IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received May 20, 2021, accepted June 22, 2021, date of publication June 30, 2021, date of current version July 9, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3093778

Aperiodic Sampled-Data H_∞ Filtering for Lipschitz Nonlinear Systems: An Impulsive System Approach

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This work was supported in part by the Scientific Research Fund of Sichuan Provincial Science and Technology Department under Grant 2021YFG0133 and Grant 2021YFG0295, and in part by the Natural Science Foundation of Shandong Province under Grant ZR2020QF051.

ABSTRACT This paper is concerned with the H_∞ filtering for Lipschitz nonlinear systems under aperiodically sampled measurements. The developed filter is a hybrid system, whose states undergo a change or reset at sampling instants. The resulting filtering error system is then modelled as a kind of nonlinear impulsive systems. By introducing a time-varying Lyapunov functional candidate, a sufficient condition for the existence of desired filter is derived to ensure the filtering error system asymptotic stability and guarantee an H_∞ performance. The optimal H_∞ performance and corresponding filter gain matrix can be obtained by solving a convex problem with linear matrix inequalities (LMIs) constrains. Two examples are given to show the effectiveness of the theoretical results, and our results are less conservative than existing ones through comparisons.

INDEX TERMS Lipschitz nonlinear systems, sampled-data filtering, H_∞ performance, impulsive system approach.

I. INTRODUCTION

As one of essential problems in control and signal processing fields, filtering (or estimation) issue has gotten extensive concern in the past few decades [1]–[3]. Given a system disturbed by Gaussian random noise, the well-known Kalman filtering scheme [4] is a powerful tool to handle the state estimation problem, and it has a wide application in many fields, such as aerospace [5], [6], robot vision [7], [8] and networked systems [9]–[11]. In reality, however, the priori information of external noises is always not available, and the celebrated Kalman filtering theory will no longer be valid. Three filtering schemes have been developed for the case of non-Gaussian noise: L_1 filtering [12]–[14], $L_2 - L_\infty$ filtering [1], [15]–[17] and H_∞ filtering [2], [18]–[22]. More detailed descriptions of the above three filtering techniques can be found in [23].

At the computer times, digital processor is always responsible for the realization of filtering or control algorithm [24], [25]. In fact, the sampled-data system is indeed a class of hybrid systems, which is composed of both continuous-time dynamic systems and sampled-data filters/controllers with discrete-time (or sampling) behaviors. There have been fruitful research results (see e.g. [19], [20], [22], [26]–[30]) in the area of sampled-data filtering for various types of dynamic systems, such as linear system [22], [27], [28], [31], stochastic systems [29], T-S fuzzy systems [19], [20], [30] and time-varying system with finite discrete jumps [21]. Up to now, three main methods have been developed for the issues of sampled-data filtering from the modeling technique perspective [32]: *discrete-time system* approach, *impulsive/hybrid system* approach and *time-delay system* approach. 1) *discrete-time system* approach [28], [31], [33]:

The associate editor coordinating the review of this manuscript and approving it for publication was Chao-Yang Chen.

Given a fixed data-rate setting, a discrete-time equivalent system is devised, and then the theory in the framework of discrete-time systems has its place in dealing with the filtering issue of sampled-data systems. One limitation of the *discrete-time system* method is that the sampling period should be fixed. Another is that discretization loses the information about the inter-sampling behavior [32]. 2) *impulsive/hybrid system* approach [20], [21], [26]: The main feature of this method is that the impulsive behavior is used to represent the sampling characteristics. The advantage of this approach lies in that it provides a natural time-domain based framework to deal with the sampled-data filtering issue [34], [35]. 3) *time-delay system* approach [27], [36]: The sampled-data filtering error system is modeled as a time-delay system, and the time-delay system theory is ready to use. Unfortunately, as mentioned in [29], [36], the main shortcoming of *time-delay system* method is that the value of the integral terms of the constructed Lyapunov functional may increases suddenly if the jumping behavior of sampled signal occurs. In addition, the input of sampled-data filter designed by the *time-delay system* method is usually a piecewise continuous rectangle signal, which means that the process of signal reforming by a zero-order hold (ZOH) device is necessary.

