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Decision-Making System Based on a Fuzzy Hierarchical Analysis Process and an Artificial Neural Network for Flow Shop Machine Scheduling Model Under Uncertainty

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ABSTRACT The management of the uncertainty existing in any production system is fundamental to define machine scheduling models that allow programming production instances attached to the real world. In this research, a generalized decision-making system is developed for the management of uncertainty existing in flow shop machine scheduling models. The system assessment the uncertainty existing in internal and external factors that influence the decision-making process of production programming experts, and that is decisive in a final machine scheduling. The system is based on the combination of the Fuzzy Hierarchical Analysis Process, a membership analysis, and an Artificial Neural Network (ANN). The system allows to concentrate the experience of experts in machine scheduling and generalize their knowledge. The efficiency of the system is verified with a Fuzzy Hierarchical Analysis Process Model, the ''ANN toolbox'' preloaded in MATLAB and variety of structures of an Artificial Neural Network. The results are validated in an industrial application and the system is contrasted against an expert. The results show the efficiency of the system as it defines and predicts the final machine scheduling of production instances; the joint assessment of variables that add uncertainty to the production system allowed to reduce delays in product deliveries.

INDEX TERMS Artificial neural network, decision-making system, flow shop, fuzzy hierarchical analysis process, machine scheduling, uncertainty.

I. INTRODUCTION

Machine scheduling is responsible for organizing, choosing and scheduling the efficient use of resources in such a way that products or services are produced within a reasonable agreement with customer demand. Flow shop machines scheduling is one of the main models that adapts strongly of the manufacturing industries [1], [2]. In Mexico, most companies in the ''leather-footwear'' manufacturing sector are characterized by having a flow shop machine scheduling model, where their main needs are related to the properties of completion times which seek to avoid late delivery of orders. In this sector, much of the production programming is based on the experience of experts who have to deal with a complex decision-making process involving external and internal alterations that affect effective machine scheduling; these alterations are considered as the uncertainty existing in the production system.

In the literature review, a dominant trend has been identified in the formulation of stochastic and deterministic solution models and procedures, to focus on the machine scheduling models [3]; these processes do not consider the inherent uncertainty of production systems. In the same way, several approaches of multi-criteria assessment methods have been identified to address decision-making processes, such as: *Analytic Hierarchy Process* (AHP),

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Preference Ranking Organization Methods for Enrichment Evaluations (PROMETHEE), *Technique for Order of Preference by Similarity to Ideal Solution* (TOPSIS) [4]–[6]; such work is based on the knowledge of experts. The AHP and its fuzzy AHPD variant stand out in the last decade [7]–[9], its main applications are oriented to the choice of manufacturing strategies, flexibility in production processes and action plans in various circumstances [10]–[16]. Additionally, the works of [17]–[20] have been identified where uncertainty is used as a tool that strengthens the solution convergence of solving methods of machine scheduling models, and not as a determining variable in the final scheduling. In the literature review, a transition is being conceived to the development of research involving *production planning models under uncertainty* [21]–[23]; it can be said that, tendency purpose is to generate models that adhere contextualized variables subjectively and that analyze the extent of belonging in which these variables really influence. This exposes an important area of research.

On the other hand, the Artificial Neural Networks (ANN) have tried to position themselves as one of the main exponents to treat decision-making processes in productive systems [24]; however, they are deficient in addressing resource allocation problems [25]–[27]. The approaches proposed by [28]–[31] show areas of opportunity; such approaches require specific adjustments to both input data and final results, evidence that their procedures are biased to show good performance. Approaches that combine ANN and AHP [32], [33] have been identified that are oriented towards generalizing the selection of alternatives; one of the important benefits of combining ANN and AHP is the ability to generalizing the knowledge experts to solve real-world problems.

Under this context, it can be said that uncertainty has been defined ambiguously and only for specific scenarios of productive systems, which has made it impossible to develop methods or tools for a generalized application. Therefore, the complexity involved in machines scheduling, in a real-world context, not only involves the development of algorithms that converge to optimal solutions, but also the development of systems that support the complex decision-making process experienced by the production programming expert. This has motivated the development of this research.

The contribution of this research is a generalized decision-making system called *Fuzzy Neuro Analytic Hierarchy Process with Extension Analysis (AHPND-"Extent")*. The system is integrated by the Fuzzy Hierarchical Analysis Process (AHPD), the principles of a membership analysis ("Extent Analysis") [34], [35] and a Multilayer Perceptron Artificial Neural Network (MPANN). The system is used to assess the uncertainty existing in the flow shop machine scheduling general model. The system makes it possible to analyze the extent to which internal and external factors influence the decision-making process experienced by production programming experts when issuing final machine scheduling. Therefore, the system concentrates the experience of experts and generalizes its knowledge to define or predict the most appropriate machine processing sequencing.

After a brief introduction to the topic discussed in Section 1, the rest of the document is organized as follows. Section 2 defines the concept of *uncertainty variables* and the *flow shop machine scheduling model under uncertainty*, referred to in this research; the fuzzy triangular numbers, the multilayer perceptron and the "Extent" Analysis are then described. Section 3 shows the decision-making system AHPND-"Extent". In Section 4, the proposed system is implemented and validated in an industrial application. In Section 5, the discussions and conclusions are stated.

II. CONCEPTUAL BASIS

A. UNCERTAINTY VARIABLES

In this research uncertainty is addressed using the *possibility theory* which deals with the possible rather than probable values of a variable, the possibility being a matter of extent of influence and not a probability of occurrence. Production programming experts play an important role in assessing the extent to which a variable that generates uncertainty influences. Therefore, from the point of view of this research, an *uncertainty variable* is a variable that is subjectively contextualized by an expert and to which the extent to which it influences a productive system can be assigned. In this research, the extent of influence of an *uncertainty variable* is quantified by a fuzzy scale and the use of fuzzy triangular numbers in the range of 0-1.

