IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received June 24, 2021, accepted July 14, 2021, date of publication July 26, 2021, date of current version August 2, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3099435

Planning of Reserve Branches to Increase Reconfiguration Capability in Distribution Systems: A Scenario-Based Convex Programming Approach

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This work was supported in part by the Coordination for the Improvement of Higher Education Personnel (CAPES)–Finance Code 001; in part by the São Paulo Research Foundation (FAPESP) under Grant 2017/02831-8, Grant 2015/21972-6, Grant 2018/20355-1, and Grant 2018/20990-9; and in part by the Brazilian National Council for Scientific and Technological Development (CNPq) under Grant 313047/2017-0 and Grant 170670/2018-0.

ABSTRACT Distribution networks are usually designed with a fixed tree-shape topology, which limits its adapting capability against failures and excessive variation of load/generation. Reconfiguration is an important tool to optimize the steady-state operating point of the distribution system under normal operating conditions, as well as increasing the network's responsiveness under interruptions by minimizing non-supplied demand. This paper studies how to increase the capabilities of reconfiguration in distribution networks through the optimal installation of reserve branches even if few branches are switchable. For this purpose, a scenario-based convex programming model is proposed to minimize the operation cost under normal operating conditions along with the energy not supplied related to network interruptions. Instead of conventional simulation adopted by previous approaches, the formulation is based on a set of linear logical constraints that represent the post-fault network reconfiguration process. Hence, classical optimization techniques can be used to solve the proposed model, which provides a suitable framework for the attainment of global optimality using efficient off-the-shelf software. Results show a decrease in the cost of energy production (2.85% and 1.08% for the 33-node and 69-node test systems) along with a decrease in the cost of energy not supplied (58.17% and 72.42% for the 33-node and 69-node test systems) due to the greater restoration capacity of the system using reserve branches.

INDEX TERMS Convex programming, post-fault reconfiguration, reconfiguration, restoration.

I. INTRODUCTION

Distribution operators generally aim to achieve three major objectives: meeting end users' demand, economically operating the network, and ensuring service continuity standards. To attend these objectives, several techniques have been used, such as expansion planning that implies the allocation of new assets in the network [1], [2], operation planning that maximizes the use of installed assets to reduce energy losses in the operation and ensure service safety criteria [3]–[5], and lastly, reliability planning, which ensures that the service provided searches for the lowest number of hours of discontinuity in the service [6]–[8]. In the framework of the three aforementioned options, the reconfiguration of the distribution network is common to all of them, so its study is of great interest to all stakeholders across the electrical sector.

Furthermore, network reconfiguration requires that new assets in reserve branches be planned by seeking to increase the system's capacity to respond to different operating conditions, both technically and economically. Reserve branches are redundant branches that form loops within the distribution network and are essentially used for reconfiguration.

The associate editor coordinating the review of this manuscript and approving it for publication was Chenghong Gu.

Since distribution systems operate in a radial pattern, reserve branches remain open when the system is operating normally and are used in case of interruptions [9], [10]. Consequently, if planned, reconfiguration ensures greater possibilities for restoration of service in the event of permanent interruptions [6], [8], [11]. Recently, the study of reserve branch planning and its operation has been directed toward increasing the resiliency of the distribution network [12]–[15].

Reconfiguration of distribution systems is carried out by changing the on/off status of the network switches (normally closed) and the connection switches (normally open) [16]. Switching must be done in such a way that the radiality of the network is maintained and all loads are satisfied. Obviously, the greater the number of switches, the greater the possibilities for reconfiguration and the better technical and economical effects [17]. In the context of intelligent networks, it is assumed that all branches have the equipment to be switched. Therefore, it is still important for the network operator to determine, from a possible set of new branches to be built, which ones increase the network capability in technical, economical, and reliable terms.

In the normal operating condition, distribution system reconfiguration has been conducted to enhance the efficiency of the system to achieve goals such as the reduction of power losses, load balancing, or voltage stability [18], [19]. However, considerable interest in reducing the economic losses due to reliability events has recently been identified by stakeholders in the electric sector. Reliability losses are defined through penalties established by regulatory bodies of the distribution network in each country. The penalties are related to the frequency and duration of the interruptions perceived by the end-users. The effects of the frequency and duration of interruptions in the network can be minimized via system restoration which can be analyzed as a particular instance of the reconfiguration problem [20].

Given the importance of the reconfiguration problem, it has been widely studied in the specialized literature; comprehensive reviews of the methods and instances of the reconfiguration problem can be found in [21]–[23]. A literature review of the reconfiguration problem for the service restoration in [24] is highlighted, in which the objective functions, constraints, and solution techniques commonly employed when reconfiguration is used to provide restoration services are summarized. Therefore, this review does not cover the planning of reserve branches. In this regard, a Tabu Search algorithm is proposed in [25] to support the decision process for the optimal allocation of reserve branches when road-related information is considered and it increases the number of candidate branches. Following the use of metaheuristics, a genetic algorithm is proposed in [26] to study the correlation between reserve capacity at substations and reserve branches along with load transfer capacity via customer outage cost. Aiming at the allocation of reserve branches to cover large distances, such that each interruption case can be restored with a pair of switches, a search algorithm based on the greedy set coverage problem is proposed in [27]. The previous approaches have great flexibility and robustness in their application, solving large instances involving the reconfiguration problem and the sizing of reserve branches for restoration service purposes; however, the application of those metaheuristics techniques makes difficult to get information about the feasible region exploration or even if the optimal solution is already obtained.

Optimization techniques are being widely used in the study of the reconfiguration problem [28] and recently adopted for the restoration problem [29] as consequence of the commercial solver development, which at the same time encourages the development of more complex and sophisticated mathematical models. For instance, a mixed-integer second-order cone programming model is used for solving the reconfiguration problem including the reliability assessment in [30], in which a multi-objective approach is proposed to obtain the best solutions for minimizing both the active power losses and reliability-related costs of the system. Another proposal based on second-order conic programming is found in [31] for the restoration service problem, whereby a set of the equation based on fictitious flow is stated to determine the post-reconfiguration topologies in the network aiming the minimization of non-supplied loads. Furthermore, a linear-programming approach for smart grid restoration is formulated in [32] including distributed generation. In this work, the post-fault network reconfiguration topologies, distributed generation requirements, and load shedding are obtained using a mixed-integer linear formulation. In addition to the self-healing actions included in [32], in [33], other actions are incorporated into a second-order conic programming model, such as modifying the tap setting of voltage regulating devices and providing reactive power from capacitor banks. The formulation determines the most efficient restoration plan by minimizing the number of de-energized nodes with the minimum number of self-healing actions. Recently, works have focused on solving the problem of restoration aiming at the resilience of the distribution network based on stochastic optimization techniques against extreme interruption events [12]–[14].

The aforementioned mathematical programming-based approaches obtain restoration schemes for specific cases of interruption in the distribution system, making use of the assets that are already installed, i.e., none of the mentioned approaches plan to increase the restoring capacity of the network. In this regard, an analytical method based on fault incidence matrices is used in [34] for the optimal planning of the sectional switches and reserve branches, in which a mixed strategy uses a genetic algorithm to determine switch allocation, while post-fault reconfiguration is accomplished via a quadratically-constrained optimization model. More recently, a mixed-integer linear programming model for the concurrent allocation of switches and reserve branches to enhance the reliability system was proposed in [10], whereby unreliability cost is estimated based on a reward-penalty scheme and the revenue lost due to the not supplied demand during network interruptions. The aforementioned approaches are the most similar to the present work. However, in [34] the optimal

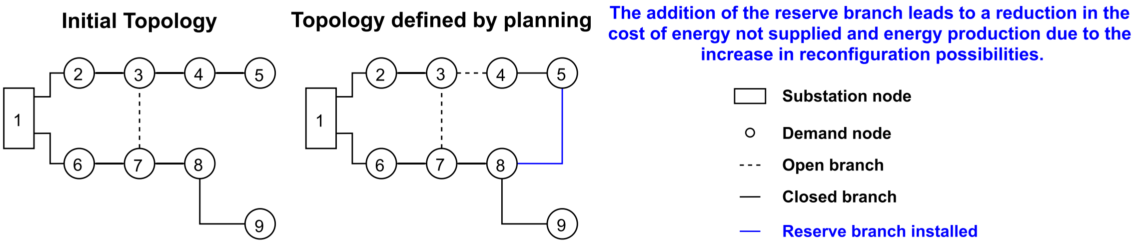


FIGURE 1. **Graphical abstract of the proposed method.**

allocation of reserve branches is done using a metaheuristic and the quality of the solution can only be evaluated in relative terms. Moreover, the work in [10] disregards the constraints related to the operation of the distribution system.