Lipschitz nonlinear system is a class of common nonlinear systems, and now has been used to describe lots of real processes, such as robotic systems [37] and circuit systems [38], [39]. The H_∞ filter design for Lipschitz nonlinear systems with periodically sampling was addressed in [33], in which an Euler approximate model approach was proposed. Unfortunately, the authors merely take the external disturbance into account, failing to consider the measurement noise which is unavoidable in practice. In addition, the *discrete-time system* approach used in [33] is often suitable to the case of periodic sampling and appears powerless to the case of non-uniform sampling. In our knowledge, there few reports on the issue of aperiodic sampled-data filtering for Lipschitz nonlinear system disturbed by both external interference and measurement noise. In addition, designing a sampled-data filter with ZOH free is believed to be a significant attempt from the perspective of simplifying the hardware circuit. Those issues will be addressed by the proposed *impulsive system* method.

In this work, we develop a ZOH free H_∞ filter for Lipschitz nonlinear systems with aperiodically sampled measurements. We follow an *impulsive system* approach, that is to say, the filtering error system is modeled as a class of nonlinear impulsive differential systems. We employ a time-varying Lyapunov function to analyze the stability and H_∞ performance of the filtering error system. A sufficient condition for the existence of desired filter is formulated in terms of LMIs. Two numerical examples are exploited to demonstrate the effectiveness of the main results. The first one is used to compare with the existing *time-delay system* approach [27]. In the second example, the designed sampled-data filter is utilized to estimate the states of a Chua's circuit system. In summary, the main contributions include: 1) An H_∞ filter is designed for Lipschitz nonlinear systems that is based on aperiodically sampled data and ZOH free. 2) In comparison with [27], our results are less conservativeness.

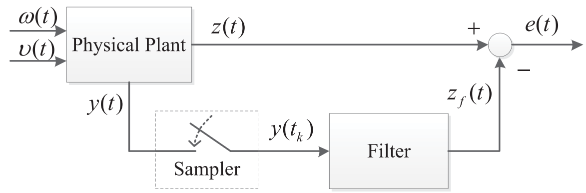

FIGURE 1. General diagram for filtering problem based on sampled-data.

The remainder of this paper is organized as follows. The problem formulation of sampled-data filtering is introduced in Section II. The main results are given in Section III. In Section IV, two numerical examples are given, and conclusions are drawn in Section V.

Notation: The sets of non-negative real and natural numbers are denoted by $\mathbb{R}_+$ and $\mathbb{N}$. The identity matrix will be denoted by I. The notion $P > 0$ (or $P < 0$), for $P \in \mathbb{R}^{n\times n}$, means that P is symmetric and positive (or negative) definite. The notation $f(\theta^-)$, for any given $\theta \in \mathbb{R}_+$, indicates the limit of $f(t)$ as t goes to θ from the left. In symmetric block matrices, we use "$\star$" as an ellipsis for the terms that are introduced by symmetry. The space of square-integrable vectors functions over $[0, \infty)$ are denoted by $L_2[0, \infty)$, and $l_2[0, \infty)$ are the set of square summable series on $\{0, 1, 2, \ldots\}$.