B. FLOW SHOP MACHINE SCHEDULING MODEL UNDER UNCERTAINTY DEFINITION

In this investigation, a *flow shop machine scheduling model under uncertainty* is defined as a flow shop model that integrates the inherent uncertainty of influential internal and external variables directly and indirectly when creating a final processing sequencing. The goal is to minimize processing completion times by focusing the analysis on the existing uncertainty.

The manufacture of footwear is one of the main productions processes of the manufacturing companies of the sector "leather-footwear" which can be represented as a *flow shop machine scheduling model under uncertainty*. Considering the experiences and subjective issues of experts in the programming of footwear production, the following *uncertainty variables* have been identified for this type of manufacturing companies:

1) Initial Dispatch Criteria (CDI): Order size (CDI-TP), Arrival Order (CDI-OLL), and Priority (CDI-P).
2) Customer Payment Criteria (CPC): Prepayment (CPC-PA), Delivery Payment (CPC-PE), and Funded Payment (CPC-PF).
3) Materials Supply Criteria (CSM): Raw Material Supply (CSM-AMP), and Workforce (CSM-FT).
4) Inventory Criteria (IC): Maquila + (CI-M+), Maquila - (CI-M-), Inventory in Process (CI-IP) Delivery

5) Time Criteria (CTE): Completion Time (CTE-TF), Due Date (CTE-DD), and Deadline (CTE-DL).

C. FUZZY TRIANGULAR NUMBERS

The AHPD incorporates fuzzy logic by replacing crisp numbers with fuzzy triangular numbers [7]. A fuzzy set can be defined as a convex function. A trapezoidal or triangular function closely approximates the convex function. Therefore, fuzzy triangular numbers are convenient in fuzzy environments due to their computational simplicity and their ability to promote the representation and process information [7], [36].

A fuzzy number M in R is a fuzzy triangular number if its membership function $\mu_M(x) : R \rightarrow [0, 1]$ is equal to:

$$\mu(x) = \left\{ \begin{matrix} \frac{x}{m-l} & -\frac{l}{m-l} & x \in [l, m] \\ \frac{x}{m-u} & -\frac{u}{m-u} & x \in [m, u] \\ 0 & \text{otherwise} & \end{matrix} \right\} \tag{1}$$

where $l \leq m \leq u$, l y u are the lower and upper limits of the support value of M respectively, and m is the modal value. The fuzzy triangular number can be defined by (l, m, u). The support of M is the set of elements $x \in R|1 < x < u$. When $l = m = u$ is not a fuzzy triangular number, by conviction. When there are two fuzzy triangular numbers $\tilde{A} = (a_1, a_2, a_3)$ y $\tilde{B} = (b_1, b_2, b_3)$, their operating laws are as follows:

$$\begin{aligned} \tilde{A} \oplus \tilde{B} &= (a_1, a_2, a_3) \oplus (b_1, b_2, b_3) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3) \end{aligned} \tag{2}$$

$$\begin{aligned} \tilde{A} \otimes \tilde{B} &= (a_1, a_2, a_3) \otimes (b_1, b_2, b_3) \\ &= (a_1 b_1, a_2 b_2, a_3 b_3) \end{aligned} \tag{3}$$

$$\tilde{A}^{-1} = (1/a_3, 1/a_2, 1/a_1) \tag{4}$$

$$\begin{aligned} \lambda \otimes \tilde{A} &= \lambda \otimes (a_1, a_2, a_3) \\ &= (\lambda a_1, \lambda a_2, \lambda a_3)\lambda > 0, \lambda \in R \end{aligned} \tag{5}$$

D. "EXTENT" ANALYSIS

The principle of uncertainty assess processes that are carried out in this research are referred in [34], [35]. Let $X = x_1, x_2, \ldots, x_n$ be a set of objects and $U = u_1, u_2, \ldots, u_m$ a set of objectives, each object is taken, and the extension analysis is performed for each objective, gi, respectively. Therefore, m "Extent" Analysis values can be obtained for each object:

$$M_{gi}^1, M_{gi}^2, \ldots, M_{gi}^m, \quad i = 1, 2, \ldots, n \tag{6}$$

where all M_{gi}^j $(j = 1, 2, \ldots, n)$ are fuzzy triangular numbers.

The steps of the "Extent" Analysis can be defined as follows:

1) THE FUZZY VALUE "SYNTHETIC EXTENT"

The fuzzy value "synthetic extent" for the i-th object is defined as:

$$S_i = \sum_{j=i}^{m} M_{gi}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1} \tag{7}$$

To obtain $\sum_{j=1}^{m} M_{gi}^j$ the operation of the Fuzzy addition of the "Extent" Analysis values m for a particular matrix is performed, so that:

$$\sum\nolimits_{j=1}^{m} M_{gi}^j = \left(\sum\nolimits_{j=1}^{m} a_j, \sum\nolimits_{j=1}^{m} b_j, \sum\nolimits_{j=1}^{m} c_j \right), \quad i = 1, 2, \ldots, n \tag{8}$$

And to obtain $\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1}$ the Fuzzy addition operation of $M_{gi}^j (j = 1, 2, \ldots n)$ is performed, so that:

$$\left[\sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{m} M_{gi}^j \right]^{-1} = \left(\frac{\mathbf{1}}{\sum_{i=1}^{n} c_i}, \frac{\mathbf{1}}{\sum_{i=1}^{n} b_i}, \frac{\mathbf{1}}{\sum_{i=1}^{n} a_i} \right) \tag{9}$$

