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Fuzzy Multi-Choice Goal Programming and Artificial Bee Colony Algorithm for Triangular and Trapezoidal Membership Functions

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ABSTRACT Multi-choice goal programming (MCGP) has been widely used to find satisfying solutions for multiple criteria/objective decision problems in which the target value of ''the more, the better,'' or ''the less, the better'' can easily be obtained. This paper proposes two new models for representing the triangular and trapezoidal membership functions, which improve the efficacy of fuzzy MCGP (FMCGP). Two real-world applications are provided in this study to demonstrate the usefulness of the proposed models. Furthermore, the same problems are resolved by using the proposed nature-inspired optimization method (NIOM) to find the differences between them. While the artificial bee colony (ABC) algorithm is a well-known NIOM technique, studies have shown that it has an excellent performance with high-quality solutions. Thus, this study initially uses the ABC algorithm to find the differences between MCGP and ABC. Finally, some insightful information is obtained from the comparison to contribute to the NIOM and MCGP fields and their respective applications.

INDEX TERMS Fuzzy, multi-choice goal programming, multiple objective decision making.

I. INTRODUCTION

In recent years, fuzzy multiple objective decision making (FMODM) has become more important for helping companies make an appropriate decision under environmental uncertainty. Chang [1]–[3] proposed a series of multi-choice goal programming (MCGP) methods that effectively solve multi-aspiration level problems to contribute to the field of multiple objective decision making (MODM). In the MCGP, the multi-aspiration level can be represented as discrete, vector, and utility functions (UF) according to decision-makers' (DM) needs. By using MCGP to solve MODM problems, both qualitative and quantitative issues can be considered simultaneously. Moreover, MCGP has been widely applied to solve many real-world decision-making problems such as supplier selection [4], [5], house selection [6], sugar and ethanol milling problem [7], selection of locations for coffee shops and renewable-energy facilities [8], [9], supply chain management problem [10], consumer choice problem [11], and catering supplier selection problem [12]. On the other hand, many advanced MCGP relevant methods were subsequently proposed including fuzzy MCGP (FMCGP) [13], multi-segment MCGP [14], multi-coefficient GP [15], MCGP with the conic scalarizing function [16], and weighted-additive fuzzy MCGP (WA-FMCGP) [17].

Membership function (MF) is usually used to quantify linguistic terms and represent a fuzzy set graphically. For example, an MF for a fuzzy set on the universe of discourse is defined as $\mu_A : x \rightarrow [0, 1]$. Further, the applications of MF can be seen in automatic control, decision-making capacity, and fuzzy logic. Chang [3] was the first to add two popular UFs (linear and S-shaped UFs) to MCGP in dealing with both qualitative and quantitative issues simultaneously. This technique enriches the fields of MCGP and MCDM.

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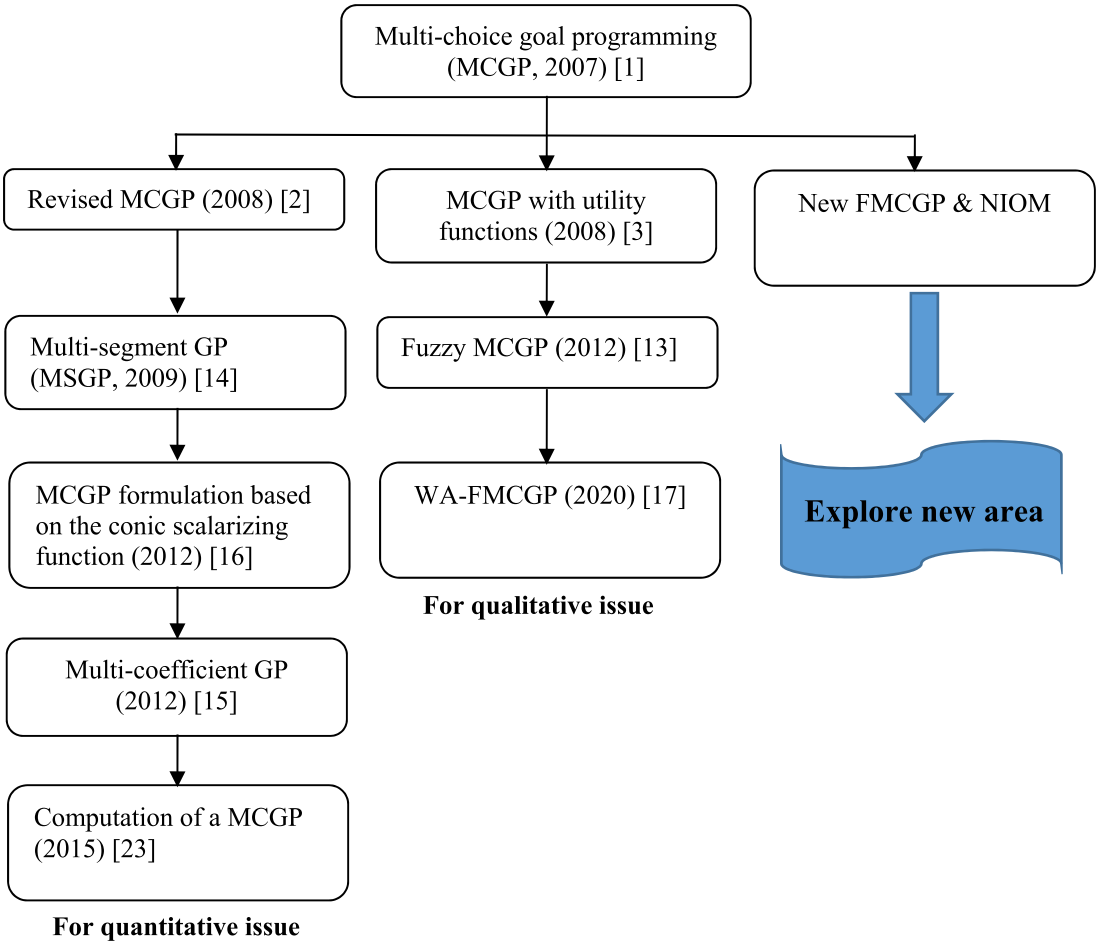


FIGURE 1. The relationship of MCGP family p of MCGP family.

Chang [3] first introduced the fuzzy method to MCGP, called the FMCGP, to formulate triangular MF and enrich MCGP for qualitative decision-making issues. However, there are two limitations on their method: (1) it only deals with triangular MF, which is less flexible in resolving practical problems, and (2) it only maximizes the values of MF (i.e., qualitative issue) in the objective function; while it may be necessary to consider both qualitative and quantitative issues in practice.

Additionally, Hocine *et al.* [17] proposed a WA-FMCGP method to formulate triangular and trapezoidal MFs. However, there is a major restriction on their method: $2n$ binary variables were required in their model to formulate n triangular MFs, which becomes time-consuming when the problem size increases. Li *et al.* [18] also proposed a large-scale group decision-making method to manage crisp, interval and triangular numbers, and [19] a generalized fuzzy number to express decision makers' preferences. On the other hand, Zheng and Chang [20] proposed an MGCP model with a trapezoidal utility function to measure senior citizens' satisfaction in topology design. Zhang *et al.* [21] derived a consensus-reaching method for social network group decision making by considering leadership and bounded confidence. Zhang *et al.* [22] proposed a consensus reaching method for group decision making with multi-granular unbalanced linguistic information. Patro *et al.* [23] also presented an MCGP model using Vandermonde's interpolating polynomial, binary variables, and a least square approximation method.

Moreover, MCGP with triangular and trapezoidal MFs has been rarely studied in the past. To fill this gap, this study proposes a novel method with two contributions: (1) it can be easily used to formulate triangular and trapezoidal MFs, which will improve the usefulness of MCGP in solving real-world problems, and (2) only $\lceil \log_2 2n \rceil$ binary variables are required to formulate n triangular MFs. In addition, given the lack of literature, this study aims at understanding the relationship between MCGP and the nature-inspired optimization method to understand the related technologies of MODM better. Therefore, the new FMCGP and the nature-inspired optimization method (NIOM) are proposed to improve the usefulness of MCGP in the field of MODM. The relationships between the MCGP family members are shown in Figure 1. Although Lingo 12 [24] or related software can be used to solve the FMCGP model, Lingo cannot guarantee to find the

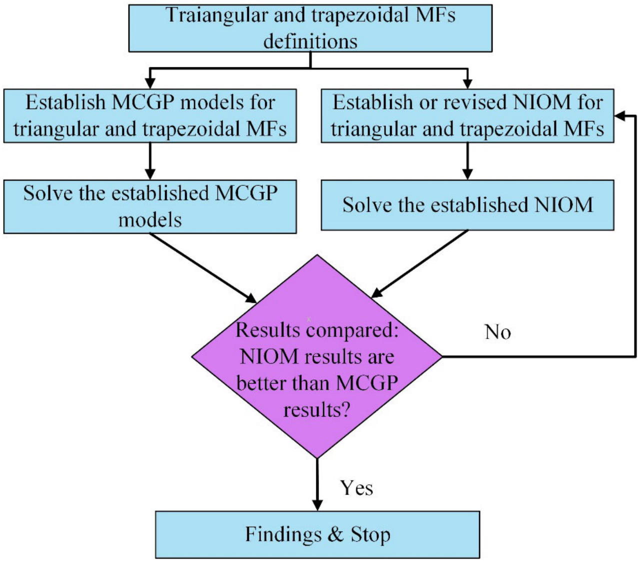


FIGURE 2. The flowchart of this study.

global optimal solution when it faces a non-linear problem. Thus, many meta-heuristics, including nature-inspired optimization methods, are proposed as global optimization search algorithms to solve FMCGP problems.