II. PROBLEM FORMULATION

Consider the following system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Gf(Hx(t)) + B_\omega \omega(t) \\ y(t_k) &= Cx(t_k) + D_\nu \nu(t_k) \\ z(t) &= Ex(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state; $y(t_k) \in \mathbb{R}^m$ is the discrete-time measured output; $z(t) \in \mathbb{R}^p$ is the signal to be estimated; $\omega(t) \in \mathbb{R}^l$ and $\nu(t_k) \in \mathbb{R}^q$ are the external disturbance and measurement noise respectively, satisfying that $\omega(t) \in L_2[0, \infty)$ and $\nu(t) \in l_2[0, \infty)$; $A \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{n\times g}$, $H \in \mathbb{R}^{g\times n}$, $B_\omega \in \mathbb{R}^{n\times l}$, $C \in \mathbb{R}^{m\times n}$, $D_\nu \in \mathbb{R}^{m\times q}$ and $E \in \mathbb{R}^{p\times n}$ are known real constant matrices; $f(\cdot) : \mathbb{R}^g \mapsto \mathbb{R}^g$ is assumed to be an Lipschitz nonlinearity with an Lipschitz constant $\beta > 0$, i.e.,

$$\| f(x) - f(y) \| \leq \beta \| \text{x-y} \|, \ \forall x, y \in \mathbb{R}^g \tag{2}$$

A typical filtering framework for system (1) based on sampled-data is shown in Fig. 1. In Fig.1, t_k is the sampling point of time, and the sampling sequences are donated as $\{t_k\}$ which are monotonically increasing with $\lim_{k\to+\infty} t_k = +\infty, k \in \mathbb{N}$. The sampling period is assumed to be time-varying and satisfies the following Assumption 1.

Assumption 1: Two adjacent sampling instants are bounded by $0 < \tau_1 \leq t_{k+1} - t_k \leq \tau_2$, *which is denoted as* $t_k \in \mathcal{T}(\tau_1, \tau_2)$, $k \in \mathbb{N}$.

To estimate the signal $z(t)$, the following sampled-data based filter is considered.

$$\begin{aligned} \dot{x}_f(t) &= Ax_f(t) + Gf(Hx(t_f)) \\ x_f(t_k) &= x_f(t_k^-) + F(y(t_k) - Cx_f(t_k^-)) \\ z_f(t) &= Ex_f(t) \end{aligned} \quad (3)$$

where $x_f(t) \in \mathbb{R}^n$ is the filter state; $y(t_k) \in \mathbb{R}^m$ is the filter input which is also the measured value at the instant t_k, $k \in \mathbb{N}$; $z_f(t) \in \mathbb{R}^p$ is the filter output (or an estimate of $z(t)$); $A \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{n\times g}$, $H \in \mathbb{R}^{g\times n}$, $C \in \mathbb{R}^{m\times n}$ and $E \in \mathbb{R}^{p\times n}$ are the same matrices as the system (1); $F \in \mathbb{R}^{n\times m}$ is the filter gain matrix to be determined.

Remark 1: Unlike the traditional sampled-data based filter (i.e., the filter input is generally realized through a ZOH, like [22], [27], [29], [33], [36]), filter (3) *is a class of impulsive systems. It just makes use of the discrete-time measured value and can save the ZOH device.*

Define $x_e(t) = x(t) - x_f(t)$ and $e(t) = z(t) - z_f(t)$, then the filtering error system can be represented by

$$\begin{aligned} \dot{x}_e(t) &= Ax_e(t) + G\Delta f(t, x, x_f) + B_\omega \omega(t) \\ x_e(t_k) &= (I - FC)x_e(t_k^-) + (-FD_\nu)\nu(t_k) \\ e(t) &= Ex_e(t) \end{aligned} \quad (4)$$

where $\Delta f(t, x, x_f) = f(Hx(t)) - f(Hx_f(t))$.

A. APERIODIC SAMPLED-DATA H_∞ FILTERING PROBLEM

Given system (1) and assume $t_k \in \mathcal{T}\{\tau_1, \tau_2\}$, design a filter (3) such that the filtering error system (4) is asymptotically stable, and for all nonzero $\omega(t) \in L_2[0, \infty)$ and nonzero $\nu(t_k) \in l_2[0, \infty)$, $k \in \mathbb{N}$, the following H_∞ performance is achieved.

$$\int_0^\infty e^{\mathrm{T}}(t)e(t)dt \le \int_0^\infty \gamma^2\omega^{\mathrm{T}}(t)\omega(t)dt + \sum_{k=1}^{k=\infty} \gamma^2\nu^{\mathrm{T}}(t_k)\nu(t_k) \quad (5)$$

III. MAIN RESULTS

The following lemma is useful in deriving the main results.