2) THE EXTENT OF POSSIBILITY THAT $M_2 \geq M_1$

The extent of possibility that $M_2 = (a_2, b_2, c_2) \geq M_1 = (a_1, b_1, c_1)$ is defined as:

$$V(M_2 \geq M_1) = sup \left\lfloor min \left(\mu_{M_1}(x), \mu_{M_2}(y) \right) \right\rfloor \tag{10}$$

Equivalent to the following expression:

$$\begin{aligned} &V(M_2 \geq M_1) \\ &= hgt\ (M_1 \cap M_2) = \mu_{M_2}(d) \\ &= \begin{cases} 1 & if\, b_2 \geq b_1 \\ 0 & if\, a_1 \geq c_2 \\ \dfrac{a_1 - c_2}{(b_2 - c_2) - (b_1 - a_1)} & otherwise \end{cases} \end{aligned} \tag{11}$$

where (d) is the ordinate of the highest point of intersection D between μ_{M_1} and μ_{M_2}. To compare M_1 and M_2 we need both values of $V\ (M_1 \geq M_2)$ y $V(M_2 \geq M_1)$

3) THE EXTENT OF POSSIBILITY THAT $M \geq M_k$

The extent of possibility that i fuzzy convex numbers are greater than k fuzzy convex numbers M_i, $(i = 1, 2, \ldots, k)$ can be defined by:

$$\begin{aligned} &V\ (M \geq M_1, M_2 \ldots M_k) \\ &\quad = V \left[(M \geq M_1) \text{ y} (M \geq M_2) \text{y} \ldots \text{y} (M \geq M_k) \right] \\ &\quad = min(M \geq M_i), \quad i = 1, 2, \ldots, k \end{aligned} \tag{12}$$

Assume that:

$$\begin{aligned} &d'\ (A_i) = minV(S_i \geq S_k) \\ &\text{for } k = 1, 2, \ldots, n; \quad k \neq i \end{aligned} \tag{13}$$

Therefore, the weight of the vector is given by:

$$W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T \tag{14}$$

where A_i, $(1, 2, \ldots, n)$ are n elements.

4) NORMALIZED VECTOR

The normalized vector is given by:

$$W = (d(A_1), d(A_2), \ldots, d\ (A_n))^T \tag{15}$$

where W is a crisp number.

E. MULTILAYER PERCEPTRON

The Multilayer perceptron can be conceptualized like a generic type of network, which can be trained to solve different non-linear problems. This is achieved by cascading discriminant nodes called neurons. A neuron with a d-dimensional input vector s and output r_j is mathematically expressed as follows:

$$r_j = f\left(a_j\right) = f\left(\sum_{i=0}^{d} w_{ji}, s_i\right) \tag{16}$$

where w_{ji} denotes the weight corresponding to the connection of the output neuron j to the input neuron i. The function f is the activation function, and a_j is the so-called post-synaptic potential. The term ''multilayer'' refers to the existence of several layers of weights in the network, an example of this architecture is shown in Fig. 1.

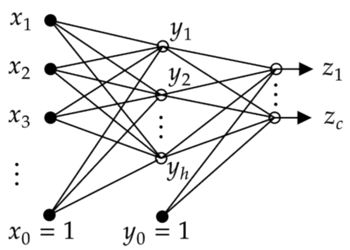

FIGURE 1. **Multilayer perceptron structure.**

In Fig. 1, there are two layers of weights, one connecting the feature vector x (input nodes) to the layer vector y (hidden nodes), and another connecting them to vector z (output nodes). Finally, to set the suitable weight values, there is a powerful algorithm for finding a minimum error solution based on the concept of gradient descent, called error back-propagation. In Section 4, we delve into the implementation of this algorithm.

III. AHPND-''EXTENT''

The AHPND-''Extent'' system proposed in this research is shown in Fig. 2. The system is based on the contributions of [7], [32]–[35], [37]. The system consists of the following stages: Stage I: Database; Stage II: Integration of an AHPD with ''Extent'' Analysis; Stage III: Integration of an MAPNN; Stage IV: Decision. Stage I and II concentrate the experience of experts; Stage III generalizes experience to define or predict final scheduling; Stage IV makes a decision on final machine scheduling.

The description of each of the stages that make up the AHPND-''Extent'' system is given below.

A. STAGE I: DATABASE

The database is created and fed by *production programming experts;* they, in turn, are responsible for deciding to correct or approve the information. The database contains information regarding various production instances and information on *uncertainty variables*.

B. STAGE II: INTEGRATION OF AHPD WITH ''EXTENT'' ANALYSIS (AHPD-''EXTENT'')

The AHPD-''Extent'' is the result of the integration of the AHPD with the ''Extent'' Analysis; allows to assess the uncertainty existing in the *uncertainty variables*, thus concentrating the knowledge of experts. The stage consists of the following steps:

1) SELECT EVALUATION CRITERIA

Experts select influential *uncertainty variables* for a given production instance.

2) SELECTION OF ALTERNATIVES

Experts select the alternatives that will be considered for selection and prioritization.

3) DEFINE THE HIERARCHICAL STRUCTURE

Experts define and approve how the general objective of the study, the evaluation criteria and the alternatives interact and influence each other.

4) PAIRED COMPARISONS

Each expert performs a fuzzy assessment for each of the interactions of the hierarchical structure, ''criterion vs criterion'' and ''criterion vs alternative'', using the Fuzzy Scale proposed by [38].