The artificial bee colony (ABC) algorithm is a well-known technique, and many studies have shown that it has an excellent performance with high-quality solutions. Thus, this study first uses the ABC algorithm to solve FMCGP problems and finds out the differences between MCGP and ABC methods. This study also fully applies the ''equation constraint'' feature in the proposed NIOM to reduce the number of required decision variables for the original model, dramatically reducing a problem's complexity. Furthermore, the role of this study is to explore a new area that enriches the field of MODM.

This paper proposes two new methods to formulate popular triangular and trapezoidal MFs. In addition, a NIOM method is also provided to enrich the related fields of MCGP. To prove the novelty of the NIOM and the proposed FMCGP methods, these same methods are used to solve the same set of MODM problems with triangular and trapezoidal MFs. A comparison of the accuracy of the solutions of both methods is further provided. To briefly demonstrate the comparison in the study, a flowchart is shown in Figure 2. First, it defines the triangular and trapezoidal MFs. Second, it establishes the MCGP and NIOM models for triangular and trapezoidal MFs. Third, it solves the MCGP and NIOM models and then compares the results. If the NIOM result is better than the MCGP, the findings are obtained, and the procedure is halted. Otherwise, the NIOM algorithm would be revised and compared again. Furthermore, this study provides more insightful information regarding the NIOM algorithm, which contributes to the MCGP and NIOM fields and their respective applications.

The remainder of this paper is organized as follows: Section 2 presents model formulations for the proposed model with triangular and trapezoidal MFs. Section 3 presents the artificial bee colony algorithm for solving triangular and trapezoidal MFs. In Section 5, we conclude our findings and recommendations for future research are provided.

II. MODEL FORMULATIONS

A. PREVIOUS MODELS

The MCGP with UF $\mu_i(y_i)$ proposed by Chang [3] allows DM to set their preferences mapping with linear UF for an MODM problem. The linear UFs can be expressed as in (1) and (2) as shown in Figure 3.

$$\mu_i(y_i) = \begin{cases} 1, & \text{if } y_i \le g_{i,\min} \\ \dfrac{g_{i,\max} - y_i}{g_{i,\max} - g_{i,\min}}, & \text{if } g_{i,\min} \le y_i \le g_{i,\max} \\ 0, & \text{if } y_i \ge g_{i,\max} \end{cases} \quad \text{for LLUF} \tag{1}$$

$$\mu_i(y_i) = \begin{cases} 1, & \text{if } y_i \ge g_{i,\max} \\ \dfrac{y_i - g_{i,\min}}{g_{i,\max} - g_{i,\min}}, & \text{if } g_{i,\min} \le y_i \le g_{i,\max} \\ 0, & \text{if } y_i \le g_{i,\min} \end{cases} \quad \text{for RLUF} \tag{2}$$

where $g_{i,\max}$ and $g_{i,\min}$ are lower and upper bounds for the ith goal, respectively.

Further, the MCGP with utility functions proposed by Chang [3] was expressed, as follows:

$$\text{Min} \sum_{i=1}^{n} [w_i(d_i^+ + d_i^-) + \beta_i f_i^-]$$

$$\text{s.t. } \lambda_i \le \frac{g_{i,\max} - y_i}{g_{i,\max} - g_{i,\min}}, \quad i = 1, 2, \ldots, n, \text{ for LLUF} \tag{3}$$

$$\lambda_i \le \frac{y_i - g_{i,\min}}{g_{i,\max} - g_{i,\min}}, \quad i = 1, 2, \ldots, n, \text{ for RLUF} \tag{4}$$

$$f_i(\mathbf{x}) - d_i^+ + d_i^- = y_i, \quad i = 1, 2, \ldots, n, \tag{5}$$

$$\lambda_i + f_i^- = 1, \quad i = 1, 2, \ldots, n, \tag{6}$$

$$g_{i,\min} \le y_i \le g_{i,\max}, \quad i = 1, 2, \ldots, n, \tag{7}$$

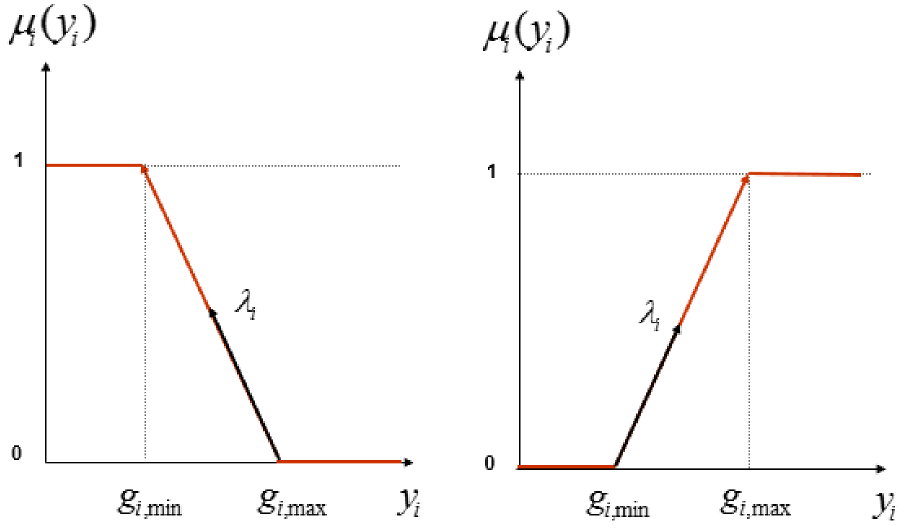
$$d_i^+, d_i^-, f_i^-, \lambda_i \ge 0, \quad i = 1, 2, \ldots, n, \tag{8}$$

$\mathbf{x} \in \mathbf{F}$, ($\mathbf{F}$ is a feasible set, while $\mathbf{x}$ is unrestricted in sign), where $w_i$and $\beta_i$are weights attached to deviational variables d_i^+, d_i^- and f_i^-; λ_i is the utility value. As described in (6), the highest possible value of the L(R)LUF (from (3) and (4)) is 1. Other variables are defined as in MCGP (see Chang [1]).

B. PROPOSED MODEL FOR TRIANGULAR MF FORMULATION

A triangular MF is widely used to solve many uncertainty problems, which can be expressed as:

$$M_i(y_i) = \begin{cases} \dfrac{y_i - g_{i1}^{\min}}{\tilde{g}_{i1} - g_{i1}^{\min}}, & \text{if } g_{i1}^{\min} \le y_i \le \tilde{g}_{i1} \\ 1, & \text{if } y_i = \tilde{g}_{i1} \\ \dfrac{y_i - \tilde{g}_{i1}}{g_{i1}^{\max} - \tilde{g}_{i1}}, & \text{if } \tilde{g}_{i1} \le y_i \le g_{i1}^{\max} \end{cases} \tag{9}$$



(a) Left linear utility function (LLUF) (b) Right linear utility function (RLUF)

FIGURE 3. The linear utility functions.

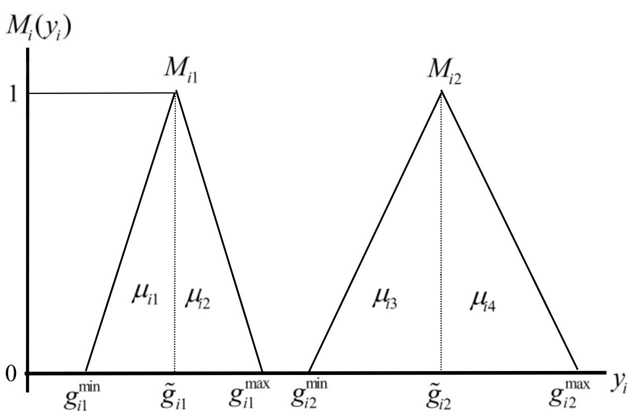


FIGURE 4. Two triangular MFs.

