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Comparison of Simple Strategies for Vehicular Platooning With Lossy Communication

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ABSTRACT This paper studies vehicle platooning with communication channels subject to random data loss. We focus on homogeneous discrete-time platoons in a predecessor-following topology with a constant time headway policy. We assume that each agent in the platoon sends its current position to the immediate follower through a lossy channel modeled as a Bernoulli process. To reduce the negative effects of data loss over the string stability and performance of the platoon, we use simple strategies that modify the measurement, error, and control signals of the feedback control loop, in each vehicle, when a dropout occurs. Such strategies are based on holding the previous value, dropping to zero, or replacing with a prediction based on a simple linear extrapolation. We performed a simulation-based comparison among a set of different strategies, and found that some strategies are favorable in terms of performance, while some others present improvements for string stabilization. These results strongly suggest that proper design of compensation schemes for the communications of interconnected multi-agent systems plays an important role in their performance and their scalability properties.

INDEX TERMS Vehicular platoon control, lossy channels, string stability, constant time-headway, networked systems.

I. INTRODUCTION

The development of new technologies in the field of traffic highway management is currently a great challenge for the transition to a more efficient mobility. One widely studied alternative for improving the efficiency of highways is that of platoons of autonomous vehicles that navigate in a coordinated fashion at a consensus speed, keeping a desired inter-vehicle distance between them [1], [2]. In a cooperative setting, it is often assumed that inter-vehicle communication allowing the interchange of information, such as speeds or positions, is present. This leads to the concept of Cooperative Adaptive Cruise Control (CACC) [3], [4]. The advantages of such cooperative schemes include reductions in vehicular traffic, reduction of fuel consumption and polluting emissions, increase of road safety, decrease of aerodynamic drag losses, among several others [5], [6]. Nevertheless, a non ideal inter-vehicle communication may have an important impact on the platoon performance [7]–[11]. In this paper we analyze the effect that lossy inter-vehicle communication channels may have on the platoon performance.

To evaluate the platoon performance and safety, the tracking errors between vehicles are usually analyzed. How these errors behave when more vehicles are added to the platoon is a key aspect of the broader concept of scalability. Indeed, the tracking errors caused by a disturbance at any vehicle may increase in amplitude along the string of vehicles, having a detrimental effect on the tracking errors and affecting said scalability. A well-studied property of platooning systems, known as string stability, ensures that disturbances are not amplified as they propagate along the string independently of the size of the of the vehicle chain [12].

In a majority of cases, string stability has been studied for platooning with perfect inter-vehicle communication [2], [13]. It has been found that the topology of the network and the spacing policy between vehicles play key roles when

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determining the string stability of the platoon [2], [14], [15]. One of the most relevant topologies, due to its simplicity, is the predecessor-following topology, in which the communication is unidirectional and each agent can obtain information only from its nearest predecessor [16]. To achieve string stability with this topology, it is necessary to consider a constant time headway spacing policy, in which the desired inter-vehicle spacing between vehicles increases with the speed of the agents [16], [17]. The most used theoretical tool for obtaining results concerning the string stability properties of platooning schemes is the frequency domain analysis. Within the aforementioned framework, string stability is ensured when the frequency response peak of a sequence of transfer functions relating the inter-vehicle spacing of consecutive vehicles, remains uniformly bounded with respect to the number of agents, ensuring disturbances are not amplified as they propagate [3], [18], [19].

When the platoon is subject to communication problems (e.g. channel noise, random delays or random data loss), the signals of interest become stochastic due to the random nature of the communication channels, and thus, the deterministic analysis of string stability must be reconsidered. Many questions regarding scalability of platooning schemes remain open in a stochastic setting, including a clear definition of string stability that is valid for stochastic scenarios including several types of communication problems. Moreover, analytical conditions for string stability in these stochastic scenarios have not been derived yet [13]. Nevertheless, important progress has been made in this regard, indicating that the second-order moments of the tracking errors should be analyzed with a string stability criteria [8], [20]–[24].

Within the recent literature, communication issues have been incorporated in the study of platooning, although the analysis of string stability is restricted [7], [22], [23] or absent [25]–[27]. Among several communication problems, random data loss is one of the most relevant and recurring. Lossy communication links can be modeled as erasure channels, which use a Bernoulli stochastic process to describe whether the transmitted data is received or lost. A key aspect for dealing with this case is defining an action to perform at the receiver end when the expected data does not arrive. This yields strategies or protocols to deal with data loss that would result in different performances. In the context of networked control systems, two of the most common and simpler protocols are those that hold the last available data of the signals of interest, and those that set their values to zero when a dropout occurs. These strategies are simply referred to as *to-hold* or *to-zero* type, and neither can be claimed superior to the other [28].

In the context of platooning, these simple strategies have also been adopted not only to replace the lost data, but also to manage the controller input or the controller output. For instance, in [21] the previous available measurement vector is used to replace the non-received position. A to-hold type strategy is applied to the plant input in [29]–[32]. In [23] the authors considered that the controller input (inter-vehicle spacing error) is equal to zero whenever there is data loss. As far as we are aware, a study where such strategies are compared and the consequences that they may have on the string stability of the platoon is not available in the literature.

In this work we study predecessor-following platoons where the inter-vehicle communication is affected by random data-loss. We adopt different strategies to deal with data loss in the context of platooning and compare them. Specifically, we use simple strategies to deal with data loss that turn a signal to zero (to-zero), or maintain the previous value of the signal (to-hold). We also use a strategy that replaces the missing measurements with a linear extrapolation, estimating the lost data using older measurements (to-extrapolate). For the platoon configuration, the signals in each vehicle managed by these strategies are: the measured input, the local error and the control signal. This yields three different groups of strategies, namely: *measurement-based*, *error-based* and *control signal-based*. We also consider all possible combinations among these basic strategies. The contribution of this work is twofold:

- We show, via numerical results, that the string stability of a platoon subject to data loss could be affected by the adopted strategy. Indeed, for the same time headway constant value and probability of successful transmission, some strategies exhibit a string stable behavior, while some others do not. For each strategy and keeping the vehicle dynamics and controllers fixed, we determine a region in the space formed by the time headway constant and probability of successful transmission, that is compatible with string stability.
- We numerically compared a set of strategies beyond the string stability criteria, and found that there are strategies that present higher tracking error variance compared with some others that yield considerably lower values. However, such strategies with low variances are not those having a broader region in the parameter space compatible with string stability.

The outline of this paper is as follows: In Section II we present the platooning configuration under study assuming perfect communication, while in Section III we incorporate the lossy channels in the framework and discuss the notion of string stability for platooning under random data-loss. In Section IV we present the set of strategies to deal with data loss analyzed in this paper. Simulation results and their analysis are provided in Section V. Finally, conclusions and future work are placed in Section VI.

A. NOTATION

We use the notation of the standard systems and control literature. Lowercase is used for real scalar signals, $w : \mathbb{Z} \to \mathbb{R}$ with specific values of the signal denoted by $w(k)$ at the discrete instants $k \in \mathbb{Z}$. Uppercase is used for scalar complex-valued $\mathcal{Z}-$transforms of signals and transfer functions, $W : \mathbb{C} \to \mathbb{C}$ with specific values denoted by $W(z)$. For the sake of brevity in the notation, where there is no place for confusion, the argument (z) will be omitted. Vectors will be denoted as $\underline{w}(k) \in \mathbb{R}^n$ and $\underline{W} \in \mathbb{C}^n$, while $\underline{w}(k)^\top$ and $\underline{W}^\top$ denote their transposes. The imaginary unit is denoted by j, with $j^2 = -1$. Boldface will be used for matrices $\boldsymbol{G} \in \mathbb{C}^{n\times m}$ and the (i, k)-th entry of $\boldsymbol{G}$ is denoted by $G_{i,k}$. The magnitude of W when $z = e^{j\omega}$, $\omega \in [0, 2\pi]$, is denoted by $|W|$ and its magnitude peak over all possible values of $\omega \in [0, 2\pi]$ is denoted as $||W||_\infty := \sup_\omega |W(e^{j\omega})|$. For $z \in \mathbb{C}$, $\Re(z)$ and $\Im(z)$ denote the real and imaginary parts of z respectively. Finally, we use $\mathcal{E}\{\cdot\}$ to denote the expectation operator.

II. PLATOONING WITH PERFECT COMMUNICATION

In this section we describe the general platoon setup under analysis (vehicle dynamics, information flow topology and spacing policy), and introduce the idea of string stability for deterministic platoons (i.e. ideal communication channels). This section motivates the present work and serves as comparison for the case where the considered platoon is affected by random data losses.

