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The Dynamic Extensions of Fuzzy Grey Cognitive Maps

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ABSTRACT Fuzzy Cognitive Map is recognized as an important model in soft computing. The Fuzzy Cognitive Map theory studies often treat uncertainty modeling and dynamic modeling separately. So far, there is no single Fuzzy Cognitive Map that can deal with uncertain data and dynamic environments simultaneously. This paper proposed an environment model to describe the link between changing weights and the dynamic environment. As the extensions of the classic Fuzzy Grey Cognitive Map, two dynamic models were designed and implemented in this work: Dynamic Fuzzy Grey Cognitive Map and Dynamic Fuzzy General Grey Map. In this work, we also analyze the characteristics of the two models and evaluate their performance using industrial control problems. The results showed that the proposed models could well handle uncertain data in dynamic environments.

INDEX TERMS Dynamic fuzzy grey cognitive map, environment model, grey system theory, fuzzy grey cognitive map, dynamic modeling.

I. INTRODUCTION

Fuzzy Cognitive Map (FCM) belongs to neural networks, the most attractive characteristics of FCM is that the FCM is explainable and support circle causalities [1], [2]. As a kind of neural network, FCM is constituted of nodes and fuzzy weights between nodes [3], and iterates using activation functions like sigmoid and tanh. The inference results of an FCM can be a fixed attractor, a limited circle or chaos [3], [4]. One can build an FCM by experts' knowledge or historical data or both of them [5]. Thus, FCM is a promising tool to construct both data-driven and knowledge-driven artificial intelligence. Till now, FCMs and their extensions have been used in many areas, such as industrial process control [6]–[9], intrusion detection [10], smart city [11], system prediction [12], system simulation [13], decision-support [14], [15], pattern recognition [16], [17], intelligence modeling [18] etc.

To improve the uncertainty modeling and dynamic modeling ability are two main research fields for FCM extensions [19]–[21]. To tackle the uncertainty modeling problem, the Fuzzy Grey Cognitive Map (FGCM) is an impressive model that combines the Grey System Theory (GST) and FCM [22], it makes the FCM can deal with the data with uncertainty like interval values [23]. The FGCM keeps the same topological structure of the FCM and uses the interval grey number (IGN) as its basic computing element rather than FCM's fuzzy number [24]. FGCMs also have their activation functions like sigmoid and tanh in the form of IGN [25]. The convergence characteristic of FGCM is similar to the counterpart of FCM: a fixed point, a limited circle or chaos [19], [26], [27]. Compared with other efficient and excellent methods that also deals with the uncertainty data, such as complex mass function [28]–[31] in the field of Dempster-Shafer (D-S) evidence theory, the FGCMs and their extensions try to exploit all information from the uncertainty data rather than choose the maximum likelihood value from data. The typical applications of FGCM include: supply chain performance and organizational culture [32], empathic buildings [33], interval-valued time series forecasting [34], process modeling [35] and so on. The Fuzzy General Grey Cognitive Map (FGGCM) was recently introduced to improve the uncertainty modeling ability of FGCM [20]. As an extension of FGCM, FGGCM uses the general grey number (GGN), rather than the interval grey number (IGN). This improvement makes the FGGCM deal with the uncertain data in interval values and the data in multiple interval values.

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The study about the dynamic extensions of FCM mainly focuses on the dynamic change of the weights of FCM under a dynamic environment. In this case, the weights in an FCM are not static anymore but are influenced by the environment. For example, a heating element affects the temperature of the water, but the temperature of the environment, the voltage, and the water's status influence the heating effect. That is, the environment has an impact on the intensity of the causal link between the heating element and the temperature of the water. Particularly, if the temperature of the water reaches 100°C, the power of the heating element will not influence the temperature anymore. Several kinds of Dynamic Fuzzy Cognitive Maps (DFCM) are proposed to deal with the dynamic causal link. However, there is not an FCM extension designed for both uncertain data and dynamic environments. This paper's aim is to fill the gap of the FCM extension under a dynamic and uncertain environment.

Literally, there are two ways to realize the dynamic and uncertainty extension of FCM. One is to extend the existing DFCM, make it can deal with the uncertain data. But till to now, the definition of DFCM is ambiguous, basically, the existing DFCM can be categorized into two groups, the DFCM base on the rules [15], [36], [37] and the DFCM base on the proportional regulation [38]–[40]. The mechanics of the dynamic for DFCMs should be clarified to realize the uncertainty extension of DFCM. Another way to get the extension model is to extend the FGCM and FGGCM directly to deal with the dynamic environment. Also, making the FGCM and FGGCM cope with the dynamic environment is the first question to get the dynamic extensions of FGCM and FGGCM. This paper proposed the Environment Model (EM) to describe the function between weights and dynamic environment. Based on the EM, a general definition of DFCM is given. And then, two new models: Dynamic Fuzzy Grey Cognitive Maps (DFGCM) and the Dynamic Fuzzy General Grey Cognitive Maps (DFGGCM), are designed to deal with uncertain data in the dynamic environment. These proposed models are evaluated using the industrial control system.

Thus, the contributions of this paper are:

- Giving a general definition for DFCM;
- Design and implement two dynamic models to deal with the dynamic environment for FGCM;
- Evaluate the new models and explore the characteristics using an industrial control system.

The rest of the paper is organized in the following way: the FGCM and FGGCM's fundamentals are reviewed in Section II. Section III shows the dynamic modeling methods for the existing DFCM, and proposed a general definition for DFCM. Section IV shows the proposed DFGCM and DFGGCM. Section V is the experiment design for validating the DFGCM and DFGGCM. Section VI shows the experiments results. Section VII discusses the experiment results and analyses the characteristics of the proposed models. And finally, Section VIII illustrates the characteristics of the proposed model and indicates the future work.

II. PRELIMINARIES

This section introduces the preliminary knowledge for the dynamic extensions of FGCM, including the basics of FGCM and FGGCM.

A. FUZZY GREY COGNITIVE MAPS

FGCM is an extension of FCM to cope with the uncertainty of the data, and it combines Grey System Theory (GST) and FCM [5]. The interval grey number (IGN) is the essential element of FGCM, denoted as $\otimes G \in [\underline{G}, \overline{G}], \underline{G} \leqslant \overline{G}$ [24]. The IGN should be written as $\otimes G \in [\underline{G}, +\infty)$ when we do not know the upper bound [41]. On the other hand, if the lower bound is unclear, it should be indicated as $\otimes G \in (-\infty, \overline{G}]$ [25]. The grey number becomes a black number, indicated as $\otimes G \in (-\infty, +\infty)$ if all the bounds are unclear [9]. A grey number becomes a white number only and only if its upper bound is equal to its lower bound [42].

The IGNs' operation rules [43] are:

$$\otimes G_1 + \otimes G_2 \in \left[\underline{G_1} + \underline{G_2}, \overline{G_1} + \overline{G_2}\right] \tag{1}$$

$$- \otimes G \in \left[-\overline{G}, -\underline{G}\right] \tag{2}$$

$$\begin{aligned}\otimes G_1 - \otimes G_2 &= \otimes G_1 + (- \otimes G_2)\\ &\in \left[\underline{G_1} - \overline{G_2}, \overline{G_1} - \underline{G_2}\right]\end{aligned} \tag{3}$$

$$\lambda \cdot \otimes G \in \left[\lambda \cdot \underline{G}, \lambda \cdot \overline{G}\right] \tag{4}$$

$$\begin{aligned}&\otimes G_1 \times \otimes G_2 \in\\ &\quad\left[\min(\underline{G_1} \cdot \underline{G_2}, \overline{G_1} \cdot \overline{G_2}, \underline{G_1} \cdot \overline{G_2}, \overline{G_1} \cdot \underline{G_2}),\right.\\ &\quad\left.\max(\underline{G_1} \cdot \underline{G_2}, \overline{G_1} \cdot \overline{G_2}, \underline{G_1} \cdot \overline{G_2}, \overline{G_1} \cdot \underline{G_2})\right]\end{aligned} \tag{5}$$

$$\frac{1}{\otimes G} \in \left[\frac{1}{\overline{G}}, \frac{1}{\underline{G}}\right] \tag{6}$$

$$\begin{aligned}\otimes G_1 \div \otimes G_2 &= \otimes G_1 \times \frac{1}{\otimes G_2}\\ &\in \left[\min(\underline{G_1}\frac{1}{\underline{G_2}}, \overline{G_1}\frac{1}{\overline{G_2}}, \underline{G_1}\frac{1}{\overline{G_2}}, \overline{G_1}\frac{1}{\underline{G_2}}),\right.\\ &\quad\left.\max(\underline{G_1}\frac{1}{\underline{G_2}}, \overline{G_1}\frac{1}{\overline{G_2}}, \underline{G_1}\frac{1}{\overline{G_2}}, \overline{G_1}\frac{1}{\underline{G_2}})\right]\end{aligned} \tag{7}$$

The degree of greyness, for convenience, will be called greyness later, is an indicator of how much uncertainty a grey number represents [44]. If a grey number is $\otimes$, and the domain is Ω, the greyness is [45]:

$$g^{\circ}(\otimes) = \frac{\mu(\otimes)}{\mu(\Omega)}. \tag{8}$$

The $\mu()$ is the function to calculates the measure or "length" of the grey number or the domain [45]. And $0 \leqslant g^{\circ}(\otimes) \leqslant 1$. If $g^{\circ}(\otimes) = 0$, $\otimes$ is a white number. If $g^{\circ}(\otimes) = 1$, $\otimes$ is a black number.

