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Distributed Algorithms for Spectral and Energy-Efficiency Maximization of K-User Interference Channels

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ABSTRACT In this paper, we propose a cooperative distributed framework to optimize a variety of rate and energy-efficiency (EE) utility functions, such as the minimum-weighted rate or the global EE, for the K-user interference channel. We focus on the single-input multiple-output (SIMO) case, where each user, based solely on local channel state information (CSI) and limited exchange information from other users, optimizes its transmit power and receive beamformer, although the framework can also be extended to the multiple-output multiple-input (MIMO) case. The distributed framework combines an alternating optimization approach with majorization-minimization (MM) techniques, thus ensuring convergence to a stationary point of the centralized cost function. Closed-form power update rules are obtained for some utility functions, thus obtaining very fast convergence algorithms. The receivers treat interference as noise (TIN) and apply the beamformers that maximize the signal-to-interference-plus-noise (SINR). The proposed cooperative distributed algorithms are robust against channel variations and network topology changes and, as our simulation results suggest, they perform close to the centralized solution that requires global CSI. As a benchmark, we also study a non-cooperative distributed framework based on the so-called ''signal-to-leakage-plus-noise ratio'' (SNLR) that further reduces the overhead of the cooperative version.

INDEX TERMS Distributed algorithms, energy-efficiency region, fairness rate, global energy efficiency, majorization minimization, SIMO systems, sum-rate maximization.

I. INTRODUCTION

Interference has been the main bottleneck of multi-user wireless communication systems for decades, and interference-management techniques are expected to continue playing a key role in future beyond 5G (B5G) networks [1]. Many interference-limited systems, such as device-to-device systems, mesh networks, or multi-cell wireless communication systems, can be modeled as a K-user interference channel (IC). In this paper, we study the K-user IC and propose distributed algorithms to optimize a variety of spectral and energy-efficiency functions.

A. RELATED WORK

Recent research in interference-limited systems aims at improving spectral and energy efficiency (EE) [1]–[3]. The EE of a user is defined as the ratio of its achievable rate to its total power consumption [4]. For a multi-user system, different EE metrics (or objective functions) have been used in [3]–[5]. Among them is the global EE, which is defined as the ratio between the total achievable rate to the total power of the system [4]. This metric does not consider the EE of each individual user and, hence, does not provide fairness among the users. To do so, we have to use a metric or utility function that reflects the EE of each user, such as, the EE region, which can be cast as a maximization of the minimum weighted EE of users [4]. In this paper, we consider the rate region and the sum rate, as the metrics for spectral efficiency, as well as the EE region and the global EE as the metrics for the energy-efficiency of the network.

To design interference-management techniques, we usually have to solve complicated non-convex, sometimes NP-hard, optimization problems [5], [6]. It is known that obtaining the global optimum of such optimization problems

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is, in general, very difficult and cannot be done in polynomial time. Hence, such optimal solutions cannot be implemented in practice, and we have to find suboptimal solutions with affordable computational costs [7]–[14]. A way to solve non-convex optimization problems is to employ iterative majorization-minimization (MM) algorithms. Each iteration of these algorithms consists of two steps: majorization and minimization [15]. In the majorization step, surrogate upper bounds for the objective and/or constraint functions are found. Then, in the minimization step, the corresponding surrogate optimization problem is solved. Under mild conditions, the MM algorithms converge to a stationary point of the considered optimization problems [13], [15], [16], and hence they have been vastly used in wireless communications [7]–[12]. For instance, the papers [8], [10] employed MM-based algorithms to enlarge the rate region of the 2-user single-input, single-output (SISO) and K-user multiple-input, multiple-output (MIMO) ICs with additive hardware distortion, respectively. The authors in [11] proposed algorithms to improve the global and sum EE of users for the downlink (DL) of a MIMO multi-cell system. The paper [12] increased the achievable rate of a multi-hop SISO relay channel with imperfect devices. The paper [7] proposed schemes to improve the spectral and energy efficiency of the K-user MIMO IC with I/Q imbalance.

The aforementioned papers proposed centralized algorithms in which a fusion center or central processing unit (CPU) with full channel state information (CSI) knowledge solves the corresponding optimization problem and then shares the transmission parameters with users. Centralized algorithms are not always feasible in current wireless communication systems specially in large-scale systems due to excessive signaling overheads caused by global CSI acquisition and sharing the solution with all users. Moreover, the corresponding optimization problems can be very complicated to solve, and finding a solution may be time consuming. Thus, distributed and computationally efficient algorithms are vital for practical scenarios and play a key role in modern wireless communications. There are different approaches to develop distributed algorithms. For instance, the authors in [17] proposed a distributed power control algorithm for high signal-to-noise-ratio (SNR) scenarios based on geometric programming. The authors of [18] showed that the maximization of the weighted sum utility for a multi-cell broadcast MIMO system is equivalent to minimizing the weighted-minimum-mean-square error (WMMSE), and proposed a distributed iterative algorithm to solve the latter problem. The algorithm has closed-form solutions for the parameters in each iteration and converges to a stationary point of the WMMSE minimization problem. Game theory has been extensively used to derive distributed algorithms, as shown in [19]–[22]. The paper [23] considered a MIMO orthogonal-frequency-division-multiple-access (OFDMA) system in which each user transmits on a resource block, and each resource block is allocated to only one user. The authors of this work studied the maximization of the global EE of the system, which falls into the category of mixed-integer programming problems, and proposed two distributed algorithms based on auction theory and stable matching to solve the problem with different computational complexities.

An alternative promising way to obtain fast solutions for complicated optimization problems can be to employ deep neural networks (DNNs) [5], [6], [24], [25]. In DNN-based algorithms, the channels are taken as the input of the network, and the transmission parameters are returned as the output. Hence, these algorithms may usually be centralized and require global CSI. In [6], the authors applied deep learning to maximize the sum-rate of the K-user SISO IC. Moreover, the global optimal solution of the weighted sum-EE optimization problem for the uplink (UL) of a multi-cell was derived in [5]. Since the model-based optimum algorithm is very slow, [5] also applied deep learning to obtain fast but approximate solutions and showing the DNN solution is very close to the global optimal solution.

The successful implementation of DNNs in various scenarios suggests that deep learning can be a promising technique for the future of wireless networks. Although deep learning is capable of solving complicated problems, its implementation has some challenges. For instance, applying deep learning requires large training data sets, which are not always available. Moreover, the DNN has to be designed and trained to avoid overfitting problems, as well as to perform robustly in scenarios that may not be present in the training data set. Additionally, it might be difficult to interpret the solutions provided by DNNs. In other words, DNN-based solutions do not necessarily show the existing trade-offs between design parameters of the model-based solutions. Furthermore, even if a parameter, e.g., number of users, number of transmit/receive antennas, power budget of a user, is changed, we have to retrain the network, and the previous solutions may not be applicable to the new parameters. As a result, fast analytical solutions, when available, are preferable to deep learning solutions, and DNN should be employed when existing analytical solutions are not available or are computationally costly.

B. CONTRIBUTIONS

In this paper, we propose computationally-efficient distributed algorithms for the K-user single-input, multiple-output (SIMO) ICs to optimize the spectral and energy efficiency of the system. For the sake of completeness, we also describe how the framework can be applied to the K-user MIMO IC. A key feature in the design of distributed algorithms is to keep the computational cost, as well as the signaling exchanges among users, as less as possible. We propose two distributed frameworks with different requirements of signaling exchange among the users, i.e., cooperative and non-cooperative distributed algorithms. In the cooperative distributed algorithm, each user updates its transmission parameters independently and shares the new parameters with other users at the end of each updating step. In the non-cooperative algorithm, each user requires only to know its own channel and obtains its transmission parameters without any signaling overhead regarding the transmission powers of other users.

We study the most common optimization problems in wireless communications such as rate region, sum-rate maximization, EE region and global-EE maximization. All of them are non-convex optimization problems, in which the objective function and/or constraints are linear functions of the achievable rates. We propose a general framework to solve these problems that employs MM combined with an alternating optimization approach to distributively solve the original coupled optimization problem. The proposed cooperative distributed algorithms dynamically adapt to possible channel variations and changes in the network. This is due to the fact that each user can employ the latest CSI when updating its transmission parameters. Hence, the algorithms can run continuously and cope with the possible variations in the system.

To further reduce the overhead of the cooperative algorithms, we propose a non-cooperative distributed framework based on the definition of the so-called "signal-to-leakage-plus-noise ratio" (SLNR) [26]. To this end, we define the virtual rate and virtual EE for each user, and derive distributed algorithms that maximize these metrics. We employ MM to obtain suboptimal, but computationally cheap, solutions for the maximization of the virtual rate and virtual EE in the K-user SIMO IC.

We compare our algorithms with the optimal solution obtained by conducting an exhaustive search as well as with the centralized algorithm proposed in [7], which converges to a stationary point[1] of each considered optimization problem. Our numerical results show that the cooperative distributed algorithms outperform the non-cooperative schemes and perform very close to the centralized algorithm and the optimal solution. To be specific, the solution of our cooperative distributed algorithm is very close to (or even the same as) the solution of the centralized algorithm for the sum-rate maximization, global-EE maximization and the EE-region optimization problems over a wide range of parameters. The reason is that both algorithms converge to a stationary point of the considered problems. Furthermore, the gap between the distributed and the optimal solutions is sometimes negligible. This is in line with the results in [5], where it was shown that the MM-based algorithm performs very close to the global optimal solution. Since the proposed power control algorithms admit closed-form solutions for the SIMO case, they perform very fast. This is in contrast with the centralized algorithm in [7], especially when the number of users grows.

The main contribution of this paper can be summarized as follows:

- We propose a cooperative distributed framework that obtains a stationary point of any optimization problem in single/multiple-antenna interference-limited systems applying TIN as decoding strategy, as long as the objective function and/or constraints are linear functions of the rates.
- To obtain fast solutions for the K-user SIMO IC, we specialize our framework to each considered utility function and find closed-form solutions for the power updating rules. These closed-form solutions make it possible to implement these distributed algorithms in large-scale networks even with low-cost user equipments.
- Our cooperative distributed algorithms have much lower computational complexities than the centralized algorithms for the K-user SIMO IC proposed in [7]. Nevertheless, both algorithms converge to a stationary point of the considered problems and perform close to each other, which implies that the proposed distributed cooperative algorithms can be applied as a fast alternative to obtain the centralized solution.

C. PAPER OUTLINE

This paper is organized as follows. Section II presents the signal model and formulates the optimization problems. Section III proposes a general framework for cooperative distributed power control algorithms. Section IV specializes the proposed cooperative distributed framework to optimize several rate and EE metrics for the K-user IC. Section V proposes non-cooperative distributed algorithms that reduce significantly the overhead of the cooperative algorithms. Finally, Section VI presents some numerical results.

