mathematical cognitive education (e.g. sequence problems involving objects, scores involving objects, encounter problems involving objects, and geometric objects), and analyzed their corresponding operations and attributes. Furthermore, we classified and built a mathematical model and, finally, formed its overall research technology route (see Fig. 1).

3) MATHEMATICAL METHOD

The local mathematical method was employed to set aside all extraneous characteristics of the research object, and only extract various quantities, the variation of quantities, as well as the relationship between them. Based on the characteristics of mathematical cognitive education tools and content attributes, this research combined mathematical methods with systematic ones to investigate the method of measuring relevant mathematical cognitive ability, evaluating the effectiveness of mathematical cognitive education tools, extracting and analyzing the object and its relationship to mathematical cognitive education content, classifying these tools, and developing advanced technology.

4) SCIENTIFIC EXPERIMENT METHOD AND SYSTEMATIC SUMMARY AND INDUCTION

Experimentation is both the source and criterion of natural science theory. For the later part of this study, the theory, method, and system were mainly verified by corresponding scientific experiments, which were designed to promote and enhance children's cognitive ability.

We summarized all the related objects of mathematical knowledge, excavated the relationships between them, and used the object-oriented technology development method

of C#. Technologies used to design and develop the system mainly included: inheritance, polymorphism, cross thread. Finally, a series of methods for supporting the system of children's mathematics education, which could explore the relationship between OAG, was formed.

In the early stages of our research, based on C#, a system was implemented to support the typical issue of "encounter problem", including: the basic vehicle of the moving object, exploration space and object storage, saving and opening of encounter scene, and some evaluation functions. This established a specific situation for children to explore, where they could set up roads, cars, and driving rules by themselves. Obviously, this engagement played a key role in the formation and accumulation of children's rich representation of mathematical principles, and helped children understand the knowledge of speed, time, and distance in mathematics. These basic functions were improved upon and expanded, and a children's mathematical inquiry system that can explore the relationship between OAG has been designed and implemented.

The above system integrated the current teaching system and covered the content of children's mathematical cognitive education in an all-round way. As a comprehensive open-inquiry system, it could provide a basic platform for future experimental analyses of children's mathematical education. In the analysis, we collected the relevant data from teaching applications (e.g. scores, questionnaires or interview data, mathematical ability measurement data), used the log or evaluation data of the inquiry system to automatically extract relevant data, and verified our proposal through comparative tests, statistical analysis of significant differences. Apart from that, we explored the potential to further improve the system.

B. RESEARCH STEPS

We conducted this study with the following steps.

First, we investigated math learning technologies employed in elementary schools: 1) We obtained information about the most frequently used math software by interviewing our cooperative schools; 2) We identified the software with search engines, such as Google and Baidu; 3) We combined the databases, such as the Web of Science, China National Knowledge Infrastructure (CNKI), and patent databases; and 4) We searched for information about such tools.

Second, after a preliminary assessment, they were downloaded and installed on the local computer, installed on the computer, whose functions were identified.

Third, knowledge units were listed in accordance with units of elementary math courses in China, which were tested against the identified software applications to examine whether these programs could effectively support math learning. In addition, the coverage range of these units in elementary math was also calculated.

Fourth, a new approach to categorizing these units was employed. With this approach, all math knowledge units in elementary education were categorized into OAG, which can eliminate the pitfalls compared with conventional ways.

We can break all these units into a comprehensive number of points making it possible to support math learning.

Fifth, an exploring system was developed based on the categorized knowledge points and the functional features of math learning tools. In the process of developing the system, different models were employed in the system based on the diverse types of knowledge points. In this study, dozens of models had been established, such as encounter models, weight models, sequence models, box models, coordinates models, evaluation models, favorite models, Renminbi(RMB, China's official currency) models, item production and web-based distribution models, congregation models; algebra, object, and abacus models,. This study focused on elaborating the most frequently used encounter model.

Finally, we selected several schools randomly to test the newly developed exploring system. In order to identify the exploring effectiveness of the system, a questionnaire survey was conducted and many interviews were administered.

Our survey mainly involves the following aspects for exploration: function, novelty, innovation, characteristics, stability, ease of use, and knowledge coverage of the system. All the math teachers from six different grades were selected with an identical number in each grade so as to maintain an equal number of participants. A majority (98%) of these participants had rich experience of teaching for at least five years, with 71% for even more than ten years. Since these experienced teachers had a better understanding of the content, objectives, and requirements of the textbooks, their participation in the survey can, to some extent, help make our results more convincing, which were further statistically analyzed.

IV. SYSTEM DESIGN AND KEY TECHNOLOGIES

A. SYSTEM DESIGN

According to our above research into the learning software for primary-school mathematics, we found that none of these tools were programmed in C#. We decided to design our exploring system based on C#, in particular, our system mainly focused on primary mathematics, which will not involve complex operations.

All in all, we tried to construct the object-oriented union model to reveal the nature of math for children. There are three innovations. First, we proposed a common base class of children's math and defined the attributes and methods of the base union class. All the objects within primary-school mathematics could be made through the inheritance of the common class. Second, we developed over 100 mathematical tools based on the programming language of C#. These tools could assist teachers of primary math to display and operate the most basic knowledge units of children's math and the relationships among them. Third, we provided a common platform, which was open and sustainable, for all these software, whereby both teachers and students could make mathematical experiments. They just need to adjust these tools according to their preferences.

1) THE THEORETICAL BASIS OF DESIGNING CMES

Our original ideas of designing inquiry-based tools for primary mathematics mainly come from mathematical experiments of evidence-based theory [53]. Experiments in mathematical research are different from that for learning tools. Firstly, their experimental purposes and auxiliary objects are different. Experiments in mathematical research are mainly used to assist researchers while the experiments in mathematics learning tools are used to assist learners in learning, so as to better understand the concepts and principles. Secondly, their experimental tools are different, which depends on the purpose of the internship and the auxiliary objects. Researchers typically use experimental tools in mathematical research, while children generally use experimental tools in mathematics learning. This suggests that the learning experimental tools should consider the characteristics of children and should be simpler and easier to understand than the research experimental tools. Few scholars have studied this area of expertise because most people believe that mathematics does not require physical and chemical tests. In fact, mathematics has its own laws, which can also be understood through experiments. On the one hand, available mathematical tools for experiments are few. On the other hand, mathematics neither has any dynamic physics experiments nor has chemical experiments that could generate new material, resulting in a less experimental study. Seen from this perspective, we can draw the conclusion that it is an excellent means for children to understand scientific laws, including laws for mathematics by experiments.

