# Robust MPPT Observer-Based Control System for Wind Energy Conversion System With Uncertainties and Disturbance

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**ABSTRACT** The problem of tracking the maximum power point for the wind energy conversion system (WECS) is taken into consideration in this paper. The WECS in this article is simultaneously affected by the uncertainties and the arbitrary disturbance that cause the WECSs to be much more challenging to control. A new method to synthesize a polynomial disturbance observer for estimating the aerodynamic torque, wind speed, and electromagnetic torque without using sensors is proposed in this paper. Unlike the previous methods, in this work, both the uncertainties and the disturbance are estimated, then estimations of the uncertainties and disturbance are transmitted to the Linear Quadratic Regulator (LQR) controller for eliminating the influences of the uncertainties and disturbance; and tracking the optimal power point of WECS. It should be noted that the uncertainties in this work are time-varying and both uncertainties and disturbance do not need to satisfy the bounded constraints. The wind speed and aerodynamic torque are arbitrary and unnecessary to fulfill the low-varying constraint or $r^{th}$ time derivative bound. On the basis of Lyapunov methodology and the sum-of-square technique (SOS), the main theorems are derived to design the polynomial disturbance observer. Finally, the simulation results are provided to demonstrate the effectiveness and merit of the proposed method.

**INDEX TERMS** WECSs, LQR, disturbance observer, uncertainties, polynomial observer, SOS.

## I. INTRODUCTION

Nowadays, air pollution and global warming caused by using fossil fuels are increasingly becoming a serious problem in the whole world that affects not only human health but also economic development. Due to this reason, discovering environmentally friendly energy resources to replace the fossil fuels is a pressing issue. Among new renewable energies developed in recent years, wind energy is considered as an efficient and free-pollution renewable resource. Recently, there is a fast-growing number of studies focusing on wind energy conversion system (WECS) [1]–[7]. For example,

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Wu *et al.* [1], the sliding mode control was applied for tracking the Maximum Power Point of the Low-Power WECS and improving the performance of the system. The new approach to synthesize the MPPT (maximum power point tracking) scheme based on dc-link voltage has been investigated for the small scale WECS with a fast-changing wind speed condition in paper [2]. Hodzic and Tai [3] have developed the novel purb and observe (P&O) algorithm to track the optimal power point and decrease the effects of harmonic distortion of the wind energy conversion system. Moreover, the MPPT algorithm for the offshore wind turbine system has been presented in [6] in which the active-rectifier d-axis current was employed to control the whole system. Although there are plenty of studies concentrating on WECS in the past few

years, up to now, studying WECS is still a promising land for researchers.

Moreover, to design the controller for tracking optimal power point, the information of wind speed and other state variables of the WECS is necessary for sending back to the controller. Regarding obtaining the wind speed information, it can be measured directly by anemometers; however, using anemometers maybe obtain an inaccurate result and it also causes to increase the cost to build the system. In order to overcome the mentioned-above limitations, the wind speed can be estimated by employing the observer [9]–[16] instead of using the anemometers. For instance, the nonlinear disturbance observer and the sliding mode control were studied to estimate the aerodynamic torque, wind speed [9], and track the optimal power point and the high-order observer form was proposed to estimate the aerodynamic torque and wind speed [10]. The polynomial observer has been proposed by Vu and Do [11] to aerodynamic torque, electromagnetic torque, and stator current in axis-d in which aerodynamic torque did not need to satisfy any constraint. Moreover, a nonlinear observer was designed for WECS to estimate the rotor speed, position, and the turbine torque in paper [12] where the frequency of the grid was time-varying and unknown. The polynomial observer combining with fault-tolerant control was studied for the WECS to eliminate the influence of faults as well as tracking the maximum power point of WECS [13]. The sliding mode observer-based nonlinear controller has been developed by Hussain and Mishra [14] to estimate the wind speed, aerodynamic torque, and rotor speed of WECS. Although the observer in previous papers can estimate the aerodynamic torque and wind speed very well, there still exist several drawbacks. For example, the aerodynamic torque and wind speed must satisfy the constraint that the first-derivative of aerodynamic torque and wind speed is equal to zero [9]; or the aerodynamic torque and wind speed still need to fulfill the constraint that the $r^{th}$ time derivative of aerodynamic torque and wind speed must be bounded [10].

In practice, the systems are always impacted by the uncertainties and the disturbance [17]–[19]. The uncertainties may origin from the parameter error and/or the modeling error. Both the uncertainties and system disturbance will make the controller design more challenging and even cause to degrade the performance of the system. The WECS is not an exception, it is also strongly affected by the disturbance and the uncertainties. Recently, there are several studies paying attention to the control of the WECS with the existence of the uncertainties and/or disturbance [9], [20]–[23]. In paper [9], the time-varying uncertainty affected WECS; however, the uncertainty of this case was only impacted to the mechanical part of WECS and the system disturbance was not considered in this work. The WECS was modeled in terms of the uncertain T-S fuzzy model and then the robust observer-based controller was designed to estimate the wind speed [20]. Nonetheless, the uncertainties considered by Sung *et al.* [20] must satisfy the bounded constraints. The high-order observer was also synthesized for the WECS with the effects of the

uncertainties to estimate the wind speed [21]. However, the uncertainties in the paper [21] were time-unvarying and this work omitted the effects of the system disturbance. The active disturbance rejection controller was synthesized for the WECS in papers [22] and [23], but these two papers only consider the existence of the disturbance and did not concern the influence of the uncertainties. With the above analyses, it is seen that the previous papers either deal with the uncertainties [9], [20], [21] or take into consideration the disturbance [22], [23]. To the best of our knowledge, up to now, there is not any paper considering the WECS system with the existence of both uncertainties and disturbance simultaneously.

In recent years, a new format to model the system called "polynomial linear system" was investigated in many papers [23]–[28]. The polynomial linear system, actually, is an extended format of the linear system. However, the difference between the polynomial linear system and the linear system is that the matrices of the polynomial linear system are varying matrices and are expressed in terms of polynomial form while the matrices of the linear system are constant. Employing the polynomial linear system to model the nonlinear system will decrease the number of linearization terms and reduce the modeling error as well. Recently, with the aid of the SOS tool of Matlab [29], solving the conditions expressed under polynomial format to design controller and observer becomes easy and efficient. Therefore, in this paper, the WECS will be represented under the framework of the polynomial linear system and then the polynomial observer is synthesized for this system.

On the basis of the above discussions, we are inspired to develop a new method for designing a robust disturbance observer-based controller to track the optimal power point of WECS that is simultaneously influenced by time-varying uncertainties and the disturbance. The contributions of this paper are presented as follows

i) Unlike the previous papers that ignored the effects of the uncertainties and disturbance of WECS system [10]–[16], or merely considered the impacts of the uncertainties [9], [20], 21] or disturbance [18], [19], the WECS system in our paper is simultaneously influenced by both the uncertainties and disturbance.

ii) The uncertainties in this paper are time-varying uncertainties that more relax than the constant uncertainties do in paper [21]. In addition, the uncertainties are unnecessary to fulfill the bounded constraints that are mandatory in the paper [20].

iii) The WECS is represented in terms of the polynomial linear system that will assist to decrease the modeling errors with respect to the method used T-S fuzzy model in [20].

iv) The wind speed and aerodynamic torque in this article are more relaxed with respect to the previous studies, because they do not need to fulfill the low-varying constraint as in paper [9], and $r^{th}$ time derivative of

aerodynamic torque and wind speed must be bounded as in paper 10], [20].

v) The robust polynomial disturbance observer-based controller that is the combination of disturbance observer and LQR controller is synthesized to estimate aerodynamic torque, wind speed, electromagnetic torque to track the maximum power point. Moreover, both the uncertainties and the disturbance are also estimated and then this information is transmitted to the LQR controller to eliminate completely the impacts of the uncertainties and the system disturbance.

