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Probabilistic Intuitionistic Fuzzy Decision Making Algorithms

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ABSTRACT In this paper, the probabilistic version of intuitionistic fuzzy decision making methods is introduced to show the duality present within probabilistic distance. Here, two types of methods, namely, probabilistic intuitionistic fuzzy TOPSIS (PI-TOPSIS) algorithm and probabilistic intuitionistic fuzzy face identification (PIFI) algorithm are proposed. The PI-TOPSIS is utilizing probabilistic distance as the separation measure; for its justification rankings and reduced information loss of the multi criteria decision making problems are compared. Our other proposed decision making method called PIFI algorithm handles Face Identification Problem. It is exemplified by better benchmark indexes of PIFI in comparison to support vector machine, naive bais classifier, fuzzy support vector machine algorithms, that, the probabilistic distance can be used as a similarity measure. The two well-known feature extraction techniques, called Local binary pattern (LBP) and Angular radial transformation (ART) are employed to extract the features in the face images. Further, it is concluded from the experimental findings, that, the proposed algorithms are adaptive in nature.

INDEX TERMS Supplier selection, intuitionistic fuzzy set, TOPSIS, face identification.

I. INTRODUCTION

Atanassov intuitionistic fuzzy set (IFS) [1] deals with uncertain, imprecise and vague information more accurately in comparison to fuzzy set [2]. In IFS uncertain information is elaborately explained, because it contains an additional component called hesitancy value beside membership non-membership values. Vague Set proposed by Gau and Buehrer in [3] is another extension of fuzzy set. Bustince and Burillo recapitulated in [4] that the definition of vague set coincides with IFS. Therefore, IFS is a generalized framework of modeling fuzziness. Moreover, the theory of intuitionistic fuzzy sets is widely applied in numerous fields such as modeling imprecision [5], pattern recognition [6], computational intelligence [7], medical diagnosis ([8], [9]), decision making ([10], [11]) and face identification [12].

The main aim of the scientific study is to show how decisions are actually made and how they can be made better. So undoubtedly decision making is a fundamental activity in the scientific study. There are two ways of decision making: (1) Decision making based on the ranking techniques; (2) Decision making not based on the ranking techniques. In a ranking technique distance of the alternatives from the left and right ideal alternatives are used for the determination of separation measure, and the measure computes dissimilarity between alternatives. The separation measure brings an ordering relation and the relation ranks the alternatives of the multi criteria decision making (MCDM) problem. In this paper a number of MCDM problems, for example supplier selection problem are dealt by the proposed PI-TOPSIS algorithm. The rankings obtained by us suggest that PEDM has resulted good separation measures. There are decision making problems which require selection of the best alternative from given alternatives. Such types of problems are not MCDM, because they don't require ranking of the alternatives in selection of the best one. Rather, the method of these non MCDM problems compute similarity of the input alternative with given alternatives and the alternative which gets highest similarity value is selected. The face identification method selects most similar image from a database image with reference image.

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Several matching methods for face and object identification systems are designed in literature [13] based on image similarity measurements. We have also proposed a PIFI algorithm in the paper for the face identification problem. The benchmark indexes of PIFI validate that a good similarity measure has resulted out of the PEDM.

A. MULTI CRITERIA DECISION MAKING PROBLEMS

Multi criteria decision making (MCDM) methods [14] provide a systematic quantitative approach of solving decision making problems in which multiple criteria are involved. The performance of these methods enhance with the improvement in the expertise of the decision makers. The various real life problems in the fields of operational research, industrial engineering, supplier selection, and management science etc are satisfactorily solved by MCDM methods proposed in the literature. The alternatives given in the problem are ranked by the ranking procedure of MCDM method and then best alternative is chosen from them. Some of the well-known ranking methods are technique for order preference by similarity to ideal solutions (TOPSIS) ([11], [15]), weighted sum model (WSM) [16], analytic hierarchy process (AHP) [17], elimination and choice translating reality (ELECTRE) [18], Grey relational analysis (GRA) [19] etc. Here every method is implemented in three steps as follows:

1) Determining relevant criteria and alternatives.
2) Attaching numerical measures which give relative importance to criteria and impacts of these criteria on alternatives.
3) Processing numerical values for ranking of the alternatives.

If uncertainty is missing, then sustainable suppliers can be easily selected by the help of classical MCDM methods (See [20], [21]). But there are situations where uncertainty and impreciseness appear during the best supplier selection. The uncertainty and impreciseness is observed in the problem because experts have different perception about significance levels of the criteria in the decision making or decisions are taken within a time constraint with limited information processing capacities, and lack of data knowledge [22]. Such situations are predominantly handled by fuzzy approach based MCDM methods [11].

The prominently employed fuzzy approach based MCDM method in the dealing of supplier selection problem (SSP) is IFS-TOPSIS, because it is several times verified that uncertainty and impreciseness in the problem is conveniently expressed by the help of IFS ([23]–[26]). The weighted IFS distance measures (WDM) are mostly utilized as separation measures in the proposal of IFS-TOPSIS method (See ([27]–[31]). The probabilistic Euclidean distance measure (PEDM) introduced in [32] is used in the paper to propose a novel probabilistic intuitionistic fuzzy TOPSIS (PI-TOPSIS) algorithm.

B. FACE IDENTIFICATION PROBLEM

Face is an important biometric identifier for recognizing humans [33]. In past several years, face identification have become pioneering and interesting research area because of its remarkable application in numerous domains. The application includes educational institutions, smart cards (passport, national ID), entertainment, driving license, law enforcement and many corporates. The face identification procedure keeps all digital images of faces as set of template images; this transformation of images preserves their unique identity. Then similarity of query image with the set of template images is computed for face identification ([34]–[37]). The query image is assigned the identity of that template image with which it obtains the highest similarity.

Several pattern classification methods are defined to deal with the face identification problem, but none of them is fullproof [22]. So, the tools based on fuzzy set theory are also exploited in the improvisation of face identification results [37]. The IFS based distance measures and similarity measures are applied in a face identification problem for the first time in [38] and the useful inferences are drawn regarding the accuracy and confidence of the recognition results. In [39], face recognition results are improved by rough set based similarity measures. An algorithm based on dice similarity measure is introduced in [12] along with detail comparisons. We have proposed a novel probabilistic intuitionistic fuzzy face identification (PIFI) algorithm using PEDM.

C. MOTIVATION AND CONTRIBUTION OF THE RESEARCH WORK

Intuitionistic fuzzy set (IFS) based distance measures are showing highly significant applications in the variety of fields such as pattern recognition ([6], [10], [40], [41]), image processing [42], decision making ([22], [23]), clustering ([32], [43]) because every measure has a capability of distinguishing diverse patterns. This capability of distance measure is keenly investigated on several type of problems ([44]–[49]). The performance of the distance measures are further improved when probability and fuzzy theory are exploited simultaneously in them. Recently an adaptive tool based on fuzzy and probability theory called Probabilistic Euclidean distance measure (PEDM) is introduced in [32] to improvise the clustering techniques. PEDM uses data driven probabilistic weights, so it is actually a probabilistic version of the IFS Euclidean distance measure.

The probability and fuzzy together models uncertainty in a better fashion. So, it has motivated us to introduce probabilistic version of intuitionistic fuzzy TOPSIS method and intuitionistic fuzzy face identification algorithm. The two algorithms introduced in the paper are Probabilistic Intuitionistic Fuzzy TOPSIS (PI-TOPSIS) algorithm and Probabilistic Intuitionistic fuzzy face identification (PIFI) algorithm. The proposed techniques give better results on the supplier selection problem (SSP) and face identification problem (FIP) in comparison to their IFS counterparts. The two algorithms are briefly described as follows:

1) **Probabilistic Intuitionistic Fuzzy TOPSIS Algorithm:** In this proposed TOPSIS algorithm, PEDM works as a separation measure. The weights p_{ij}, q_{ij} and ρ_{ij} used in the PEDM are computed directly from the dataset using values of the alternatives provided in the data. The proposed PI-TOPSIS is an adaptive algorithm because of its separation measure. In order to verify the adaptiveness property in the proposed TOPSIS algorithm, we have solved certain well-known MCDM problems such as supplier selection problem with the help of the method. As rankings obtained by our method matches with the already evaluated rankings on these problems, hence adaptiveness claim of the method gets verified.
2) **Probabilistic Intuitionistic Fuzzy Face Identification Algorithm:** In the paper, the second proposal is a novel probabilistic intuitionistic fuzzy face identification (PIFI) algorithm. Here, PEDM is used as a similarity measure, that is, it is a structural classifier measuring the similarity between important points. The eyes, nose and different angles of facial components are some of the important points for face identification [50]. The degree of importance of each facial points is defined by weights p_{ij}, q_{ij} and ρ_{ij} corresponding to its membership value, non-membership value and hesitancy value respectively. The two feature extraction techniques utilized in the paper are [51]: Local binary pattern (LBP) and Angular radial transformation (ART). The experimentation is carried out on two face datasets, namely, ORL face dataset and Yale face dataset.

The rest of the paper is organized as follows: In section II, the preliminaries related to the work are discussed. Section III proposes Probabilistic Intuitionistic Fuzzy TOPSIS (PI-TOPSIS) algorithm. In section IV, the proposed method is explored on the supplier selection problem. Section V discusses about the proposed probabilistic intuitionistic fuzzy face identification (PIFI) algorithm. In section VI, the proposed algorithm is implemented on ORL and yale face dataset. Finally, the conclusion and future work is stated in section VII.