5) OBTAINING EIGENVECTORS

For each expert, the fuzzy values *''synthetic extent''* corresponding to each comparison are obtained.

6) ALTERNATIVES RANKING

For each expert, the ''d Value '' is calculated and the priority vectors corresponding to each comparison are defined.

7) MATRIX OF PRIORITIES

For each expert, priority vectors are concentrated to determine the global priority vector of each alternative.

C. STAGE III: INTEGRATION OF AN MPANN

The AHPND-''Extent'' is the result of the integration of the AHPD-''Extent'' with an MPANN and is implemented to generalize knowledge captured by experts. The stage consists of the following steps:

1) INPUT VALUES

The priority vectors of the paired comparisons *''criterion vs. criterion''* obtained in the ''AHPD-Extent'' are considered.

2) NUMBER OF NODES IN THE HIDDEN LAYER

The number of nodes in the hidden layer is a function of the number of input nodes (IN), the number of output nodes (ON) and the number of samples (NM) [32], [39]:

$$NHN1 = (IN \times ON)^{1/2} \tag{17}$$

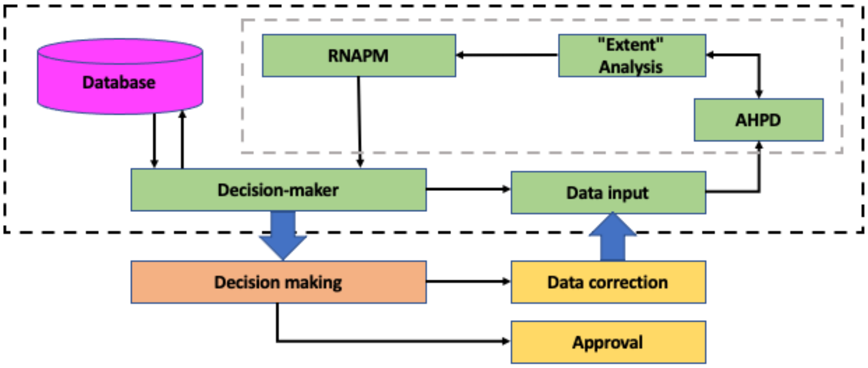

FIGURE 2. Decision-making system for a flow shop machine scheduling model under uncertainty.

TABLE 1. Production instance to be programmed in a shoe manufacturing company.

		Workstations									
		Number of machines									
		3	20	8	10						
Order (pairs)	Job: Models	P.T. *Cutting* (h)	P.T. *Stitching* (h)	P.T. *Mounting* (h)	P.T. *Adornment* (h)	Total P.T. (h)	Total Estimated (days)	Total Real (days)	Arrival Order	Due Date (days)	Deadline (days)
1710	Model A	4.8	6.4	13.7	12.8	37.7	4.2	5.7	1	15	18
1620	Model B	4.5	6.8	25.0	12.2	48.4	5.4	5.8	4	20	23
1530	Model C	2.0	5.1	12.8	13.2	33.0	3.7	4.6	2	10	13
1700	Model D	7.9	8.1	14.3	12.8	43.1	4.8	5.8	3	15	17
Total 6570		19.2	26.3	65.7	51.0	162.2	18.0				

$$NHN2 = \frac{1}{2}(IN + ON) \tag{18}$$

$$NHN3 = \frac{1}{2}(IN + ON) + (SN)^{1/2} \tag{19}$$

$$NHN4 = 2(IN) \tag{20}$$

3) OUTPUT VALUES

The global priority vectors of each alternative corresponding to each expert are used as reference values to train the MPANN.

4) SAMPLES PARTITION

The assessment of the experts are divided to be used in the training and validation process of the MPANN.

5) MODEL TRAINING

Training designs are a function of the number of hidden nodes, the transfer function, the training algorithm, different epochs, and a performance criterion.

6) MPANN VALIDATION

The sample assigned for validation is used. The results obtained by "AHPD-Extent", the "toolbox ANN" app preloaded in MATLAB and the training designs of the MPANN with better performance are compared.

D. STAGE IV: DECISION

At this stage, the results obtained by the AHPND-"Extent" system are verified by experts. If this is the case, appropriate adjustments are made and the process is repeated. Once the results are approved, the trained MPANN is used to program machines from production instances of the production system.

IV. RESULTS

A. CHARACTERIZATION OF THE APPLICATION

The proposed system was used to solve a real instance of a "shoe manufacturing" company structured as a *flow shop machine scheduling model under uncertainty*. A processing flow was considered to start in the *cutting area, followed by the stitching area, mounting area, and ends in the adornment area.* Five *production programming experts* Exp_1, Exp_2, Exp_3, Exp_4, and Exp_5 were considered. The production programming instance of Table 1 was solved, where it is observed: the demand for four footwear models, the number of machines per operation and the estimated time of processing in hours, the total estimated time of completion in days, the total real-time of completion in days, the arrival order, and the delivery times defined by the customer in days.