This work proposes a mixed-integer nonlinear model, suitable to be rewritten as a mixed-integer linear model using known linearization schemes to solve the reconfiguration problem in both normal and interruption operating states. Because the occurrence of interruptions is inevitable and to increase the reconfiguration capacity of the system, the mathematical model determines investments in reserve branches that allow the distribution system to anticipate the occurrence of interruptions using a scenario-based approach in which the investments corresponds to the first-stage stochastic decisions, i.e., *here and now* decisions. On the other hand, the post-fault reconfiguration topologies for all cases of branch interruption, as well as the topology of minimal active power losses for the operating state without interruptions, correspond to second-stage stochastic decisions, i.e., *wait and see* decisions. Furthermore, the effect of the distribution system is included in the formulation through a set of operational constraints for all operating states. Fig. 1 graphically summarizes the proposal of this manuscript.

Consequently, the main contributions of this work are twofold:

1) From a modeling perspective, the reconfiguration of distribution system problem considering post-fault reconfiguration is addressed by a new mathematical formulation based on algebraic expressions. Novel features include 1) considering the investment in reserve branches to improve reconfiguration and restoration service capability and 2) providing post-fault reconfiguration topologies for all cases of branch interruptions. The resulting model is suitable for classical optimization techniques with well-known properties in terms of solution quality and convergence. 3) The model allows considering that few branches of the network are switchable; other works perform preprocessing or simulation to incorporate this feature.
2) From a methodological point of view, a mixed-integer linear programming formulation is proposed to jointly consider economic, operational, and reliability aspects in the distribution network reconfiguration problem. The applicability of the proposed approach is supported by its effectiveness to optimally solve case studies.

The remainder of this paper is organized as follows. Section II presents the problem formulation. In Section III, numerical results from two cases of study are reported and analyzed. Relevant conclusions are drawn in Section IV.

II. PROBLEM FORMULATION

Typically, the reconfiguration of distribution systems aims to determine network topology by operating switches in a way that minimizes network operating costs. For this purpose, the reconfiguration model is formulated according to the operating limits of the system, such as voltage deviation, current limits through branches, substation capacity, and, for protection reasons, providing radial operating topologies [30]. However, to include reliability issues, a radially-operated distribution system and sustained interruptions due to single-branch outages are modeled, as done in [30], [35].

A non-linear programming model that considers both the aforementioned reliability characteristics along with the particularities of the reconfiguration problem is presented in this section seeking to fill some of the gaps in [10], [34]. Using some characteristics from stochastic programming, a scenario-based formulation is proposed to emulate the interruption in each branch of the system, while investment on reserve branches is made to increase the restoration capacity of the network. The resulting mixed-integer nonlinear program model is challenging to solve [36], wherefore the original model is recast as a mixed-integer linear program to achieve computational tractability. It is important to note that mixed-integer linear programming guarantees finite converge to the optimum, while providing a measure of the distance to optimality along the solution process.

A. LOGIC CONSTRAINTS FOR POST-FAULT RECONFIGURATION

After the occurrence of a fault in the distribution system, the first circuit breaker located upstream of the fault trips and all the downstream demands are curtailed. Consequently, to reduce the non-supplied energy, the system topology must be reconfigured through the operation of switches and circuit breakers. Thus, the fault is isolated opening the first switch upstream of the fault. Then, the circuit breaker is closed seeking that all the load demands between the circuit breaker and the switch are restored. Finally, the complete service is reestablished by the closure of the corresponding switch after clearing the isolated fault. In such a distribution system, each branch connected to a substation is equipped with a circuit breaker without a recloser at the output of the substation.

Under this scheme, work [35] characterizes the load node interruptions in two classes: repair-and-switching interruptions and switching-only interruptions. Repair-and-switching interruptions are characterized as being the longest lasting and the affected nodes only will be re-energized when damage will be repaired. On the other hand, switching-only interruptions are associated with the process of post-fault reconfiguration, which involves the isolation and restoration of the affected loads to another feeder if possible. For the calculation of both types of nodal interruptions, it is necessary to determine the influence of branch interruptions on the system, which depends on the expected rates and durations of both repair-and-switching and switching-only branch parameters as well as the radial topology of the distribution network.

To determine the investments in reserve branches along with the post-fault topological reconfiguration for service restoration, the effects of branch interruptions should be evaluated. For this purpose, the evaluation of a set of possible interruptions in the branches is necessary since the network topology is not known *a priori*. Thus, to consider the previous issues within the reconfiguration distribution network as well as service restoration problem, the following assumptions are adopted:

- Only individual branch interruptions are considered, where each interruption is defined by a failure rate and power outage duration [30], [35]. As a novelty, not all branches have their protection equipment for reliability as assumed in [10], [35]. Differently from [30], preprocessing is not necessary to reduce the network in zones with a reduced number of switchable branches.
- The individual branch interruptions are represented by a set of operating states S. A special state s_0 represents the normal operation of the network, i.e., operation without interruptions. Therefore, an element of the state set, different from s_0, coincides with the one branch ij, i.e., $s = ij$, to represent that this particular branch is the interruption under analysis. For this purpose, an interruption in the branch ij is representing with the indicator parameter $y^{UI}_{ij,t,s} = 1; \forall ij \in B, \forall t \in T, \forall s \in S|ij = s$, otherwise $y^{UI}_{ij,t,s} = 0; \forall ij \in B, \forall t \in T, \forall s \in S|ij \neq s$. B is the set of branches, and T is the set of planning stages.
- Because of the existence of meshed-radially operated networks that includes some branches open in the normal operating state, i.e. reserve branches, it is necessary to identify if the branch on failure is part of the optimal topology in normal operation. If so, the network suffers a "real" interruption that may lead to the reconfiguration of the network. Otherwise, interruption state do not produce any change in the network topology. Only real interruptions are included in the post-fault restoration. Boolean variable $y^{RI}_{ij,t,s}$ is used to indicate if the interruption of branch ij is a real one, i.e. if $y^{RI}_{ij,t,s} = 1$ then the interruption is a real one, otherwise $y^{RI}_{ij,t,s} = 0$. Through constraints (1)–(3) are defined the value of the variable $y^{RI}_{ij,t,s}$ as follow:

$$y^{UI}_{ij,t,s} + y^{B}_{ij,t,s_0} - 2y^{RI}_{ij,t,s} \leq 1; \quad \forall ij \in B,\ \forall t \in T,\ \forall s_0,\ s \in S|ij = s \tag{1}$$

$$y^{UI}_{ij,t,s} \geq y^{RI}_{ij,t,s}; \quad \forall ij \in B,\ \forall t \in T,\ \forall s \in S|ij = s \tag{2}$$

$$y^{B}_{ij,t,s_0} \geq y^{RI}_{ij,t,s}; \quad \forall ij \in B,\ \forall t \in T,\ \forall s_0,\ s \in S|ij = s \tag{3}$$

Thus, an interruption is real if the branch under interruption is part of the normal operating state. Binary variables y^{B}_{ij,t,s_0} are used to indicate if a branch ij is part of the normal operating topology at stage t, and operating state s, i.e. $y^{B}_{ij,t,s_0} = 1$, otherwise $y^{B}_{ij,t,s_0} = 0$ and the branch ij is a reserve branch. Furthermore, variables y^{B}_{ij,t,s_0} are relating to the power flow direction variables $y^{B+}_{ji,t,s_0}, y^{B-}_{ji,t,s_0}$ as follows: $y^{B}_{ij,t,s_0} = y^{B+}_{ji,t,s_0} + y^{B-}_{ij,t,s_0}$.

Using the above definitions it is possible to formulate a set of linear logical conditions to assess the effect of the post-fault reconfiguration and its impact in reliability indices. The following conditions are based on [37]:

Condition-1: If the interruption in the branch ij is a real interruption, then the nodes connected with them are directly affected by the interruption. Substation nodes are excluding by definition [35]. Thus, the above Boolean constraint is expressed as follow:

$$y^{RI}_{ij,t,s} \rightarrow \left(\Pi_{i,t,s} \wedge \Pi_{j,t,s}\right); \quad \forall ij \in B,\ \forall t \in T,\ \forall s \in S|\,(ij = s) \wedge \left(i, j \notin N^S\right) \tag{4}$$

Boolean prepositions $\Pi_{i,t,s}$ indicate if node i is affected by a real interruption in the operating state. This relationship can be recast as:

$$\Pi_{i,t,s} \geq y^{RI}_{ij,t,s}; \quad \forall ij \in B,\ \forall t \in T,\ \forall s \in S|\,(ij = s) \wedge \left(i, j \notin N^S\right) \tag{5}$$

$$\Pi_{j,t,s} \geq y^{RI}_{ij,t,s}; \quad \forall ij \in B,\ \forall t \in T,\ \forall s \in S|\,(ij = s) \wedge \left(i, j \notin N^S\right) \tag{6}$$

$$\Pi_{i,t,s} = 0; \quad \forall i \in N^S,\ \forall t \in T,\ \forall s \in S. \tag{7}$$

where N^S is the set of substations. Besides, $\Pi_{i,t,s} = 1$ if the demand node i is affected by the interruption at stage t, and operating state s; otherwise $\Pi_{i,t,s} = 0$.