As a fuzzy number is a kind of special case of a grey number [46], the FGCM uses IGNs instead of fuzzy numbers. Like the nodes and weights in FCM, the nodes mean concepts and variables, the weights mean the causalities, but in FGCM, both weights and nodes are in the form of IGN. The relationship between FGCM and FCM is shown in Fig. 1.

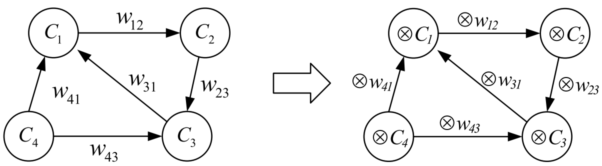

FIGURE 1. The relationship between the FCM and the FGCM [20].

A simple FGCM is shown in Fig. 1, assume the initial vector is

$$\begin{aligned}\otimes\vec{C}_0 &= \left(\otimes C_0^{[1]}, \otimes C_0^{[2]}, \otimes C_0^{[3]}, \otimes C_0^{[4]}\right)\\ &= \left(\left[\underline{C_0^{[1]}}, \overline{C_0^{[1]}}\right], \left[\underline{C_0^{[2]}}, \overline{C_0^{[2]}}\right],\right.\\ &\quad \left.\left[\underline{C_0^{[3]}}, \overline{C_0^{[3]}}\right], \left[\underline{C_0^{[4]}}, \overline{C_0^{[4]}}\right]\right). \end{aligned} \tag{9}$$

The weight matrix is

$$\otimes w = \begin{pmatrix} 0 & \otimes w_{12} & 0 & 0 \\ 0 & 0 & \otimes w_{23} & 0 \\ \otimes w_{31} & 0 & 0 & 0 \\ \otimes w_{41} & 0 & \otimes w_{43} & 0 \end{pmatrix}. \tag{10}$$

The FGCM iterates like FCMs [24]:

$$\begin{aligned}\otimes\vec{C}_{t+1} &= S\left(\otimes\vec{C}_t \cdot \otimes w\right)\\ &= S\left(\otimes\vec{C}'_{t+1}\right)\\ &= \left(S\left(\otimes C_t^{[1]'}\right), S\left(\otimes C_t^{[2]'}\right),\right.\\ &\quad \left. \dots, S\left(\otimes C_t^{[n]'}\right)\right), \end{aligned} \tag{11}$$

in which, $S(\otimes)$ is one of the activation functions like sigmoid, and tanh, etc. The sigmoid function in the form of IGN is

$$S(\otimes G) = \left[\frac{1}{1+\mathrm{e}^{-\lambda \underline{G}}}, \frac{1}{1+\mathrm{e}^{-\lambda \overline{G}}}\right]. \tag{12}$$

The tanh in the form of IGN is:

$$\tanh(\otimes G) = \left[\frac{\mathrm{e}^{\lambda\underline{G}} - \mathrm{e}^{-\lambda\underline{G}}}{\mathrm{e}^{\lambda\underline{G}} + \mathrm{e}^{-\lambda\underline{G}}}, \frac{\mathrm{e}^{\lambda\overline{G}} - \mathrm{e}^{-\lambda\overline{G}}}{\mathrm{e}^{\lambda\overline{G}} + \mathrm{e}^{-\lambda\overline{G}}}\right]. \tag{13}$$

λ is a positive parameter to set the steepness of the activation function [47]. After several iterations, a FGCM may reach a limit cycle, a steady-state, or chaos [24].

B. FUZZY GENERAL GREY COGNITIVE MAPS

FGGCM is the extension of FGCM, it aims to improve the uncertainty modeling capability of FGCM because FGCM does not exploit all uncertainty processing ability of GST [20]. FGGCM uses the general grey number (GGN) rather than the IGN as its basic element.

The GGN is the union of IGNs, defined as $g^{\pm} \in \bigcup_{i=1}^{n}[\underline{a}_i, \overline{a}_i]$. For example, $[0, 1.2] \cup [1.5, 2] \cup [3, 5] \cup 6$ is a GGN, $[0, 1]$ is not only an IGN but also a special case of GGN. The operations between general grey numbers are different from that of interval grey numbers. The IGNs use their upper and lower bounds to do the mathematical operation [48], but the GGNs use kernel and greyness to do the mathematical operation. Both GGN and IGN have a kernel that represents the most likely crisp value of the grey number locates. The kernel of an IGN can be calculated as the expectation of the crisp value, and the kernel of a GGN can be calculated using Eq. (14) and Eq. (15) [45], [49].

$$\hat{g} = \frac{1}{n}\left(\sum_{i=1}^{n}\hat{a}_i\right). \tag{14}$$

$$\hat{g} = \sum_{i=1}^{n} p_i\hat{a}_i. \tag{15}$$

In Eq. (14) and Eq. (15), $\hat{a}_i$ is the kernel of $[\underline{a}_i, \overline{a}_i]$, is calculate as the expectation of the crisp value. If the distribution of the GGN is unknown, use Eq. (14) to calculate the kernel of the GGN. If we know distribution of a GGN, and the probability of $g^{\pm} \in [\underline{a}_i, \overline{a}_i], (i = 1, 2, \dots, n)$ is p_i, and $\sum_{i=1}^{n} p_i = 1, p_i > 0, i = 1, 2, \dots, n$, use Eq. (15).

Assume Ω is the domain, $\mu()$ is the measure, $g^{\pm} \in \bigcup_{i=1}^{n}[\underline{a}_i, \overline{a}_i]$, then the greyness of the GGN is

$$g^{\circ}(g^{\pm}) = \frac{1}{|\hat{g}|}\sum_{i=1}^{n}|\hat{a}_i|\mu(\otimes_i)/\mu(\Omega). \tag{16}$$

Using the kernel and greyness of the GGN, a GGN can be represented as $\hat{\otimes}_{(g^{\circ})}$, called the grey number's simplified form. $\hat{\otimes}$ is the kernel, and g° is the greyness. The operating rules of the GGN [50] are shown as follows:

$$\hat{g}_{1(g_1^{\circ})} = \hat{g}_{2(g_2^{\circ})} \Longleftrightarrow \hat{g}_1 = \hat{g}_2 \text{ and } g_1^{\circ} = g_2^{\circ} \tag{17}$$

$$\hat{g}_{1(g_1^{\circ})} + \hat{g}_{2(g_2^{\circ})} = (\hat{g}_1 + \hat{g}_2)_{(w_1 g_1^{\circ} + w_2 g_2^{\circ})} \tag{18}$$

where

$$w_1 = \frac{|\hat{g}_1|}{|\hat{g}_1| + |\hat{g}_2|}, \quad w_2 = \frac{|\hat{g}_2|}{|\hat{g}_1| + |\hat{g}_2|}$$

$$-\hat{g}_{(g^{\circ})} = (-\hat{g})_{(g^{\circ})} \tag{19}$$

$$\hat{g}_{1(g_1^{\circ})} - \hat{g}_{2(g_2^{\circ})} = (\hat{g}_1 - \hat{g}_2)_{(w_1 g_1^{\circ} + w_2 g_2^{\circ})} \tag{20}$$

where

$$w_1 = \frac{|\hat{g}_1|}{|\hat{g}_1| + |\hat{g}_2|}, \quad w_2 = \frac{|\hat{g}_2|}{|\hat{g}_1| + |\hat{g}_2|}$$

$$k \cdot \hat{g}_{(g^{\circ})} = (k \cdot \hat{g})_{(g^{\circ})}, k \in \mathbb{R} \tag{21}$$

$$\hat{g}_{1(g_1^{\circ})} \times \hat{g}_{2(g_2^{\circ})} = (\hat{g}_1 \times \hat{g}_2)_{(\max(g_1^{\circ}, g_2^{\circ}))} \tag{22}$$

$$\frac{1}{\hat{g}_{1(g_1^{\circ})}} = \left(\frac{1}{\hat{g}_1}\right)_{(g_1^{\circ})}, \quad (\hat{g}_1 \neq 0) \tag{23}$$

$$\frac{\hat{g}_{1(g_1^{\circ})}}{\hat{g}_{2(g_2^{\circ})}} = \left(\frac{\hat{g}_1}{\hat{g}_2}\right)_{(\max(g_1^{\circ}, g_2^{\circ}))}, \quad (\hat{g}_2 \neq 0) \tag{24}$$

$$(\hat{g}_{(g^{\circ})})^k = (\hat{g})^k_{(g^{\circ})}, \quad k \in \mathbb{R} \tag{25}$$

The FGGCM uses the GGN to replace the IGN of the FGCM. In this way, FGGCM extends the uncertainty modeling ability of the FGCM. The sigmoid and tanh activation functions are proven in [20] and shown in Eq. (26) and Eq. (29).

