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Event-Triggered Adaptive Neural Network Backstepping Sliding Mode Control for Fractional Order Chaotic Systems Synchronization With Input Delay

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ABSTRACT This paper investigates the fractional order chaotic systems synchronization with input delay. To reduce the utilization of communication resources and achieve system synchronization, an adaptive neural network backstepping sliding mode controller is proposed based on event-triggered scheme without Zeno behavior. To avoid ''explosion of complexity'' and obtain fractional derivatives for virtual control functions continuously, the fractional order dynamic surface control (DSC) technology is introduced into the controller. The sliding term is introduced to enhance robustness. The Pade delay approximation method is used to handle the input delay, which can reduce the analysis complexity of fractional order chaotic systems with input delay. The unknown nonlinear functions and uncertain disturbances are approximated by the RBF neural network. An observer is used for state estimation of the fractional order system. By applying the Lyapunov stability theory, we can prove that the all closed-loop signals are bounded. Examples and simulations prove the feasibility of the proposed control method.

INDEX TERMS Fractional order chaotic systems synchronization, event-triggered, dynamic surface control, neural network, observer, input delay.

I. INTRODUCTION

In practice, most coupled dynamic systems have complex dynamics. The dynamical systems have the chaotic dynamics characteristics whenever its evolution sensitively depends on the initial conditions. It is observed that chaotic dynamics exist in many naturally occurring systems [1]–[3]. It is an ideal treatment of actual chaotic systems to describe chaotic systems through integer-order calculus. Fractional calculus has unique memory properties and the ability to accurately model the system [4]. Therefore, the fractional order dynamic systems can more truly reflect the situation of the system itself and present the physical phenomena reflected by the system, so the use of fractional calculus can more accurately describe the chaotic phenomenon [5], [6].

The chaotic systems synchronization is an important research field of chaotic system dynamics and it plays an important role in the field of control and industrial applications, especially in the application of physical systems, electrical circuits and secure communication [7]–[9]. To date, the chaotic systems synchronization has made many progresses in the field of integer order control, such as adaptive control [10]–[12], linear matrix inequality (LMI) control [2], [13], sliding mode control (SMC) [14]–[16] and so on. Up to present, there are some progresses in the research on the synchronization problem of fractional order chaotic systems. In [17], a fractional order adaptive synchronization controller was proposed for a new four-scroll chaotic systems. In [18], the authors studied adaptive terminal sliding mode synchronization for fractional chaotic systems with uncertainty and nonlinearity. In [19], the authors studied the fractional order chaotic systems with randomly occurring uncertainties and proposed a feedback controller based on LMI control to reach chaos synchronization. To deal with the uncertain of nonlinear systems, the universal approximation theories of neural networks(NNs) and fuzzy logic systems can be employed to approach the unknown nonlinear functions [20]. For example, by introducing radial

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basis function (RBF) into the backstepping control method, [21] developed an adaptive NN controller for an uncertain fractional order chaotic systems synchronization. [22] introduced fuzzy neural network technology into sliding mode controller to approximate those uncertain terms and unknown parameters. For fractional multi-agent systems with unknown uncertainty, [23] designed a distributed controller based on NN control method, the NN was used to approximate the system unknown uncertainty. Based on RBFNN, an adaptive backstepping control method was proposed by [24] for the consensus problem of fractional order nonlinear multi-agent systems. In most practical applications, the system states cannot be fully accessed. In this case, observer-based control is usually required. For example, [13] introduced a disturbance observer to estimate disturbance and uncertainty and [25] designed a state observer to obtain the system unmeasurable state for fractional order chaotic systems.

Due to the limited information processing and transmission speed, the time delay is common in actual engineering systems. The time delay phenomenon can easily cause the stability of the controlled system to decrease [26]. Therefore, it is important and necessary to consider the time delay problem of nonlinear systems. For example, Pade approximation method was introduced to cope with small delay and the designed controller was robust to the uncertain input delay to some degree in [27] and [28]. [29] applied Pade approximation technology to nonlinear systems to deal with input delay, and designed backstepping controller based on fuzzy logic system. [30] considered the control problem of fractional order systems with time-varying delays. [31] studied two different fractional order chaotic systems synchronization with time delay and introduced the delayed function to solve time delay.

Furthermore, the above-mentioned work input control signal must be continuously updated, which may cause unnecessary resource consumption. In terms of reducing the utilization of communication resources, the event-triggered control scheme has proven to be superior. It has been shown in [32] that the proposed dynamic triggering mechanism, wherein the threshold involves an internal dynamic variable, can allow for the larger minimum interevent times than a static counterpart. Based on event-triggered scheme, [33], [34] designed an adaptive backstepping controller to ensure that the tracking error converges in finite time for fractional order system and gave a proof process to avoid the Zeno behavior. [35] studied the problem of adaptive consensus tracking control for uncertain nonlinear fractional-order multi-agent systems with unmeasurable states and unknown nonlinearities and proposed an event-triggered mechanism with a decreasing threshold function. Aiming at the fractional order chaotic systems, [36] proposed impulsive control and event-triggered control to achieve systems synchronization. [37] investigated the adaptive networked synchronization communication for nonlinear uncertain fractional order chaotic systems based on the event-triggered mechanism. And the author proposed a novel event-triggered mechanism by combining two types of event-triggered conditions.

Based on the previous discussion, this paper designs an adaptive neural network backstepping sliding mode controller to reach fractional order chaotic systems synchronization. A dynamic event-triggered scheme is considered to reduce the number of transmissions of control input signals. It should be pointed out that the theoretical results obtained in this paper is not a simple extension from integer-order systems to fractional systems. We use some properties of the Caputo fractional derivative and the integral inequality to overcome the adverse effects from the incorporation of weakly singular kernels in fractional derivative. Compared with the current research, this work has the following contributions:

1) Aiming at the fractional order chaotic systems synchronization, an event-triggered adaptive neural network backstepping sliding controller is proposed for the first time.

2) In comparison with [12], [21], the sliding mode control technology is introduced into the proposed method to enhance robustness. An event-triggered scheme without Zeno phenomenon is proposed, which can reduce the frequency of network governance. The fractional order dynamic surface control technology is introduced into the controller to avoid ''explosion of complexity'' and obtain fractional derivatives for virtual control functions continuously. The state observer is used to estimate system states. Compared with the previous works in [36], the RBF neural network is developed to estimate uncertain parts. And in our design, the proposed fractional update laws estimate the unknown parameters and the upper limit of the approximation errors.

3) Compared with the previous works in [33] and [36], we consider that fractional order chaotic systems synchronization problem contains input delay, and introduce Pade delay approximation method into fractional order control system to solve input delay problem. By using this method, the original system can be converted to the system without input delay.

The rest of the paper is organized as follows. Section II introduces basic theory about fractional calculus and the fractional order chaotic systems model. In Section III, First, we construct an observer to estimate the system state, then we propose an event-triggered adaptive backstepping sliding mode controller and finally analyze the stability and Zeno behavior. In Section IV, the effectiveness of the proposed control method is proved by example. In Section V, we summarize the paper and give some conclusions.

II. PRELIMINARIES

A. FRACTIONAL CALCULUS

The Caputo fractional derivative [4] is defined as

$$
{}_0^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{1+\alpha-n}} d\tau \tag{1}
$$

where $n \in N$ and $n-1 < \alpha \leq n$, $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ is the Gamma function.

The Laplace transfer of equation (1) is given as

$$\int_0^{\infty} e^{-st}\,{}_0^C D_t^{\alpha} f(t) = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0) \tag{2}$$

where $F(s)$ is the Laplace transform of $f(t)$, and $\alpha \in (0, 1]$.

Definition 1 [4]: The Mittag–Leffler function is given as

$$E_{\alpha,\gamma}(\varsigma) = \sum_{k=0}^{\infty} \frac{\varsigma^k}{\Gamma(\alpha k + \gamma)} \tag{3}$$

where ς is complex number and $\alpha, \gamma > 0$. Its Laplace transform is given by

$$L\left(t^{\gamma-1} E_{\alpha,\gamma}(\varsigma)\right) = \frac{s^{\alpha-\gamma}}{s^{\alpha}+a} \tag{4}$$

Lemma 1 [38]: For a complex number β and two real numbers α, v satisfying $\alpha \in (0, 1)$ and

$$\frac{\pi\alpha}{2} < v < \min\{\pi, \pi\alpha\} \tag{5}$$

For all integer $n \geq 1$, we can obtain

$$E_{\alpha,\beta}(\varsigma) = -\sum_{j=1}^{\infty} \frac{1}{\Gamma(\beta - \alpha j)} + o\left(\frac{1}{|\varsigma|^{n+1}}\right) \tag{6}$$

when $|\varsigma| \to \infty$, $v \leq |\arg(\varsigma)| \leq \pi$.

Lemma 2 [38]: Let $\alpha \in (0, 2)$ and β be an arbitrary real number, and for $\forall \upsilon > 0$ such that $(\pi\alpha/2) < \upsilon \leq \min\{\pi, \pi\alpha\}$, one has

$$\left|E_{\alpha,\beta}(\varsigma)\right| \leq \frac{\mu}{1+|\varsigma|} \tag{7}$$

where $\mu > 0$, $\upsilon \leq |\arg(\varsigma)| \leq \pi$, and $|\varsigma| \geq 0$.

Lemma 3 [39]: Let $x(t) = [x_1(t), \ldots, x_n(t)] \in R^n$ be a vector of continuous and differentiable function. And then, the following relationship holds

$$\frac{1}{2} D^{\alpha}\left(x^T(t) P x\right) \leq x^T(t) P D^{\alpha} x(t) \qquad \forall \alpha \in (0, 1), \forall t > t_0 \tag{8}$$

where $P = diag(p_1, p_2, \ldots, p_n)$ and $p_i > 0$, $i = 1, 2, \ldots, n$.

Lemma 4 [40]: For any $x, y \in R^n$, the following inequality relationship holds

$$x^T y \leq \frac{c^a}{a}\|x\|^a + \frac{1}{bc^b}\|y\|^b \tag{9}$$

where $a > 1$, $b > 1$, $c > 0$, and $(a-1)(b-1) = 1$.

Lemma 5 [41]: The following inequality relationship holds

$$D^{\alpha}(af(t) + bg(t)) = aD^{\alpha} f(t) + bD^{\alpha} g(t) \tag{10}$$

where $a, b \in R$.

Lemma 6 [42]: The following inequality relationship holds

$$D^{\alpha}\left(D^{\beta} x(t)\right) = D^{\beta}\left(D^{\alpha} x(t)\right) = D^{\alpha+\beta} x(t) = \dot{x}(t) \tag{11}$$

where $x(t) = [x_1(t), \ldots, x_n(t)] \in R^n$, $\alpha, \beta \in R^*$ and $\alpha + \beta = 1$.