Condition-2: If two adjacent nodes i, j are connected in the normal operating state topology, then they are equally affected by each interruptions in the network. This logical constraint is expressed as:

$$y^{B}_{ij,t,s_0} \rightarrow \left(\Pi_{i,t,s} \leftrightarrow \Pi_{j,t,s}\right); \quad \forall ij \in B,\ \forall t \in T,\ \forall s \in S|\,(ij \neq s) \wedge \left(i, j \notin N^S\right) \tag{8}$$

Mathematically, it can be recast as:

$$y^{B}_{ij,t,s_0} + \Pi_{j,t,s} - \Pi_{i,t,s} \leq 1; \quad \forall ij \in B,\ \forall t \in T,\ \forall s_0,\ s \in S|\,(ij \neq s) \wedge \left(i,j \notin N^S\right) \tag{9}$$

$$y^{B}_{ij,t,s_0} + \Pi_{i,t,s} - \Pi_{j,t,s} \leq 1; \quad \forall ij \in B,\ \forall t \in T,\ \forall s_0,\ s \in S|\,(ij \neq s) \wedge \left(i,j \notin N^S\right) \tag{10}$$

Condition-3: If a demand node i is not affected by a branch interruption, then its connection in the post-fault reconfiguration topology must be guaranteed. Constraints (11) express that condition:

$$\neg\Pi_{i,t,s} \rightarrow \Psi_{i,t,s}; \quad \forall i \in N \backslash N^S,\ \forall t \in T,\ \forall s \in S \tag{11}$$

where, Boolean preposition $\Psi_{i,t,s}$ indicates if a demand node i is restored. This logical constraint can be recast algebraically as:

$$1 - \Pi_{i,t,s} \leq \Psi_{i,t,s}; \quad \forall i \in N \backslash N^S,\ \forall t \in T,\ \forall s \in S. \tag{12}$$

where N is the set of all nodes. Hence, $\Psi_{i,t,s} = 1$ if the demand node i is restored at stage t, and operating state s; otherwise $\Psi_{i,t,s} = 0$.

Condition-4: If the interruption on branch ij is a real interruption and that branch does not have a switch, then the nodes connected to the branch cannot be restored.

$$\left(y^{RI}_{ij,t,s} \wedge \neg\psi_{ij}\right) \rightarrow \left(\neg\Psi_{i,t,s} = \neg\Psi_{j,t,s}\right); \quad \forall ij \in B,\ \forall t \in T,\ \forall s \in S|\,(ij = s) \wedge \left(i,j \notin N^S\right) \tag{13}$$

where, parameter ψ_{ij} indicates if a branch ij has a switch, i.e., $\psi_{ij} = 1$; otherwise $\psi_{ij} = 0$. Consequently, the above constraint is mathematically equivalent to:

$$\Psi_{i,t,s} \leq \left(1 - y^{RI}_{ij,t,s}\right); \quad \forall ij \in B,\ \forall t \in T,\ \forall s \in S|\,(ij = s) \wedge \left(\psi_{ij} = 0\right) \wedge \left(i,j \notin N^S\right) \tag{14}$$

$$\Psi_{j,t,s} \leq \left(1 - y^{RI}_{ij,t,s}\right); \quad \forall ij \in B,\ \forall t \in T,\ \forall s \in S|\,(ij = s) \wedge \left(\psi_{ij} = 0\right) \wedge \left(i,j \notin N^S\right) \tag{15}$$

Condition-5: If a branch ij without a switch is part of the normal state operation topology and is disconnected in the post-fault reconfiguration topology, then both nodes connected to it should not be restored.

$$\left(\neg\psi_{ij} \wedge y^{B}_{ij,t,s_0} \wedge \neg y^{B}_{ij,t,s}\right) \rightarrow \left(\neg\Psi_{i,t,s} \wedge \neg\Psi_{j,t,s}\right); \quad \forall ij \in B,\ \forall t \in T,\ \forall s \in S|\,(ij \neq s) \wedge \left(i,j \notin N^S\right) \tag{16}$$

This condition can be recast as follow:

$$y^{B}_{ij,t,s_0} - y^{B}_{ij,t,s} + \Psi_{j,t,s} \leq 1; \quad \forall ij \in B,\ \forall t \in T,\ \forall s_0,\ s \in S|\,(ij \neq s) \wedge \left(i,j \notin N^S\right) \tag{17}$$

$$y^{B}_{ij,t,s_0} - y^{B}_{ij,t,s} + \Psi_{i,t,s} \leq 1; \quad \forall ij \in B,\ \forall t \in T,\ \forall s_0,\ s \in S|\,(ij \neq s) \wedge \left(i,j \notin N^S\right) \tag{18}$$

Condition-6: If a branch ij without a switch is part of the normal state operation topology and is connected in the post-fault reconfiguration topology, then both nodes connected to it must be restored. This logical condition is set as:

$$\left(\neg\psi_{ij} \wedge y^{B}_{ij,t,s_0} \wedge \neg y^{B}_{ij,t,s}\right) \rightarrow \left(\neg\Psi_{i,t,s} \wedge \neg\Psi_{j,t,s}\right); \quad \forall ij \in B,\ \forall t \in T,\ \forall s \in S|\,(ij \neq s) \wedge \left(i,j \notin N^S\right) \tag{19}$$

Mathematically, this can be expressed as:

$$y^{B}_{ij,t,s_0} + y^{B}_{ij,t,s} - \Psi_{j,t,s} \leq 1; \quad \forall ij \in B,\ \forall t \in T,\ \forall s_0,\ s \in S|\,(ij \neq s) \wedge \left(i,j \notin N^S\right) \tag{20}$$

$$y^{B}_{ij,t,s_0} + y^{B}_{ij,t,s} - \Psi_{j,t,s} \leq 1; \quad \forall ij \in B,\ \forall t \in T,\ \forall s_0,\ s \in S|\,(ij \neq s) \wedge \left(i,j \notin N^S\right) \tag{21}$$

Briefly, $\Pi_{i,t,s}$ identifies if a node is affected in each operating state, either because it is directly connected with the real failure branch, as expressed in (5)–(7), or because it belongs to the same feeder of the real failure branch-related with the operating state, determined on a recurring way by (9) and (10). On the other hand the logical constraints related to reconfiguration are expressed in (11)–(21). This set of constraints together with the set of investment and radiality constraints, which are presented below, determine the reconfiguration topologies for all operating states. Furthermore, as a result of the restoration optimization process, $\Psi_{i,t,s}$ determines if a node is affected by a switching-only interruption or a repair-and-switching interruption. Hence, $\Pi_{i,t,s}$ and $\Psi_{i,t,s}$ can be used along with rates and duration interruptions of branches to assess nodal and system reliability indices equivalent to [35], as follows:

$$CIF_{i,t} = \sum_{\substack{ij \in B| \\ ij = s}} \lambda_{ij}\Pi_{i,t,s}; \quad \forall i \in N,\ \forall t \in T,\ \forall s \in S. \tag{22}$$

$$CID_{i,t} = \sum_{\substack{ij \in B| \\ ij = s}} \left(\tau^{RS}_{ij} - \tau^{SO}_{ij}\right)\lambda_{ij}\left(1 - \Psi_{i,t,s}\right) + \sum_{\substack{ij \in B| \\ ij = s}} \tau^{SO}_{ij}\lambda_{ij}\Pi_{i,t,s}; \quad \forall i \in N,\ \forall t \in T,\ \forall s \in S. \tag{23}$$

$$SAIFI_t = \frac{\sum_{i \in N} NC_i CIF_{i,t}}{\sum_{i \in N} NC_i}; \quad \forall t \in T \tag{24}$$

$$SAIDI_t = \frac{\sum_{i \in N} NC_i CID_{i,t}}{\sum_{i \in N} NC_i}; \quad \forall t \in T \tag{25}$$

$$ENS_t = \sum_{i \in N} CID_{i,t} \sum_{l \in LL} \frac{\Delta_l LL_l P^D_{i,t}}{8760}; \quad \forall t \in T. \tag{26}$$

where variables $CIF_{i,t}$ assess the customer interruption frequency at each planning stage t and variables $CID_{i,t}$ measure the customer interruption duration at each stage t. Hence, variables $SAIFI_t$ allow to assess the system average interruption frequency index at each planning stage t, and variables $SAIDI_t$ allow for the assessment of the system average interruption duration index. Finally, variables ENS_t enable to assess the energy not supplied (ENS). For the calculation of the last three indices, the number of users per node NC_i should be included. Besides, the above indices can be used in the optimization problem either in the objective function or as part of constraints on compliance with electrical service quality standards established by regulatory authorities [38].

Note that the aforementioned formulation takes into account that not all branches are switchable, which is closer to reality, where few branches are used for reconfiguration. Additionally, the constraints work recursively, so no preprocessing of the topology is needed to determine how non-switchable branches are affected at each interruption.