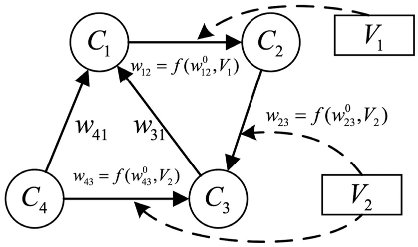

FIGURE 2. The exsiting DFCM model.

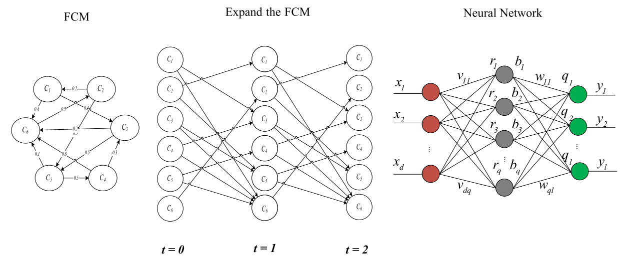

FIGURE 3. The relationship between FCM and neural network.

The sigmoid function is [20]

$$S(g^{\pm}) = \frac{1}{1+\mathrm{e}^{-\lambda g^{\pm}}}. \tag{26}$$

$S(g^{\pm})$ is a new GGN, the kernel is

$$\hat{S}(g^{\pm}) = \frac{1}{1+\mathrm{e}^{-\lambda \hat{g}}}, \tag{27}$$

the greyness is

$$S^{\circ}(g^{\pm}) = \frac{1}{1+\mathrm{e}^{-\lambda \hat{g}}} g^{\circ}. \tag{28}$$

The hyperbolic tangent function in the form of GGN is:

$$\tanh(g^{\pm}) = \frac{\mathrm{e}^{\lambda g^{\pm}} - \mathrm{e}^{-\lambda g^{\pm}}}{\mathrm{e}^{\lambda g^{\pm}} + \mathrm{e}^{-\lambda g^{\pm}}}. \tag{29}$$

The kernel is

$$\tanh(\hat{g}) = \frac{\mathrm{e}^{\lambda \hat{g}} - \mathrm{e}^{-\lambda \hat{g}}}{\mathrm{e}^{\lambda \hat{g}} + \mathrm{e}^{-\lambda \hat{g}}}, \tag{30}$$

the greyness is

$$(\tanh(g^{\pm}))^{\circ} = g^{\circ} \tag{31}$$

Like Eq. (12) and Eq. (13), in Eq. (26) to Eq. (29), the λ is a positive parameter to set the steepness of the curve or surface.

III. DYNAMIC MODELING METHODS FOR FUZZY COGNITIVE MAPS

The existing Dynamic Fuzzy Cognitive Maps (DFCM) improve their dynamic modeling ability by introducing environment variables. In the iteration process, the weight of DFCM will be affected by environment variables. The DFCM model is shown in Fig. 2.

DFCM introduces the external environment variable set V to FCM so that the original static weights in FCM become an adjustable function related to its initial value and environment control variable V. V can directly affect the causal relationship between concept nodes so that the reasoning process can reflect the impact of the external environment.

At present, the function that how V influences the weights includes two main methods, one is a rule-based method [15], [36], [37], as shown in Eq. (32).

$$W = \begin{cases} a, & \text{criterion 1 is committed} \\ b, & \text{criterion 2 is committed} \\ c, & \text{criterion 3 is committed} \end{cases} \tag{32}$$

Another is proportional regulation method [38]–[40]

$$\begin{aligned} w_{ij}(t+1) &= f\left\{w_{ij}(t), V(t), \alpha(t)\right\} \\ &= \begin{cases} -1 & w_{ij}(t+1) < -1 \\ w_{ij}(t)\,(1+\alpha(t)V(t)) & -1 \leq w_{ij}(t+1) \leq 1 \\ 1 & w_{ij}(t+1) > 1 \end{cases} \end{aligned} \tag{33}$$

w_{ij} at the current step is proportional to the value of the previous step. The proportion is affected by the influence factor $\alpha(t)$ and the environment variable $V(t)$, which reflects that the influence of environment variables on the system is continuous and cumulative.

The adjustment method of the weight of DFCM should not be limited to the above two ways. If we expand the reasoning process of FCM, as shown in Fig. 3, FCM can be regarded as a neural network with the same weight of each layer. If we expand a DFCM, the weights of each layer of FCM will be different, which is closer to the neural network. The function of the environment variable set is to give the weight matrix of DFCM at each iteration step.

The dynamic of the external environment results in the dynamic of the data. It is not enough to use the static weight matrix to describe and fit the data affected by the high dynamic of the environment in each iteration. Therefore, the weight matrix changes dynamically are used in each iteration, which reflects that the casual relationship will change due to the influence of the dynamic environment. In order to describe the influence that dynamic environment exerts on the weights of DFCM, we propose the environment model (EM) to calculate the weight matrix of each iteration of DFCM. This paper will call the DFCM that uses the environment model as the General Dynamic Fuzzy Cognitive Maps (GDFCM). Accordingly, the DFCM using the rule-based method and the DFCM using the proportional regulation method can be seen as two special cases of GDGCM. Unlike the other DFCMs that are formulated as a quaternion $\{C, W, f, V\}$, the GDFCM is formulated as a quintuple $\{C, W, f, V, EM\}$, as shown in Fig. 4. For convenience, thereafter, we will not distinguish the DFCM and GDFCM, both of them are referred to as GDFCM.

The input of the environment model is environment variables and the weights at $t-1$ iteration. The input can also include the node value in GDFCM if the node value itself participates in the weights' changing in GDFCM. The output of

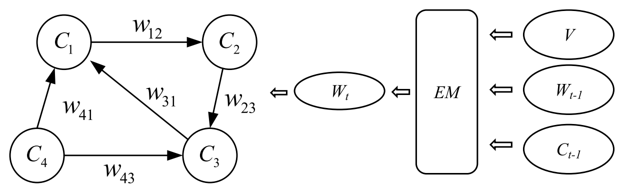

FIGURE 4. The concept graph of GDFCM.

the environment model is the weight matrix of each iteration of GDFCM. Thus, an environment model is a multiple-input-multiple-output system that connects the environment effect and the reasoning process of GDFCM.

The GDFCM can run online, that is, the GDFCM can collect real-time data of environment variables ($V(t)$). The environment model translates the environment variables into the weights of GDFCM, and the GDFCM gives the reasoning results in real-time. The GDFCM can also run offline, in this situation, the system is simulated, and the EM is used to modeling the dynamic environment. The choice of environment model can be varied, and the purpose is to make GDFCM accurately fit the data from the dynamic environment. Specifically, the environment model can be the above-mentioned rule-based method and proportional regulation method. In the framework of the GDFCM, the rule-based method is called the Rule-based Environment Model (RBEM), and the proportional regulation method is called Proportional Regulation Environment Model (PREM). Other interpretable models can also be used. To ensure the interpretability of the model, it is better to use the models which are interpretable, such as Multivariate Regression, Logistic Regression, FCM, Decision Tree and Random Forest, Support Vector Machine, etc. Besides, suppose the above models still cannot fit the data from the dynamic environment well, in that case, black-box models, such as Deep Neural Network, Convolutional Neural Network, can be used instead. After all, using interpretable models to simulate and model the dynamic environment is sometimes challenging. The GDFCM with an unexplainable environment model will become a "grey box" model. The environment model is not easy to interpret, but the inner FCM still retains the interpretable causality or correlation between nodes.

The weights of each iteration step of GDFCM are different, closer to the structure of Deep Neural Network. Generally, the model is more complex, which can deal with more complex and highly dynamic data. Unlike Deep Neural Network, the GDFCM is interpretable. Algorithm 1 shows the GDFCM's pseudocode.