Particularly, an example of two triangular MFs is demonstrated in Figure 4, which can be represented by $M_{i1} = \mu_{i1} \cup \mu_{i2}$ and $M_{i2} = \mu_{i3} \cup \mu_{i4}$, where μ_{ij} is a sub-triangular MF. To improve the usefulness of MCGP, a new triangular MF should be added. Therefore, Figure 4 can be intuitively formulated, as follows:

(P1) New MCGP with triangular MF

$$\text{Min} \sum_{i=1}^{n} (\alpha_i d_i^+ + \beta_i d_i^-) + \sum_{i=1}^{n} \sum_{j=1}^{4} w_{ij} e_{ij}^-$$

$$\text{s.t.}\ f_i(y_i) - d_i^+ + d_i^- = y_{i1} z_{i1} z_{i2} + y_{i2} z_{i1}(1 - z_{i2}) + y_{i3}(1 - z_{i1}) z_{i2} + y_{i4}(1 - z_{i1})(1 - z_{i2}),$$
$$i = 1, 2, .., n \tag{10}$$

$$\mu_{i1} = 1 - \frac{\tilde{g}_{i1} - y_{i1}}{\tilde{g}_{i1} - g_{i1}^{\min}}, \quad \mu_{i2} = 1 - \frac{y_{i2} - \tilde{g}_{i1}}{g_{i1}^{\max} - \tilde{g}_{i1}},$$
$$i = 1, 2, \ldots, n \tag{11}$$

$$\mu_{i1} + e_{i1}^- = 1, \quad \mu_{i2} + e_{i2}^- = 1, \; i = 1, 2, \ldots, n \tag{12}$$

$$\mu_{i3} = 1 - \frac{\tilde{g}_{i2} - y_{i3}}{\tilde{g}_{i2} - g_{i2}^{\min}}, \quad \mu_{i4} = 1 - \frac{y_{i4} - \tilde{g}_{i2}}{g_{i2}^{\max} - \tilde{g}_{i2}},$$
$$i = 1, 2, \ldots, n \tag{13}$$

$$\mu_{i3} + e_{i3}^- = 1, \quad \mu_{i4} + e_{i4}^- = 1, \; i = 1, 2, \ldots, n \tag{14}$$

$X \in F$, (F is a feasible set), where d_i^+ and d_i^- are positive and negative deviational variables attached to $|f_i(y) - \tilde{g}_i|$; α_i and β_i are the weights attached to d_i^+ and d_i^-; w_{ij} is the weight attached to e_{ij}^-; $f_i(y)$ is the i^{th} objective function; z_{ij} is the binary variable; μ_{ij} is the MF of $\tilde{g}_{ij}$; e_{ij}^- is the negative deviational variable used to force the value of μ_{ij} approaching 1 (i.e., the maximum value of MF). $g_{ij}^{\max}(g_{ij}^{\min})$ is upper (lower) bound of y_{ij}; $y_{i1} \in [g_{i1}^{min}, \tilde{g}_{i1}]$, $y_{i2} \in [\tilde{g}_{i1}, g_{i1}^{max}]$, $y_{i3} \in [g_{i2}^{min}, \tilde{g}_{i2}]$, and $y_{i4} \in [\tilde{g}_{i2}, g_{i2}^{max}]$ are additional variables.

Proposition 1: P1 and Figure 4 are equivalent in the sense that they have the same optimal solutions.

Proof:

(i) If the value of $y_{i1} \in [g_{i1}^{\min}, \tilde{g}_{i1}]$, then $\mu_{i1} = 1 - \frac{\tilde{g}_{i1} - y_{i1}}{\tilde{g}_{i1} - g_{i1}^{\min}}$ (from (11)). This forces $z_{i1} = z_{i2} = 1$ (from (10)) and the value of μ_{i1} approaching 1 (from (12)).

(ii) If the value of $y_{i1} \in [\tilde{g}_{i1}, g_{i1}^{\max}]$, then $\mu_{i2} = 1 - \frac{y_{i2} - \tilde{g}_1}{g_{i1}^{\max} - \tilde{g}_{i1}}$ (from (11)). This forces $z_{i1} = 1$, $z_{i2} = 0$ (from (10)) and the value of μ_{i2} approaching 1 (from (12)).

(iii) If the value of $y_{i2} \in [g_{i2}^{\min}, \tilde{g}_{i2}]$, then $\mu_{i3} = 1 - \frac{\tilde{g}_{i2} - y_{i3}}{\tilde{g}_{i2} - g_{i2}^{\min}}$ (from (13)). This forces $z_{i1} = 0$, $z_{i2} = 1$ (from (10)) and the value of μ_{i3} approaching 1 (from (14)).

(iv) If the value of $y_{i2} \in [\tilde{g}_{i2}, g_{i2}^{\max}]$, then $\mu_{i4} = 1 - \frac{y_{i4} - \tilde{g}_{i2}}{g_{i2}^{\max} - \tilde{g}_{i2}}$ (from (13)). This forces $z_{i1} = z_{i2} = 0$ (from (10)) and the value of μ_{i4} approaching 1 (from (14)).

This is essentially the same as P1=Figure 4 and completes the proof of Proposition 1.

By referring to Chang [25], the quadratic binary term $z_{i1}z_{i2}$ in (10) can be linearized, as follows:

Let $x_i = z_{i1}z_{i2}$, where x_i satisfy the following inequalities.

$$(z_{i1} + z_{i2} - 2) + 1 \leq x_i \leq (2 - z_{i1} - z_{i2}) + 1, \quad (15)$$

$$x_i \leq z_{i1}, \quad (16)$$

$$x_i \leq z_{i2}, \quad (17)$$

$$x_i \geq 0, \quad (18)$$

The management implication of P1 is that two MFs (M1 and M2) represent the aspiration levels on the right-hand side of (10) as "choosing one from them." For example, the two MFs can be used to map two types of customer satisfaction in the MODM model. This provides an optimal choice of MFs for MODM problems. As seen in P1, plus 4 sign constraints (from (18)) are required to deal with 2 triangular MFs, 2 binary variables (from (10)), and 16 auxiliary constraints (from (15)-(17)). Accordingly, to deal with the number of n triangular MFs, $\lceil \log_2 2n \rceil$ binary variables, $8n$ auxiliary constraints, plus $2n$ sign constraints are required. This handling is better than the model of Hocine *et al.* [17], where $2n$ binary variables are needed in their model to deal with the number of n triangular MFs. In order to reduce the auxiliary constraints in P1, Proposition 2 is introduced, as follows:

Proposition 2: Given the two infinite sets $K = \{1, \ldots, n\}$ and $J = \{1, \ldots, \lceil \log_2 n \rceil\}$, extra continuous variables are denoted as $c_k (k \in K)$, which can be constructed by adding a logarithmic number of binary variables $b_j (j \in J)$ and the following constraints.

$$f(x) = \sum_{k \in K} d_k c_k, \quad (19)$$

$$\sum_{k \in K} c_k = 1, \quad (20)$$

$$\sum_{k \in J^+(j)} c_k = b_j, \quad \forall j \in J, \quad (21)$$

where $J^+(j) = \{k \in K : j \in \sigma(B(k)); B : K \rightarrow \{0, 1\}^J$ is any injective function; $\sigma(B(k))$ is the support of vector $B(k)$; c_k is the continuous variable; b_j is the binary variable; the number of $j = \lceil \log_2 k \rceil$; $f(x)$ contains k possible values in the set $D = \{d_1, \ldots, d_k\}, d_k \in \mathbb{R}$.

Example 1: A company is manufacturing two products, y_1 and y_2. For product y_1, there are two types of customers, namely general customers (GC) and VIP customers (VIPC), with "approximate" demands of 30 and 50, respectively. These demands can be represented by different MFs, as shown in Figure 4. The maximum allowable negative and positive deviations for GC and VIPC from their goals are set as 4 and 5, respectively. Further, (G1) is denoted as $y_1 \cong 30$ or 50 and (G2) as $y_2 \cong 15$ or 30. The selling profit for the product y_1 (y_2) is 10 (12) dollars. The information about these two products is shown in Table 1. However, due to limitations such as political ones, the company must select only one of its customers for each product. A profit of at least 450 dollars from the product is expected. Weights w_1 and w_2 are given as 0.4 and 0.3, respectively. The available amount of resources for products y_1 and y_2 is shown in Table 2. The objective is to find the best degree of achievement of the two fuzzy goals.

TABLE 1. Related information about products.

Product	Customer	Demands	Profits ($)
y_1	GC	30	10
	VIPC	50	
y_2	GC	15	12
	VIPC	30	

TABLE 2. The amount of resource consumption for each product.

Resource	y_1	y_2	$y_1 y_2$	Available amount
S_1	$4y_1^2$	$7y_2^2$	0	5000
S_2	3	5	0	380
S_3	1	3	0	120
S_4	0	0	1	400

Based on the proposed FMCGP and P1 models, the problem can be formulated as follows.

(M1)

$$\text{Min} \sum_{i=1}^{2} (d_i^+ + d_i^-) + 0.4 \sum_{i=1}^{4} e_i^- + 0.3 \sum_{i=5}^{8} e_i^-$$

$$\text{s.t. } y_1 - d_1^+ + d_1^- = x_1 z_1 z_2 + x_2 z_1 (1 - z_2) + x_3 (1 - z_1) z_2 + x_4 (1 - z_1)(1 - z_1), \quad (22)$$

$$\mu_1 = 1 - (\frac{x_1 - 30}{4}), \quad \mu_2 = 1 - (\frac{30 - x_2}{4}),$$

$$\mu_1 + e_1^- = 1, \quad \mu_2 + e_2^- = 1, \text{ for GC}, \quad (23)$$

$$\mu_3 = 1 - (\frac{x_3 - 50}{5}), \quad \mu_4 = 1 - (\frac{50 - x_4}{5}),$$

$$\mu_3 + e_3^- = 1, \quad \mu_4 + e_4^- = 1, \text{ for VIPC}, \quad (24)$$

$$y_2 - d_2^+ + d_2^- = x_5 z_3 z_4 + x_6 z_3 (1 - z_4) + x_7 (1 - z_3) z_4 + x_8 (1 - z_3)(1 - z_4), \quad (25)$$

$$\mu_5 = 1 - (\frac{x_5 - 15}{4}), \quad \mu_6 = 1 - (\frac{15 - x_6}{4}),$$

$$\mu_5 + e_5^- = 1, \quad \mu_6 + e_6^- = 1, \text{ for GC}, \quad (26)$$

$$\mu_7 = 1 - (\frac{x_7 - 30}{5}), \quad \mu_8 = 1 - (\frac{30 - x_8}{5}),$$

$$\mu_7 + e_7^- = 1, \quad \mu_8 + e_8^- = 1, \text{ for VIPC}, \quad (27)$$

$$10y_1 + 12y_2 \geq 450, \quad (28)$$

$$4y_1^2 + 7y_2^2 \leq 5000, \quad (29)$$

$$3y_1 + 5y_2 \leq 380, \quad (30)$$

$$y_1 + 2y_2 \leq 120, \quad (31)$$

$$y_1 y_2 \leq 400, \quad (32)$$

$$z_1, z_2, z_3 \text{ and } z_4 \text{ are binary variables} \quad (33)$$

$$d_i^+, e_i^- \geq 0, \quad (34)$$

$$28 \leq x_1 \leq 32, \quad 28 \leq x_2 \leq 32, \quad 47.5 \leq x_3 \leq 52.5, \quad 47.5 \leq x_4 \leq 52.5, \quad (35)$$

$$13 \le x_5 \le 17, \quad 13 \le x_6 \le 17, \; 27.5 \le x_7 \le 32.5, \; 27.5 \le x_8 \le 32.5, \tag{36}$$