As claimed by an article [53], the majority of current mathematics teaching is in the development stage of "Presented Theorem, Inference Verification, and Applications". Moreover, it is deficient in the stage of "Observation, Measurement or Experiment, Rules Discovery, and Hypothesis Making," mainly because of the lack of experimental tools, particularly the hands-on experimental tools to facilitate the learning of the students themselves. It was mentioned that some objects or issues in mathematical teaching should be placed in a dynamic lab environment so that students can freely explore and discover mathematical concepts and laws, accumulate a dynamic representation of the concepts, and finally form a correct understanding of mathematical principles.

More importantly, the inquiry-based software of the typical "encounter problems" should maintain the criteria "Independence, Inquiry, and Co-operation" which is included in the new curriculum standard of primary mathematics in China. The mathematical simulation model of learning, in turn, could be used to achieve an in-depth integration of Information Technology and the primary-school mathematics curriculum, which will greatly cultivate their innovative spirit and practical abilities. The inquiry-based learning theory also emphasizes that learners have their own internal structure for learning and stressed that a problem-solving model should be constructed according to the specific situation, based on their original system of

concepts. In addition, it attaches importance to integrity, situation as well as application of the theory, which is the starting point for the construction of these tools. Moreover, mathematical experiments within an exploring theory also raise the concept of dynamic mathematics learning.

2) THE DESIGN OF CMES

The design of CMES was advanced, as in Fig. 2, by collaborating with the teachers of LFDEEE in primary mathematics, and by classifying, abstracting, and analyzing a dozen of textbooks (published by PEPC) of primary-school mathematics. Then, based on the logical integrity of the content of primary-school mathematics, the design of inquiry learning support software was advanced (see Fig. 2).

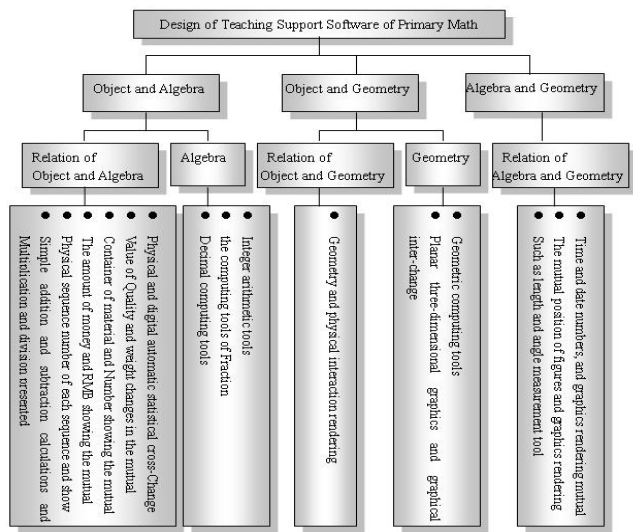


FIGURE 2. Design of exploring system for children’s math.

For example, Geometer’s Sketchpad is actually a tool for geometric computing, which can not only calculate area, perimeter, but also do mapping, graphics, and cutting. It automatically generates tracking marks and it can also be used for quizzical experiments. Besides, an inquiry learning tool for encounters provides another examples, as it is actually a mutual rendering tool for digital position information and graphics.

The author proposes the “multilevel model architecture”, as well as the Five Vertical and Three Horizontal (FVTH) strategy, aiming to explore the system of OAG. “Five Vertical” means that the model can be divided into five layers vertically, while “Three Horizontal” indicates that the model can be separate from three layers horizontally. As a matter of fact, the five vertical layers refer to the bottom scalable logic architecture, the mathematical abstract object layer, the logic layer, the entity representation layer, and the user interface operation layer in an down-up order. While the three horizontal ones include the mathematical evaluation layer, the language description layer, and the layers for evaluation feedback. Particularly, we also put forward a series of model

constructions related to OAG cognitive system, such as the construction of encounter model, sequence basket model, physical basket model, evaluation automation model, object geometry corresponding model, image management model, geometric operation model.

In the process of creating a primary-school mathematics exploring tool for encounter problems, the design and presentation of dynamic representation should be focused on. Additionally, the students’ hands-on abilities and exploring abilities should be paid more attention. In specific model, the author built a logical structure for a software model for primary-school mathematics learning(see the schematics in Fig. 3).

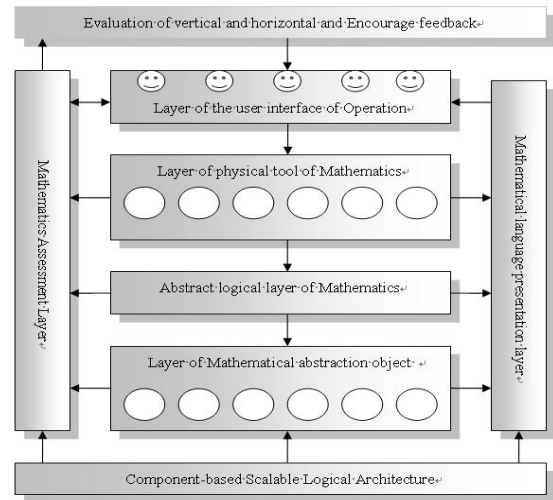


FIGURE 3. Logical structure of exploring system for children’s math.

On this basis, the author built a design of root class for mathematical object and their derived classes of encounter problems of primary-school mathematics(see in Fig. 4).

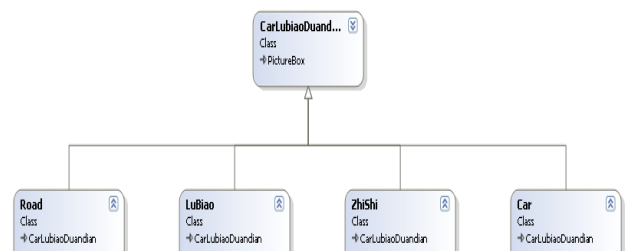


FIGURE 4. Class design of car–road model in exploring system.