B. INVESTMENT AND OPERATIONAL CONSTRAINTS

Increasing the reconfiguration capacity of distribution systems implies investments in the network to expand the number of loops present in it. Based on [39], [40], here it is proposed a model that determines the investments in branches, construction or reinforcement, the best topology of operation in the normal state, as well as the best restoration topology for each stage t into the planning horizon. Expressions (27)–(36) constrain investment and operational variables as follows:

$$\sum_{t \in T} x_{ij,t} \leq 1; \quad \forall ij \in BA \tag{27}$$

$$y^B_{ij,t,s} \leq \sum_{p=1}^{t} x_{ij,p}; \quad \forall ij \in BA | \psi_{ij} = 0, \quad \forall t \in T, \ \forall s \in S | s \neq ij \tag{28}$$

$$y^B_{ij,t,s} = \sum_{p=1}^{t} x_{ij,p}; \quad \forall ij \in BA | \psi_{ij} = 1, \quad \forall t \in T, \ \forall s \in S | s \neq ij \tag{29}$$

$$y^B_{ij,t,s} \leq 1; \quad \forall ij \in BF | \psi_{ij} = 0, \ \forall t \in T, \ \forall s \in S | s \neq ij \tag{30}$$

$$y^B_{ij,t,s} = 1; \quad \forall ij \in BF | \psi_{ij} = 1, \ \forall t \in T, \ \forall s \in S | s \neq ij \tag{31}$$

$$y^B_{ij,t,s} = y^{B+}_{ij,t,s} + y^{B-}_{ij,t,s}; \quad \forall ij \in B, \ \forall t \in T, \ \forall s \in S | s \neq ij \tag{32}$$

$$y^B_{ij,t,s_0} = y^{B+}_{ij,t,s_0} + y^{B-}_{ij,t,s_0}; \quad \forall ij \in B, \ \forall t \in T, \ s_0 \in S \tag{33}$$

$$x_{ij,t} \in \{0,1\}; \quad \forall ij \in BA, \ \forall t \in T \tag{34}$$

$$y^B_{ij,t,s} \in \{0,1\}; \quad \forall ij \in B, \ \forall t \in T, \ \forall s \in S \tag{35}$$

$$y^{B-}_{ij,t,s}, y^{B+}_{ij,t,s} \in \{0,1\}; \quad \forall ij \in B, \ \forall t \in T, \ \forall s \in S. \tag{36}$$

where variables $x_{ij,t} = 1$ represent the addition of a branch ij at stage t; otherwise $x_{ij,t} = 0$. Note that the set of branches B has been divided into two subsets: one to indicate the branches to be added (BA) and the other to indicate the fixed branches (BF).

Expressions (27) ensure that a maximum of one addition is performed for each branch throughout the planning horizon. Expressions (28) and (29) guarantee that a branch can be used once its corresponding investment has already been made. It is considered that some addition branches are fixed and, therefore, their operation is limited as expressed in (29). The operation of fixed branches is limited through (30) and (31).

The operating state of a branch at each planning period is represented by two binary variables (32), as proposed in [41]. If $y^{B+}_{ij,t,s}$ or $y^{B-}_{ij,t,s}$ is equal to 1, then branch ij is in operation, whereas, if both are equal to 0, then branch ij is out of operation. Furthermore, the direction of the flow through a particular branch ij is modeled by the values of variables $y^{B+}_{ij,t,s}$ and $y^{B-}_{ij,t,s}$. Thus, the combination $y^{B+}_{ij,t,s} = 1$ and $y^{B-}_{ij,t,s} = 0$ is used to identify that node i is upstream of node j and, hence, the flow is from i to j. Conversely, the combination $y^{B+}_{ij,t,s} = 0$ and $y^{B-}_{ij,t,s} = 1$ is used to identify that node i is downstream of node j and, hence, the flow is from j to i. Expressions (33) define which branches are part of the topology in normal operating state, while the set of constraints (34)–(36) defines the binary nature of the decision variables.

C. TOPOLOGY CONSTRAINTS

This work aims to build a meshed network but capable of being reconfigured under different operating conditions to operate radially. This is because in distribution systems the radial topology offers operational advantages such as low operation cost and the simplest analysis and coordination of the system [20]. Furthermore, this proposal contemplates the construction of a radial operation topology where some nodes may not be connected to the network in the event of interruptions. Consequently, expressions (37)–(39), together with (32), guarantee the radial operation for the reconfiguration problem of distribution systems.

$$\sum_{ji \in B} y^{B+}_{ji,t,s} + \sum_{ij \in B} y^{B-}_{ij,t,s} = 0; \quad \forall i \in N^S, \ \forall t \in T, \ \forall s \in S \tag{37}$$

$$\sum_{ji \in B} y^{B+}_{ji,t,s} + \sum_{ij \in B} y^{B-}_{ij,t,s} = \Psi_{i,t,s}; \quad \forall i \in N \backslash N^S, \ \forall t \in T, \ \forall s \in S | P^D_{i,t} > 0 \tag{38}$$

$$\sum_{ji \in B} y^{B+}_{ji,t,s} + \sum_{ij \in B} y^{B-}_{ij,t,s} \leq \Psi_{i,t,s}; \quad \forall i \in N \backslash N^S, \ \forall t \in T, \ \forall s \in S | P^D_{i,t} = 0. \tag{39}$$

Expressions (37) ensure that, for those branches connecting load nodes with substations, the substation node is the sending node. Expressions (38) guarantee that each load node is the receiving node of a single branch. Finally, expressions (39) model transfer nodes, i.e., nodes without demand that are used to connect two load nodes. The parameter $P_{i,t}^{D}$ indicates whether the node i has demand or not at stage t.

It is worth noting that the proposed reconfiguration constraints ensure the radial operation of the network considering reserve branches planning and post-fault reconfiguration, extending the work presented in [28], which is suitable for a given network topology. This extension allows the optimization process to generate optimal topologies for different operating states aimed at the smallest number of loads affected by interruptions as well as the best topology under a normal operation state. Additionally, expressions (37)–(39) ensure the connection of unaffected nodes for each operating state.

D. REPRESENTATION OF THE OPERATING STATE IN THE DISTRIBUTION SYSTEM

Expressions (40)–(46) represent the ac power flow model for a radially-operated distribution network based on the set of recursive equations proposed in [42]:

$$\sum_{ki\in B} P_{ki,t,s} - \sum_{ij\in B}\left(P_{ij,t,s} + R_{ij} I_{ij,t,s}^{sqr}\right) + P_{i,t,s}^{F} + P_{i,t,s}^{S} = P_{i,t}^{D}\Psi_{i,t,s};\quad \forall i\in N,\ \forall t\in T,\ \forall s\in S \tag{40}$$

$$\sum_{ki\in B} Q_{ki,t,s} - \sum_{ij\in B}\left(Q_{ij,t,s} + X_{ij} I_{ij,t,s}^{sqr}\right) + Q_{i,t,s}^{F} + Q_{i,t,s}^{S} = Q_{i,t}^{D}\Psi_{i,t,s};\quad \forall i\in N,\ \forall t\in T,\ \forall s\in S \tag{41}$$

$$V_{i,t,s}^{sqr} = V_{j,t,s}^{sqr} + 2\left(R_{ij}P_{ij,t,s} + X_{ij}Q_{ij,t,s}\right) - Z_{ij}^{2} I_{ij,t,s}^{sqr} + \Delta_{ij,t,s}^{V};\quad \forall ij\in B,\ \forall t\in T,\ \forall s\in S \tag{42}$$

$$-b^{V}(1-y_{ij,t,s}^{B}) \le \Delta_{ij,t,s}^{V} \le b^{V}(1-y_{ij,t,s}^{B});\quad \forall ij\in B,\ \forall t\in T,\ \forall s\in S \tag{43}$$

$$V_{j,t,s}^{sqr} I_{ij,t,s}^{sqr} = P_{ij,t,s}^{2} + Q_{ij,t,s}^{2};\quad \forall ij\in B,\ \forall t\in T,\ \forall s\in S \tag{44}$$

$$P_{i,t,s}^{F} \le P_{i,t}^{D};\quad \forall i\in N,\ \forall t\in T,\ \forall s\in S | s=s_0 \tag{45}$$

$$Q_{i,t,s}^{F} = \tan(\cos^{-1}(pf_i))P_{i,t,s}^{F};\quad \forall i\in N,\ \forall t\in T,\ \forall s\in S | s=s_0 \tag{46}$$

In this formulation, the active and reactive power flows through branches are represented by the variables $P_{ij,t,s}$ and $Q_{ij,t,s}$, respectively. Likewise, the square of the current is represented by variables $I_{ij,t,s}^{sqr}$ and the square of the voltage at the nodes i is represented by variables $V_{i,t,s}^{sqr}$. Note that variables $V_{i,t,s}^{sqr}$ and $I_{ij,t,s}^{sqr}$ represent $V_{i,t,s}^{2}$ and $I_{ij,t,s}^{2}$, respectively.

Accordingly, equations (40) and (41) respectively ensure the active and reactive power balances at each node, i.e., Kirchhoff's first law. Expressions (42)–(44) correspond to Kirchhoff's second law. Note the presence of variable $\Psi_{i,t,s}$ in (40) and (41), indicating if each demand node should be satisfied. As expressions (12) guarantee, in normal operation $\Psi_{i,t,s} = 1$ for all demand nodes and therefore all demands must be supplied. In interruptions states, the reconfiguration decides whether the demands is supplied or not. Additionally, nodal load shedding at normal operating state is limited by (45) and (46).