IV. THE PROPOSED DYNAMIC EXTENSIONS OF FUZZY GREY COGNITIVE MAPS

In this section, two dynamic extensions of FGCM are proposed based on the mechanism of the environment model. By adding an environment model to control the change of the grey weights, FGCMs become the Dynamic Fuzzy grey Cognitive maps (DFGCM). And transfer the IGN to the GGN, the Dynamic Fuzzy General Grey Cognitive maps (DFGGCM) is obtained.

Algorithm 1 General Dynamic Fuzzy Cognitive Maps (GDFCM)

Data: initial vector $\vec{C}_0$, environment variables V, environment model $EM(\cdot)$
Result: output vector $\vec{C}_m$
$t = 0$;
while $t < m$ **do**

$$W(t) = EM(V, W(t-1), C(t-1), t);$$
$$\vec{C}_{t+1} = S\left(\vec{C}_t \cdot W(t)\right) = S\left(\vec{C}'_{t+1}\right)$$
$$= \left(S\left(C_t^{[1]'}\right), S\left(C_t^{[2]'}\right), \ldots, S\left(C_t^{[n]'}\right)\right),$$
$$t = t + 1;$$

$S\left(C_t^{[i]'}\right)$ is calculated by activation functions like sigmoid and tanh, etc.;
end

Algorithm 2 Dynamic Fuzzy Grey Cognitive Map (DFGCM)

Data: initial vector in the form of IGN $\otimes\vec{C}_0$, initial weights in the form of IGN $\otimes W_0$, environment variables in the form of IGN $\otimes V$, environment model $EM(\otimes)$
Result: output vector $\vec{C}_m$ (IGN)
$t = 0$;
while $t < m$ **do**

$$\otimes W(t) = EM(\otimes V, \ \otimes W(t-1), \otimes C(t-1), t);$$
$$\otimes\vec{C}_{t+1}$$
$$= S\left(\otimes\vec{C}_t \cdot \otimes W(t)\right) = S\left(\otimes\vec{C}'_{t+1}\right)$$
$$= \left(S\left(\otimes C_t^{[1]'}\right), S\left(\otimes C_t^{[2]'}\right), \ldots, S\left(\otimes C_t^{[n]'}\right)\right);$$
$$t = t + 1;$$

$S\left(\otimes C_t^{[i]'}\right)$ is calculated according to Eq. (12), and Eq. (13);
end

A. DYNAMIC FUZZY GREY COGNITIVE MAP

With the proposal of the environment model, it is easy to extend the FGCM to Dynamic Fuzzy Grey Cognitive Map (DFGCM). The principle is to use the interval grey number to replace the fuzzy number of GDFCM. So the basic structure of DFGCM is the same as the GDFCM, which contains an environment model and an FGCM whose grey weights will change at each iteration step.

Note that for the DFGCM, the output of the environment model must be in the form of IGNs, and the input can be fuzzy numbers or IGNs. As shown in Algorithm 2, at the beginning of the iteration, the environment model calculates the weights

$W(t)$ firstly according to the environment variables and the weights at the last step, and then the inner FGCM iterate once according to $W(t)$ and the nodes values of the previous iteration.

B. DYNAMIC FUZZY GENERAL GREY COGNITIVE MAPS

Like the DFGCM, the Dynamic Fuzzy General Grey Cognitive Maps (DFGGCM) is obtained by exploiting the general grey number as the basic element. The DFGGCM also has an environment model to reflect the dynamic environment. In DFGGCM, the output of the environment model should be in the form of the GGN, and there is also no limit on the input; it can be real numbers, interval grey numbers, or general grey numbers.

Algorithm 3 Dynamic Fuzzy General Grey Cognitive Map (DFGGCM)

Data: initial vector in the form of GGN $\otimes\vec{C}_0$, initial weights in the form of GGN $\otimes W_0$, environment variables in the form of GGN $\otimes V$, environment model $EM(\otimes)$
Result: output vector $\vec{C}_m$ (GGN)
$t = 0$;
while $t < m$ **do**

$$\otimes W(t) = EM(\otimes V, \otimes W(t-1), \otimes C(t-1), t);$$
$$\otimes\vec{C}_{t+1} = S\left(\otimes\vec{C}_t \cdot \otimes W(t)\right) = S\left(\otimes\vec{C}'_{t+1}\right) = \left(S\left(\otimes C_t^{[1]'}\right), S\left(\otimes C_t^{[2]'}\right), \ldots, S\left(\otimes C_t^{[n]'}\right)\right);$$
$$t = t + 1;$$

$S\left(\otimes C_t^{[i]'}\right)$ is calculated according to Eq. (26), (27), (28), or Eq. (29), (30), (31);

end

Algorithm 3 are pseudocodes of the DFGGCM. DFGGCM also calculates the weights in the form of the GGN at the beginning of each iteration. Then the inner FGGCM does the iteration using the calculated weights.

With the DFGCM and DFGGCM, the dynamic and uncertain system can be modeled at the same time. The following part of this paper will illustrate the applications of DFGCM and DFGGCM through an industrial process control system as an example.

V. EXPERIMENTS DESIGN

In this section, two series of experiments were designed. The first series of experiments were conducted under RBEMs, and the second was under PREMs. Firstly, we will introduce the background of the experiment in Section V-A and then the experimental setting under the RBEM (Section V-B) and PREM (Section V-C) are shown. The relevant data are collected from [6]–[8], [20], [51]–[53].

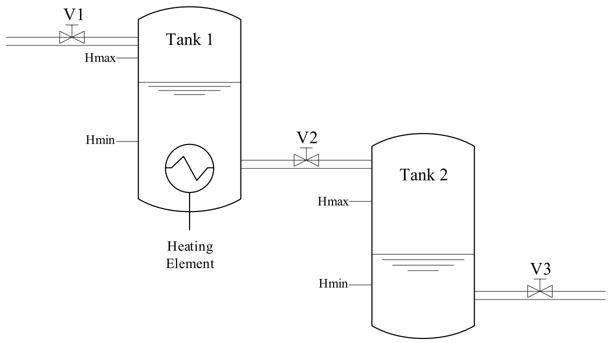

FIGURE 5. The industrial process control system [20].

A. EXPERIMENTS BACKGROUND

In 1998, Stylios and Groumpos [51], [52] discussed the industrial process control problem by FCM firstly. After that, the industrial process control problem was exploited to validate algorithms relevant to FCM. In 2006, two unsupervised learning methods for FCM were validated by this control system [54]. In 2011, another FCM's training algorithm, the Extended Great Deluge Algorithm (EGDA) is also validated by this system [55]. The FGCM was exploited to analyze this problem in 2014 [9]. And recently, the FGGCM was also validated by this system [20]. Because it was widely applied in the field of FCM, we also use the industrial control problem as the background to validate the proposed DFGCM and DFGGCM.

Fig. 5 shows how the industrial process control system works. The system contains 2 tanks, 3 valves, and a heating element in tank 1 [40]. The aim is to keep the height of the liquid and temperature of the liquid in both 2 tanks at a predetermined level by control the 3 valves open and close, and the power of the heating element.

TABLE 1. The FCM nodes' definitions of the industrial process control system.

Node	Definition
C_1	The liquid height in tank 1
C_2	The liquid height in tank 2
C_3	The valve 1's state
C_4	The valve 2's state
C_5	The valve 3's state
C_6	The liquid temperature in tank 1
C_7	The liquid temperature in tank 2
C_8	The heating element's state.

Stylios and Groumpos [51] have constructed the FCM for the industrial control problem, which contains 8 nodes as shown in table 1. The node values were normalized linearly to the range of [0, 1] before processing by the FCM.

The weight matrix is shown in Eq. (34), as shown at the bottom of the next page.