The remains of this paper are structured as follows. The mathematical model of WECS and the problem description are presented in Section 2. Section 3 provides the method to design LQR controller. The polynomial disturbance observer synthesis for WECSs is shown in Section 4. The stability analysis of the closed-loop WECS system is mentioned in Section 5. Section 6 will provide the simulation. Finally, the conclusions are presented in Section 7.

*Notations:* $\Lambda^T$ and $\Lambda^{-1}$ indicate the transpose and inverse of matrix $\Lambda$, respectively. $\Lambda > 0$ ($\Lambda < 0$) means that $\Lambda$ is the positive(negative) definite matrix. $I$ is an identity matrix.

## II. SYSTEM DESCRIPTION AND PROBLEM STATEMENT
### A. MATHEMATICAL MODEL OF WECS
In practice, the power obtained by the wind turbine (WT) is described in the following equation:

$$P_a = \frac{1}{2}\rho\pi R^2 C_p(\lambda, \beta) v^3 \qquad (1)$$

in which $P_a$ is the aerodynamics power, $v$ indicates the wind speed; $\rho$ and $R$ denote the WT rotor radius and the air density, respectively. $C_p(\lambda, \beta)$ is for the power coefficient which is a nonlinear function dependent on the tip-speed ratio $\lambda$ and the pitch angle $\beta$ of the blades.

The tip-speed ratio $\lambda$ is calculated as follows

$$\lambda = \frac{\omega_r R}{v} \qquad (2)$$

where $\omega_r$ is the rotor speed of the WT.

Let us define $T_a$ to be the aerodynamic torque of WT, then substituting (1) into (2), one obtains

$$T_a = \frac{P_a}{\omega_r} = \frac{1}{2}\rho\pi R^3 \frac{C_p(\lambda, \beta)}{\lambda} v^2$$

Denote $C_q(\lambda, \beta) = C_p(\lambda, \beta)/\lambda$ that is the torque coefficient, then

$$T_a = \frac{P_a}{\omega_r} = \frac{1}{2}\rho\pi R^3 C_q(\lambda, \beta) v^2 \qquad (3)$$

From (1), it is obvious that the power $P_a$ is proportional to the power coefficient $C_p$, therefore, the optimal power point is obtained when the $C_p$ is achieved its maximum value at an optimal tip-speed ratio $\lambda_{opt}$. With a specific blade angle, this optimal ratio is constant.

**TABLE 1.** *Parameters of WECS.*

| No. | Parameters | Meaning |
|---|---|---|
| 1 | $i_d$ | Stator currents in $d$-axis |
| 2 | $i_q$ | Stator currents in $q$-axis |
| 3 | $v_d$ | Stator voltages in $d$-axis |
| 4 | $v_q$ | Stator voltages in $q$-axis |
| 5 | $P$ | Number of pole pairs |
| 6 | $R_s$ | Nominal stator resistance |
| 7 | $L$ | Nominal stator inductances |
| 8 | $B_v$ | Equivalent viscous friction coefficient |
| 9 | $J$ | Equivalent rotor inertia |
| 10 | $\psi_m$ | Magnet flux linkage |
| 11 | $T_e$ | Electromagnetic torque |
| 12 | $T_a$ | aerodynamic torque |

The maximum power is obtained by tracking the optimal reference of turbine rotor speed

$$\omega_{r,ref} = \frac{\lambda_{opt}}{R} v \qquad (4)$$

The turbine is connected to the generator via a gearbox with ratio $n_{gb}$, thus the relations of speed and torque between two sides are determined as

$$n_{gb} = \frac{\omega}{\omega_r} = \frac{T_a}{T_{gs}} \qquad (5)$$

in which $\omega$ is the mechanical angular speed of the generator, and $T_{gs}$ is the equivalent aerodynamic torque applied to the generator.

The mathematical model of a permanent magnet synchronous generator (PMSG) is presented as

$$\begin{cases} J\dfrac{d\omega}{dt} = -B_v\omega + T_{gs} - T_e \\ \dfrac{di_q}{dt} = -\dfrac{R_s}{L}i_q - P\omega i_d - \dfrac{\psi_m P}{L}\omega + \dfrac{1}{L}v_q \\ \dfrac{di_d}{dt} = -\dfrac{R_s}{L}i_d + P\omega i_q + \dfrac{1}{L}v_d \end{cases} \qquad (6)$$

where the parameters are explained in Table 1.

The electromagnetic torque can be calculated by,

$$T_e = Ki_q \qquad (7)$$

in which $K = 3/2 \, \psi_m P$.

Combining equations (5), (6), and (7), the dynamic equations of a PMSG are expressed as:

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{B_v}{J}\omega - \dfrac{1}{J}T_e + \dfrac{1}{J.n_{gb}}T_a \\ \dfrac{dT_e}{dt} = -\dfrac{R_s}{L}T_e - PK\omega i_d - \dfrac{\psi_m PK}{L}\omega + \dfrac{K}{L}v_q \\ \dfrac{di_d}{dt} = -\dfrac{R_s}{L}i_d + \dfrac{P}{K}\omega T_e + \dfrac{1}{L}v_d \end{cases} \qquad (8)$$

Assume that WECS system is impacted by the uncertainties and the system disturbance, then (8) is modified in the following for

$$
\begin{cases}
\dfrac{d\omega}{dt} = -\dfrac{B_v}{J}\omega - \dfrac{1}{J}T_e + \dfrac{1}{J.n_{gb}}T_a \\[2mm]
\dfrac{dT_e}{dt} = -\left[\dfrac{R_s}{L} + \Delta a_{Te}\,(t)\right]T_e - [PK\omega + \Delta a_{id}\,(t)]\,i_d \\[2mm]
\quad -[\dfrac{\psi_m PK}{L} + \Delta a_\omega(t)]\omega + \dfrac{K}{L}v_q + \gamma_1 d(t) \\[2mm]
\dfrac{di_d}{dt} = -[\dfrac{R_s}{L} + \Delta b_{id}\,(t)]i_d + [\dfrac{P}{K}\omega + \Delta b_{Te}\,(t)]T_e \\[2mm]
\quad + \Delta b_\omega(t)\omega + \dfrac{1}{L}v_d + \gamma_2 d(t)
\end{cases}
\tag{9}
$$

where $d(t)$ is the disturbance, $\gamma_1$ and $\gamma_2$ are the coefficient of the disturbance; $\Delta a_{Te}\,(t)$, $\Delta a_{id}\,(t)$, $\Delta a_\omega(t)$, $\Delta b_{id}\,(t)$, $\Delta b_{Te}\,(t)$, $\Delta b_\omega(t)$ are the time-varying uncertainties that may come from the modeling error or parameter errors.

*Remark 1:* The uncertainties in this paper are time-varying, it means that it is a more general case with respect to the constant uncertainties in the paper [17].

### B. PROBLEM STATEMENT
It is assumed that the WECS system is simultaneously influenced by the time-varying uncertainties and the system disturbance. It will cause difficulties to design a controller for WECS. In addition, the upper/lower bound values of the uncertainties do not know, hence the previous methods in papers [9]–[16], [20]–[23] are unable to apply to design the controller for this case. Additionally, the WECS in this work also assumes that

1. Both $\omega$ and $i_d$ are measured.
2. $i_q$ is unavailable, therefore $T_e$ is not known either.
3. Wind speed $v$ and aerodynamic torque $T_a$ are unavailable.