Lemma 7 [43]: The following inequality holds

$$0 \leq |a| - \frac{a^2}{\sqrt{a^2+b^2}} \leq b \tag{12}$$

where $a \in R$ and $b > 0$.

Lemma 8 [4], [33]: In fractional-order nonlinear system, if the α-order derivative of Lyapunov function $V(t, x)$ satisfying

$$D^{\alpha} V(t, x) \leq -CV(t, x) + D$$

we can obtain

$$V(t, x) \leq V(0) E_{\alpha}(-Ct^{\alpha}) + \tfrac{D\mu}{C}, \ t \geq 0 \tag{13}$$

where $0 < \alpha < 1$, $C > 0$ and $D \geq 0$. Then, $V(t, x)$ is bounded on $[0, t]$ and fractional order systems are stable, where μ is defined in Lemma 2.

B. SYSTEMS MODEL

In synchronization task, there are two dynamic systems, called the master system and the slave system. From a control point of view, the task is to design a controller that obtains signals from the master system to adjust the behavior of the slave system. This article describes the fractional order master system as

$$\begin{cases} D^{\alpha} x_1(t) = x_2 + f_1(x) \\ D^{\alpha} x_i(t) = x_{i+1} + f_i(x) \\ D^{\alpha} x_n(t) = f_n(x) \\ v = x_1 \end{cases} \tag{14}$$

The fractional order slave system with input delay is described as

$$\begin{cases} D^{\alpha} y_1(t) = y_2 + g_1(y) + d_1(t) \\ D^{\alpha} y_i(t) = y_{i+1} + g_i(y) + d_i(t) \\ D^{\alpha} y_n(t) = u(t-\tau) + g_n(y) + d_n(t) \\ \mu = y_1 \end{cases} \tag{15}$$

where $i = 2, \ldots, n-1$, $\alpha \in (0, 1)$; $x = (x_1, x_2, \ldots, x_n)^T \in R^n$ and $y = (y_1, y_2, \ldots, y_n)^T \in R^n$ are the system state vectors, $u(t-\tau)$ is the control input of the system with delay, μ and v are the systems outputs. $f_i(x)$ and $g_i(y)$ are unknown nonlinear functions. $d_i(t)$ is the external disturbance, $|d_i| \leq d_i^*$.

Defining the synchronization error as $z_i = y_i - x_i$. According to Lemma 5, the designed synchronization error system can be defined as

$$\begin{cases} D^{\alpha} z_1(t) = z_2 + h_1(z) + d_1(t) \\ D^{\alpha} z_i(t) = z_{i+1} + h_i(z) + d_i(t) \\ D^{\alpha} z_n(t) = u(t-\tau) + h_n(z) + d_n(t) \\ \vartheta = z_1 \end{cases} \tag{16}$$

where $z = (z_1, z_2, \ldots, z_n)^T \in R^n$, $h_i(z) = g_i(y) - f_i(x)$ is the new nonlinear function by the difference between the nonlinear functions $g_i(y)$ and $f_i(x)$.

To solve the input delay problem in the system (16), the Pade delay approximation method is extended to fractional order systems, which converts the original system into system without input delay. Then we can have

$$L\{u(t-\tau)\} = e^{-\tau s} L\{u(t)\} = \frac{e^{-\tau s/2}}{e^{\tau s/2}} L\{u(t)\} \tag{17}$$

where $L\{u(t)\}$ is the Laplace transform of $u(t)$ and s is Laplace variable.

According to the Taylor formula, $e^{-\tau s/2}/e^{\tau s/2}$ can be approximately equal to $(1-\tau s/2)/(1+\tau s/2)$. Therefore, we introduce a variable $m(t)$ to satisfy the following equation.

$$\frac{1-\tau s}{1+\tau s} L\{u(t)\} = L\{m(t)\} - L\{u(t)\} \tag{18}$$

so that

$$u - \frac{\tau \dot{u}}{2} = m + \frac{\tau \dot{m}}{2} - u - \frac{\tau \dot{u}}{2} \tag{19}$$

Then, we can obtain

$$\dot{m} = -\eta m + 2\eta u \tag{20}$$

where $\eta = 2/\tau$.

$$D^{\alpha}\left(D^{1-\alpha} m\right) = -\eta m + 2\eta u \tag{21}$$

we set

$$D^{1-\alpha} m(t) = n(t) \tag{22}$$

so that, we have

$$\begin{aligned} D^{\alpha} n(t) &= -\eta m(t) + 2\eta u(t) \\ D^{1-\alpha} m(t) &= n(t) \end{aligned} \tag{23}$$

Introducing the above transformation into the synchronization error system, the synchronization error system (16) can be written as

$$\begin{cases} D^{\alpha} z_1(t) = z_2 + h_1(z) + d_1(t) \\ D^{\alpha} z_i(t) = z_{i+1} + h_i(z) + d_i(t) \\ D^{\alpha} z_n(t) = m(t) - u(t) + h_n(z) + d_n(t) \\ \vartheta = z_1 \end{cases} \tag{24}$$

Rewrite the system (24) by

$$\begin{aligned} D^{\alpha} z &= Az + K\vartheta + \sum_{i=1}^{n} B_i \left[h_i(z) + d_i\right] + B(m-u) \\ \vartheta &= Cz \end{aligned} \tag{25}$$

where $A = \begin{bmatrix} -k_{11} & & \\ \vdots & I_{n-1} & \\ -k_{1n} & 0 & \ldots & 0 \end{bmatrix}$, $K = \begin{bmatrix} k_{11} \\ \vdots \\ k_{1n} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$, $B_i = [0 \ldots 1 \ldots 0]^T$, $C = [1 \ldots 0 \ldots 0]$ and given a positive matrix $Q^T = Q$, there exists a positive matrix $P^T = P$ satisfying

$$A^T P + PA = -2Q \tag{26}$$

Remark 1 [27]: Due to the increasing complexity of the engineering environment, input delay may bring some deviations, which leads to an uncertain η. In the analysis process, the variable $m(t)$ is introduced in the system. When deviation in η occurs, $m(t)$ will be uncertain (i.e., the time-varying case). Since the designed controller $u(t)$ can be used to eliminate such item, it is robust to the uncertain input delay to some degree.

Remark 2 [28]: Pade approximation method is introduced to cope with the small unknown delay. Since Pade approximation has some limitations in handling delay, the proposed scheme can not work in long-delay case. Relaxed control design for systems with long delay and output constraint deserves further investigation.

Control Objectives: This paper aims to take the system (14) as the driving system, the system (15) to respond to the system, construct the synchronization error system (16), and design the fractional order controller based on the state observer to make the synchronization error converge as close to the origin as possible.

III. MAIN RESULTS

A. OBSERVER DESIGN

Assumption 1: The unknown function $h_i(x)$, $i = 1, \ldots, n$ can be expressed as

$$h_i(z|\theta_i) = \theta_i^T \varphi_i(z), \quad 1 \le i \le n \tag{27}$$

where θ_i is the ideal constant vector, $\varphi_i(z)$ is the basis function vector and Gaussian basis function is used in this paper.

Assuming that the states of the system are not available. In this case, the system states need to be estimated by an observer, and the observer is designed as

$$\begin{aligned} D^{\alpha}\hat{z} &= A\hat{z} + K\vartheta + \sum_{i=1}^{n} B_i \hat{h}_i\left(\hat{z}|\theta_i\right) + B(m-u) \\ \hat{\vartheta} &= C\hat{z} \end{aligned} \tag{28}$$

where $\hat{z} = \left(\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_n\right)^T$ is the estimated value of $z = (z_1, z_2, \ldots, z_n)^T$.

Define the state observation error $e = z - \hat{z}$, from (25) and (28), we can obtain

$$D^{\alpha} e = Ae + \sum_{i=1}^{n} B_i \left[h_i(\hat{z}) - \hat{h}_i\left(\hat{z}|\theta_i\right) + \Delta h_i + d_i\right] \tag{29}$$

where $\Delta h_i = h_i(z) - h_i(\hat{z}_i)$.

According to Assumption 1, we can obtain

$$\hat{h}_i\left(\hat{z}|\theta_i\right) = \theta_i^T \varphi_i(\hat{z}) \tag{30}$$

Defining the vector of optimal parameters as

$$\theta_i^* = \arg\min_{\theta_i \in \Omega_i} \left[\sup_{\hat{z}_i \in U_i} \left|\hat{h}_i\left(\hat{z}|\theta_i\right) - h_i(\hat{z})\right|\right] \tag{31}$$

where $1 \le i \le n$, Ω_i and U_i are compact regions for θ_i, z_i and $\hat{z}_i$.

Defining error of the optimal approximation and parameters estimation as

$$\begin{aligned} \varepsilon_i &= h_i\left(\hat{z}\right) - \hat{h}_i\left(\hat{z}\left|\theta_i^*\right.\right) \\ \tilde{\theta}_i &= \theta_i^* - \theta_i, i = 1, 2, \ldots, n \end{aligned} \tag{32}$$

Assumption 2: The optimal approximation error remain bounded, there exists positive constants ε_{i0} and θ_i^*, satisfying $\left|\varepsilon_{i,l}\right| \le \varepsilon_{i0}$, $|\theta_i| \le \theta_i^*$.

Assumption 3: There exists a set of known constants γ_i, the following relationship holds

$$\left|h_i(z) - h_i\left(\hat{z}\right)\right| \le \gamma_i \left\|z - \hat{z}\right\| \tag{33}$$

Combining (29) and (30) together gives rise to

$$\begin{aligned} D^\alpha e &= Ae + \sum_{i=1}^{n} B_i \left[h_i\left(\hat{z}\right) - \hat{h}_i\left(\hat{z}\left|\theta_i\right.\right) + \Delta h_i + d_i\right] \\ &= Ae + \sum_{i=1}^{n} B_i \left[\varepsilon_i + \Delta h_i + \tilde{\theta}_i^T \varphi_i\left(\hat{z}\right) + d_i\right] \\ &= Ae + \Delta h + \kappa + \sum_{i=1}^{n} B_i \left[\tilde{\theta}_i^T \varphi_i\left(\hat{z}\right)\right] \end{aligned} \tag{34}$$

where $\kappa = [\varepsilon_1 + d_1, \ldots, \varepsilon_n + d_n]$, $\Delta h = [\Delta h_1, \ldots, \Delta h_n]$.