II. SYSTEM MODEL AND PROBLEM STATEMENT

A. SIGNAL MODEL

In this paper, we consider the K-user IC, which consists of K transceiver pairs employing the same resource blocks (see Fig. 1). For instance, this model can represent the UL channel in a multi-cell wireless system with single-antenna users and multi-antenna base stations (BS). Our main focus is on the SIMO case for which we obtain fast solutions, but for the sake of completeness, we also briefly discuss the extension of our work to the MIMO case. To this end, we first state the more general rate and EE functions of the K-user MIMO IC, and then specialize these expressions to the K-user SIMO IC. Without loss of generality, we consider that all users are symmetric, i.e., they have N_t transmit antennas and N_r receive antennas. Hence, the received signal at the k-th user is

$$\begin{aligned}\mathbf{y}_k &= \sum_{i=1}^{K} \mathbf{H}_{ki}\mathbf{x}_i + \mathbf{n}_k \\ &= \underbrace{\mathbf{H}_{kk}\mathbf{x}_k}_{\text{desired signal}} + \underbrace{\sum_{i=1,i\neq k}^{K} \mathbf{H}_{ki}\mathbf{x}_i}_{\text{interference}} + \underbrace{\mathbf{n}_k}_{\text{noise}}, \end{aligned} \tag{1}$$

where $\mathbf{x}_i$ is the transmit signal of user i, $\mathbf{H}_{ki}$ is the channel matrix between transmitter i and receiver k, and $\mathbf{n}_k$ is the

[1] A stationary point of an optimization problem satisfies the Karush-Kuhn-Tucker (KKT) conditions.

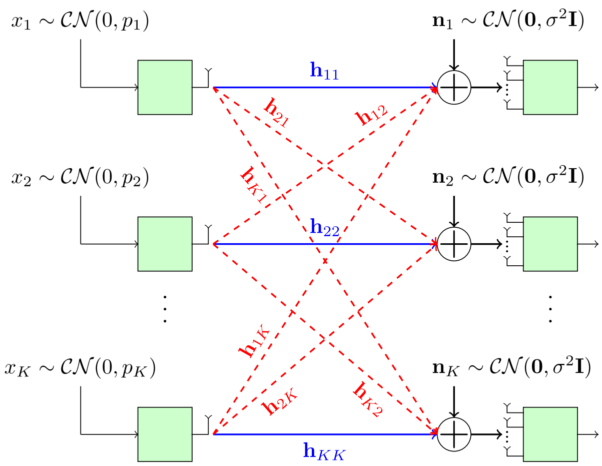


FIGURE 1. The K-user SIMO IC.

noise at receiver k. We represent the covariance matrix of the transmitted signal of user k by $\mathbf{P}_k = \mathbb{E}\{\mathbf{x}_k\mathbf{x}_k^H\}$. Moreover, we assume that the additive noise is a proper zero-mean complex Gaussian signal with covariance $\sigma^2\mathbf{I}$. Treating interference as noise (TIN), the achievable rate of user k is

$$R_k = \log_2 \left|\mathbf{I} + \mathbf{D}_k^{-1}\left(\mathbf{P}_{\bar{k}}\right)\mathbf{S}_k\left(\mathbf{P}_k\right)\right| \tag{2}$$

$$= \log_2 \left|\mathbf{D}_k\left(\mathbf{P}_{\bar{k}}\right) + \mathbf{S}_k\left(\mathbf{P}_k\right)\right| - \log_2 \left|\mathbf{D}_k\left(\mathbf{P}_{\bar{k}}\right)\right|, \tag{3}$$

where $\mathbf{S}_k$ is the covariance matrix of the desired signal at receiver k

$$\mathbf{S}_k\left(\mathbf{P}_k\right) = \mathbf{H}_{kk}\mathbf{P}_k\mathbf{H}_{kk}^H. \tag{4}$$

Furthermore, $\mathbf{D}_k$ is the covariance matrix of the interference-plus-noise term at the receiver side given by

$$\mathbf{D}_k\left(\mathbf{P}_{\bar{k}}\right) = \underbrace{\sigma^2\mathbf{I}}_{\text{noise covariance}} + \underbrace{\sum_{i=1, i\neq k}^{K} \mathbf{H}_{ki}\mathbf{P}_i\mathbf{H}_{ki}^H}_{\text{interference covariance}}, \tag{5}$$

where $\mathbf{P}_{\bar{k}} = \{\mathbf{P}_i\}_{i=1,\neq k}^K$. As can be observed through (2) and (3), R_k is concave in $\mathbf{P}_k$ and convex in $\mathbf{P}_i$ for $i \neq k$.

In this paper, we also consider metrics based on the EE of a user, which is defined as the ratio of its achievable rate to its total power consumption [4]. There can be different metrics that capture the overall EE of the K-user IC such as the global EE and the EE region. The global EE of a system is defined as the ratio of total achievable rate to the total power consumption of the system [3]–[5]:

$$GEE = \frac{\sum_{k=1}^K R_k}{\sum_{k=1}^K \left(\eta_c \text{Tr}(\mathbf{P}_k) + P_c\right)}, \tag{6}$$

where η_c and P_c are, respectively, the power efficiency and the constant circuit power consumption of each transmitter, which are assumed to be equal for all users without loss of generality. The global EE does not consider the EE of each individual user and may not meet any fairness metric. To address this issue, we also consider the minimum weighted EE or, more generally, the EE region.

1) SIMO CASE

In this case, the transmit signal of user k is scalar x_k, and hence, we only need to optimize its transmission power p_k rather than its transmit covariance matrix. Thus, $\mathbf{S}_k(p_k)$ and $\mathbf{D}_k(\mathbf{p}_{\bar{k}})$ can be simplified as

$$\mathbf{S}_k(p_k) = p_k\mathbf{h}_{kk}\mathbf{h}_{kk}^H, \quad \mathbf{D}_k(\mathbf{p}_{\bar{k}}) = \sigma^2\mathbf{I} + \sum_{i=1, i\neq k}^{K} p_i\mathbf{h}_{ki}\mathbf{h}_{ki}^H, \tag{7}$$

where $\mathbf{p}_{\bar{k}}$ denotes the power vector of all users except user k, i.e., $\{p_i\}_{i=1,i\neq k}^K$. Note that we represent the channel vector between transmitter i and receiver k by $\mathbf{h}_{ki} \in \mathbb{C}^{N_r\times 1}$ to be consistent with the notation used in the paper. Additionally, in the SIMO case, R_k simplifies to

$$R_k = \log_2 \left|\mathbf{I} + p_k\mathbf{D}_k^{-1}(\mathbf{p}_{\bar{k}})\mathbf{h}_{kk}\mathbf{h}_{kk}^H\right|$$
$$= \log_2 \left(1 + p_k\mathbf{h}_{kk}^H\mathbf{D}_k^{-1}(\mathbf{p}_{\bar{k}})\mathbf{h}_{kk}\right) = \log_2\left(1 + p_k\tilde{d}_k(\mathbf{p}_{\bar{k}})\right), \tag{8}$$

where $\tilde{d}_k(\mathbf{p}_{\bar{k}}) = \mathbf{h}_{kk}^H\mathbf{D}_k^{-1}(\mathbf{p}_{\bar{k}})\mathbf{h}_{kk}$ is a positive real scalar, which is independent of p_k. Hereafter, we drop the dependency of $\tilde{d}_k$ to $\mathbf{p}_{\bar{k}}$ to simplify the notation. Note that $p_k\tilde{d}_k$ is the signal-to-interference-plus-noise ratio (SINR). The rate expression in (8) is the maximum achievable rate that the channels can support for any beamforming scheme at the receiver side, so this rate can be seen as an upper bound for the actual rates. Notice that for any given set of transmit powers, the design of the receive beamformers can decouple into K independent problems. Therefore, each user can obtain its optimal max SINR beamformer independently by using only local CSI. In order to achieve the rates in (8) the receivers have to apply the optimal beamformer. These rate expressions would be simplified to the rate of SISO systems if the receiver uses a given beamforming scheme, for example, matched filtering [5, Eq. (2)]. However, matched filtering is in general suboptimal in this multi-cell scenario and therefore the resulting rates would be lower than (8), as will be shown in the numerical results.

B. PROBLEM STATEMENT

The proposed distributed optimization framework allows us to consider different utility functions such as the minimum-weighted rate, minimum-weighted EE, weighted sum rate and the global EE. These utility functions yield optimization problems in which the objective function and/or constraints are linear functions of the achievable rates. For instance, the weighted-rate maxmin problem can be written as

$$\max_{\mathbf{p}\in\mathcal{P}} r, \quad \text{s.t.} \quad R_i \geq \alpha_i r \quad \text{for} \quad i = 1, \cdots, K, \tag{9}$$

where the α_i^{-1}s are the corresponding weights. As can be observed through (9), the constraints are linear functions of the rates. Thus, we formulate all these utility-maximization problems as the following general optimization problem

$$\max_{\mathbf{p}\in\mathcal{P}} f_0(\mathbf{p}), \quad \text{s.t.} \quad f_i(\mathbf{p}) \geq 0 \quad \forall i, \tag{10}$$

where $f_i(\mathbf{p})$ for all i are linear functions of achievable rates. Moreover, $\mathcal{P}$ is the feasibility set of users' powers $\mathbf{p}$ defined for the SIMO case as

$$\mathcal{P} = \{\mathbf{p} : 0 \leq p_k \leq P_k, k = 1, 2, \cdots, K\}, \tag{11}$$

where P_k is the power budget of user k. When all f_is for $i = 0, \cdots, K$ are concave in $\mathbf{p}$, the optimization problem (10) is a convex problem and can be solved efficiently. Unfortunately, this is not the case in general when $f_i(\mathbf{p})$ for $i = 0, \cdots, K$ are functions of the rates.

C. CENTRALIZED ALGORITHM

In the centralized algorithm, there is a CPU that collects all required CSI from all users, solves (10) and then sends the transmission powers to be applied by the users during the next coherence block over which the CSI is assumed to remain constant. To this end, the CPU needs the network global CSI, which consists of K^2 vectors (i.e., N_rK^2 complex scalars) for the SIMO case, and K^2 matrices (i.e., $N_tN_rK^2$ complex scalars) for the MIMO case. Moreover, to solve the centralized problem, the CPU also needs high computational resources, especially in large-scale networks. Note also that the centralized solution needs to be updated at each coherence block, or when the network topology changes.

Finding the global optimal solution of (10) may be difficult. For this reason, in this paper, we use as a benchmark a suboptimal centralized algorithm proposed in [7] for the K-user MIMO IC. The centralized algorithm in [7] is based on MM, and hence its convergence to a stationary point of the optimization problems is ensured. For the sake of completeness, we provide a brief summary of this algorithm, and refer the readers to [7] for more details.