The five experts considered the following *uncertainty variables* as selection criteria: CDI: CDI-TP, CDI-OLL; CPC: CPC-PA, CPC-PF; CSM: CSM-AMP; CTE: CTE-TF, CTE-DD, and CTE-DL. The objective was to define the best sequencing order of processing in such a way that *lateness* $\tilde{L}_j$ and *tardiness* $\tilde{D}_j$ optimization criteria are minimized, prior to

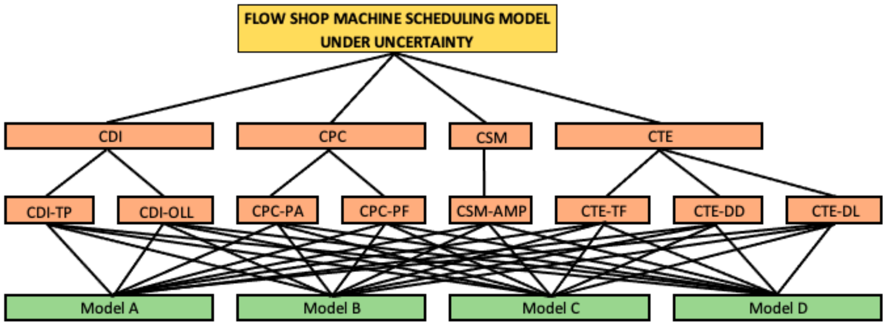

FIGURE 3. Hierarchical structure of the AHPD-"Extent" for the production programming instance in Table 1.

TABLE 2. Fuzzy matrix of Exp_1 resulting from the paired comparison of "selection criteria vs selection criteria".

	CDI-TP	CDI-OLL	CPC-PA	CPC-PF	CSM-AMP	CTE-TF	CTE-DD	CTE-DL
CDI-TP	$\tilde{1}$	$\tilde{3}^\wedge-1$	$\tilde{3}^\wedge-1$	$\tilde{3}$	$\tilde{3}$	$\tilde{3}^\wedge-1$	$\tilde{5}^\wedge-1$	$\tilde{3}^\wedge-1$
CDI-OLL	$\tilde{3}$	$\tilde{1}$	$\tilde{3}^\wedge-1$	$\tilde{3}$	$\tilde{3}$	$\tilde{3}^\wedge-1$	$\tilde{3}^\wedge-1$	$\tilde{3}^\wedge-1$
CPC-PA	$\tilde{3}$	$\tilde{3}$	$\tilde{1}$	$\tilde{5}$	$\tilde{3}$	$\tilde{3}^\wedge-1$	$\tilde{3}^\wedge-1$	$\tilde{3}^\wedge-1$
CPC-PF	$\tilde{3}^\wedge-1$	$\tilde{3}^\wedge-1$	$\tilde{5}^\wedge-1$	$\tilde{1}$	$\tilde{3}^\wedge-1$	$\tilde{5}^\wedge-1$	$\tilde{5}^\wedge-1$	$\tilde{5}^\wedge-1$
CSM-AMP	$\tilde{3}^\wedge-1$	$\tilde{3}^\wedge-1$	$\tilde{3}^\wedge-1$	$\tilde{3}$	$\tilde{1}$	$\tilde{5}^\wedge-1$	$\tilde{5}^\wedge-1$	$\tilde{5}^\wedge-1$
CTE-TF	$\tilde{3}$	$\tilde{3}$	$\tilde{3}$	$\tilde{5}$	$\tilde{5}$	$\tilde{1}$	$\tilde{3}^\wedge-1$	$\tilde{1}$
CTE-DD	$\tilde{5}$	$\tilde{3}$	$\tilde{3}$	$\tilde{5}$	$\tilde{5}$	$\tilde{3}$	$\tilde{1}$	$\tilde{1}$
CTE-DL	$\tilde{3}$	$\tilde{3}$	$\tilde{3}$	$\tilde{5}$	$\tilde{5}$	$\tilde{1}$	$\tilde{1}$	$\tilde{1}$

uncertainty assessment existing in *uncertainty variables*.

$$Lateness\tilde{L}_j = \tilde{C}_j - \tilde{d}_j \tag{21}$$

$$Tardiness\tilde{D}_j = max\left\{\tilde{C}_j - \tilde{d}_j, 0\right\} \tag{22}$$

where:

$\tilde{L}_j$ *It is a measure of the lateness in the processing completion time of Model j*

$\tilde{D}_j$ *It is a measure of the tardiness in the processing completion time of Model j*

$\tilde{C}_j = \tilde{s}_j + \tilde{p}_j$ *It is the processing completion time of Model j*

$\tilde{s}_j$ *It is the processing start time of Model j*

$\tilde{p}_j$ *It is the time needed for processing Model j*

$\tilde{d}_j$ *It is the time limit by for completing Model j*

B. CAPTURE EXPERT KNOWLEDGE

After selecting the *uncertainty variables*, the experts approved the hierarchical structure shown in Fig. 3.

The paired comparisons were made with a questionnaire based on the hierarchical structure and the Fuzzy Scale of [38]. The selection criteria were assessed with AHPD-"Extent". Based on the consideration of 5 experts, 8 selection criteria and 5 alternatives, a total of 45 matrices were obtained. For practical purposes and with a view to simplifying the procedure of the proposed system, some examples of Exp_1 estimation are presented. The matrix of fuzzy paired comparisons established for "selection criteria vs selection criteria" and "order size vs alternatives" are shown in Tables 2 and 3. Table 4 shows the uncertainty assessment obtained from the interaction "Order size vs alternative"; the vector w shows the extent of influence of the selection criterion "Order size" for each alternative.

The estimation of the uncertainty assessment was performed as follows.