II. PRELIMINARIES

In this section, basic concepts related to IFS and its distance measure are reviewed.

A. INTUITIONISTIC FUZZY SETS

In fuzzy set, only membership function $\mu(x), x \in X$ is used for completely defining the set, whereas intuitionistic fuzzy set (IFS) [1] is generated by membership function $\mu(x)$ as well as non-membership function $\nu(x)$. An intuitionistic fuzzy set A in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ is of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{1}$$

Here μ_A: $X \to [0,1]$ and ν_A: $X \to [0,1]$ simultaneously assigns membership value and non-membership value respectively to each element $x \in X$ with respect to A, if

$$0 \le \mu_A(x) + \nu_A(x) \le 1. \tag{2}$$

If $\nu_A(x) = 1 - \mu_A(x)$ for x in X, then set A reduces to fuzzy set.

In an intuitionistic fuzzy set, the hesitation degree $\pi_A(x)$ arises due to incomplete information about an element x of X in A, which is defined as:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), \quad \text{where } 0 \le \pi_A(x) \le 1. \tag{3}$$

In the continuation, the pair $\langle \mu_A(x), \nu_A(x)\rangle$ is called intuitionistic fuzzy value (IFV).

B. PROBABILISTIC EUCLIDEAN DISTANCE MEASURE (PEDM) BETWEEN IFSs

Lohani et. al proposed a weighted IFS distance measure, named as probabilistic Euclidean distance measure (PEDM) in [32]. It is an adaptive measure, where the probabilistic weights p_{ij}, q_{ij} and ρ_{ij} corresponding to the membership value, non-membership value and hesitancy value respectively are data driven. Let A_1, A_2 are IFSs in X having membership and non-membership values $\mu_{A_1}(.)$, $\mu_{A_2}(.)$ and $\nu_{A_1}(.)$, $\nu_{A_2}(.)$, respectively. PEDM is defined between IFSs A_1 and A_2 as follows:

$$d_{PE}(A_1, A_2) = \Big[\frac{1}{2n}\sum_{i=1}^{n} p_{12}(\mu_{A_1}(x_i) - \mu_{A_2}(x_i))^2 + q_{12}(\nu_{A_1}(x_i) - \nu_{A_2}(x_i))^2 + \rho_{12}(\pi_{A_1}(x_i) - \pi_{A_2}(x_i))^2\Big]^{1/2} \tag{4}$$

Here, corresponding to feature i, $p_{12} \in [p'(A_{12}), p''(A_{12})]$, $q_{12} \in [q'(A_{12}), q''(A_{12})]$ and $\rho(A_1, A_2)$ are the weights associated to membership, non-membership and hesitancy part respectively. The intervals $[p'(A_{12}), p''(A_{12})]$ and $[q'(A_{12}), q''(A_{12})]$ are the confidence intervals and $\rho(A_1, A_2)$ is the correlation coefficient between A_1 and A_2. The intervals are computed as follows:

$$\begin{aligned} p'(A_{12}) &= \max(p_{\min}(A_1), p_{\min}(A_2)), \quad p''(A_{12}) \\ &= \min(p_{\max}(A_1), p_{\max}(A_2)) \\ q'(A_{12}) &= \max(q_{\min}(A_1), q_{\min}(A_2)), \quad q''(A_{12}) \\ &= \min(q_{\max}(A_1), q_{\max}(A_2)). \end{aligned}$$

III. PROPOSED PROBABILISTIC INTUITIONISTIC FUZZY TOPSIS (PI-TOPSIS) ALGORITHM

The weighing criteria selected for alternatives play an important role in the MCDM methods. The expert assign values to each weighing criterion, so opinion of decision maker (expert) impacts the final result of the decision making method. If decision makers lack experience or lack rational judgment, then true importance of the criterion in decision making will not be reflected by weights. In such situations, we cannot find a ranking that meets the actual needs. Therefore, for effective weighing of all criterion's, we have proposed a PI-TOPSIS algorithm based on PEDM. Here, intuitionistic fuzzy weighted averaging (IFWA) operator is used to valuate opinions of all decision makers regarding the importance of alternatives and criteria. The PI-TOPSIS algorithm requires both positive-ideal and negative-ideal solution. The complete flowchart of PI-TOPSIS algorithm explaining each step is shown in Figure 1.

Let $A = \{A_1, A_2, \ldots A_m\}$ be the set of the alternatives and $C = \{C_1, C_2, \ldots C_n\}$ be the set of the criteria. Z_{ij} $(i = 1, 2, \ldots, m; j = 1, 2, \ldots n)$ are the performance values of the alternatives A_i $(i = 1, 2, \ldots, m)$ under the criterion $C_j (j = 1, 2, \ldots, n)$ which are shown in Table 1.

TABLE 1. Performance of *m* alternatives over *n* criteria.

Alternative	Criterion's			
	C_1	C_2	$\ldots$	C_n
A_1	Z_{11}	Z_{12}	$\ldots$	Z_{1n}
A_2	Z_{21}	Z_{22}	$\ldots$	Z_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	Z_{m1}	Z_{m2}	$\ldots$	Z_{mn}

The implementation of proposed PI-TOPSIS requires following steps:

Step 1: Assign weights to decision makers in correspondence to their opinions. Here, we have assumed p decision makers in the decision board. The opinions of decision makers are expressed in terms of linguistic variables, which are modeled by intuitionistic fuzzy numbers (IFN).

Let us grade the l^{th} decision maker by an IFN $D_l = [\mu_l, \nu_l, \pi_l]$. The weight corresponding to the opinion of l^{th} decision maker is computed below:

$$\lambda_l = \frac{\left(\mu_l + \pi_l\left(\frac{\mu_l}{\mu_l+\nu_l}\right)\right)}{\sum\limits_{l=1}^{p}\left(\mu_l + \pi_l\left(\frac{\mu_l}{\mu_l+\nu_l}\right)\right)} \text{ such that } \sum_{l=1}^{p}\lambda_l = 1. \tag{5}$$

Step 2: Construct aggregated intuitionistic fuzzy decision matrix using evaluation criteria of all the decision makers.

$Z^{(l)} = (z_{ij}^{(l)})_{m\times n}$ be an intuitionistic fuzzy decision matrix of each decision maker. Here, $\lambda = \{\lambda_1, \lambda_2, \ldots \lambda_p\}$ is the collection of weights allocated to p decision makers, where $\sum\limits_{l=1}^{p}\lambda_l = 1$, $\lambda_l \in [0, 1]$. The IFWA operator (see [52]) rates the alternatives given to decision makers for constructing the intutionistic fuzzy decision matrix, $Z = (z_{ij})_{m\times n}$, where $z_{ij} = (\mu_{A_i}(x_j), \nu_{A_i}(x_j), \pi_{A_i}(x_j))(i = 1, 2, \ldots m; j = 1, 2, \ldots n)$.

$$\begin{aligned} z_{ij} &= IFWA_\lambda(z_{ij}^{(1)}, z_{ij}^{(2)}, \ldots . z_{ij}^{(p)}) \\ &= \lambda_1 z_{ij}^{(1)} \oplus \lambda_2 z_{ij}^{(2)} \oplus \lambda_3 z_{ij}^{(3)} \oplus \ldots .. \oplus \lambda_p z_{ij}^{(p)} \\ &= \left[1 - \prod_{l=1}^{p}(1-\mu_{ij}^{(l)})^{\lambda_l}, \prod_{l=1}^{p}(\nu_{ij}^{(l)})^{\lambda_l}, \right. \\ &\quad \left. \times \prod_{l=1}^{p}(1-\mu_{ij}^{(l)})^{\lambda_l} - \prod_{l=1}^{p}(\nu_{ij}^{(l)})^{\lambda_l}\right] \end{aligned} \tag{6}$$

The aggregated decision matrix can be defined as:

$$Z = \begin{bmatrix} (\mu_{A_1}(x_1), \nu_{A_1}(x_1), \pi_{A_1}(x_1)) & \ldots & (\mu_{A_1}(x_n), \nu_{A_1}(x_n), \pi_{A_1}(x_n)) \\ (\mu_{A_2}(x_1), \nu_{A_2}(x_1), \pi_{A_2}(x_1)) & \ldots & (\mu_{A_2}(x_n), \nu_{A_2}(x_n), \pi_{A_2}(x_n)) \\ \vdots & \vdots & \vdots \\ (\mu_{A_m}(x_1), \nu_{A_m}(x_1), \pi_{A_m}(x_1)) & \ldots & (\mu_{A_m}(x_n), \nu_{A_m}(x_n), \pi_{A_m}(x_n)) \end{bmatrix}$$

$$Z = \begin{bmatrix} z_{11} & z_{12} & z_{13} & \ldots & z_{1m} \\ z_{21} & z_{22} & z_{23} & \ldots & z_{2m} \\ z_{31} & z_{32} & z_{33} & \ldots & z_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & z_{n3} & \ldots & z_{nm} \end{bmatrix}$$

Step 3: Calculate weights corresponding to each criterion.

Since all the criteria cannot be equally important, so a weighted decision matrix is constructed in which the perspective of decision makers regarding the importance of each criterion is exploited.