The $T_a$ is arbitrary signals, thus, the approaches in papers [9], [10], [20] are failed to design a controller for WECS. Owing to these reasons, in this paper a polynomial observer is proposed to estimate electromagnetic torque $T_e$, aerodynamic torque $T_a$, the time-varying uncertainties, and the system disturbance $d(t)$ without employing the sensors. The information of these signals will be transmitted to the LQR controller for tracking maximum power point purposes.

### III. LINEAR QUADRATIC OPTIMAL CONTROL DESIGN
To track the maximum power point of WECS, we need to design the controller to track the optimal reference of turbine rotor speed.

Let us define that $\tilde{\omega}$ is the tracking error of rotor speed, $\tilde{T}_e$ is the tracking error of the electromagnetic torque of the generator, and $u_{qc}$ and $u_{dc}$ are the compensating terms of

control input, then the system (9) can be rewritten as follows:

$$
\begin{cases}
\dfrac{d\tilde{\omega}}{dt} = -\dfrac{B_v}{J}\tilde{\omega} - \dfrac{1}{J}\tilde{T}_e \\[2mm]
\dfrac{d\tilde{T}_e}{dt} = -\dfrac{R_s}{L}\tilde{T}_e - \dfrac{\psi_m PK}{L}\tilde{\omega} + \dfrac{K}{L}\left(v_q - u_{qc}\right) \\[2mm]
\dfrac{di_d}{dt} = -\dfrac{R_s}{L}i_d + \dfrac{1}{L}\left(v_d - u_{dc}\right)
\end{cases}
\tag{10}
$$

where

$$
\tilde{\omega} = \omega - \omega_{ref}; \; \omega_{ref} = \omega_{r,ref}.n_{gb} = \dfrac{\lambda_{opt}}{R}v.n_{gb}
$$

$$
\tilde{T}_e = T_e - T_{e,ref}; \; T_{e,ref} = \dfrac{1}{n_{gb}}T_a - B_v\omega_{ref} - J\dot{\omega}_{ref}
$$

$$
u_{qc} = \dfrac{R_s}{K}T_{e,ref} + \dfrac{L}{K}\dot{T}_{e,ref} + \psi_m P\omega_{ref} + PL\omega i_d
$$
$$
\quad + \dfrac{L}{K}[\Delta a_{Te}\,(t)\,T_e + \Delta a_{id}\,(t)\,i_d + \Delta a_\omega\,(t)\,\omega - \gamma_1 d\,(t)]
$$

$$
u_{dc} = -\dfrac{PL}{K}\omega T_e + L[\Delta b_{id}\,(t)\,i_d - \Delta b_{Te}\,(t)\,T_e
$$
$$
\quad - \Delta b_\omega\,(t)\,\omega - \gamma_2 d(t)]
$$

For the sake of simplification, we assign two slack terms which include both time-varying uncertainties and the disturbance in the following forms:

$$
\phi_1\,(t) = [\Delta a_{Te}\,(t)\,T_e + \Delta a_{id}\,(t)\,i_d + \Delta a_\omega\,(t)\,\omega - \gamma_1 d\,(t)]
$$
$$
\phi_2\,(t) = [\Delta b_{id}\,(t)\,i_d - \Delta b_{Te}\,(t)\,T_e - \Delta b_\omega\,(t)\,\omega - \gamma_2 d(t)]
$$

and then

$$
u_{qc} = \dfrac{R_s}{K}T_{e,ref} + \dfrac{L}{K}\dot{T}_{e,ref} + \psi_m P\omega_{ref} + PL\omega i_d + \dfrac{L}{K}\phi_1\,(t)
$$
$$
u_{dc} = -\dfrac{PL}{K}\omega T_e + L\phi_2\,(t)
\tag{11}
$$

where $\omega_{ref}$ and $T_{e,ref}$ are the speed reference of the generator and the reference of electromagnetic torque.

*Remark 2:* It is seen that two variables $\phi_1\,(t)$ and $\phi_2\,(t)$ consist of the information of both the time-varying uncertainties and disturbance, therefore, when the $\phi_1\,(t)$ and $\phi_2\,(t)$ are estimated, it means that the information of the uncertainties and disturbance are also estimated.

Denote $x = \begin{bmatrix}\tilde{\omega} & \tilde{T}_e & i_d\end{bmatrix}^T$, $u = \begin{bmatrix}v_q & v_d\end{bmatrix}^T$, $u_c = \begin{bmatrix}u_{qc} & u_{dc}\end{bmatrix}^T$, then the system (10) can be expressed under the state-space framework as follows,

$$
\dot{x} = Ax + B\,(u - u_c)
\tag{12}
$$

in which

$$
A = \begin{bmatrix}
-\dfrac{B_v}{J} & \dfrac{1}{J} & 0 \\[2mm]
-\dfrac{\psi_m PK}{L} & -\dfrac{R_s}{L} & 0 \\[2mm]
0 & 0 & -\dfrac{R_s}{L}
\end{bmatrix}, \;
B = \begin{bmatrix}
0 & 0 \\[1mm]
\dfrac{K}{L} & 0 \\[1mm]
0 & \dfrac{1}{L}
\end{bmatrix}.
$$

In this section, Linear Quadratic Regulator (LQR) controller is selected to control the system (12) in order to make the vector of state variables approach zero. It means that the tracking errors also converge towards zero.

To design the LQR controller, firstly, let us consider the cost function as follows:

$$J(x,u) = \int_0^\infty \left( x^T Z x + u^T R u \right) \quad (13)$$

in which $Z \in \mathbb{R}^{3 \times 3}$ and $R \in \mathbb{R}^{2 \times 2}$ denote the positive-definite weigh-matrices. The matrices $Z$ and $R$ are typically selected as the following diagonal matrices:

$$Z = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}, R = \tau \begin{bmatrix} t_1 & 0 \\ 0 & t_2 \end{bmatrix} \quad (14)$$

with

$$q_i = \frac{1}{t_{si}(x_{imax})^2}, t_i = \frac{1}{(u_{imax})^2}, \rho > 0 \quad (15)$$

where $t_{si}$ is considered as expected settling time of $x_i$, $\tau$ is a control parameter, and $x_{imax}$ and $u_{imax}$ are upper bounds of $|x_i|$ and $|u_i|$, respectively.

The optimal controller form for system (12) is

$$u = u_c + K_u x \quad (16)$$

where $K_u$ is the optimal controller gain. The optimal LQR controller gain is obtained by the following steps:

*Step 1:* Solving the following algebraic Riccati Equation to obtain the positive-definite matrix

$$P_u A + A^T P_u - P_u B T_c^{-1} B^T P_u + Z = 0 \quad (17)$$

*Step 2:* Determine the optimal controller gain by following the formula

$$K_u = -R^{-1} B^T P_u. \quad (18)$$

## IV. OBSERVER SYNTHESIS
In this section, a polynomial disturbance observer will be synthesized to estimate the aerodynamic torque $T_a$, wind speed, electromagnetic torque $T_e$, the uncertainties, and the system disturbance $d(t)$. However, before starting designing the polynomial disturbance observer, the system mathematical model of the WECS needs to be modified in the following steps.