As shown in (3), the rates can be written as a difference of two concave functions, which enables us to apply an MM approach. Each iteration of MM algorithms consists of two steps: majorization and minimization. The majorization step finds a suitable upper bound or surrogate optimization problem, which is then solved in the minimization step. Exploiting the MM approach, the main idea of the centralized algorithm is to approximate the convex part of the rate by a linear function, which is also known as the convex-concave procedure (CCP). Hence, the surrogate function for the rates of the SIMO case in each iteration of the MM algorithm is

$$\begin{aligned} R_i \geq \tilde{R}_i = r_{1,i}(\mathbf{p}) - r_{2,i}(\mathbf{p}_{\bar{i}}^{(l-1)}) \\ + \sum_{k=1,k\neq i}^{K} \frac{\partial r_{2,i}(\mathbf{p}_{\bar{i}}^{(l-1)})}{\partial p_k}(p_k - p_k^{(l-1)}), \end{aligned} \tag{12}$$

where $\mathbf{p}^{(l-1)} = [p_1^{(l-1)}, p_2^{(l-1)}, \cdots, p_K^{(l-1)}]$ denotes the user powers at the previous, $(l-1)$-th, iteration. Additionally, the derivative of $r_{2,i}(\mathbf{p}_{\bar{i}})$ with respect to p_k in (12) is

$$\frac{\partial r_{2,i}(\mathbf{p}_{\bar{i}})}{\partial p_k} = \frac{1}{\ln 2}\mathbf{h}_{ik}^H \mathbf{D}_i^{-1} \mathbf{h}_{ik}. \tag{13}$$

We can apply MM by employing the surrogate function in (12) to obtain a stationary point of (10) iteratively. Note that we also employ Dinkelbach-based algorithms in order to solve each surrogate optimization problem for energy-efficiency functions, as indicated in [7].

III. COOPERATIVE DISTRIBUTED FRAMEWORK

A. SIMO CASE

To solve (10) distributively, we combine an alternating optimization approach that updates the power of a single user at each iteration while fixing the powers of the other users, and an MM-based approach that allows for a closed-form power update at each iteration. More specifically, we first fix $\mathbf{p}_{\bar{k}}$ and solve (10) for p_k, which yields the following optimization problem

$$\max_{0\leq p_k \leq P_k} f_0(p_k), \quad \text{s.t.} \quad f_i(p_k) \geq 0 \quad \forall i. \tag{14}$$

Note that the users powers are updated one by one to ensure that the algorithm falls into MM and converges to a stationary point of the original problem. There can be different approaches to select which user updates its power at a given iteration. For example, we can choose a user randomly according to a uniform distribution. In this paper, we consider a round-robin fashion to choose users since it simplifies the protocol. That is, we first update the power of user $k = 1$ by solving (14) at the first epoch. Then, we use the updated power of user 1 to obtain a new power for user 2 by solving (14) for $k = 2$ in the next epoch. We continue this procedure until $k = K$, which concludes one iteration. In other words, each iteration consists of K epochs, and in each epoch, we solve (14) for a single user. Note that the powers can be updated in a continuous fashion, independently of how often each user updates its local CSI. In this way, the power control algorithm is adaptive and is able to track slow channel variations.

As indicated, R_k is concave in p_k while R_i for $i \neq k$ is convex in p_k. Thus, the optimization problem (14) is non-convex in general. Following the MM approach, to solve (14) we lower-bound R_i by an affine function as

$$\begin{aligned} R_i \geq \tilde{R}_i^{(l,k)} &= R_i(\mathbf{p}^{(l,k)}) + \frac{\partial R_i(\mathbf{p}^{(l,k)})}{\partial p_k}(p_k - p_k^{(l-1)}) \\ &= R_i^{(l,k)} - a_{ik}^{(l)}(p_k - p_k^{(l-1)}), \end{aligned} \tag{15}$$

where $R_i^{(l,k)} = R_i(\mathbf{p}^{(l,k)})$, and $\mathbf{p}^{(l,k)}$ denotes the vector of user powers at the k-th epoch of the l-th iteration. Additionally,

$a_{ik}^{(l)}$ can be derived as

$$a_{ik}^{(l)} = -\frac{\partial R_i\left(\mathbf{p}^{(l,k)}\right)}{\partial p_k} = \frac{1}{\ln 2}\mathbf{h}_{ik}^H \boldsymbol{\phi}_i\left(\mathbf{p}^{(l,k)}\right)\mathbf{h}_{ik}, \quad (16)$$

where

$$\boldsymbol{\phi}_i(\mathbf{p}) = \mathbf{D}_{\bar{i}}^{-1}(\mathbf{p}_{\bar{i}})\mathbf{S}_i(p_i)\left(\mathbf{I} + \mathbf{D}_{\bar{i}}^{-1}(\mathbf{p}_{\bar{i}})\mathbf{S}_i(p_i)\right)^{-1}\mathbf{D}_{\bar{i}}^{-1}(\mathbf{p}_{\bar{i}}). \quad (17)$$

In order to compute the lower bound in (15), transmitter k needs to know only $R_i^{(l,k)}$ and $a_{ik}^{(l)}$, which are scalar values. More precisely, transmitter k has to be informed about at most two scalar parameters from other users in addition to $\tilde{d}_k^{(l)}$ in order to solve (14). Depending on the utility function, it might happen that transmitter k requires even less network information as will be shown in the following subsections. There is no sharing of CSI among users for solving (14), and the signaling overheads are thus reduced. Furthermore, the parameter $a_{ik}^{(l)}$ in (16) has to be computed at receiver i. To this end, receiver i needs to know $\mathbf{h}_{ik}$.

To summarize, the algorithm works as follows. At the beginning of each epoch, a user is chosen to update its power (e.g., user k) following a round-robin protocol. The other users (receivers) compute $a_{ik}^{(l)}$ and $R_i^{(l,k)}$, and send them to user k, which then solves (14) and updates its power. If the power changes, the user informs the other users about the new power. Otherwise, there will be no signaling exchange at the end of the epoch. We discuss further the solutions of (14) for different utility functions in the following subsections.

B. EXTENSION TO MIMO

The proposed procedure can be easily extended to include MIMO systems. That is, we approximate R_i with a lower-bound affine function as

$$R_i \geq \tilde{R}_i^{(l,k)} = -\mathrm{Tr}\left(\mathbf{A}_{ik}^{(l)}(\mathbf{P}_k - \mathbf{P}_k^{(l)})\right) + R_i^{(l,k)}, \quad (18)$$

where $\mathbf{A}_{ik}^{(l)}$ is

$$\mathbf{A}_{ik}^{(l)} = -\frac{\partial R_i\left(\{\mathbf{P}_i^{(l,k)}\}_{i=1}^K\right)^H}{\partial \mathbf{P}_k} = \frac{\mathbf{H}_{ik}^H \boldsymbol{\phi}_i\left(\{\mathbf{P}_i^{(l,k)}\}_{i=1}^K\right)\mathbf{H}_{ik}}{\ln 2}, \quad (19)$$

where $\boldsymbol{\phi}_i^{(l)}$ is given by (17), and $\{\mathbf{P}_i^{(l,k)}\}_{i=1}^K$ is the set of the transmit covariance matrices of the users at the k-th epoch of the l-th iteration. The main difference between the MIMO and SIMO cases is that the optimization variable is a matrix in the MIMO case rather than a scalar in the SIMO case. Thus, the derivative of the rates with respect to the covariance matrices, i.e., $\mathbf{A}_{ik}^{(l)}$, is a matrix in the MIMO case, while in the SIMO case, the derivative, i.e., $a_{ik}^{(l)}$, is a scalar. Apart from that, the framework remains unchanged. Similarly, convergence to a stationary point of the original problem is guaranteed in the MIMO case as well. Finally, note that, in the MIMO case, the feasibility set of the covariance matrices is

$$\mathcal{P}_M = \left\{\{\mathbf{P}_k\}_{k=1}^K : \mathrm{Tr}(\mathbf{P}_k) \leq P_k,\ \mathbf{P}_k \succcurlyeq \mathbf{0},\ \forall k\right\}. \quad (20)$$

IV. SPECIALIZING THE COOPERATIVE DISTRIBUTED ALGORITHM TO SOME UTILITY FUNCTIONS

A. WEIGHTED-SUM-RATE MAXIMIZATION

The weighted-sum-rate maximization problem can be written as

$$\max_{\mathbf{p}\in\mathcal{P}} \sum_{k=1}^K \alpha_k R_k, \quad (21)$$

where α_ks are the weights or priorities assigned to the users. In the following, we first consider the SIMO case and then present the extension to the MIMO case.

1) SIMO CASE

Similar to (14), we first fix $\mathbf{p}_{\bar{k}}$ and solve (21) over p_k as

$$\max_{0\leq p_k \leq P_k} \alpha_k R_k + \sum_{i=1, i\neq k}^K \alpha_i R_i. \quad (22)$$

Remember that R_k is concave in p_k, and R_i for $i \neq k$ is convex in p_k, which means that (22) is non-convex. Resorting again to the MM approach, the corresponding surrogate optimization problem is

$$\max_{0\leq p_k\leq P_k} \alpha_k R_k - \sum_{i=1,i\neq k}^K \alpha_i a_{ik}^{(l)}(p_k - p_k^{(l-1)}) + \sum_{i=1,i\neq k}^K \alpha_i R_i^{(l,k)}. \quad (23)$$

Replacing R_k and simplifying (26), we have

$$\max_{0\leq p_k\leq P_k} \alpha_k \log_2\left(1 + p_k \tilde{d}_k^{(l)}\right) - a_k^{(l)} p_k, \quad (24)$$

where $a_k^{(l)} = \sum_{i=1,i\neq k}^K \alpha_i a_{ik}^{(l)}$. The solution of (24) can be simply found as

$$p_k^{(\star)} = \max\left\{0, \min\left(P_k, \frac{\alpha_k/\ln 2}{a_k^{(l)}} - \frac{1}{\tilde{d}_k^{(l)}}\right)\right\}. \quad (25)$$

As it can be verified through (25), the users require to know only two scalar parameters from the system, i.e., $a_k^{(l)}$ and $\tilde{d}_k^{(l)}$, which makes this algorithm scalable.

The closed-form solution in (25) admits the following interpretation: The parameter $a_k^{(l)}$ depends on the interfering links of user k, and $\tilde{d}_k^{(l)}$ depends on the direct link of user k. Therefore, user k might be switched off if either it has a weak direct link (small $\tilde{d}_k$) or causes strong interference (large $a_k^{(l)}$). In other words, the algorithm yields a threshold for switching off a user. Additionally, user k transmits with maximum power if it does not interfere or causes low interference to other users (small $a_k^{(l)}$). We summarize the weighted sum-rate power control method in Algorithm I.

2) MIMO CASE

Replacing the rates by the lower bound in (18) results in the following surrogate optimization problem

$$\max_{\mathbf{P}_k\in\mathcal{P}_M} \alpha_k R_k - \sum_{i=1,i\neq k}^K \alpha_i \mathrm{Tr}\left(\mathbf{A}_{ik}^{(l)}(\mathbf{P}_k - \mathbf{P}_k^{(l)})\right) + \sum_{i=1,i\neq k}^K \alpha_i R_i^{(l)}, \quad (26)$$

Algorithm 1 Distributed Algorithm for Sum-Rate Maximization

Initialization
Set $l = 1$, $p_k = p_k^{(0)}$ for all k
Repeat
  For $k = 1, \cdots, K$
    Fix p_i for $i \neq k$ and obtain $a_k^{(l)}, \tilde{d}_k^{(l)}$
    $p_k^{(l)} = p_k^{(\star)}$, where $p_k^{(\star)}$ is given by (25)
  End (For)
  $l = l + 1$
End (Repeat)

which can be rewritten as

$$\max_{\mathbf{P}_k \in \mathcal{P}_M} \alpha_k \log_2 \left|\mathbf{D}_k^{(l)} + \mathbf{H}_{kk}\mathbf{P}_k\mathbf{H}_{kk}^H\right| - \text{Tr}\left(\mathbf{A}_k^{(l)}\mathbf{P}_k\right), \quad (27)$$

where $\mathbf{A}_k^{(l)} = \sum_{i=1,i\neq k}^K \alpha_i \mathbf{A}_{ik}^{(l)}$, and $\mathbf{D}_k^{(l)}$ are constant and independent of $\mathbf{P}_k$. The optimization problem (27) is convex and can be solved efficiently.