Let, l^{th} decision maker assigns weight $w_j^{(l)} = [\mu_j^{(l)}, \nu_j^{(l)}, \pi_j^{(l)}]$ to the criterion x_j, where $w_j = (\mu_j, \nu_j, \pi_j)$ $(j = 1, 2, \ldots, n)$.

The weights assigned to criterion x_j by p decision makers is aggregated by IFWA operator as follows:

$$\begin{aligned} w_j &= IFWA_\lambda(w_j^{(1)}, w_j^{(2)}, \ldots . w_j^{(p)}) \\ &= \lambda_1 w_j^{(1)} \oplus \lambda_2 w_j^{(2)} \oplus \lambda_3 w_j^{(3)} \oplus \ldots .. \oplus \lambda_p w_j^{(p)} \\ &= \left[1 - \prod_{l=1}^{p}(1-\mu_j^{(l)})^{\lambda_l}, \prod_{l=1}^{p}(\nu_j^{(l)})^{\lambda_l}, \right. \\ &\quad \left. \times \prod_{l=1}^{p}(1-\mu_j^{(l)})^{\lambda_l} - \prod_{l=1}^{p}(\nu_j^{(l)})^{\lambda_l}\right]. \end{aligned} \tag{7}$$

Step 4: Construction of aggregated weighted intuitionistic fuzzy decision matrix.

The aggregated weighted intuitionistic fuzzy decision matrix is determined by help of weights ($W = [w_1, w_2, w_3, \ldots . w_n]$) and the aggregated intuitionistic fuzzy decision matrix as follows (for details see [1]):

$$\begin{aligned} Z \oplus W = \{x, &\mu_{A_i}(x).\mu_W(x), \\ &+ \nu_W(x) - \nu_{A_i}(x).\nu_W(x) \mid x \in X\} \end{aligned} \tag{8}$$

$$\begin{aligned} \pi_{A_iW(x)} = 1 &- \nu_{A_i}(x) - \nu_w(x) - \mu_{A_i}(x).\mu_w(x) \\ &+ \nu_{A_i}(x).\nu_w(x) \end{aligned} \tag{9}$$

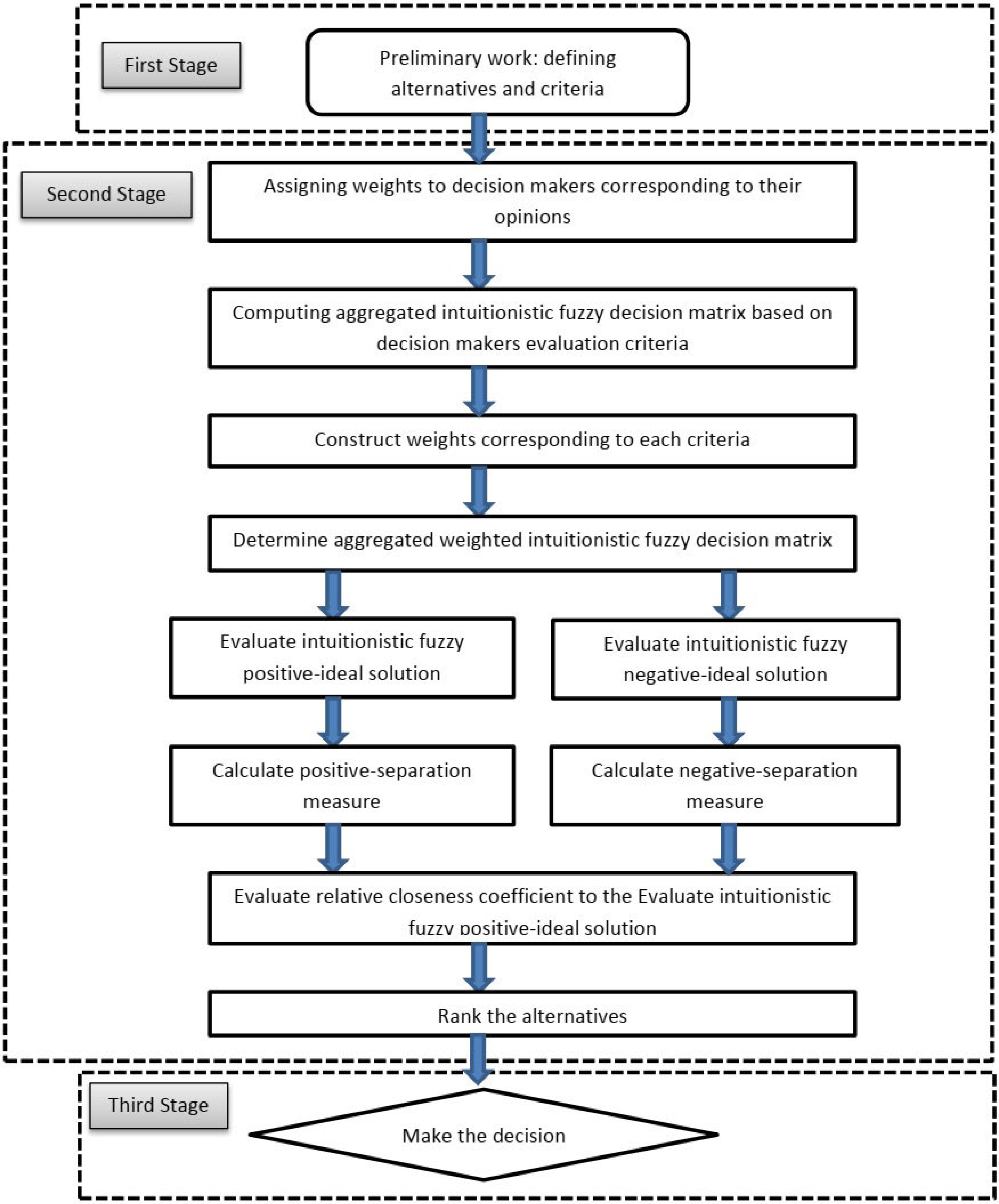


FIGURE 1. Flowchart of the probabilistic intuitionistic fuzzy TOPSIS (PI-TOPSIS) algorithm.

The aggregated weighted intuitionistic fuzzy decision matrix $Z' = (z'_{ij})_{m\times n}$, where $z'_{ij} = (\mu'_{ij}, \nu'_{ij}, \pi'_{ij}) = (\mu_{A_i}(x_j), \nu_{A_i}(x_j), \pi_{A_i}(x_j))$ is the ij^{th} entry of Z'. The matrix Z' is computed as follows:

$$Z' = \begin{bmatrix} (\mu_{A_1W}(x_1), \nu_{A_1W}(x_1), \pi_{A_1W}(x_1)) & \cdots & (\mu_{A_1W}(x_n), \nu_{A_1W}(x_n), \pi_{A_1W}(x_n)) \\ (\mu_{A_2W}(x_1), \nu_{A_2W}(x_1), \pi_{A_2W}(x_1)) & \cdots & (\mu_{A_2W}(x_n), \nu_{A_2W}(x_n), \pi_{A_2W}(x_n)) \\ \vdots & \ddots & \vdots \\ (\mu_{A_mW}(x_1), \nu_{A_mW}(x_1), \pi_{A_mW}(x_1)) & \cdots & (\mu_{A_mW}(x_n), \nu_{A_mW}(x_n), \pi_{A_mW}(x_n)) \end{bmatrix}$$

$$Z' = \begin{bmatrix} z'_{11} & z'_{12} & z'_{13} & \cdots & z'_{1m} \\ z'_{21} & z'_{22} & z'_{23} & \cdots & z'_{2m} \\ z'_{31} & z'_{32} & z'_{33} & \cdots & z'_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z'_{n1} & z'_{n2} & z'_{n3} & \cdots & z'_{nm} \end{bmatrix}$$

Step 5: Calculate intuitionistic fuzzy positive-ideal solution (A^+) and intuitionistic fuzzy negative-ideal solution (A^-). Let J_1 represents the benefit criterion and J_2 represents the cost criterion. The positive ideal A^+ and negative ideal A^- are defined as follows:

$$A^+ = (\mu_{A+W}(x_j), \nu_{A+W}(x_j)) \quad (10)$$

$$A^- = (\mu_{A-W}(x_j), \nu_{A-W}(x_j)) \quad (11)$$

$$\mu_{A^+W}(x_j) = ((\max_i \mu_{A_i.W}(x_j) \mid j \in J_1), \times (\min_i \mu_{A_i.W}(x_j) \mid j \in J_2)) \quad (12)$$

$$\nu_{A^+W}(x_j) = (\min_i \nu_{Ai.W}(x_j) \mid j \in J_1), \times ((\max_i \nu_{A_i.W}(x_j) \mid j \in J_2)) \quad (13)$$

$$\mu_{A^-W}(x_j) = ((\min_i \mu_{Ai.W}(x_j) \mid j \in J_1), \times (\max_i \mu_{A_i.W}(x_j) \mid j \in J_2)) \quad (14)$$

$$\nu_{A^-W}(x_j) = ((\max_i \nu_{Ai.W}(x_j) \mid j \in J_1), \times (\min_i \nu_{A_i.W}(x_j) \mid j \in J_2)) \quad (15)$$