The quadratic terms of (22) and (25) can be linearized using (15)-(18) or Proposition 2. To solve this problem, Lingo 12 [24] is used to obtain the local optimal solutions (objective function value (OFV) = 15.2086426) as $(y_1, y_2, \mu_1, \mu_2, \mu_3, \mu_4, z_1, z_2, z_3, z_4) = (17.0996122, 23.3923452, 1, 0.5, 1, 1, 1, 0, 0, 0)$ and the degree of achievement of two fuzzy goals where $\tilde{g}_{y1}$ and $\tilde{g}_{y2}$ is 50.00% and 50.00%, respectively ($\tilde{g}_{y1} = 1-(30-28)/4) =$ 50.00%; $\tilde{g}_{y2} = 1-(30-27.5)/5 = 50.00\%$). A profit of \$451.7043 is also obtained.

Based on Proposition 2, (22) and (25) can be replaced, as follows:

$$y_1 - d_1^+ + d_1^- = x_1c_1 + x_2c_2 + x_3c_3 + x_4c_4, \tag{37}$$
$$c_1 + c_2 + c_3 + c_4 = 1, \tag{38}$$
$$c_2 + c_4 = z_1, \tag{39}$$
$$c_3 + c_4 = z_2, \tag{40}$$
$$y_2 - d_2^+ + d_2^- = x_5c_5 + x_6c_6 + x_7c_7 + x_8c_8, \tag{41}$$
$$c_1 + c_2 + c_3 + c_4 = 1, \tag{42}$$
$$c_2 + c_4 = z_1, \tag{43}$$
$$c_3 + c_4 = z_2, \tag{44}$$

where c_i, $\forall i$ are continous variables, and z_i, $\forall i$ are binary variables.

This problem is solved again using Lingo 12 [24] to obtain the same solutions.

C. PROPOSED MODEL FOR TRAPEZOIDAL MF FORMULATION

A trapezoidal MF can be expressed as (37).

$$M_i(y_i) = \begin{cases} 0, & \text{if } y_i \le g_i^{\min} \text{or } y_i \ge g_i^{\max} \\ \dfrac{y_i - g_i^{\min}}{\tilde{g}_{ib} - g_i^{\min}}, & \text{if } g_i^{\min} \le y_i \le \tilde{g}_{ib} \\ 1, & \text{if } \tilde{g}_{ib} \le y_i \le \tilde{g}_{ic} \\ \dfrac{y_i - \tilde{g}_{ic}}{g_i^{\max} - \tilde{g}_{ic}}, & \text{if } \tilde{g}_{ic} \le y_i \le g_i^{\max} \end{cases} \tag{45}$$

An example of two trapezoidal MFs is demonstrated in Figure 5, which can be represented by $M_{i1} = \mu_{i1} \cup R_{i1} \cup \mu_{i2}$ and $M_{i2} = \mu_{i3} \cup R_{i2} \cup \mu_{i4}$. To improve the usefulness of MCGP, a new trapezoidal MF should be added. Therefore, Figure 5 can be intuitively formulated, as follows:

(P2) The new MCGP with trapezoidal MF

$$\text{Min} \sum_{i=1}^{n} (\alpha_i d_i^+ + \beta_i d_i^-) + \sum_{i=1}^{n}\sum_{j=1}^{6} w_{ij} e_{ij}^-$$
$$\text{s.t. } f_i(y_i) - d_i^+ + d_i^- = y_{i1}z_{i1}z_{i2}z_{i3} + y_{i2}z_{i1}z_{i2}(1-z_{i3}) + y_{i3}z_{i1}(1-z_{i2})z_{i3} + y_{i4}z_{i1}(1-z_{i2})(1-z_{i3}) + y_{i5}(1-z_{i1})z_{i2}z_{i3} + y_{i6}(1-z_{i1})z_{i2}(1-z_{i3}),$$
$$i = 1, 2, \ldots, n, \tag{46}$$

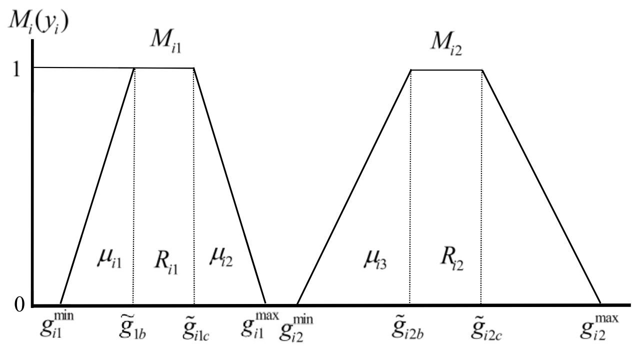


FIGURE 5. Two trapezoidal MFs.

$$\mu_{i1} = 1 - \frac{\tilde{g}_{i1b} - y_{i1}}{\tilde{g}_{i1b} - g_{i1}^{\min}}, \quad \mu_{i2} = 1 - \frac{y_{i2} - \tilde{g}_{i1c}}{g_{i1}^{\max} - \tilde{g}_{i1c}},$$
$$i = 1, 2, \ldots, n, \tag{47}$$
$$\mu_{i1} + e_{i1}^- = 1, \quad \mu_{i2} + e_{i2}^- = 1, \; i = 1, 2, \ldots, n, \tag{48}$$
$$R_{i1} = z_{i1}(1 - z_{i2})z_{i3}, \quad R_{i1} + e_{i3}^- = 1,$$
$$i = 1, 2, \ldots, n, \tag{49}$$
$$\mu_{i3} = 1 - \frac{\tilde{g}_{i2b} - y_{i4}}{\tilde{g}_{i2b} - g_{i2}^{\min}}, \quad \mu_{i4} = 1 - \frac{y_{i5} - \tilde{g}_{i2c}}{g_{i2}^{\max} - \tilde{g}_{i2c}},$$
$$i = 1, 2, \ldots, n, \tag{50}$$
$$\mu_{i3} + e_{i4}^- = 1, \quad \mu_{i4} + e_{i5}^- = 1, \; i = 1, 2, \ldots, n, \tag{51}$$
$$R_{i2} = (1 - z_{i1})z_{i2}(1 - z_{i3}), \quad R_{i2} + e_{i6}^- = 1,$$
$$i = 1, 2, \ldots, n, \tag{52}$$
$$g_{i1}^{\min} \le f_i(y_i) \le g_{i2}^{\max}, \quad i = 1, 2, \ldots, n,$$
$$X \in F, \; (F \text{ is a feasible set}) \tag{53}$$

where d_i^+ and d_i^- are positive and negative deviational variables attached to $|f_i(y_i) - \tilde{g}_i|$; α_i and β_i are the weights attached to d_i^+ and d_i^-; w_{ij} is the weight attached to e_{ij}^-; $f_i(y_i)$ is the objective function; z_{ij} is the binary variable; μ_{ij} is the MF; e_{ij}^- is the negative deviational variable used to force μ_{ij} and R_{ij} approaching 1 (i.e., maximum value of MF). $g_i^{\max}(g_i^{\min})$ is upper (lower) bound of y_i; $y_{i1} \in [g_{i1}^{min}, \tilde{g}_{i1b}]$, $R_{i1} \in [\tilde{g}_{i1b}, \tilde{g}_{i1c}]$, $y_{i2} \in [\tilde{g}_{i1c}, g_{i1}^{max}]$, $y_{i3} \in [g_{i2}^{min}, \tilde{g}_{i2b}]$, $R_{i2} \in [\tilde{g}_{i2b}, \tilde{g}_{i2c}]$, and $y_{i4} \in [\tilde{g}_{i2c}, g_{i2}^{max}]$ are additional variables.

Proposition 3: P2 and Figure 5 are equivalent in the sense that they have the same optimal solutions.

Proof:

(i) If the value of $y_i \in [g_{i1}^{\min}, \tilde{g}_{i1b}]$, then $\mu_{i1} = 1 - \frac{\tilde{g}_{i1b} - y_{i1}}{\tilde{g}_{i1b} - g_{i1}^{\min}}$ (from Eq.(47)). This forces $z_{i1} = z_{i2} = z_{i3} = 1$ (from (46)) and the value of μ_{i1} approaching 1 (from (48)).