B. GLOBAL-ENERGY-EFFICIENCY MAXIMIZATION

In this subsection, we maximize the global EE by optimizing over the transmit powers

$$\max_{\mathbf{p}\in\mathcal{P}} GEE = \frac{\sum_{k=1}^K R_k}{\sum_{k=1}^K (\eta_c p_k + P_c)}. \quad (28)$$

We consider the SIMO and MIMO cases separately in the following.

1) SIMO CASE

To derive a distributed algorithm, we first fix $\mathbf{p}_{\bar{k}}$ similar to (14) and solve (28) over p_k as

$$\max_{0\leq p_k \leq P_k} GEE. \quad (29)$$

This optimization problem is nonconvex and has a fractional structure. Hence, we employ MM and the Dinkelbach algorithm to solve it. We first apply MM to (29) by approximating R_i with the lower-bound in (15). The resulting problem is

$$\max_{0\leq p_k\leq P_k} \frac{\log_2\left(1 + p_k\tilde{d}_k^{(l)}\right) - a_k^{(l)}(p_k - p_k^{(l-1)})}{\eta_c p_k + P_c + p_{t,k}^{(l)}}, \quad (30)$$

where $a_k^{(l)} = \sum_{i=1,i\neq k}^K a_{ik}^{(l)}$, $p_{t,k}^{(l)} = \sum_{i=1,i\neq k}^K \left(\eta_c p_i^{(l,k)} + P_c\right)$, and $\tilde{d}_k^{(l)}$ are constant and independent of p_k. The optimization problem (30) is not convex, but we can obtain its global optimal solution by the Dinkelbach algorithm. To this end, we solve the following surrogate optimization problem

$$\max_{0\leq p_k\leq P_k} \log_2\left(1 + p_k\tilde{d}_k^{(l)}\right) - a_k^{(l)}p_k - \mu^{(m,l)}\eta_c p_k, \quad (31)$$

where $\mu^{(m,l)}$ is a constant given by

$$\mu^{(m,l)} = \frac{\log_2\left(1 + p_k^{(m-1,l)}\tilde{d}_k^{(l)}\right) - a_k^{(l)}p_k^{(m-1,l)}}{\left(\eta_c p_k^{(m-1,l)} + P_c + p_{t,k}^{(l)}\right)}, \quad (32)$$

where $p_k^{(m-1,l)}$ is the solution of (31) at the $(m-1)$th iteration of the inner loop. Moreover, $p_k^{(0,l)} = p_k^{(l)}$. The optimization problem (31) is convex and its solution can be found as

$$p_k^{(\star)} = \max\left\{0, \min\left(P_k, \frac{\alpha_k/\ln 2}{a_k^{(l)} + \mu^{(m,l)}\eta_c} - \frac{1}{\tilde{d}_k^{(l)}}\right)\right\}. \quad (33)$$

Note that the k-th transmitter requires to know only the scalar parameters $a_k^{(l)}$, $p_{t,k}^{(l)}$, and $\tilde{d}_k^{(l)}$ as can be verified through (32) and (33). We summarize the global EE power control method in Algorithm II.

Algorithm 2 Distributed Algorithm for GEE Maximization

Initialization
Set M, $l = 1$ and $p_k = p_k^{(0)}$ for all k
Repeat
  For $k = 1, \cdots, K$
    Fix p_i for $i \neq k$ and obtain $a_k^{(l)}, \tilde{d}_k^{(l)}$
    $m = 1$
    While $t > \epsilon$ and $m \leq M$ **do**
      Compute $\mu^{(m,l)}$ from (32)
      $p_k^{(m,l)} = p_k^{(\star)}$, where $p_k^{(\star)}$ is given by (33)
      $t = \log_2\left(1 + p_k^{(m,l)}\tilde{d}_k^{(l)}\right) - a_k^{(l)}p_k^{(m,l)}$
      $-\mu^{(m,l)}\left(\eta_c p_k^{(m,l)} + P_c + p_{t,k}^{(l)}\right)$
      $m = m + 1$
    End (While)
  End (For)
    $l = l + 1$
End (Repeat)

2) MIMO CASE

The corresponding optimization problem can be obtained by substituting the rates by the lower bound in (18) as

$$\max_{\mathbf{P}_k\in\mathcal{P}} \frac{\log_2\left|\mathbf{D}_k^{(l)} + \mathbf{H}_{kk}\mathbf{P}_k\mathbf{H}_{kk}^H\right| - \text{Tr}\left(\mathbf{A}_k^{(l)}\mathbf{P}_k\right)}{\eta_c\text{Tr}(\mathbf{P}_k) + P_c + p_{t,k}^{(l)}}, \quad (34)$$

where $\mathbf{A}_k^{(l)} = \sum_{i=1,i\neq k}^K \mathbf{A}_{ik}^{(l)}$, $p_{t,k}^{(l)} = \sum_{i=1\neq k}^K \left(\eta_c\text{Tr}(\mathbf{P}_i^{(l)}) + P_c\right)$, and $\mathbf{D}_k^{(l)}$ are constant and independent of $\mathbf{P}_k$. The optimization problem (34) is not convex, but we can obtain its global optimal solution by the Dinkelbach algorithm, which is an iterative algorithm. That is, we solve the following surrogate optimization problem

$$\max_{\mathbf{P}_k\in\mathcal{P}} \log_2\left|\mathbf{D}_k^{(l)} + \mathbf{H}_{kk}\mathbf{P}_k\mathbf{H}_{kk}^H\right| - \text{Tr}\left(\mathbf{A}_k^{(l)}\mathbf{P}_k\right) - \mu^{(m,l)}\left(\eta_c\text{Tr}(\mathbf{P}_k) + P_c + p_{t,k}^{(l)}\right), \quad (35)$$

where $\mu^{(m,l)}$ is a constant and given by

$$\mu^{(m,l)} = \frac{\log_2\left|\mathbf{D}_k^{(l)} + \mathbf{H}_{kk}\mathbf{P}_k^{(m-1,l)}\mathbf{H}_{kk}^H\right| - \text{Tr}\left(\mathbf{A}_k^{(l)}\mathbf{P}_k^{(m-1,l)}\right)}{\left(\eta_c\text{Tr}(\mathbf{P}_k^{(m-1,l)}) + P_c + p_{t,k}^{(l)}\right)}, \quad (36)$$

where $\mathbf{P}_k^{(m-1,l)}$ is the solution of (35) at $(m-1)$th iteration. Moreover, $\mathbf{P}_k^{(0,l)} = \mathbf{P}_k^{(l)}$. The optimization problem (35) is convex and can be solved efficiently. We iteratively solve (35) and update μ until a convergence criterion is met.

C. MAX-MIN WEIGHTED RATE

1) SIMO CASE

The max-min weighted rate problem is written in (9). Similar to (14), we first fix $\mathbf{p}_{\bar{k}}$ and solve (9) over p_k as

$$\max_{0\leq p_k\leq P_k} r, \quad \text{s.t.} \quad R_i \geq \alpha_i r \quad \forall i. \tag{37}$$

R_i can be lower-bounded as in (15), which results in

$$\max_{0\leq p_k\leq P_k} r, \quad \text{s.t. } \log_2\left(1+p_k\tilde{d}_k^{(l)}\right) \geq \alpha_k r \tag{38a}$$

$$R_i^{(l,k)} - a_{ik}^{(l)}(p_k - p_k^{(l-1)}) \geq \alpha_i r \quad \forall i \neq k, \tag{38b}$$

The optimization problem (38) is convex and can be solved efficiently. Here, we propose a fast scheme to obtain its global optimal solution by solving a sequence of feasibility problems. That is, we fix r as $r^{(t)}$ and easily verify whether $r^{(t)}$ is feasible or not. Then, the optimal solution for (38), i.e., $r^{(\star)}$, can be derived by a bisection. A lower bound for $r^{(\star)}$ is $r^{(l)} = \min_i \{\frac{R_i^{(l,k)}}{\alpha_i}\}$. An upper bound can be chosen to ensure that $r^{(\star)} < r^{(u)}$ as, e.g., $r^{(u)} = \max_i \{\frac{R_i^{(l,k)}}{\alpha_i}\}$. Now consider the feasibility problem

$$\text{find } 0 \leq p_k \leq P_k, \tag{39a}$$

$$\text{s.t. } \log_2\left(1+p_k\tilde{d}_k^{(l)}\right) \geq \alpha_k r^{(t)} \tag{39b}$$

$$R_i^{(l,k)} - a_{ik}^{(l)}(p_k - p_k^{(l-1)}) \geq \alpha_i r^{(t)} \quad \forall i \neq k. \tag{39c}$$

The problem is feasible for $r^{(t)}$ if and only if

$$\frac{2^{\alpha_k r^{(t)}}-1}{\tilde{d}_k^{(l)}} \leq \min\left\{P_k, p_k^{(l-1)} + p'\right\}, \tag{40}$$

where

$$p' = \min_{\forall i\neq k}\left\{\frac{R_i^{(l,k)} - \alpha_i r^{(t)}}{a_{ik}^{(l)}}\right\}. \tag{41}$$

If $r^{(t)}$ is feasible, we set $r^{(l)} = r^{(t)}$. Otherwise, we set $r^{(u)} = r^{(t)}$. We summarize the procedure in Algorithm III.

The closed-form solutions in (40) and (41) can be interpreted as follows. The optimal value for the parameter p' is actually the power modification for user k. If $R_k^{(l,k)} > \alpha_k r^{(l)}$, another user has the minimum weighted rate. Hence, user k has to reduce its power to improve the minimum weighted rate of other users. In this case, the optimal value for p' is negative, which implies that the rates of other users, and consequently, the minimum weighted rate is improved.

If $R_k^{(l,k)} = \alpha_k r^{(l)} < \min_{\forall i\neq k}\left\{R_i^{(l,k)}\right\}$, user k has the minimum weighted rate among the users, and its rate has to be increased in order to improve the minimum weighted rate. This can only be possible by increasing its transmission power, which decreases the rates of other users. In this case, the optimal value for p' is positive.