By equations (7), (8) and (9) we obtain:

$$S1 = (8, 10, 13) \otimes \left(\frac{1}{29.75}, \frac{1}{23.2}, \frac{1}{17.92}\right) = (0.27, 0.43, 0.73)$$

$$S2 = (2.5, 2.67, 4) \otimes \left(\frac{1}{29.75}, \frac{1}{23.2}, \frac{1}{17.92}\right) = (0.08, 0.11, 0.22)$$

$$S3 = (1.92, 2.53, 2.75) \otimes \left(\frac{1}{29.75}, \frac{1}{23.2}, \frac{1}{17.92}\right) = (0.06, 0.11, 0.15)$$

$$S4 = (5.5, 8, 10) \otimes \left(\frac{1}{29.75}, \frac{1}{23.2}, \frac{1}{17.92}\right) = (0.18, 0.34, 0.56)$$

By equations (10), (11) and (12) we obtain:

$$V(S1 \geq S2) = 1$$
$$V(S1 \geq S3) = 1$$
$$V(S1 \geq S4) = 1$$
$$V(S2 \geq S1) = -0.169$$
$$V(S2 \geq S3) = 1$$
$$V(S2 \geq S4) = 0.14$$
$$V(S3 \geq S1) = -0.56$$
$$V(S3 \geq S2) = 0.92$$
$$V(S3 \geq S4) = -0.15$$
$$V(S4 \geq S1) = 0.77$$
$$V(S4 \geq S2) = 1$$
$$V(S4 \geq S3) = 1$$

TABLE 3. Fuzzy matrix of the Exp_1 resulting from the paired comparison "order size vs alternatives.

Criteria: Size of order				
Model	Model A	Model B	Model C	Model D
Model A	$\tilde{1}$	$\tilde{3}$	$\tilde{5}$	$\tilde{1}$
Model B	$\tilde{3}^{\wedge}-1$	$\tilde{1}$	$\tilde{1}$	$\tilde{3}^{\wedge}-1$
Model C	$\tilde{5}^{\wedge}-1$	$\tilde{1}$	$\tilde{1}$	$\tilde{3}^{\wedge}-1$
Model D	$\tilde{1}$	$\tilde{3}$	$\tilde{3}$	$\tilde{1}$

TABLE 4. Estimation of the extent of influence of the selection criterion "order size" based on the Exp_1.

Criteria: Size of order					"Synthetic" value			w
Model	Model A	Model B	Model C	Model D	S1	S2	S3	
Model A	$\tilde{1},\tilde{1},\tilde{1}$	$\tilde{2},\tilde{3},\tilde{4}$	$\tilde{4},\tilde{5},\tilde{6}$	$\tilde{1},\tilde{1},\tilde{2}$	0.27	0.43	0.73	0.96
Model B	$\tilde{4}^{\wedge}-1,\tilde{3}^{\wedge}-1,\tilde{2}^{\wedge}-1$	$\tilde{1},\tilde{1},\tilde{1}$	$\tilde{1},\tilde{1},\tilde{2}$	$\tilde{4}^{\wedge}-1,\tilde{3}^{\wedge}-1,\tilde{2}^{\wedge}-1$	0.08	0.11	0.22	-0.16
Model C	$\tilde{6}^{\wedge}-1,\tilde{5}^{\wedge}-1,\tilde{4}^{\wedge}-1$	$\tilde{2}^{\wedge}-1,\tilde{1},\tilde{1}$	$\tilde{1},\tilde{1},\tilde{1}$	$\tilde{4}^{\wedge}-1,\tilde{3}^{\wedge}-1,\tilde{2}^{\wedge}-1$	0.06	0.11	0.15	-0.57
Model D	$\tilde{2}^{\wedge}-1,\tilde{1},\tilde{1}$	$\tilde{2},\tilde{3},\tilde{4}$	$\tilde{2},\tilde{3},\tilde{4}$	$\tilde{1},\tilde{1},\tilde{1}$	0.18	0.34	0.56	0.77

TABLE 5. Summary of the sequencing obtained by the five experts.

Model	Alternatives Ranking					Weights
	Exp_1	Exp_2	Exp_3	Exp_4	Exp_5	
Model A	0.2597	0.2705	0.2653	0.31795	0.2425	0.2712
Model B	0.1905	0.1542	0.1855	0.15728	0.1667	0.1708
Model C	0.3314	0.3557	0.3464	0.26664	0.3519	0.3304
Model D	0.2184	0.2196	0.2028	0.25813	0.2389	0.2276
Sequencing	CADB	CADB	CADB	ACDB	CADB	CADB

Finally, by equation (13):

$$\begin{aligned} d'(A_1) &= minV\,(S1 \geq S2, S3, S4) \\ &= min\,(1, 1, 1) = 1 \\ d'(A_2) &= minV\,(S2 \geq S1, S3, S4) \\ &= \min\,(-0.169, 1, 0.14) = -0.169 \\ d'(A_3) &= minV\,(S3 \geq S1, S2, S4) \\ &= min\,(= -0.56, 0.92, -0.15) = -0.56 \\ d'(A_4) &= minV\,(S4 \geq S1, S2, S3) \\ &= min\,(0.77, 1, 1) = 0.77 \end{aligned}$$

Therefore, the weight of the vector is given by (14):

$$W' = (1, -0.169, -0.56, 0.77)^T$$

The normalized vector is given by (15), where W is a crisp number:

$$W = (0.96, -0.16, \; -0.57, 0.77)^T$$

The final results for the weights of the alternatives and the corresponding sequences, based on the assessment of the five experts, are shown in Table 5. It can be observed that the order of processing is CADB

C. GENERALIZE EXPERT KNOWLEDGE

The training of the MPANN was carried out with the assessment of Exp_1, Exp_2, Exp_3, and Exp_4, and the validation with the assessment of the Exp_5. The priority vectors of "criterion vs criterion" were considered as input values and the global priority vectors of each alternative corresponding to each expert as output values.