(ii) If the value of $y_i \in [\tilde{g}_{i1b}, \tilde{g}_{i1c}]$, then $R_{i1} = 1$ (from (49)). This forces $z_{i1} = 1$, $z_{i2} = 0$, $z_{i3} = 1$ (from (46)).

(iii) If the value of $y_i \in [\tilde{g}_{i1c}, g_{i1}^{\max}]$, then $\mu_{i2} = 1 - \frac{y_{i2} - \tilde{g}_{i1c}}{g_{i1}^{\max} - \tilde{g}_{i1c}}$ (from (47)). This forces $z_{i1} = z_{i2} = 1$, $z_{i3} = 0$ (from (46)) and the value of μ_{i2} approaching 1 (from (48)).

(iv). If the value of $y_i \in [g_{i2}^{\min}, \tilde{g}_{i1b}]$, then $\mu_{i3} = 1 - \frac{\tilde{g}_{i2b} - y_{i4}}{\tilde{g}_{i2b} - g_{i2}^{\min}}$ (from (50)). This forces $z_{i1} = 1$, $z_{i2} = 0$, $z_{i3} = 1$ (from (46)) and the value of μ_{i3} approaching 1 (from (51)).

(iv) If the value of $y_i \in [\tilde{g}_{i2b}, \tilde{g}_{i2c}]$, then $R_{i2} = 1$ (from (52)). This forces $z_{i1} = 0$, $z_{i2} = 1$, $z_{i3} = 0$ (from (46)).

(v) If the value of $y_i \in [\tilde{g}_{i2c}, g_{i2}^{\max}]$, then $\mu_{i4} = 1 - \frac{y_{i5} - \tilde{g}_{i2c}}{g_{i2}^{\max} - \tilde{g}_{i2c}}$ (from (50)). This forces $z_{i1} = 0$, $z_{i2} = 1$, $z_{i3} = 1$ (from (46)) and the value of μ_{i4} approaching 1 (from (51)).

This is essentially the same as P2=Figure 5 and completes the proof of Proposition 3.

Example 2: Eating disorders or imbalanced diets have recently become a serious health problem worldwide. Many chronic diseases such as atherosclerosis, heart disease, and stroke are associated with imbalanced diets. Basal metabolic rate (BMR) is the rate at which the body uses energy while at rest to maintain vital functions such as breathing and body temperature. In this example, the revised Harris-Benedict equation from the American college of sports medicine is used to determine a person's daily energy expenditure in calories. The BMR is calculated as follows:

$$\begin{aligned}\text{BMR(male)} &= (13.397 \times \text{weight(kg)}) \\ &\quad + (4.799 \times \text{height(cm)}) \\ &\quad - (5.677 \times \text{age}) + 66 \\ \text{BMR(female)} &= (9.247 \times \text{weight(kg)}) \\ &\quad + (3.098 \times \text{height(cm)}) \\ &\quad - (4.33 \times \text{age}) + 447.593\end{aligned}$$

The basic summation of calories to be burned off is BMR plus 400 Kcal, to account for daily activities such as walking, thinking, breathing, and other important functions.

A 26-year-old man, Sam, is selected as the analytical target whose height and weight are 168cm and 77kg, respectively. Sam's basic consumption of calories is calculated as follows:

$$\begin{aligned}&13.397 \times 77 + 4.799 \times 168.5.677 \\ &\quad \times 26 + 66 + 400 = 2090 \text{ Kcal/day} \\ &1254 \text{ Kcal (carbohydrate)} + 522.5 \text{ Kcal (protein)} \\ &\quad + 313.5 \text{ Kcal (fat)} = 2090 \text{ Kcal /day}\end{aligned}$$

The acceptable distribution range of nutrients is shown in Table 3. On the other hand, the range of target values for carbohydrate, protein, and fat is depicted in Figures 6-8. Twenty-seven types of food have been chosen from four categories (grains, meat, vegetable, egg), where each food has nine types of nutrients, as shown in Table 4. The objective is to find the best degree of carbohydrate, protein, and fat achievement for Sam.

1) GOALS

Maximize the degree of achievement of carbohydrate, protein, and fat

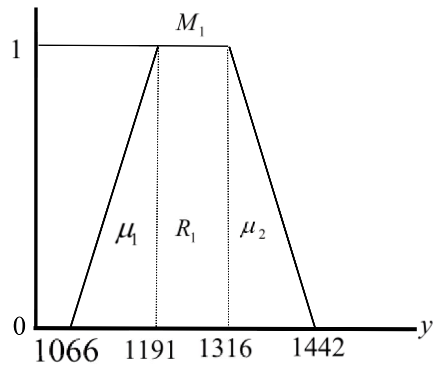


FIGURE 6. A carbohydrate MF.

2) DECISION VARIABLES

M_1: the degree of achievement of carbohydrate
M_2: the degree of achievement of protein
M_3: the degree of achievement of fat

3) PARAMETERS

x_1: total intake amount of carbohydrate (Kcal)
x_2: total intake amount of protein (Kcal)
x_3: total intake amount of fat (Kcal)
x_4: total intake amount of vitamin A (μg)
x_5: total intake amount of vitamin C (mg)
x_6: total intake amount of vitamin B1 (mg)
x_7: total intake amount of vitamin B2 (mg)
x_8: total intake amount of calcium (mg)
x_9: total intake amount of iron (mg)
x_j^U: upper bound of total intake of the nutrient j
x_j^L: lower bound of total intake of the nutrient j
G_i: i^{th} grain is selected if $G_i = 1$, otherwise $G_i = 0$ where $G_i \in \{0, 1\}$
M_i: i^{th} meat is selected if $M_i = 1$, otherwise $M_i = 0$ where $M_i \in \{0, 1\}$
V_i: i^{th} vegetable is selected if $V_i = 1$, otherwise $V_i = 0$ where $V_i \in \{0, 1\}$
E_i: i^{th} egg is selected if $E_i = 1$, otherwise $E_i = 0$ where $V_i \in \{0, 1\}$
N_{ij}: i^{th} food ($i = 1, \ldots, 27$) and j^{th} nutrients ($j = 1, \ldots, 9$)

The objective function maximizes the degree of achievement of carbohydrate, protein, and fat, as follows:

$$\text{Max } M_1(y) + M_2(y) + M_3(y)$$

Furthermore, the constraints of the problem are described as follows:

1. Nutrients $x_1, \ldots, x_9$ must fall within a certain interval, as shown in Table 4. Therefore, the constraint $x_j^L \leq x_j \leq x_j^U$ ($j = 1, \ldots, 9$) should be obeyed. Assuming his personal needs, his nutrition may have some non-linear limitations as in $2x_6^2 + 3x_7^2 \leq 27$ and $x_6 x_7 x_9 \leq 120$.

TABLE 3. Nutrient requirements (Sam: Gender: Male; Age: 26; Height: 168cm; Weight: 77Kg).

Nutrients	Target Value	Range of Target Value
Carbohydrate (x_1)	1254 Kcal	1066-1442
Protein (x_2)	522 Kcal	458-594
Fat (x_3)	313 Kcal	164-462
Vitamin A (x_4)	600	600-3000
Vitamin C (x_5)	100	100-2000
Vitamin B1 (x_6)	1.2	1.2-2
Vitamin B2 (x_7)	1.3	1.3-2
Calcium (x_8)	1000	1000-2500
Iron (x_9)	10	10-40

TABLE 4. Food nutrients.

		Carbohydrate (Kcal)	Protein (Kcal)	Fat (Kcal)	Vitamin				Calcium (mg)	Iron (mg)
					A (μg)	C (mg)	B_1 (mg)	B_2 (mg)		
Grains										
	No.1	300.0	3.9	1.80	78.75	2.0	0.12	0.050	39.00	1.00
	No.2	400.0	24.2	7.70	58.50	10.0	0.24	0.190	108.00	1.50
	No.3	450.0	6.0	0.60	22.80	9.0	0.13	0.060	12.40	0.90
	No.4	280.0	12.4	6.30	29.70	5.0	0.09	0.130	47.00	1.40
	No.5	200.0	14.2	2.30	15.00	3.2	0.13	0.150	24.50	1.50
	No.6	170.0	2.8	3.10	141.60	1.0	0.06	0.020	6.00	0.30
Meat										
	No.7	2.9	30.6	15.90	17.40	30.0	0.06	0.140	9.00	1.20
	No.8	23.5	18.4	13.60	249.30	28.0	0.15	0.090	23.00	1.40
	No.9	20.7	24.5	24.70	186.60	24.5	0.29	0.140	56.50	2.20
	No.10	18.5	32.2	14.30	270.90	15.0	0.12	0.250	71.00	2.40
	No.11	10.6	40.7	23.10	36.30	63.0	0.25	0.150	26.00	1.70
	No.12	12.5	50.9	40.30	53.70	4.0	0.11	0.270	123.00	1.50
	No.13	7.1	18.4	30.00	1065.90	13.0	0.14	0.130	94.00	0.50
	No.14	6.7	29.5	10.70	135.00	9.0	0.14	0.180	18.00	0.80
	No.15	4.9	36.2	10.20	0.90	0.0	0.03	0.060	7.00	0.90
Vegetable										
	No.16	1.2	1.3	1.50	96.83	5.7	0.03	0.050	52.22	0.70
	No.17	2.5	6.2	0.30	27.00	48.0	0.02	0.050	86.00	1.00
	No.18	4.9	5.1	8.10	60.00	80.0	0.08	0.150	30.00	1.20
	No.19	4.7	5.3	0.20	1.50	22.0	0.03	0.040	26.00	0.30
	No.20	1.18	0.9	0.08	95.60	12.5	0.04	0.060	10.70	0.77
	No.21	2.9	9.0	13.30	324.00	45.0	0.07	0.140	294.00	3.50
	No.22	5.6	7.5	4.10	510.00	65.0	0.20	0.260	87.00	4.10
	No.23	6.6	0.4	0.05	64.90	67.0	0.01	0.015	10.75	0.15
	No.24	13.1	8.6	6.80	63.30	24.0	0.33	0.170	42.00	1.70
Egg										
	No.25	5.5	63.8	15.00	609.90	10.0	0.28	0.560	102.00	1.80
	No.26	3.8	128.2	20.00	319.80	11.0	0.06	0.280	51.00	1.30
	No.27	1.7	100.0	30.00	31.10	3.5	0.03	0.030	67.50	0.80