Constructed Lyapunov function:

$$V_0 = \frac{1}{2} e^T P e \tag{35}$$

According to Lemma 3, we can obtain

$$\begin{aligned} D^\alpha V_0 &\le \frac{1}{2} e^T \left(PA^T + AP\right) e + e^T P (\kappa + \Delta h) \\ &\quad + \sum_{i=1}^{n} e^T P B_i \left[\tilde{\theta}_i^T \varphi_i\left(\hat{z}\right)\right] \\ &\le -e^T Q e + e^T P (\kappa + \Delta h) + e^T P \sum_{i=1}^{n} B_i \tilde{\theta}_i^T \varphi_i\left(\hat{z}\right) \end{aligned} \tag{36}$$

By Lemma 4 and Assumption 3, we obtain

$$\begin{aligned} & e^T P (\kappa + \Delta h) \\ &\le \left|e^T P \kappa\right| + \left|e^T P \Delta h\right| \\ &\le \frac{1}{2}\|e\|^2 + \frac{1}{2}\|P\kappa\|^2 + \frac{1}{2}\|e\|^2 + \frac{1}{2}\|P\|^2\|\Delta h\|^2 \\ &\le \|e\|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 + \frac{1}{2}\|P\|^2 \sum_{i=1}^{n} |\Delta h_i|^2 \\ &\le \|e\|^2 + \frac{1}{2}\|e\|^2\|P\|^2 \sum_{i=1}^{n} {\gamma_i}^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 \\ &\le \|e\|^2 \left(1 + \frac{1}{2}\|P\|^2 \sum_{i=1}^{n} {\gamma_i}^2\right) + \frac{1}{2}\left\|P\kappa^*\right\|^2 \end{aligned} \tag{37}$$

and

$$\begin{aligned} & e^T P \sum_{i=1}^{n} B_i \tilde{\theta}_i^T \varphi_i\left(\hat{z}\right) \\ &\le \frac{1}{2} e^T P^T P e + \frac{1}{2} \sum_{i=1}^{n} \tilde{\theta}_i^T \varphi_i\left(\hat{z}\right) \varphi_i^T\left(\hat{z}\right) \tilde{\theta}_i \\ &\le \frac{1}{2} \lambda_{\max}^2(P) \|e\|^2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i \end{aligned} \tag{38}$$

where $\kappa^* = \left[\varepsilon_{10} + d_1^*, \ldots, \varepsilon_{n0} + d_n^*\right]$.

By equations (36), (37) and (38), we obtain

$$D^\alpha V_0 \le -q_0\|e\|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i \tag{39}$$

where $q_0 = \lambda_{\min}(Q) - \left(1 + \frac{1}{2}\|P\|^2 \sum_{i=1}^{n} \gamma_i^2 + \frac{1}{2}\lambda_{\max}^2(P)\right)$.

B. CONTROLLER DESIGN

Theorem 1: Consider the master system (14) and the slave system (15), construct the synchronization error system (24) and design state observer. Suppose that Assumptions 1-3 hold. The following designs can ensure that all the signals remain semi-global uniformly ultimately bounded in the closed-loop system and the synchronization errors can converge to near zero, the error variables

$$\begin{cases} S_1 = z_1 \\ S_i = \hat{z}_i - v_i, i = 2, \ldots, n \end{cases} \tag{40}$$

the intermediate controllers

$$\begin{cases} \alpha_1 = -\left[c_1 S_1 + \theta_1^T \varphi_1\left(\hat{z}\right) + sign\left(S_1\right)\delta_1\right] \\ \alpha_i = -\left[S_{i-1} + c_i S_i + \theta_i^T \varphi_i\left(\hat{z}\right) + sign\left(S_i\right)\delta_i - \dfrac{\alpha_{i-1} - v_i}{\lambda_i}\right] \end{cases} \tag{41}$$

the parameter adaptive laws

$$\begin{cases} D^\alpha \theta_i = \sigma_i \varphi_i\left(\hat{z}\right) S_i - \rho_i \theta_i \\ D^\alpha \delta_i = r_i \left|S_i\right| - \eta_i \delta_i \end{cases} \tag{42}$$

the event-triggered controller

$$\begin{cases} \alpha_n(t) = S_{n-1} + c_n S_n + \theta_n^T \varphi_n\left(\hat{z}\right) + sign\left(S_n\right)\delta_n - \dfrac{\alpha_{n-1} - v_n}{\lambda_n} \\ \bar{u}(t) = -\alpha_n - \dfrac{S_n(\kappa_1\alpha_n)^2}{\sqrt{(S_n\kappa_1\alpha_n)^2 + \kappa_2^2}} - \dfrac{S_n M_1^2}{\sqrt{(S_n M_1)^2 + \kappa_2^2}} \\ u(t) = \bar{u}(t_k), \forall\left[t_k, t_{k+1}\right), k \in N^* \end{cases} \tag{43}$$

where $i = 2, \ldots, n-1$, $c_i, \sigma_i, \rho_i, r_i, \eta_i$ are design constant parameters, $c_i > 0$, $\sigma_i > 0$, $\rho_i > 0$, $r_i > 0$, $\eta_i > 0$. $\tilde{\delta}_i = \delta_i^* - \delta_i$ is the upper bound estimation error. S_i is defined as the sliding mode surface. $sign(S_i)$ is the sliding term to enhance robustness. t_k denotes the update time of controller.

Proof: In this section, we combine backstepping control, DSC technology and Lyapunov method to design virtual control laws, control input and fractional parameter adaptive laws.

Step 1: Definde the error variable

$$S_1 = \vartheta = z_1 \tag{44}$$

According to $e_2 = z_2 - \hat{z}_2$, the Caputo fractional derivative of S_1 is given by

$$\begin{aligned} D^\alpha S_1 &= D^\alpha z_1 \\ &= z_2 + h_1(z) + d_1 \\ &= \hat{z}_2 + h_1(z) + d_1 + e_2 \\ &= \hat{z}_2 + \theta_1^T \varphi_1(\hat{z}) + \tilde{\theta}_1^T \varphi_1(\hat{z}) + d_1 + \varepsilon_1 + \Delta h_1 + e_2 \end{aligned} \tag{45}$$

Define the second error surface S_2 and the output error w_2 of a fractional order filter

$$\begin{aligned} S_2 &= \hat{z}_2 - v_2 \\ w_2 &= v_2 - \alpha_1 \end{aligned} \tag{46}$$

Substituting it into (45) generates

$$\begin{aligned} D^\alpha S_1 &= S_2 + w_2 + \alpha_1 + \theta_1^T \varphi_1(\hat{z}) + \tilde{\theta}_1^T \varphi_1(\hat{z}) + d_1 \\ &\quad + \varepsilon_1 + \Delta h_1 + e_2 \end{aligned} \tag{47}$$

Constructing Lyapunov function:

$$V_1 = V_0 + \frac{1}{2}S_1^2 + \frac{1}{2\sigma_1}\tilde{\theta}_1^T\tilde{\theta}_1 + \frac{1}{2r_1}\tilde{\delta}_1^2 \tag{48}$$

And we can obtain the following Caputo fractional derivative $D^\alpha V_1$:

$$\begin{aligned} D^\alpha V_1 &\le D^\alpha V_0 + S_1 D^\alpha S_1 + \frac{1}{\sigma_1}\tilde{\theta}_1^T D^\alpha \tilde{\theta}_1 + \frac{1}{r_1}\tilde{\delta}_1 D^\alpha \tilde{\delta}_1 \\ &\le -q_0\|e\|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 + \frac{1}{2}\sum_{i=1}^n \tilde{\theta}_i^T\tilde{\theta}_i \\ &\quad + S_1\left(S_2 + w_2 + \alpha_1 + \theta_1^T\varphi(\hat{z}) + \tilde{\theta}_1^T\varphi(\hat{z})\right) + S_1 e_2 \\ &\quad + S_1(\varepsilon_1 + \Delta h_1 + d_1) - \frac{1}{\sigma_1}\tilde{\theta}_1^T D^\alpha\theta_1 - \frac{1}{r_1}\tilde{\delta}_1 D^\alpha \delta_1 \\ &\le -q_0\|e\|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 + S_1\left(S_2 + w_2 + \alpha_1 + \theta_1^T\varphi_1\right) \\ &\quad + S_1 e_2 + S_1(\varepsilon_1 + \Delta h_1 + d_1) + \tilde{\theta}_1^T\left(\varphi_1 S_1 - \frac{1}{\sigma_1}D^\alpha\theta_1\right) \\ &\quad - \frac{1}{r_1}\tilde{\delta}_1 D^\alpha\delta_1 + \frac{1}{2}\sum_{i=1}^n \tilde{\theta}_i^T\tilde{\theta}_i \end{aligned} \tag{49}$$

Designing the virtual controller α_1 together with the parameter adaptation laws θ_1 and δ_1 as follows:

$$\alpha_1 = -\left[c_1 S_1 + \theta_1^T\varphi_1(\hat{z}) + sign(S_1)\delta_1\right] \tag{50}$$

$$D^\alpha\theta_1 = \sigma_1\varphi_1(\hat{z})S_1 - \rho_1\theta_1 \tag{51}$$

$$D^\alpha\delta_1 = r_1|S_1| - \eta_1\delta_1 \tag{52}$$

Substituting (49),(50) and (51) into (48) produces

$$\begin{aligned} D^\alpha V_1 &\le -q_0\|e\|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 + S_1\left(S_2 - c_1 S_1 - \theta_1^T\varphi_1 \right. \\ &\quad \left. - sign(S_1)\delta_1 + \theta_1^T\varphi_1\right) + S_1(\varepsilon_1 + \Delta h_1 + d_1) \\ &\quad + S_1 e_2 + S_1 w_2 + \tilde{\theta}_1^T\left(\varphi_1 S_1 - \frac{1}{\sigma_1}(\sigma_1\varphi_1 S_1 - \rho_1\theta_1)\right) \\ &\quad - \frac{1}{r_1}\tilde{\delta}_1(r_1|S_1| - \eta_1\delta_1) + \frac{1}{2}\sum_{i=1}^n \tilde{\theta}_i^T\tilde{\theta}_i \\ &\le -q_0\|e\|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 - c_1 S_1^2 + S_1 S_2 \\ &\quad - |S_1|\delta_1 + S_1 e_2 + S_1 w_2 + S_1(\varepsilon_1 + \Delta h_1 + d_1) \\ &\quad + \frac{\rho_1}{\sigma_1}\tilde{\theta}_1^T\theta_1 - \tilde{\delta}_1|S_1| + \frac{\eta_1}{r_1}\tilde{\delta}_1\delta_1 + \frac{1}{2}\sum_{i=1}^n \tilde{\theta}_i^T\tilde{\theta}_i \end{aligned} \tag{53}$$

where $\varepsilon_1 + \Delta h_1 + d_1 = \Delta_1$, and $S_1\Delta_1 \le |S_1\Delta_1| \le |S_1||\Delta_1| \le |S_1|\delta_1^* = |S_1|\left(\tilde{\delta}_1 + \delta_1\right)$.