A. PLATOON SETUP

The problem of interest in this work is based on a standard deterministic platooning setup. However, much of the setup can be used for an arbitrary collection of systems. In particular, we consider a collection of $N + 1 \in \mathbb{N}$ identical agents, each modeled by a feedback system composed of a discrete-time LTI plant, $G(z)$, and its local LTI controller, $C(z)$.

We denote as $y_i(k)$ the position of the i-th agent for $i = 0, \ldots, N$, at the discrete time instant k, where the indexes reflect the ordering of the vehicles within the string. The leader is labeled as $i = 0$ and moves independently, setting the desired trajectory for the platoon. We consider a homogeneous platoon, with every agent having the same dynamical model and local controller, and all vehicles moving in a straight line as depicted in Fig. 1.

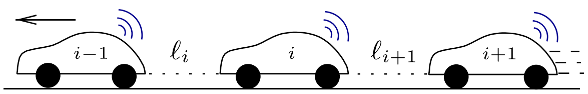

FIGURE 1. Three consecutive agents of the platoon configuration.

Remark 1: A more realistic scenario would be a heterogeneous platoon, that is, a platoon formed by vehicles with different dynamics. However, homogeneous platooning has been widely used in the literature (see for instance [10], [14], [17], [18], [25]) since this simplified representation of the platoon is, naturally, the first approach for introducing the basic properties of scalability and convergence of the system. Additionally, with an appropriate controller design, a homogeneous platoon may be a feasible situation in some applications [18], [26]. A vast part of the recent literature considers this assumption for platoon analysis and real world oriented applications [33].

Each agent is able to communicate its current position to its nearest follower through a wireless communication channel. This communication is unidirectional, which locks the information flow, and corresponds to a predecessor-follower topology where the vehicle with index i receives as an input, or reference, the position of the vehicle with index $i-1$. It is also possible to assume that the predecessor position can be estimated by the follower through ranging sensors and its own position, however this requires the predecessor to be close enough, limiting the maximum distance between vehicles. Also, the ranging sensors present their own limitations which could also be considered as lost measurements.

A first control goal in platooning aims to having the separation between agents equal to predefined desired references $r_i(k)$. Therefore, we will define as a performance signal the tracking error $\zeta_i(k)$, computed as the difference between the measured vehicle separation (by the i-th agent), $\ell_i(k) = y_{i-1}(k) - y_i(k)$ and the desired separation $r_i(k)$, that is,

$$\zeta_i(k) = \ell_i(k) - r_i(k). \tag{1}$$

For safety reasons, in order to avoid possible collisions due to unexpected disturbances, the inter-vehicle distances should increase with the speed of the platoon. A way to implement this spacing policy is to consider desired inter-vehicle separations $r_i(k)$ with the form

$$r_i(k) = \epsilon_i + h(y_i(k) - y_i(k-1)), \tag{2}$$

where ϵ_i is a constant corresponding to the minimum desired separation, $y_i(k) - y_i(k-1)$ represents the discrete time rate of change of the position (speed) of the vehicle with index i, and h is a constant parameter known as the time headway constant, which allows to modify the desired inter-vehicle separation $r_i(k)$ along with the speed of the agent. In the platooning literature, this setting is known as a constant time-headway spacing policy [10], [16], [17], [22]. We will consider that the agents have no volume and that $\epsilon_i = 0$ for all $i = 1, \ldots, N$ in accordance to similarly themed works [11], [27]. These assumptions are adopted for simplicity in the exposition, and should have no impact on the controller design nor on the stability properties of the platoon. Moreover, we will consider that the agents at the time instant $k = 0$ start at rest and in the desired spatial formation and ordering.

With this considerations, the tracking errors can be written as $\zeta_i(k) = e_i(k)$, where

$$e_i(k) = y_{i-1}(k) - v_i(k) \tag{3}$$

with $v_i(k) = (h+1)y_i(k) - hy_i(k-1)$. It is worth to mention that $\zeta_i(k)$ and $e_i(k)$ are the same signals if the communication is perfect. However, when the channel is lossy, this will not be the case, which motivates the use of different notation for these error signals.

The second control goal considers the tracking of a constant speed by the collection of agents. We use u_i to denote the control output signal. With this in mind we will translate the mathematical description to the frequency domain.

To implement a reference depending on the speed, an alternative is to use a control scheme using two degrees of freedom [5], where a block $H(z)$ is incorporated in the feedback loop and represents the transfer function between the position y_i and the feedback signal v_i introduced in (3) to construct the tracking error. Thus, the control loop of each vehicle (see Fig. 2) is described by the plant modeling the vehicle dynamics, $G(z)$, the local stabilizing controller, $C(z)$, and the transfer function $H(z)$ closing the feedback loop and given by

$$H(z) = (1+h) - \frac{h}{z}, \tag{4}$$

allowing to include the speed of the i-th agent as a part of the spacing policy. Note that the reference $r_i(k)$ is not explicit within Fig. 2, however, said scheme represents the same problem as the one described earlier.

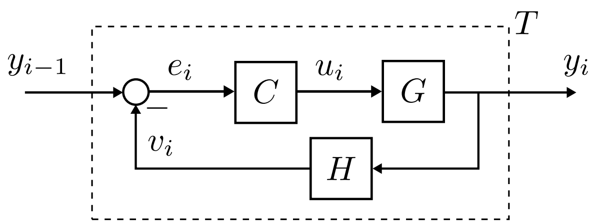

FIGURE 2. Local control loops at each vehicle assuming perfect communications.

Usually, the vehicle dynamics are modeled as a first or second order system [13], [18], with the product $C(z)G(z)$ having at least double integral action, that is, two poles at $z = 1$. This is necessary for the agents to achieve the tracking of a constant speed reference with zero error in steady state, a second important goal in platooning. It is common to consider that one integrator is in the plant $G(z)$, since this is a simple model for the inertia of the agent. To obtain our results, in subsequent sections we assume that the model of the plant $G(z)$ is a simple integrator (see e.g. [34], [35]), and the controller $C(z)$ also includes integral action.

The dynamics from the position of the $(i-1)$-th vehicle to the i-th vehicle are given in the frequency domain by

$$Y_i(z) = T(z)Y_{i-1}(z), \tag{5}$$

where the complementary sensitivity function of this two degree-of-freedom loop, $T(z)$, is given by

$$T(z) = \frac{G(z)C(z)}{1 + G(z)H(z)C(z)}. \tag{6}$$

The lead vehicle is assumed to move with a constant speed in regular operation. However, in the simulation section, we consider a period of acceleration that brings the vehicle from rest to a given speed. Also, a braking maneuver (with constant deceleration) is added to the leader's reference. Moreover, $H(z)$ is such that $T(z)$ is a stable transfer function and there are no unstable cancellations in the product $G(z)H(z)C(z)$.

B. STRING STABILITY WITH PERFECT COMMUNICATION

A conventional string stability definition for the deterministic case considers that a platoon is string stable if disturbances at particular agents are not amplified as they propagate along the string, independently of the string size $N+1$. It will be considered string unstable otherwise. This is sometimes formalized using the $\mathcal{H}_\infty$ norm of a sequence of transfer functions relating disturbances and inter-vehicle spacings. In fact, for the platooning configuration presented above, it is only required that the transfer function from the output of an agent to the output of the follower, satisfies the condition $||T(z)||_\infty \leq 1$ to guarantee string stability in the deterministic perfect communication case.

In the following, we will study the string stability of a platoon with ideal inter-vehicle communication, arising from (5), and provide an analytical condition on the vehicle dynamics to guarantee it. In order to do so, we first study the collective platoon dynamics. For simplicity we will omit the argument (z) whenever it is safe to do so without causing confusion.

The string stability property is unavoidable in the continuous time case if the constant $h = 0$, and there exists an infimum value of this parameter that provides string stability [17]. Consequently, as we are considering the discrete time case, we must obtain conditions over this parameter to guarantee string stability.

If we consider that the lead vehicle moves independently and its trajectory is set by $Y_0 = GD_0$, where D_0 is a certain reference, using straightforward computations, we have that the effect of D_0 on the n-th inter-vehicle spacing is simply given by

$$E_n = T^{n-1}SD_0 \tag{7}$$

where E_n denotes the $\mathcal{Z}$-transform of the n-th error and S is a stable transfer function that does not depend on the number of agents, $N+1$.

The following lemma, taken from [36], implies that the strategy considered above is string unstable whenever $h = 0$.