The initial vector is

$$C_{initial} = \begin{pmatrix}0.2 & .01 & 0.55 & 0.58 & 0 & 0.05 & 0.2 & 0.1\end{pmatrix}. \quad (35)$$

To show the reasoning mechanism of DFGCM and DFGGCM clearly, assume that the voltage of the heating element (V) will change when the system runs. Influenced

TABLE 2. The weights in IGN and GGN forms.

weights	IGN	GGN
$\otimes w_{13}$	$[0.01, 0.41]$	$0.21_{(0.20)}$
$\otimes w_{14}$	$[0.28, 0.48]$	$0.38_{(0.10)}$
$\otimes w_{24}$	$[0.60, 0.80]$	$0.70_{(0.10)}$
$\otimes w_{25}$	$[0.50, 0.70]$	$0.60_{(0.10)}$
$\otimes w_{31}$	$[0.51, 1]$	$0.755_{(0.245)}$
$\otimes w_{41}$	$[-0.90, -0.70]$	$-0.80_{(0.10)}$
$\otimes w_{42}$	$[0.70, 0.90]$	$0.8_{(0.10)}$
$\otimes w_{48}$	$[-0.16, 0.34]$	$0.09_{(0.25)}$
$\otimes w_{52}$	$[-0.52, -0.32]$	$-0.42_{(0.1)}$
$\otimes w_{67}$	$[0.5, 0.7]$	$0.6_{(0.1)}$
$\otimes w_{73}$	$[0.3, 0.5]$	$0.4_{(0.1)}$
$\otimes w_{76}$	$[0.43, 0.63]$	$0.53_{(0.1)}$
$\otimes w_{83}$	$[0.05, 0.55]$	$0.3_{(0.25)}$

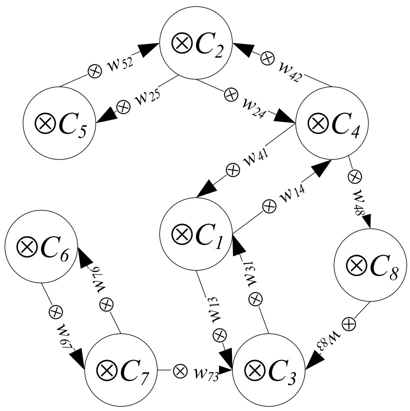

FIGURE 6. The topological graph of FGCM for the industrial process problem [20].

by the changing voltage, the effect of the heating element to valve 2 (w_{84}) will change in a kind of pattern. The environment model is constructed to describe such a pattern. This paper uses RBEM and PREM to show how DFGCM and DFGGCM work.

To test the DFGCM's and DFGGCM's compatibility with DFCM, a comparison between DFCM, DFGCM, and DFGGCM should be done. The structure of the DFCM, DFGCM, and DFGGCM is the same as each other. The initial weights of DFCM are the same as the weights of Stylios' FCM (Eq. (34)), and the initial weights of DFCM and DFGGCM are referred to as those in the FGCM built by Salmeron [53]. The weights are shown in Table 2. At the same time, they are transferred to the form of GGN, also listed in Table 2. And the basic graph of the FGCM is shown in Fig.6.

B. RULE-BASED ENVIRONMENT MODEL

This part describes the experimental settings under the RBEM. Firstly, the RBEM of DFCM is given in Section V-B1, and then the counterpart of the DFGCM and DFGGCM is shown in Section V-B2.

1) RULE-BASED ENVIRONMENT MODEL OF THE DYNAMIC FUZZY COGNITIVE MAP

The RBEM of DFCM is assumed to be

$$w_{84} = \begin{cases} 0, & 0.0 < V \leq 0.2 \\ 0.15, & 0.2 < V \leq 0.7 \\ 0.3, & 0.7 < V \leq 1.2 \\ 0.45, & 1.2 < V \leq 1.5 \\ 0, & 1.5 < V \end{cases} \tag{36}$$

That is to say, with the increase of the voltage, the influence of the heating element on the valve will increase step by step. However, when the voltage increases to a specific limit, the heating element will fail and not exert any influence on the valve.

Suppose that the time step represented by each iteration of DFCM is t, and the voltage sequence in a certain period of time is as follow 6 stages.

(1) In the first stage, there is no effect caused by insufficient voltage. In this stage,

$$V_1 = [0, 0.1, 0.15, 0.18, 0.19, 0.19, 0.19],$$

and according to Eq. (36), $w_{84} = 0$;

(2) The second stage is the low impact stage caused by insufficient voltage. In this stage,

$$V_2 = [0.21, 0.3, 0.4, 0.45, 0.5, 0.6, 0.65],$$

and according to Eq. (36), $w_{84} = 0.15$;

(3) In the third stage, the voltage is normal,

$$V_3 = [0.71, 0.75, 0.78, 0.95, 1.1, 1.15, 1.2],$$

and $w_{84} = 0.3$;

(4) The fourth stage is the high influence stage caused by high voltage. The voltage at each step is

$$V_4 = [1.21, 1.25, 1.3, 1.35, 1.4, 1.45, 1.5],$$

and $w_{84} = 0.45$;

$$W = \begin{pmatrix} 0 & 0 & 0.21 & 0.38 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.70 & 0.60 & 0 & 0 & 0 \\ 0.76 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.80 & 0.80 & 0 & 0 & 0 & 0 & 0 & 0.09 \\ 0 & -0.42 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0.4 & 0 & 0 & 0.53 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{34}$$

(5) In the fifth stage, the voltage is too high, and the heating element does not have any influence on the temperature.

$$V_5 = [1.51, 1.6, 1.7, 1.8, 1.7, 1.6, 1.7],$$

thus $w_{84} = 0$.

(6) The sixth stage is the dynamic influence stage caused by the unstable voltage. In this stage,

$$V_6 = [1.7, 1.2, 0.8, 0.2, 0.1, 1.4, 1.6, 1.3, 1.7, 0.5, 0.25, 0.15, 0.9, 1],$$

w_{84} can be calculated according to Eq. (36).

2) RULE-BASED ENVIRONMENT MODEL OF THE DYNAMIC FUZZY GREY COGNITIVE MAP AND DYNAMIC FUZZY GENERAL GREY COGNITIVE MAP

Assuming that the measured voltage value is an interval, then the DFGCM and DFGGCM can be used for modeling. The DFGCM and DFGGCM share a same RBEM in the form of GGN, as shown in Eq. (37) and Eq. (38).

$$\hat{w}_{84} = \begin{cases} 0, & 0.0 < \hat{V} \leq 0.2 \\ 0.15, & 0.2 < \hat{V} \leq 0.7 \\ 0.3, & 0.7 < \hat{V} \leq 1.2 \\ 0.45, & 1.2 < \hat{V} \leq 1.5 \\ 0, & 1.5 < \hat{V} \end{cases} \quad (37)$$

$$(w_{84})^{\circ} = \max((w_{84})^{\circ}, V^{\circ}) = \max(0.25, V^{\circ}) \quad (38)$$

During the operation process, the change of the voltage is also in six stages.

(1) In the first stage, there is no influence caused by insufficient voltage:

$$\otimes V_1 = ([0.0, 0.0], [0.0, 0.2], [0.1, 0.2], [0.09, 0.27], [0.09, 0.29], [0.04, 0.34], [0.15, 0.23])\,.$$

Transfer it into the form of GGN:

$$\otimes V_1 = \left(0.0_{(0.0)}, 0.1_{(0.1)}, 0.15_{(0.05)}, 0.18_{(0.09)}, 0.19_{(0.1)}, 0.19_{(0.15)}, 0.19_{(0.04)}\right).$$

In this stage, $V^{\circ} < (w_{84})^{\circ}$ is always exist, so $\otimes w_{84} = 0_{(0.25)}$. (2) The second stage is the low impact stage caused by insufficient voltage:

$$\otimes V_2 = ([0.11, 0.31], [0.2, 0.4], [0.3, 0.5], [0.15, 0.75], [0.2, 0.8], [0.3, 0.9], [0.35, 0.95])$$

In the form of GGN:

$$\otimes V_2 = \left(0.21_{(0.1)}, 0.3_{(0.1)}, 0.4_{(0.1)}, 0.45_{(0.3)}, 0.5_{(0.3)}, 0.6_{(0.3)}, 0.65_{(0.3)}\right)$$

In the first half of this stage, $V^{\circ} < (w_{84})^{\circ}$, so $\otimes w_{84} = 0.15_{(0.25)}$. And in the second half, $V^{\circ} > (w_{84})^{\circ}$, so $\otimes w_{84} = 0.15_{(0.3)}$.