Further, we can obtain

$$\begin{aligned} D^\alpha V_1 &\le -q_0\|e\|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 - c_1 S_1^2 + S_1 S_2 \\ &\quad - |S_1|\delta_1 + S_1 e_2 + S_1 w_2 + S_1\delta_1^* + \frac{\rho_1}{\sigma_1}\tilde{\theta}_1^T\theta_1 \\ &\quad - |S_1|\tilde{\delta}_1 + \frac{\eta_1}{r_1}\tilde{\delta}_1\delta_1 + \frac{1}{2}\sum_{i=1}^n \tilde{\theta}_i^T\tilde{\theta}_i \\ &\le -q_0\|e\|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 - c_1 S_1^2 + S_1\chi_2 \\ &\quad - |S_1|\delta_1 + S_1 e_2 + S_1 w_2 + |S_1|\left(\tilde{\delta}_1 + \delta_1\right) \\ &\quad + \frac{\rho_1}{\sigma_1}\tilde{\theta}_1^T\theta_1 - |S_1|\tilde{\delta}_1 + \frac{\eta_1}{r_1}\tilde{\delta}_1\delta_1 + \frac{1}{\tau}\sum_{i=1}^n \tilde{\theta}_i^T\tilde{\theta}_i \\ &\le -q_0\|e\|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 - c_1 S_1^2 + S_1 S_2 + S_1 e_2 \\ &\quad + S_1 w_2 + \frac{\rho_1}{\sigma_1}\tilde{\theta}_1^T\theta_1 + \frac{\eta_1}{r_1}\tilde{\delta}_1\delta_1 + \frac{1}{2}\sum_{i=1}^n \tilde{\theta}_i^T\tilde{\theta}_i \end{aligned} \tag{54}$$

According to Lemma 4, we have

$$S_1 e_2 \le \frac{1}{2}|S_1|^2 + \frac{1}{2}|e_2|^2 \tag{55}$$

$$S_1 w_2 \le \frac{1}{2}|S_1|^2 + \frac{1}{2}|w_2|^2 \tag{56}$$

Substituting (55) and (56) into (54), we arrive at

$$\begin{aligned} D^\alpha V_1 &\le -q_1\|e\|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 + \frac{1}{2}\sum_{i=1}^n \tilde{\theta}_i^T\tilde{\theta}_i - c_1 S_1^2 \\ &\quad + S_1 S_2 + \frac{\rho_1}{\sigma_1}\tilde{\theta}_1^T\theta_1 + \frac{\eta_1}{r_1}\tilde{\delta}_1\delta_1 + |S_1|^2 + \frac{1}{2}|w_2|^2 \end{aligned} \tag{57}$$

where $q_1 = q_0 - 1/2$.

By using DSC technique, we can obtain the state variable v_2 as

$$\lambda_2 D^\alpha v_2 + v_2 = \alpha_1, \; v_2(0) = \alpha_1(0) \tag{58}$$

according to (58), we have

$$\begin{aligned} D^\alpha w_2 &= D^\alpha v_2 - D^\alpha\alpha_1 \\ &= -\frac{v_2 - \alpha_1}{\lambda_2} - D^\alpha\alpha_1 \\ &= -\frac{w_2}{\lambda_2} + B_2 \end{aligned} \tag{59}$$

where λ_2 represents a constant, B_2 is a continuous function of variables $S_1, S_2, w_2, \theta_1, \delta_1, S_3, w_3$.

Step 2: Define the error surface S_3 and the output error of the fractional order filter

$$\begin{aligned} S_3 &= \hat{z}_3 - v_3 \\ w_3 &= v_3 - \alpha_2 \end{aligned} \tag{60}$$

And the Caputo fractional derivative of S_2 is given by

$$\begin{aligned} D^\alpha S_2 &= \hat{z}_3 + \theta_2^T \varphi_2 + \tilde{\theta}_2^T \varphi_2 \\ &\quad + k_{12} e_1 + \varepsilon_2 + \Delta h_2 + d_2 - D^\alpha v_2 \\ &= S_3 + w_3 + \alpha_2 + \theta_2^T \varphi_2 + \tilde{\theta}_2^T \varphi_2 \\ &\quad + k_{12} e_1 + \varepsilon_2 + \Delta h_2 + d_2 - D^\alpha v_2 \end{aligned} \tag{61}$$

We construct the Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2} S_2^2 + \frac{1}{2\sigma_2} \tilde{\theta}_2^T \tilde{\theta}_2 + \frac{1}{2r_2} \tilde{\delta}_2^2 + \frac{1}{2} w_2^2 \tag{62}$$

According to Lemma 3 and the solution of (61), we can obtain the fractional derivative $D^\alpha V_2$:

$$\begin{aligned} D^\alpha V_2 &\leq D^\alpha V_1 + S_2 D^\alpha S_2 + \frac{1}{\sigma_2} \tilde{\theta}_2^T D^\alpha \tilde{\theta}_2 + \frac{1}{r_2} \tilde{\delta}_2 D^\alpha \tilde{\delta}_2 \\ &\quad + w_2 D^\alpha w_2 \\ &\leq -q_1 \|e\|^2 + \frac{1}{2} \left\| P\kappa^* \right\|^2 - c_1 S_1^2 + S_1 S_2 \\ &\quad + S_2 \left(S_3 + w_3 + \alpha_2 + \theta_2^T \varphi_2 + \tilde{\theta}_2^T \varphi_2 - D^\alpha v_2 \right) \\ &\quad + k_{12} S_2 e_1 + S_2 \left(\varepsilon_2 + \Delta h_2 + d_2 \right) + \frac{\rho_1}{\sigma_1} \tilde{\theta}_1^T \theta_1 \\ &\quad + \frac{\eta_1}{r_1} \tilde{\delta}_1 \delta_1 + |S_1|^2 + \frac{1}{2} |w_2|^2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i \\ &\quad + \frac{1}{\sigma_2} \tilde{\theta}_2^T D^\alpha \tilde{\theta}_2 + \frac{1}{r_2} \tilde{\delta}_2 D^\alpha \tilde{\delta}_2 - \frac{w_2^2}{\lambda_2} + B_2 w_2 \\ &\leq -q_2 \|e\|^2 + \frac{1}{2} \left\| P\kappa^* \right\|^2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i - c_1 S_1^2 \\ &\quad + S_2 \left(S_1 + S_3 + \alpha_2 + \theta_2^T \varphi_2 + \tilde{\theta}_2^T \varphi_2 - D^\alpha v_2 \right) \\ &\quad + S_2 \Delta_2 - \frac{1}{\sigma_2} \tilde{\theta}_2^T D^\alpha \theta_2 - \frac{1}{r_2} \tilde{\delta}_2 D^\alpha \delta_2 + \frac{\rho_1}{\sigma_1} \tilde{\theta}_1^T \theta_1 \\ &\quad + \frac{\eta_1}{r_1} \tilde{\delta}_1 \delta_1 + |S_1|^2 + |S_2|^2 - \frac{w_2^2}{\lambda_2} + B_2 w_2 \\ &\quad + \frac{1}{2} |w_2|^2 + \frac{1}{2} |w_3|^2 \end{aligned} \tag{63}$$

where $q_2 = q_1 - k_{12}^2/2$ and note that $k_{12} S_2 e_1 \leq |S_2|^2/2 + k_{12}^2 |e_1|^2/2$. Further, we have

$$\begin{aligned} D^\alpha V_2 &\leq -q_2 \|e\|^2 + \frac{1}{2} \left\| P\kappa^* \right\|^2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i - c_1 S_1^2 \\ &\quad + S_2 \left(S_1 + S_3 + \alpha_2 + \theta_2^T \varphi_2 + \tilde{\theta}_2^T \varphi_2 - \frac{\alpha_1 - v_2}{\lambda_2} \right) \\ &\quad + |S_2| \left(\tilde{\delta}_2 + \delta_2 \right) - \frac{1}{\sigma_2} \tilde{\theta}_2^T D^\alpha \theta_2 - \frac{1}{r_2} \tilde{\delta}_2 D^\alpha \delta_2 \\ &\quad - \frac{w_2^2}{\lambda_2} + B_2 w_2 + \frac{\rho_1}{\sigma_1} \tilde{\theta}_1^T \theta_1 + \frac{\eta_1}{r_1} \tilde{\delta}_1 \delta_1 + |S_1|^2 \\ &\quad + |S_2|^2 + \frac{1}{2} |w_2|^2 + \frac{1}{2} |w_3|^2 \end{aligned} \tag{64}$$

where $\varepsilon_2 + \Delta h_2 + d_2 = \Delta_2$, and $S_2 \Delta_2 \leq |S_2 \Delta_2| \leq |S_2| |\Delta_2| \leq |S_2| \delta_2^* \leq |S_2| \left(\tilde{\delta}_2 + \delta_2 \right)$.

Selecting the second virtual controller α_2 together with the parameter adaptation laws as follows:

$$\alpha_2 = -\left[c_2 S_2 + S_1 + \theta_2^T \varphi_2 \left(\hat{z} \right) + sign\left(S_2 \right) \delta_2 - \frac{\alpha_1 - v_2}{\lambda_2} \right] \tag{65}$$

$$D^\alpha \theta_2 = \sigma_2 \varphi_2 \left(\hat{z} \right) S_2 - \rho_2 \theta_2 \tag{66}$$

$$D^\alpha \delta_2 = r_2 |S_2| - \eta_2 \delta_2 \tag{67}$$

Substituting (65),(66) and (67) into (64) produces

$$\begin{aligned} D^\alpha V_2 &\leq -q_2 \|e\|^2 + \frac{1}{2} \left\| P\kappa^* \right\|^2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i - c_1 S_1^2 \\ &\quad + S_2 \left(S_1 + S_3 - c_2 S_2 - S_1 - \theta_2^T \varphi_2 - sign\left(S_2 \right) \delta_2 \right. \\ &\quad + \frac{\rho_1}{\sigma_1} \tilde{\theta}_1^T \theta_1 + \frac{\eta_1}{r_1} \tilde{\delta}_1 \delta_1 + |S_1|^2 + |S_2|^2 + \frac{1}{2} |w_2|^2 \\ &\quad + \frac{1}{2} |w_3|^2 - \frac{w_2^2}{\lambda_2} + B_2 w_2 \\ &\leq -q_2 \|e\|^2 + \frac{1}{2} \left\| P\kappa^* \right\|^2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i - c_1 S_1^2 \\ &\quad - c_2 S_2^2 + S_2 S_3 - |S_2| \delta_2 + \theta_2^T \varphi_2 S_2 \\ &\quad + |S_2| \left(\tilde{\delta}_2 + \delta_2 \right) - \tilde{\theta}_2^T \varphi_2 S_2 + \frac{\rho_2}{\sigma_2} \tilde{\theta}_2^T \theta_2 - |S_2| \tilde{\delta}_2 \\ &\quad + \frac{\eta_2}{r_2} \tilde{\delta}_2 \delta_2 + \frac{\rho_1}{\sigma_1} \tilde{\theta}_1^T \theta_1 + \frac{\eta_1}{r_1} \tilde{\delta}_1 \delta_1 + |S_1|^2 + |S_2|^2 \\ &\quad + \frac{1}{2} |w_2|^2 + \frac{1}{2} |w_3|^2 - \frac{w_2^2}{\lambda_2} + B_2 w_2 \end{aligned} \tag{68}$$