Step : Calculate the probabilistic separation measures $\tilde{d}_2^+$ and $\tilde{d}_2^-$. The PEDM evaluates $\tilde{d}_2^+$ and $\tilde{d}_2^-$ for each alternative from (A^+) and (A^-) as follows:

$$\tilde{d}_2^+ = \Big[\frac{1}{2n}\sum_{i=1}^{n} p_{j+}(\mu_{A_jW}(x_i) - \mu_{A^+W}(x_i))^2 + q_{j+}(\nu_{A_iW}(x_i) - \nu_{A^+W}(x_i))^2 + \rho(A_jW, A^+W)(\pi_{A_jW}(x_i) - \pi_{A^+W}(x_i))^2\Big]^{1/2} \quad (16)$$

$$\tilde{d}_2^- = \Big[\frac{1}{2n}\sum_{i=1}^{n} p_{j-}(\mu_{A_jW}(x_i) - \mu_{A^-W}(x_i))^2 + q_{j-}(\nu_{A_iW}(x_i) - \nu_{A^-W}(x_i))^2 + \rho(A_jW, A^-W)(\pi_{A_jW}(x_i) - \pi_{A^-W}(x_i))^2\Big]^{1/2} \quad (17)$$

Step 7: Evaluate relative closeness coefficient (RC_i, $1 \le i \le m$) of alternative A_i corresponding to A^+ as follows:

$$RC_i = \frac{\tilde{d}_2^-}{\tilde{d}_2^+ + \tilde{d}_2^-} \quad \text{where } 0 \le RC_i \le 1 \quad (18)$$

Step 8: Rank the alternatives.

Alternatives are ranked in descending order. Select the alternative with the maximum value of the relative closeness (RC_i) for the best alternative.

IV. IMPLEMENTATION OF PROPOSED PI-TOPSIS ALGORITHM ON SUPPLIER SELECTION PROBLEM

This section is divided into three subsections. The description of an automobile sector dataset is briefly discussed in the section IV-A. The results obtained by the proposed PI-TOPSIS algorithm is evaluated in section IV-B. The comparative analysis of the proposed algorithm with other intuitionistic fuzzy TOPSIS (IF-TOPSIS) method is discussed in section IV-C.

A. AUTOMOBILE COMPANY DATASET

An automobile company [23] is desired to choose the most suitable supplier for one of the key elements in its production process. After preliminary evaluation, five suppliers A_1, A_2, A_3, A_4 and A_5 remain for further evaluation. A committee of three decision makers (DM) D_1, D_2 and D_3 is formed in order to evaluate the suppliers and to select the most suitable supplier. The four benefit criteria for selection process are as follows:

1) Product quality (C_1)
2) Relationship closeness (C_2)
3) Delivery performance (C_3)
4) Price (C_4)

The Tables 2, 3, 4 and 5 provides the complete description about automobile company dataset. Table 2 describes about the intuitionistic fuzzy value (given in form of IFN) corresponding to the linguistic terms used for importance the decision maker (DM) in the selection of the supplier/alternative. The rating given by for three DM's with respect to each criterion C_i $(i = 1, 2, 3, 4)$ is shown in Table 3. Table 4 describes about the different rating which can be given to the alternatives in the form of linguistic terms and their corresponding IFN. The rating given by each DM corresponding to each criterion for every supplier is given in Table 5.

The hierarchical structure of the supplier selection problem taken from automobile company is shown in the Fig. 2.

TABLE 2. Criteria rating in terms of IFNs.

Linguistic terms	IFNs
Very important(VI)	(0.90,0.10)
Important(I)	(0.75,0.20)
Medium(M)	(0.50,0.45)
Unimportant(UI)	(0.35,0.60)
Very unimportant(VUI)	(0.10,0.90)

TABLE 3. Criteria rating by decision makers.

Criteria	DM_1	DM_2	DM_3
C_1	VI	VI	I
C_2	I	I	I
C_3	I	I	M
C_4	M	I	M

TABLE 4. Linguistic ratings of suppliers in terms of IFNs.

Linguistic terms	IFN
Extremely good(EG)/ Extremely high(EH)	[1.00,0.00]
Very very good(VVG)/ Very very high(VVH)	[0.90,0.10]
Very good(VG)/ Very high(VH)	[0.80,0.10]
Good(G)/ High(H)	[0.70,0.20]
Medium Good(MG)/ Medium High(MH)	[0.60,0.30]
Fair(F)/ Medium (M)	[0.50,0.40]
Medium Bad(MB)/ Medium Low(ML)	[0.40,0.50]
Bad(B)/ Low(L)	[0.25,0.60]
Very Bad(VB)/ Very Low(VL)	[0.10,0.75]
Very Very Bad(VVB)/ Very Very Low(VVL)	[0.10,0.90]

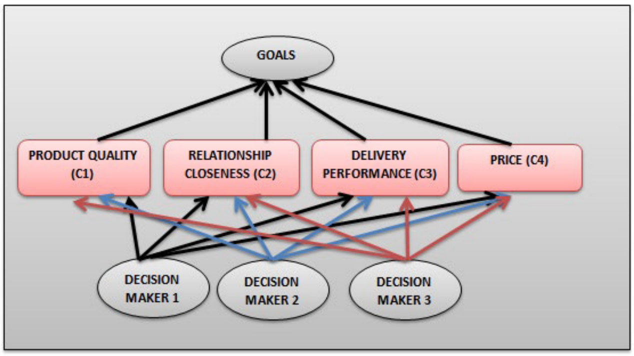


FIGURE 2. Hierarchical structure of supplier selection problem (SSP).

The figure gives the pictorial representation of the problem containing DM and criterion for reaching the goal.

B. EVALUATION OF ALTERNATIVES BY PROPOSED PROBABILISTIC INTUITIONISTIC FUZZY TOPSIS ALGORITHM

The steps used in solving supplier selection problem by proposed PI-TOPSIS algorithm are summarized below:

Step 1: Expertise of decision makers is converted into weights. Using Eq.(5), decision makers are assigned weights in Table 6. The highest weightage is given given to first DM.

Step 2: Aggregate intuitionistic fuzzy decision matrix. Use Eq.(6) over the alternative ratings given in Table 5 by taking their numerical values from Table 4.

$$\begin{bmatrix} (0.728, 0.170, 0.103), & (0.626, 0.272, 0.101) & (0.780, 0.118, 0.102), & (0.700, 0.200, 0.100) \\ (0.596, 0.302, 0.103), & (0.605, 0.292, 0.103) & (0.644, 0.254, 0.101), & (0.578, 0.321, 0.101) \\ (0.882, 0.100, 0.018), & (0.780, 0.118, 0.102) & (0.769, 0.128, 0.103), & (0.769, 0.128, 0.103) \\ (0.663, 0.236, 0.101), & (0.538, 0.361, 0.101) & (0.746, 0.151, 0.104), & (0.644, 0.254, 0.101) \\ (0.562, 0.337, 0.101), & (0.462, 0.438, 0.100) & (0.668, 0.231, 0.101), & (0.526, 0.374, 0.101) \end{bmatrix}$$

TABLE 5. Rating of suppliers.

Criteria	Suppliers	Decision Makers DM_1	DM_2	DM_3
C_1 (Product Quality)	A_1	G	VG	G
	A_2	MG	G	F
	A_3	VVG	VG	VG
	A_4	MG	G	G
	A_5	F	MG	MG
C_2 (Relationship closeness)	A_1	MG	G	MG
	A_2	F	MG	G
	A_3	VG	G	VG
	A_4	F	F	MG
	A_5	MB	F	F
C_3 (Delivery performance)	A_1	VG	G	VG
	A_2	G	MG	MG
	A_3	VG	VG	G
	A_4	VG	G	G
	A_5	G	G	MG
C_4 (Price)	A_1	H	H	H
	A_2	MH	M	MH
	A_3	VH	VH	H
	A_4	H	MH	MH
	A_5	M	MH	M

TABLE 6. Weights assigned to decision makers.

	DM_1	DM_2	DM_3
Linguistic terms	Very important	Medium	Important
Weight	0.4061	0.2375	0.3563

Step 3: Use Eq.(7) in criteria weights evaluation to explain the perspective of decision makers about criteria.

$$W_{\{x_1,x_2,x_3,x_4\}} = \begin{bmatrix} (0.861, & 0.118, & 0.021) \\ (0.750, & 0.200, & 0.050) \\ (0.680, & 0.267, & 0.053) \\ (0.576, & 0.371, & 0.053) \end{bmatrix}^T$$

Step 4: Use criteria weights and aggregated intuitionistic fuzzy decision matrix in Eq.(8) to compute aggregated weighted intuitionistic fuzzy decision matrix.

$$Z' = \begin{bmatrix} (0.627, 0.268, 0.106), & (0.470, 0.418, 0.112) \\ (0.513, 0.384, 0.103), & (0.454, 0.433, 0.113) \\ (0.760, 0.206, 0.034), & (0.585, 0.294, 0.121) \\ (0.571, 0.326, 0.103), & (0.404, 0.489, 0.108) \\ (0.484, 0.415, 0.101), & (0.346, 0.550, 0.103) \end{bmatrix}$$

$$\begin{matrix} (0.530, 0.353, 0.116), & (0.403, 0.497, 0.100) \\ (0.438, 0.454, 0.109), & (0.333, 0.573, 0.094) \\ (0.523, 0.361, 0.116), & (0.443, 0.452, 0.106) \\ (0.507, 0.378, 0.115), & (0.371, 0.531, 0.098) \\ (0.454, 0.436, 0.110), & (0.303, 0.606, 0.091) \end{matrix}\Bigg]$$

Step 5: Calculate intuitionistic fuzzy positive-ideal solution (A^+) and intuitionistic fuzzy negative-ideal solution (A^-) using Eqs. (12)-(15). Here $J_1 = \{C_1, C_2, C_3\}$ is the benefit criterion and $J_2 = \{C_4\}$ is the cost criterion. A^+ and A^- are obtained as follows:

$$\begin{aligned} A^+ &= \{(0.760, 0.206, 0.034), (0.585, 0.294, 0.121), \\ &\quad \times (0.530, 0.353, 0.116), (0.303, 0.606, 0.091)\} \\ A^- &= \{(0.484, 0.415, 0.101), (0.346, 0.550, 0.103), \\ &\quad \times (0.438, 0.454, 0.109), (0.443, 0.452, 0.106)\} \end{aligned}$$

Step 6: Calculate positive and negative probabilistic separation measures using Eq.(16) and Eq.(17) (see Table 7).