Algorithm 3 Distributed Algorithm for MWRM

Initialization
Set $l=1$, and $p_k = p_k^{(0)}$ for all k
Repeat
  For $k = 1, \cdots, K$
    Fix p_i for $i \neq k$ and obtain $a_k^{(l)}$, $\tilde{d}_k^{(l)}$
    Obtain $p_k^{(l)}$ by solving (39), i.e., checking inequality (40), and conducting a bisection
  End (For)
  $l = l+1$
End (Repeat)

2) MIMO CASE

Approximating R_i with the lower-bound in (15) gives us the following convex problem

$$\max_{\{\mathbf{P}_k\}\in\mathcal{P}} r, \tag{42a}$$

$$\text{s.t. } \log_2\left|\mathbf{D}_k^{(l)} + \mathbf{H}_{kk}\mathbf{P}_k\mathbf{H}_{kk}^H\right| - \log_2\left|\mathbf{D}_k^{(l)}\right| \geq \alpha_k r \tag{42b}$$

$$R_i^{(l)} - \text{Tr}\left(\mathbf{A}_{ik}^{(l)}(\mathbf{P}_k - \mathbf{P}_k^{(l)})\right) \geq \alpha_i r \quad \forall i \neq k. \tag{42c}$$

D. ENERGY-EFFICIENCY REGION

The energy-efficiency-region problem can be written as

$$\max_{\mathbf{p}\in\mathcal{P}} e, \quad \text{s.t.} \quad E_i = \frac{R_i}{\eta_c p_i + P_c} \geq \lambda_i e \quad \forall i, \tag{43}$$

where λ_is are the corresponding rate-profile coefficients, satisfying $\sum_{i=1}^K \lambda_i = 1$. In the following, we present the cooperative distributed algorithm for the SIMO and MIMO cases.

1) SIMO CASE

Similar to (14), we first fix $\mathbf{p}_{\bar{k}}$ and solve (43) over p_k as

$$\max_{0\leq p_k\leq P_k} e, \quad \text{s.t.} \quad E_i = \frac{R_i}{\eta_c p_i + P_c} \geq \lambda_i e \quad \forall i. \tag{44}$$

Using a lower-bound for R_i as (15), the surrogate optimization problem becomes (45) shown at the bottom of the next page. The optimization problem (45) is not convex; however, its global optimal solution can be found by solving a sequence of feasibility problems. That is, we fix e as $e^{(t)}$ and solve the convex feasibility problem in (46), shown at the bottom of the next page. The optimal solution of (45), $e^{(\star)}$ and $p_k^{(\star)}$, can be found by conducting a bisection over $e^{(t)}$. Since the constraints (46b) are linear, the problem can be simplified as

$$\text{find } 0 \leq p_k \leq p_{\max}, \tag{47a}$$

$$\text{s.t. } \log_2\left(1+p_k\tilde{d}_k^{(l)}\right) - \lambda_k e^{(t)}\left(\eta_c p_k + P_c\right) \geq 0, \tag{47b}$$

where $p_{\max} = \min\left\{P_k, p_k^{(l)} + p'\right\}$, and

$$p' = \min_{\forall i\neq k}\left\{\frac{EE_i^{(l)} - \lambda_i e^{(t)}}{a_{ik}^{(l)}\left(\eta_c p_i^{(l)} + P_c\right)}\right\}. \tag{48}$$

Theorem 1: The problem in (47) *is feasible for $e^{(t)}$ if and only if*

$$\log_2\left(1+p^{(t)}\tilde{d}_k^{(l)}\right)-\lambda_k e^{(t)}\left(\eta_c p^{(t)}+P_c\right)\geq 0, \tag{49}$$

where

$$p^{(t)}=\max\left(0,\min\left\{\frac{1}{\lambda_k e^{(t)}\eta_c}-\frac{1}{\tilde{d}_k^{(l)}},p_{\max}\right\}\right). \tag{50}$$

Proof: Let us define $f(p)$ as

$$f(p)=\log_2\left(1+p_k\tilde{d}_k^{(l)}\right)-\lambda_k e^{(t)}\left(\eta_c p_k+P_c\right). \tag{51}$$

It can be easily verified that $f(0)<0$. The derivative of $f(p)$ with respect to p is

$$\frac{\partial f(p)}{\partial p}=\frac{\tilde{d}_k^{(l)}}{1+\tilde{d}_k^{(l)}p}-\lambda_k e^{(t)}\eta_c. \tag{52}$$

If $\tilde{d}_k^{(l)}\leq\lambda_k e^{(t)}\eta_c$, $f(p)$ is decreasing for $p\geq 0$, and $f(p)$ attains its maximum value at $p=0$, which implies that (47) is not feasible.

Now we consider $\tilde{d}_k^{(l)}>\lambda_k e^{(t)}\eta_c$. In this case, $f(p)$ attains its maximum at

$$p^{(\star)}=\frac{1}{\lambda_k e^{(t)}\eta_c}-\frac{1}{\tilde{d}_k^{(l)}}. \tag{53}$$

In other words, $f(p)$ is increasing for $p\leq p^{(\star)}$ and decreasing for $p>p^{(\star)}$. Thus, $f(p)$ has its maximum at $p^{(t)}$ for $0\leq p\leq P_k$. If the maximum of $f(p)$ is negative, then (47) is not feasible. Otherwise, (47) is feasible. □

2) MIMO CASE

The corresponding optimization problem is

$$\max_{\{\mathbf{P}_k\}\in\mathcal{P}_M} \quad e, \tag{54a}$$

$$\text{s.t. } E_k=\frac{\log_2\left|\mathbf{D}_k^{(l)}+\mathbf{H}_{kk}\mathbf{P}_k\mathbf{H}_{kk}^H\right|}{\eta_c\text{Tr}(\mathbf{P}_k)+P_c}\geq\lambda_k e, \tag{54b}$$

$$\frac{R_i^{(l,k)}-\text{Tr}\left(\mathbf{A}_{ik}^{(l)}(\mathbf{P}_k-\mathbf{P}_k^{(l,k)})\right)}{\eta_c\text{Tr}(\mathbf{P}_i^{(l)})+P_c}\geq\lambda_i e \quad \forall i\neq k. \tag{54c}$$

Algorithm 4 Distributed Algorithm for MWEE Maximization

Initialization
Set $l=1$, and $p_k=p_k^{(0)}$ for all k
Repeat
 For $k=1,\cdots,K$
  Fix p_i for $i\neq k$ and obtain $a_k^{(l)}$, $\tilde{d}_k^{(l)}$
  Obtain $p_k^{(l)}$ by solving (47), i.e., employing Theorem 1, and conducting a bisection
 End (For)
 $l=l+1$
End (Repeat)

We can obtain the global optimal solution of (54) by solving a sequence of feasibility problems. That is, we fix e as e^c and solve the convex feasibility problem in (55), shown at the bottom of this page. The global optimal solution of (54), $e^\star$ and $\mathbf{P}_k^\star$, can be found by conducting a bisection over e^c.

E. BEAMFORMING OPTIMIZATION AT RECEIVER SIDE

In the previous subsections, we propose power optimizations for the transmitters by considering the maximum achievable rates in (8). In this section, we propose beamforming vectors at the receiver sides to achieve the rates in (8) for the K-user SIMO IC. The received signal after the beamforming is

$$y_k'=\mathbf{w}_k^H\mathbf{y}_k=\mathbf{w}_k^H(\mathbf{h}_{kk}x_k+\mathbf{d}_k) \tag{56}$$

where $\mathbf{d}_k$ and $\mathbf{w}_k$ are, respectively, the interference plus noise and beamforming vector at receiver k. Thus, the rate of user k is

$$R_k=\log_2\left(1+\frac{p_k|\mathbf{w}_k^H\mathbf{h}_{kk}|^2}{\mathbf{w}_k^H\mathbf{D}_k\mathbf{w}_k}\right), \tag{57}$$

where $\mathbf{D}_k=\mathbb{E}\left\{\mathbf{d}_k\mathbf{d}_k^H\right\}$. Since the rate of user $\bar{k}$ is independent of $\mathbf{w}_k$, we choose $\mathbf{w}_k$ such that the rate of user k is maximized, i.e.,

$$\max_{\mathbf{w}_k} \ R_k. \tag{58}$$

$$\max_{0\leq p_k\leq P_k} \ e, \quad \text{s.t.} \quad \log_2\left(1+p_k\tilde{d}_k^{(l)}\right)-\lambda_k e^{(t)}\left(\eta_c p_k+P_c\right)\geq 0, \tag{45a}$$

$$R_i^{(l,k)}-a_{ik}^{(l)}(p_k-p_k^{(l-1)})-\lambda_i e^{(t)}\left(\eta_c p_i^{(l,k)}+P_c\right)\geq 0 \quad \forall i\neq k. \tag{45b}$$

$$\text{find } 0\leq p_k\leq P_k, \quad \text{s.t.} \quad \log_2\left(1+p_k\tilde{d}_k^{(l)}\right)-\lambda_k e^{(t)}\left(\eta_c p_k+P_c\right)\geq 0, \tag{46a}$$

$$R_i^{(l)}-a_{ik}^{(l)}(p_k-p_k^{(l)})-\lambda_i e^{(t)}\left(\eta_c p_i^{(l,k)}+P_c\right)\geq 0 \quad \text{for} \quad \forall i\neq k. \tag{46b}$$

$$\text{find } \{\mathbf{P}_k\}\in\mathcal{P}_M, \quad \text{s.t.} \quad \log_2\left|\mathbf{D}_k^{(l)}+\mathbf{H}_{kk}\mathbf{P}_k\mathbf{H}_{kk}^H\right|-\lambda_k e^c\left(\eta_c\text{Tr}(\mathbf{P}_k)+P_c\right)\geq 0, \tag{55a}$$

$$R_i^{(l)}-\text{Tr}\left(\mathbf{A}_{ik}^{(l)}(\mathbf{P}_k-\mathbf{P}_k^{(l)})\right)-\lambda_i e^c\left(\eta_c\text{Tr}(\mathbf{P}_i^{(l)})+P_c\right)\geq 0 \quad \text{for} \quad \forall i\neq k. \tag{55b}$$

Since logarithm is a monotone function, the optimization problem (58) is equivalent to

$$\max_{\mathbf{w}_k} \frac{p_k|\mathbf{w}_k^H\mathbf{h}_{kk}|^2}{\mathbf{w}_k^H\mathbf{D}_k\mathbf{w}_k}. \quad (59)$$

The closed-form solution of (59) is the eigenvector corresponding to the largest eigenvalue of $\mathbf{D}_k^{-1}(\mathbf{h}_{kk}\mathbf{h}_{kk}^H)$. Note that since the solution is scale-invariant, we choose $\mathbf{w}_k$ such that $\mathbf{w}_k^H\mathbf{w}_k = 1$.

V. SLNR-BASED NON-COOPERATIVE DISTRIBUTED ALGORITHM

In the distributed framework described in Sec. III, the users or BSs have to cooperate to update their transmission powers. However, it is not always possible to have the required level of cooperation among the users. Thus, in this section we propose an alternative suboptimal distributed algorithm in which each user needs to know only its own channels, and there is no need to exchange any information among the users. This non-cooperative framework is based on the so-called SLNR, which is defined in [26] as

$$\mathbf{L}_k \triangleq \mathbf{J}_k(p_k)^{-1}\mathbf{S}_k(p_k), \quad (60)$$

where $\mathbf{S}_k(p_k) = p_k\mathbf{h}_{ik}\mathbf{h}_{ik}^H$ is defined as in (7), and $\mathbf{J}_k(p_k)$ is the aggregated pseudo-interference-plus-noise (PIN), which is generated by user k in the system as

$$\mathbf{J}_k(p_k) = \underbrace{\sigma^2\mathbf{I}}_{\text{noise covariance}} + \underbrace{p_k\sum_{i=1,i\neq k}^{K}\mathbf{h}_{ik}\mathbf{h}_{ik}^H}_{\text{pseudo-interference covariance}}. \quad (61)$$

Note that, in order to construct $\mathbf{L}_k$ in (60), each receiver must have the same number of antennas. Moreover, $\mathbf{L}_k$ depends only on the channels from user k. We define the virtual rate for user k as

$$R'_k = \log_2\left|\mathbf{I}+\mathbf{J}_k^{-1}\mathbf{S}_k\right| = \underbrace{\log_2|\mathbf{J}_k+\mathbf{S}_k|}_{\triangleq r'_{1,k}} - \underbrace{\log_2|\mathbf{J}_k|}_{\triangleq r'_{2,k}}. \quad (62)$$

The virtual rate is a difference of two concave functions in p_k, i.e., $r'_{1,k}$ and $r'_{2,k}$ are concave. Additionally, we define the virtual energy efficiency of user k as

$$E'_k = \frac{R'_k}{\eta_c p_k + P_c}. \quad (63)$$

The non-cooperative approach maximizes either the virtual rate and/or the virtual EE. Intuitively, we want to maximize the ratio of the desired signal to the undesired signal provoked by each transmitter, and hence the signaling overhead is reduced significantly.