By employing Young's inequality, we have $w_2 B_2 \leq \frac{w_2^2 B_2^2}{2\mu} + 2\mu$. Then, we have

$$\begin{aligned} D^\alpha V_2 &\leq -q_2 \|e\|^2 + \frac{1}{2} \left\| P\kappa^* \right\|^2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i - c_1 S_1^2 - c_2 S_2^2 \\ &\quad + S_2 S_3 + \frac{\rho_2}{\sigma_2} \tilde{\theta}_2^T \theta_2 + \frac{\eta_2}{r_2} \tilde{\delta}_2 \delta_2 + \frac{\rho_1}{\sigma_1} \tilde{\theta}_1^T \theta_1 + \frac{\eta_1}{r_1} \tilde{\delta}_1 \delta_1 \\ &\quad + |S_1|^2 + |S_2|^2 + \frac{1}{2} |w_2|^2 + \frac{1}{2} |w_3|^2 - \frac{w_2^2}{\lambda_2} + \frac{w_2^2 B_2^2}{2\mu} + 2\mu \end{aligned} \tag{69}$$

Step i: According to Theorem 1, design the error surface S_{i+1} and the output error of a fractional order filter.

$$\begin{aligned} S_{i+1} &= \hat{z}_{i+1} - v_{i+1} \\ w_{i+1} &= v_{i+1} - \alpha_i \end{aligned} \tag{70}$$

The intermediate control function α_i and the update laws are designed as

$$\alpha_i = -\left[S_{i-1} + c_i S_i + \theta_i^T \varphi_i\left(\hat{z}\right) + sign\left(S_i\right)\delta_i - \frac{\alpha_{i-1} - v_i}{\lambda_i}\right] \tag{71}$$

$$D^\alpha \theta_i = \sigma_i \varphi_i\left(\hat{z}\right) S_i - \rho_i \theta_i \tag{72}$$

$$D^\alpha \delta_i = r_i \left|S_i\right| - \eta_i \delta_i \tag{73}$$

Employing DSC technique, v_i can be obtain as

$$\lambda_i D^\alpha v_i + v_i = \alpha_{i-1}, v_i\left(0\right) = \alpha_{i-1}\left(0\right) \tag{74}$$

by (74), we have

$$D^\alpha w_i = -\frac{w_i}{\lambda_i} + B_i \tag{75}$$

where λ_i represents a constant and $B_i = -D^\alpha \alpha_{i-1}$.

Selecting the Lyapunov function candidate as follows:

$$V_i = V_{i-1} + \frac{1}{2}S_i^2 + \frac{1}{2\sigma_i}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2r_i}\tilde{\delta}_i^2 + \frac{1}{2}w_i^2 \tag{76}$$

We can obtain the following fractional derivative $D^\alpha V_i$:

$$\begin{aligned} D^\alpha V_i \le & -q_i\|e\|^2 - \sum_{m=1}^{i} c_m S_m^2 + S_i S_{i+1} + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i \\ & + \sum_{m=1}^{i}\frac{\rho_m}{\sigma_m}\tilde{\theta}_m^T\theta_m + \sum_{m=1}^{i}\frac{\eta_m}{r_m}\tilde{\delta}_m\delta_m \\ & + \sum_{m=1}^{i}\left|S_m\right|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 \\ & + \sum_{m=2}^{i}\left(\frac{B_m^2}{2\mu} - \frac{1}{\lambda_m} + \frac{1}{2}\right)w_m^2 + \frac{1}{2}w_{i+1}^2 + 2\mu\left(i-1\right) \end{aligned} \tag{77}$$

where $q_i = q_{i-1} - k_{1i}^2/2$ and $w_i B_i \le \frac{w_i^2 B_i^2}{2\mu} + 2\mu$.

Step n: At this step, we will design the event-triggered controller. Because of input delay in system, we define the new error variable S_n and the output error of a fractional order filter

$$\begin{aligned} S_n &= \hat{z}_n - v_n + \frac{1}{\eta}n\left(t\right) \\ w_n &= v_n - \alpha_{n-1} \end{aligned} \tag{78}$$

Constructing the Lyapunov function as follows:

$$V_n = V_{n-1} + \frac{1}{2}S_n^2 + \frac{1}{2\sigma_n}\tilde{\theta}_n^T\tilde{\theta}_n + \frac{1}{2r_n}\tilde{\delta}_n^2 + \frac{1}{2}w_n^2 \tag{79}$$

We can obtain the Caputo fractional derivative of V_n as follows:

$$\begin{aligned} D^\alpha V_n \le & D^\alpha V_{n-1} + S_n D^\alpha S_n + \frac{1}{\sigma_n}\tilde{\theta}_n^T D^\alpha\tilde{\theta}_n \\ & + \frac{1}{r_n}\tilde{\delta}_n D^\alpha\tilde{\delta}_n + w_n D^\alpha w_n \\ \le & -q_{n-1}\|e\|^2 - \sum_{i=1}^{n-1} c_i S_i^2 + S_{n-1}S_n + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i \\ & + \sum_{i=1}^{n-1}\frac{\rho_i}{\sigma_i}\tilde{\theta}_i^T\theta_i + \sum_{i=1}^{n-1}\frac{\eta_i}{r_i}\tilde{\delta}_i\delta_i + \sum_{i=1}^{n-1}\left|S_i\right|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 \\ & + \sum_{i=2}^{n-1}\left(\frac{B_i^2}{2\mu} - \frac{1}{\lambda_i} + \frac{1}{2}\right)w_i^2 + \frac{1}{2}w_n^2 + 2\mu\left(n-2\right) \\ & + S_n D^\alpha S_n - \frac{1}{\sigma_n}\tilde{\theta}_n^T D^\alpha\theta_n - \frac{1}{r_n}\tilde{\delta}_n D^\alpha\delta_n + w_n D^\alpha w_n \end{aligned} \tag{80}$$

Combining (77), (79) and the Caputo fractional derivative of S_n together leads to

$$\begin{aligned} D^\alpha V_n \le & -q_{n-1}\|e\|^2 - \sum_{i=1}^{n-1} c_i S_i^2 + S_{n-1}S_n + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i \\ & + \sum_{i=1}^{n-1}\frac{\rho_i}{\sigma_i}\tilde{\theta}_i^T\theta_i + \sum_{i=1}^{n-1}\frac{\eta_i}{r_i}\tilde{\delta}_i\delta_i + \sum_{i=1}^{n-1}\left|S_i\right|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 \\ & + \sum_{i=2}^{n-1}\left(\frac{B_i^2}{2\mu} - \frac{1}{\lambda_i} + \frac{1}{2}\right)w_i^2 + \frac{1}{2}w_n^2 + 2\mu\left(n-2\right) \\ & + S_n\left(m - u + k_{1n}e_1 + \theta_n^T\varphi_n + \tilde{\theta}_n^T\varphi_n - D^\alpha v_n + \frac{1}{\eta}\left(-\eta m + 2\eta u\right)\right) + S_n\left(\varepsilon_n + \Delta h_n + d_n\right) \\ & - \frac{1}{\sigma_n}\tilde{\theta}_n^T D^\alpha\theta_n - \frac{1}{r_n}\tilde{\delta}_n D^\alpha\delta_n + w_n D^\alpha w_n \\ \le & -q_{n-1}\|e\|^2 - \sum_{i=1}^{n-1} c_i S_i^2 + S_{n-1}S_n + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i \\ & + \sum_{i=1}^{n-1}\frac{\rho_i}{\sigma_i}\tilde{\theta}_i^T\theta_i + \sum_{i=1}^{n-1}\frac{\eta_i}{r_i}\tilde{\delta}_i\delta_i + \sum_{i=1}^{n-1}\left|S_i\right|^2 + \frac{1}{2}\left\|P\kappa^*\right\|^2 \\ & + \sum_{i=2}^{n-1}\left(\frac{B_i^2}{2\mu} - \frac{1}{\lambda_i} + \frac{1}{2}\right)w_i^2 + \frac{1}{2}w_n^2 + 2\mu\left(n-2\right) \\ & + S_n\left(u + k_{1n}e_1 + \theta_n^T\varphi_n + \tilde{\theta}_n^T\varphi_n - D^\alpha v_n\right) \\ & + S_n\left(\varepsilon_n + \Delta h_n + d_n\right) - \frac{1}{\sigma_n}\tilde{\theta}_n^T D^\alpha\theta_n \\ & - \frac{1}{r_n}\tilde{\delta}_n D^\alpha\delta_n + w_n D^\alpha w_n \end{aligned} \tag{81}$$

The event-triggered controller $\bar{u}\left(t\right)$ and the parameter adaptation laws can be given by

$$D^\alpha \theta_n = \sigma_n \varphi_n\left(\hat{z}\right) S_n - \rho_n \theta_n \tag{82}$$

$$D^\alpha \delta_n = r_n \left|S_n\right| - \eta_n \delta_n \tag{83}$$

$$\alpha_n = S_{n-1} + c_n S_n + \theta_n^T \varphi_n\left(\hat{z}\right) + sign\left(S_n\right)\delta_n - \frac{\alpha_{n-1} - v_n}{\lambda_n} \tag{84}$$

$$\bar{u}\left(t\right) = -\alpha_n - \frac{S_n(\kappa_1\alpha_n)^2}{\sqrt{(S_n\kappa_1\alpha_n)^2 + \kappa_2^2}} - \frac{S_n M_1^2}{\sqrt{(S_n M_1)^2 + \kappa_2^2}} \tag{85}$$

The actual controller and the triggering condition for the sampling instants are as follows:

$$u(t) = \bar{u}(t_k), \forall [t_k, t_{k+1})$$
$$t_{k+1} = \inf\{t \in R \mid |\Delta(t)| \geq \kappa_1 |u(t)| + M_1\} \tag{86}$$

where $\Delta(t) = \bar{u}(t) - u(t)$ is the event sampling error, $0 < \kappa_1 < 1$, M_1 is a positive constant, $t_k, k \in N^*$ is the controller update time.