TABLE 7. Separation measures and RC coefficient of each supplier.

Suppliers	$\tilde{d}_2^+$	$\tilde{d}_2^-$	RC
A_1	0.0055	0.0018	0.5450
A_2	0.0014	0.0096	0.2494
A_3	0.0028	0.0034	0.8719
A_4	0.0049	0.0015	0.2368
A_5	0.0092	0.0015	0.1397

Step 7: Calculate relative closeness (RC) coefficient with A^+ by Eq.(18). The Table 7 shows the final RC values. The highest RC value 0.8719 is obtained corresponding to supplier A_3.

Step 8: Rank the suppliers in the descending order of RC_i values. So, suppliers are ranked as $A_3 > A_1 > A_2 > A_4 > A_5$. Our result shows that Supplier A_3 is the best alternative among all alternatives.

The positive and negative separation measures and relative closeness (RC) coefficient values of Table 8 can be compared with their probabilistic counterparts by the help of Table 7. The adaptive proposed PI-TOPSIS algorithm delivers that ranking which exact matches with the rankings obtained from other standard methods (see [11], [23]). On the other hand ranking obtained by non-adaptive IF-TOPSIS method differs with those of [23] and [11]. Hence, our proposed PI-TOPSIS algorithm properly functions on the supplier selection problem.

C. A COMPARATIVE ANALYSIS OF PROPOSED PI-TOPSIS ALGORITHM WITH OTHER IF- TOPSIS ALGORITHMS

The overall comparison of proposed PI-TOPSIS algorithm with certain IF- TOPSIS methods is discussed in the section. The proposed PI-TOPSIS algorithm is utilized for decision making in the problems of automobile company [23], portfolio selection [26] and credit risk evaluation [53]. The IF-TOPSIS algorithm of Shen *et al.* [53] is executed in six steps as follows: (1) obtain performance data in the form of intuitionistic fuzzy numbers; (2) identify positive ideal solution and negative ideal solution; (3) calculate positive intuitionistic fuzzy decision matrix and negative intuitionistic fuzzy decision matrix; (4) construct the composite intuitionistic fuzzy decision matrix; (5) determine the optimal weight for each criterion by using maximizing deviation method; (6) rank the alternatives on the basis of distances obtained by the exploitation of optimal alternative based weighted intuitionistic fuzzy distance measure. Joshi *et al.* [26] uses non-optimally derived weighted distance measure in their TOPSIS method and the criteria weights are determined by means of intuitionistic fuzzy entropy measure. Finally, alternatives are ranked on the basis of relative closeness coefficient. Boran *et al.* [23] exploited non- optimal Euclidean distance measure in the TOPSIS such that all components of the measure are assigned equal weights in the ranking process. In the proposed PI-TOPSIS algorithm, the adaptive probabilistic Euclidean distance measure is employed to rank the alternatives.

TABLE 8. Separation measures and RC coefficient for IF-TOPSIS method in [23].

Alternatives	S^+	S^-	RC
A_1	0.092	0.110	0.546
A_2	0.131	0.082	0.385
A_3	0.074	0.175	0.702
A_4	0.124	0.075	0.375
A_5	0.174	0.074	0.300

In Table 9, the rankings obtained by proposed PI-TOPSIS algorithm is compared with the rankings obtained by the methods given in [23], [26], [53], respectively. On automobile company dataset [23], and portfolio selection dataset [26], the rankings obtained by proposed PI-TOPSIS algorithm exactly matches with the rankings given in the paper [23], [26]. Shen *et al.* dealt with the credit risk evaluation dataset through extended IF-TOPSIS method (see [53]). However, the pair (0.5, 0.6) under criterion 'condition (C_5)' is claimed in credit risk evaluation dataset [53] as an intuitionistic fuzzy number, which contradicts that their pair sum should be less than or equal to one. Hence, none of IF-TOPSIS method guarantee reliable ranking on that credit risk evaluation dataset. The non-membership value of negative ideal solution on (C_5) should be less than 0.5. The values of C_5 in credit risk evaluation dataset suggest that the non-membership value should be

TABLE 9. A comparison between the proposed PI-TOPSIS algorithm with certain IF-TOPSIS methods.

MCDM Methods	Original Result	Proposed PI-TOPSIS Result
Shen et al. [53]	$A_2 > A_3 > A_5 > A_1 > A_4$	$A_2 > A_3 > A_1 > A_5 > A_4$
Joshi et al. [26]	$A_3 > A_1 > A_4 > A_2$	$A_3 > A_1 > A_4 > A_2$
Boran et al. [23]	$A_3 > A_1 > A_2 > A_4 > A_5$	$A_3 > A_1 > A_2 > A_4 > A_5$

in between 0.2 and 0.4. Thus, pairs (0.5, 0.2), (0.5, 0.3) and (0.5, 0.4) are used in both extended IF-TOPSIS method and proposed PI-TOPSIS method, which results same ranking $A_2 > A_3 > A_1 > A_5 > A_4$.

The results obtained by the proposed PI-TOPSIS algorithm are interpretated as follows:

- The proposed PI-TOPSIS algorithm is validated over the three different MCDM problems and these problems are dealt in [26], [53] and [23]. In all the three problems, our method yields the same best and the worst alternatives as those obtained in [26], [53] and [23]. It implies that the proposed method easily adapts with the situation given in the problems, henceforth the method is adaptive in nature.
- The dataset of the MCDM problem contains information regarding the alternatives. The ranking method which explores more information or in other words information lost in the method is less, then high superiority of the best alternative over the worst alternative will be delivered while ranking the alternatives. The larger variation in the relative coefficient (RC) values of the best alternative and worst alternative imply high superiority of best alternative over the worst alternative. The proposed PI-TOPSIS algorithm gives maximum difference between the RC values of the best alternative and worst alternative (See Table 10). Hence in comparison to the methods given in [26] and [23], our method has lesser information loss.

TABLE 10. A comparison between relative coefficient (RC) values of MCDM methods.

Methods	Best RC	Worst RC	Difference of RC
Joshi et al. [26]	0.9220	0.2570	0.6650
Proposed PIFI	0.9338	0.0616	0.8722
Boran et al. [23]	0.7020	0.3000	0.4020
Proposed PIFI	0.8719	0.1397	0.7322

V. FACE IDENTIFICATION PROBLEM

A face image is generally represented by feature vectors and a single numerical value is associated with each feature. Identification of two face images becomes complex process when the images are distorted and modified (such as uncertainties in feature due to many non stochastic reasons of calibration, noise, temperature, resolution, repeatability and light effect etc). There are several feature based techniques [51] proposed in literature which extract features from such images. The feature based techniques are relatively more robust to position variations in the input space. The three primary and prominent features extracted from face images using feature based approaches include their color, texture and shape. The two set of these features are extracted from face images by Local Binary Pattern (LBP) and Angular Radial Transformation (ART) techniques are briefly discussed in section V-A. The novel Probabilistic Intuitionistic Fuzzy Face Identification (PIFI)Algorithm is proposed in section V-B

A. FEATURE EXTRACTION BASED ON LBP AND ART

1) LOCAL BINARY PATTERN

Local Binary Pattern (LBP) ([54], [55]) is a prominent texture feature in face identification. The feature distinguishes one face from the other with the help of structural arrangement of a region in the face image. The structural arrangement is efficiently recapitulated by LBP, because each pixels of local structure of an image is compared with the neighboring pixels [56]. The computational cost of LBP is low, and its tolerance regarding the change in monotonic illumination is also not high. Mathematical formulation of LBP is represented as follows:

$$\mathrm{LBP}(x, y) = \sum_{P=0}^{P-1} s(i_P - i_c)2^P \tag{19}$$

where, for any given pixel (x, y), Let i_c and i_P are the values of central pixels and P surrounding pixels respectively in circle neighborhood with a radius R. The function $s(x)$ is defined as:

$$s(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \tag{20}$$

2) ANGULAR RADIAL TRANSFORMATION

The humans have capability of identifying faces on the basis of their shapes also, so shape-based features are exploited in the face identification. There are mainly two categories of shape based features: contour-based approach and region-based approach. The features used in contour-based approach are extracted from the outer boundary, whereas region-based approach extracts features from the entire region [57]. We have several region-based approaches, such as Zernike Moments (ZMs), Angular Radial Transform (ART), Geometric Moments, Moment Invariants [58], etc. The complex images are comfortably described by the ART features [59], as these features have robustness to noise and scaling, invariance to rotation and compact size. Mathematically, ART is calculated by the formula given below:

$$F_{nm} = \sum_{i=0}^{P-1}\sum_{j=0}^{P-1} f(x_i, y_j)V^*_{nm}(x_i, y_j)\Delta x_i \Delta y_j, \quad x_i^2 + y_j^2 \leq 1 \tag{21}$$

where $f(x_i, y_j)$ is the image intensity function in Cartesian coordinate and $V^*_{nm}(x_i, y_j)$ is ART basis function. The coordinates (x_i, y_j) lies in the unit disk, where

$$x_i = \frac{2i + 1 - P}{D}, \quad y_j = \frac{2j + 1 - P}{D}, \quad \times\, i, j = 1, 2, \ldots, P - 1. \tag{22}$$