### A. MODIFIED WECS SYSTEM
The WECS system model (9) is able to be rewritten in the following form

$$\begin{bmatrix} \dot{\omega} \\ \dot{T}_e \\ \dot{i}_d \end{bmatrix} = \left[ \begin{bmatrix} -\frac{B_v}{J} & -\frac{1}{J} & 0 \\ -\frac{\psi_m PK}{L} & -\frac{R_s}{L} & PK\omega \\ 0 & \frac{P}{K}\omega & -\frac{R_s}{L} \end{bmatrix} \right. \\ \left. + \begin{bmatrix} 0 & 0 & 0 \\ -\Delta a_\omega(t) & -\Delta a_{Te}(t) & -\Delta a_{id}(t) \\ -\Delta b_\omega(t) & \Delta b_{Te}(t) & \Delta b_{id}(t) \end{bmatrix} \right] \begin{bmatrix} \omega \\ T_e \\ i_d \end{bmatrix} \\ + \begin{bmatrix} 0 & 0 \\ \frac{K}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} v_q \\ v_d \end{bmatrix} + \begin{bmatrix} \frac{1}{Jn_{gb}} \\ 0 \\ 0 \end{bmatrix} T_a + \begin{bmatrix} 0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix} d(t) \quad (19)$$

Suppose that the Electromagnetic torque $T_e$ is not measured, thus, the output of the system (19) is represented by

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega \\ T_e \\ i_d \end{bmatrix} \quad (20)$$

From (19) and (20), the mathematical model of WECS is rewritten in terms of the polynomial linear system

$$\begin{cases} \dot{\tilde{x}} = [A(\omega) + \Delta A(t)]\tilde{x} + Bu + DT_a + Hd(t) \\ y = C\tilde{x} \end{cases} \quad (21)$$

where

$$\tilde{x} = \begin{bmatrix} \omega & T_e & i_d \end{bmatrix}^T, u = \begin{bmatrix} v_q & v_d \end{bmatrix}^T,$$

$$A(\omega) = \begin{bmatrix} -\frac{B_v}{J} & -\frac{1}{J} & 0 \\ -\frac{\psi_m PK}{L} & -\frac{R_s}{L} & PK\omega \\ 0 & \frac{P}{K}\omega & -\frac{R_s}{L} \end{bmatrix},$$

$$\Delta A(t) = \begin{bmatrix} 0 & 0 & 0 \\ -\Delta a_\omega(t) & -\Delta a_{Te}(t) & -\Delta a_{id}(t) \\ -\Delta b_\omega(t) & \Delta b_{Te}(t) & \Delta b_{id}(t) \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \frac{K}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} \frac{1}{Jn_{gb}} & 0 & 0 \end{bmatrix}^T, \text{ and } H = \begin{bmatrix} 0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix}.$$

$\gamma_1$ and $\gamma_2$ are the disturbance coefficient.

*Assumption 1:* suppose that the uncertainty satisfies the matching condition $\Delta A(t)\tilde{x} = H\vartheta(t)$ where $\vartheta(t)$ is an arbitrary signal with an appropriate dimension.

Under Assumption 1, the system is modified as follows

$$\begin{cases} \dot{\tilde{x}} = A(\omega)\tilde{x} + Bu + DT_a + H[\vartheta(t) + d(t)] \\ y = C\tilde{x} \end{cases} \quad (22)$$

Let us define $\varphi(t) = [\vartheta(t) + d(t)]$ then we

$$\begin{cases} \dot{\tilde{x}} = A(\omega)\tilde{x} + Bu + DT_a + H\varphi(t) \\ y = C\tilde{x} \end{cases} \quad (23)$$

The system (23) can be modified as follows

$$\begin{cases} \dot{\tilde{x}} = A(\omega)\tilde{x} + Bu + [D \; H] \begin{bmatrix} T_a \\ \varphi(t) \end{bmatrix} \\ y = C\tilde{x} \end{cases} \quad (24)$$

Let us define:

$$\tilde{D} = [D \; H] \text{ and } \tilde{\varphi} = \begin{bmatrix} T_a \\ \varphi(t) \end{bmatrix}$$

then (24) becomes

$$\begin{cases} \dot{\tilde{x}} = A(\omega)\tilde{x} + Bu + \tilde{D}\tilde{\varphi} \\ y = C\tilde{x} \end{cases} \quad (25)$$

From (19)-(25), it can see that the system (21) has been modified to the polynomial system (25)

## B. DISTURBANCE POLYNOMIAL OBSERVER SYNTHESIS

In this section, the modified model (25) is used to replace the system (21) to design the disturbance for estimating the unknown state $T_e$, aerodynamic torque $T_a$, uncertainties, and the disturbance $d(t)$.

The polynomial disturbance observer form for the system (25) is

$$\begin{cases} \dot{\hat{\tilde{x}}} = X(\omega)\hat{\tilde{x}} + Wu + R(\omega)y + E\dot{y} \\ \hat{y} = C\hat{\tilde{x}} \\ \hat{\tilde{\varphi}} = (C\tilde{D}(\omega))^{-1}\dot{y} - S(\omega)\hat{\tilde{x}} - J(\omega)u \end{cases} \quad (26)$$

where $\hat{\tilde{x}}$, $\hat{y}$ and $\hat{\tilde{\omega}}$ are the estimation of $\tilde{x}$, $y$ and $\tilde{\varphi}(t)$, respectively. Polynomial matrix $X(\omega) \in \mathbb{R}^{3\times3}$, $W \in \mathbb{R}^{3\times1}$, polynomial matrix $R(\omega) \in \mathbb{R}^{3\times1}$, $E \in \mathbb{R}^{3\times1}$, polynomial matrix $S(\omega) \in \mathbb{R}^{1\times3}$, and $J \in \mathbb{R}^{1\times2}$ are the observer gains which will be determined later.

The estimation of $T_a$ and $\varphi(t)$ are computed by the following formulas

$$\hat{T}_a = [\hat{\tilde{\varphi}}]^T \begin{bmatrix} I \\ 0 \end{bmatrix}, \hat{\varphi}(t) = [\hat{\tilde{\varphi}}]^T \begin{bmatrix} 0 \\ I \end{bmatrix}$$

It should be noted that the system (21) and the observer (26) consist of the polynomial matrices. Therefore, the LMI technique in paper [17]–[19] is unable to apply to find the observer gains. Because of this reason, in this work, the Sum-Of-Square (SOS) tool is used to design the polynomial disturbance observer (26). To help readers easily understand, the definitions of SOS are presented by the following two propositions.

*Proposition 1 [30]:* for a given arbitrary function $h(x(t))$, this function is called Sum-Of-Square if only if it can be decomposed to the form $h(x(t)) = \sum_{i=1}^{n}[b_i(x(t))]^2$, in which $b_i(x(t))$ is expressed in the polynomial form in $x(t)$. If the function $h(x(t))$ is an SOS, we can conclude that $h(x(t)) \geq 0$, unfortunately, the converse is not guaranteed.

*Proposition 2 [30]:* Let us consider a polynomial symmetric matrix $\Pi(x) \in \mathfrak{R}^{n\times n}$ in $x$ and a vector $v \in \mathfrak{R}^n$ does not depend on $x$, it concludes that then $\Pi(x) \geq 0$ for all $x$ if only if $v^T\Pi(x)v$ is expressed under the SOS framework.

*Theorem 1:* The estimation error of the system (25) with the observer (26) converge towards zero asymptotically if there exist matrice $W \in \mathbb{R}^{3\times1}$, $E \in \mathbb{R}^{3\times1}$, $J \in \mathbb{R}^{1\times2}$, the polynomial matrices $X(\omega) \in \mathbb{R}^{3\times3}$, $R(\omega) \in \mathbb{R}^{3\times1}$, $S(\omega) \in \mathbb{R}^{1\times3}$, and a symmetric matrix $Q \in \mathbb{R}^{3\times3}$ such that the following conditions satisfy

$$A(\omega) - R(\omega)C - ECA(\omega) - X(\omega) = 0 \quad (27)$$
$$B - W - ECB = 0 \quad (28)$$
$$\tilde{D} - EC\tilde{D} = 0 \quad (29)$$
$$S(\omega) - (C\tilde{D})^{-1}CA(\omega) = 0 \quad (30)$$
$$J - (C\tilde{D})^{-1}CB = 0 \quad (31)$$
$$v_1^T(Q - \epsilon_1 I)v_1 \text{ is SOS} \quad (32)$$
$$-v_2^T\left(X^T(\omega)Q + QX(\omega) - \epsilon_2(\omega)I\right)v_2 \text{ is SOS} \quad (33)$$

where $v_1$, $v_2$ are two vectors which are independent on $\omega$, $\epsilon_1$ is positive scalar and $\epsilon_2(\omega)$ is positive and $\epsilon_2(\omega) \neq 0$ with $\omega \neq 0$.