The training designs were developed in the MATLAB based on the input nodes (8 evaluation criteria), the hidden nodes defined by the equations (17), (18), (19), (20) and the output nodes (4 models to be processed). For each design, different hidden nodes and different transfer functions ("sigmoid tangent" and "logsigmoid") considered. The designs were trained with a *feed-forward backpropagation algorithm*, different epochs were considered, the mean square error was used as performance criteria. Likewise, the parameters were established empirically after a meticulous experimentation stage, where it was observed that adding more nodes in the hidden layer implies a higher computational cost and the results are not significant. The designs are shown in Table 6.

The validation of the MPANN was carried out with the assessment of the Exp_5, and the training of the system was contrasted with the results given by an AHPD, the app "tool box ANN" preloaded in MATLAB and the structures of N1, N2, and N3 with better performance. Table 7 shows the concentration of the results obtained. The processing sequencing for each of the methods evaluated is practically the same: CADB.

TABLE 6. MPANN training designs.

Design	Architecture			Training				
	Input Nodes	Hidden Nodes	Output Nodes	Training Fuction	Transfer Function	Learning Rate	Epochs	Performance
N1	8	6	4	TRAINGDM	TANSIG	0.08	600	0.000348
	8	6	4	TRAINGDM	TANSIG	0.08	800	0.000315
	8	6	4	TRAINGDM	TANSIG	0.08	1000	0.000281
	8	6	4	TRAINGDM	TANSIG	0.08	1500	0.000179
	8	6	4	TRAINGDM	TANSIG	0.08	2000	7.92E-05
	8	6	4	TRAINGDM	LOGSIG	0.08	600	0.000722
	8	6	4	TRAINGDM	LOGSIG	0.08	800	0.000693
	8	6	4	TRAINGDM	LOGSIG	0.08	1000	0.000643
	8	6	4	TRAINGDM	LOGSIG	0.08	1500	0.000416
	8	6	4	TRAINGDM	LOGSIG	0.08	2000	0.000128
N2	8	8	4	TRAINGDM	TANSIG	0.08	600	0.000321
	8	8	4	TRAINGDM	TANSIG	0.08	800	0.000218
	8	8	4	TRAINGDM	TANSIG	0.08	1000	0.00021
	8	8	4	TRAINGDM	TANSIG	0.08	1500	5.85E-05
	8	8	4	TRAINGDM	TANSIG	0.08	2000	4.56E-05
	8	8	4	TRAINGDM	LOGSIG	0.08	600	0.000619
	8	8	4	TRAINGDM	LOGSIG	0.08	800	0.000439
	8	8	4	TRAINGDM	LOGSIG	0.08	1000	0.000257
	8	8	4	TRAINGDM	LOGSIG	0.08	1500	0.000198
	8	8	4	TRAINGDM	LOGSIG	0.08	2000	0.000108
N3	8	16	4	TRAINGDM	TANSIG	0.08	600	0.000591
	8	16	4	TRAINGDM	TANSIG	0.08	800	0.000392
	8	16	4	TRAINGDM	TANSIG	0.08	1000	0.000269
	8	16	4	TRAINGDM	TANSIG	0.08	1500	0.000164
	8	16	4	TRAINGDM	TANSIG	0.08	2000	0.000109
	8	16	4	TRAINGDM	LOGSIG	0.08	600	0.000582
	8	16	4	TRAINGDM	LOGSIG	0.08	800	0.000457
	8	16	4	TRAINGDM	LOGSIG	0.08	1000	0.000417
	8	16	4	TRAINGDM	LOGSIG	0.08	1500	0.000363
	8	16	4	TRAINGDM	LOGSIG	0.08	2000	0.000301

TABLE 7. MPANN validation using AHPD, MATLAB App and N1, N2 and N3.

Models	AHPD	App	N11	N12	N21	N22	N31	N32
Model A	0.2712	0.2671	0.3051	0.2427	0.2452	0.2840	0.2959	0.2605
Model B	0.1708	0.1587	0.1852	0.1726	0.1542	0.1724	0.16875	0.1900
Model C	0.3304	0.3443	0.3555	0.3552	0.3092	0.3198	0.32067	0.3111
Model D	0.2276	0.1659	0.2527	0.2034	0.2054	0.2030	0.21858	0.2039
Sequencing	CADB	CADB	CADB	CADB	CADB	CADB	CADB	CADB

D. VALIDATION OF AHPND-"EXTENT" RESULTS

The validation of the results produced by the proposed system was carried out by comparing the production schedule defined by AHPND-"Extent" versus a "regular schedule" that is normally done by a production programming expert; this can be seen in Fig. 4.

The results of the optimization criteria related to the number of late deliveries were as follows.

By equations (10), (11), and (12) we obtain:

1) LATE DELIVERIES GENERATED WITH AHPND-"EXTENT"

Regarding the "Due date":

$$\tilde{L}_A = (5+6) - 15 = -4$$
$$\tilde{D}_A = max\,\{(11-15), 0\} = 0$$
$$\tilde{L}_B = (17+6) - 20 = 3$$
$$\tilde{D}_B = max\,\{(23-20), 0\} = 3$$
$$\tilde{L}_C = (0+5) - 10 = -5$$
$$\tilde{D}_C = max\,\{(5-10), 0\} = 0$$