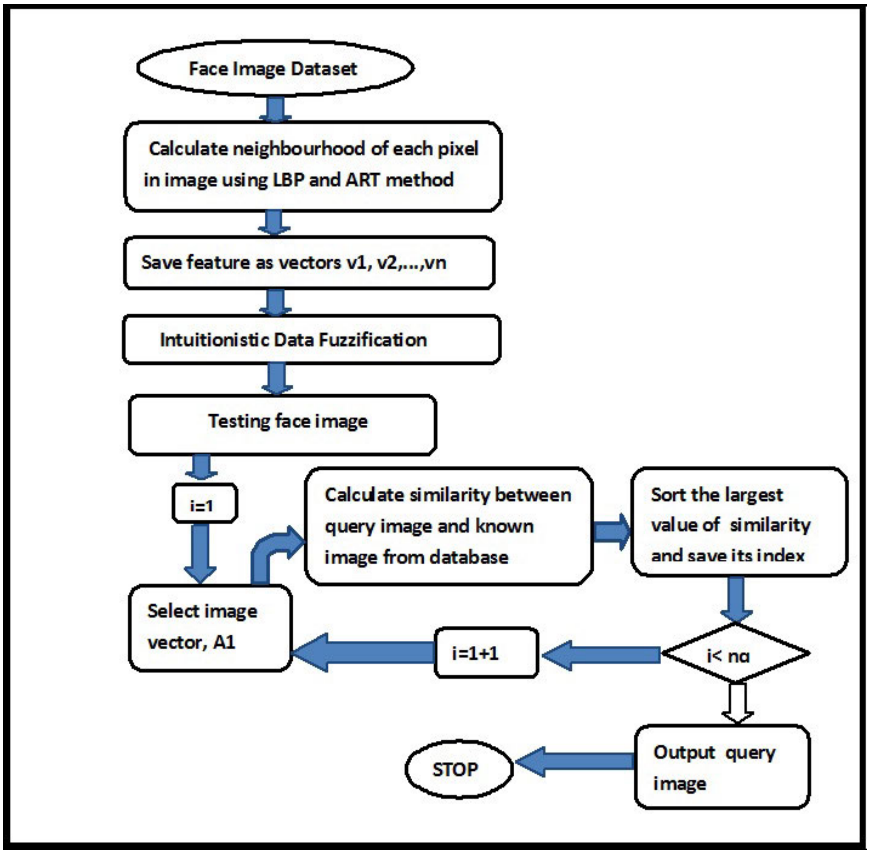


FIGURE 3. Flowchart illustrating proposed PIFI algorithm for face identification.

$$D = \begin{cases} P & \text{for inner circular disk contained in the square image} \\ P\sqrt{2} & \text{for outer circular disk containing in the whole square image} \end{cases} \tag{23}$$

$$\Delta x_i = \frac{2}{D} \text{ and } \Delta y_j = \frac{2}{D} \tag{24}$$

B. PROPOSED PROBABILISTIC INTUITIONISTIC FUZZY FACE IDENTIFICATION (PIFI) ALGORITHM

In this section, we will describe a face identification algorithm using two set of feature vector extraction methods, namely local binary pattern (LBP) and angular radial transformation (ART). Automatic face identification problem is a decision making problem which tries to find the identification of given face images according to the stored database. The training set generally simulate the available database for the face identification. Therefore, in this paper the training set comprises of feature extracted from the known face images of the different peoples. Then, the face identifier matches the test image with the most similar feature vector among the training set. Here, we want to identify a person whose image is given to the system in the form of test image. Let $g(x, y)$ be the value of the pixel located at the point (x, y) in the digital image of size $P \times Q$ (P rows, Q columns), where $1 \leq x \leq P$ and $1 \leq y \leq Q$. Let, $g_1, g_2, \ldots, g_n$ be the face image in the given face database.

In the training phase, the feature vectors are extracted from the face image in the training set. Let v_j be the training face image vector corresponding to the person j which has the pixel resolution of $P \times Q$. The average gray value $f_j(x, y)$ for each pixel is calculated by the LBP and ART formula given in Eqs. (19) and (21). In the testing phase (or identification phase), we give the test face image i of the person. Similar to the training phase, here also we calculate the feature vector v_k of the person using LBP and ART method. For identifying face image vector v_i, we calculate the similarities between the vector v_i with all the feature vectors v_j's of the training set. In this paper, we have calculated similarities between the face images feature vectors using probabilistic Euclidean distance measure PEDM (Eq. (29)) between IFS. The most similar identity will be considered as the output of the proposed face identifier. The similarity values will be ordered in the descending way and then stores the label of the face image having the highest similarity values with the face image database.

The proposed probabilistic intuitionistic fuzzy face identification (Proposed PIFI) algorithm (See Figure 3) is implemented in following steps as follows:

Step 1. Conversion of face image to face dataset Read the grey values of face images g_i, where $0 \leq g_i(x, y) \leq 255$, $i = 1, 2, \ldots n$. Compute the average value of the neighborhood for each value $g_i(x, y)$ using LBP feature (Eq.(19))

and ART feature (Eq.(21)). The values are stored in the matrix $X = [f_i]_{P\times Q}$.

Step 2. Normalization of the face dataset: The gray value of the face dataset is normalized as follows:

$$f_i' = a + \frac{f_i - f_{\min}}{f_{\max} - f_{\min}}(b-a) \tag{25}$$

where $f_{\min}$ and $f_{\max}$ are the minimum and maximum values respectively of the dataset X; a and b are the parameters used for the mapped dataset values. In our case, the values of face image f_i are mapped in [0, 1].

Step 3. Intuitionistic fuzzification of dataset: The face datasets are real number's, therefore a process to transform them into IFS need to be applied. For the allocation of membership value and non-membership value to both LBP and ART features, the process used in [38] is applied. The membership value is computed as follows:

$$\mu_i = -4f_i'^2 + 4f_i' \tag{26}$$

The non-membership value is allocated by using generalized intuitionistic fuzzy generator [60]. Yager's intuitionistic fuzzy complement calculates the non- membership value as follows:

$$\nu_i = (1-\mu_i^\alpha)^{\frac{1}{\alpha}} \tag{27}$$

The parameter α is always positive and is tuned in the interval [0, 1]. The hesitancy value is then deduced using the following formula as

$$\pi_i = 1 - \mu_i - (1-\mu_i^\alpha)^{\frac{1}{\alpha}} \tag{28}$$

Hence, the IFS representation of the data is $x_i = (\mu_i, \nu_i, \pi_i)$.

Step 4. Testing face image: The testing/query face image is taken from the dataset and probabilistic similarity measure procedure [32] is applied to calculate the similarity between the faces. The following similarity measure PEDM (Probabilistic Euclidean Distance Measure) is used to calculate similarity score:

$$\begin{aligned} s_{PE}(A_1,A_2) = \Big[\frac{1}{2m}\sum_{i=1}^{m} & p_{12}(\mu_{A_1}(x_i)-\mu_{A_2}(x_i))^2 \\ & + q_{12}(\nu_{A_1}(x_i)-\nu_{A_2}(x_i))^2 \\ & + \rho_{12}(\pi_{A_1}(x_i)-\pi_{A_2}(x_i))^2\Big]^{1/2} \end{aligned} \tag{29}$$

Here, A_1 is the query face image and A_2 is the known image from the face database. The symbol $\mu_{A_1}(x_i)$, $\nu_{A_1}(x_i)$, and $\pi_{A_1}(x_i)$ represents the membership value, non- membership value, and hesitancy value respectively of the i th feature of data point x_i for query image A_1. Similarly, $\mu_{A_2}(x_i)$, $\nu_{A_2}(x_i)$, and $\pi_{A_2}(x_i)$ represents the membership value, non- membership value, and hesitancy value respectively represents the membership value of the i th feature of data point x_i for known face image A_2 from the database. The weights p_{ij}, q_{ij} and ρ_{ij} in Eq. (29) are assigned corresponding to the membership value, non-membership value and hesitancy value respectively are data driven. Therefore, this measure is adaptive in nature.

Step 5. Storing query face image: For each query face image i, $1 \leq i \leq n_q$, store the label i and its similarity $s_{PE}(A_1, A_2)$.

Step 6. Output query face image: The recognition principle of maximum degree of similarity is used to decide which images is similar to which one.

Step 7. Stop.