The model class includes four categories of objects: vehicles, roads, road signs, and instructions. Instruction includes arrows and their data prompt. These four categories of objects are inherited from the general category of the car road model root class. In this way, it can facilitate different objects to be called under the same operation, simultaneously, in addition to a time object, which can act on all objects. The model reveals the correlation between design objects (see Fig. 5).

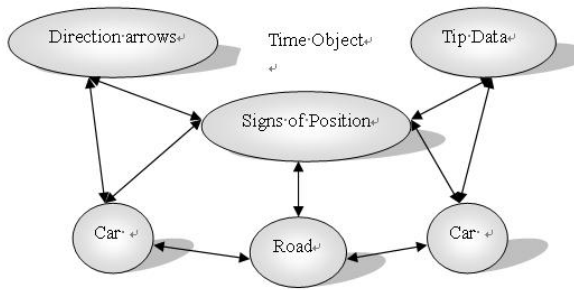


FIGURE 5. Relation of classes in car-road model in exploring system.

The car road model was modeled as $A = F(R, C, L, T, Z)$, where:

- A = Action, namely, car road model to perform the action;
- R Road factor;
- C Car factor;
- L Position factor in road;
- T Time factor;
- Z A directing factor.

Among them, the specific relationship is as follows: $R = R(p, s, l, t)$, where:

- p = A hint picture;
- s The journey length;
- l Length of the image;
- t Distance of road to the top;

as well as $C = C(p, t, r, d, v)$, where:

- p = A hint picture;
- t The ratio of car position at the road's total length from the left side;
- r The binding path;
- d Driving direction;
- v Speed;

and $L = L(p, t, r)$, where:

- p = A hint picture;
- t The ratio of car position at the road's total length from the left side;
- r Binding road;

and $T = T(v, s, b)$ where:

- v = Automotive speed;
- s The driving step value;
- b The legend;

and also $Z = Z(p, c1, c2)$ where:

- p = A hint picture;
- c1 One object of the class of car, signs, road, and position;
- c2 One object of the class of car, signs, road, and position;

and finally $A = A(n, s, f, fs, ts)$ where:

- n = No action;
- s To stop;

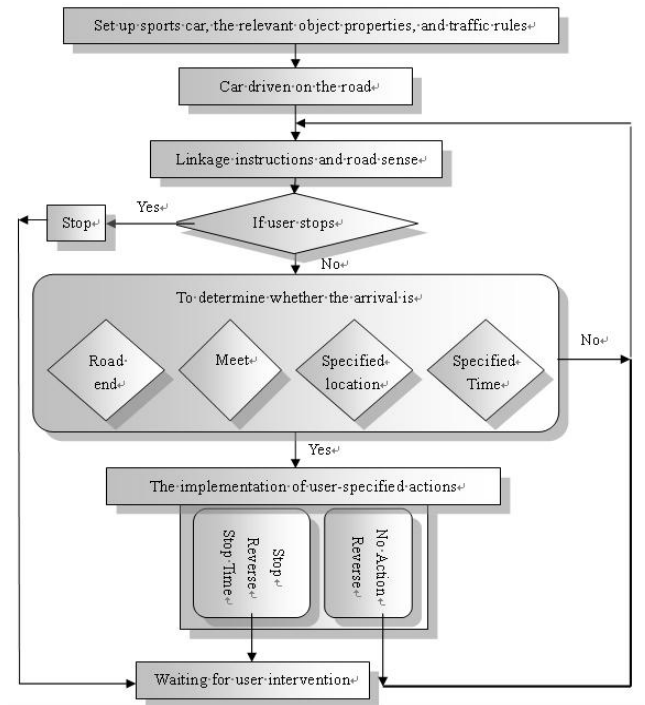


FIGURE 6. Flow diagram of car-road model in exploring system.

- f The U-turn running;
- fs The U-turn to stop;
- ts Time to stop.

Both the signs L on the road and the action execution time T can be set arbitrarily, which ensures that experimenters could control the car in all scenes and make further exploration. In this system, action execution time refers to the virtual time of vehicle driving. When the driving time reaches the specified time, it will automatically execute the driving actions set by the user, such as stop, U-turn driving, or U-turn stop. If it's two cars driving, you can set the execution action after the arrival time for each car independently, or stop it altogether, so that children can explore and observe their distance. It is noteworthy that the virtual time is different from the real time. Additionally, the virtual time is always consistent with the virtual speed and the virtual driving distance. The user can control the car to run to the designated position or to run for a certain period of time. The user may also set actions for the car to perform in advance. For example, you can ask the car to turn back when it reaches the middle of the road or ask the car to stop at a certain time. The execution action A in the above formula includes both turning driving f and turning stop fs. Indeed, starting to drive after the turning stops is equivalent to realizing the turning driving f in two steps. In other words, children can take a step-by-step approach to solve any kind of linear encounter problems and that the car can be stopped at the right time to enable them to analyze the situation. The specific flow diagram of the car road model is displayed in Fig. 6.

B. KEY TECHNOLOGIES

The CMES is a complex and comprehensive support software system for elementary mathematics teaching and learning, which involves multi-disciplinary research and development. The CMES is a hybrid application of online and offline modes. It is mainly offline applications with online updates and exercise publishing, as well as other functions, that can be used in the classroom. The system is configured by explorers, but it requires teachers to demonstrate its configuration or operation for the first time. Children can then configure their specific parameters to explore further.

The key technologies of CMES mainly include:

1) THE OVERALL CONSTRUCTION OF CMES AND RELATED TECHNOLOGY

The system should integrate the content from different grades of elementary mathematics teaching, include all the objects and their relationships, and be accessible to more external mathematical objects. Therefore, the overall system and its embedded design technology are of great significance.

2) THE RELATION ENGINES OF CMES

The relation engines of exploring system include five types: relation engine of object and algebra, of object and geometry, of algebra and geometry, and of algebra and that of geometry.

3) INFORMATION SEARCH ENGINE (INCLUDING TYPE IDENTIFICATION OF OBJECT) AND RECOGNITION OF NAMED ENTITY

In the transformation between objects and graphics, it is often necessary to find the shape similar to the specified object, or to find the object similar to the specified figure. This technology can ensure that the corresponding results can be retrieved.

4) ANSWER INFORMATION EXTRACTION AND DIVERSIFICATION EVALUATION

There are many forms of diversified evaluation. In addition to evaluating the commonly-used objective questions, it can also be used for the evaluation of classification, sequence, connection, possibility, overall arrangement, which is of vital importance as well.