Lemma 1 (Schur Complement Lemma [40]): Given a symmetric block matrix S, the following conditions are equivalent.

$$(i) \qquad S = \begin{bmatrix} S_{11} & S_{12} \\ \star & S_{22} \end{bmatrix} < 0; \quad (6)$$

$$(ii) \quad S_{11} < 0, S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{12} < 0; \quad (7)$$

$$(iii) \quad S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^{\mathrm{T}} < 0. \quad (8)$$

A sufficient condition for the existence of filter (3) is provided by the following theorem.

Theorem 1: Given scalars $\gamma > 0$ and $\beta > 0$, there exists a filter (3) *such that the filtering error system* (4) *is asymptotically stable with a guaranteed H_∞ performance* (5)*, if there exist real matrices $P_i > 0$, $\bar{F}$ and scalars $\epsilon_{ij} > 0$, $i, j = 1, 2$, satisfying the following LMIs*:

$$\Pi_{ij} = \begin{bmatrix} \Pi_{ij}^{(11)} & P_iG & P_iB_\omega \\ \star & -\epsilon_{ij}I & 0 \\ \star & \star & -\gamma^2 I \end{bmatrix} < 0, \quad i, j = 1, 2 \quad (9)$$

$$\begin{bmatrix} -P_1 & 0 & P_2 - C^{\mathrm{T}}\bar{F}^{\mathrm{T}} \\ \star & -\gamma^2 I & -D_\nu^{\mathrm{T}}\bar{F}^{\mathrm{T}} \\ \star & \star & -P_2 \end{bmatrix} < 0 \quad (10)$$

where

$$\Pi_{ij}^{(11)} = A^{\mathrm{T}}P_i + P_iA + \frac{1}{\tau_j}(P_1 - P_2) + \epsilon_{ij}\beta^2 H^{\mathrm{T}}H + E^{\mathrm{T}}E$$

In this case, a desired filter gain matrix F in (3) *can be given by $F = P_2^{-1}\bar{F}$.*

Proof: Define two piecewise linear functions $\rho(t)$, $\rho_1(t) : [t_0, \infty) \to \mathbb{R}_+$:

$$\rho(t) = \frac{t_{k+1} - t}{t_{k+1} - t_k}, \quad \rho_1(t) = \frac{1}{t_{k+1} - t_k}, \quad t \in [t_k, t_{k+1}), k \in \mathbb{N} \quad (11)$$

It is obviously that there exists a function $\rho_2(t) \in [0, 1]$ such that

$$\rho_1(t) = \frac{1 - \rho_2(t)}{\tau_1} + \frac{\rho_2(t)}{\tau_2} \quad (12)$$

Inspired by [41], we consider a time-varying Lyapunov functional as following

$$V(t) = x_e^{\mathrm{T}}(t)P(t)x_e(t) \quad (13)$$

where $P(t) = \tilde{\rho}(t)P_1 + \rho(t)P_2$, $P_i > 0, i = 1, 2$, $\tilde{\rho}(t) = 1 - \rho(t)$.