*Proof:*
From Eq. (25), it infers that

$$\dot{y} = C\dot{\tilde{x}} = CA(\omega)\tilde{x} + CBu + C\tilde{D}(\omega)\tilde{\varphi} \quad (34)$$

Let us assign the estimation error as follows:

$$e(t) = \tilde{x}(t) - \hat{\tilde{x}}(t) \quad (35)$$

Taking the derivative two sides of Eq. (35), one obtains

$$\dot{e}(t) = \dot{\tilde{x}} - \dot{\hat{\tilde{x}}} \quad (36)$$

Substituting (25) and (26) into (36) yields

$$\dot{e}(t) = (\omega)\tilde{x} + Bu + \tilde{D}(\omega) \\ - \left[X(\omega)\hat{\tilde{x}} + Wu + R(\omega)y + E\dot{y}\right] \quad (37)$$

From (34) and (37), we have

$$\dot{e}(t) = X(\omega)e(t) + [A(\omega) \\ - R(\omega)C - ECA(\omega) - X(\omega)]\tilde{x} \\ + [B - W - ECB]u + [\tilde{D} - EC\tilde{D}]\tilde{\varphi} \quad (38)$$

From Theorem 1, if the conditions (27)-(29) are satisfied then (38) is equivalent to

$$\dot{e}(t) = X(\omega)e(t) \quad (39)$$

The Lyapunov function is selected as follows:

$$V(e(t)) = e^T(t)Qe(t) \quad (40)$$

The condition (32) of Theorem 1 means that the matrix $Q > 0$. Hence, It can conclude that $V(e(t)) > 0$.

Taking the derivative both sides of Eq.(40)

$$\dot{V}(e(t)) = \dot{e}^T(t)Qe(t) + e^T(t)Q\dot{e}^T(t) \quad (41)$$

Substituting (39) into (41), it yields

$$\dot{V}(e(t)) = e^T(t)[N^T(\omega)Q + QN(\omega)]e(t) \quad (42)$$

It is seen that if the condition (33) of Theorem 1 holds, then $N^T(\omega)Q + QN(\omega) < 0$, hence, it infers that $\dot{V}(e(t)) < 0$ and the estimation error $e(t)$ converges towards zero asymptotically.

Denote the estimation error of the disturbance

$$e_{\tilde{\varphi}}(t) = \tilde{\varphi} - \hat{\tilde{\varphi}} \quad (43)$$

Substituting (26) into (43) obtains

$$e_{\tilde{\varphi}}(t) = \tilde{\varphi} - (C\tilde{D}(\omega))^{-1}\dot{y} - S(\omega)\hat{\tilde{x}} - J(\omega)u \quad (44)$$

Combining (34) and (44), we have

$$e_{\tilde{\varphi}}(t) = \tilde{\varphi} - (C\tilde{D})^{-1}\left[CA(\omega)\tilde{x} + CBu + C\tilde{D}\tilde{\varphi}\right] \\ - S(\omega)\hat{\tilde{x}} - J(\omega)u \\ = [S(\omega) - (C\tilde{D})^{-1}CA(\omega)]\tilde{x} + [J(\omega) - (C\tilde{D})^{-1}CB] \\ \times u + S(\omega)e(t) \quad (45)$$

If the conditions (30) and (31) of Theorem 1 fulfill, then (45) is written in the following form

$$e_{\tilde{\varphi}}(t) = S(\omega) e(t) \tag{46}$$

From Eq. (46), it is clear that if $e(t) \to 0$ when $t \to \infty$ then we can conclude that $e_{\tilde{\varphi}}(t) \to 0$.

The proof of Theorem 1 is completed.

From the condition (33), it is seen that this is a nonlinear polynomial matrix inequality (BPMI) which is hard to solve by SOS Tool in MATLAB. Owing to this reason, the condition (33) must be transformed to the Polynomial Linear Matrix Inequality (PLMI) which is presented in the next steps.

*Theorem 2:* The estimation error $e(t)$ with the observer (26) approach zero asymptotically if there exist the matrices $W \in \mathbb{R}^{3 \times 1}, E \in \mathbb{R}^{3 \times 1}, J \in \mathbb{R}^{1 \times 2}$, the polynomial matrices $X(\omega) \in \mathbb{R}^{3 \times 3}, R(\omega) \in \mathbb{R}^{3 \times 1}, S(\omega) \in \mathbb{R}^{1 \times 3}$, and a symmetric matrix $Q \in \mathbb{R}^{3 \times 3}$ to ensure the following conditions satisfy

$$v_1^T (Q - \epsilon_1 I) v_1 \text{ is SOS} \tag{47}$$
$$-v_2^T (\Xi(\omega) + \epsilon_2(\omega) I) v_2 \text{ is SOS} \tag{48}$$

in which

$$\begin{aligned}\Xi(\omega) &= [(I - \tilde{D}(C\tilde{D})^{-1}C)A(\omega)]^T Q \\ &\quad + Q[(I - \tilde{D}(C\tilde{D})^{-1}C)A(\omega)] \\ &\quad - C^T (\bar{R}(\omega))^T - Q\bar{R}(\omega) C \end{aligned} \tag{49}$$
$$\bar{R}(\omega) = QR(\omega) \tag{50}$$

$v_1$ and $v_2$ are the vectors that are independent on $\omega$. $\epsilon_1$ is the positive scalar and $\epsilon_2(\omega) > 0$ with $\omega \neq 0$.

And the observer gains are calculated

$$E = \tilde{D}(C\tilde{D})^{-1} \tag{51}$$
$$R(\omega) = Q^{-1}\bar{R}(\omega) \tag{52}$$
$$X(\omega) = A(\omega) - R(\omega) C - ECA(\omega) \tag{53}$$
$$W = B - ECB \tag{54}$$
$$S(\omega) = (C\tilde{D})^{-1}CA(\omega) \tag{55}$$
$$J = (C\tilde{D})^{-1}CB \tag{56}$$

*Proof:*

From the condition (29) of Theorem 1, it infers that

$$E = \tilde{D}(C\tilde{D})^{-1} \tag{57}$$

From (27), we have

$$\begin{aligned}X(\omega) &= A(\omega) - R(\omega) C - ECA(\omega) \\ &= (I\text{-}EC) A(\omega) - R(\omega) C \end{aligned} \tag{58}$$

Substituting (57) into (58) yields

$$X(\omega) = \left(I - \tilde{D}(C\tilde{D})^{-1}C\right) A(\omega) - R(\omega) C \tag{59}$$

From (33), it implies that

$$X^T(\omega) Q + QX(\omega) < 0 \tag{60}$$

Substituting (59) into (60), one obtains

$$\begin{aligned}&[(I - \tilde{D}(C\tilde{D})^{-1}C)A(\omega) \\ &\quad - R(\omega) C]^T Q + Q[(I - \tilde{D}(C\tilde{D})^{-1}C)A(\omega) - R(\omega) C] \\ &= [(I - \tilde{D}(C\tilde{D})^{-1}C)A(\omega)]^T Q - C^T (R(\omega))^T Q \\ &\quad + Q[(I - \tilde{D}(C\tilde{D})^{-1}C)A(\omega)] - QR(\omega) C < 0 \end{aligned} \tag{61}$$

Define

$$\bar{R}(\omega) = QR(\omega) \tag{62}$$

then (61) becomes

$$\begin{aligned}&[(I - \tilde{D}(C\tilde{D})^{-1}C)A(\omega)]^T Q - C^T (\bar{R}(\omega))^T \\ &\quad + Q[(I - \tilde{D}(C\tilde{D})^{-1}C)A(\omega)] - \bar{R}(\omega) C < 0 \end{aligned} \tag{63}$$

From (63), it is obvious that the PBMI (33) has been converted to the PLMI successfully by Theorem 2. The observer gains will be easily determined by resolving the PLMIs conditions of Theorem by SOS tool of Matlab. The proof is completed.