Expressions (42) model branch voltage drops through auxiliary variables $\Delta_{ij,t,s}^{V}$, which are bounded in (43). Expressions (44) establish the relationship between active and reactive power flows $P_{ij,t,s}$ and $Q_{ij,t,s}$, squared current $I_{ij,t,s}^{sqr}$, and squared of the voltage $V_{j,t,e}^{sqr}$. Note that (44) contains nonlinear terms that can be linearized as described in [43]. Based on the limited range within nodal voltage magnitudes lie in practice, the product $V_{j,t,e}^{sqr}, I_{ij,t,s}^{sqr}$ in (44) can be linearized as follows:

$$V_{j,t,e}^{sqr} I_{ij,t,s}^{sqr} \approx V_{j,t,e}^{est} I_{ij,t,s}^{sqr};\quad \forall ij\in B,\ \forall t\in T,\ \forall s\in S \tag{47}$$

where $V_{j,t,e}^{est}$ is an estimated squared voltage. Furthermore, the quadratic terms in the right-hand side of (44) can be linearized using a piecewise approximation f using a number Λ of discretizations, as described in [43], [44], resulting in (48).

$$V_{j,t,e}^{est} I_{ij,t,s}^{sqr} = f(P_{ij,t,s}, max\left\{\overline{SL}_{ij}\right\}, \Lambda) + f(Q_{ij,t,s}, max\left\{\overline{SL}_{ij}\right\}, \Lambda);\quad \forall ij\in B,\ \forall t\in T,\ \forall s\in S. \tag{48}$$

The piecewise linearization is used to determine the operating point for the normal operating state. However, to reduce the computational effort, the *disflow* model in [45] that disregards the current calculation is used to set the operating point for each of the interruption states.

E. OPERATIONAL LIMITS

Expressions (49)–(52) set the acceptable ranges of the operational variables taking into account the operational state of the branches:

$$\underline{V}^{2} \le V_{i,t,s}^{sqr} \le \overline{V}^{2};\quad \forall i\in N,\ \forall t\in T,\ \forall s\in S \tag{49}$$

$$0 \le I_{ij,t,s}^{sqr} \le \overline{I}_{ij}^{2} y_{ij,t,s}^{B};\quad \forall ij\in B,\ \forall t\in T,\ \forall s\in S \tag{50}$$

$$-\overline{SL}_{ij} y_{ij,t,s}^{B} \le P_{ij,t,s} \le \overline{SL}_{ij} y_{ij,t,s}^{B};\quad \forall ij\in B,\ \forall t\in T,\ \forall s\in S \tag{51}$$

$$-\overline{SL}_{ij} y_{ij,t,s}^{B} \le Q_{ij,t,s} \le \overline{SL}_{ij} y_{ij,t,s}^{B};\quad \forall ij\in B,\ \forall t\in T,,\ \forall s\in S \tag{52}$$

These expressions represent the limits for voltages (49), currents (50), active power flows (51) and reactive power flows (52).

F. OBJECTIVE FUNCTION

The objective of the proposed model is to minimize the present value of the total cost, which includes the investment cost and the operating cost. The latter includes energy production, load shedding, and restoration costs. Here, the restoration cost is expressed in terms of the energy non-supplied. Note, however, that the proposed modeling framework is general enough to accommodate other standard reliability metrics formulated in (22)–(26).

Mathematically, the economic goal of the proposed optimization is formulated as:

$$\begin{aligned} Minimize c^{PV} = & \sum_{t\in T} \frac{(1+I_f)^{-(t-1)k}}{I_f} c_t^I \\ & + \sum_{t\in T} \left[(1+I_f)^{-(t-1)k} \left(c_t^E + c_t^{Sh} + c_t^{ENS} \right) \right] \\ & + \frac{(1+I_f)^{-(|T|-1)k}}{I_f} \left(c_{|T|}^E + c_{|T|}^{Sh} + c_{|T|}^{ENS} \right). \end{aligned} \tag{53}$$

The present value of the total cost c^{PV} comprises three terms. The first one represents the present value of the investments cost, while the second and third terms characterize the present value of the operating cost under the hypothesis of a perpetual or infinite planning horizon [46] along with a number of years k into each stage [39]. Note that the third term depends on costs at the last planning stage. Individual cost terms in (53) are cast as follows:

$$c_t^I = RR^B \sum_{ij\in BA} C_{ij}^{BA} x_{ij,t}; \quad \forall t \in T \tag{54}$$

$$c_t^E = PVF \sum_{l\in LL} \Delta_l LL_l \left(\sum_{i\in N^S} C^E P_{i,t,s}^S \right); \quad \forall t \in T \tag{55}$$

$$c_t^{Sh} = PVF \sum_{l\in LL} \Delta_l LL_l \left(\sum_{i\in N^S} C^{Sh} P_{i,t,s}^F \right); \quad \forall t \in T \tag{56}$$

$$c_t^{ENS} = PVF \left(C^{ENS} ENS_t \right); \quad \forall t \in T \tag{57}$$

In (54), the amortized cost of the investment at each stage is formulated as the sum of the costs associated with the addition and replacement of branches. The capital recovery rate for branches is computed as $RR^B = \frac{I_f(1+I_f)^{\eta^L}}{(1+I_f)^{\eta^L}-1}$, while the present value factor into each stage is computed as $PVF = \left(1-(1+I_f)^{-k}\right)/I_f$, respectively. The costs of energy production, load shedding, and ENS at each stage are modeled in (55)–(57), respectively.

Note that the model presented above is a mixed-integer linear programming formulation that can be solved using off-the-shelf software guaranteeing feasibility and optimality. It is important to clarify that the model considers multiple planning stages and within each of these periods, the same set of interruption scenarios is considered to determine the post-fault reconfiguration topologies, along with a single normal operation scenario that represents the maximum demand case for a typical day as done in [47]. The latter consideration requires making an adjustment to the operating costs to avoid their overestimating, which is done using the parameters Δ_l and LL_l in expressions (26), (55)–(57). To increase the accuracy of the model, it can be modified using the index ll to include more than one scenario for the normal operating state.

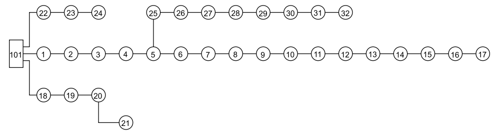


FIGURE 2. Initial topology of the 33-nodes test system.

III. NUMERICAL RESULTS

The proposed model has been applied in a 33-node test system adapted from [48] and a 69-node test system adapted from [49]. To investigate the reduction of the energy production and the energy not supplied, the planning of the systems is carried out without and with the addition of reserve branches. To analyze the performance of the proposed formulation and its dependence on the set of interruptions, two cases are studied: Case A and Case B; the former includes the outage of a limited number of branches, while Case B assesses the interruption of all branches of the system.

Simulations were performed on a DELL PowerEdge T430 computer with a 2.20 GHz processor and 65GB of RAM using CPLEX 12.6 [50] and AMPL [51]. For easy interpretation of the figures presented in this section, it must be taken into account that rectangles represent substations nodes, circles represent load nodes; continuous lines represent existent branches, while dashed lines represent candidate branches for addition. In the figures related to post-fault topologies, green circles indicate affected-fault load nodes while red circles indicate the non-restored load nodes or unused transfer nodes.

A. 33-NODE TEST SYSTEM

This system consists of 33 nodes, one substation (node 101), 32 load nodes, and 33 branches already constructed, as shown in Fig. 2; the nominal voltage is 12.66 kV and the lower and upper voltage limits are 0.95 and 1.05 p.u., respectively. The total load at first stage corresponds to 4,109.40 kW and 1,990.27 kVAr while at the second stage the total load is 8,218.80 kW and 3,980.55 kVAr. The complete data of this system can be found in [52]. Following, three case studies are presented. The base case where the system operation is evaluated without considering investments in reserve branches, i.e., the initial topology presented in Fig. 2 is fixed and without post-fault reconfiguration possibilities. The second case (Case A) enables investments in reserve branches considering interruptions in a limited set of branches, which for this system corresponds to interruptions in branches 101-1, 101-18, and 101-22. The third case (Case B) allows investments in reserve branches in the same way as case study A, but considers interruptions in all branches of the system. It is important to mention it is assumed that all branches are switchable for this test system.

Table 1 shows a comparison between the objective function for each of the case studies of the 33-node system. When comparing case A to the base case, there is a 2.36% reduction in energy production and a 37.52% reduction in the expected cost of ENS; on the other hand, the reduction of case B for the same metric, compared to the base case, is 2.89%

TABLE 1. Present value of costs (10^3 $) for 33-node test system.