2. The amount of nutrients is made up of selected foods. Therefore, it can be expressed as:

$$x_j = \sum_{i=1}^{6} G_i N_{ij} + \sum_{i=7}^{16} M_i N_{ij} + \sum_{i=17}^{24} E_i N_{ij} + \sum_{i=25}^{27} E_i N_{ij} \times (j = 1, \ldots, 9).$$

Based on the proposed FMCGP and P2 models, this problem can easily be formulated as follows:

(M2)

$$\text{Min} \sum_{i=1}^{3} (d_i^+ + d_i^-) + \sum_{i=1}^{9} w_i e_i^-$$

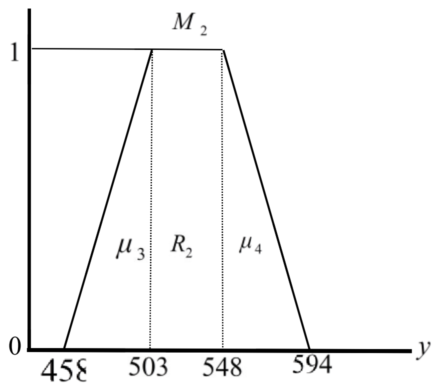


FIGURE 7. A protein MF.

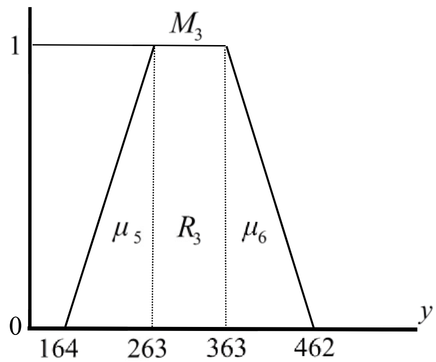


FIGURE 8. A fat MF.

$$\text{s.t.}\ f_1(y) - d_1^+ + d_1^- = y_1 z_1 z_2 + y_2 z_1(1-z_2) + y_3(1-z_1)z_2, \quad \text{for carbohydrate}, \tag{54}$$

$$\mu_1 = 1 - \frac{1191 - y_1}{1191 - 1066}, \quad \mu_2 = 1 - \frac{y_2 - 1316}{1442 - 1316}, \tag{55}$$

$$\mu_1 + e_1^- = 1, \quad \mu_2 + e_2^- = 1, \tag{56}$$

$$R_1 = (1 - z_1)z_2, \quad R_1 + e_3^- = 1, \tag{57}$$

$$f_2(y) - d_2^+ + d_2^- = y_4 z_3 z_4 + y_5 z_3(1-z_4) + y_6(1-z_3)z_4 \quad \text{for protein}, \tag{58}$$

$$\mu_3 = 1 - \frac{503 - y_4}{503 - 458}, \quad \mu_4 = 1 - \frac{y_5 - 548}{594 - 548}, \tag{59}$$

$$\mu_3 + e_4^- = 1, \quad \mu_4 + e_5^- = 1, \tag{60}$$

$$R_2 = (1 - z_3)z_4, \quad R_2 + e_6^- = 1, \tag{61}$$

$$f_3(y) - d_3^+ + d_3^- = y_7 z_5 z_6 + y_8 z_5(1-z_6) + y_9(1-z_5)z_6 \quad \text{for fat}, \tag{62}$$

$$\mu_5 = 1 - \frac{263 - y_7}{263 - 164}, \quad \mu_6 = 1 - \frac{y_8 - 363}{462 - 363}, \tag{63}$$

$$\mu_5 + e_7^- = 1, \quad \mu_6 + e_8^- = 1, \tag{64}$$

$$R_3 = (1 - z_5)z_6, \quad R_3 + e_9^- = 1, \tag{65}$$

$$x_j = \sum_{i=1}^{6} G_i N_{ij} + \sum_{i=7}^{16} M_i N_{ij} + \sum_{i=17}^{24} V_i N_{ij} + \sum_{i=25}^{27} E_i N_{ij}, \quad j = 1, \ldots, 9, \tag{66}$$

$$2x_6^2 + 3x_7^2 \le 27 \tag{67}$$

$$x_6 x_7 x_9 \le 120 \tag{68}$$

$$x_j^L \le x_j \le x_j^U, \quad \text{range of nutrient intake} \tag{69}$$

$$Z_i \in \{0, 1\}, \quad G_i \in \{0, 1\}, \ M_i \in \{0, 1\}, \quad V_i \in \{0, 1\}, \quad E_i \in \{0, 1\} \tag{70}$$

where $f_1(y) = x_1; f_2(y) = x_2; f_3(y) = x_3$.

The model is then solved using Lingo [24] to obtain a satisfied solution: ($OFV = 2.77870707$) as ($x_1, x_2, x_3, \mu_1, R_1, \mu_2, \mu_3, R_2, \mu_4, \mu_5, R_3, \mu_6, z_1, z_2, z_3, z_4, z_5, z_6 M_1(y), M_2(y), M_3(y)$) = (1191.0, 479.9, 235.9, 1, 1, 1, 0.4866, 0, 1, 0.7262, 0, 1, 0, 1, 1, 1, 1, 1, 1, 0.497826087, 0.726262626). This means that the demand for carbohydrates is fully met. However, the degree of achievement of protein and fat is only 48.666667% and 72.6262626%, respectively:

$$M_1(y) = 1 - (1191 - 1191)/(1191 - 1066) = 1.0 = 100\%$$

$$M_2(y) = 1 - (503 - 479.9)/(503 - 458) = 0.48666667 = 48.666667\%$$

$$M_3(y) = 1 - (263 - 235.9)/(263 - 164) = 0.72662626 = 72.6262626\%$$

III. SOLVING M1 AND M2 BY ARTIFICIAL BEE COLONY ALGORITHM

The ABC algorithm is used to solve M1 and M2 and demonstrate the usefulness of the proposed methods. The ABC algorithm is a swarm-based metaheuristic algorithm introduced by Karaboga [26] for optimizing numerical problems. Satisfactory ABC-based algorithm results have been reported when applied to the numerical test functions of optimization problems [27], [28]. Moreover, a honeybee swarm inspires the emergence of ABC algorithm by its intelligent foraging behavior, and has been transformed into artificial bees postulated in three groups: employed bees, onlooker bees, and scout bees. An employed bee exploits a food source; an onlooker bee decides to search a food source; and a scout bee performs a random search for a new food source. When an employed bee abandons a food source, it becomes a scout bee. Meanwhile, as employed bees and onlooker bees perform the exploitation process, the scout bees control the exploration process.

The proposed ABC algorithm uses four variables as parameters: (1) *SN* refers to the number of food sources, (2) *limit* denotes a predefined number that if the food source cannot be improved in successive *limit* iterations, the employed bee becomes a scout bee to explore a new solution, (3) $G_{\max}$ is the number of maximal allowable iterations, and (4) *Penalty* is the penalty of a solution if equations are violated. Both numbers of employed bees and onlooker bees are set as *SN*. The pseudo-code of the proposed ABC algorithm is shown in Figure 9. It begins with a population of randomly generated food sources (initialize each solution X_i in the first population by random, $i = 2, \ldots, SN$). In order to obtain a good initial solution, we set the initialize solution X_1 in the first population using the solution obtained by FMCGP. Assume

```
ABC (SN, limit, G_max, Penalty)
{
    Initialize solution X_1 in the first population using the solution obtained by FMCGP;
    Initialize each solution X_i in the first population by random, i = 2,...., SN;
    G=0;
    Calculate the fitness of each solution fit_i = fit(X_i, Penalty), i = 1,2,....,SN;
    While (G ≤ G_max) {
      G=G+1;
      //Place the employed bees on their food sources;
      For i = 1,2,....,SN {
         Obtain new solution Y_i based X_i;
         If fit(Y_i, Penalty) ≥ fit(X_i, Penalty) replace X_i by Y_i;
      }
      //Place the onlooker bees on the food sources depending on their nectar amounts
      For i = 1,2,....,SN {
         selects a food source X_j depending on its probability value p_j calculated by p_j = fit_j / Σ_{j=1}^{SN} fit_j,
          Obtain new solution Y_j based on x_j;
          If fit(Y_i, Penalty) > fit(X_j, Penalty) replace X_j by Y_j;
      }
      // Send the scouts to the search area for discovering new food sources;
      For i = 1,2,....,SN {
          If (food source X_i is not changed in successive limit times)
             Regenerate X_i randomly;
      }
      // Memorize the best food source found so far
      For i = 1,2,....,SN {
           If (fit(X_i, Penalty) ≥ fit(X_best, Penalty)) X_best = X_i;
      }
    }
    Output the best food source found so far (X_best);
}
```

FIGURE 9. **The pseudo-code of the proposed ABC algorithm.**

that the food source is $X_i (X_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,D}])$ and $j \in \{1, \ldots, D\}$, where D is the number of decision variables in the solution; $x_{i,j} = x_{\min,j} + rand(0, 1)(x_{\max,j} - x_{\min,j})$, where $x_{\min,j}$ and$x_{\max,j}$ are the minimum and maximum values of the $x_{i,j}$, and $rand(0, 1)$ gives a random real value between 0 and 1. For simplicity, we set $x_{\min,j}$ and $x_{\max,j}$ to be 0 and 1, respectively, so all variables will range from 0 to 1.