VI. IMPLEMENTATION OF PROPOSED PIFI ALGORITHM ON FACE DATASET

In this section, we implement the proposed PIFI algorithm on the face identification problem. The focus of our experiments is on comparing the performance of our proposed PIFI algorithm with other classical and fuzzy version of classification algorithms, namely, support vector machine (SVM) [61], naivye bais classfier (NBC) [62] and fuzzy support vector machine (FSVM) [63]. Accuracies are obtained by the standard 10-fold cross-validation repeated 10 times for each dataset. The 80% of data samples are used for training phase and the remaining 20% for the testing phase. The optimal values of the parameters have been obtained using the exhaustive search method [64]. The value of α in PIFI algorithm is explored in the set $\{\alpha : \alpha = 0.05 + 0.05(k-1), 1 \leq k \leq 20, k \in N\}$. In SVM, NBC and FSVM, all samples are normalized between 0 and 1. The SVM and FSVM parameters are set as follows: C is explored in the grids $\{2^i | i = -10, -9, \ldots, 9, 10\}$. Plus, Gaussian kernel is applied to trade with the nonlinear cases, i.e., $K(x_1, x_2) = \exp(-||x_1 - x_2||^2/\sigma^2)$ and $\sigma \in \{2^{\sigma_{\min}:\sigma_{\max}}\}$ with $\sigma_{\min} = -10, \sigma_{\max} = 10$.

Six performance indicators [65] including recognition rate (RR), specificity, precision, sensitivity/recall, F1-Score, G-Mean are used to compare the performance of proposed PIFI algorithm with other algorithms. The true positive rate or sensitivity is the ratio of classified positive images over all positive images, while the true negative rate or specificity is the ratio of correctly classified negative images over all negative images. Generally, performance of binary class problem is compared by confusion matrix; where one versus all method is used for multi-class problem.

A. EXPERIMENTAL DATASETS

For experimentation purpose ORL and Yale datasets are explored and these datasets are discussed below:

1) **ORL face dataset [66]:** The Cambridge (ORL) face dataset contains 40 images of different human faces. The 10 different images in varying light, different times, facial details (glasses/no glasses) and facial expressions (smiling/non-smiling, open/closed eyes) of each human face is taken in the dataset. The images of the humans are in upright, frontal positions and all of them are taken against a dark homogeneous

FIGURE 4. Sample face images of ORL dataset.

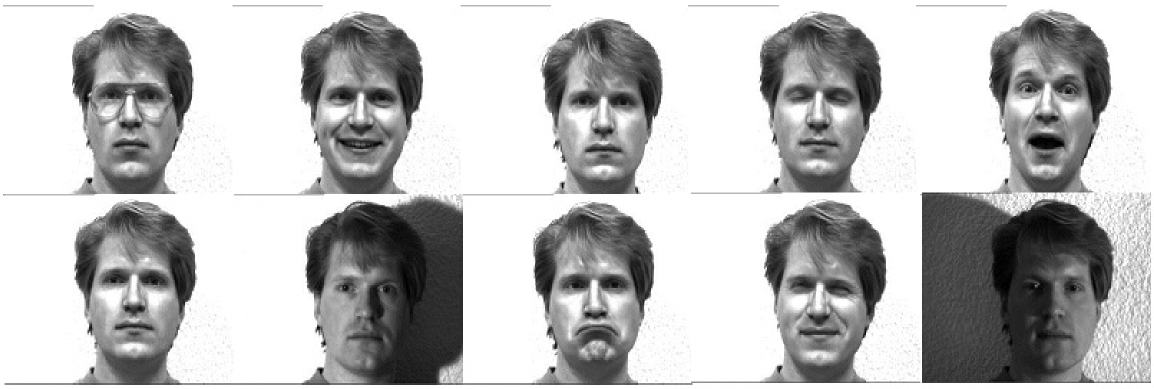


FIGURE 5. Sample face images of yale dataset.

background. Some samples of faces image is shown in Figure 4.

2) **Yale face dataset [67]:** The Yale dataset contains 15 images of different human faces. Then 11 images in different illuminations, face expressions, and small occlusion (by glasses) of each human face is taken in the dataset. The resolution of each face image is 320 × 243. The variations of face images of Yale face dataset are shown in Figure 5.
3) **Tool used for experimental results:** The entire experiments are performed using MATLAB 2018 under a desktop PC with 3.40 GHz frequency and 16-GB RAM.

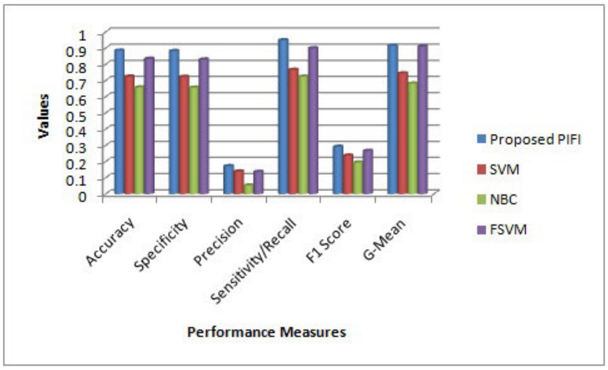
B. EXPERIMENTAL RESULTS AND DISCUSSION

The results obtained using proposed PIFI algorithm as well as other classical and fuzzy version of machine learning algorithm are examined on the basis of performance measures. Tables 11 and 12 shows the results of the ORL face dataset using LBP and ART features respectively. Also, the results of the yale face dataset using LBP and ART features are given in Tables 13 and 14 respectively. Figures 6 and 7 shows the performance on ORL and Yale datasets respectively for both LBP and ART features. The performance of Proposed PIFI algorithm is compared with other three algorithms, namely, SVM, NBC and FSVM in each figure. The bar graph of six benchmark indexes such as, Accuracy, Specificity, Precision, Sensitivity/Recall, F1 Score and G-Mean are plotted for comparison.

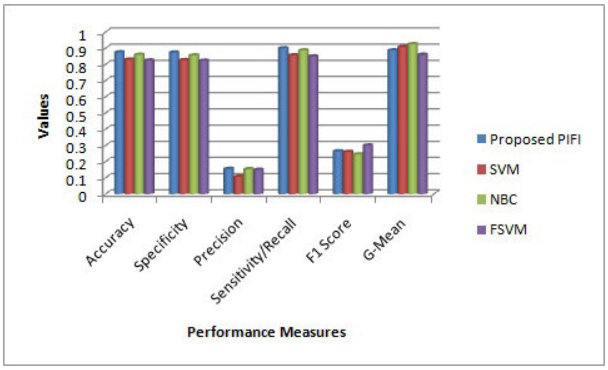
TABLE 11. Performance measures on ORL dataset using LBP features.

Method	Proposed PIFI	SVM	NBC	FSVM
Recognition Rate	0.8850	0.7250	0.6587	0.8350
Specificity	0.8833	0.7237	0.6571	0.8308
Precision	0.1726	0.1395	0.0534	0.1372
Sensitivity/Recall	0.9500	0.7670	0.7250	0.9000
F1 Score	0.2921	0.2379	0.1952	0.2667
G-mean	0.9157	0.7450	0.6823	0.9112

In Figure 6, the performance of IFS based proposed PIFI algorithm with the classical algorithm (SVM and NBC), and fuzzy set based algorithm (FSVM) on ORL dataset using LBP and ART feature. For LBP feature, the highest recognition rate/accuracy of 0.8850 is given by proposed PIFI algorithm and the lowest recognition rate of 0.6587 is achieved by NBC classifier. Similarly for ART features, the RR of 0.8750 is obtained by the proposed PIFI algorithm which is highest among all the algorithms/classifier. For yale dataset (using LBP feature), the higher recognition rate of 0.8121 is attained using proposed PIFI algorithm in comparison to SVM and NBC and FSVM classifiers (RR are 0.7433



(a) Using LBP feature



(b) Using ART feature

FIGURE 6. Performance analysis of benchmark index on ORL dataset.

TABLE 12. Performance measures on ORL dataset using ART features.

Method	Proposed PIFI	SVM	NBC	FSVM
Recognition Rate	0.8750	0.8300	0.8600	0.8250
Specificity	0.8744	0.8270	0.8564	0.8231
Precision	0.1552	0.1081	0.1539	0.1500
Sensitivity/Recall	0.9000	0.8571	0.8867	0.8500
F1 Score	0.2647	0.2609	0.2454	0.3000
G-mean	0.8871	0.9082	0.9254	0.8608

TABLE 13. Performance measures on yale dataset using LBP features.

Method	Proposed PIFI	SVM	NBC	FSVM
Recognition Rate	0.8121	0.7433	0.6606	0.7700
Specificity	0.8117	0.7393	0.6590	0.7679
Precision	0.2368	0.1976	0.1300	0.1822
Sensitivity/Recall	0.8182	0.8333	0.6833	0.6934
F1 Score	0.3673	0.3448	0.3077	0.3514
G-mean	0.7985	0.7639	0.6646	0.7980

TABLE 14. Performance measure on yale dataset using ART features.

