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The Method of Insulator Defect Recognition Based on Group Theory

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ABSTRACT The auto insulator defect recognition method is more efficient and reliable than manual method, and it has lots of application. The paper presents an insulator recognition method, and meanwhile it attempts to explore the intrinsic characteristics among different images. Then, based on that many transformations of the image have the structure of matrix Lie group;the paper proposes a method that can recognize the defect of insulator using the Lie group method. The simulation results show the effective of the method.

INDEX TERMS Defects, image recognition, Lie group, group theory.

I. INTRODUCTION

Unmanned Aerial Vehicles (UAV) are widely used to inspect the transmission lines, they can capture a large number of aerial images under very harsh environment. There are lots of studies to overcome the problem of automatic identification of insulator faults.

The common insulator segmentation methods include threshold processing, Hoff transform, region segmentation and morphology segmentation method. For example,the maximum inter class variance method can be used to segment the image, the Hough transform can be used to detect the ellipse of insulator string. and the ellipse parameter can be applied to judge the problem of insulator string dropping, and so on [1].Meanwhile, the insulator skeleton can be extracted by morphological algorithm and connected domain to identify and locate the defect [2].

The insulator recognition based on machine vision is mainly based on feature extraction and classification by classifier [3].For example,Reddy uses discrete orthogonal transform (dost) and some intelligent classification algorithms to detect the defect of insulators [4].

The deep learning method has two kinds of method. One kind of method is to locate defects directly on the original image, the another kind of method is to extract the insulator in the entire image firstly, and then it detects defect [5]. Paper [6] adopts the second kind of method, it locates insulators firstly, and then it identifies defects.

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Oberweger uses discriminative training of local gradient based descriptors and a subsequent voting scheme for localization, and he uses an automatic extraction of the individual insulator caps and check them for faults by using a descriptor with elliptical spatial support [7].

For Group theory has the characteristics of clear physical meaning and easy implementation in fault classification and image transformation. Its research has been widely concerned. The paper focuses on the classification method based on group theory.

The related researches of group representation and classification include two catalogues, one is group signal representation and detection, another one is deep learning and object tracking based on group theory.

In the first kind of researches, Wang *et al.* [8] proposed an assumption of unified signal representations using Mendeleev's Periodic Table of Elements based on group theory. Puschel and Moura [9] and Sandryhaila *et al.* [10] proposed 'Algebraic Signal Processing Theory'.

In the works of Goodall [11], they review basic techniques of Fourier analysis on a finite abelian group. Gu [12] proposed that the observable quantity can be represented by a generalized function with compact support on Lie group G. The product of two observable quantities is the convolution of the corresponding generalized function.With the help of the unitary representation of G, one can express the generalized function on G by the linear operator on the representation space of G, so that the group can be represented in the relation between form and usual operator form of Hilbert space.

In compressed sensing, Bubacarrl *et al.* [13] studied the problem of signal recovery based on its linear sketches knowledge of a given set of groups.

In the second kind of researches, Kondor and Shubhendu [14] presents some of the key features of convolutional neural networks (CNNs) and analyses how it can be applied in more wide area. For example, A feed forward network is equivariant to the group action if and only if it respects its notion of convolution,and so on. Cohen *et al.* [15] and Kondor *et al.* [16] introduced a method to recognize images on the sphere, the data set G = SO(3), the group of three-dimensional rotations. Ebata *et al.* [17] had analyzed a semi simple Lie groups. Sengupta [18] had studied Riemannian symmetric spaces. In Electromagnetic Imaging problems, Ghodgaonkar and Ismail [19] proposed that the group representation theory to solve problems of electromagnetic imaging.

Tuzel *et al.* [20] had present an object classify algorithm using covariance matrices based on Riemannian manifold. Tuzel *et al.* [21] also presented an object detection method based on the Lie algebra and Lie group structure.

So, it is natural to represent and classify the computer vision measurement processing using group theory.

Contributions of this paper include:

1) It analyzes the classification and recognition methods based on group theory, and gives some success examples of the group representation and classification theory.

2) It proposes an insulator defect detection method based on group theory.

3) It discusses the principle of defect detection methods based on group theory.

The structure of paper is as follows: In Section 2, the manuscript presents mathematical formulation. In Section 3, it presents the methodology. Experiments and simulation are given in Section 4. The Section 5 is conclusion.

II. RELATED WORKS

Classical algebra is concerned with the properties of elements, and Modern algebra is concerned with the structure and operational properties of algebras. Correspond, a single test gives scale of a quantity in a certain place and time, more general test also should consider the measurement processing representation and classification.

Group

A group is present by **G**. Its definition is introduced in paper conditions [22]:

Permutation Groups

A permutation means rearrangement. [22].

So, if g is first rearrangement and h is second rearrangement, the results is represented by g * h [22].

Some Feature of Abelian Group

Abelian invariants of the finitely presented group G represents the canonical decomposition of the abelianization G/[G,G] of G. And the G should be a finitely presented group [22].