Lemma 1: Let T be a real rational scalar function of $z \in \mathbb{C}$. Suppose that $T(1) = 1$ and also that T is stable. Then

$$\int_0^\pi \ln\left|T(e^{j\omega})\right| \frac{d\omega}{1-\cos(\omega)} \geq \pi T'(1).$$

■

Indeed, for T given in (6) and $h = 0$, that is $H(z) = 1$, it is straightforward to show that $T'(1) = 0$, due to the two integrators of GC of the open loop. Since $GC = \tilde{G}\tilde{C}/(z-1)^2$ with $\tilde{G}(1) \neq 0$ and $\tilde{C}(1) \neq 0$ we have that

$$\frac{d}{dz}(T) = \frac{G'C + GC'}{(1+GC)} - \frac{GC(G'C+GC')}{(1+GC)^2} \tag{8}$$

$$= \frac{(G'C+GC')}{(1+GC)^2}. \tag{9}$$

Now, $(1+GC)^{-1}$ has two zeros at $z = 1$ (due to the stability of the closed loop), and therefore $(1+GC)^{-2}$ has four zeros at $z = 1$. Moreover, $(G'C + GC')$ has at most three poles at $z = 1$ which yields that $T'(1) = 0$. Lemma 1 now implies that $||T||_\infty > 1$ when $h = 0$. Notice that this is also valid for more complex transfer functions G than a simple integrator, as long as GC has double integration.

As an illustrative example, Fig. 3 shows the magnitude plots of $|T(e^{j\omega})|$, $\omega \in (0, 2\pi)$ for fixed G, a stabilizing C, and different values of the parameter h. We see that for values of $h < 3.4$, $||T||_\infty > 1$, and for $h > 3.4$, T satisfies the condition for string stability, that is, $||T||_\infty \leq 1$. This indicates the existence of an infimal value of h that ensures string stability in this case. This *infimal* value of h, obtained numerically, will be used in the platoon setup simulations in section V. In Fig. 4 we can observe the trajectories of a platoon of $N = 50$ agents for two different values of the time headway constant h, and how the inter-vehicle spacings reveal the undesirable effect of string instability when $h = 3$ and a well behaved disturbance propagation when $h = 4$. It should be noted that the price to pay for this is larger inter-vehicle spacings whenever the vehicles travel at higher velocities.

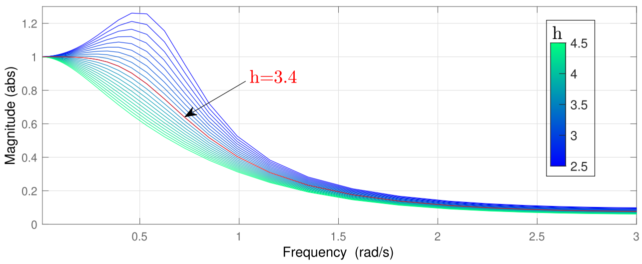

FIGURE 3. Bode plots of $|T(e^{j\omega})|$, $\omega \in (0, 2\pi)$ for $h \in [2.5, 4.5]$.

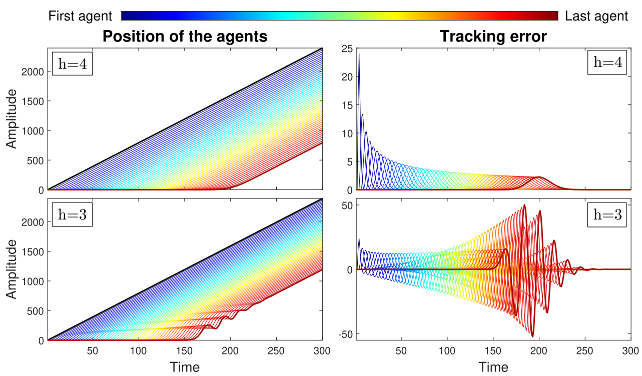

FIGURE 4. String stable ($h = 4$) and unstable ($h = 3$) behavior for the position and tracking error of a platoon of $N = 50$ agents with perfect communication.

Remark 2: Most of the numerical results are presented through figures that use a color code, as in Fig. 4, to help the reader to identify the dynamic response of the vehicles of the platoon. The first agent is represented by the curve with the blue color at the beginning of the color bar. As the agent index increases along the chain of vehicles, so does in the color bar. The last vehicle is represented with the red color at the end of the color code.

III. PLATOONING WITH IMPERFECT COMMUNICATION

In this section we describe the framework where the wireless communication channel of each vehicle in Section II is considered to be affected by random data-loss.

When lossy channels are in place, the platooning scheme becomes the one in Fig. 5, where $f(\cdot)$ is a possibly non-linear function that represents the feedback loop in each vehicle, that now depends not only on the vehicle dynamics G, the linear controller C, and the constant time headway h, but also on the strategies to deal with data-loss, which are described in Section IV, and also on the channel model, which is described next.

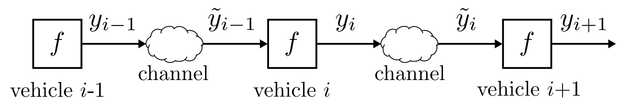

FIGURE 5. Platoon with inter-vehicle communication subject to data loss.

A. COMMUNICATION CHANNEL MODEL

The lossy inter-agent communication channels are modeled as erasure channels, which are defined by a Bernoulli process $\theta_i \in \{1, 0\}$. This process is such that, when $\theta_i(k) = 1$, the position of the predecessor vehicle with index $i-1$, namely $y_{i-1}(k)$, is received successfully at the vehicle with index i. When $\theta_i(k) = 0$, the data is considered lost. The channel model for the i-th vehicle can be described by

$$\tilde{y}_{i-1}(k) = \theta_i(k) y_{i-1}(k), \tag{10}$$

where $y_{i-1}(k)$ is the channel input and $\tilde{y}_{i-1}(k)$ corresponds to the channel output.

We assume that $\theta_i(k)$, with $i = 1, \dots, N$ are mutually independent and identically distributed (i.i.d.) processes with successful communication probability p. We also assume that $\theta_i(k)$ is an argument of the feedback control function f, since each vehicle is able to determine at a given instant k whether the data from the predecessor arrives or not.

The type of model described above is widely used in the context of networked control systems [28], [37]. It should be noted that, in practice, the transmitted data being lost is not equivalent to receiving a zero value as a measurement. The Bernoulli variable $\theta_i(k)$ in the channel model should be interpreted as an indicator for data loss, but the value that will be considered in the receiver depends on the adopted protocol which should be activated by $\theta_i(k)$.

B. PERFORMANCE CRITERIA

1) TRACKING ERROR DEFINITION

When the data from the predecessor is lost, the i-th vehicle cannot determine the current tracking error $\zeta_i(k)$ defined in (1) as in the deterministic case. It is then necessary to distinguish between two types of errors: the *local error* and the *true error*.

The local error corresponds to the error that each vehicle measures locally using the available data. The local controller in each vehicle defines its control signal to track its predecessor based on the local error, which is denoted by $e_i(k)$, as in the deterministic case and given by given in (3) (see also Fig. 2). This error is affected by the communication impairment, which means that the local controller is tracking the received predecessor position, which could not be the true position. Thus, the true tracking error corresponds to $\zeta_i(k)$ in (1). The local controllers do not have access to $\zeta_i(k)$, nevertheless, to globally evaluate whether a platoon shows an appropriate behaviour or not, it is necessary to use the true error $\zeta_i(k)$ as a performance signal.

2) PERFORMANCE EVALUATION

Since the channels have a random nature, the signals of interest become stochastic processes, and thus, to evaluate whether a controller achieves a reasonable performance or not, it is necessary to consider a suitable performance criteria. In stochastic feedback control systems it is a basic criteria to consider controllers that guarantee mean square stability (MSS) of the feedback loop [38], [39]. Moreover, controllers achieving zero mean in steady state for the error signal are also part of the design criteria. In addition, a small error variance is desirable, which reduces the probability of having outcomes far from the error mean. Unfortunately, since the platooning framework turns to a stochastic scenario, it is in general not possible to guarantee that every vehicle will perform in an appropriate fashion. In practice, this implies that the vehicles could have an erratic behaviour or even collide. However, reducing the variance will also reduce the probability that such poor behaviour occurs, or at least it should make it negligible.

Hence, to analyze different strategies to deal with data loss we consider as a minimum for a reasonable performance, controllers that achieve mean square stability and also ensure that

$$\lim_{k\to\infty} \mu_{\zeta_i}(k) = 0, \ \ \forall i \in \{1, \cdots N\}$$

where $\mu_{\zeta_i}(k) \triangleq \mathcal{E}\{\zeta_i(k)\}$ corresponds to the mean of the tracking error. Additionally, we can examine the variance of the errors,

$$\sigma_{\zeta_i}(k) \triangleq \mathcal{E}\left\{(\zeta_i(k) - \mathcal{E}\{\zeta_i(k)\})^2\right\},$$

and determine which strategy achieves a lower variance.