To help readers easily understand, the procedure to determine the observer gains of the disturbance observer (26) is briefly presented as follows.

*Step 1:* Resolve the conditions (48) and (50) by SOS tool in Matlab to get $\bar{R}(\omega)$ and $Q$

*Step 2:* Compute the observer gains from (51)-(56).

*Step 3:* Based on the observer gains in *Step 2*, constructing the observer (26) to estimate the $\hat{\tilde{x}}(t)$, and $\hat{\tilde{\varphi}}(t)$

*Step 4:* The estimation $\hat{T}_a$ and $\hat{\varphi}(t)$ (that consist of uncertainties and disturbance) are estimated by the following formulas

$$\hat{T}_a = [\hat{\tilde{\varphi}}]^T \begin{bmatrix} I \\ 0 \end{bmatrix},$$
$$\hat{\varphi}(t) = [\hat{\tilde{\varphi}}]^T \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

## V. OBSERVER-BASED CONTROL SCHEME AND CLOSED-LOOP STABILITY ANALYSIS

### A. OBSERVER-BASED CONTROLLER

With the estimated information of speed reference, aerodynamic torque, time-varying uncertainties, and the disturbance, the tracking error and compensating terms become

$$\hat{\tilde{\omega}} = \omega - \hat{\omega}_{ref}; \hat{\omega}_{ref} = \sqrt{\frac{\hat{T}_a}{k_{opt}}}$$

$$\hat{\tilde{T}}_e = \hat{T}_e - \hat{T}_{e,ref}; \hat{T}_{e,ref} = \frac{1}{n_{gb}}\hat{T}_a - B_v\hat{\omega}_{ref} - J\dot{\hat{\omega}}_{ref}$$

$$\hat{u}_{qc} = \frac{R_s}{K}\hat{T}_{e,ref} + \frac{L}{K}\dot{\hat{T}}_{e,ref} + \psi_m P\hat{\omega}_{ref} + PLi_d + \frac{L}{K}\hat{\phi}_1(t)$$

$$\hat{u}_{dc} = -\frac{PL}{K}\omega\hat{T}_e + L\hat{\phi}_2(t). \tag{64}$$

Under Assumption 1, the estimation of uncertainties and disturbance $\hat{\phi}_1(t), \hat{\phi}_2(t)$ are computed as follows

$$\hat{\phi}_1(t) = \gamma_1\hat{\varphi}(t), \quad \hat{\phi}_2(t) = \gamma_2\hat{\varphi}(t)$$

The controller framework (16) is written as follows

$$u = \hat{u}_c + K_u \hat{x} \qquad (65)$$

in which

$$\hat{x} = \begin{bmatrix} \hat{\omega} & \hat{T_e} & i_d \end{bmatrix}^T, \hat{u}_c = \begin{bmatrix} \hat{u}_{qc} & \hat{u}_{dc} \end{bmatrix}^T.$$

Then, from (31), the following equations are achieved,

$$\hat{x} = x + U\tilde{e}$$
$$\hat{u}_c = u_c + V\tilde{e} \qquad (66)$$

where

$$\tilde{e} = \begin{bmatrix} e_{T_a} & \dot{e}_{T_a} & \ddot{e}_{T_a} & e_{T_e} & e_{\phi_1} & e_{\phi_2} \end{bmatrix}^T$$

$$U = \begin{bmatrix} u_1 & 0 & 0 & 0 & 0 \\ u_2 & u_3 & 0 & u_4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$V = \begin{bmatrix} v_1 & v_2 & v_3 & 0 & v_4 & 0 \\ 0 & 0 & 0 & v_5 & 0 & v_6 \end{bmatrix},$$

$$u_1 = \frac{1}{\sqrt{k_{opt}}\left(\sqrt{T_a} + \sqrt{\hat{T}_a}\right)}, u_2 = \frac{1}{n_{gb}} - B_v u_1 + Jf_2,$$

$$u_3 = -Jf_1, f_1 = \frac{1}{2\sqrt{k_{opt}T_a}}, f_2 = \frac{\dot{\hat{T}}_a}{2\sqrt{T_a\hat{T}_a}}u_1; f_3 = \frac{f_1}{2T_a}$$

$$u_4 = -1$$

$$f = \left(\frac{T_a + \hat{T}_a + \sqrt{T_a\hat{T}_a}}{2T_a\hat{T}_a} + \frac{\ddot{\hat{T}}_a}{\dot{\hat{T}}_a}\right)f_2 u_1,$$

$$v_1 = \psi_m P u_1 + \frac{L}{K}f_3 + \frac{R_s}{K}u_2, v_2 = u_3 + \frac{L}{K}f_4,$$

$$v_3 = \frac{L}{K}f_1, v_4 = \frac{L}{K}, v_5 = \frac{PL}{K}\omega$$

$$e_{T_a} = T_a - \hat{T}_a, e_{T_e} = T_e - \hat{T}_e, e_{\phi_1} = \phi_1 - \hat{\phi}_1,$$

$$e_{\phi_2} = \phi_2 - \hat{\phi}_2.$$

### B. STABILITY ANALYSIS

From (65) and (66), we can have

$$u = u_c + K_u x + L\tilde{e} \qquad (67)$$

where

$$L = U + K_u V,$$

From (65) and (12), one obtains

$$\dot{x} = (A + BK_u)x + BN\tilde{e} \qquad (68)$$

It is seen that the system (68) is considered a closed-loop system with an observer-based controller.

*Lemma 1[30]:* For a given system with the following form

$$\begin{cases} \dot{z} = f(z, y) \\ \dot{y} = s(y) \end{cases} \qquad (69)$$

suppose that the system $\dot{y} = s(y)$ has an asymptotically stable equilibrium at $y = 0$. If $\dot{z} = f(z, 0)$ has an asymptotically

stable equilibrium at $z = 0$, then (69) has an asymptotically stable equilibrium at $(z, y) = (0, 0)$.

*Theorem 3:* Under Lemma 1, LQR controller design in Section 2 and observer design in Section 2, the tracking error $x$ and estimation error $\tilde{e}$ with observer (26) and controller (65) converge towards zero asymptotically.

*Proof:*

If there exist the observer gains of the observer (26) that satisfy the conditions of Theorem 2 in Section 3 then disturbance observer (26) successfully estimates the state $T_e$, aerodynamic torque $T_a$, time-varying, and system disturbance. It means that the estimation error $\tilde{e} \to 0$ when $t \to \infty$. Due to this reason, the system (68) is modified

$$\dot{x} = (A + BK_u)x \qquad (70)$$

According to the LQR controller design procedure in Section 2, it is clear that system (70) is asymptotically stable equilibrium at zero. Therefore, under Lemma 1, it is concluded that the closed-loop control system (68) with disturbance observer-based controller is stable at the tracking error $x(t)$ and estimation error $\tilde{e}(t)$ is asymptotically stable at zero.