Lie Group

A Lie group G has the feature of a smooth differentiable manifold. In the paper, focus on real matrix groups: group elements are represented as matrices in $R_{n\times n}$ [22].

Lie Algebra

The Lie algebra g is the tangent space of the identity of G. This tangent space is represented by G1, . . ., Gk [22].

Exponential Map and Logarithm

The exponential map tracks in the differential direction along the group manifold in the algebra. The logarithm map is continuous near the identity. Note that for most groups, including all groups with compact subgroups such as rotations, neither exp nor log is injective [22].

Adjoint Representation

An element $\mathbf{x} \in$ group $\mathbf{G}$, and $a \in$ Lie algebra g, the adjoint representation is $Adj_{\mathbf{x}} : g \rightarrow g$,and follow the equation (1) [22], and the G in $R_{n\times n}$:

$$\begin{aligned} &\mathbf{x} \in \mathbf{G} \\ &a \in g \\ &Adj_{\mathbf{X}} : g \rightarrow g \\ &Adj_{\mathbf{X}(a)} = \mathbf{X} \cdot a \cdot (\mathbf{X})^{-1} \in g \end{aligned} \tag{1}$$

Group action on R^n

A Group action $\mathbf{X}(v)$ on the vector space R_n (equivalently the projective space P_{n-1}) is followed the equation (2) [22].:

$$\begin{aligned} &\mathbf{X} \in \mathbf{G} \\ &\mathbf{X} : (\mathbf{R})^n \rightarrow (\mathbf{R})^n \\ &\mathbf{X}(v) = \mathbf{X} \cdot \mathbf{v} \end{aligned} \tag{2}$$

SO(2)

SO(2) is the group of rotations in the 2D plane, it satisfies the equation (3).

$$\begin{aligned} &\mathbf{S} \in \mathbf{SO}(2) \subset \mathbf{R}^{2\times 2} \\ &\mathbf{X}^{-1} = \mathbf{X}^T \end{aligned} \tag{3}$$

The Lie algebra so(2) is corresponded by differential rotation as follow matrix [22].:

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

The exponential map from so(2) to SO(2) is simply a 2D rotation, it satisfies the equation (4).

$$exp(alg(\theta)) = exp\begin{pmatrix} 0 & -\theta \\ \theta & 0 \end{pmatrix} = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \tag{4}$$

The logarithm and the adjoint representation follow the equation (5) and equation(6).

$$\mathbf{X} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \in \mathbf{SO(2)}, \quad a^2 + b^2 = 1 \tag{5}$$

$$\begin{aligned} Adj_X(alg(\theta)) &= X \cdot alg(\theta) \cdot X^{-1} \\ &= \begin{pmatrix} 0 & -\theta \\ \theta & 0 \end{pmatrix} \\ &= alg(\theta) \\ &\Rightarrow Adj_X = \mathbf{I} \end{aligned} \tag{6}$$

The related works of the image represent and classification include

1) region definition and its mapping, for example, the recognition object or same properties parameters.

2) Classification method based on the feature of measurement set or its mapping.

Related work 1: the classification based on metric of manifold or group mapping.

Step 1:selecting metric of detection region (2D,3D or 4D), let point be **p**, and its map function be Φ, then make $F(\mathbf{p})$ a certain representation of a point of this region, the measurement set **I** is some properties of this point, as it is shown in equation(7);

$$F(\mathbf{p}) = \Phi(\mathbf{I}, \mathbf{p}) \tag{7}$$

Step 2: When we transform one form to another form, the classifier should be made by a metric of super space (for example, map the data from original point to tangent space of manifold).let f be classifying function.

$$sign[F(\ldots)] = sign[\sum_{\cdots}^{\cdots} f(..)]$$

Example 1 (The Method of Mapping to Riemannian):

In [20],a *Wide* × *Height* × *d* dimension feature image **F** can be mapped from *I*(a multi-dimensional matrix **I**, and its coordinates *x*,*y*). And Φ is a certain function. Give equation (8) [20].

$$F(x, y) = \Phi(\mathbf{I}, x, y) \tag{8}$$

Select $R \subset F$ is a rectangular region, the d-dimensional points inside R is represented by $z_{i, i=1\ldots S}$. The covariance matrix $\mathbf{C}_R$ is equation (9) [20]. The mean of the points is μ.

$$C_R = \frac{1}{S-1}\sum_{i=1}^{S}(z_i - \mu)(z_i - \mu)^T \tag{9}$$

To detect the object, one can define different mapping $\Phi(I, x, y)$, for example,

$$[x \quad y \ldots \sqrt{I_x^2 + I_y^2} \quad |I_{xx}| \quad |I_{yy} \quad arctan\frac{|I_x|}{|I_y|}]^T$$

where the pixel location is x and y; intensity derivatives are, $I_x; I_{xx}$.