Another performance requirement on the context of platooning corresponds to string stability, which is discussed in the next section for stochastic scenarios.

C. STRING STABILITY WITH LOSSY COMMUNICATION CHANNELS

Due to the stochastic nature of the communication channels, the tracking errors in this framework are not deterministic signals, and thus the string stability notion in Section II-B cannot be adopted. Indeed, in Fig. 6 it is shown how the tracking errors behave when lossy channels are in place and for different values of the successful transmission probability p and time-headway constant h. It is clear that the erratic behaviour of the error is not compatible with the standard notion of string stability, since the uniform convergence observed in Fig. 4 is not present in this case, as expected.

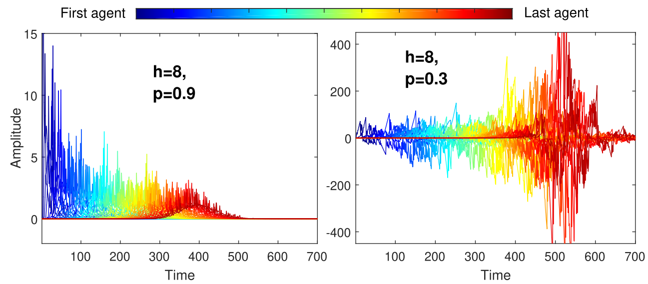

FIGURE 6. Tracking errors for different values of h and p when considering a platoon of $N = 50$ agents with imperfect inter-vehicle communications.

The notion of string stability for a stochastic framework has not been completely addressed in the current literature, although some progress has been made (see e.g. [8], [20], [23]). From these works it can be inferred that, in the stochastic scenario, the notion of string stability should be applied to the moments of the tracking errors, rather than to the tracking error signals. Thus, we consider the following definition

Definition 1: The platoon subject to data loss given in Fig. 5 is said to be compatible with string stability if the mean $\mu_{\zeta_i}(k)$ and the variance $\sigma_{\zeta_i}(k)$ of the tracking errors exhibit a string stable behaviour.

Clearly, a first condition for a platoon to be compatible with string stability can be obtained from the deterministic analysis in which a transfer function between two vehicles must satisfy a frequency domain restriction. Even though this does not guarantee string stability for the stochastic case, it would be a necessary condition.

Additionally, it is clear that the first and second moments of each tracking error must converge to a constant value when $k \to \infty$. In particular, the mean of the tracking errors must converge to zero since the local controller is designed to do so. The convergence of the second order moments implies that the local controller must be designed to guarantee MSS for the tracking errors. Since the system is subject to random data loss, MSS is not achieved, in general, with a controller designed only to ensure internal stability. This imposes an extra degree of complexity. The MSS requirement, together with the string stable behaviour on the second order moments of the errors, are the main challenges in these type of problems, and have limited the analysis in the literature to mostly numerical results, rather than analytical expressions.

IV. STRATEGIES TO DEAL WITH DATA LOSS

In control systems subject to measurement loss, the controller must have a protocol to decide what to do when a measurement does not arrive. Various strategies could be adopted to mitigate the potential negative effect on the control system.

Inspired by simple strategies like those in [28], in this section we propose and analyze some simple strategies to deal with data loss within the current platooning setup. We divide such strategies in three groups depending on the signal that is modified by the protocol. We also use three different types of numbering symbols in order to identify such protocols. This aims to clearly identify these basic protocols in Section IV-D, where several combinations of these basic strategies are presented.

Remark 3: The platooning setup in this paper considers that G is a simple integrator, and that the measurements represent position. If the platoon setup is modified, these strategies should be modified accordingly.

A. MEASUREMENT BASED

These strategies maintain the closed loop system T invariant, and focus on a pre-treatment of the received measurement. Specifically, this type of strategies consist of replacing the lost measurement $y_{i-1}(k)$ with the value $\hat{y}_{i-1}(k)$, which is computed with a predefined protocol. In the sequel we present three different protocols, whose block diagrams are depicted in Fig. 7.

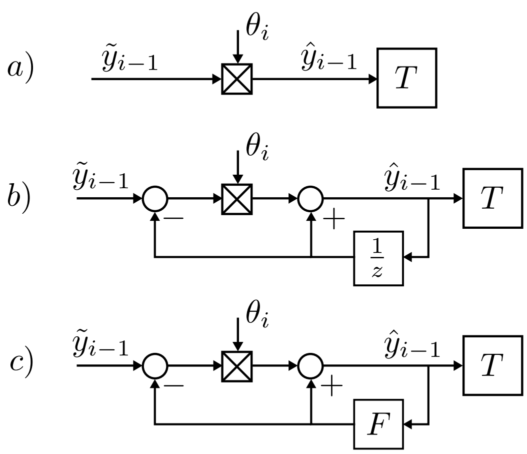

FIGURE 7. Measurement-based strategies: a) To-zero type, b) To-hold type, c) To-extrapolate type.

1) STRATEGY a)

The first strategy is given by default in our setup since it is inherited from the erasure channel model and consists of replacing the missing data with a zero value. That is:

$$\hat{y}_{i-1}(k) = \begin{cases} y_{i-1}(k) & \text{if } \theta_i(k) = 1, \\ 0 & \text{if } \theta_i(k) = 0. \end{cases} \quad (11)$$

Thus, strategy a can be implemented by

$$\hat{y}_{i-1}(k) = y_{i-1}(k)\, \theta_i(k). \quad (12)$$

This to-zero type strategy yields an exaggerated error in the local control loop of the follower when a loss occurs. This should yield an overreacted movement that could be desirable in a worst case scenario, for instance, whenever the predecessor moves backwards and its position signal is not arriving at the follower. On the contrary, in stationary navigation it is highly expected that this protocol will not work properly. This strategy is only added for completeness, and it represents the non-use of a proper measurement protocol when the erasure channel model described in Section IV is adopted. Notice that, if the transmitted data were to be acceleration rather than position, this strategy is quite reasonable. A diagram with the aforementioned strategy is given in Fig. 7.*a*)

2) STRATEGY b)

In this case, the missing measurement is replaced by the previously received predecessor position. Thus, from the point of view of the follower, the predecessor has temporarily stopped. This should yield a preventive speed reduction of the follower. In other words:

$$\hat{y}_{i-1}(k) = \begin{cases} y_{i-1}(k) & \text{if } \theta_i(k) = 1, \\ \hat{y}_{i-1}(k-1) & \text{if } \theta_i(k) = 0. \end{cases} \quad (13)$$

Thus, strategy b can be implemented by

$$\hat{y}_{i-1}(k) = \left[\tilde{y}_{i-1}(k) - \hat{y}_{i-1}(k-1)\right] \theta_i(k) + \hat{y}_{i-1}(k-1), \quad (14)$$

which is depicted in Fig. 7.*b*). This is a hold-type strategy with a conservative purpose: to avoid collisions when the predecessor stops and it is not perceived by the follower due to communication dropouts.

3) STRATEGY c)

This strategy consists of using previously received data to replace the lost measurement $y_{i-1}(k)$ according to a suitable estimation $\hat{y}_{i-1}(k)$. Sophisticated algorithms to construct $\hat{y}_{i-1}(k)$ can be proposed, however this is out of the scope of this paper since we are focusing on simpler protocols. Hence, in our case, we construct the estimates based on a linear extrapolation. The reasoning behind this protocol is that the platoon has reached its equilibrium state, and thus, y_{i-1} behaves as a ramp signal. This yields

$$\hat{y}_{i-1}(k) = \begin{cases} y_{i-1}(k) & \text{if } \theta_i(k) = 1, \\ \hat{y}_{i-1}(k-1) + \Delta(k) & \text{if } \theta_i(k) = 0, \end{cases} \quad (15)$$

where $\Delta(k)$ corresponds to an estimate of the travelled distance during the time period between $k-1$ and k. This distance can be estimated as

$$\Delta(k) = \hat{y}_{i-1}(k-1) - \hat{y}_{i-1}(k-2). \quad (16)$$

Given the fact that $\tilde{y}_{i-1}(k) = y_{i-1}(k)\theta_i(k)$, we can write the protocol as

$$\hat{y}_{i-1}(k) = \tilde{y}_{i-1}(k) - \left[2\hat{y}_{i-1}(k-1) - \hat{y}_{i-1}(k-2)\right]\theta_i(k) + 2\hat{y}_{i-1}(k-1) - \hat{y}_{i-1}(k-2). \quad (17)$$

Fig. 7.*c*) represents the proposed strategy, where

$$F(z) = (2z-1)/z^2.$$

This proposed protocol exploits previously received data to construct the measurement extrapolation using a second order FIR filter. Certainly, higher order or more sophisticated estimators to calculate $\hat{y}_{i-1}(k)$ can be used, but they are out of the scope of this work.