## VI. RESULTS AND DISCUSSION

To prove the effectiveness of the proposed method, in this section, the system WECS with parameters illustrated in Table 1. The structure of the WECS with a robust polynomial disturbance observer-based controller is described in Fig. 1. Suppose that the system is impacted by the time-varying uncertainties (71) and the random system disturbance is shown in Fig. 2. The waveform of the wind speed is shown in Fig. 3.

$$\Delta A(t) = \begin{bmatrix} 0 & 0 & 0 \\ 6\sin(\omega) & -3\cos(\omega) & -3\sin(2\omega) \\ 4\sin(\omega) & 2\cos(\omega) & 2\sin(2\omega) \end{bmatrix} \qquad (71)$$

*Discussion 1:* It is seen that in this simulation, the uncertainties (71) are time-varying and we do not know the bound range of the uncertainties. Moreover, the disturbance in Fig. 1 and the wind speed in Fig. 2 are arbitrary and do not need to satisfy the bounded constraint or the first-order equal to zero.

Employing the SOS tool of Matlab, the observer gains are obtained as follows

$$Q = \begin{bmatrix} 0.8252 & 2.971 * 10^{-8} & 0.636 * 10^{-3} \\ 2.971 * 10^{-8} & 3.068 * 10^{-8} & 3.61 * 10^{-9} \\ 0.636 * 10^{-3} & 3.61 * 10^{-9} & 0.8492 \end{bmatrix}$$

$$\bar{R}(\omega) = \begin{bmatrix} \bar{R}_{11} & \bar{R}_{12} \\ \bar{R}_{21} & \bar{R}_{22} \\ \bar{R}_{31} & \bar{R}_{32} \end{bmatrix}$$

$$\bar{R}_{11} = 0.5069\omega^2 + 0.507 \times 10^{-3}\omega + 0.3154$$
$$\bar{R}_{12} = 4.114 \times 10^{-6}\omega^2 2.793 \times 10^{-3}\omega + 0.0043$$
$$\bar{R}_{21} = 2.227 \times 10^{-8}\omega^2 - 2.249 \times 10^{-5}\omega - 0.00162$$

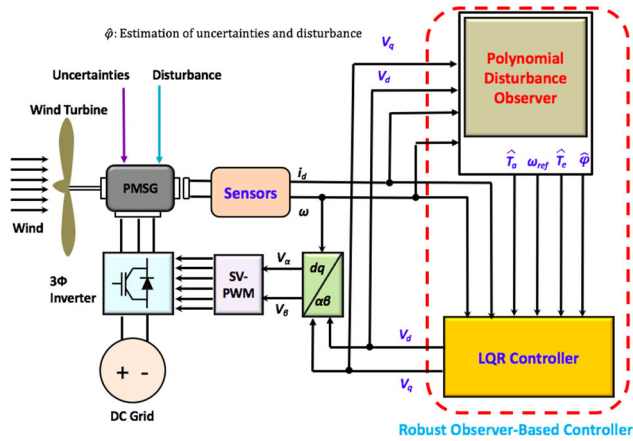Following the procedure for synthesizing the polynomial disturbance observer in Theorem 2, the observer gains are

**FIGURE 1.** Structure of the WECS with Robust disturbance observer-based Controller.

**TABLE 2.** WECS parameters.

| Symbols | Parameters | Values | Unit |
|---------|-----------|--------|------|
| $P_{rated}$ | Rated power | 5 | kW |
| $P$ | Pole pairs | 14 | - |
| $R_s$ | Stator resistance | 0.3676 | $\Omega$ |
| $L$ | Stator inductance | 3.55 | mH |
| $\psi_m$ | Magnet flux linkage | 0.2867 | V· s/rad |
| $J$ | Mechanical inertia | 7.856 | kg· m$^2$ |
| $B$ | Viscous friction coefficient | 0.002 | kg· m$^2$/s |
| $R$ | Rotor radius | 1.84 | m |
| $\rho$ | Air density | 1.25 | kg/m$^3$ |
| $\gamma_1$ | Disturbance coefficient | 3 | - |
| $\gamma_2$ | Disturbance coefficient | 2 | - |

computed as follows.

$$R(\omega) = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \\ R_{31} & R_{32} \end{bmatrix}$$

$R_{11} = 0.614\omega^2 + 6.433 \times 10^{-4}\omega + 0.384$
$R_{12} = 4.548 \times 10^{-4}\omega^2 3.36 \times 10^{-3}\omega + 4.708 \times 10^{-4}$
$R_{21} = 0.131\omega^2 - 733.095\omega - 52809.933$
$R_{22} = -10.202\omega^2 - 669.247\omega + 1180.209$
$R_{31} = -4.553 \times 10^{-4}\omega^2 3.285 \times 10^{-3}\omega + 5$
$R_{32} = 0.596\omega^2 + 1.530 \times 10^{-4}\omega + 0.596$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1.5 \\ 0 & 1.5 \end{bmatrix}, X(\omega) = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}$$

$X_{11} = -0.614\omega^2 6.433 \times 10^{-4}\omega - 0.384$
$X_{12} = 0$
$X_{13} = 4.548 \times 10^{-4}\omega^2 + 0.003\omega - 0.0047$



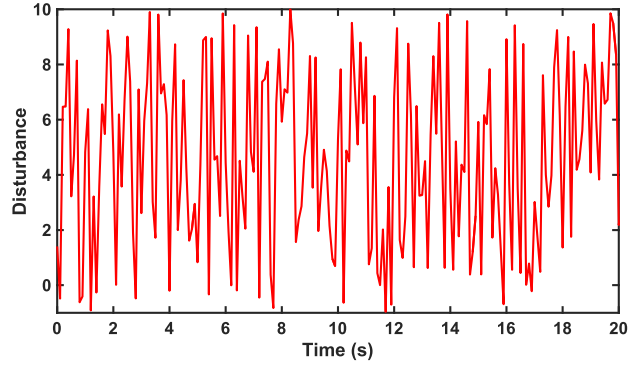**FIGURE 2.** The disturbance.

$X_{21} = -0.131\omega^2 + 733.09\omega + 46002.64$
$X_{22} = 3.48\omega - 103.55$
$X_{23} = 10.202\omega^2 + 584.957\omega - 1024.885$
$X_{31} = 4.553 \times 10^{-4}\omega^2 + 0.003\omega - 0.005$
$X_{32} = 0$
$X_{33} = -0.596\omega^2 1.53 \times 10^{-4}\omega - 0.596$

$$W = 10^3 \times \begin{bmatrix} 0 & 0 \\ 1.696 & -0.4225 \\ 0 & 0 \end{bmatrix}$$

$$S(\omega) = \begin{bmatrix} -1/500 & -1 & 0 \\ 0 & 1.16\omega & -51.77 \end{bmatrix},$$

$$J = \begin{bmatrix} 0 & 0 \\ 0 & 140.8451 \end{bmatrix}$$

Based on the LQR controller in Section 3, using Matlab to calculate the optimal controller gain of the LQR controller with

$$Z = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } R = 10^{-2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

one obtains

$$K_\mu = \begin{bmatrix} -35.8703 & 9.9394 & -0.0000 \\ 0.0000 & -0.0000 & 9.6392 \end{bmatrix}$$

With the obtained observer and controller gains, WECS with the proposed observer and the controller is simulated in MATLAB/Simulink and the simulation results are illustrated in Figs. 2-7.