When $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$, in view of (11) and (12), the time derivative of $V(t)$ along the trajectories of filtering error system (4) is given by

$$\begin{aligned} \dot{V}(t) &= \dot{x}_e^{\mathrm{T}}(t)P(t)x_e(t) + x_e^{\mathrm{T}}(t)\dot{P}(t)x_e(t) + x_e^{\mathrm{T}}(t)P(t)\dot{x}_e(t) \\ &= x_e^{\mathrm{T}}(t)\{A^{\mathrm{T}}P(t) + P(t)A + \rho_1(t)(P_1 - P_2)\}x_e(t) \\ &\quad + 2x_e^{\mathrm{T}}(t)P(t)G\Delta f(t, x, x_f) + 2x_e^{\mathrm{T}}(t)P(t)B_\omega\omega(t) \end{aligned} \quad (14)$$

By using (2), it is easy to check that the following inequality holds.

$$\epsilon(t)\{\beta^2 x_e^{\mathrm{T}}(t)H^{\mathrm{T}}Hx_e(t) - \Delta f^{\mathrm{T}}(t, x, x_f)\Delta f(t, x, x_f)\} \ge 0 \quad (15)$$

where $\epsilon(t) = \tilde{\rho}(t)\{\tilde{\rho}_2(t)\epsilon_{11} + \rho_2(t)\epsilon_{12}\} + \rho(t)\{\tilde{\rho}_2(t)\epsilon_{21} + \rho_2(t)\epsilon_{22}\}$, $\epsilon_{ij} > 0$, $i, j = 1, 2$.

Combining (14) and (15), it yields

$$\dot{V}(t) \leqslant \varphi^{\mathrm{T}}(t)\Pi(t)\varphi(t) - e^{\mathrm{T}}(t)e(t) + \gamma^2\omega^{\mathrm{T}}(t)\omega(t) \quad (16)$$

where

$$\varphi(t) = \mathrm{col}\{x_e(t),\ \Delta f(t, x, x_f),\ \omega(t)\}$$

$$\Pi(t) = \begin{bmatrix} \Pi(t)^{(11)} & P(t)G & P(t)B_\omega \\ \star & -\epsilon(t)I & 0 \\ \star & \star & -\gamma^2 I \end{bmatrix}$$

$$\begin{aligned}\Pi(t)^{(11)} &= A^{\mathrm{T}}P(t) + P(t)A + \rho_1(t)(P_1 - P_2)\\ &\quad +\epsilon(t)\beta^2 H^{\mathrm{T}}H + E^{\mathrm{T}}E\end{aligned}$$

Considering inequality (9) and by using the convex combination technique, it can be verified that

$$\begin{aligned}\Pi(t) = \tilde{\rho}(t)\{\tilde{\rho}_2(t)\Pi_{11} + \rho_2(t)\Pi_{12}\}\\ +\rho(t)\{\tilde{\rho}_2(t)\Pi_{21} + \rho_2(t)\Pi_{22}\} < 0,\end{aligned} \quad (17)$$

and then there holds

$$\dot{V}(t) < -e^{\mathrm{T}}(t)e(t) + \gamma^2\omega^{\mathrm{T}}(t)\omega(t). \quad (18)$$

Integrating both sides of (18) with respect to t over the time interval $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$, we have

$$\begin{aligned}\int_{t_k}^{t_{k+1}^-} \dot{V}(t)dt &= V(t_{k+1}^-) - V(t_k)\\ &< \int_{t_k}^{t_{k+1}^-}(-e^{\mathrm{T}}(t)e(t) + \gamma^2\omega^{\mathrm{T}}(t)\omega(t))dt.\end{aligned} \quad (19)$$

At the instant: $t_{k+1}^- \to t_{k+1},\ k \in \mathbb{N}$, we can obtain

$$\begin{aligned}&V(t_{k+1}) - V(t_{k+1}^-)\\ &= x_e^{\mathrm{T}}(t_{k+1})P(t_{k+1})x_e(t_{k+1}) - x_e^{\mathrm{T}}(t_{k+1}^-)P(t_{k+1}^-)x_e(t_{k+1}^-)\\ &= \zeta^{\mathrm{T}}(t_{k+1}^-, t_{k+1})\Xi\zeta(t_{k+1}^-, t_{k+1}) + \gamma^2\nu^{\mathrm{T}}(t_{k+1})\nu(t_{k+1}),\end{aligned} \quad (20)$$