This strategy is designed for an equilibrium behaviour, and thus, cases where a transient response due to disturbances or a change of leader speed is present may affect the platooning performance, at least momentarily. Also, since F is of second order, this protocol may not work properly in cases where two or more consecutive losses in the channel are frequent. Opposite to Strategy b), this strategy considers that data loss and disturbances are not so frequent, which is not a preventive strategy.

B. LOCAL ERROR BASED

Since the local errors are the inputs to the controllers, we can incorporate strategies that manipulate the local errors, and thus, indirectly manipulate the controller output in case of data loss. Also, since controllers are in general dynamical systems and hence with memory, how the controller input is defined when measurements are missing would have an impact on future control inputs. We propose two types of strategies based on the local errors.

1) STRATEGY 1)

In this case, a zero-type strategy is adopted. Thus, the error is set to zero if no measurement is available. The controller input $\hat{e}_i(k)$ is given by

$$\hat{e}_i(k) = \begin{cases} e_i(k) & \text{if } \theta_i(k) = 1, \\ 0 & \text{if } \theta_i(k) = 0. \end{cases} \tag{18}$$

The reasoning behind this strategy is, given the fact that the controller has integral action, setting the error to zero would mimic a stationary behaviour, yielding as a control signal the one in the controller memory due to the integral action.

This protocol can be implemented using

$$\hat{e}_i(k) = e_i(k)\theta_i(k), \tag{19}$$

and this strategy is depicted in Fig.8.1).

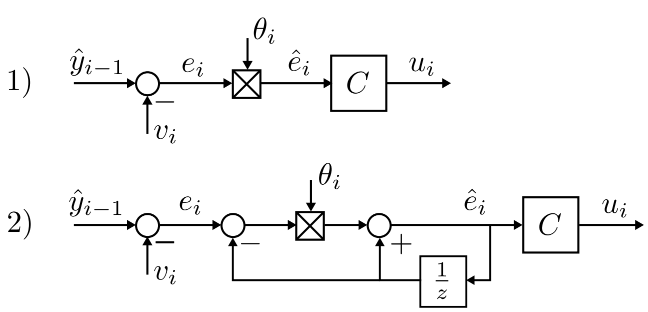

FIGURE 8. Error-based strategies: 1) To-zero type, 2) To-hold type.

2) STRATEGY 2)

In this case a hold-type strategy for the error signal is used, maintaining the previous error when a dropout occurs. That is

$$\hat{e}_i(k) = \begin{cases} e_i(k) & \text{if } \theta_i(k) = 1, \\ \hat{e}_i(k-1) & \text{if } \theta_i(k) = 0. \end{cases} \tag{20}$$

This is a risky strategy in which several losses in a row would keep a constant error that, given the controller integral action, could motivate a stronger control action to reduce the error, possibly bringing a follower dangerously closer to its predecessor. At first, this risky strategy does not seem reasonable in our setup, however, in a real application, the quality of communications could improve if both vehicles are closer.

This protocol can be implemented using

$$\hat{e}_i(k) = \left[e_i(k) - \hat{e}_i(k-1)\right]\theta_i(k) + \hat{e}_i(k-1), \tag{21}$$

whose block diagram representation is shown in Fig. 8. 2).

C. CONTROL SIGNAL BASED

These strategies seek to directly manipulate the behavior of the plant when there is data loss, managing the controller output to generate the new control signal $\hat{u}_i(k)$, reaching the actuator. We consider two cases.

1) STRATEGY i)

Here we use a zero-type strategy, in which the control input is set to zero whenever a lost measurement is detected. Thus we have

$$\hat{u}_i(k) = \begin{cases} u_i(k) & \text{if } \theta_i(k) = 1, \\ 0 & \text{if } \theta_i(k) = 0. \end{cases} \tag{22}$$

This protocol can be implemented using

$$\hat{u}_i(k) = u_i(k)\theta_i(k), \tag{23}$$

leading to the scheme in Fig 9.*i*). Since the agent dynamics G are modeled as a simple integrator, this protocol tends to stop the vehicle when a measurement loss occurs.

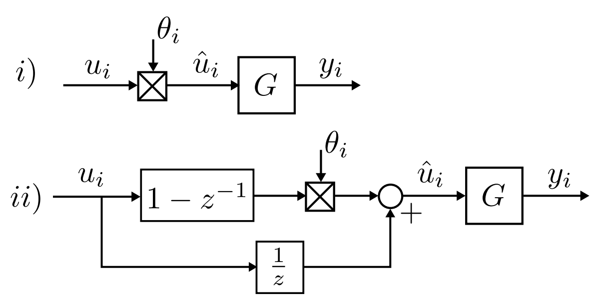

FIGURE 9. Control signal-based strategies: i) To-zero type, ii) To-hold type.

2) STRATEGY ii)

This strategy aims to maintain the behavior of the controller when a loss occurs. For this, the last control action is used,

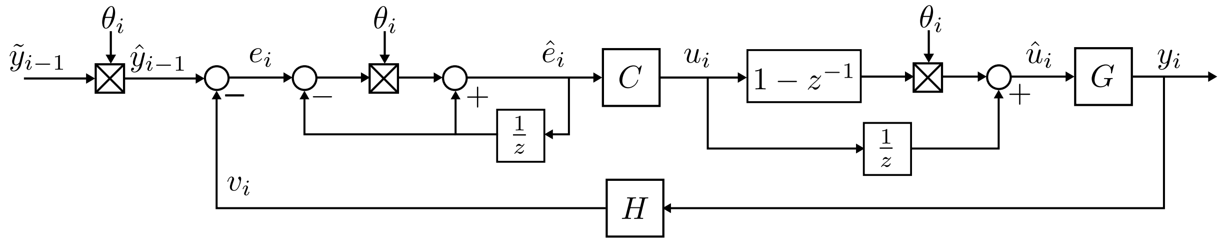

FIGURE 10. Example of a block diagram for the combined strategy *a*.2.*ii*.

whenever data loss is detected. This yields

$$\hat{u}_i(k) = \begin{cases} u_i(k) & \text{if } \theta_i(k) = 1, \\ u_i(k-1) & \text{if } \theta_i(k) = 0. \end{cases} \tag{24}$$

We can also write

$$\hat{u}_i(k) = [u_i(k) - u_i(k-1)]\,\theta_i(k) + u_i(k-1), \tag{25}$$

which is depicted in Fig.9.*ii*). This protocol is expected to yield an acceptable platoon behavior for a stationary case, when it is reasonable to consider that u_i has reached steady state.

D. MIXED STRATEGIES

Here we use the basic strategies and combine them to create new enhanced ones to deal with data loss. It is important to note that, since the lossy channel model yields the strategy a) by default, it is clear that the use of one of such measurement-based strategies is unavoidable. Thus, we consider all possible combinations that include one measurement-based strategy. Such combinations are listed in the first three columns of Table 1, where we have used the nomenclature previously introduced for the 3 types of basic strategies. The block diagram of these strategies is composed by the corresponding block diagrams in Figs. 7, 8 and 9. For instance, in Fig. 10 the strategy $a.2.ii$ is depicted, which is the combination of the individual strategies a, 2 and ii.

TABLE 1. Combinations of simple strategies.

a	b	c		
$a.1$	$b.1$	$c.1$	$\rightarrow$	$x.1$
$a.2$	$b.2$	$c.2$	$\rightarrow$	$x.2$
$a.i$	$b.i$	$c.i$		
$a.ii$	$b.ii$	$c.ii$		
$a.1.i$	$b.1.i$	$c.1.i$	$\rightarrow$	$x.1.i$
$a.1.ii$	$b.1.ii$	$c.1.ii$	$\rightarrow$	$x.1.ii$
$a.2.i$	$b.2.i$	$c.2.i$	$\rightarrow$	$x.2.i$
$a.2.ii$	$b.2.ii$	$c.2.ii$	$\rightarrow$	$x.2.ii$

In Table 1 we also added the measurement-based strategies to be employed individually, giving a total of 27 different strategies. However, instead of analyzing the 27 strategies, we can analyze a reduced number by noticing that some combinations are redundant. Indeed, from (18) it is clear that, when a loss occurs, the term $\hat{y}_{i-1}(k)$ is not getting into the feedback loop. On the other hand, since the measurement-based strategies satisfy $\hat{y}_{i-1}(k) = y_{i-1}(k)$ for $\theta_i(k) = 1$, the local error $e_i(k)$ corresponds to the true error regardless of the chosen measurement-based strategy. Hence, the effect of any measurement-based strategy is cancelled by the zero-type local-error strategy. This implies that the strategies $\{(a.1), (b.1), (c.1)\}$ yield exactly the same platoon behaviour. To simplify our notation, we name any of these cases as an $x.1$ strategy, emphasizing that the measurement-based protocols are not important in these cases. The same conclusion is valid for the group of strategies $\{(a.1.i), (b.1.i), (c.1.i)\}$ and $\{(a.1.ii), (b.1.ii), (c.1.ii)\}$, which are named $x.1.i$ and $x.1.ii$ respectively. A similar analysis can be done if a hold-type error-based strategy is used, yielding the protocols $x.2$, $x.2.i$ and $x.2.ii$. The last column in Table 1 refers to the set of equivalent strategies which are in the same row. Considering the equivalent combined strategies, the set of possible different cases is reduced from 27 to 15.