5) OPEN SYSTEM

Both the framework and content of the system are open, meaning that users can import their own content into the framework to update the system. As for the diversified evaluation in the system, once the user updates the content, it can give immediate evaluation automatically. Even students with different scores can get separate incentive comments, which are customized in advance by the teachers. There are three ways for external cognitive tools to enter the system. To be specific, one is Flash mathematical cognitive animation, which can run directly by pressing the button “external program” in the system. The second way is to use the

question button to integrate external questions or web pages into the system. The third way is through the mathematics textbook unit menu. In the menu of CMES, the tools that can be used in each unit of the course are listed together. For one thing, teachers can use these tools to teach. For another, students can use these tools to explore. The basic knowledge of mathematics is gradually learned in the process of teachers’ teaching and students’ exploration. The unit menu of this textbook is closely related to the specific textbooks used by students. Textbooks are not only a main source for students to acquire basic knowledge of mathematics, but also the main supporting resources of this system. Textbooks are generally static, dead and integral, while inquiry is dynamic, live, and can be displayed dynamically step by step, which is more conducive to the progressive and in-depth learning content. This system does not exclude teachers and textbooks. On the contrary, they complement each other, and each has its own advantages. If you open the associated configuration file of CMES, type the name of the external program, and give a corresponding name inside this file, and you can use the external program. Furthermore, there will be a fourth way in the future. The fourth way will provide the API of CMES for the second encoding function, which can easily embed the foreign program as an object entity.

6) NETWORK COMMUNICATION MIDDLEWARE

The transmission of messages between the system and the user uses network middleware. The information of evaluation results for students can be transmitted to the teacher’s machine, and then teacher’s customized comments can also be automatically transmitted to the student’s machine. Furthermore, teachers can update the question bank automatically through the system update deployment release.

Many of these technologies of CMES are very difficult to implement, such as the overall construction and its related component embedding technology. Besides, the relation engines among OAG have their own challenges. Nevertheless, in a specific design, especially for the application of a specific field in the system, we can focus on some key technologies, or even use other methods to replace key

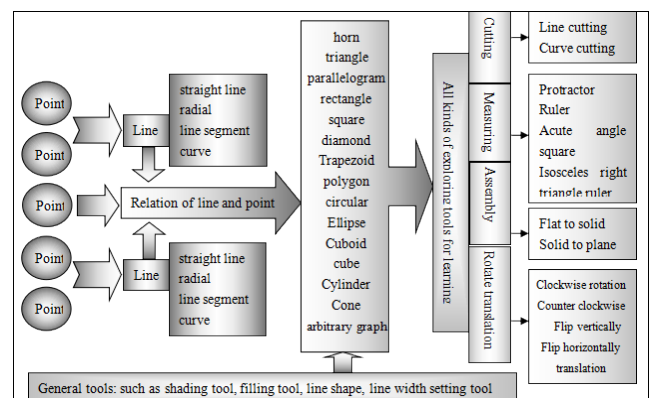


FIGURE 7. Relation operation and general tools of geometry.

technologies, such as geometry relationship engine. Since the existing Geometer's Sketchpad or MPLAB is very advanced, we can temporarily use the existing technology to replace the corresponding development and its implementation. Below is the graph of the relation operation and general tools of geometry. Due to layout limitations, other designs will not be involved here.

V. THE IMPLEMENTATION OF CMES

The object-oriented C# was chosen as the programming language to make the exploring system. This research realized various forms of computer rendering through GDI and technology within the objects of each class, and successfully displayed most relationships between topics within children's math. The structure interface, the base class, and its derived class object were also realized. The whole system is a scalable hierarchical open system, and the core of the system is the executive engine of operation, which is mainly used to support children's exploration of the relationships between mathematical objects.

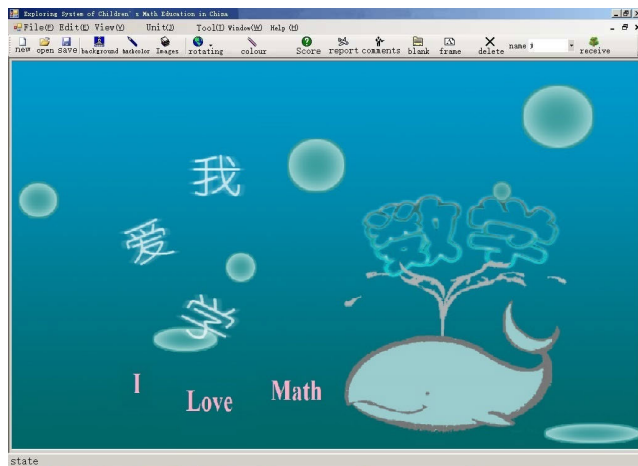


FIGURE 8. The main interface of exploring system for children's math.

With the adoption of the object-oriented development, the system had good re-usability. In class module interface, the user could absorb a variety of external resources or pictures of primary mathematics into this system through operation. This gives it a strong opening character. In particular, Microsoft Visual Studio was chosen as our specific platform for development. As for model development, we adopted not only the object-oriented classes, inheritance, encapsulation, but also the overloading, delegation, generics, and reflection, which enormously increased the flexibility, re-usability, and maintainability of the system. The key technologies to support the system can be listed as follows.

A. OVERALL GENERAL TOOLS

In order to explore different mathematical objects, we designed and developed general tools that do not depend on specific mathematical contents in the system. There are more than 22 interfaces in this part, including the interface

of inquiry practice, auxiliary question setting, score reporting, score receiving, three-level comment display, and more. These tools play a basic supporting role in the whole inquiry system. These general tools include a series of tools, such as exploration experiment space, free writing and drawing space, the tools for rotation, coloring, graphics and text box, deletion and selection, comment, question scoring, score receiving, question addition and deletion, animation playing, website publishing and updating, and a tool for new creation and saving of exploration space.

B. EXPLORATION OF THE RELATIONSHIP BETWEEN OBJECTS AND ALGEBRA

The relationship between objects and algebra is one of the essential relationships. In order to explore the relationship between the physical objects and digital numbers, we designed and realized 104 tools in total in this technology. This part mainly focus on object algebra corresponding cognitive exploration tool (Arabic numbers, abacus numbers, and different colors), object favorites, digital favorites, basket tools, RMB basket cognitive tool, balance tool, sequences of different kinds, transaction arrangement exploration, the evaluation tool, the inquiry of the digital coding, charts of different kinds, the possibility and evaluation, possibility inquiry tool, and so on.