where

$$\begin{aligned}\zeta(t_{k+1}^-, t_{k+1}) &= \mathrm{col}\{x_e(t_{k+1}^-), \nu(t_{k+1})\},\\ \Xi &= \begin{bmatrix}\Xi_{11} & \Xi_{12}\\ \star & \Xi_{22}\end{bmatrix},\\ \Xi_{11} &= (I - FC)^{\mathrm{T}}P_2(I - FC) - P_1,\\ \Xi_{12} &= -(I - FC)^{\mathrm{T}}P_2(FD_\nu),\\ \Xi_{22} &= -\gamma^2 I + (FD_\nu)^{\mathrm{T}}P_2(FD_\nu).\end{aligned}$$

Setting $\bar{F} = P_2F$ for inequality (10), by using the Schur Complement Lemma 1, it can be verified that $\Xi < 0$. Further, the following inequality can be easily obtained by (20).

$$V(t_{k+1}) - V(t_{k+1}^-) < \gamma^2\nu^{\mathrm{T}}(t_{k+1})\nu(t_{k+1}). \quad (21)$$

Combining (19) and (21), one has

$$\begin{aligned}V(t_{k+1}) - V(t_k) &< \gamma^2\nu^{\mathrm{T}}(t_{k+1})\nu(t_{k+1})\\ &\quad + \int_{t_k}^{t_{k+1}}(-e^{\mathrm{T}}(t)e(t) + \gamma^2\omega^{\mathrm{T}}(t)\omega(t))dt.\end{aligned} \quad (22)$$

When $\omega(t) \equiv \mathbf{0}$ and $\nu(t_k) \equiv \mathbf{0}$, inequality (22) is simplified into

$$V(t_{k+1}) - V(t_k) < -\int_{t_k}^{t_{k+1}} e^{\mathrm{T}}(t)e(t)dt \le 0, k \in \mathbb{N} \quad (23)$$

Due to $V(t_k) > 0$, $k \in \mathbb{N}$, so inequality (23) implies that $\lim_{k\to\infty} V(t_k) = 0$, that is $\lim_{t\to\infty}\|x_e(t)\| = 0$, which means that the filtering error system (4) is asymptotically stable.

Calculating $\sum_{k=0}^{k=\infty}$ for both sides of (22), we have, under the initial condition $x_e(t) = \mathbf{0}$,

$$\begin{aligned}\int_0^\infty(-e^{\mathrm{T}}(t)e(t) + \gamma^2\omega^{\mathrm{T}}(t)\omega(t))dt + \gamma^2\sum_{k=1}^{k=\infty}\nu^{\mathrm{T}}(t_k)\nu(t_k)\\ > V(\infty) - V(0) \ge 0,\end{aligned} \quad (24)$$

which means that the H_∞ performance (5) is achieved and the proof is completed. □

Remark 2: When $P = P_1 = P_2$, the utilized time-varying Lyapunov function (13) *reduces to the common Lyapunov function $V(t) = x_e^{\mathrm{T}}(t)Px_e(t)$. This means that the time-varying Lyapunov function* (13) *is a generalization of the common Lyapunov function, and thus can potentially reduce the conservatism of the obtained results.*

Remark 3: Note that conditions in Theorem 1 are LMIs and can be efficiently solved by using interior-point. The computational cost is proportional to $\mathcal{N}_{\mathrm{var}}^3\mathcal{N}_{\mathrm{row}}$ with the number of scalar decision variables $\mathcal{N}_{\mathrm{var}}$ and the total row size $\mathcal{N}_{\mathrm{row}}$ of the LMIs. As for Theorem 1, the values of $\mathcal{N}_{\mathrm{var}}$ and $\mathcal{N}_{\mathrm{row}}$ are $n(n+1)+ nm+5$ and $4n+4(2n+1)+q$ respectively.