In order to classify, a suitable representation should be used in the tangent space. For its symmetric characteristic, the orthonormal coordinates of a tangent vector **y** in the tangent space at point **X** is given by equation [20]:

$$vec_x(y) = vec_I(X^{-1/2} y X^{-1/2})$$

where I is the identity matrix. The vector operator is described as follow, and it relates the Riemannian metric on the tangent space to the canonical metric defined in $\mathbb{R}_m$ [20].

$$< y, y >_X = ||vec_X(y)||_2^2$$

The classification processing is described in the following [20].

Input: Measurement set $\mathbf{X}_i, y_{i_{i=1\ldots N}}, \mathbf{X}_i \in \mathcal{M}, y_i \in 0, 1$

step 1: weight $\omega_i = 1/N, i = 1 \ldots N, F(\mathbf{X} = 0, and, p(\mathbf{X}_i) = 1/2$

step 2: Repeat for $l = 1 \ldots L$.

-Compute weights and the response values

$$z_i = \frac{y_i - p(\mathbf{X}_i)}{p(\mathbf{X}_i)(1 - p(\mathbf{X}_i)}, \omega_i = p(\mathbf{X}_i)(1 - p(\mathbf{X}_i))$$

- Compute weighted mean of points

$$\mu_l = argmin_{\mathbf{X} \in \mathcal{M}} \sum_{i=1}^{N} \omega_i d^2(\mathbf{X}_i, \mathbf{X})$$

$$\mu^{t+1} = exp_{\mu^t}[1/N \sum_{i=1}^{N} log_{\mu^t}(\mathbf{X}_i]$$

Here, a Riemannian manifold $\mathcal{M}$ is a differentiable manifold in which each tangent space has an inner product $<;>_{(\mathbf{X} \in \mathcal{M})}$, which varies smoothly from point to point

-Map the data points to tangent space at μ_l

$$\mathbf{x}_i = vec_{\mu_l}(log_{\mu_l}(\mathbf{X}_i))$$

-Fit the function $g_l(x)$ by weighted least-square regression of z_i to x_i using weights ω_i.

-Update $F(\mathbf{X}) \leftarrow F(\mathbf{X}) + 1/2 f_l(\mathbf{X})$ using

$$\begin{aligned} p(\mathbf{X}) &\leftarrow \frac{e^{F(X)}}{e^{F(X)} + e^{-F(X)}} \\ f_l(\mathbf{X} &= g_l(vec_{\mu_l}(log_{\mu_l}(X))) \end{aligned}$$

step 3: Store $F = \mu_l, g_{l_{l=1\ldots L}}$ and Output the classifier

$$sign[F(\mathbf{X})] = sign[\sum_{l=1}^{L} f_l(\mathbf{X}])$$

Related work 2: the tracking (predefine and dynamic classification) based on Lie group.

Step 1:

It is reasonable that measurement point p from image coordinate p_{mg} to object coordinates p_{obj} forms a Lie algebra, it can be represented by maxtrix **M**.

In its initial point p_0, and its infinite variation is δ or Δ. Considering the measurement set tracking transform processing.(x,y is coordinate of image point p, **I** is the identity element of the group, and ω is the $m \times d$ matrix). They are shown in equation (10)-(13).

$$\Delta \mathbf{M}_{p_0+\Delta p} \approx \Delta \mathbf{M}_{p_0} + \frac{\partial \Delta \mathbf{M}}{\partial p} \Delta p \tag{10}$$

$$[x_{img} \quad y_{img} \quad 1]^T = \mathbf{M}[x_{obj} \quad y_{obj} \quad 1]^T \tag{11}$$

$$exp(\mathbf{M}) = \sum_{n=0}^{\infty} \frac{1}{n!} \mathbf{M}^n \tag{12}$$

$$log(\mathbf{M}) = \sum_{n=1}^{\infty} \frac{-1^{n-1}}{n} (\mathbf{M} - \mathbf{I})^n \tag{13}$$

Step 2: Using some quota to ensure the transformation. For example, judging by the squared geodesic distances J_a.

The matrix of initial observations is $\mathbf{X}$ and the matrix of mappings of motions is $\mathbf{Y}$ of Lie algebra. And the matrix of regression coefficient is Ω.

Example 2 (Tracking Using Lie Group Method):

Based on assumption of affine motion of image, [21] presents an example using affine motion group, and uses matrix Lie group structured transformation to do object tracking.

Lie group is a differentiable manifold, its tangent space forms a Lie algebra $\mathbf{g}$ to the identity element $\mathbf{I}$. Normally, using the geodesic that is their minimum length.

An affine transformation A(2) is given by a 3*3 matrix $\mathbf{M}$ in equation (14). [21].

$$M = \begin{bmatrix} \mathbf{A} & b \\ 0 & 1 \end{bmatrix} \tag{14}$$

where $\mathbf{A}$ is a nonsingular 2*2 matrix and b $\in \Re^2$. The set of all affine transformations form a matrix Lie Group [21].

$$[x_{img} \quad y_{img} \quad 1]^T = \mathbf{M}[x_{obj} \quad y_{obj} \quad 1]^T$$

The inverse $\mathbf{M}^{-1}$ is also an affine motion matrix and it transforms the image coordinates to the object coordinates [21].