Besides, each tool can be divided into several types so as to cover the contents of elementary mathematics teaching completely. For example, the above-mentioned sequence tools can be divided into seven types: "double chart sequence, double color sequence, equal difference sequence, sum sequence, equal ratio sequence, square sequence, and difference equal difference sequence". In addition, the possibility inquiry tools include cube throwing, coin throwing, and rotary table pointer possibility inquiry. Another example, the balance tool can enhance students' understanding of objects and weight numbers, while the tool for object number corresponding inquiry will enable students to experience the group number changes caused by object movement in real time.

C. THE EXPLORATION OF ALGEBRA

The relationships within algebra are also of great significance. To illustrate the relationships among digital numbers and algebra, we designed and realized 23 tools in total in this technology. This part mainly includes the tools for "factor inquiry, operation inquiry, drawer principle exploration, common multiple and evaluation, fraction to decimal and percentage, fraction graphic representation interface, shopping interface, fraction to fraction interface, sketchpad interface, touch ball, drag in or move out (possibility inquiry) tool".

D. EXPLORATION OF THE RELATIONSHIP BETWEEN OBJECTS AND GEOMETRY

Also, the relationship between objects and geometry remains essential. This technology, through which children can explore the relationship between physical objects and

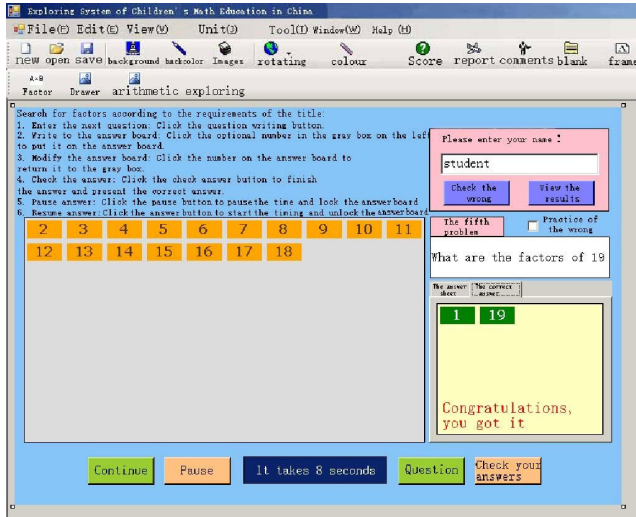


FIGURE 9. Factor inquiry interface of exploring system.

geometric figures, was further promoted. There are 24 interfaces in total, such as display 3D cube interface and graphic manipulation interface. Still, this part mainly involves the interface for “object favorites and geometry favorites and the corresponding presentation of objects and geometry, learning classification (method 1, method 2, independent operation exploration), classification basket and classifiable objects and graphics, evaluation status, grading classification, evaluation exploration of classification basket, classification basket attribute, mirror symmetry, and axis symmetry”.

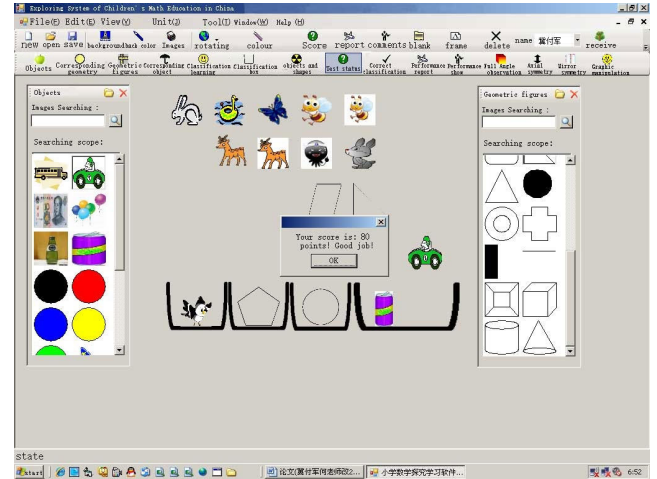


FIGURE 11. Inquiry interface of classified evaluation basket.

F. EXPLORATION OF THE RELATIONSHIP BETWEEN ALGEBRA AND GEOMETRY

For this part, there are 17 interfaces, including: “date display, time display, clock recognition, dial scale recognition, clock recognition, time and scene correspondence, time cognition clock face exploration, time cognition scene exploration, adding road, car, road sign, and distance indication, road attribute, car attribute, road sign attribute, driving rule setting, distance prompt attribute”.

It’s simply impossible to list all of the corresponding interfaces of mathematical objects and their relationship, therefore, we just took encounter problems as an example to briefly introduce its interface and application here. More and more researcher have begun to study this problem in recent years [57]–[59]. They put forward many solutions about how to make students approach mastery when learning this problem. However, most of these solutions were based on PowerPoint or Flash. These solutions would not let teachers easily or voluntarily control the parameters of all encounter problems. Our encounter tool of moving is based on C#, so that teachers could easily set all parameters regarding motion. The core GUI of the car road model of the moving exploration tool is shown in Fig. 12.

Through this GUI, teachers could set parameters of road, car, special position, rules, and prompt. Under the rules menu with a label button, they could choose moving a single car or a double car. Each kind of situation has a few parameters to be set. Users are able to perform all kinds of orders, for instance, “which double cars to run, what action a car should do, and when the car should take action”. Apart from that, they could choose by which condition (time or length of driving) the car should take action. In short, there are five kinds of actions for the car: “stop driving; continue driving; reverse driving; stop and reverse; and time stop”. The last action, time stop, would control both cars. Fig. 13 not only shows the driving situation, but also displays the entities of the car road model.