Remark 4: The optimal H_∞ performance γ^* and corresponding filter gain matrix F^* can be obtained by solving the following optimization problem:

$$\begin{aligned}(\gamma^*, F^*) = \arg \inf_{P_1>0, P_2>0, \bar{F}, \epsilon_{ij}>0, i,j=1,2}(\gamma^2)\\ \text{s.t. } (9), (10)\end{aligned} \quad (25)$$

Constrains in (25) *are LMIs, hence they are easy to solve by using the existing LMI Toolbox.*

When $G = 0$, nonlinear systems (1) reduce to an linear system. In this case, Theorem 1 reduces to the following corollary.

Corollary 1: Consider the filtering error system (4) *($G = 0$) and assume time sequences $t_k \in \mathcal{T}\{\tau_1, \tau_2\}$, $k \in \mathbb{N}$. Given a scalar $\gamma > 0$, there exists a filter* (3) *($G = 0$) such that the filtering error system* (4) *($G = 0$) is asymptotically stable with a guaranteed H_∞ performance* (5)*, if there exist real matrices $P_i > 0$, $i = 1, 2$ and $\bar{F}$ satisfying* (10) *and the following LMIs*:

$$\Pi_{ij} = \begin{bmatrix}\Pi_{ij}^{(11)} & P_iB_\omega\\ \star & -\gamma^2 I\end{bmatrix} < 0, \quad i,j = 1,2 \quad (26)$$

where $\Pi_{ij}^{(11)} = A^{\mathrm{T}}P_i + P_iA + \frac{1}{\tau_j}(P_1 - P_2) + E^{\mathrm{T}}E$.

In this case, a desired filter gain matrix F in (3) *can be given by $F = P_2^{-1}\bar{F}$.*

Proof: The proof is omitted since it is quite similar to that of Theorem 1. □

Remark 5: For the case of linear system (i.e., $G = 0$), the optimal H_∞ performance γ^* and corresponding filter gain matrix F^* can be obtained by solving the following optimization problem:

$$(\gamma^*, F^*) = \arg \inf_{P_1>0, P_2>0, \bar{F}}(\gamma^2) \text{ s.t. } (10), (26) \quad (27)$$

TABLE 1. Results of the comparison with time-delay system approach.

	τ_2	$\pi/45$	$\pi/35$	$\pi/25$	$\pi/15$		$\gamma^* < 0.2$
γ^*	Theorem 2 [27]	0.2668	0.3344	0.4641	0.7366	τ_m	47ms
	Remark 5.	0.2301	0.3134	0.4487	0.7185		62ms

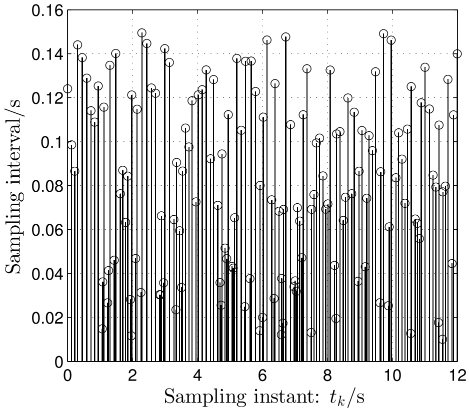

FIGURE 2. Sampling instant and sampling interval distribution.

IV. EXAMPLES

In this section, two examples are given to demonstrate the effectiveness of the proposed filter design method. We first consider the linear system case and compare the proposed method with the result in [27].

Example 1: Consider the following system given in [27].

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ -16 & -4.8 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 16 \end{bmatrix} \omega(t).\\ y(t_k) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t_k) + 0.1\nu(t_k).\\ z(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{aligned} \tag{28}$$

To show the effectiveness of proposed method, we compare it with the existing result [27] from the following two perspectives: 1) calculating the minimum H_∞ performance under the same sampling interval (for the convenience of comparison, we assume that $0.0001 = \tau_1 < t_{k+1} - t_k < \tau_2$); 2) comparing the maximum value of sampling interval τ_m (when $0.00001 = \tau_1 < t_{k+1} - t_k < \tau_2 = \tau_m$) with the constraint $\gamma^* < 0.2$. The detailed results are listed in Table 1, from which it is worth noticing that the proposed approach in this paper not only achieves a better H_∞ performance in the same sampling interval, but it also can gain a longer sampling interval with the same constraint $\gamma^* < 0.2$.