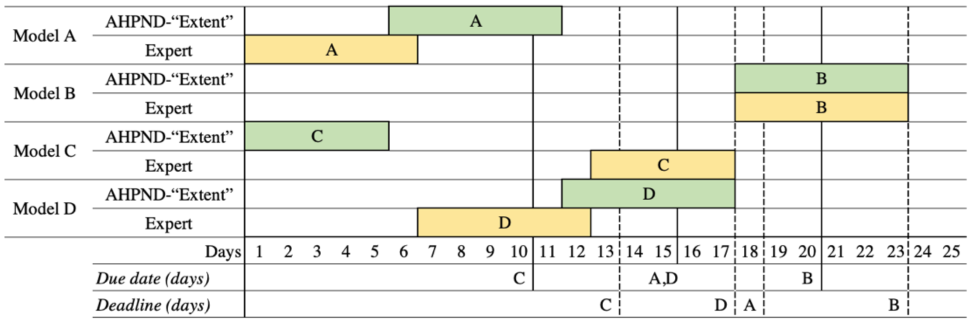

FIGURE 4. AHPND-"Extent" production sequencing vs Empirical Model.

$$\tilde{L}_D = (11+6) - 15 = 2$$
$$\tilde{D}_D = max\,\{(17-15), 0\} = 2$$

Regarding the "Deadline":

$$\tilde{L}_A = (5+6) - 18 = -7$$
$$\tilde{D}_A = max\,\{(11-18), 0\} = 0$$
$$\tilde{L}_B = (17+6) - 23 = 0$$
$$\tilde{D}_B = max\,\{(23-23), 0\} = 0$$
$$\tilde{L}_C = (0+5) - 13 = -8$$
$$\tilde{D}_C = max\,\{(5-13), 0\} = 0$$
$$\tilde{L}_D = (11+6) - 17 = 0$$
$$\tilde{D}_D = max\,\{(17-17), 0\} = 0$$

2) LATE DELIVERIES GENERATED WITH EXPERT

Regarding the "Due date":

$$\tilde{L}_A = (0+6) - 15 = -9$$
$$\tilde{D}_A = max\,\{(6-15), 0\} = 0$$
$$\tilde{L}_B = (17+6) - 20 = 3$$
$$\tilde{D}_B = max\,\{(23-20), 0\} = 3$$
$$\tilde{L}_C = (12+5) - 10 = 7$$
$$\tilde{D}_C = max\,\{(17-10), 0\} = 7$$
$$\tilde{L}_D = (6+6) - 15 = -3$$
$$\tilde{D}_D = max\,\{(12-15), 0\} = 0$$

Regarding the "Deadline":

$$\tilde{L}_A = (0+6) - 18 = -12$$
$$\tilde{D}_A = max\,\{(6-18), 0\} = 0$$
$$\tilde{L}_B = (17+6) - 23 = 0$$
$$\tilde{D}_B = max\,\{(23-23), 0\} = 0$$
$$\tilde{L}_C = (12+5) - 13 = 4$$
$$\tilde{D}_C = max\,\{(17-13), 0\} = 4$$
$$\tilde{L}_D = (6+6) - 17 = -5$$
$$\tilde{D}_D = max\,\{(12-17), 0\} = 0$$

The scheduling of machines thrown by the proposed system AHPND-"Extent", regarding the delivery date "Due date", generated a delay in the delivery for three days for model B and two days for model D; regarding the delivery date "Deadline", did not generate a delay. On the other hand, the scheduling of machines suggested by the "Expert", regarding the delivery date "Due date", generated a delay in the delivery for three days for model B and seven days for model C; regarding the delivery date "Deadline", caused a delay in delivery for four days for model C. It can therefore be concluded that the sequencing resulting from the scheduling of machines based on the AHPND-"Extent" decision system proposed in this research has satisfactory results.

V. DISCUSSION AND CONCLUSION

Given the lack of clarity in the definition of uncertainty existing in a production system, this research defines the concept of *uncertainty variables* in a machine scheduling context and defines the *flow shop machine scheduling model under uncertainty*. The aim is to focus on the assessment of the existing uncertainty and then defines the best order of processing sequencing that minimizes the number of late deliveries.

The selection of the order of processing in a *flow shop machine scheduling model under uncertainty* and consistency with the manufacturing objectives is a problem of fuzzy multicriteria decision-making that needs the intervention of the "subjectivity of experts", "uncertainty assessment methods" and "artificial intelligence techniques". This research develops the AHPND-"Extent" which is a decision-making system based on a Fuzzy Hierarchical Analysis Process, a membership analysis and an Artificial Neural Network. Uncertainty is addressed by the extent of influence and is quantified by a Fuzzy Scale and the use of fuzzy triangular numbers in the range of 0-1.

The AHPND-"Extent" system is validated in a real production instance of a shoe manufacturing company. In an ordinary instance of production, it has been identified that despite the fact that there is a definite demand and the productive capacity to meet it, it is not possible to finish the production on the established dates; which leads to penalties or cancellations by clients, translated into large monetary losses. The scheduling obtained by the AHPND-"Extent" and a scheduling normally carried out by an expert are contrasted in terms of late deliveries. The results show the efficiency of the proposed system since the joint assessment of variables that add uncertainty to the production system allowed to reduce late deliveries.

System performance depends on four factors. First, *uncertainty variables* are defined by experts and defined by internal and external factors that influence fluctuations in the achievement of manufacturing objectives. Second, the database must be updated as the subjectivity existing in the *uncertainty variables* changes. Third, as the number of MPANN training data samples increases, performance will improve and with it, the learning that defines and predicts final processing sequencing. Fourth, as long as *uncertainty variables* prevail and the existing subjectivity does not undergo significant changes, MPANN can still be used to predict new processing sequences.

The scope of this work can be extended in different directions: to attend to machine scheduling model under uncertainty, to consider instances where the optimization criteria are restrictive and complex. The above are current issues which we are working with.

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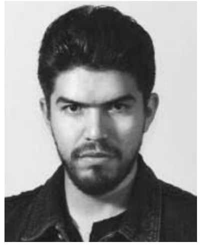

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