	Base Case	Case A	Case B
Investment	0.00	524.97	665.26
Energy Production	34,268.75	33,458.04	33,450.87
ENS	5,277.07	2,238.24	2,075.47
Total cost	376,575.84	319,726.29	319,402.36

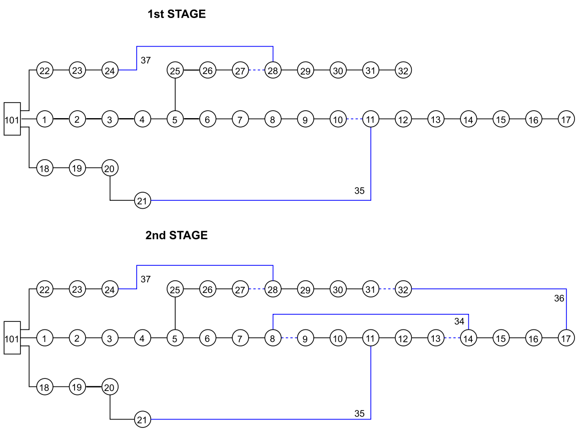


FIGURE 3. Investment topology for the 33-node test system - case A.

and 58.17%, respectively. Comparing Cases A and B, it can be seen that the expected value of non-supplied energy is 33.05% lower in Case B, but the investment values are 200% higher; this result shows the conflicting relationship between investment and reliability. Following, the investments and post-fault reconfiguration topologies for the set of interruptions considered in case studies A and B are described.

1) INVESTMENTS AND POST-FAULT RECONFIGURATION TOPOLOGIES FOR 33-NODE TEST SYSTEM CASE A

Four operation states are considered in Case A: one without permanent interruptions (normal operation) and three representing the interruption of one network feeder, i.e., faults for each of the branches connected directly to the substation (101-1, 101-18, and 101-22). The investment topology is shown in Fig. 3, whereby it states the installation of the reserve branches 11-21 and 24-28 at the first planning period and the installation of the reserve branches 8-14 and 17-32 at the second planning period. Furthermore, the optimal normal operation topology for the first planning period defines the opening of the branches 10-11 and 27-28, and the closing of the branches 24-28 and 11-21. At the second planning period, the optimal normal operation topology defines the opening of the branches 8-9, 13-14, 27-28, and 31-32, and the closing of the branches 8-14 and 17-32.

The post-fault reconfiguration scheme is shown in Fig. 4a for the interruption of branch 101-1. For the first planning period, the restoration service is obtained through the closure of the branch 10-11. Moreover, for the second planning period, the post-fault reconfiguration scheme proposes the switching of four branches (8-9, 27-28, 1-2, 6-7); the first two branches are indicated to be closed and the last two should be opened. In this case, total service restoration was not possible and node 1 is not re-energized, leading to an ENS equal to 117 kVA. Post-fault reconfiguration topology related to an interruption on the branch 101-18 is shown in Fig. 4b. In this reconfiguration topology at the first planning period, branch 10-11 is indicated to be closed. For the second planning period, five branches (8-9, 27-28, 31-32, 5-25, and 8-14) are proposed to be switched; the first three are closed and the last two are opened. Moreover, in this case, the full-service restoration is achieved at both planning periods. The last operating state corresponds to an interruption on the branch 101-22, whereby post-fault reconfiguration topology is shown in Fig. 4c for both stages of planning. At the first planning period, the restoration is obtained through the closure of the branch 27-28. Furthermore, at the second planning period, the post-fault reconfiguration topology states the opening of branch 22-23 and the closure of branch 27-28. In this case, the total restoration of the out-of-service area is not possible and node 22 is disconnected until the fault is repaired. The total ENS is 1,990.27 kVA for this interruption.

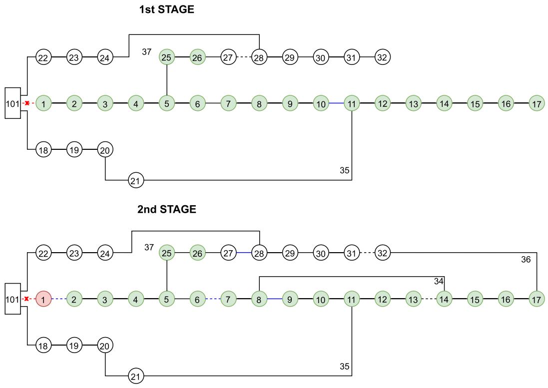
2) INVESTMENTS AND POST-FAULT RECONFIGURATION TOPOLOGIES FOR 33-NODE TEST SYSTEM CASE B

Thirty-seven operation states are considered in Case B: one without permanent interruptions (normal operation) and thirty-six representing the interruption of one network branch. The investment topology is shown in Fig. 5, whereby it states the installation of the reserve branches 11-21, 17-32, and 24-28 at the first planning period, while defines the installation of the reserve branches 7-20 and 8-14 at the second planning period. Moreover, the optimal normal operation topology for the first planning period defines the opening of the branches 9-10, 27-28, and 31-32, and the closing of the branches 11-21, 17-32, and 24-28. At the second planning period, the optimal normal operation topology defines the opening of the branches 11-21, 17-32, and 24-28 and the closing of the branches 10-11, 27-28, 31-32, 7-20 and 8-14.

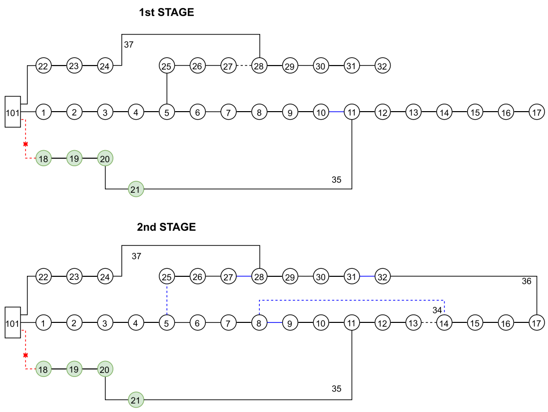
The thirty-seven post-fault reconfiguration topologies are summarized in Table 2 and Table 3, in which, for each branch interruption, the whole de-energized portion was successfully restored. It should be noted that the operating decisions have been separated for each planning stage in Table 2 and 3 to show the influence on demand growth on the post-fault reconfiguration topologies.

B. 69-NODE TEST SYSTEM

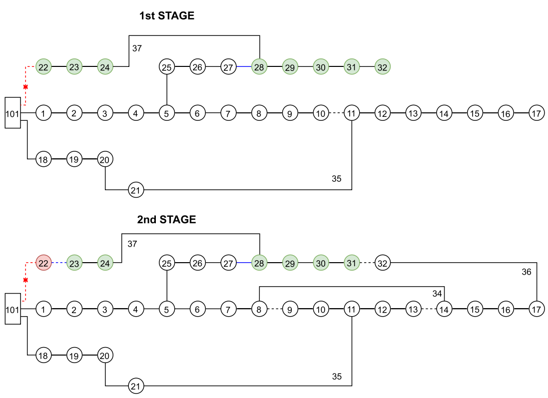
This test system consists of 69 load nodes and one substation (node 0), with 69 branches already constructed, as show in Fig. 6. In this system, nominal voltage is equal to 12.66 kV and the lower and upper voltage magnitude limits are equal to 0.95 and 1.05 p.u., respectively. The total load system corresponds to 1,1107.90 kW of active power and 897.93 kVAr of reactive power (more details are available in [49]). For this test system, only one planning period is taking into account



(a) Restoration scheme for the interruption on branch 101-1



(b) Restoration plan for the interruption on branch 101-18



(c) estoration plan for the interruption on branch 101-22

FIGURE 4. **Restoration scheme for the interruption on 33-node test for case A.**

and it is defined that few branches can be switched from the initial topology (0-1, 7-8, 20-21, 2-28, 3-36, 4-47, 9-53, and 59-60) along with the proposed reserve branches.

Three case studies are presented; the base case, without considering investments in reserve branches, whereby the fixed initial topology is shown in Fig. 2. The second case (Case A), which enables investments in reserve branches considering interruptions in a limited set of branches; that set corresponds to interruptions in branches 9-10, 30-31, 53-54, and 57-58. The third case (Case B) allows investments in reserve branches in the same way as Case A, but considers interruptions in all branches of the system.