Next is the practical application of the program. For example, here’s a Maths question: “Suppose a road had a

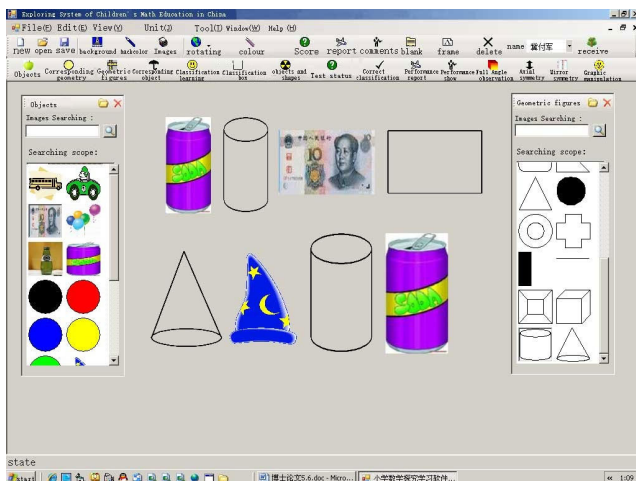


FIGURE 10. Inquiry interface of objects and geometry of exploring system.

E. THE EXPLORATION OF GEOMETRY

There are 12 interfaces in this part, including the interface for “geometry favourite, measuring cup, protractor, ruler, quarter isosceles triangle ruler, 30 degree acute angle rectangular ruler, reading ruler (practice, feedback), area grid and its coloring, translation, meter rotation, flipping”.

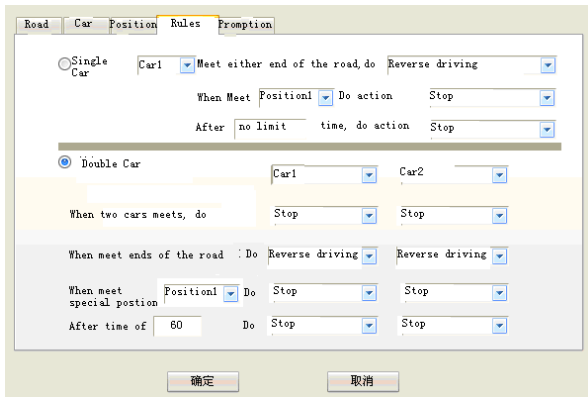


FIGURE 12. Setting of car-road model in exploring system.

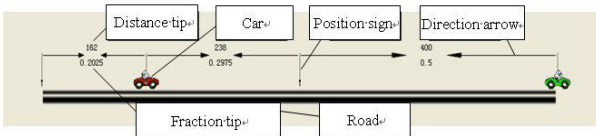


FIGURE 13. Entities of car-road model in exploring system. Note that most of the entities contained in the GUI, such as cars or roads, are adjustable in size and movable in position. In Figure 13, for the learning of children's encounter problems, the "Distance tip" element is indeed more important. In fact, there are two elements hidden in the attributes of the car entity and not displayed, that is, "speed" and "driving time", which can be displayed through software settings. Another element in this figure is also very important, that is, the "Position sign", because in the setting of Figure 12, it is equivalent to setting it to "stop sign". Therefore, when two cars reach this position, they will execute the stop action and stop driving.

length of 1000 meters, a red car started running from its left side with a speed of 8 m/s. At the same time, a green car started from the road's right side, moving towards the red car with speed of 12 m/s. When the two vehicles met, how far had the red car driven?"

The following instructions are given to solve the raised question.

First, click the road button and car button to generate the objects of a road and two cars. Then, right-click the roads or cars to select properties to set the path length or speed of their vehicles and their starting position.

Second, establish the driving rules, select pairs of cars, and set the motion state of the two cars (e.g., that they must stop when meeting).

Third, set the property of prompt between left side and red car by clicking the prompt button.

Finally, click the drive button to figure out the real dynamic driving conditions, with prompt digitally changing in real time. Meanwhile, students could stop the movement of the two main vehicles at any time and view the distance between them.

Teachers could inspire students to think and encourage them to explore "why the two vehicles met at 400 m from the left end of the road; If the red car accelerated to the speed of 40 m/s, where would they meet? If the green car's speed

changed to 6 m/s, where would it meet the red one?" Teachers might motivate students to further explore this software to verify whether the results of the experiment accord with their expectation and predictions. The system could promote students to think over this inquiry exploration thoroughly.

After laying a solid foundation for questions of motion, teachers could direct students to summarize relative formulas through our platform, such as " $S = vt$ ". Furthermore, teachers should give instruction that if two parameters of distance, time, and speed in the formula are known, then the third one could be calculated. For the opposite movement question raised above, the rule summarized by students is that distance always equals the sum of both rates multiplied by time. We could calculate the time spent for the two cars from the beginning to their meeting. Since we knew the speeds of both cars and total distance, and the result was $1000/(8+12) = 50$, and we also knew the red car's rate was 8 m/s, we could also figure out the distance it had driven. The result was $50 * 8 = 400$. This is the answer to the above question. By using this tool, students can constantly change the speed of two cars, observe the location where the two cars encounter, and think about the relationship between the speed ratio of the two cars and the location where they meet. Therefore, through this kind of continuous simulation, students will manage to accumulate a large number of correct images gradually so as to better grasp the key points.

Particularly, the encounter tool was only one of the massive exploring system with 118 independent tools that covered most objects and relationships of primary mathematics. We put this system into trial use at three primary schools in province of Guangdong, with more than a thousand students and teachers participating in our experiment. Also, an experimental design consistent with Randomized Educational Trials (RET) was arranged for the feedback of the efficiency.

VI. QUANTITATIVE RESULTS AND DISCUSSION

CMES was first trained and explained in three primary schools in Guangdong Province. Subsequently, the mathematics teachers from 6 different grades participated in the training and tried to introduce CMES into the classroom, and then, three teachers demonstrated the course of applying CMES. Finally, relevant data and information of both the teachers being trained and their application of CMES for teaching were reflected in questionnaires.

The total sample of the survey was 91 teachers (including interviewees and questionnaire respondents). In the upper-level class, each student had one machine, and some classes, but not all, participated from each grade level. From each grade, the research team chose one or two classes randomly, according to the requirements of RET. There were more than 1000 students in total using the CMES (some grades only had one class). In order to ensure the grade balance of the investigated teachers, 15 to 16 teachers were selected for each grade from Grade 1 to Grade 6. Considering that the elder teachers are familiar with the teaching content, with accurate teaching needs and clear curriculum objectives,

and are more concerned about the characteristics of teaching objects in primary mathematics, the vast majority of the teachers surveyed (98%) were those with rich experience for at least five years. Even more than 71% of these participants were those who have been teaching for more than ten years, as shown in Fig. 14.