The simulation results of the WECS are illustrated in Figs. 4-7. The mechanical angular speed $\omega$, the estimation $\hat{\omega}$ and their tracking error $\tilde{\omega}$ are shown in Fig. 4. The real aerodynamic torque $T_a$, estimated signal $\hat{T}_a$ and their estimation error $e_{Ta}$ are presented in Fig. 5. The estimation performance of electromagnetic torque (including real $T_e$, the estimation $\hat{T}_e$, and estimation error $e_{Te} = T_e - \hat{T}_e$) and estimation performance of stator currents $i_d$ in $d$-axis (including real current $i_d$, estimated $\hat{i}_d$, and estimation error $e_{id} = i_d - \hat{i}_d$) are demonstrated in Fig. 6 and Fig. 7, respectively. Fig. 4 shows that the mechanical angular speed $\omega$ can be tracked the signal reference very well and the tracking
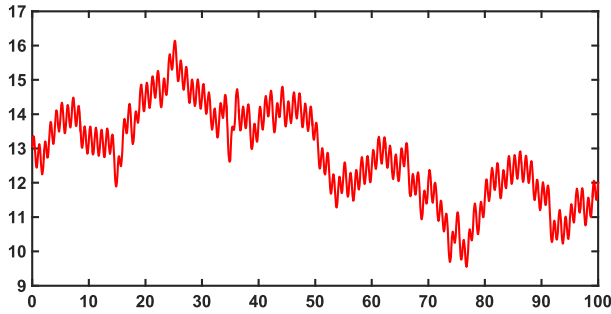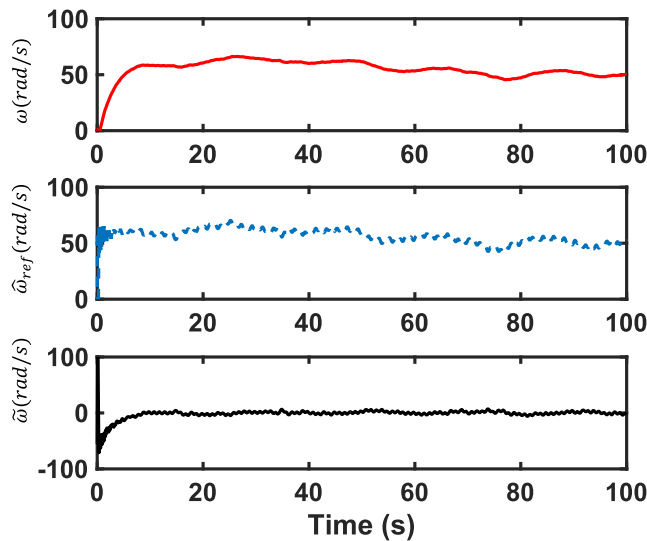
**FIGURE 3.** Wind speed waveform.
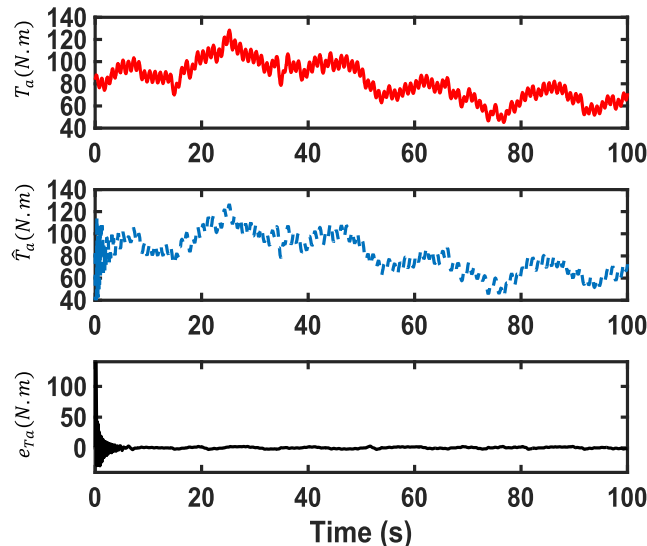


**FIGURE 4.** Speed tracking performance.



**FIGURE 5.** Estimation of the electromagnetic torque $T_a$.



**FIGURE 6.** Estimation of aerodynamic torque $T_e$.
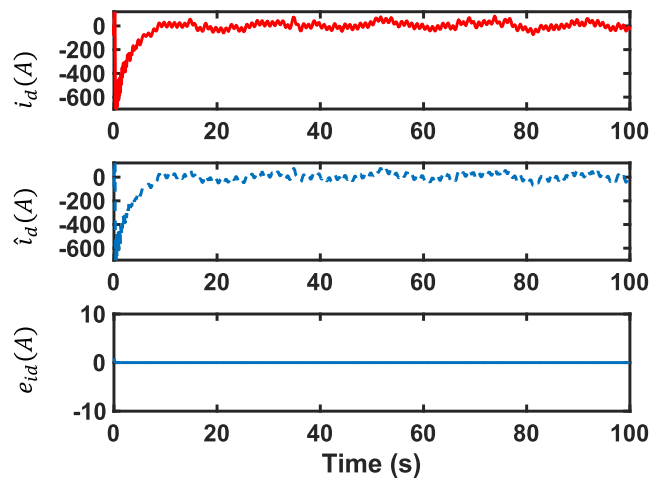


**FIGURE 7.** Estimation of the stator currents $i_d$ in $d$-axis.

error converges towards zero. In addition, the simulation results in Figs. 5-7 show that the estimation of aerodynamic torque $\hat{T}_a$, the estimations of electromagnetic torque $\hat{T}_e$, and estimation of stator currents $\hat{i}_d$ in $d$-axis are able to approach real states $T_a$, $T_e$, and $i_d$, respectively; and the estimation

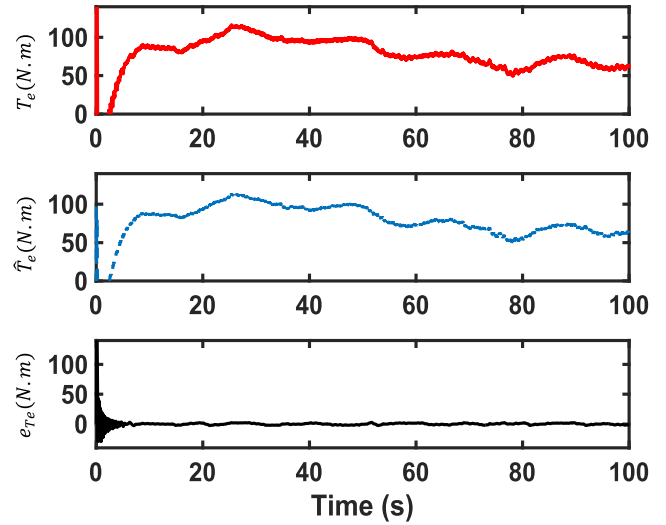errors $e_{Ta}$, $e_{Te}$, and $e_{id}$ converge towards zero asymptotically as well. With these results, it is obvious that the proposed polynomial disturbance observer and the LQR controller still work efficiently to estimate $T_a$, $T_e$, and $i_d$, of WECS system even with the effects of the time-varying uncertainties and disturbance. In addition, both the time-varying uncertainties and the disturbance are estimated by the observer and send back to the LQR controller to eliminate the impacts of the uncertainties and the disturbance; and track the optimal power point very well.

*Discussion 2:* It should be noted that the method in [13] was proposed for WECS with the effects of the faults, while in this paper, WECS system is affected both time-varying uncertainties and disturbance. Therefore, the method in [13] is impossible to apply for WECS in this work.

## VII. CONCLUSION

A new approach to design a polynomial disturbance observer-based controller to track the maximum power point of WECS has been studied in this article. The system WECS is impacted

by the time-varying uncertainties and the disturbance. The robust polynomial disturbance observer is designed to estimate the aerodynamic torque, electromagnetic torque, wind speed, uncertainties as well as disturbance. The estimation information of these parameters, uncertainties and the disturbance is transmitted to the LQR controller to eliminate the influences of both uncertainties and disturbance and track the maximum power point. The conditions for the polynomial disturbance observer expressed under the polynomial framework are derived in main theorems. Finally, the obtained simulation results have proved that the robust disturbance observer-based controller can operate efficiently and the proposed method of this paper is successful to control the WECS.

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