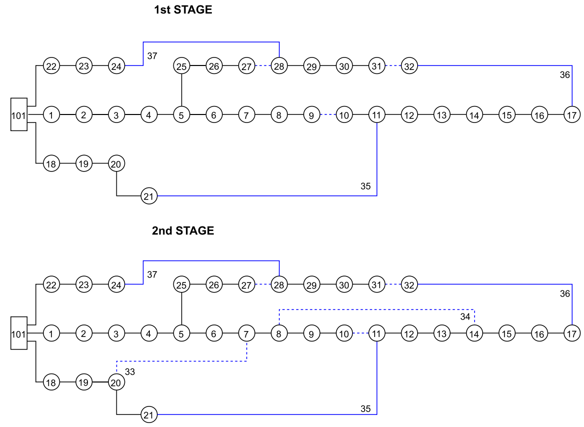


FIGURE 5. **Investment topology for the 33-nodes test system - case B.**

TABLE 2. **Post-fault reconfiguration topologies for 33-node test system case B at first stage.**

Fault	Closed branches	Opened branches
101-1	-/-	101-1, 9-10
1-2	-/-	1-2, 27-28
2-3	-/-	2-3, 9-10
3-4	-/-	3-4, 9-10
4-5	-/-	4-5, 27-28
5-6	-/-	5-6, 9-10
6-7	-/-	6-7, 9-10
7-8	-/-	7-8, 9-10
8-9	-/-	8-9, 9-10
*9-10	-/-	-/-
10-11	-/-	10-11, 9-10
11-12	-/-	11-12, 31-32
12-13	-/-	12-13, 31-32
13-14	-/-	13-14, 31-32
14-15	-/-	14-15, 31-32
15-16	-/-	15-16, 31-32
16-17	-/-	16-17, 31-32
101-18	-/-	101-18, 9-10
18-19	-/-	18-19, 9-10
19-20	-/-	19-20, 9-10
20-21	-/-	20-21, 9-10
101-22	-/-	101-22, 27-28
22-23	-/-	22-23, 27-28
23-24	-/-	23-24, 27-28
5-25	-/-	5-25, 27-28
25-26	-/-	25-26, 27-28
26-27	-/-	26-27, 27-28
*27-28	-/-	27-28
28-29	23-24	28-29, 27-28, 31-32
29-30	-/-	29-30, 31-32
30-31	-/-	30-31, 31-32
*31-32	-/-	31-32
**7-20	-/-	7-20
**8-14	-/-	8-14
11-21	-/-	11-21, 9-10
17-32	-/-	17-32, 31-32
24-28	-/-	24-28, 27-28

*opened branch at normal operation topology
**reserve branch constructed at second planning stage

It is important to point out that, for the fault cases, all branches are allowed to be switched, disregarding constraints

TABLE 3. Post-fault reconfiguration topologies for 33-node test system case B at second stage.

Fault	Closed branches	Opened branches
101-1	11-21	101-1, 31-32, 7-20
1-2	14-15	1-2, 31-32, 7-20
2-3	-/-	2-3, 7-20
3-4	-/-	3-4, 7-20
4-5	-/-	4-5, 7-20
5-6	-/-	5-6, 7-20
6-7	-/-	6-7, 10-11
7-8	-/-	7-8, 10-11
8-9	-/-	8-9, 10-11
9-10	-/-	9-10, 10-11
*10-11	-/-	-/-
11-12	-/-	11-12, 8-14
12-13	-/-	12-13, 8-14
13-14	-/-	13-14, 31-32
14-15	-/-	14-15, 31-32
15-16	-/-	15-16, 31-32
16-17	-/-	16-17, 31-32
101-18	15-16	101-18, 31-32, 7-20
18-19	-/-	18-19, 10-11, 31-32
19-20	-/-	19-20, 7-20
20-21	-/-	20-21, 8-14
101-22	22-23	101-22, 27-28
22-23	101-22	22-23, 27-28
23-24	5-6	23-24, 27-28, 7-20
5-25	-/-	5-25, 27-28
25-26	-/-	25-26, 27-28
26-27	-/-	26-27, 27-28
*27-28	-/-	27-28
28-29	8-9, 13-14, 24-28	28-29, 10-11, 31-32, 8-14
29-30	-/-	29-30, 31-32
30-31	-/-	30-31, 31-32
*31-32	-/-	-/-
*7-20	-/-	-/-
*8-14	-/-	-/-
11-21	-/-	11-21, 10-11
17-32	-/-	17-32, 31-32
24-28	13-14	24-28, 27-28, 8-14

*opened branch at normal operation topology
**reserve branch constructed at second planning stage

(29) and (31), although logical constraints of section II-A prevent nodes connected to fixed branches from being energized if they are affected by an interruption. In this case, the switching only allows the feasibility of the solution from the mathematical point of view, but the real switching that is described in each case of interruption only consists of the branches that have switches.

Table 4 shows a comparison between the components of the objective function for each of the case studies of the 69-node system; it is important to highlight that there is no cost of load shedding in the solution found for the normal state operation. In this case, a 1.08% reduction in energy production and a 66.01% reduction in the expected cost value of ENS can be observed when comparing case A to the base case. Whereas, the reduction of case B for the same metrics compared to the base case is 0.88% and 72.42%,

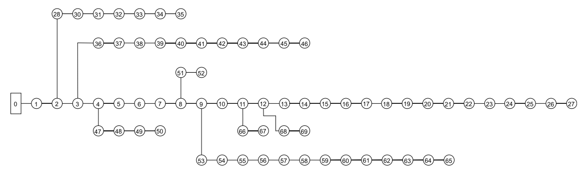


FIGURE 6. Initial topology for the 69-node test system.

TABLE 4. Present values of costs (10^3 \$).

	Base Case	Case A	Case B
Investment	0	82.63	137.71
Energy Production	39,373.74	38,947.44	39,027.03
ENS	38,044.27	12,931.01	10,491.52
Total cost	77,418.01	51,961.08	49,656.26

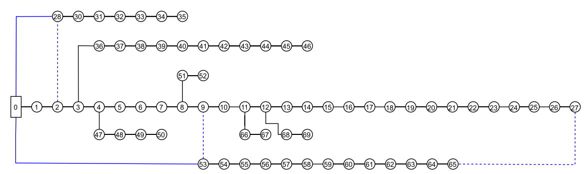


FIGURE 7. Investment topology for the 69-node test system - case A.

respectively. Comparing cases A and B, it can be seen that the expected cost value of ENS is 18.87% lower in Case B, as a consequence of the increase in investment (around 66.66%). Following, the investments and post-fault reconfiguration topologies for the set of interruptions considered in case studies A and B are presented.

1) INVESTMENTS AND POST-FAULT RECONFIGURATION TOPOLOGIES FOR 69-NODE TEST SYSTEM CASE A

The case study A for the 69-node test system considers five operation states. The first one corresponds to the normal operation state in which no interruptions are considered, while the remaining operation states correspond to the presence of permanent interruptions on several network branches (9-10, 30-31, 53-54, and 57-58). The investment topology is shown in Fig. 7, whereby it is proposed the installation of three branches (27-65, 0-28, and 0-53). For the normal operation topology, branches 2-28, 9-53, and 27-65 are opened.

In case of interruption on branch 9-10, the post-fault reconfiguration scheme proposes the switching of three branches (27-65, 7-8, and 20-21), in which the first branch is closed and the last two are opened. In this case, total restoration was not possible and the nodes 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 51, 52, 66, 67, 68, and 69 are not re-energized, totalizing an ENS equal to 294.96 kVA. When considering an interruption on branch 30-31, the post-fault reconfiguration topology consists of the opening of branches 0-28. The total ENS for this fault is 37.17 kVA, which represents the repair of nodes 28, 29, 30, 31, 32, 33, 34, and 35.

Moreover, for the interruption in branch 53-54, the post-fault reconfiguration topology states the opening of branches

TABLE 5. Post-fault reconfiguration topologies for 69-node test system case B (part one).

Fault	Closed branches	Opened branches
0-1	4-47, 15-46	0-1, 7-8
1-2	15-46, 27-65	0-1, 7-8, 3-36
2-3	15-46, 27-65	0-1, 7-8, 3-36
3-4	15-46, 27-65	0-1, 7-8, 3-36
4-5	15-46, 27-65	0-1, 7-8, 3-36
5-6	15-46, 27-65	0-1, 7-8, 3-36
6-7	15-46, 27-65	0-1, 7-8, 3-36
7-8	9-53	7-8
8-9	27-65	7-8, 20-21
9-10	27-65	7-8, 20-21
10-11	27-65	7-8, 20-21
11-12	27-65	7-8, 20-21
12-13	27-65	7-8, 20-21
13-14	27-65	7-8, 20-21
14-15	27-65	7-8, 20-21
15-16	27-65	7-8, 20-21
16-17	4-47, 27-65	7-8, 20-21, 0-47
17-18	27-65	7-8, 20-21
18-19	27-65	7-8, 20-21
19-20	27-65	7-8, 20-21
20-21	2-28, 4-47, 11-43, 27-65	7-8, 20-21, 0-28, 0-47
21-22	9-53	20-21, 0-53
22-23	-/-	20-21
23-24	-/-	20-21
24-25	-/-	20-21
25-26	9-53	0-1, 20-21
26-27	11-43	7-8, 20-21
*2-28	9-53	0-1
28-29	11-43, 27-65	0-1, 3-36, 0-28
29-30	4-47, 11-43, 27-65	0-1, 3-36, 0-28, 0-53
30-31	4-47, 9-53	0-1, 0-28, 0-47
31-32	11-43, 27-65	3-36, 59-60, 0-28
32-33	4-47,	0-1, 0-28
33-34	9-53, 11-43	0-1, 7-8, 0-28
34-35	4-47, 9-53, 11-43	0-1, 7-8, 0-28, 0-53
3-36	11-43	3-36
36-37	4-47, 27-65	20-21, 3-36, 0-47

*opened branch at normal operation topology

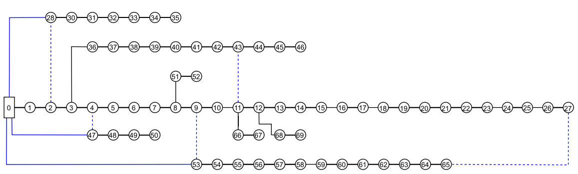


FIGURE 8. Investment topology the 69-node test system- case B.