A. VIRTUAL RATE MAXIMIZATION

The virtual rate maximization problem for user k is

$$\max_{0\leq p_k\leq P_k} R'_k = \log_2|\mathbf{J}_k+\mathbf{S}_k| - \log_2|\mathbf{J}_k|. \quad (64)$$

This optimization problem is not convex; however, we can again apply MM and use an affine upper bound for $r'_{2,k}$ as

$$r'_{2,k}(p_k) \leq \tilde{r}'^{(l)}_{2,k}(p_k) = r'_{2,k}(p_k^{(l)}) + a_k^{(l)}(p_k - p_k^{(l)}), \quad (65)$$

where $a_k^{(l)}$ is the derivative of $r'_{2,k}$ with respect to p_k at $p_k^{(l)}$ and can be derived by replacing $p_k^{(l)}$ in

$$\frac{\partial r'_{2,k}(p_k)}{\partial p_k} = \frac{1}{\ln 2}\sum_{i=1,i\neq k}^{K}\mathbf{h}_{ik}^H\mathbf{J}_k^{-1}\mathbf{h}_{ik}. \quad (66)$$

Finally, a stationary point of (64) can be derived by iteratively solving

$$\max_{0\leq p_k\leq P_k} f_k(p_k) \quad (67)$$

and updating $p_k^{(l)}$, where

$$f_k(p_k) = \log_2\left|\sigma^2\mathbf{I}+p_k\sum_{i=1}^{K}\mathbf{h}_{ik}\mathbf{h}_{ik}^H\right| - a_k^{(l)}(p_k - p_k^{(l)}) - r'_{2,k}(p_k^{(l)}). \quad (68)$$

The optimization problem (67) is convex since its objective function $f_k(p_k)$ is concave in p_k. The following theorem gives its solution.

Theorem 2: The optimal solution of (67) *is* $p_k^{(\star)} = \min\{P_k, p^{(t)}\}$*, where* $p^{(t)}$ *is the solution of*

$$\sum_{i=1}^{K}\mathbf{h}_{ik}^H\left(\sigma^2\mathbf{I}+p_k\sum_{i=1}^{K}\mathbf{h}_{ik}\mathbf{h}_{ik}^H\right)^{-1}\mathbf{h}_{ik} = a_k^{(l)}\ln 2. \quad (69)$$

Proof: The function $f_k(p_k)$ is concave, continuous and differentiable in p_k. Thus, it maximizes at $\frac{\partial f_k(p_k)}{\partial p_k} = 0$, which is denoted by $p^{(t)}$. Moreover, $f_k(p_k)$ is strictly increasing for $p_k < p^{(t)}$. Thus, $f_k(p_k)$ is maximized at $p_k = p_k^{(\star)}$ for $0 \leq p_k \leq P$. □

Note that there is no closed-form solution for (69); however, its optimal solution can easily be found by employing a bisection over p_k.

1) MIMO SYSTEMS

In this case, the affine upper bound for $r'_{2,k}$ is

$$\tilde{r}'^{(l)}_{2,k}(\mathbf{P}_k) = r'_{2,k}\left(\mathbf{P}_k^{(l)}\right) + \mathrm{Tr}\left(\mathbf{A}_k^{(l)^H}(\mathbf{P}_k - \mathbf{P}_k^{(l)})\right), \quad (70)$$

where $\mathbf{A}_k^{(l)}$ is the derivative of $r'_{2,k}$ with respect to $\mathbf{P}_k$ at $\mathbf{P}_k^{(l)}$ and can be derived by replacing $\mathbf{P}_k^{(l)}$ in

$$\frac{\partial r'_{2,k}(\mathbf{P}_k)}{\partial \mathbf{P}_k} = \frac{1}{\ln 2}\sum_{i=1}^{K}\mathbf{H}_{ik}^H\mathbf{J}_k^{-1}\mathbf{H}_{ik}. \quad (71)$$

We can derive a stationary point of (64) by replacing $r'_{2,k}$ with the lower (70) and updating $\mathbf{P}_k^{(l)}$.

B. VIRTUAL EE MAXIMIZATION

Using the SNLR definition, the virtual EE optimization problem is

$$\max_{0\leq p_k\leq P_k} E'_k = \frac{\log_2|\mathbf{J}_k+\mathbf{S}_k| - \log_2|\mathbf{J}_k|}{\eta_c p_k + P_c}. \tag{72}$$

This optimization problem is nonconvex, but a stationary point can be found by applying MM and the Dinkelbach algorithm. We first apply MM and approximate $r'_{2,k}$ by the affine upper bound $\tilde{r}'_{2,k}$ in (65), which results in

$$\max_{0\leq p_k\leq P_k} \tilde{E}'_k = \frac{\log_2|\mathbf{J}_k+\mathbf{S}_k| - \tilde{r}'_{2,k}(p_k)}{\eta_c p_k + P_c}. \tag{73}$$

Now, we apply Dinkelbach algorithm and solve

$$\max_{0\leq p_k\leq P_k} \log_2\left|\sigma^2\mathbf{I} + p_k\sum_{i=1}^{K}\mathbf{h}_{ik}\mathbf{h}_{ik}^H\right| - \tilde{r}'^{(l)}_{2,k}(p_k) - \mu^{(m,l)}\left(\eta_c p_k + P_c\right), \tag{74}$$

where $\mu^{(m,l)}$ is a constant given by

$$\mu^{(m,l)} = \tilde{E}'_k\left(p_k^{(m-1,l)}\right), \tag{75}$$

where $p_k^{(m-1,l)}$ is the solution of (74) at the $(m-1)$th iteration. We iteratively solve (74) and update $\mu^{(m,l)}$ until a convergence criterion is met. This algorithm converges to the global optimal solution of (77). Furthermore, the whole algorithm converges to a stationary point of (72). The following theorem characterizes the optimal solution of (74).

Theorem 3: The optimal solution of (74) *is* $p_k^{(\star)} = \min\{P_k, p^{(t)}\}$*, where* $p^{(t)}$ *is the solution of*

$$\sum_{i=1}^{K}\mathbf{h}_{ik}^H\left(\sigma^2\mathbf{I} + p_k\sum_{i=1}^{K}\mathbf{h}_{ik}\mathbf{h}_{ik}^H\right)^{-1}\mathbf{h}_{ik} = (a_k^{(l)} + \eta_c\mu^{(m,l)})\ln 2. \tag{76}$$

Proof: It can be proved similar to Theorem 2 by replacing $a_k^{(l)}\ln 2$ with $(a_k^{(l)} + \eta_c\mu^{(m,l)})\ln 2$. □

The solution of (76) can be found by a bisection over p_k.

1) MIMO CASE

Substituting $\tilde{r}'_{2,k}$ in (65), we have

$$\max_{\{\mathbf{P}_k\}\in\mathcal{P}_M} \tilde{E}'_k = \frac{\log_2|\mathbf{J}_k+\mathbf{S}_k| - \tilde{r}'_{2,k}(\mathbf{P}_k)}{\eta_c \mathrm{Tr}(\mathbf{P}_k) + P_c}. \tag{77}$$

We can obtain its global optimal solution by the Dinkelbach algorithm; however, due to the space restriction, we cannot provide the detailed solution in the paper.

VI. NUMERICAL RESULTS

In this section, we provide some numerical examples by conducting Monte Carlo simulations[2] and averaging the results over 100 channel realizations. Each entry of every channel realization is drawn from a zero-mean complex proper Gaussian distribution with unit variance, i.e., $\mathcal{CN}(0, 1)$. We assume that the power budget of the users is the same, i.e., $P_k = P$ for all k. We stop the MM algorithms when the improvement in the objective of the corresponding optimization problem in the new step of the algorithm is less than 0.001%, or after a maximum number of 50 iterations. Note that MM-based algorithms depend on the initial point, and therefore the performance might change by considering different initial points. The algorithms are initialized as $p_k^{(0)} = P$ for the rate optimization and $p_k^{(0)} = 0.3P$ for the EE optimization. For the simulations, we define the SNR as the ratio of the power budget to σ^2, i.e., $\text{SNR} = \frac{P}{\sigma^2}$.

The following algorithms are compared in this section:

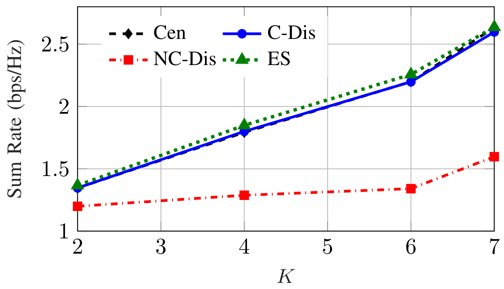
- **Cen**: The centralized algorithm in [7].
- **C-Dis**: The proposed cooperative distributed algorithm.
- **NC-Dis**: The proposed non-cooperative distributed algorithm.
- **ES**: The solution obtained by conducting the exhaustive search (ES).
- **C-BF**: Our proposed cooperative distributed algorithm with max-SINR beamforming.
- **C-MF**: The proposed cooperative distributed algorithm with matched filtering beamforming, employed in [5].

Since, to the best of our knowledge, there is no other work that studies all the considered optimization problems, we use as a reference the solution obtained by conducting an exhaustive search over the transmission powers, which can be considered as a good approximation for the optimal solution. Unfortunately, the exhaustive search requires a high computational cost, especially when the number of users is large, or when multi-antenna receivers are used. Therefore, we consider the exhaustive search only for SISO systems. We consider a uniform quantization over transmission powers with step size 0.02 when $P = 1$. Note that, as indicated, our proposed algorithm converges to a stationary point of the considered problems, which meets the first order optimality conditions and performs very close to optimum solution especially for SISO systems, as verified by our simulations. We also compared our proposed beamforming with the fixed beamforming obtained by employing matched filter [5]. It is worth emphasizing that the NC-Dic, C-BF and C-MF algorithms are identical in the K-user SISO IC since there is only one antenna at receivers. Thus, we consider only NC-Dis in the K-user SISO IC.