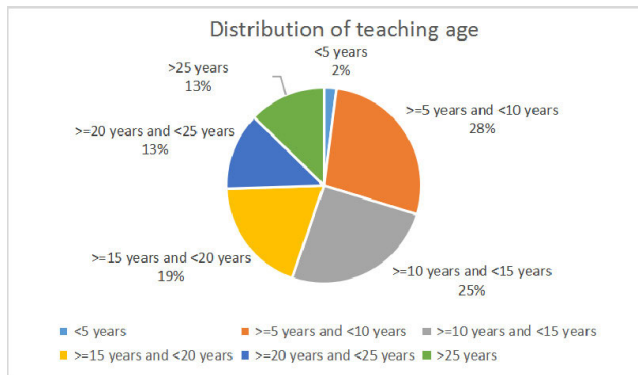


FIGURE 14. Teaching age distribution of the teachers surveyed.

According to the survey, 89% of these mathematics teachers thought the system was very useful or the most useful.

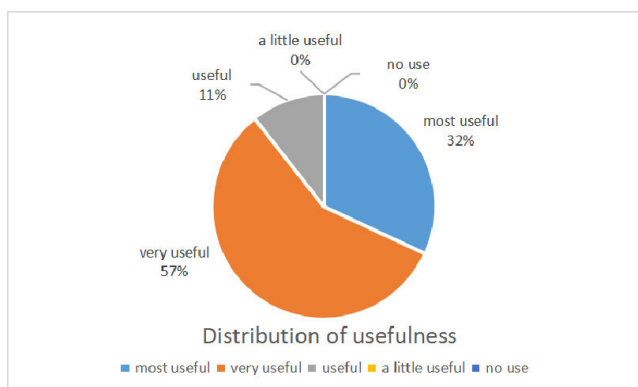


FIGURE 15. Distribution of usefulness.

Regarding the novelty of the system, 77% of the teachers added that they had never used such system before.

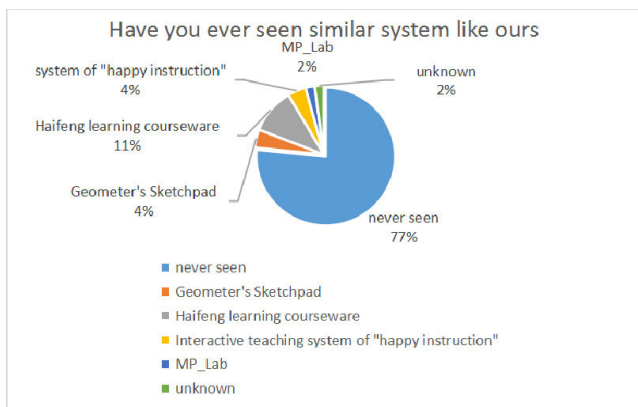


FIGURE 16. Similar software survey.

As for content coverage, 72% of them believed that the system could cover 80% of the teaching content in the curriculum. There was no feedback of below 50% for that.

With respect to the characteristics and creativity of the system, 40% of the teachers expressed that the system was characterized by openness, independence, inquiry, systematicness, and diversification. Besides, 25% of the teachers considered that the system was unique in its scoring function and feedback. Meanwhile 19% regarded the system to be controllable, changeable, simple and practical. Still, the rest of the respondents, 16%, believed that the system enjoyed an edge in its vivid and dynamic presentation to comprehend a math problem.

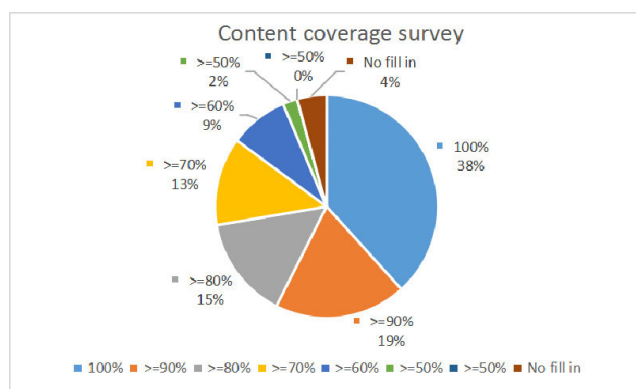


FIGURE 17. Content coverage survey.

At the same time, the participating teachers also make suggestions for improving the system in terms of supplementation of primary mathematics knowledge content and promoting software function. For example, they hope to add the contents of modern primary mathematics textbooks of different versions and Mathematical Olympiad, published by various press, such as Jiangsu Education Edition, Beijing Normal University Edition, Institute of Psychology, and Chinese Academy of Sciences, to expand and deduce the three-dimensional graphic formula. They also express a strong desire to perfect the software installation, PowerPoint media support, question system, and interface.

Considering that the exploring system aims to improve students' cognitive ability, we will give a case to help illustrate its efficiency in practical application. In the experimental period of our project, a teacher, who had a rich teaching experience for more than 20 years, adopted our system to explain "volume and volume unit" to students in Grade 5(92 students from two classes), and assigned the homework as usual. After correcting the homework that night, much to her surprise, almost 99% of the students were right in terms of volume, which had never occurred in previous years. According to her experience, there was only less than 50% students who could give the right answer previously. In addition, a comparative survey was conducted before and after the application of the CMES. As shown by results, after using the exploring

system, the children's interest in mathematics increased from 90% to 99%. During the interview, almost all the children said that they liked this exploring system, feel "very interesting", and hoped to "have a try next time".

Take the model of encounter problem as example. It implements the development of the encounter problem of primary mathematics, and is used for virtual encounter situations, which resolves in the abstract difficulties in primary mathematics learning. Beyond that, it provides an inquiry-based learning tool for primary mathematics in China, filling up a weak spot among current primary mathematics supporting tools.

CMES provides children with a large number of visual resources which are embedded in the toolbar of different exploring tools. Most exploring tools are interactive. Children can easily operate through them. The system provides two ways to start exploring, in which one is to provide the entrance of exploring tools for teaching material unit learning, and the other is to offer the entrance of exploring tools based on the division of OAG.