59-60 and 0-53 and the closure of branch 27-65. In this case, the nodes 53, 54, 55, 56, 57, 58, and 59 are not restored and the amount of ENS is 46.54 kVA. Finally, for a permanent fault on branch 57-58, the restoring topology determines the opening of branch 0-28. In this case, the total restoration of the out-of-service area is not possible and the nodes 53, 54, 55, 56, 57, 58, and 59 are disconnected until the fault is repaired. The total ENS is 46.54 kVA for this interruption.

TABLE 6. Post-fault reconfiguration topologies for 69-node test system case B (part two).

Fault	Closed branches	Opened branches
37-38	27-65	3-36, 0-53
38-39	4-47, 9-53, 27-65	20-21, 3-36, 0-47, 0-53
39-40	4-47	3-36, 0-47
40-41	-/-	3-36
41-42	-/-	3-36
42-43	9-53, 27-65	3-36, 59-60, 0-53
43-44	9-53	3-36, 0-53
44-45	-/-	3-36
45-46	4-47	3-36, 0-47
*4-47	11-43	3-36
47-48	-/-	0-47
48-49	27-65	0-47, 0-53
49-50	9-53, 11-43, 27-65	7-8, 20-21, 0-47, 0-53
8-51	27-65	7-8, 20-21
51-52	27-65	7-8, 20-21
*9-53	-/-	-/-
53-54	27-65	59-60, 0-53
54-55	27-65	59-60, 0-53
55-56	27-65	59-60, 0-53
56-57	27-65	59-60, 0-53
57-58	27-65	59-60, 0-53
58-59	2-28, 11-43, 27-65	0-1, 3-36, 59-60, 0-53
59-60	9-53, 27-65	0-1, 59-60
60-61	-/-	59-60
61-62	-/-	59-60
62-63	4-47	59-60, 0-47
63-64	9-53, 11-43	0-1, 7-8, 59-60
64-65	4-47, 9-53, 11-43	0-1, 7-8, 59-60, 0-53
11-66	27-65	7-8, 20-21
66-67	27-65	7-8, 20-21
12-68	27-65	7-8, 20-21
68-69	27-65	7-8, 20-21
*11-43	4-47, 9-53, 27-65	0-1, 20-21, 0-47
*27-65	9-53	0-53
0-28	2-28, 4-47, 11-43	0-1, 7-8, 0-28
0-47	2-28, 4-47, 27-65	0-1, 7-8, 0-47
0-53	4-47, 9-53	0-1, 0-53

*opened branch at normal operation topology

2) INVESTMENTS AND POST-FAULT RECONFIGURATION TOPOLOGIES FOR 69-NODE TEST SYSTEM CASE B

Seventy-seven operation states are considered in Case B; one without permanent interruptions (normal operation) and seventy-six representing the interruption of one network branch. The investment topology is shown in Fig. 8, whereby it states the installation of the branches 11-43, 27-65, 0-28, 0-47, and 0-53. Besides, the optimal normal operation topology defines the opening of branches 2-28, 4-47 and 9-53. The 77 post-fault reconfiguration topologies are summarized in Table 5 and Table 6 for each branch interruption. For Case A, there are de-energized nodes in most of the outages, as a consequence of few switchable branches in the network, which is similar to the operation of a real distribution network.

IV. CONCLUSION

To enhance the capacity of distribution networks to different load/generation changes and faults, it is important to increase the number of loops within the system with the installation of reserve branches. In this regard, this paper has presented a scenario-based linear programming model that allows determining investments in branches that increase the capacity of the network (reserve branches) to cope with interruptions, while making it possible to improve the steady-state operation of the distribution system.

Unlike other models that generally use simulation or fictitious flows, a set of logical constraints is proposed in this paper to allows the assessment of the impact of restoration by evaluating reliability indices such as the cost of energy non supplied. The model allows obtaining both the reconfiguration topologies in the normal operating state, as well as the restoration topologies for each of the fault scenarios considered, which for this study are based on the N − 1 criterion.

The results obtained using two test systems show a decrease in power production cost (2.85% and 1.08% for the small and larger test systems), along with a decrease in the cost of energy non-supplied (58.17% and 72.42% for the small and larger test system); both achievements are due to the higher system reconfiguration capability using reserve branches.

Future work may consider stochastic aspects related to demand and branch reliability parameters. Other fields of application can be opened up by considering active network paradigms such as intermittent distributed generation, storage systems, electric vehicles, and demand-side management.

APPENDIX
NOMENCLATURE

Sets and Indexes:

B	Set of indices *ij, ji, ki* of branch types. $B = BA \cup BF$ where BA and BF denote added branch and existing fixed branch, respectively.
LL	Set of indices l of load levels.
N	Set of indices i, j of nodes. $N^S \subseteq N$ where N^S are substation nodes.
S	Set of indices s of operating states.
T	Set of indices t of planning stages.

Parameters:

Δ_{ll}	Duration of load level l.
η^B	Lifetime of branches.
λ_{ij}	Unitary failure rate of branch ij.
τ_{ij}^{RS}, τ_{ij}^{SO}	Durations of the repair-and-switching and switching-only interruptions associated with the failure of branch ij
ψ_{ij}	Indicator of a switching in branch ij.
Λ	Number of discretizations used in the piece-wise linearization function f.
b^V	Upper bounds for the absolute value of $\Delta_{ij,t}^V$.
C^E	Cost coefficient for the energy supplied by the substation.
C^{ENS}, C^{Sh}	Cost coefficients for load shedding and expected energy non supplied under branch outages.
C_{ij}^{BA}	Cost coefficients for the installation of branches ij.
I_f	Interest rate.
$\overline{I}_{ij}$	Maximum current of branch ij.
k	Years into each stage.
LL_l	Demand factor of load level l.
$P_{i,t}^D$, $Q_{i,t}^D$	Active and reactive power demands at node i and stage t.
PVF	Present value factor into each planning stage.
R_{ij}, X_{ij}, Z_{ij}	Unitary resistance, reactance, and impedance in branch ij.
RR^B	Capital recovery rate for investments in branches.
$\overline{SL}_{ij}$	Apparent power capacity of branch ij.
$\underline{V}$, $\overline{V}$	Lower and upper voltage limits.
$V_{i,t}^{est}$	Estimated squared voltage at node i, and stage t.
$y_{ij,t,s}^{UI}$	Binary parameter to identify if there is an interruption in the branch ij.
NC_i	Number of customers at node i.

Continuous Variables:

$\Delta_{ij,t}^V$	Square of voltage drop in branch ij and stage t.
c_t^E, c_t^I, c_t^{Sh}	Costs of production, investment, and load shedding at stage t.
c^{PV}	Present value of the total cost.
$CIF_{i,t}$	Customer interruption frequency at node i and stage t.
$CID_{i,t}$	Customer interruption duration at node i and stage t.
ENS_t	Energy not supplied at stage t.
$I_{ij,t,s}$	Current through branch ij, at stage t, and operating state s
$I_{ij,t}^{sqr}$	Squared of the current through branch ij, at stage t, and operating state s.
$P_{i,t,s}^S$, $Q_{i,t,s}^S$	Active and reactive power injections at substation node i, at stage t, and operating state s.
$P_{ij,t,s}$, $Q_{ij,t,s}$	Active and reactive power flows through branch ij, load level b, at stage t, and operating state s.
$SAIFI_t$	System average interruption frequency index at stage t.
$SAIDI_t$	System average interruption duration index at stage t.
$V_{i,t,s}$	Voltage at node i, at stage t, and operating state s.

$V_{i,t,s}^{sqr}$ Squared voltage at node i, at stage t, and operating state s.

$P_{i,t,s}^{F}$ Active power shedding at node i, stage t operating state s.

$Q_{i,t,s}^{F}$ Reactive power shedding at node i, stage t operating state s.

Binary Variables:

$\Pi_{i,t,s}$ Logic variable which indicates if node i is affected by an interruption event at stage t and operating state s.

$\Psi_{i,t,s}$ Logic variable which indicates if demand node i is restored at stage t and operating state s.

$x_{ij,t}$ Investment variables for branch ij at stage t.

$y_{ij,t,s}^{B}$ Operational variable for branch ij, at stage t, and operating state s.

$y_{ij,t,s}^{B+}$, $y_{ij,t,s}^{B-}$ Forward and backward directions of branch ij, at stage t, and operational state s.

$y_{ij,t,s}^{RI}$ Logic variables indicating if branch ij has a real interruption at stage t, and operational state s.

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