C. STABILITY ANALYSIS

From (86), the following equation can be arrived at

$$\Delta(t) = \bar{u}(t) - u(t) = \beta_1(t)\kappa_1 u(t) + \beta_2(t) M_1 \tag{87}$$

where $\beta_1(t)$, $\beta_2(t)$ are time-varying parameters satisfying $|\beta_1(t)| \leq 1$ and $|\beta_2(t)| \leq 1$. Accordingly, one can obtain

$$u(t) = \frac{\bar{u}(t) - \beta_2(t) M_1}{1 + \beta_1(t)\kappa_1} \tag{88}$$

Thus, substituting (88) into (81) produces

$$\begin{aligned} D^\alpha V_n \leq& -q_{n-1}\|e\|^2 - \sum_{i=1}^{n-1} c_i S_i^2 + S_{n-1}S_n + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i \\ &+ \sum_{i=1}^{n-1}\frac{\rho_i}{\sigma_i}\tilde{\theta}_i^T\theta_i + \sum_{i=1}^{n-1}\frac{\eta_i}{r_i}\tilde{\delta}_i\delta_i + \sum_{i=1}^{n-1}|S_i|^2 + \frac{1}{2}\|P\kappa^*\|^2 \\ &+ \sum_{i=2}^{n-1}\left(\frac{B_i^2}{2\mu} - \frac{1}{\lambda_i} + \frac{1}{2}\right)w_i^2 + \frac{1}{2}w_n^2 + 2\mu(n-2) \\ &+ S_n\left(\frac{\bar{u}(t) - \beta_2(t)M_1}{1+\beta_1(t)\kappa_1} + k_{1n}e_1 + \theta_n^T\varphi_n + \tilde{\theta}_n^T\varphi_n\right. \\ &\left. - D^\alpha v_n\right) + S_n(\varepsilon_n + \Delta h_n + d_n) - \frac{1}{\sigma_n}\tilde{\theta}_n^T D^\alpha\theta_n \\ &- \frac{1}{r_n}\tilde{\delta}_n D^\alpha\delta_n + w_n D^\alpha w_n \end{aligned} \tag{89}$$

where $\varepsilon_n + \Delta h_n + d_n = \Delta_n$, and $S_n\Delta_n \leq |S_n\Delta_n| \leq |S_n||\Delta_n| \leq |S_n|\delta_n^* \leq |S_n|\left(\tilde{\delta}_n + \delta_n\right)$.

Further, we can obtain

$$\begin{aligned} D^\alpha V_n \leq& -q_{n-1}\|e\|^2 - \sum_{i=1}^{n-1} c_i S_i^2 + S_{n-1}S_n + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i \\ &+ \sum_{i=1}^{n-1}\frac{\rho_i}{\sigma_i}\tilde{\theta}_i^T\theta_i + \sum_{i=1}^{n-1}\frac{\eta_i}{r_i}\tilde{\delta}_i\delta_i + \sum_{i=1}^{n-1}|S_i|^2 + \frac{1}{2}\|P\kappa^*\|^2 \\ &+ \sum_{i=2}^{n-1}\left(\frac{B_i^2}{2\mu} - \frac{1}{\lambda_i} + \frac{1}{2}\right)w_i^2 + \frac{1}{2}w_n^2 + 2\mu(n-2) \\ &+ S_n\left(\frac{\bar{u}(t) - \beta_2(t)M_1}{1+\beta_1(t)\kappa_1} + \alpha_n\right) - k_n S_n^2 - S_{n-1}S_n \\ &- |S_n|\delta_n + k_{1n}S_n e_1 + \tilde{\theta}_n^T S_n\varphi_n + |S_n|\left(\delta_n + \tilde{\delta}_n\right) \\ &- \frac{1}{\sigma_n}\tilde{\theta}_n^T(\sigma_n\phi_n S_n - \rho_n\theta_n) - \frac{1}{r_n}\tilde{\delta}_n(r_n|S_n| - \eta_n\delta_n) \\ &+ w_n D^\alpha w_n \\ \leq& -q_{n-1}\|e\|^2 - \sum_{i=1}^{n} c_i S_i^2 + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i \\ &+ \sum_{i=1}^{n}\frac{\rho_i}{\sigma_i}\tilde{\theta}_i^T\theta_i + \sum_{i=1}^{n}\frac{\eta_i}{r_i}\tilde{\delta}_i\delta_i + \sum_{i=1}^{n-1}|S_i|^2 + \frac{1}{2}\|P\kappa^*\|^2 \\ &+ \sum_{i=2}^{n-1}\left(\frac{B_i^2}{2\mu} - \frac{1}{\lambda_i} + \frac{1}{2}\right)w_i^2 + \frac{1}{2}w_n^2 + 2\mu(n-2) \\ &+ S_n\left(\frac{\bar{u}(t) - \beta_2(t)M_1}{1+\beta_1(t)\kappa_1} + \alpha_n\right) + k_{1n}S_n e_1 \\ &+ w_n\left(-\frac{w_n}{\lambda_n} + B_n\right) \end{aligned} \tag{90}$$

By employing Young's inequality, we have

$$w_n B_n \leq \frac{w_n^2 B_n^2}{2u} + 2u \tag{91}$$

Further, we can obtain

$$\begin{aligned} D^\alpha V_n \leq& -q_n\|e\|^2 - \sum_{i=1}^{n} c_i S_i^2 + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2}\|P\kappa^*\|^2 \\ &+ \sum_{i=1}^{n}\frac{\rho_i}{\sigma_i}\tilde{\theta}_i^T\theta_i + \sum_{i=1}^{n}\frac{\eta_i}{r_i}\tilde{\delta}_i\delta_i + \sum_{i=1}^{n-1}|S_i|^2 + \frac{1}{2}S_n^2 \\ &+ \sum_{i=2}^{n}\left(\frac{B_i^2}{2\mu} - \frac{1}{\lambda_i} + \frac{1}{2}\right)w_i^2 + 2\mu(n-1) \\ &+ S_n\left(\frac{\bar{u}(t) - \beta_2(t)M_1}{1+\beta_1(t)\kappa_1} + \alpha_n\right) \\ \leq& -q_n\|e\|^2 + \frac{1}{2}\|P\kappa^*\|^2 + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i - \sum_{i=1}^{n} c_i S_i^2 \\ &+ \sum_{i=1}^{n}\frac{\rho_i}{\sigma_i}\tilde{\theta}_i^T\theta_i + \sum_{i=1}^{n}\frac{\eta_i}{r_i}\tilde{\delta}_i\delta_i + \sum_{i=1}^{n}|S_i|^2 + \frac{2\kappa_2}{1-\kappa_1} \\ &+ \sum_{i=2}^{n}\left(\frac{B_i^2}{2\mu} - \frac{1}{\lambda_i} + \frac{1}{2}\right)w_i^2 + 2\mu(n-1) \end{aligned} \tag{92}$$

where $q_n = q_{n-1} - k_{1n}^2/2$.

According to Lemma 4, the following equation can be given

$$\begin{aligned} \tilde{\theta}_l^T\theta_l &\leq -\frac{1}{2}\tilde{\theta}_l^T\tilde{\theta}_l + \frac{1}{2}\theta_l^{*T}\theta_l^* \\ \tilde{\delta}_l\delta_l &\leq -\frac{1}{2}\tilde{\delta}_l^2 + \frac{1}{2}\tilde{\delta}_l^{*2} \end{aligned} \tag{93}$$

Substituting (93) into (92) produces

$$\begin{aligned} D^\alpha V_n \leq& -q_n\|e\|^2 + \frac{1}{2}\|P\kappa^*\|^2 + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i - \sum_{i=1}^{n} c_i S_i^2 \\ &- \frac{1}{2}\sum_{i=1}^{n}\frac{\rho_i}{\sigma_i}\tilde{\theta}_i^T\tilde{\theta}_i - \frac{1}{2}\sum_{i=1}^{n}\frac{\eta_i}{r_i}\tilde{\delta}_i^2 + \frac{1}{2}\sum_{i=1}^{n}\frac{\rho_i}{\sigma_i}\theta_i^{*T}\theta_i^* \\ &+ \frac{1}{2}\sum_{i=1}^{n}\frac{\eta_i}{r_i}\delta_i^{*2} + \sum_{i=1}^{n}|S_i|^2 + \frac{2\kappa_2}{1-\kappa_1} \end{aligned}$$

$$+\sum_{i=2}^{n}\left(\frac{B_i^2}{2\mu}-\frac{1}{\lambda_i}+\frac{1}{2}\right)w_i^2+2\mu\,(n-1)$$
$$\leq -q_n\|e\|^2-\sum_{i=1}^{n}c_iS_i^2-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{\rho_i}{\sigma_i}-1\right)\tilde{\theta}_i^T\tilde{\theta}_i$$
$$-\frac{1}{2}\sum_{i=1}^{n}\frac{\eta_i}{r_i}\tilde{\delta}_i^2+\sum_{i=2}^{n}\left(\frac{B_i^2}{2\mu}-\frac{1}{\lambda_i}+\frac{1}{2}\right)w_i^2+\xi \tag{94}$$

note that $\xi = \frac{1}{2}\sum_{i=1}^{n}\left(\frac{\rho_i}{\sigma_i}\theta_i^{*T}\theta_i^*+\frac{\eta_i}{r_i}\delta_i^{*2}+2|S_i|^2\right)+\frac{1}{2}\|P\kappa^*\|^2+2\mu\,(n-1)+\frac{2\kappa_2}{1-\kappa_1}$.

Define

$$C=\min\left\{2q_{n-1}/\lambda_{\min}(P),\ 2c_i,\ (\rho_i-\sigma_i),\ \eta_i\left(\frac{B_i^2}{2\mu}-\frac{2}{\lambda_i}+\frac{1}{2}\right)\right\} \tag{95}$$

Combining (95) into (94) together leads to

$$D^\alpha V_n \leq -CV_n+\xi \tag{96}$$

Along with (96) and Lemma 8, we can obtain

$$V_n \leq V(0)\,E_\alpha\left(-Ct^\alpha\right)+\frac{\xi\mu}{C}, \tag{97}$$

Then we can obtain

$$\lim_{t\to\infty}|V_n(t)| \leq \frac{\xi\mu}{C} \tag{98}$$

Since $\frac{1}{2}|S(t)|^2 \leq V_n(t)$, we can obtain

$$\lim_{t\to\infty}|S(t)| \leq \sqrt{\frac{2\xi\mu}{C}} \tag{99}$$

Then we can summarize that all the signals remain bounded in the closed-loop system and the synchronization errors converge to near zero.