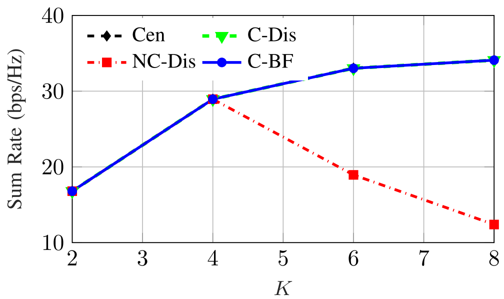
A. SUM-RATE MAXIMIZATION

Figure 2 shows the average sum rate of the K-user IC as a function of the number of users, K, for SNR = 20 dB, $N_r = 5$ and SNR = 0 dB, $N_r = 1$. As can be observed, the proposed cooperative distributed and centralized algorithms perform similarly, and their solutions are close to the ES solutions in the SISO case. It is worth emphasizing that the computational complexity of our proposed cooperative distributed algorithm is much lower than the other algorithms. Additionally, it can be observed that these algorithms

[2]The simulation codes will be available on our repository at https://github.com/SSTGroup.



(a) $N_r = 1$ and SNR$= 0\,$dB.



(b) $N_r = 5$ and SNR$= 20\,$dB.

FIGURE 2. The average sum rate versus the number of users for the *K*-user SISO and SIMO ICs.

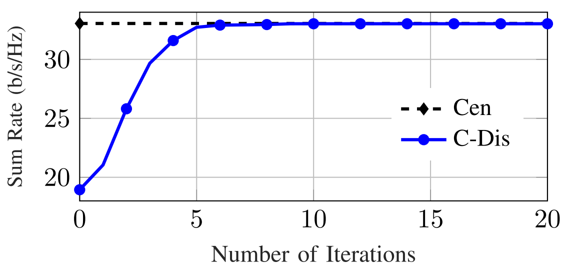
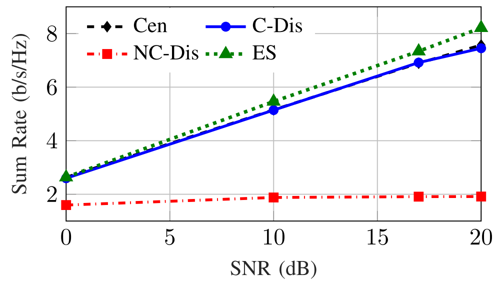


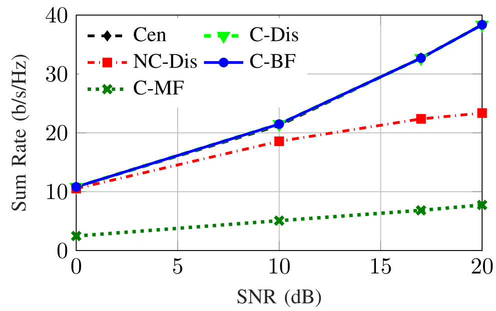
FIGURE 3. The average sum rate versus the number of iterations for the 6-user 1 × 5 SIMO IC with SNR = 20 dB.

outperform the non-cooperative algorithm for large K ($K > 4$). It seems that for a small number of users, the total level of interference is not significant, and the users do not benefit from cooperation. Moreover, we observe that the sum rate for the centralized and cooperative distributed algorithms increases with K. This is in contrast with the non-cooperative algorithm for SIMO case in which the sum rate decreases for large K ($K > 4$) as shown in Fig. 2b. The reason for this behavior is that we do not consider any quality-of-service constraint for individual user rates in the distributed and centralized algorithms. Thus, the users with a weak direct channel and/or strong interference channels might be switched off to reduce the aggregated network interference. However, in the non-cooperative algorithm, all users transmit simultaneously, which results in severe interference for large K. We also observe that the optimal max-SINR beamforming algorithm performs very close to the maximum rates in (8) and significantly outperforms the matched filter beamformer.

Figure 3 shows the average sum-rate versus the number of iterations for the 6-user 1×5 SIMO IC with SNR $= 20\,$dB. As can be observed, the cooperative distributed algorithm



(a) 7-user SISO IC.



(b) 7-user 1×6 SIMO IC.

FIGURE 4. The average sum rate versus SNR for the 7-user SISO and SIMO ICs.

converges very fast and reaches to 99% of its final value after only 5 iterations. Hence, the cooperative distributed algorithm can be efficiently implemented with affordable information exchange. It is worth emphasizing that the centralized algorithm is also iterative. In Fig. 3, we show only the final value of the centralized algorithm since our main focus is on the signaling exchange for the distributed algorithm.

In Fig. 4, we consider the effect of SNR on the average sum rate of the 7-user IC with $N_r = 1, 6$. As can be observed, the centralized and cooperative distributed algorithms have a similar performance and outperform the non-cooperative algorithm. Moreover, the centralized and cooperative distributed algorithms perform very close to the ES solution for the SISO case. Additionally, the proposed beamforming algorithm performs very close to the rates in (8). However, the performance of the matched filter algorithm is even worse than the non-cooperative distributed algorithm. The performance gap between the non-cooperative algorithm and the other algorithms increases with SNR. We also observe in Fig. 2 a similar behavior when the number of users increases.

Figure 5 shows the average sum rate of the K-user 4×4 MIMO IC as a function of the number of the users, K, for SNR $= 10\,$dB. As can be observed, the overall behavior is similar to the SIMO case. In other words, the proposed cooperative distributed and centralized algorithms perform similarly since they both converge to a stationary point of the considered problem. Moreover, these algorithms outperform the SLNR-based algorithm.

B. GLOBAL ENERGY EFFICIENCY

Figure 6 shows the effect of K on the average global EE of the K-user IC with $P_c = 10$, $N_r = 3$, SNR $= 10\,$dB

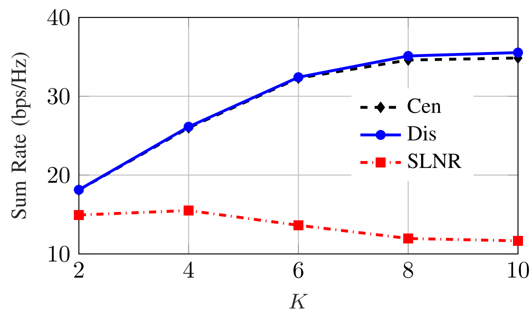
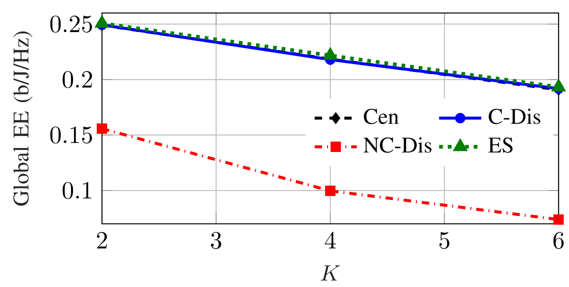
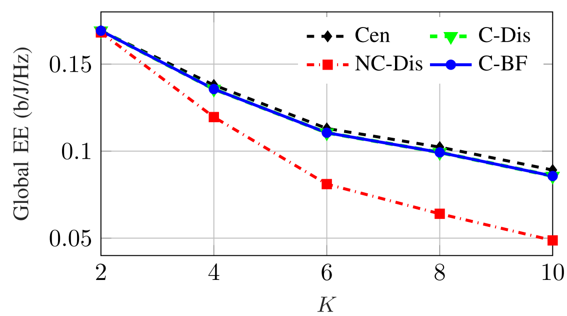


FIGURE 5. The average sum rate versus K for the K-user 4×4 MIMO IC and SNR = 10 dB.



(a) $N_r = 1$ and SNR= 0 dB.



(b) $N_r = 3$ and SNR= 10 dB.

FIGURE 6. The average global EE versus the number of cells for the K-user SISO and SIMO ICs with $P_c = 10$.

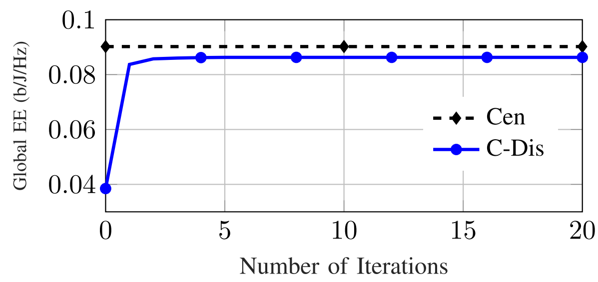


FIGURE 7. The average global EE versus the number of iterations for the 10-user 1×3 SIMO IC with $P_c = 10$ and SNR = 10 dB.

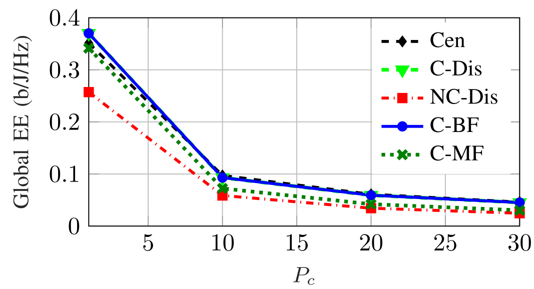


FIGURE 8. The average global EE versus P_c for the 6-user 1×3 SIMO IC with SNR = 10 dB.

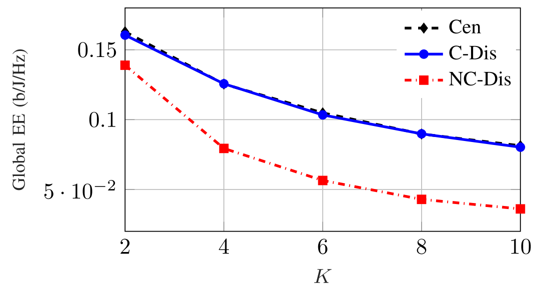


FIGURE 9. The average global EE versus K for the K-user 3×3 MIMO IC and SNR = 10 dB.

and $P_c = 10$, $N_r = 1$, SNR = 0 dB. As can be observed, the cooperative distributed algorithm performs very closely to the centralized solution and outperforms the non-cooperative algorithm. Additionally, the cooperative distributed and centralized algorithms perform very close to the ES solutions for the SISO case. Furthermore, the average global EE decreases with K as expected.

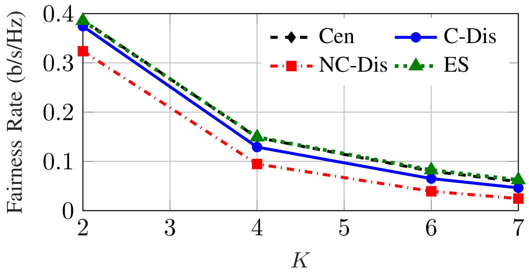
Figure 7 shows the average global EE versus the number of iterations for the 10-user 1×3 SIMO IC with $P_c = 10$ and SNR = 10 dB. As can be observed, the cooperative distributed algorithm converges very fast and reaches to 99% of its final value after only 2 iterations.

Figure 8 shows the effect of P_c on the average global EE for the 6-user 1×3 SIMO IC with SNR = 10 dB. As can be observed, the cooperative distributed and centralized algorithms perform similarly and outperform the non-cooperative algorithm. Moreover, our proposed beamforming is very close to the upper bound performance and outperforms the algorithm with matched filter.