I is the observed images and t is the time index. The transformation matrix $\mathbf{M}_t$ is given the observations up to time$t$, $I_{0...t}$, and the initial transformation M_0. The transformations is modeled incrementally by equation (15) [25].

$$\mathbf{M}_t = \mathbf{M}_{t-1} \cdot \delta \mathbf{M}_t \tag{15}$$

The tracking algorithm is below:

Input: Location of target at time t - 1 is $\mathbf{M}_{t-1}$ and the current observation is I_t, maximum iteration number is K.

step 1: k = 1, and, $\mathbf{M}_t = \mathbf{M}_{t-1}$

step 2: Repeat

-$\delta \mathbf{M}_t = f(o_t(\mathbf{M}_t^{-1}))$

-$\mathbf{M}_t = \mathbf{M}_t \cdot \delta \mathbf{M}_t$

-$k = k + 1$

step 3: $\delta \mathbf{M}_t = \mathbf{I}$ or $k = K$

To compare the validation of this method, should measure the mean squared geodesic error (MSGE) using equation (16).

$$MSGE = \frac{1}{n_{te}} \sum_{j=1}^{n_{te}} \rho^2[f(o_o^j), \Delta \mathbf{M}_j] \tag{16}$$

Related work 3: using deep learning network based on group theory to classify

Example 3 (Using Lie Group Method in Deep Learning Network to Identity an Object:)

Huang, *et al.* [26] use Lie group represent skeletal data.

A body skeleton S = (V, E), where $V = v_1, \ldots, v_N$ is the set of body joints, and $E = e_1, \ldots, e_M$ is the set of edges.

The body moving skeleton can be represented with a curve on the Lie group $SO_3 \times \ldots \times SO_3$, and it is differentiable. As the tangent spaces are equipped with the inner product, the Riemannian metric on SO_n can be defined by the Frobenius inner product [26],it is described in equation(17).

$$< \mathbf{A}_1, \mathbf{A}_2 >= trace(\mathbf{A}_1^T \mathbf{A}_2), \mathbf{A}_1, \mathbf{A}_2 \in T_{\mathbf{R}_0}, SO_n \tag{17}$$

The logarithm map $log_{\mathbf{R}_0}$ and exponential map $exp_{\mathbf{R}_0}$ at $\mathbf{R}_0$ on SO_n associated with the Riemannian metric can be expressed in terms of the usual matrix logarithm log and exponential exp, it satisfies equation (18) and equation(19) [26].

$$log_{\mathbf{R}_0}(\mathbf{R}_1) = log(\mathbf{R}_1 \mathbf{R}_0^T), \quad \mathbf{R}_0, \ \mathbf{R}_1 \in SO_n \tag{18}$$

$$exp_{\mathbf{R}_0}(\mathbf{A}_1) = exp^{\mathbf{A}_1 \mathbf{R}_0^T}, \quad \mathbf{A}_1 \in T_{\mathbf{R}_0} SO_n \tag{19}$$

Then, the Lie group and its deep learning network architecture form LieNet.

Generally, group theory focuses on the structure of algebra, not only the individual characteristic solutions, but also the actual solution set of current engineering problems. They are not only suitable for the representation and search of solution set, but also can solve fault identification, motion planning, field solution determination, uncertain or unknown problems with the help of differential geometry, tensor, Lie group and other concepts.

At present, the difficulties of application group theory include the following aspects: 1) modeling, the solution set representation and manifold construction building; 2) The method of solving group theory and its simplification; 3) The relationship between solution sets and practical problems, for example: explain phenomena, predict faults, analyze trends, summarize laws, etc.

III. METHODS

There are two steps to recognize the defects of insulator, the first step is the recognition of insulator, the second step is the recognition of the defects of insulator.

A. THE RECOGNITION OF INSULATOR

YOLO uses the end-to-end detection method, which turns the object detection into a regression problem to solve. It directly obtains the object coordinates and classification probability from the image pixel data. Yolo algorithm uses deep neural network to obtain image features and locate the target.

The speed of image detection can meet the requirements of online detection. It is based on R-CNN framework.

1) The YOLO4 method

The selection of YOLO4 architecture is described as follow: its backbone uses CSPDarknet53; Its neck uses SPP,PAN; its head uses YOLOv3.

The selection of BoF and BoS of YOLO4 includes: activations, bounding box regression bus, data augmentation, regularization method, normalization of the network activations by their mean and variance, and skip connections.