Furthermore, the calculation of the fitness of food sources is shown in Figure 8 (later explained). A typical iteration of ABC proceeds as follows. While the employed bees and onlooker bees are both placed in their food sources, the latter is concerned with their nectar volume. On the other hand, scout bees sent to the search area to discover new food sources X_i do not change in successive limits (*limit*). An onlooker's selection is contingent on the probability associated with a certain food source: $p_i = fit_i / \sum_{j=1}^{SN} fit_j$, where fit_i is the fitness value of solution i; SN is the number of food sources. The expression $y_{i,j} = x_{i,j} + \phi_{i,j}(x_{i,j} - x_{k,j})$ is used to produce a prospect of food position $Y_i = [y_{i,1}, y_{i,2}, \ldots, y_{i,D}]$ from the old one $X_i = [x_{i,1}, x_{i,1}, \ldots, x_{i,D}]$ in the memory, where $j \in [1, 2, \ldots, D]$ and $k \in [1, 2, \ldots, SN]$are randomly chosen indices; k must be different from i; $D$is the number of variables; and $\phi_{i,j}$ is a random number in the range $[-1, 1]$. Finally, the best food source (X_{best}) found is recorded to the memory. This process repeats until it satisfies the termination

```
fit(x_k, Penalty) {
  For each variable in x_k (k = 1,...,n) {
     If the corresponding variable (Var_k) for x_k is a continuous variable Var_k = V_lower + (V_upper - V_lower) · x_k
    Else if the corresponding variable Var_k for x_k is a binary variable {
      If (x_k < 0.5) Var_k = 0;
      Else Var_k = 1;
    }
  }
  Calculate the OFV in the model;
  For each equation EQ in the model {
    If (EQ is violated) OFV = OFV + Penalty(1+degree of violation);
  }
  Return 1.0 / OFV ;
}
```

FIGURE 10. **The pseudo-code of the fitness calculation.**

condition (the number of iteration$G$equals the maximum number of iteration $G_{\max}$) [29], [30].

To apply the ABC algorithm in solving an FMCGP problem, a solution (food source) is represented by D continuous values, where D is the number of the necessary decision variables in the model. The continuous values range from 0.0 to 1.0 ($x_{\min,j}$ and $x_{\max,j}$ set to be 0 and 1, respectively). For each continuous variable in the model, its value can be determined by $v_{lower} + (v_{upper} - v_{lower}) \cdot [*]$, where v_{lower} and v_{upper} are the lower and upper bound of the variable and $[*]$ is the value obtained for the variable from the proposed ABC algorithm. For each binary variable in the model, if the value obtained from the proposed ABC algorithm is smaller than 0.5, the binary variable is set to 0; otherwise, the binary variable is set to 1. Although many decision variables exist in the FMCGP, many of them are deducible when parts of the decision variables are given (many of the equations equal (=) one) which significantly reduce the complexity of the problem.

As shown in Figure 10, after values of variables of the FMCGP model are obtained, the *OFV* can then be calculated directly. For each equation, if the violation happens, a hefty penalty is added to the *OFV*, computed by a constant plus the degree of violation. Further, the fitness value of a solution is computed as $1.0/OFV$ – the larger the value, the better the solution.

Moreover, the proposed algorithm is implemented using Visual C++ 2015 under Windows 10 operating system. The parameter *SN* and *limit* are set to 20 and 100, respectively, to be consistent with the default as in Karaboga and Basturk [31]. The *penalty* is set to 100,000. In addition, the number of iterations is set to 500,000 to balance the computing time and solution quality.

A. SOLVING M1 BY ABC ALGORITHM

In solving the M1 through the proposed ABC algorithm, a solution has the following 14 variables: $x_1, x_2, \ldots, x_8, y_1$, and y_2 are non-negative variables with given ranges, while z_1, z_2, z_3, and z_4 are binary variables. If the values of the 14 decision variables are given, the values $d_1^+, d_2^+, d_1^-, d_2^-, e_1^+, e_2^+, e_1^-, e_2^-, u_1, u_2, \ldots, u_8$ can be computed using the equal ("=") function in (22)-(27) for ABC. For example, if the values of decision variables x_1, x_2, x_3, y_1, z_1, and z_2 are given in (22), the non-zero one (d_1^+ or d_1^-) can be easily computed because either d_1^+ or d_1^- is zero. In (23), given the values of x_1 and x_2, the values of μ_1 and μ_2 can be computed directly by $\mu_1 = 1 - (\frac{x_1-30}{4})$ and $\mu_2 = 1 - (\frac{30-x_2}{4})$, respectively. After obtaining the values of μ_1 and μ_2, the values of e_1^- and e_2^- can also be obtained by $e_1^- = 1 - \mu_1$ and $e_2^- = 1 - \mu_2$, respectively. After all values of variables of the FMCGP model are obtained, the *OFV* can be calculated directly. Further, a penalty is added to avoid the violation of (28)-(32) and greatly enlarge the *OFV* resulting in a smaller fitness value of the solution. For example, if the left side value is 449.8 in the (28), the penalty for (28) is 100,000 + 100,000* (450-449.8).

The best solution is taken among 10 calculations of the M1 by the proposed ABC algorithm. The detailed best solutions obtained by Lingo 12 [24] and the proposed ABC algorithm for M1 are shown in Table 5. A prominent result is shown by the ABC ($OFV = 0.9150181$) compared to that of Lingo 12 ($OFV = 15.2080426$). Furthermore, the solution obtained by the ABC is $(y_1, y_2, \mu_1, \mu_2, \mu_3, \mu_4, z_1, z_2, z_3, z_4) = (28.0010294, 14.2849002, 1.0, 0.5002614, 1.0, 1.0, 1, 0, 1, 1)$ and the degree of achievement of two fuzzy goals $\tilde{g}_{y1}$ and $\tilde{g}_{y2}$ is 50.026143% and 100.00%

TABLE 5. The detailed solutions obtained by lingo and for M1

Variables	Lingo*	ABC
OFV	15.2080426	0.9150181
y_1	17.0996122	28.0010294
y_2	23.3923452	14.2849002
x_1	30.0000000	30.0000000
x_2	28.0000000	28.0010457
x_3	50.0000000	50.0000000
x_4	50.0000000	50.0000000
x_5	15.0000000	15.0000000
x_6	15.0000000	15.0000000
x_7	30.0000000	30.0000000
x_8	27.5000000	29.9998913
z_1	1.0000000	1.0000000
z_2	0.0000000	0.0000000
z_3	0.0000000	1.0000000
z_4	0.0000000	1.0000000
d_1^-	10.9003878	0.0000163
d_1^+	0.0000000	0.0000000
d_2^-	4.1076548	0.7150998
d_2^+	0.0000000	0.0000000
e_1	0.0000000	0.0000000
e_2	0.5000000	0.4997386
e_3	0.0000000	0.0000000
e_4	0.0000000	0.0000000
e_5	0.0000000	0.0000000
e_6	0.0000000	0.0000000
e_7	0.0000000	0.0000000
e_8	0.0000000	0.0000217
μ_1	1.0000000	1.0000000
μ_2	0.5000000	0.5002614
μ_3	1.0000000	1.0000000
μ_4	1.0000000	1.0000000
μ_5	1.0000000	1.0000000
μ_6	1.0000000	1.0000000
μ_7	1.0000000	1.0000000
μ_8	0.5000000	0.9999783

*Because the solution obtained by Global Solver in Lingo is not feasible, the feasible local optimal solution is used.

satisfied, respectively.

$$\tilde{g}_{y1} = 1-(30-x_2)/4 = 1-(30-28.0010457)/4 = 50.0261425\%$$

$$\tilde{g}_{y2} = 1-(x_5-15)/4 = 1-(15.0000000-15)/4 = 100\%$$

B. SOLVING M2 BY ABC ALGORITHM

In solving the M2 by the proposed ABC algorithm, a solution has the following 42 variables: y_1 to y_9 are continuous decision variables, while z_1 to z_6, G_1 to G_6, M_7 to M_{15}, V_{16} to V_{24}, and E_{25} to E_{27} are binary decision variables. If the value of the above decision variables are given, the value of x_1 to x_9, R_1 to R_2, u_1 to u_6, and e_1 to e_9 can be obtained by the (54)-(66) in the model. After all values of variables of the FMCGP model are obtained, the *OFV* can be calculated directly. A penalty is added to avoid the violation of (67)-(68) and greatly enlarge the *OFV* resulting in a smaller fitness value of the solution. As shown in Table 6, the *OFV* obtained by Lingo 12 [24] and the proposed ABC algorithm is 2.7870707 and 2.6809091, respectively. Consequently,

TABLE 6. The detailed solutions obtained by lingo and for M2.