To ensure that the proposed control method can avoid Zeno phenomenon, the proof is as follows:

By $\Delta(t)=\bar{u}(t)-u(t)$ and $u(t)=\bar{u}(t_k)$, we have

$$D^\alpha(\Delta(t))=D^\alpha(\bar{u}(t)-u(t))=D^\alpha(\bar{u}(t)-\bar{u}(t_k))=D^\alpha(\bar{u}(t)) \tag{100}$$

According to (1), we can obtain

$$\begin{aligned}D^\alpha|\Delta(t)| &= \frac{1}{\Gamma(n-\alpha)}\int_0^t\frac{|\Delta(\tau)|^{(n)}}{(t-\tau)^{1+\alpha-n}}d\tau\\ &=\frac{1}{\Gamma(n-\alpha)}\int_0^t\frac{(sign(\Delta(\tau))\Delta(\tau))^{(n)}}{(t-\tau)^{1+\alpha-n}}d\tau\\ &=\frac{sign(\Delta(t))}{\Gamma(n-\alpha)}\int_0^t\frac{\Delta^{(n)}(\tau)}{(t-\tau)^{1+\alpha-n}}d\tau\\ &= sign(\Delta(t))\,D^\alpha(\Delta(t))\end{aligned} \tag{101}$$

Combining (100) into (101) together leads to

$$D^\alpha(|\Delta(t)|)=sign(\Delta(t))\,D^\alpha(\Delta(t)) \leq \left|D^\alpha\bar{u}(t)\right| \tag{102}$$

According to equation (85), we can conclude that $\bar{u}(t)$ is α-th differentiable and $D^\alpha(\bar{u}(t))$ a bounded function. Therefore, we get that there exist $\varsigma > 0$ such that $|D^\alpha(\bar{u}(t))| \leq \varsigma$. From $\Delta(t_k)=0$ and $\lim_{t\to t_{k+1}}\Delta(t)=m_1$. Thus, there exists t^* such that $t^* \geq m_1/\varsigma$. Therefore, there exists $t^* \geq 0$ such that $\forall k \in N^*$, $\{t_{k+1}-t_k\} \geq t^*$, the Zeno phenomenon will not occur.

Remark 3: The system matrix A is Hurwitz stable by appropriate selection of a vector K. It is shown that the changing trends of z and $\hat{z}$ coincide well and the boundedness of e is well validated. It should be noted that the observer and controller are designed synchronously in this paper, which does not satisfy the separation principle. That is to design the observer to be bounded while ensuring that the controller is bounded.

Remark 4: It should be noted that the parameters κ_1, M_1 that will affect the synchronization accuracy of the system and the minimum trigger interval. According to (86), it is easy to know that when κ_1 and M_1 change, the trigger interval and the number of trigger will change. However, inappropriate parameters will affect the synchronization accuracy. According to equations (94) and (99), it can be concluded that if κ_1 changes, ξ will change accordingly, thereby affecting the synchronization error accuracy. Therefore, it is very necessary to select the appropriate parameters.

IV. SIMULATIONS

In this section, we use the following examples to verify the validity of the proposed method.

A. EXAMPLE 1

we consider the fractional order Duffing-Holmes chaotic system [44], and select the master system as follows:

$$\begin{cases}D^\alpha x_1 = x_2\\ D^\alpha x_2 = x_1-ax_2-x_1^3+b\cos t\end{cases} \tag{103}$$

where $\alpha=0.98$, $a=0.25$, $b=0.3$, $x(0)=[0.2, 0.2]$.

The slave system is as follows:

$$\begin{cases}D^\alpha y_1 = y_2\\ D^\alpha y_2 = y_1-ay_2-y_1^3+b\cos t\\ \qquad +\Delta f(t,y)+d(t)+u(t-\tau)\end{cases} \tag{104}$$

where τ=0, $y(0)=[0.1,-0.2]$, the nonlinear function is $\Delta f(t,y)=0.1\sin(t)\sqrt{y_1^2+y_2^2}$, $d(t)=0.1\sin(t)$. According to Theorem 1, we design the controller as follows

$$\bar{u}(t)=-\alpha_2-\frac{S_2(\kappa_1\alpha_2)^2}{\sqrt{(S_2\kappa_1\alpha_2)^2+\kappa_2^2}}-\frac{S_2M_1^2}{\sqrt{(S_2M_1)^2+\kappa_2^2}} \tag{105}$$

$$u(t)=\bar{u}(t_k),\quad \forall\in[t_k,t_{k+1}) \tag{106}$$

where $\alpha_2=S_1+c_2S_2+\theta_2^T\varphi_2(\hat{z})+sign(S_2)\,\delta_2-\frac{\alpha_1-v_2}{\lambda_2}$ and $\alpha_1=-c_1S_1$. Choose the design parameters as $c_1=15$, $c_2=35$, $\theta(0)=[0.01,\ldots,0.01]^T$,$\delta(0)=0.1$, $\rho=\eta=40$,

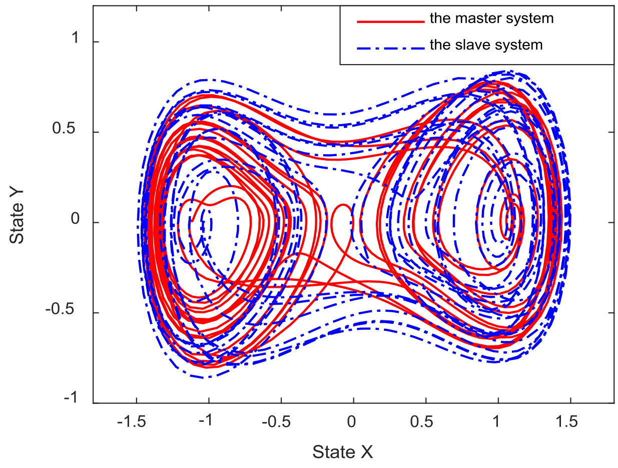

FIGURE 1. Fractional order Duffing-Holmes system and its slave system.

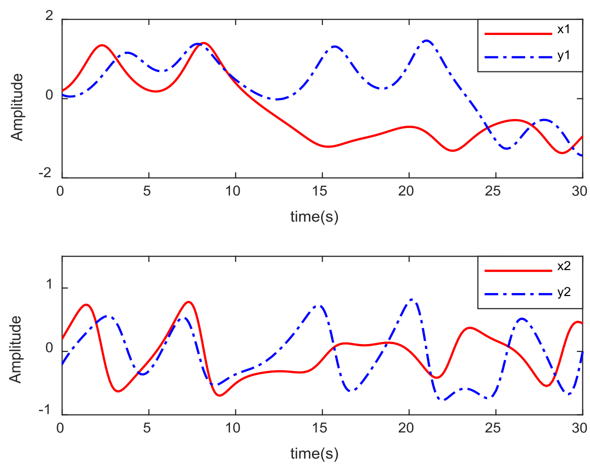

FIGURE 2. The state trajectories of the fractional Duffing-Holmes system and its slave system.

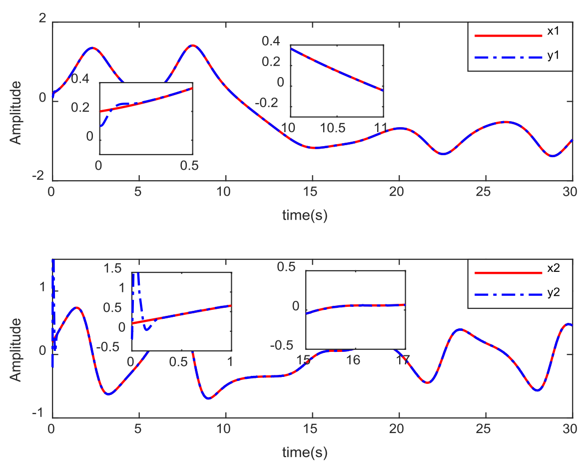

FIGURE 3. The synchronization trajectories of the master system and its slave system in Example 1.

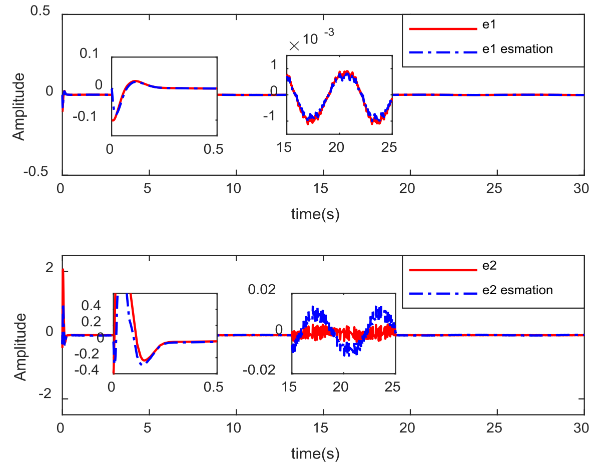

FIGURE 4. The trajectories of the synchronization errors and its estimation in Example 1.

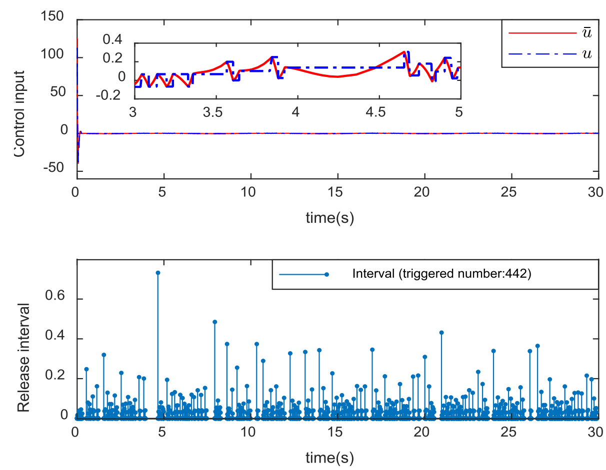

FIGURE 5. The trajectories of $\bar{u}$, u and release interval in Example 1.

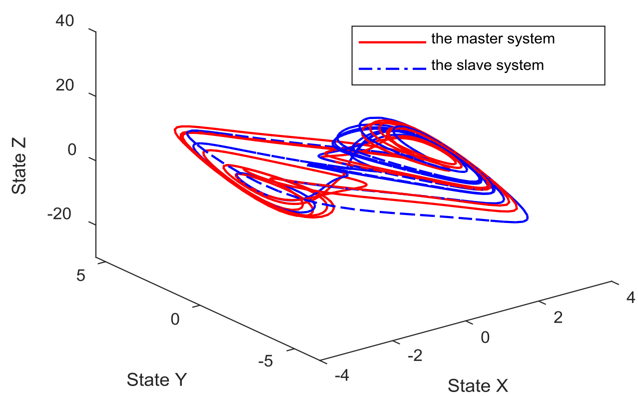

FIGURE 6. Fractional order Arneodo system and its slave system.