2) The bounding box, class prediction, and Feature extractor

The bounding box has four coordinates, t_x, t_y, t_w, t_h. When there is an offset from the top left corner of the image by (c_x, c_y) and the bounding box prior has width and height p_w, p_h, then has equation(20):

$$\begin{aligned} b_x &= \sigma(t_x) + c_x, \\ b_y &= \sigma(t_y) + c_y, \\ b_w &= p_w e^{t_w}, \\ b_h &= p_h e^{t_h} \end{aligned} \tag{20}$$

The loss uses squared error loss, and the YOLOv3 uses logistic regression to predict. In training, it uses binary cross entropy loss for the independent class predictions. In YOLOv3, the loss function includes two important parts, one is the location error, the other is the classification error, it is shown in equation(21).

$$COST = Cost_{xy} + Cost_{wh} + Cost_{confid} + Cost_{cls} \tag{21}$$

YOLOv3 predicts boxes using a similar concept to feature pyramid networks.

YOLO 3 uses successive 3 × 3 and 1 × 1 convolutional layer but now has some shortcut connections as well and is significantly larger. It has 53 convolutional layers. And the Darknet-53 performs on par with state-of-the-art classifiers but with fewer floating point operations and more speed.

3) the object detector

There are two parts in a detector: one is a backbone which is pre-trained on ImageNet and the another is a head which is used to predict classes and bounding boxes of objects. And the CSPDarknet53 is better compared to CSPResNext50 in terms of detecting objects on the MS COCO dataset.

4) the insulator detection

Using two datasets, one is the Insulator Data Set - Chinese Power Line Insulator Dataset (CPLID) provide normal insulator images captured by UAVs and synthetic defective insulator images. The other is baidu Insulator Dataset. The comparison of different method and dataset is shown in Table.1.

B. THE RECOGNITION OF DEFECTS OF INSULATOR BASED ON GROUP THEORY

1) IMAGE MAPPING USES LIE GROUP

The main processing is based on equation (1)-(6), (8), (12)-(18).

TABLE 1. A recognition comparison with different datasets.

Methods	Dataset	accuracy	time
Yolo 3	CPLID	89.2%	0.25s
Yolo 4	CPLID	95.2%	0.20s
Yolo 3	Baidu	86.2%	0.21s
Yolo 4	Baidu	93.2%	0.19s

There are lots of generic Lie group models for computer vision. It has good pattern recognition capabilities for the primary visual cortex in the complicated figure, especially in noisy, low contrast images with overlapping structures. For the 2-D object (2D translations) is itself isomorphic to the plane, the 3-D object (3D rotations) is a three-dimensional manifold called SO(3), and they are all Group.

A Lie group G is a group which is a differentiable manifold with smooth group operations. The Lie algebra g for G is defined as the tangent vector space at the identity of G. G and g are related via the exponential and log maps, exp: $g \to G$ and log: $G \to g$.

The simple principle of recognition defects based on group theory is that the normal object can act like points in the motion group, but defects cannot.

The points $x \in R^3$ move themself according to the rule $x \to R\mathbf{x} + r$. Any g describes the positional and orientational relationship between two reference frames.

An n-dimensional real matrix Lie algebra is defined by a basis consisting of real matrices X_i for i = 1, ..., n that is closed under the matrix commutator, it is shown in equation (22). The C_{ij}^k is the structure constants of the Lie algebra [33].

$$[X_i, X_j] = \sum_{i=1}^{n} C_{ij}^k X_k \tag{22}$$

Its exponential map is shown in equation(23).

$$g(\mathbf{x}) = exp\mathbf{X} \tag{23}$$

where $X = \sum_{i=1}^{n} x_i X_i$ and $x = [x_i, \ldots, x_n]^T$.

The logarithm map is shown in equation(24).

$$logg(\mathbf{x}) = \mathbf{X} \tag{24}$$

where $g \in G$ for the case when G = SE(3) or SO(3).

It is clear that a combination of Euler angles and Cartesian coordinates can be used to respectively parameterize R and **t**.

The Jacobians for general n-dimensional unimodular Lie groups is shown in equation(24). Let $q = [q_1, q_2, \ldots, q_n]^T$ denote any parametrization of a unimodular Lie group G. $J_r(q)$ and $J_l(q)$ are Jacobian matrices.

$$J_1(\mathbf{q}) = [(g^{-1}\frac{\partial g}{\partial q_1})^{v} \cdot (g^{-1}\frac{\partial g}{\partial q_n})] \tag{25}$$

The specific implementation process is as follows:

(1) Convert from Lie matrix to Lie tangent space:

Normal image is defined by the three basis vectors (0 0 1); (0 1 0); (1 0 0). These three axes constitute the basis vectors for the triangle's tangent space v_i and c_i. v and c denote the

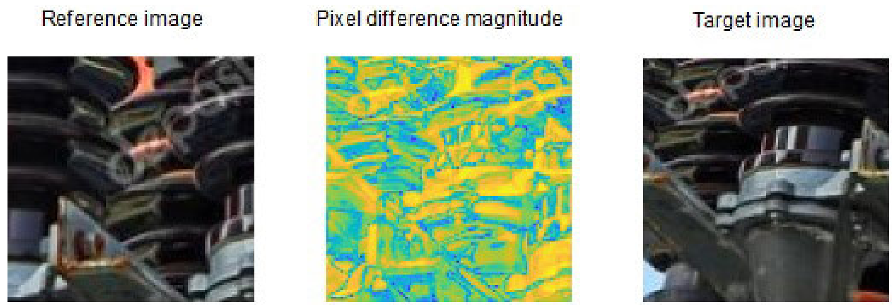


FIGURE 1. The Lie group method to map the image data.

vertex and texture coordinates for a given geometric point, respectively.