V. SIMULATION RESULTS

In this section we perform a comparison of the results obtained by simulating the formation control strategies proposed and their effect in the string stability properties of the system.

For our simulation analysis purposes, we are more interested in the influence of the adopted compensation strategies for data loss, rather than the controller design, over the string stability properties of the platoon, thus, we consider any admissible stabilizing controller. In particular, we consider that each vehicle is characterized by

$$G(z) = \frac{1}{z-1}, \quad C(z) = \frac{(1/(1+h))z}{(z-1)(z+0.7)},$$

with $H(z)$ given in (4). The controller $C(z)$ of this example was synthesized using Matlab, and is such that allows to stabilize the internal loops in the ideal case, that is, with perfect communications. Moreover, $C(z)$ allows to satisfy the classic string stability condition, that is, the transfer function $T(z)$ between two vehicles is such that $||T(z)||_\infty \leq 1$. In the general case, the poles of $T(z)$ depend on h. The chosen controller also depends on h to remove the dependence of the closed loop poles on this parameter (by canceling the dynamics of $H(z)$ in the product $G(z)H(z)C(z)$). This is a common strategy to facilitate the computation of the infimum value of h that is compatible with string stability in deterministic scenarios [17]. In this particular case, such infimum value is

$h = 3.4$, obtained in a numerical fashion as seen in Fig. 3 (a general derivation of this infimum can be seen in [23]).

To analyze the strategies in Table 1, we first numerically evaluate their performance and then we delve only into those strategies having a behavior compatible with string stability, discarding the cases that do not achieve it. Given the insight discussed in Section III-C, we designed experiments to analyze the mean and variance convergence of the tracking error for different probability of loss p and time-headway constant h. The results were obtained using a Monte Carlo simulation with 5×10^5 realizations.

A. DISCARDED STRATEGIES

We perform a set of experiments to observe the overall behaviour of each strategy in Table 1 and found that some strategies seem to not be compatible with string stability. Indeed, a deeper analysis shows that the platoon consistently exhibits a bad performance using such strategies for a wide set of values of h and p. We also modified the controller and such behaviour remains. This suggests that, under our platooning setup, string stability is not possible with such strategies.

More specifically, we identified two types of platoon behaviours that are classified as not compatible with string stability. In the first case, each vehicle in the platoon is not able to achieve mean square stability, and therefore, the mean and the variance of the errors grow unbounded over time. This bad performance is depicted in Fig. 11, where the strategy a) is used with favorable conditions for string stability, that is, a high probability of successful transmission $p = 0.98$ and a large time-headway $h = 20$. Similar results were obtained for higher values of h and p. Certainly, if mean square stability cannot be achieved, there is no way to achieve a string stable behaviour. Strategies that exhibit this performance are $\{(a), (a.i), (a.ii)\}$.

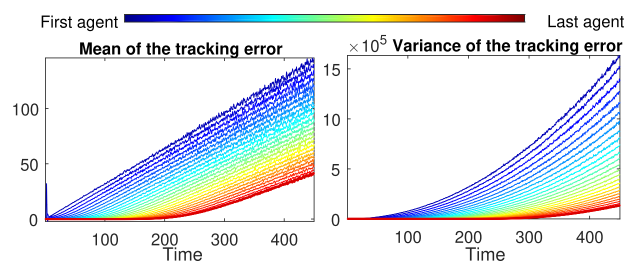

FIGURE 11. Mean and variance behaviour of the strategies that are not compatible with mean square stability (Simulation parameters: $N = 25$, $h = 20$, $p = 0.98$).

On the other hand, the second type of behaviour corresponds to one where the means and the variances converge to non-zero constant values. This behavior is indeed stable in the mean square sense, and could be compatible with string stability in some scenarios, however, in our setup, we design the controller to achieve zero error in steady state for a deterministic case. Consequently, it is expected that the mean of the error of each vehicle converges to zero for the stochastic case. This behaviour is depicted in Fig. 12, where the

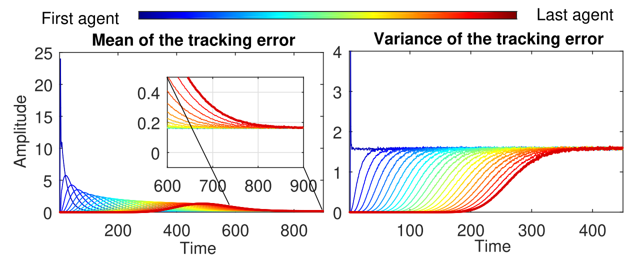

FIGURE 12. Mean and variance behaviour of the strategies that do not converge to zero (Simulation parameters: $N = 25$, $h = 20$, $p = 0.98$).

strategy b) is employed. The simulation results show that the stationary mean of the error increases as p decreases, which is undesirable. We discard strategies that always exhibit this performance, which are $\{(x.1.i), (x.2.i), (b), (b.i), (b.ii), (c.i)\}$, where the use of x indicates that it can be replaced by any measurement-based protocol.

Finally, the remaining strategies that are compatible with string stability are:

$$x.1, \quad x.1.ii, \quad x.2, \quad x.2.ii, \quad c, \quad c.ii.$$

B. PLATOON PERFORMANCE

To discuss the platoon performance, we run simulations where the lead vehicle speeds up with a constant acceleration and then it keeps moving with constant speed. As mentioned before, the platoon uses a predecessor follower topology, being the lead vehicle's position, the only deterministic signal in the platoon. Moreover, all the vehicles in the platoon start from rest and move in a straight line.

We are interested in comparing the behavior of the six principal combined strategies. To do so, in Figs. 13, 14, 15 and 16, we present the mean and variance of the tracking errors for each strategy (respectively labeled on the upper right side of each plot) for different values of h and p. It is important to notice that such figures are not always in the same amplitude scale.

First, we consider $h = 5$ and $p = 0.85$, in Figs. 13 and 14, and observe a convergent behaviour for all the strategies under study. Therefore, we focus on analysing the mean and variances of the errors. Notice that, both the mean and variance uniformly converge to zero as the number of vehicles and time grow, which is compatible with string stability.

Concerning the mean of the tracking error, we see a similar behavior among the strategies. Although, strategies $x.1$ and $x.1.ii$ are the ones with slightly higher amplitude. On the other hand, the differences in performance among strategies can be appreciated in the variance of the tracking error. In comparison, strategy $x.1.ii$ has the highest amplitude, followed by $x.1$, which suggests that replacing the error with a zero value is not a suitable strategy in terms of performance, as it was possible to predict. Also, holding the previous control signal shows a poor performance too ($x.2.ii$, $x.1.ii$, $c.ii$). Strategies $x.2$ and c on the other hand exhibit the best performances. Moreover, the amplitude difference of these two strategies

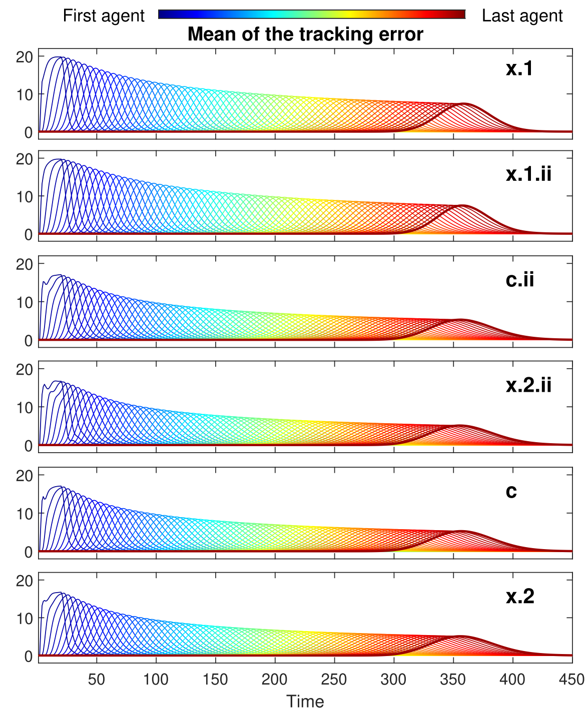

FIGURE 13. String stable behavior of the mean of the tracking error (Simulation parameters: $N = 70$, $h = 5$, $p = 0.85$).