$\sigma = r = 1$. $\kappa_1 = 0.5$, $\kappa_2 = 2$, $M_1 = 0.5$, $K = [40, 1600]$. Figure. 1-Figure. 5 show the simulation results of Example 1. Figures. 1-2 show that the states trajectories of the master system and the slave system, which can be seen from the figure that there is difference in the states trajectories between the master system and the slave system. Figure. 3 shows the states synchronization trajectories of the master system and the slave system. Figure. 4 shows the synchronization errors and the estimated errors. It can be seen that the controller designed by the method in this paper has satisfactory control

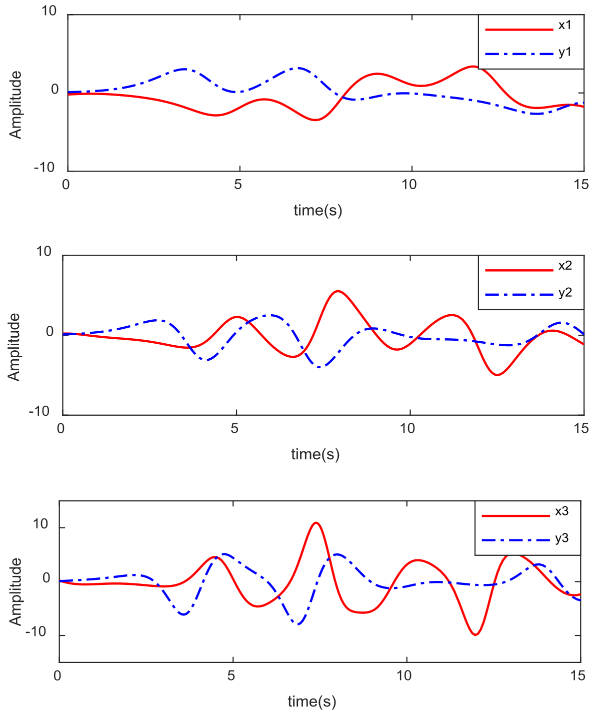

FIGURE 7. The state trajectories of the fractional order Arneodo system and its slave system.

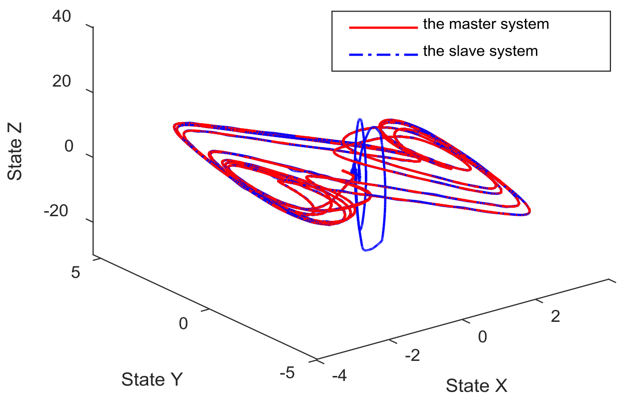

FIGURE 8. The synchronization trajectories of the fractional order Arneodo system in Example 2.

performance. Figure. 5 shows the trajectories of $\bar{u}$, u and release interval and it can illustrate the boundness of $\bar{u}$ and u in Example 1.

B. EXAMPLE 2

Consider the fractional order Arneodo chaotic system [45] and the master system is as follows:

$$\begin{cases} D^{\alpha}x_1(t) = & x_2(t) \\ D^{\alpha}x_2(t) = & x_3(t) \\ D^{\alpha}x_3(t) = & f(x) \end{cases} \tag{107}$$

where $\alpha = 0.97, f(x) = x_1^3(t) + 5.5x_1(t) - 3.5x_2(t) - 0.8x_3(t)$ is the unknown function, the initial conditions are given as $x(0) = [-0.2, 0.2, 0.2]$.

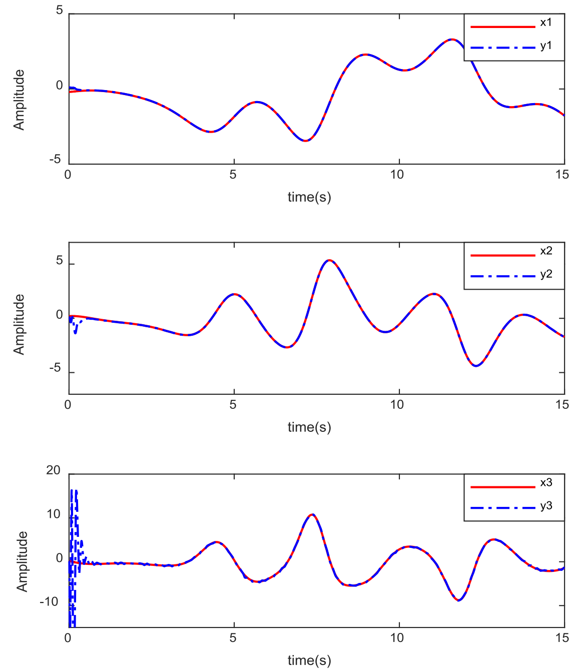

FIGURE 9. The state synchronization trajectories of the master system and its slave system in Example 2.

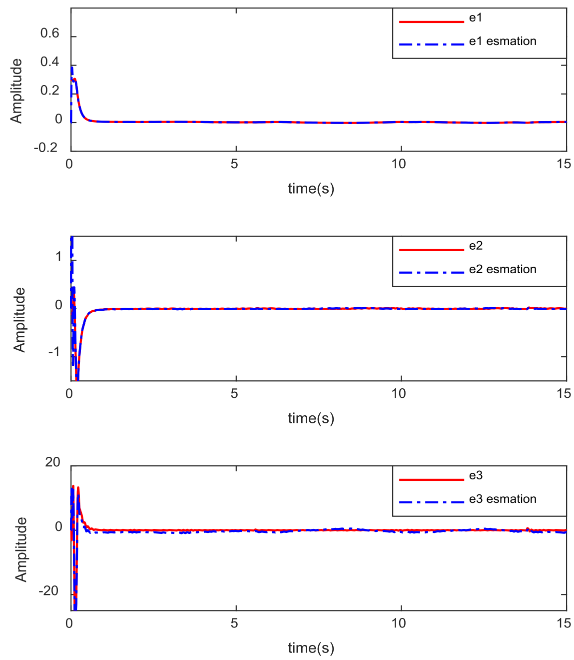

FIGURE 10. The synchronization error trajectories of the master system and its slave system in Example 2.

The slave system is given as

$$\begin{cases} D^{\alpha}y_1(t) = y_2(t) \\ D^{\alpha}y_2(t) = y_3(t) \\ D^{\alpha}y_3(t) = u(t-\tau) + g(y) + \Delta g(t,y) + d(t) \end{cases} \tag{108}$$

where $\tau = 0.01$ is input delay, $y(0) = [0.1, 0.1, 0.1]$, $\Delta g(t,y) = \sin\left(\sqrt{y_1^2 + y_2^2}\right)$.and $g(y) = y_1^3(t) + 5.5y_1(t) - 3.5y_2(t) - 0.8y_3(t)$ are the system nonlinearity.

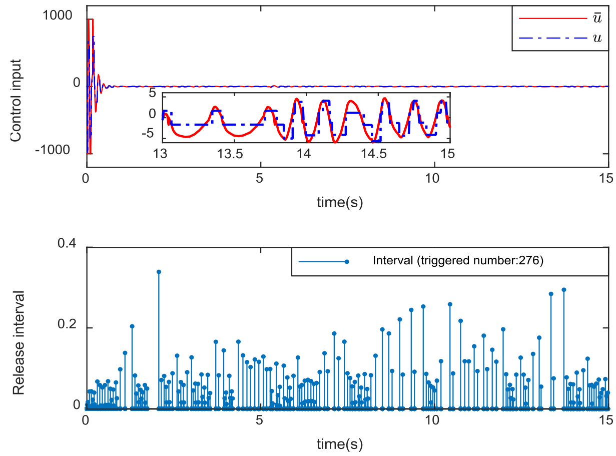

FIGURE 11. The trajectories of $\bar{u}$, u and release interval in Example 2.

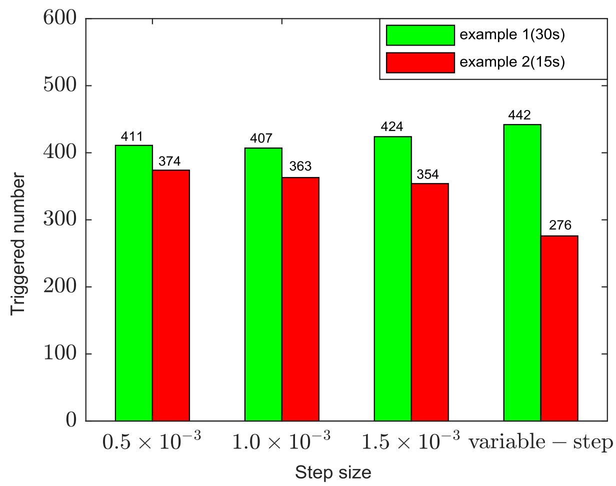

FIGURE 12. The triggered number of different sampling step size.

$d(t) = 0.1 \sin(t)$ is the external disturbance. Choose the design parameters as $c_1 = 20, c_2 = c_3 = 10, \sigma = r = 1, \rho = \eta = 40, \kappa_1 = 0.7, \kappa_2 = 2, M_1 = 2, K = [60, 1200, 8000]$.

Figure. 6-Figure. 11 show the simulation results of Example 2. Figures. 6-7 show that the states trajectories of the master system and the slave system. Figures. 8-9 show the states synchronization trajectories of the master system and the slave system. Figure. 10 shows the synchronization errors and the estimated errors. Figure. 11 shows the trajectories of $\bar{u}$, u and release interval in Example 2. We have simulated the number of event triggers with different sampling time steps (see Figure. 12), in which variable-step sampling is used for simulation in this paper. It can be seen from the simulation results that different sampling times have some effects on the number of event triggers. Therefore, the appropriate sampling time will reduce the number of triggers to a certain extent.

V. CONCLUSION

This paper investigates the synchronization problem for fractional order chaotic systems with input delay and nonlinear dynamics, and designs an event-triggered adaptive neural network backstepping sliding mode controller. An event-triggered scheme is considered to reduce the number of transmissions of control input signals. The fractional order dynamic surface control technology is introduced into the controller to avoid "explosion of complexity" and obtain fractional derivatives for virtual control functions continuously. The sliding term is introduced, which can enhance robustness. The nonlinear functions and external disturbance of systems are approximated by the RBF neural network and the state observer is designed for states estimation of system. By utilizing the Pade delay approximation method, the original systems can be converted into systems without input delay. Under the Lyapunov stability theory, the stability of systems is ensured by the proposed controller. Example and simulation results show the feasibility and effectiveness of the proposed results. In addition, the method proposed in this paper can be applied to the synchronization of high-dimensional fractional order chaotic systems, so the results of this paper have great theoretical significance, especially in the field of communication security.

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