Moreover, CMES is mainly used in the learning for new courses. In fact, after teachers get familiar with the corresponding points of all the tools in CMES, as long as the teaching content contains the points, they can use the CMES in vast fields, such as course introduction, new lesson learning, classroom practice, course review, homework, and learning exploration. The main reason lies in that CMES provides an open platform for both teachers and students, where resources and exercises can be added freely. It is unnecessary to use the system throughout the whole class all the way since most skilled teachers can distinguish the simple points from the difficult ones. So they can apply this system to explore when explaining difficult points, because it provides necessary scenes or tools for students to explore these difficult points. From the current practice, there are no behavioral problems, no interest or other problems appearing. On the contrary, it plays an obvious role in promoting children's interest in mathematics.

In conclusion, our exploring system offers a useful platform to practice the theory of mathematical experiments exploration [25], whose application shows the effectiveness and reliability of the solution.

VII. CONCLUSION

On the one hand, this paper not only studies the open, exploring space based on the relationships among OAG, but also investigates the storage, object presentation, unified operation, relationship linkage presentation, generation, open absorption, and update technologies of the open inquiry space. On the other hand, it studies the inquiry technologies of various relations of object and algebra, relations of objects and geometry, and relations of algebra and geometry placed in the open inquiry space. Finally, a mathematical cognitive system for children was designed to explore the essential relationships among OAG.

A. THIS PAPER PROPOSES A GENERAL, NORMALIZED, SUSTAINABLE AND IN-DEPTH EXPLORATION SYSTEM OF MATHEMATICAL COGNITIVE SPACE WHICH HAS THE FUNCTION OF OPEN ABSORPTION AND RENEWAL

In this open exploration space, all cognitive objects implement the unified operating procedures. The same operation also includes the progressive function level of the cognitive system. The higher the level, the more parameters can be set and the more functions it possesses. Therefore, it can satisfy the needs of children from different grades with the same concept and different depth of content. Moreover, it also proposes an open exploration space, which separates all cognitive tools into mathematical object elements, relationships, and cognitive functions, and then integrates them into the same open exploration space again. It not only allows the secondary development of mathematical objects, but also the integration of external cognitive tools into the system, which is of pioneering significance. This is also one of the core contributions of this work. Yet, because there are many new free tools on the Internet, we can't study them one by one, so there is still room for improvement in the integration of tools in this paper.

B. THIS PAPER PUTS FORWARD A MAP OF THE MATHEMATICAL OBJECTS AND THEIR RELATIONSHIPS IN MATHEMATICAL COGNITIVE EDUCATION, AS WELL AS A SET OF COGNITIVE METHODS AND TOOLS THAT CAN COVER THE MOST KNOWLEDGE CONTENT OF MATHEMATICAL COGNITIVE EDUCATION AND HAVE VARIOUS CHARACTERISTICS TYPICAL OF MULTIPLE GROUPS OF OBJECT-ALGEBRA RELATIONSHIP, OBJECT-GEOMETRY RELATIONSHIP, AND ALGEBRA-GEOMETRY RELATIONSHIP

In addition, the internal structure of the base class in the mathematical cognitive education system is proposed. All the other mathematical objects are derived from the base class, and the derived class can automatically inherit and obtain the relevant functions. The base class in the plan is an original model that can be derived and extended. At present, since many cognitive tools cannot be derived and reused due to their independence, which insulates them from the essential relations of things, numbers, and shapes that should be possessed by external mathematical objects. Through the derivation of basic objects and the interaction between the formulations of mathematical objects, this system attempts to solve this problem so as to make the breakthrough of cognitive tools' interoperability. This method is therefore one of the core contributions of this topic.

C. THIS PAPER PROPOSES A SET OF METHODS AND MODELS FOR CHILDREN'S MATHEMATICAL EDUCATION TOOLS, AIMED AT EXPLORING THE RELATIONSHIPS AMONG OAG

Relevant evaluation technology has been provided to enable children learning math to explore the relationships among

OAG, supporting both the horizontal and vertical evaluation of unit knowledge points. Currently, there are numerous kinds of cognitive tools. Although the research on the relationship between geometry and algebra is relatively mature, that for algebra and geometry is not advanced. In addition, there are many developers who focus on technological development, which can be unified into children's mathematical cognitive education system. As mentioned before, this study also proposes a multilevel model architecture, FVTH, which also makes pioneering contributions to this paper.

This research breaks through the limitation of the original encounter system while realizing the innovation of the OAG inquiry integration system. This platform can present and operate all kinds of integrated tools, which can be used to operate, explore, and evaluate all the objects, algebra, geometry, and their relationships. This study also includes the application of the system in the field of mathematics cognitive education, resulting in innovative researches in teaching practice. The new technology of this system, characterized by the exploration of OAG and their relations, enables students equipped with a computer or a tablet to carry out the practice of innovation compared to the conventional classroom mathematics teaching mode. It is unimaginable for conventional teaching, which relies on teachers and PowerPoint.

It's generally acknowledged that the creativity of the critical breakthrough depends mainly on intuitive thinking, or image thinking [54]. The inquiry-based encounter tool provides the rich object appearance and relationship representation of encounter problems. These appearances and representations could be seen as the thinking material of the image thinking and intuitive thinking, which are the core components of creative thinking. Thus, it lays a solid foundation for cultivating creative thinking for primary students.

It is noteworthy that our work raises a solution to the problem of inadequacy of math software for children, rather than list all possible problems/exercises in Maths that they might encounter. Although this exploring system cannot establish mathematical principles, it succeeds in providing effective support for the learning and comprehending of mathematical principles. Based on the coverage of CMES, after finishing primary mathematics learning, we can continue to carry out our research for junior high school [55].

Next, we will spare no effort to further improve and optimize this system. Additionally, we would advance its learning mode as well as the effects of learning applications. The formation of the full content of primary-school mathematics supporting software supplemented by the scientific guidance of teachers, is expected to solve the bottleneck problems ("The volume of the water in a barrel depends on its shortest plank"). Meanwhile, it would be conducive to comprehensively defusing the difficulties in primary mathematics learning, highlighting the focus of learning, boosting students' hands-on operational capability and capacity of self-exploration, improving the overall quality of students, and increasing the quality and efficiency of primary mathematics learning. From the practical point of view, a monitor

mechanism for error-correcting on mathematics could be constructed, thereby, aiming to figure out their mistakes in time, and make intensive practice to enhance the relevant weak points, which would greatly improve their competence and creativity in math. It is universally acknowledged that mathematics is fundamental to all disciplines. Undoubtedly, the cultivation of mathematical thinking and capabilities would significantly contribute to the innovation and prosperity of the whole world.

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