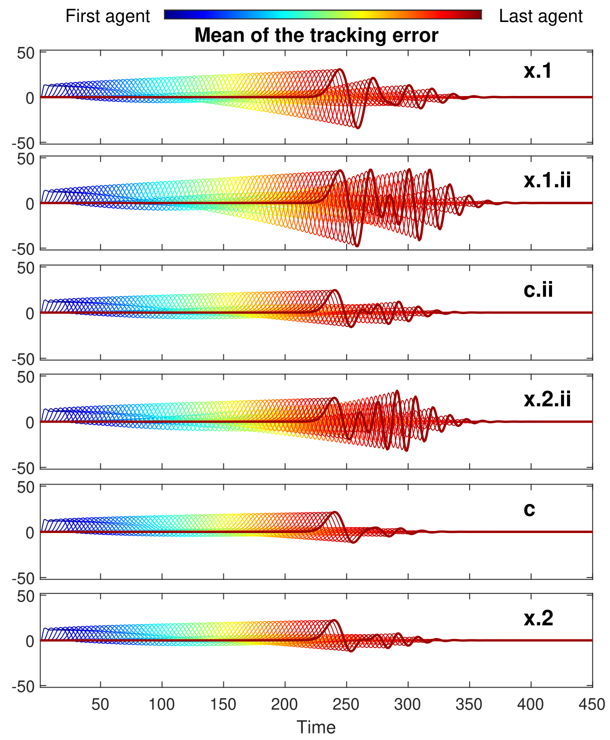

FIGURE 15. String unstable behavior of the mean of the tracking error (Simulation parameters: $N = 70$, $h = 3.2$, $p = 0.95$).

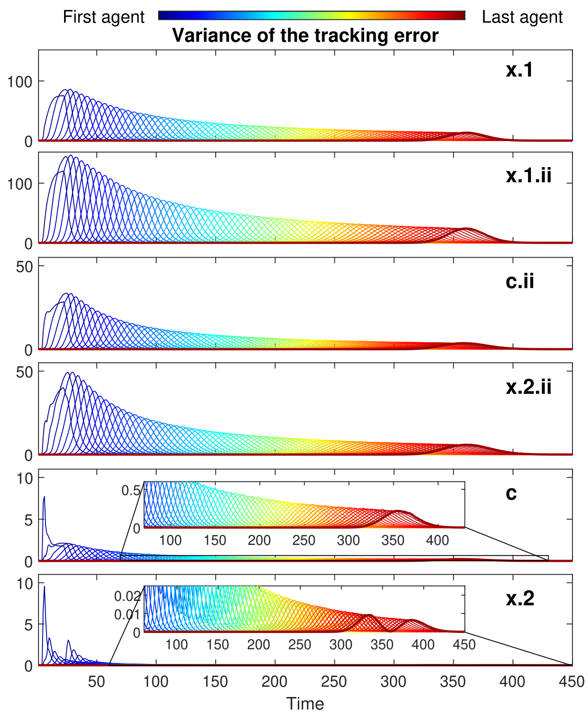

FIGURE 14. String stable behavior of the variance of the tracking error (Simulation parameters: $N = 70$, $h = 5$, $p = 0.85$).

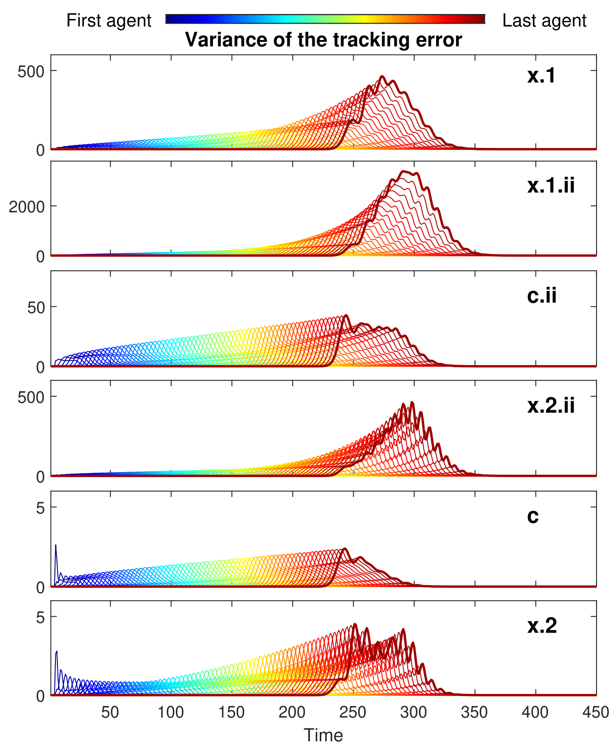

FIGURE 16. String unstable behavior of the variance of the tracking error (Simulation parameters: $N = 70$, $h = 3.2$, $p = 0.95$).

compared with the remaining ones is significant. Between these two strategies, $x.2$ is the one with minimum variance. This suggests that holding the previous error strategy is the best option for the setup considered in this paper, regardless of the adopted measurement-based strategy. Extrapolating the missing position seems also a suitable option. We also notice that the variance of the tracking error in strategy $x.2$ has a particular behavior that can be appreciated as a double peak. Nonetheless, this behavior is still compatible with string stability. On the other hand, we can observe that adding strategy ii has a detrimental effect on the performance of the strategies. In fact, strategies $c.ii$, $x.1.ii$ and $x.2.ii$ present higher error variances compared to c, $x.1$ and $x.2$, respectively. This suggests that holding the previous control signal is not a good option in terms of performance.

On the other hand, Figs. 15 and 16 present the results of a string unstable behavior for all strategies. For the values of $h = 3.2$ and $p = 0.95$, we can see the mean and variance of the tracking error of each vehicle converging to zero as time grows large, implying that each local control loop is mean square stable. However, the string unstable behavior can be observed in the increase of both peak amplitudes and oscillations along the string. It is important to recall that even though the mean and variance vanish, it does not mean that the platoon is string stable.

Although in this case the platoons are not string stable, we still can compare the performances of each strategy, as mitigating the effects of string instability may be enough to enable platoons of a smaller and fixed size (no new vehicles merge the platoon). As a matter of fact, having the same parameters and platoon length, we can evaluate the behaviour of each strategy and conclude that they are ranked almost in the same order than in the stable case, except for the strategy showing the best performance. Moreover, strategy c presents lower error variance compared to $x.2$. Nevertheless, these two strategies are considerable better than the remaining ones. The worst case is still strategy $x.1.ii$, which presents a poor behaviour in both the mean and the variance.

For completeness, we tested a different reference that contains a change in speed and a braking zone to determine whether this affects the string stable behaviour. In this experiment, the lead vehicle has two periods with constant acceleration, at the beginning of the movement, and at instant $k = 300$. Finally, at instant $k = 550$, the leader brakes to a stop with a constant deceleration. In this experiment we found that, independently of the perturbations, the mean and variance behavior is consistent with the ones showed previously for every strategy, suggesting that perturbations may not play a major role on the string stability property in these kind of setups. To illustrate this, in Fig. 17 we present the mean of the position, and the mean and variance of the tracking errors for a platoon with strategy $a.1$, for a stable behaviour. A string unstable behaviour is depicted in Fig. 18. In both cases, the braking yields higher peaks due to the amplitude of the deceleration applied, compared to the two acceleration maneuvers.