(3) In the third stage, the voltage is normal:

$$\otimes V_3 = ([0.61, 0.81], [0.65, 0.85], [0.68, 0.88], [0.65, 1.25], [0.8, 1.4], [0.85, 1.45], [0.9, 1.5])$$

In the form of GGN:

$$\otimes V_3 = \left(0.71_{(0.1)}, 0.75_{(0.1)}, 0.78_{(0.1)}, 0.95_{(0.3)}, 1.1_{(0.3)}, 1.15_{(0.3)}, 1.2_{(0.3)}\right)$$

In the first half of this stage, $V^{\circ} < (w_{84})^{\circ}$, so $\otimes w_{84} = 0.3_{(0.25)}$. And in the second half, $V^{\circ} > (w_{84})^{\circ}$, so $\otimes w_{84} = 0.3_{(0.3)}$

(4) The fourth stage is the high effect stage caused by high voltage.

$$\otimes V_4 = ([1.11, 1.31], [1.15, 1.35], [1.2, 1.4], [1.05, 1.65], [1.1, 1.7], [1.15, 1.75], [1.2, 1.8])$$

In the form of GGN:

$$\otimes V_4 = \left(1.21_{(0.1)}, 1.25_{(0.1)}, 1.3_{(0.1)}, 1.35_{(0.3)}, 1.4_{(0.3)}, 1.45_{(0.3)}, 1.5_{(0.3)}\right)$$

In the first half of this stage, $V^{\circ} < (w_{84})^{\circ}$, so $\otimes w_{84} = 0.45_{(0.25)}$. And in the second half, there is $V^{\circ} > (w_{84})^{\circ}$, so $\otimes w_{84} = 0.45_{(0.3)}$

(5) The fifth stage is the no effect stage caused by high voltage.

$$\otimes V_5 = ([1.41, 1.61], [1.5, 1.7], [1.6, 1.8], [1.5, 2.0], [1.4, 2.0], [1.3, 1.9], [1.4, 2.0])$$

In the form of GGN:

$$\otimes V_5 = \left(1.51_{(0.1)}, 1.6_{(0.1)}, 1.7_{(0.1)}, 1.75_{(0.25)}, 1.7_{(0.3)}, 1.6_{(0.3)}, 1.7_{(0.3)}\right)$$

In the first half of this stage, $V^{\circ} < (w_{84})^{\circ}$, so $\otimes w_{84} = 0_{(0.25)}$. And in the second half, $V^{\circ} \geq (w_{84})^{\circ}$, so $\otimes w_{84} = 0_{(0.3)}$
(6) The sixth stage is the dynamic influence caused by voltage instability.

$$\otimes V_6 = ([1.6, 1.8], [1.1, 1.3], [0.3, 1.3], [0.1, 0.3], [0.05, 0.15], [1.1, 1.7], [1.35, 1.85], [0.9, 1.7], [1.65, 1.75], [0.0, 1.0], [0.05, 0.45], [0.1, 0.2], [0.8, 1.0], [0.9, 1.1])$$

In the form of GGN:

$$\otimes V_6 = \left(1.7_{(0.1)}, 1.2_{(0.1)}, 0.8_{(0.5)}, 0.2_{(0.1)}, 0.1_{(0.05)}, 1.4_{(0.3)}, 1.6_{(0.25)}, 1.3_{(0.4)}, 1.7_{(0.05)}, 0.5_{(0.5)}, 0.25_{(0.2)}, 0.15_{(0.05)}, 0.9_{(0.1)}, 1.0_{(0.1)}\right)$$

In this stage, $\otimes w_{84}$ is calculated by Eq. (37) and Eq. (38).

C. PROPORTIONAL REGULATION ENVIRONMENT MODEL

In this part, the PREM for the DFCM, DFGCM, and DFGGCM is described.

1) PROPORTIONAL REGULATION ENVIRONMENT MODEL OF THE DYNAMIC FUZZY COGNITIVE MAP

To build a PREM for DFCM, the $\alpha(t)$ and w_{84} is set as follows:

$$\alpha(t) = \begin{cases} -0.1 & 0 \leq V(t) \leq 0.7 \\ 0 & 0.7 < V(t) \leq 1.2 \\ 0.1 & 1.2 < V(t) \leq 1.5 \\ -0.1 & 1.5 < V(t) \end{cases} \quad (39)$$

$$w_{84}(t+1) = \begin{cases} 0 & w_{84}(t+1) < 0 \\ w_{84}(t)\,(1+\alpha(t)V(t)) & 0 \leq w_{84}(t+1) \leq 0.45 \\ 0.45 & w_{84}(t+1) > 0.45 \end{cases} \quad (40)$$

2) PROPORTIONAL REGULATION ENVIRONMENT MODEL OF THE DYNAMIC FUZZY GREY COGNITIVE MAP AND THE DYNAMIC FUZZY GENERAL GREY COGNITIVE MAP

Now consider the PREM for DFGCM and the DFGGCM. Because DFGCM and DFGGCM use the grey number as their basic element, Eq. (33) should be transferred to the form of grey number:

$$\begin{aligned} \otimes w_{ij}(t+1) &= f\left\{\otimes w_{ij}(t), \otimes V(t), \otimes\alpha(t)\right\} \\ &= \otimes w_{ij}(t)\,(1+\otimes\alpha(t)\otimes V(t)) \end{aligned} \quad (41)$$

If the node values get out of their predetermined range in the process of reasoning, they will be truncated. The predetermined boundary will be used instead of the calculated values if it locates out of the predetermined range.

If any variable in the formula is a general grey number, only DFGGCM can be used for reasoning. If all variables are interval grey numbers, then either DFGCM or DFGGCM can be used. In this part, to explain the usage of DFGCM and DFGGCM at the same time, the above variables are set as interval grey numbers, so the PREM for DFGCM is expressed as Eq. (42).

$$\alpha(t) = \begin{cases} [-0.15, -0.05] & 0 \leq \hat{V}(t) \leq 0.7 \\ 0 & 0.7 < \hat{V}(t) \leq 1.2 \\ [0.05, 0.15] & 1.2 < \hat{V}(t) \leq 1.5 \\ [-0.15, -0.05] & 1.5 < \hat{V}(t) \end{cases} \quad (42)$$

The PREM for DFGGCM is shown in Eq. (43) and Eq. (44).

$$\alpha(t) = \begin{cases} -0.1_{(0.05)} & 0 \leq \hat{V}(t) \leq 0.7 \\ 0 & 0.7 < \hat{V}(t) \leq 1.2 \\ 0.1_{(0.05)} & 1.2 < \hat{V}(t) \leq 1.5 \\ -0.1_{(0.05)} & 1.5 < \hat{V}(t) \end{cases} \quad (43)$$

$$\otimes w_{84}(t+1) = \otimes w_{84}(t)\,(1+\otimes\alpha(t)\otimes V(t)) \quad (44)$$

VI. RESULTS

The results of the experiments above are shown in this part.

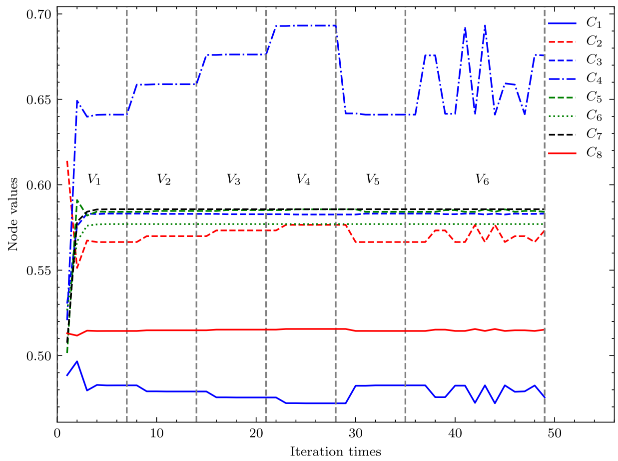

FIGURE 7. The results of GDFCM.

A. THE RESULTS OF RULE-BASED ENVIRONMENT MODEL

The results of GDFCM, DFGCM, and DFGGCM under the rule-based environment are shown in this part.

1) THE RESULTS OF GENERAL DYNAMIC FUZZY COGNITIVE MAPS

Using the environment model in Section V-B1, initial vector was Eq. (35), the initial weights were those in Eq. (34), ran the GDFCM (Algorithm 1). The results of GDFCM are shown in Fig. 7.

With the change of voltage value in each stage, the related nodes in the GDFCM also change. Among them, the values of C_1, C_2, C_4, and C_8 are affected most saliently by voltage variation due to the value of w_{84} changes according to the voltage. The node C_4 is directly affected by it, which has the most significant change and reaches the stable value the fastest, followed by the other nodes that are indirectly affected by w_{84}. Also, because the value of w_{84} is fixed in each phase in $V_1 \sim V_5$, the affected nodes reach a new stable state. And the whole GDFCM reaches a stable state, especially in the phase of V_3, the w_{84} is equal to the value in [51]. That is, the system in the phase of V_3 reaches the desired status. On the other hand, at the V_6 stage, the voltage is dynamic, so w_{84} changes dynamically, leading to the value change of corresponding nodes, which means that the system is unstable under the dynamic voltage.

2) THE RESULTS OF DYNAMIC FUZZY GREY COGNITIVE MAPS

Using the environment model in Section V-B2, initial vector was also Eq. (35), the initial weights were in the Table 2, ran the DFGCM (Algorithm 2). The results of DFGCM is shown in Fig. 8.