So, a matrix can transform a vector in world space to tangent space.And the basis vectors for the tangent space matrix will be referred to as T (tangent), B (binormal) and N(normal).

And the different Lie group matrix has different Lie tangent space.

(2) Convert from transform to Lie matrix to Lie tangent space

The SO_2 follows equation (1)-(6).

(3) Convert from Lie tangent space to Lie matrix

The tangent space matrix only needs to be recalculated if the polygon has been rotated. It should not be recalculated after a translation since the axes of the tangent space coordinate system will still have the same direction after such a transformation.

(4) Convert from Lie tangent space to transform

This is similar to 2).

(5) Other calculation:

Compute the adjoint matrix of a Lie tangent vector;

Apply the Lie bracket to two Lie tangent vectors;

Linear interpolation between transforms.

2) THE DEFECTS RECOGNITION METHOD

Based on Lie Group theory, there is a general method to classify the measurement sets in the following.

a: A METHOD TO CLASSIFY THE MEASUREMENT SETS

step 1) mapping the data to its feature space

step 2) calculating a certain super space distance or metric

step 3) Lie group method to ensure classification standard.

step 4) use method (in the related work) to check each other optional.

Suppose the **D** is the observed data set and t is the time index. The pseudo language of this algorithm demo is in the following(The meaning of the symbol in the algorithm is shown in Section "the related works"):

Input: the measurement data set in time 't' is $\mathbf{D}_t$.

step 1: Map the data points to tangent space at μ_l

$$\mathbf{x}_i = vec_{\mu_l}(log_{\mu_l}(\mathbf{X}_i))$$

step 2: Calculate the Riemannian metric on the tangent space to the canonical metric defined in $\mathbb{R}_m$.

$< y, y >_X = ||vec_X(y)||_2^2$ or the mean squared geodesic error (MSGE) in equation (26).

$$MSGE = \frac{1}{n_{te}} \sum_{j=1}^{n_{te}} \rho^2[f(o_o^j), \Delta\mathbf{M}_j] \tag{26}$$

step 3: checking the data sets using Equation (1)-(6)

To do defects recognition, we should do some preprocessing work.

The following is work of the preprocessing to recognize the defects of insulator.

b: THE PREPROCESSING TO RECOGNIZE THE DEFECTS OF INSULATOR

Suppose the **I** is the object image and t is the time index. The pseudo language of this algorithm demo is in the following:

Input: the **I** is the object image and the time index 't'.

step 1: input the test image $\mathbf{I_n}$, n is data points.

step 2: Calculating and identifying target image set $\mathbf{T_m}$,m is new data points.

step 3: Transform color space into monochromatic feature image **BF**: $T_m \rightarrow M_m$;

step 4: Select Lie Group, for example, the SO_2;

step 5: Calculating cost function, for example equation (25) and (17)-(19);

Repeat for I = 1...L. if it convergence, break the loop

–Create a delta vector of n variables. $DeltaT_n$

–Update the monochromatic image set $\mathbf{T_m}$

–Update the image sets

–Compute the residuals $\mathbf{E_n}$

–Compute the gauss-newton step.

Step 6: output the Image set

Then we can use the method of characteristic point classification and extraction to recognize the defects.

IV. SIMULATIONS

A. MEASUREMENT SET MAPPING USING LIE GROUP

The main Lie group include: 'so2', 'se2', 'sim2', 'aff2', 'so3', 'rxso3', 'uv3', 'rxuv3','se3', 'sim3', 'sl3'

The result of insulator image mapping is shown in Fig.1. It is obvious that the detail of picture can be aligned by the Lie group method.

The comparison of this method with other method is shown in Tab.2. In insulator partitioning, moment based method,