C. STRING STABILITY ANALYSIS

As seen before, analysing the convergence of the statistics of the tracking error, we can determine whether the platoon is compatible with string stability given certain values of h and p. However, in the previous experiments we consider

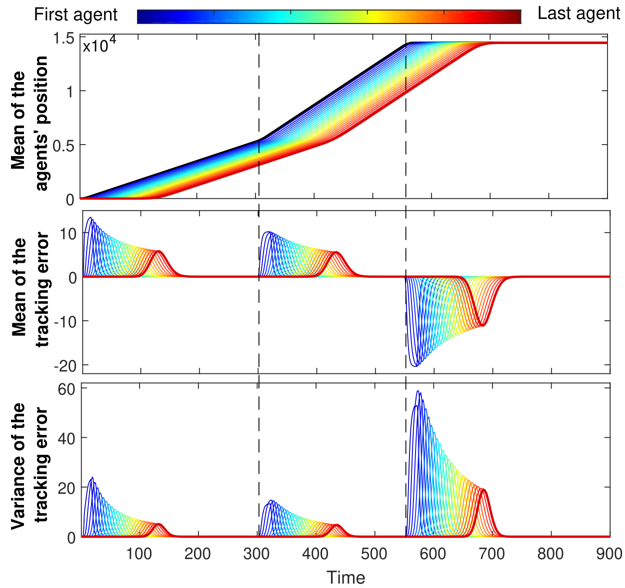

FIGURE 17. String stable behaviour for a reference with changes in the speed and breaking zone (Simulation parameters: strategy $a.1$, $N = 25$, $h = 5$ and $p = 0.9$).

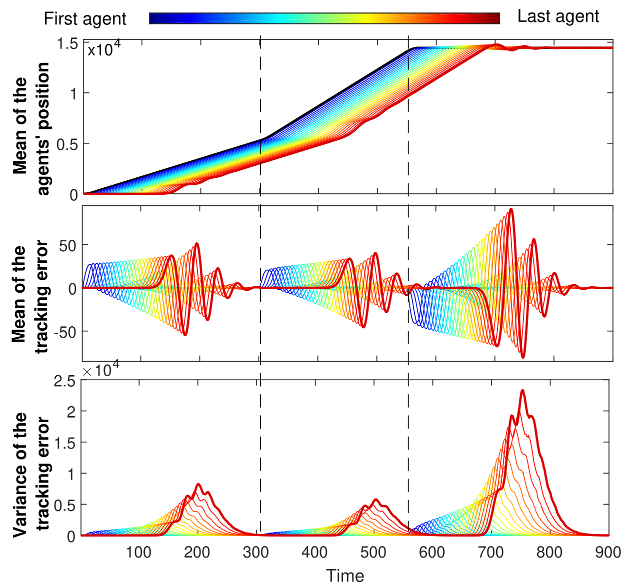

FIGURE 18. String unstable behaviour for a reference with changes in the speed and braking zone (Simulation parameters: strategy $a.1$, $N = 25$, $h = 5$ and $p = 0.5$).

values for h and p where all strategies yields stable platoons, or unstable platoons. We are now interested in determining the values of h and p where some strategies exhibit a stable behaviour and some others do not, which would reveal that the chosen strategy affects the string stabilization property of the interconnected system with respect to the characteristic parameters of the spacing policy and the communication channels.

Therefore, we test for string stability using different values of time headway (from $h = 0$ to $h = 15$) and probabilities

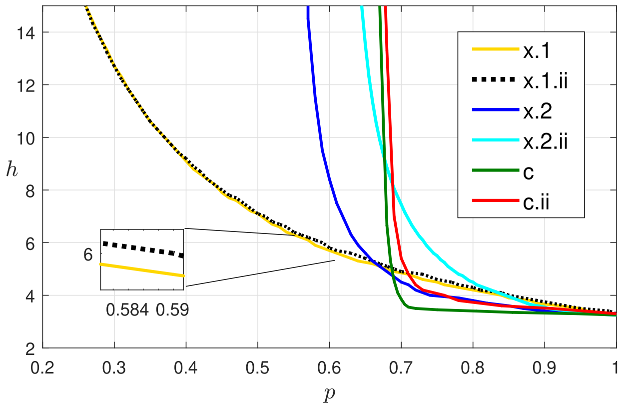

FIGURE 19. Approximations of the boundary of the string stability region for the strategies under consideration.

(from $p = 0$ to $p = 1$). The results of this heuristic-based method allow us to determine and approximation of a region in the parameter space formed by h and p which is compatible with string stability for all the strategies.

In Fig. 19 we show the six curves corresponding to boundary of the aforementioned region, for each strategy under analysis. The area above each curve corresponds to the region compatible with string stability and the area below a curve is the string unstable region. Generally speaking, if the losses increase (p decreases), a greater separation distance is required to ensure string stability (h increases), which is to be expected. In Fig. 19 we can also see that each curve may have a vertical asymptote, which could represent infimum values of the probability of successful communication for which the platoon would be string unstable whenever p is less than them and regardless of how large the value of h is. An infimum value of p has been reported in the context of networked control systems subject to data loss for mean square stabilization problems (see e.g. [40]), an aspect that may also be a characteristic for stochastic string stabilization problems as our results suggest. Strategies $x.1$ and $x.1.ii$ are the strategies with the smaller value of p at which the platoon can still be stabilized in the string sense. On the contrary, c and $c.ii$ are such that they cannot achieve a string stable behaviour unless the channel quality is better, measured by the size of p, when compared with the other strategies. Nevertheless, notice that all strategies can achieve string stability with a probability about $p = 0.7$, which means that an average of 30% of the transmitted data is lost. That represents a really poor channel in real applications.

Strategies $x.1$ and $x.1.ii$ are the ones that allow a greater probability of losses and therefore present a broader region of stability. Nevertheless, these two strategies have also the highest amplitude peak for the mean and variance of the tracking error. Although these two strategies have quite similar stability regions, they perform differently, as explained before. Between the two strategies with the best performance for the error variance, c and $x.2$; strategy $x.2$ has the broader stability region. Strategy c on the other hand seems to depend mainly on the value of h rather than the changes in p.

It is also worth to notice that adding the strategy ii, does not improve the string stability region. Indeed $c.ii$, $x.1.ii$ and $x.2.ii$ present a reduced region for string stability compared to c, $x.1$ and $x.2$, respectively. Hence, holding the previous control input seems to be a non-suitable alternative since it also has an undesirable effect on the error variance. In Table 2 we present a summary of the most relevant comparisons between strategies.

TABLE 2. Summary of some comparisons made between strategies for the simulation setup.

	$x.1$	$x.1.ii$	$x.2$	$x.2.ii$	c	$c.ii$
Strategies with higher average amplitude on the statistics of the error	✓	✓				
Strategies with lower average amplitude on the statistics of the error			✓		✓	
Strategies compatible with string stability even for $p < 0.5$	✓	✓				
Strategies with narrower string stability region				✓	✓	✓

D. OVERALL DISCUSSION

In this paper we study a set of simple strategies to deal with data loss and found that choosing such strategies must be carefully done. In particular, an initial design considers that the controller C is able to achieve string stability for the ideal communication case. However, channels suffering data dropouts may turn the platoon string unstable, mean square unstable, or not achieving an acceptable performance in terms of the mean and variance of the tracking errors.

An appropriate strategy to deal with data loss should be chosen. We found that some combination of such strategies may cancel some others, which is the case of measurement-based strategies that are irrelevant when they are used in combination with an error-based strategy. In other cases, simulation results show that some combinations do not achieve mean square stability or zero mean for the tracking error for any combination of the parameters h and p. In terms of performance, replacing the missing measurement with a simple estimation or using a strategy where the controller input is hold, are the best options. On the other hand, replacing the local error with a zero yields a broader region compatible with string stabilization. Additionally, simulation results suggest that holding the previous control signal has a detrimental effect on both performance and string stabilization. Certainly, these statements are valid for the current framework and simulation setups, and cannot be generalized to other topologies, controller or plant models. However, given the results of our exploration, we can confidently infer that the string stability, and moreover, the performance of a platoon with lossy communication channels, depends on the chosen data loss compensation strategy.

VI. CONCLUSION AND FUTURE WORK

In this article we studied platooning with random communication losses in the inter-vehicle channels that link two adjacent followers. We considered a predecessor-following topology for a homogeneous platoon described in the discrete-time domain. To reduce the impact of data loss on the platoon response, we use a set of simple strategies to deal with missing data based on well-known techniques such as holding the previous value, replacing by zero, or replacing by a prediction based on a linear extrapolation. We compared a set of different strategies through simulated experiments. In each case, we studied the convergence of the mean and variance of the tracking error to determine if the system is compatible with string stability. From our results it can be inferred that the strategy to deal with data loss may affect the string stabilizability of the platoon. We also noticed that the performance of the platoon, measured by the tracking error variance of the vehicles, also depends on the chosen strategy and, in general, the best option for performance is not the one that presents favorable conditions for string stability, which could hint towards a certain type of trade-off condition.

As future work, we consider the development of analytical expressions to determine whether a given strategy yields string stability or not. It is also in our interest to study the effect of data loss in other platooning topologies beyond the predecessor-following framework, such as leader-following and bidirectional topologies. Considering heterogeneous platoons and more complex dynamical models are also part of the future work.

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