The kernel of the DFGCM's results changes in the same way as the GDFCM, while the greyness degree of the DFGCM's results changes at each stage. C_1°, C_2°, C_4°, and C_8° changes more obvious. In the middle of the first 5 phases, the greyness degree becomes larger because $(w_{84})^\circ$ has

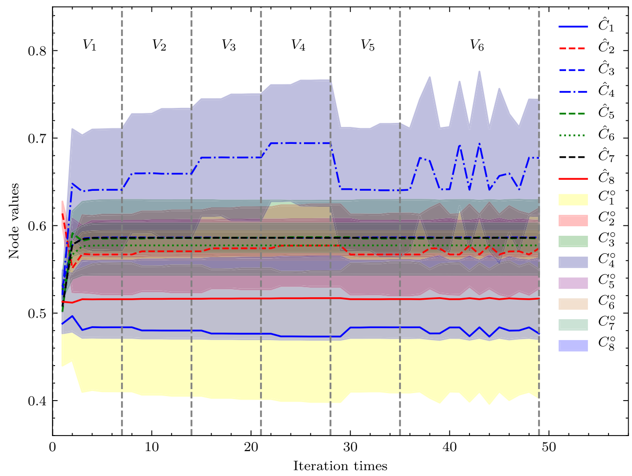

FIGURE 8. The results of DFGCM.

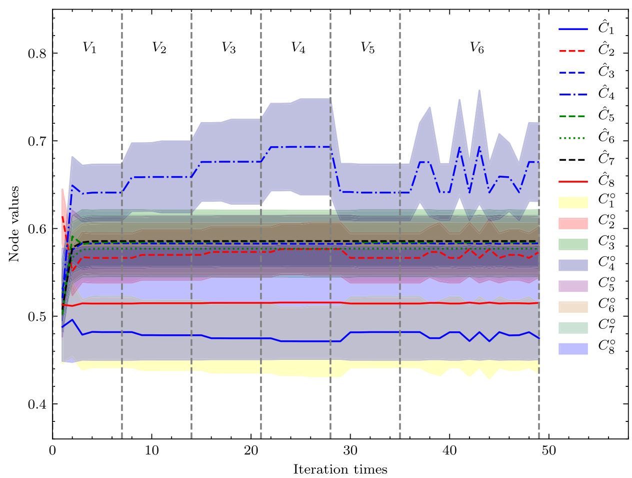

FIGURE 9. The results of DFGGCM.

changed simultaneously in the middle of these phases as designed in Section V-B2. The greyness is shown as the range of the value of each phase. The kernel of each node is shown in the middle of the range where the node values are most likely located. In the DFGCM model, the system is modeled with the greyness caused by the uncertain observations of the voltage and the DFGCM model.

3) THE RESULTS OF DYNAMIC FUZZY GENERAL GREY COGNITIVE MAPS

Using the environment model in Section V-B2, the initial vector was shown in Eq. (35), the initial weights were the same values shown in Table 2, ran the DFGGCM (Algorithm 3). The results of DFGGCM are summarized in Fig. 9.

The kernel and greyness degree of DFGGCM's results shows the same pattern as the DFGCM's, but there are some discrepancies between them. Compared to the results in Fig. 8 and Fig. 9, in the process of reasoning, the greyness of DFGGCM is slightly smaller than that of DFGCM. This is because that the FGGCM can model the system more precisely than FGCM [20], the greyness can be kept rather than amplified during the inference process.

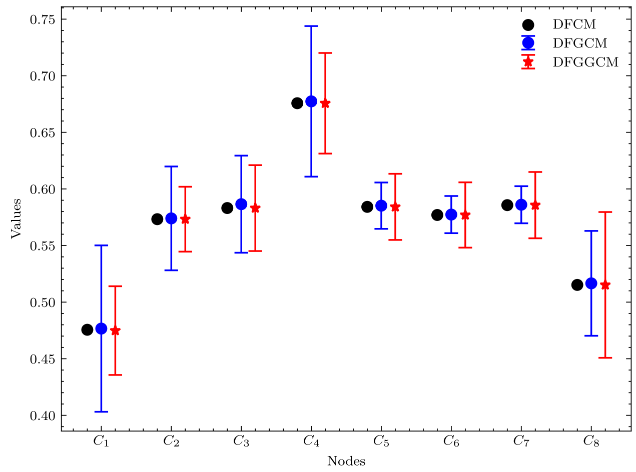

FIGURE 10. The final node values of GDFCM, DFGCM, and DFGGCM.

4) THE COMPATIBILITY UNDER THE RULE-BASED ENVIRONMENT MODEL

Just as FGGCM can be compatible with FCM and FGCM, DFGGCM can also be compatible with GDFCM and DFGCM. To illustrate the compatibility, the final reasoning states of GDFCM, DFGCM, and DFGGCM are shown in Fig. 10.

The kernels of DFGGCM and DFGCM's nodes are almost equal to that of DFCM's, and the greyness degree of DFGCM's nodes is larger a bit than that of DFGGCM's nodes, which support the results in the Section VI-A3.

B. THE RESULTS UNDER THE PROPORTIONAL REGULATION ENVIRONMENT MODEL

The results of GDFCM, DFGCM, and DFGGCM under the PREM are shown in this part.

1) THE RESULTS OF GENERAL DYNAMIC FUZZY COGNITIVE MAPS

As shown in Fig. 11, the trend of the values of the nodes of GDFCM under the proportional regulation environment model is consistent with that of GDFCM under the Rule-Based environment model. The trend of nodes values of GDFCM under the PREM are smoother than that of the GDFCM under the RBEM, especially in the V_6 stage. The GDFCM under the RBEM is affected by environment variables directly, and the values of related nodes fluctuate sharply, while the values of the nodes of GDFCM under the PREM have little fluctuation.

The weight based on the PREM is greatly affected by the weight of the previous time and has specific inertia, which does not mean that the GDFCM under PREM has stronger robustness, but the difference is caused by different characteristics of different systems. In short, the GDFCM under PREM is more suitable for modeling the inertial and continuous

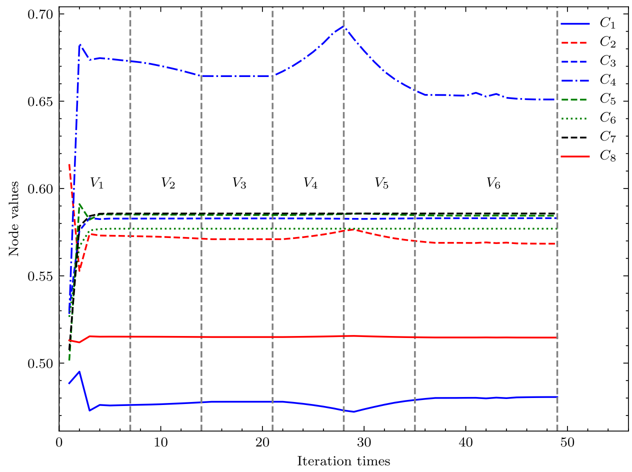

FIGURE 11. The results of GDFCM under the proportional regulation environment model.

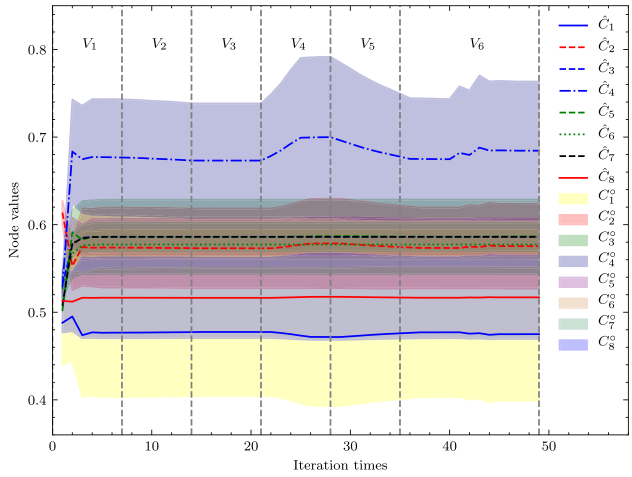

FIGURE 12. The results of DFGCM under PREM.

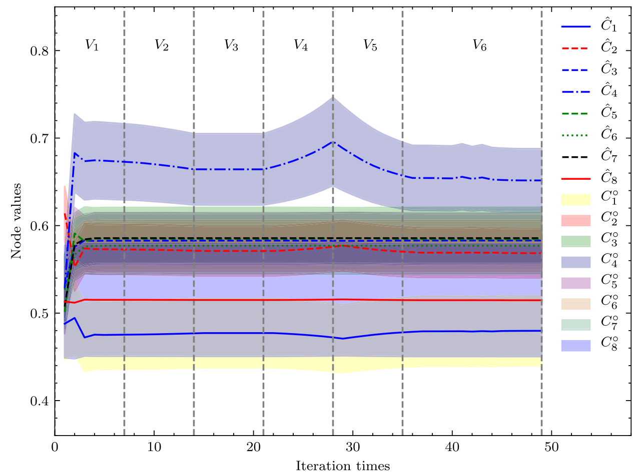

FIGURE 13. The results of DFGGCM under PREM.