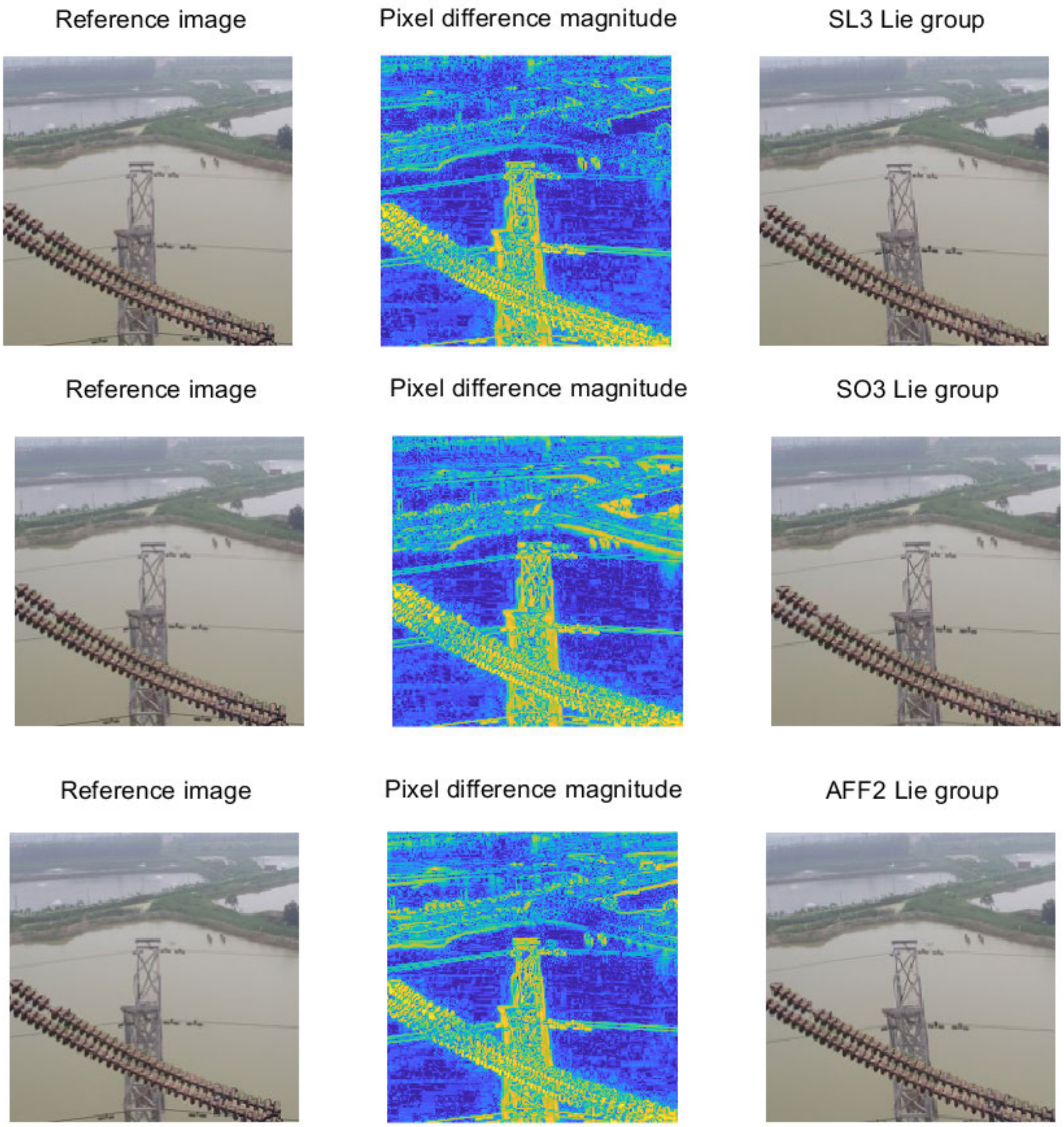


FIGURE 2. The different lie group comparing when mapping the image data.

TABLE 2. The comparison of different picture preprossing method.

preprossing type	algorithm	Average runtime
Image Partitioning [7]	Moment-based	12ms
Image Partitioning [7]	Hough-based	430ms
Image Partitioning [7]	Bounding box	0s
Image compression	Bilinear difference	15ms
Image clipping	Adaptive clipping algorithm	100ms
Image align	Lie Group	300ms

TABLE 3. The comparison of different group mapping method.

Group type	Average runtime
SL3	310ms
SO3	300ms
Aff2	302ms
SE3	320ms
SIM3	340ms
UV3	360ms

the Hough based approach and the orientation of the bounding box are three main methods.

In image compression, the Bilinear difference is a classical method. And the adaptive clipping algorithm is normal used algorithm.

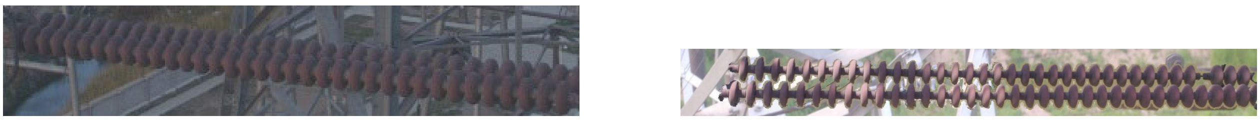
The image aligning algorithm based on Lie Group includes image compression and image clipping. And the experiments results show that it can fit the demand of insulator recognition.

The comparing result of insulator image mapping by SL3,SO3 and aff2 is shown in Fig.2. In Fig.2, there are little difference in the target picture to do picture align based on the equations in the paper.

The comparing results is shown in Tab.3.

(a) example1. (b) example2.

FIGURE 3. **The recognition of insulator using deep learning method.**



(a) example1. (b) example2.