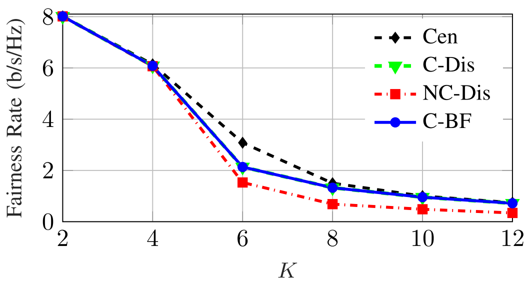
Figure 9 shows the effect of K on the average global EE of the K-user 3×3 MIMO IC for SNR = 10 dB. As can be observed, the distributed algorithm performs very closely to the centralized algorithm and outperforms the SLNR-based algorithm. Additionally, the average global EE decreases with K as expected.

C. RATE REGION

Figure 10 shows the effect of the number of users on the average fairness rate of the K-user IC with $N_r = 5$, SNR = 10 dB and $N_r = 1$, SNR = 0 dB. As can be observed in the SIMO case, the centralized algorithm outperforms the cooperative distributed algorithm for a moderate number of users. However, the cooperative distributed algorithm performs close to the centralized algorithm for a low and large number of users. Additionally, in the SISO case, there is a relatively considerable performance gap between our distributed cooperative and the ES solutions. This is in contrast with the results in Figs. 2-4, where the cooperative distributed algorithm performs similarly to the centralized algorithm and the ES solutions for the sum-rate maximization. This is due to the fact that the optimization problem for the sum-rate



(a) $N_r = 1$ and SNR$= 0$ dB.



(b) $N_r = 5$ and SNR$= 20$ dB.

FIGURE 10. **The average fairness rate versus the number of users for the *K*-user SISO and SIMO ICs.**

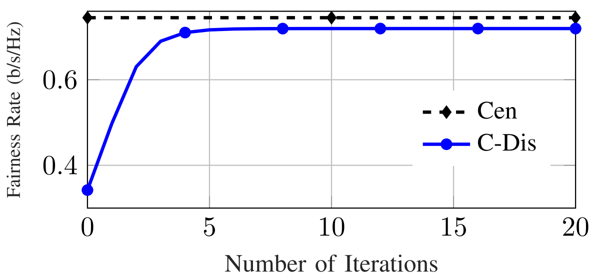


FIGURE 11. **The average fairness rate versus the number of iterations for the 12-user 1 × 5 SIMO IC with SNR = 20 dB.**

maximization is simpler than the rate-region-optimization problem since there is no rate function in the constraints for the sum-rate maximization. However, in the rate-region problem, the rate of each user appears in each constraint, and the approximations made by the distributed algorithm degrade the performance. Note that this performance degradation can be reduced if we conduct a randomization on the initial point of the distributed algorithm and choose the best solution.

Figure 11 shows the fairness rate versus the number of iterations for the 12-user 1×5 SIMO IC with SNR $= 20$ dB. We can observe that the cooperative distributed algorithm converges very fast and reaches to 99% of its final value after 5 iterations, which makes this algorithm very efficient for distributed implementations.

Figure 12 shows the average fairness rate of 7-user 1×6 SIMO IC versus the SNR. As can be observed, the centralized algorithm outperforms the cooperative distributed algorithm, which is in line with the results in Fig. 10 for a moderate number of users. Moreover, the proposed max-SINR beamforming design performs identical to the rate in (8), but the performance of matched filter is even worse than the non-cooperative design. We also observe that the performance gap between the centralized and the cooperative distributed algorithms increases with the SNR.

Figure 13 shows the effect of SNR on the average fairness rate of the 4-user 6×6 MIMO IC. As can be observed, the

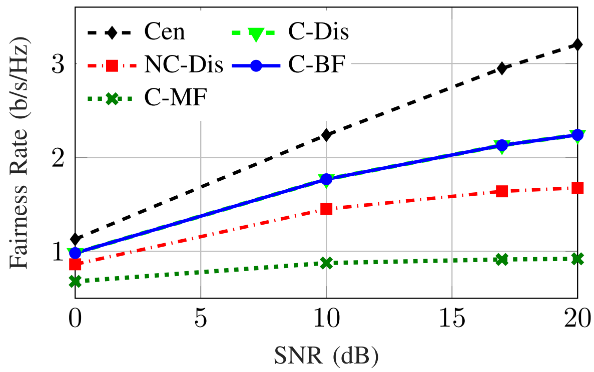


FIGURE 12. **The average fairness rate versus SNR for the 7-user 1 × 6 SIMO IC.**

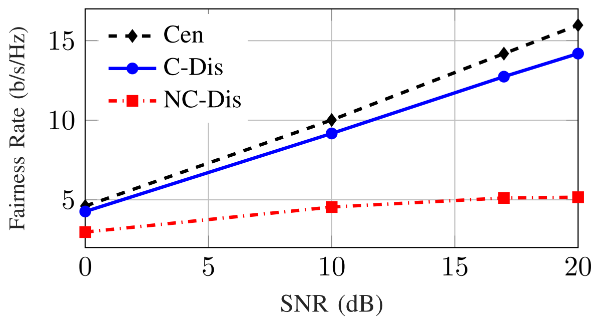
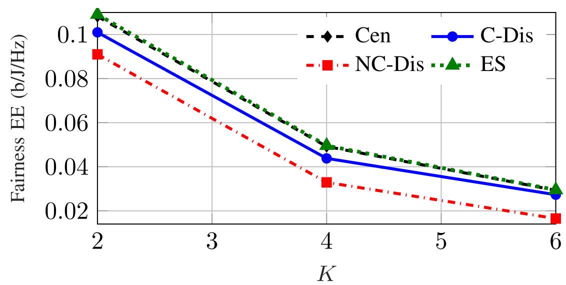
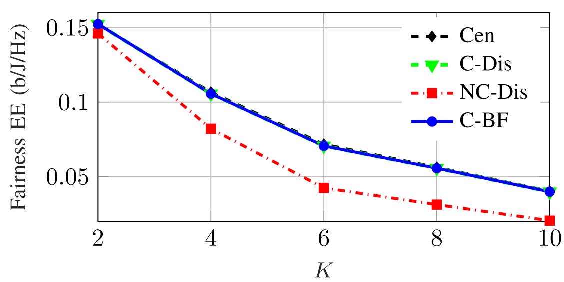


FIGURE 13. **The average fairness rate versus SNR for the 4 × 4 MIMO *K*-user IC.**



(a) $N_r = 1$ and SNR$= 0$ dB.



(b) $N_r = 3$ and SNR$= 10$ dB.

FIGURE 14. **The average fairness EE versus the number of users for the *K*-user SISO and SIMO ICs with $P_c = 10$.**

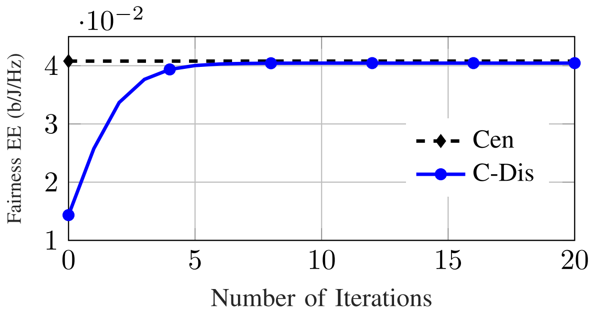


FIGURE 15. The average fairness EE versus the iterations for the 10-user 1×3 SIMO IC with $P_c = 10$ and SNR = 10 dB.

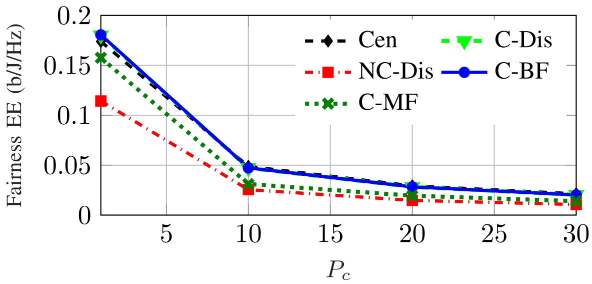


FIGURE 16. The average fairness EE versus P_c for a 6-user 1×3 IC with SNR = 10 dB.

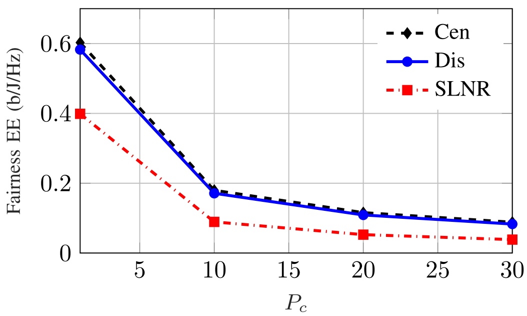


FIGURE 17. The average fairness EE versus P_c for the 4-user 3×3 MIMO IC.

centralized algorithm outperforms the distributed algorithm, and the performance gap between the centralized and distributed algorithm increases with SNR.

D. ENERGY-EFFICIENCY REGION

Figure 14 shows the average fairness EE for the K-user 1×3 SIMO IC and with SNR = 10 dB, and the K-user SISO IC with SNR = 0 dB versus the number of users. As it can be observed, the cooperative distributed algorithm performs very similar to the centralized algorithm and the ES solutions.

In Fig. 15, we show the average fairness EE versus the number of iterations for the 10-user 1×3 SIMO IC with $P_c = 10$ and SNR = 10 dB. We can observe that the cooperative distributed algorithm converges very fast and reaches to 99% of its final value after 5 iterations.

Figure 16 shows the average fairness EE as a function of P_c for the 6-user 1×3 SIMO IC with SNR = 10 dB. As can be observed, the cooperative distributed algorithm performs very close to the centralized algorithm. Additionally, our beamforming design performs very close to the rates in (8) and outperforms the matched filter design.

Finally, Fig. 17 shows the average fairness EE as a function of P_c for the 4-user 3×3 MIMO ICs. As can be observed, the distributed algorithm performs very close to the centralized algorithm and outperforms the non-cooperative algorithm.

VII. CONCLUSION

In this paper, we have proposed two distributed power control frameworks for the K-user IC when interference is treated as noise at the receivers. Our cooperative distributed algorithm can be applied to any optimization problem in which the objective and/or constraint functions are linear functions of the achievable rates of users. Such optimization problems include weighted-sum-rate maximization, global EE maximization, rate-region optimization and EE-region optimization among others. The proposed cooperative distributed algorithm requires a small amount of information exchange between users. Additionally, the resulting optimization problems sometimes have closed-form power updating rules for the K-user SIMO IC. Through numerous simulation results over a large range of parameters, we have observed that our cooperative algorithm performs close to the centralized algorithm and even close to the optimal solution, especially for the weighted-sum-rate, EE region and global EE maximization problems. The reason is that, our cooperative distributed algorithm obtains a stationary point of the considered optimization problems and meets the same optimality conditions as the centralized algorithm. Furthermore, our cooperative algorithm converges very fast requiring only a few power updating rounds, which implies that this algorithm can be efficiently implemented in practical scenarios. We also proposed a non-cooperative distributed algorithm, which further reduces the amount of information exchange between BSs required by the cooperative algorithms. In the non-cooperative algorithm, users need to know only their own channels. As expected, the cooperative distributed algorithm outperforms the non-cooperative distributed algorithm.

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