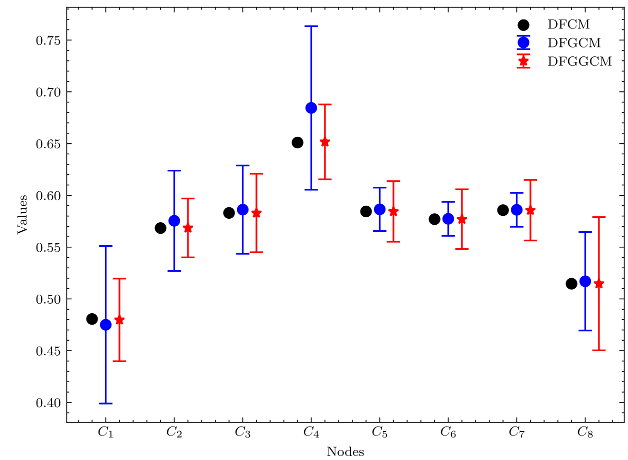

FIGURE 14. The final node values of GDFCM, DFGCM, and DFGGCM.

system. Simultaneously, the GDFCM under RBEM is better for modeling the dynamic system in which rules can describe the causal relationship.

2) THE RESULTS OF DYNAMIC FUZZY GREY COGNITIVE MAPS

Compared with the results of the DFGCM under RBEM (Fig. 8), the trends of the node values of DFGCM under PREM (Fig. 12) are almost the same as that of DFGCM under RBEM. As the relationship between GDFCM under RBEM and GDFCM under PREM, the trend of node values of DFGCM under the PREM are smoother than that of the DFGCM under the RBEM, especially in the V_6 stage, which reflects the system modeled by the DFGCM are more robust. The greyness of the nodes also means that the system are modeled in a way that can deal with uncertain data.

3) THE RESULTS OF DYNAMIC FUZZY GENERAL GREY COGNITIVE MAPS

The results of the DFGGCM under PREM (Fig. 13) show the same trend with the DFGGCM under RBEM (Fig. 9), but the value of the node of DFGGCM under PREM fluctuates more slightly than DFGGCM under RBEM. As a result, the overall greyness degree of DFGGCM under PREM is smaller than that of the DFGCM under PREM.

4) THE COMPATIBILITY UNDER THE PROPORTIONAL REGULATION ENVIRONMENT MODEL

Similarly, the final results of GDFCM, DFGCM, and DFGGCM based on PREM are shown in Fig. 14. The kernels of DFGGCM and DFGCM's nodes are almost equal with that of GDFCM's, and the greyness degree of DFGCM's nodes values are appropriately comparable to that of DFGGCM.

VII. DISCUSSION

In this study, the DFGCM and DFGGCM are proposed as the dynamic extensions of FGCM. To verify these two models' availability and compatibility, we used an industrial process control problem as the background, constructed a GDFCM, DFGCM, and DFGGCM under RBEM and PREM, compared the results of each other to show the different characteristics between different models. The results show the both DFGCM and DFGGCM can be effectively used as the extensions of the

FGCM, when the approachable data are multiple intervals, one should use the DFGGCM, and when the approachable data are intervals, one can use both DFGGCM and DFGCM.

According to the results of GDFCM, DFGCM, and DFGGCM, no matter under RBEM (Fig 7, 8, 9) or PREM (Fig 11, 12, 13), the kernel of the DFGCM's and DFGGCM's nodes values showed the same trend with the nodes' values of the DFCM. The kernels of the node values of the DFGCM and the DFGGCM are approximately equal to the nodes' values of the DFGCM (Fig. 10, Fig. 14), which demonstrate the compatibility of DFGCM and DFGGCM to the GDFCM, and the compatibility of DFGGCM to the DFGCM.

The dynamic modeling ability can also be proved by the experiments above. Compared the results of DFCM, DFGCM and DFGGCM under different environment models, i.e., compared Fig. 7 with Fig. 11, compared Fig. 8 with Fig. 12, and compared Fig. 9 with Fig. 13, the trend of node value (GDFCM) or kernel of node value (DFGCM and DFGGCM) under RBEM show a more evident fluctuation than that under PREM. Because different environment models reflect the different dynamic properties of the system, the PREM show a system that has inertia, and the RBEM show a system not influenced by the last state, reflecting the node values or the kernel of node values, they show different degrees of fluctuation.

The FGCM and FGGCM strength over FCM is that they can deal with the uncertainty of the data. In DFGCM and DFGGCM, this characteristic is also conserved. From Fig. 8 and Fig. 9, especially in the $V_2 \sim V_5$ stage, at the medium of each stage, the node values' degrees of greyness expanded slightly. The greyness of voltage increased from 0.1 to 0.3 at the medium of each stage, as shown in Section V-B2. The voltage's degree of greyness influenced the weights of DFGCM and DFGGCM, so the node values' degree of greyness changed correspondingly. However, such influence can be weakened by the PREM. As shown in Fig. 12 and Fig. 13, the change of greyness still exists, but not obvious, which is also due to the inertia of the PREM.

Finally, compared Fig. 8 with Fig. 9, and compared Fig. 12 with Fig. 13, in most cases, the greyness of nodes of DFGGCM is slightly smaller than that of DFGCM, which is consistent with the data provided in [20]. Like the relationship between FGCM and FGGCM, the smaller greyness of DFGGCM is caused by the different calculation processes of greyness between DFGCM and DFGGCM. In general, the greyness calculation method of DFGGCM is explicit and direct, while the greyness calculation process of DFGCM is implicit and indirect.

To summarize, the results support the following characteristics of DFGCM and DFGGCM:

(1) Both DFGCM and DFGGCM are compatible with the DFCM, and DFGGCM is compatible with the DFGCM.

(2) DFGCM and DFGGCM have the same topological structure: both of them are constituted by a basic FGCM (FGGCM) model and an environment model.

(3) The introduction of the environment model makes the DFGCM and DFGGCM can deal with the dynamic characteristics of the system.

(4) The DFGCM and DFGGCM's uncertainty modeling abilities are inherited from the FGCM and FGGCM.

(5) The DFGCM can deal with the uncertainty data in the form of IGN and fuzzy number, which cannot deal with the data in the form of GGN. The DFGGCM can deal with the uncertainty data in the form of GGN, IGN, and fuzzy number.

(6) The greyness of nodes of DFGGCM is slightly smaller than that of DFGCM, which is consistent with the FGCM and FGGCM.

The proposals of DFGCM and DFGGCM fill in the blank of dynamic research of FGCM and make the FGCM be able to deal with the system's dynamic property. The DFGCM and DFGGCM can express a more complex system in which the causal link changes according to the environment or internal factors so that they can be used in a broader range of applications.

VIII. CONCLUSION AND FUTURE WORK

The FGCM and FGGCM are extensions of FCM to improve the uncertainty modeling ability, but the dynamic modeling ability is the weakness of the FGCM and FGGCM. In this paper, the gap between the uncertainty modeling and dynamic modeling of FGCM was filled by the proposal of DFGCM and DFGGCM.

Firstly, a general definition of DFCM was given by the proposed Environmental Model. Secondly, the DFGCM and DFGGCM were proposed according to the mechanism of the environmental model. The DFGCM and DFGGCM were validated with PREM and RBEM under the background of the industrial process control systems. Finally, the characteristics of DFGCM and DFGGCM were discussed according to the experiment results.

To summarize, the main findings are as follows:

- A general definition for DFCM was given.
- The DFGCM and DFGGCM were proposed and validated by the industrial control problem.
- The compatibility of DFGCM, DFGGCM, and DFCM are proven in experiments.

The DFGCM can deal with the uncertainty data in the interval values under the dynamic environment. The DFGGCM can deal with the uncertain data in interval grey numbers and the multiple interval values under a dynamic environment. Thus, when using FCM-relevant algorithms to model the highly dynamic systems, if all the data are crisp, use GDFCM. If there are interval values, then both DFGCM and DFGGCM can be exploited; if encountering the multiple interval values, only DFGGCM can be used.

The future work will focus on the construction process and learning algorithms for proposed models. The construction of DFGCM and DFGGCM can be daunting, especially when the system is large and the causal link between nodes cannot be determined. The environment model can also be completed to express the high dynamic of the system. One acceptable method to construct the DFGCM and DFGGCM is to use some heuristic learning algorithms, such as Particle Swarm Optimization (PSO) and Evolution Algorithm (EA), to learn their weights and corresponding environment models from data, which will be the future work of this issue.

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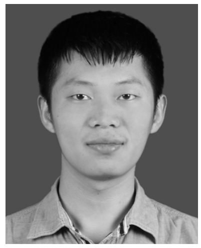

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