FIGURE 4. **The figure of the segment of the insulator.**

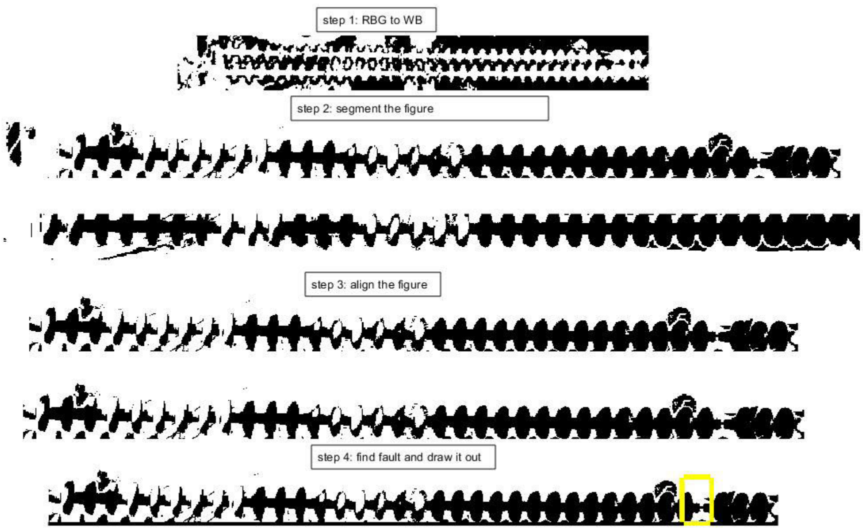


FIGURE 5. **The fault detection processing in segment method.**

B. RECOGNITION AND FAULT DETECTION USING GRAPH MAPPING AND DEEP LEARNING EXAMPLE

1) THE RECOGNITION OF INSULATOR USING DEEP LEARNING METHOD

First using YOLO 4 deep learning method to recognize the insulator. Yolo redefines object detection as a regression problem. It applies a single convolutional neural network (CNN) to the whole image, divides the image into grids, and predicts the class probability and bounding box of each grid. The results are shown in Fig.3.

And then we save the identified area as another measurement set to be processed. And the segment of the insulator is shown in Fig.4.

2) THE COMPARING OF DIFFERENT FAULT DETECTION METHOD

1) fault detection method using segment method

first step: Color space conversion.

Second step: Binarization, conversion to black and white image.

Third step: Determine and divide the boundary of identification area

Fourth step: Fault detection and output the results.

The whole process is shown in Fig.5.

2) faults detect method based the group theory

step 1: use the Lie group method, for example, the method in the Section “ measurement set mapping using Lie Group”

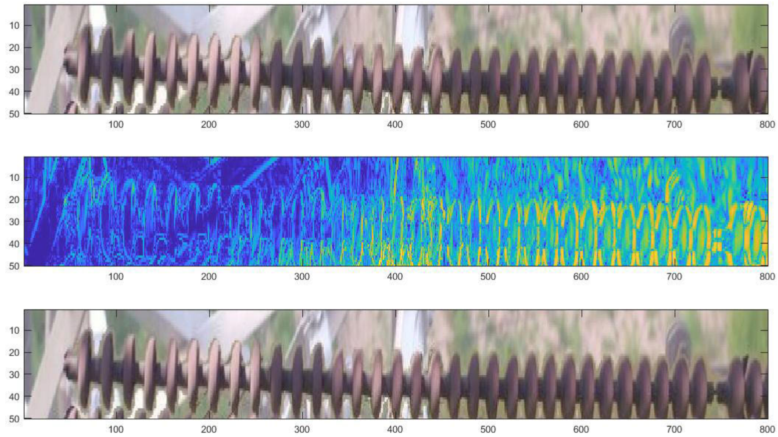


FIGURE 6. The first step of the fault detection based on group theory.

TABLE 4. A recognition comparison of insulator defect.

Methods	accuracy	time
template matching	94.2%	0.20s
Group theory method	97%	0.12s
LOF with EGL [7]	93.25%	0.22s
GLCM [7]	61.8%	

The processing of this stage is shown in Fig.6.

Step 2: Fault detection.

3) the different method can detect the fault of insulator; it is obvious that the second method is more direct and visually. The comparison is shown in Table 4.

V. CONCLUSION

Considering translation invariance and symmetry of image transformation, the paper proposes an insulator and its defects recognition method based on the group theory.

Meanwhile, the experiments and simulations show the effective of this method.

The advantages of our method are:

1) The physical meaning of Lie group it that it concerned with the structure and operational properties of algebras. So, the normal solution sets have almost same structure.

2) Many surfaces or shape faults can be detected, for example, the insulator and its defects can be detected.

3) It analyzes the classification and recognition methods based on group theory, and gives an demo to apply the group theory into defects recognition application.

The disadvantages of our method are:

1) Internal defects are difficult to detect, for example, material defect and structure defect of content. These defects are not visually represented or characterized

2) Non homology topology is difficult to deal with.

And so on.

The future works include:

1) Equivalent method to detect internal defects based on group theory;

2) High dimensional deep learning method.

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