Variables	Lingo*	ABC
OFV	2.7870707	2.6809091
y_1	1191.0000000	1191.0000000
y_2	1316.0000000	1316.0000000
y_3	1300.0000000	1307.9000000
y_4	479.9000000	485.2000000
y_5	548.0000000	548.0000000
y_6	503.2500000	514.4829102
y_7	235.9000000	234.7500000
y_8	363.0000000	363.0000000
y_9	263.2500000	288.1586914
x_1	1300.0000000	1307.9000000
x_2	479.9000000	485.2000000
x_3	235.9000000	234.7500000
x_4	2940.0800000	2935.1500000
x_5	290.4000000	399.7000000
x_6	2.0600000	2.0600000
x_7	2.4400000	2.4550000
x_8	1069.2200000	1113.7500000
x_9	22.6500000	22.9500000
d_1^-	0.0000000	0.0000000
d_1^+	0.0000000	0.0000000
d_2^-	0.0000000	0.0000000
d_2^+	0.0000000	0.0000000
d_3^-	0.0000000	0.0000000
d_3^+	0.0000000	0.0000000
u_1	1.0000000	1.0000000
u_2	1.0000000	1.0000000
u_3	0.4866667	0.6044444
u_4	1.0000000	1.0000000
u_5	0.7262626	0.7146465
u_6	1.0000000	1.0000000
e_1	0.0000000	0.0000000
e_2	0.0000000	0.0000000
e_3	0.0000000	0.0000000
e_4	0.5133333	0.3955556
e_5	0.0000000	0.0000000
e_6	1.0000000	1.0000000
e_7	0.2737374	0.2853535
e_8	0.0000000	0.0000000
e_9	1.0000000	1.0000000
$z_1 \sim z_6$	0/1/1/1/1/1	0/1/1/1/1/1
$R_1 \sim R_3$	1/0/0	1/0/0
$G_1 \sim G_6$	1/1/0/1/1/0	1/1/0/1/1/0
$M_7 \sim M_{15}$	1/1/1/1/1/1/1/1/1	1/1/1/1/1/1/1/1/1
$V_{16} \sim V_{24}$	1/0/0/1/0/1/0/0/0	0/1/0/1/0/1/0/1/0
$E_{25} \sim E_{27}$	0/1/0	0/1/0

* Solution obtained by Global Solver used in the Lingo.

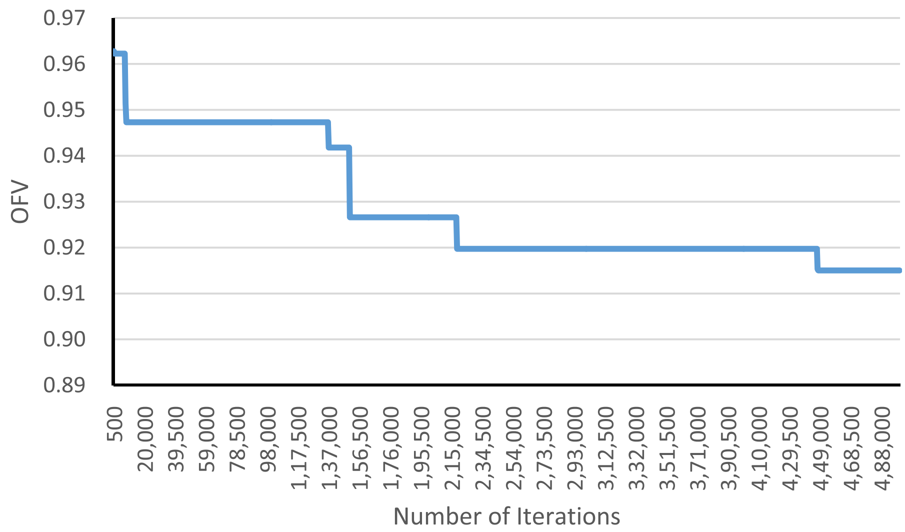


FIGURE 11. Evolution of best solution obtained for M1.

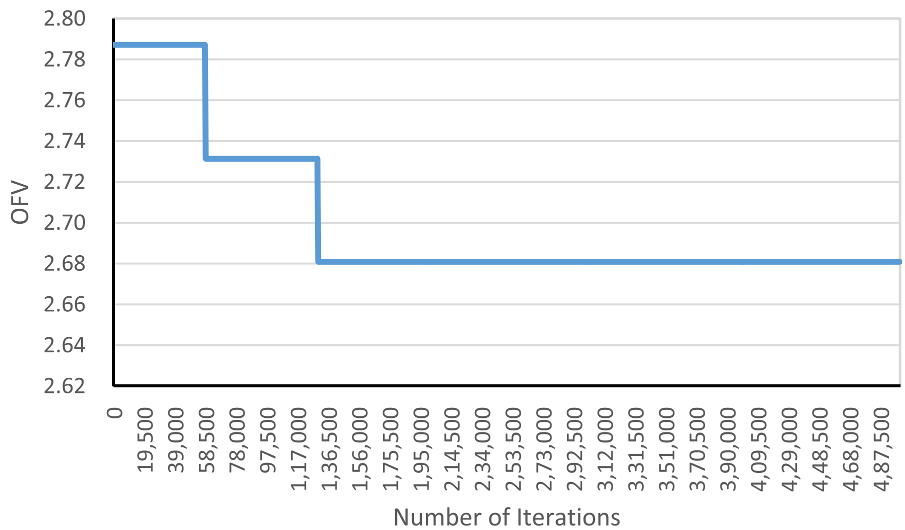


FIGURE 12. Evolution of best solution obtained for M2.

we can imply that the solution obtained by the ABC algorithm for M2 is better than that of Lingo 12. The solution obtained by the ABC is $(x_1, x_2, x_3, \mu_1, R_1, \mu_2, \mu_3, R_2, \mu_4, \mu_5, R_3, \mu_6, z_1, z_2, z_3, z_4, z_5, z_6, M_1(y), M_2(y), M_3(y)) = (1307.9, 485.2, 234.75, 1, 1, 1, 0.6044444, 0, 1, 0.7146465, 0, 1, 1, 0, 0)$. This means that the demand for carbohydrates is fully met, although the degree of achievement of protein and fat is 60.444444% and 71.464646%, respectively. Further, the degree of protein achievement is much better, while the degree of achievement of fat is closer to that of Lingo.

$$M_1(y) = 1 - (1191 - 1191)/(1191 - 1066) = 1$$
$$M_2(y) = 1 - (503 - 485.20)/(503 - 458) = 0.60444444 = 60.444444\%$$
$$M_3(y) = 1 - (263 - 234.75)/(263 - 164) = 0.71464646 = 71.464646\%$$

To show the convergence trend of the proposed ABC algorithm for M1 and M2, the *OFV* obtained with the "number of iterations" is displayed in Figures 11 and 12, respectively. As shown, the solution improvement rate decreases over an increasing number of iterations, and there is no more improvement in the best solution obtained after a certain number of iterations. Thus, using more iterations may not enhance the solution quality. In addition, the ABC algorithm can obtain more precise solutions than that obtained by the MCGP method.

According to the results of M1 and M2, the proposed ABC algorithm may obtain better solutions than the immediate solutions from Lingo 12 [24]. The reason the proposed ABC algorithm acquires better solutions may be as follows: (1) the ABC algorithm is a global optimization search algorithm; (2) many equations equal (=) one in the fuzzy MCGP and after giving the values of parts of the decision variables in these equations, the remaining variables could be deduced, which significantly reduces the complexity of the problem.

Moreover, this fills in the gap between the MCGP and NIOM methods. In the future, we can integrate MCGP and NIOM as a new method to enrich the field of MCDM techniques for solving qualitative and quantitative issues.

IV. CONCLUSION AND FUTURE RESEARCH

With the current status of available knowledge, this study is the first to investigate the relationship between MCGP and NIOM, aiming with two contributions between them and further improve the usefulness of MCGP. Therefore, new models are proposed to enrich FMCGP with triangular and trapezoidal MFs and improve the usefulness of MCGP for solving more MODM problems. NIOM is also provided to solve the same problem and know the differences between them. In addition, a comparison of the two methods is exhibited to understand their differences better and enrich the knowledge of both MCGP and NIOM. Furthermore, the new algorithms provided improve the efficacy of NIOM in solving fuzzy decision/management problems. Meanwhile, the proposed ABC algorithm uses many equal ("=") equations in the fuzzy multi-choice model. After giving the values of parts of the decision variables in these equations, the remaining variables could be deduced in the NIOM, which significantly reduces the complexity of the fuzzy MCGP model. In addition, the proposed methods could easily be used to solve multiple objective problems while considering qualitative and quantitative issues at the same time [20].

Moreover, several possible future directions exist for this research. In the future, other NIOMs, such as particle swarm optimization, firefly algorithm, bacterial foraging-inspired algorithm, genetic algorithm, and cuckoo search algorithm, can also be used to solve the FMCGP problems with non-linear constraints and MFs. Another exciting potential research direction is applying the proposed NIOM algorithm in solving more complex FMCGP problems with non-linear constraints and MFs in real applications. Finally, the proposed NIOM algorithm and new MCGP methods can be used to solve multi-attribute decision-making problems [32] while considering qualitative and quantitative issues simultaneously.

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