Method	Proposed PIFI	SVM	NBC	FSVM
Recognition Rate	0.7939	0.7700	0.6909	0.7667
Specificity	0.7922	0.7679	0.6859	0.7500
Precision	0.2195	0.2284	0.1487	0.2222
Sensitivity/Recall	0.8182	0.8333	0.7667	0.7342
F1 Score	0.3462	0.3704	0.3077	0.3636
G-mean	0.7985	0.7788	0.7180	0.8660

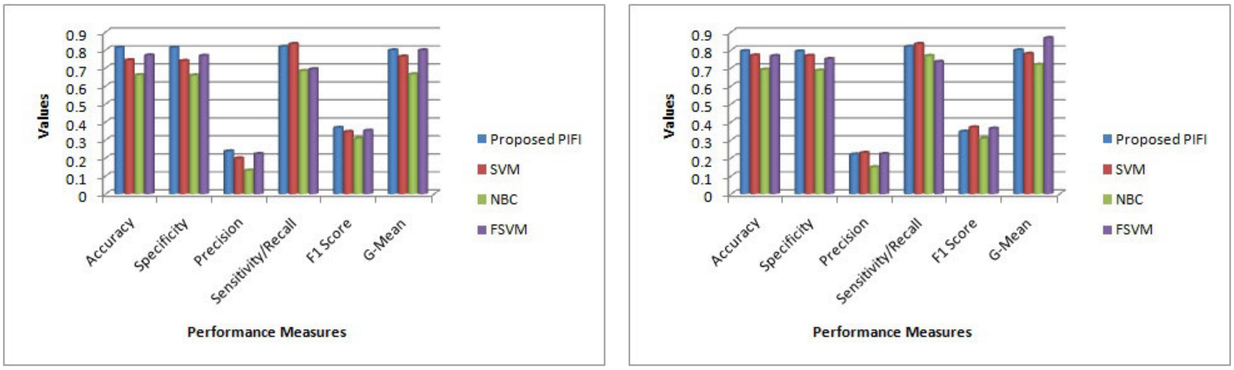
0.6606, 0.7700 respectively). The RR and other benchmark measuring indexes values are shown using bar graph in the Figure 7 for yale dataset. The results show the efficiency of our proposed algorithm.

Specificity measures the proportion of actual negative images which are correctly identified as such. The largest specificity value 0.8833 is achieved by the proposed PIFI algorithm among all the algorithms discussed in the section (see Figure 6 (Using LBP feature)). Precision which gives the correct identification of face images is recorded highest by the proposed algorithm for Yale dataset (with LBP feature) as 0.2368 (see Figure 7 (Using LBP feature)). Sensitivity/ Recall measures the proportion of actual face images that are correctly identified as such. For ORL, the highest sensitivity of 0.9500 (using LBP feature) is observed for the proposed PIFI algorithm but for Yale datasets the highest sensitivity value is recorded as 0.8333 (for both LBP and ART features) by the SVM classifier. The highest sensitivity of both ORL and Yale dataset are clearly depicted in the Figures 6 and 7.

F1 score which measure the assessment accuracy of the proposed PIFI algorithm (using LBP feature) is obtained with value equal to 0.3673. Finally, G-Mean assess how well a measure can balance the performance between the classes. The highest value using LBP feature is obtained by proposed algorithm (which is equal to 0.9157) and with ART feature the best value obtained is 0.9254 using NBC classifier. Hence, the benchmark indexes values concludes the better performance of the proposed PIFI algorithm among the existing classical (SVM and NBC) and fuzzy based algorithm (FSVM) (see Figures 6 and 7).

1) ROLE OF DISTANCE MEASURE IN PROPOSED PIFI ALGORITHM

In order to study the impact of the distance measure on the proposed PIFI algorithm, an analysis is done on the ORL face dataset (Using LBP feature). Here, we have taken three distance measure between the IFSs, namely, Probabilistic Euclidean Distance Measure (PEDM), Euclidean distance measure between IFS (IFDM), and Euclidean distance measure between Fuzzy Sets (FDM). The PEDM is represented by $s_{PE}(A_1, A_2)$ in Eq.(29). If the weights corresponding to all the components of PEDM (see Eq.(29)) are equal, i.e., p_{ij}, q_{ij}, ρ_{ij} are equal to one then it will reduce to IFDM [48]. The mathematical equation of IFDM is given



(a) Using LBP feature

(b) Using ART feature

FIGURE 7. Performance analysis of benchmark index on yale dataset.

as follows:

$$s_{IFE}(A_1, A_2) = \left[\frac{1}{2m}\sum_{i=1}^{m}(\mu_{A_1}(x_i) - \mu_{A_2}(x_i))^2 + (\nu_{A_1}(x_i) - \nu_{A_2}(x_i))^2 + (\pi_{A_1}(x_i) - \pi_{A_2}(x_i))^2\right]^{1/2} \quad (30)$$

Szmidt and Kacprzyk [48] has shown that the exclusion of the hesitancy value $\pi(.)$ from the distance measure in Eq.(30) reduces it to fuzzy sets based distance measurs. Therefore if we take the hesitancy value in Eq.(30) to be equal to zero, it will become Euclidean distance measure between Fuzzy Sets (FDM) [68] which is defined as:

$$s_{FE}(A_1, A_2) = \left[\frac{1}{m}\sum_{i=1}^{m}(\mu_{A_1}(x_i) - \mu_{A_2}(x_i))^2\right]^{1/2} \quad (31)$$

The proposed PIFI algorithm depends on the parameter α. We have explored the functioning of the PIFI algorithm on the ORL dataset for the values of α in the interval [0.05, 1]. The graphical description of LBP features based performance analysis is given in Fig.8. The following information can be easily inferred from Fig.8.

1) If $\alpha = 1$, then Eqs. (28) and (27) gives $\pi_i = 0$ and $\nu_i = 1 - \mu_i$ respectively. Hence, IFS reduces to FS. Therefore, the variants Euclidean distance measure say PEDM, IFDM and FDM does not change the accuracy of the face identification algorithm when α values are near to 1 (in our case $0.90 < \alpha \leq 1$).
2) If α approaches nearer to 0.05, then role of non-membership value significantly diminishes in FDM, IFDM and PEDM. Now, in this situation the accuracy of FDM depends only on the membership component of FS, while in IFDM and PEDM accuracy depends just on the two components (membership value and hesitancy value). Therefore, the accuracy of FDM, IFDM and PEDM are inadequate in the range $0.05 < \alpha \leq 0.35$. Hence, α should not be selected from this range in the face identification algorithm V-B.
3) If $0.35 < \alpha \leq 0.90$, then all the three components (membership, non-membership and hesitancy) contributes in the distance measure. The accuracy of face identification algorithm using PEDM and IFDM significantly improves in this range in comparison to FDM. So, IFS describes ORL dataset in a better way. Since, the weights used in PEDM are data driven, so its performance is further enhanced in comparison to IFDM.

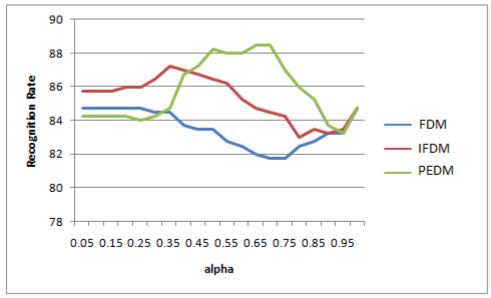


FIGURE 8. Comparison of recognition rate (RR) on ORL dataset using LBP features.

The observations about PIFI algorithm are summarized as follows:

- From the thorough investigation of α, it is concluded that only in the IFS environment recognition rate(RR) of the PIFI algorithm has been improved by PEDM.
- As data driven weights p_{ij}, q_{ij} and ρ_{ij} are used in the PEDM, and its usage in PIFI algorithm has resulted adaptiveness in the algorithm. Thus, PEDM has been utilized in the PIFI in comparison to equally likely approach based IFDM and FDM.
- The performance measures of PIFI are recognition rate, specificity, precision, sensitivity/recall, F1 score, G-mean. The values of performance measures of PIFI have indicated its better performance over SVM, NBC and FSVM identification algorithms (see Tables 11-14).

VII. CONCLUSION

The Probabilistic Euclidean distance measure (PEDM) has improved face identification results through PIFI algorithm and worked well on supplier selection problem due to the introduction of PI-TOPSIS algorithm. The proposed probabilistic version of the two intuitionistic fuzzy decision making methods have introduced adaptive intuitionistic fuzzy decision making algorithms. The proposed PI-TOPSIS algorithm efficiently handles other multi criteria decision making problems of automobile company, portfolio selection and credit risk evaluation. Also, the proposed probabilistic intuitionistic fuzzy identification (PIFI) algorithm has implemented PEDM as similarity measure on face identification problem for both LBP and ART features. The recognition rate of proposed adaptive algorithm has been high in comparison to SVM, NBC and FSVM algorithms. The better performance of proposed PIFI algorithm on both ORL and Yale datasets has been also validated through standard benchmark measuring indexes.

A. LIMITATIONS AND FUTURE WORK

In the paper, Proposed PIFI algorithm has been applied only to grayscale images. Development of suitable methods for applying the proposed algorithm to most recent and challenging coloured images dataset for face identification would be an interesting scope for future research. Also, a rigorous experimentation of the methods on different types of MCDM needs to be carried out to on different types of problems. Moreover, the use of Pythagorean fuzzy interactive Hamacher power aggregation operators for assessment of express service quality with entropy weight in place of p_{ij} and q_{ij} will be a good direction for the future research.

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