Remark 6: It should be noted that the proposed filter is ZOH free. On the contrary, the filter in [27] is realized with the aid of the ZOH device. In other words, the proposed filter can save the ZOH device resource.

Example 2: In this example, we design a sampled-data filter to estimate the states of the famous Chua's circuit [38]. The system parameters of Chua's circuit are borrowed from [39] as following

$$A = \begin{bmatrix} -3.2 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}, \quad G = H^{\mathrm{T}} = C^{\mathrm{T}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

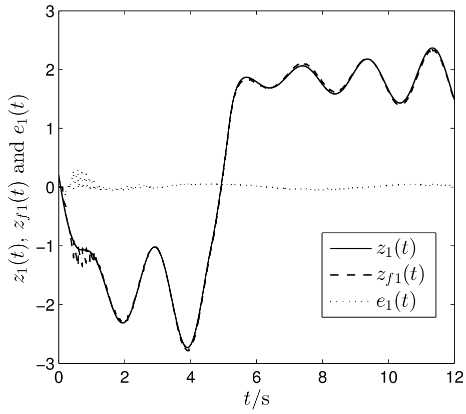

FIGURE 3. Trajectories of $z_1(t)$, $z_{f1}(t)$ and $e_1(t)$.

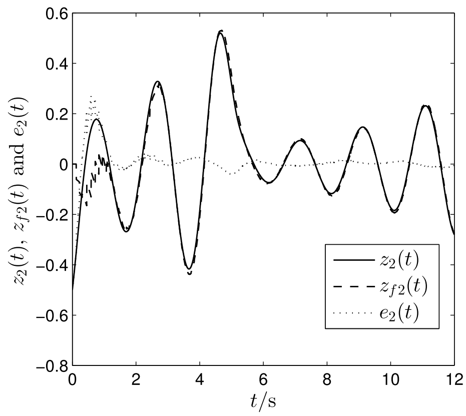

FIGURE 4. Trajectories of $z_2(t)$, $z_{f2}(t)$ and $e_2(t)$.

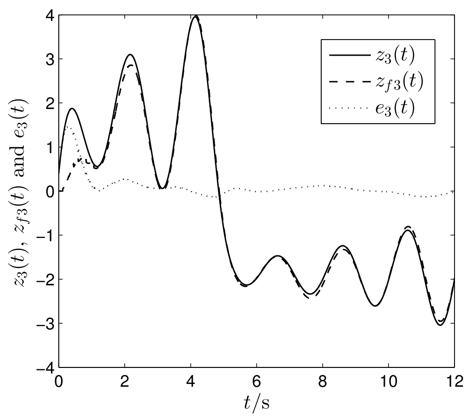

FIGURE 5. Trajectories of $z_3(t)$, $z_{f3}(t)$ and $e_3(t)$.

$$B_\omega = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad D_\nu = 1, \; E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{29}$$

nonlinearity function $f(Hx(t)) = 2.95(|x_1 + 1| - |x_1 - 1|)$.

We assume that the sampling instant sequences satisfy $t_k \in \mathcal{T}\{0.01, 0.15\}$. By solving the problem in Remark 4, we obtain the filter gain matrix $F^* = [0.9999\ 0.2997\ -0.2560]^{\mathrm{T}}$ and the minimum H_∞ performance $\gamma^* = 3.8534$.

To verify the results, assume that the external disturbance and measurement noise are respectively $\omega(t) = e^{-0.5t}$ and $\nu(t_k) = 0.05sin(t_k)$. Moreover, the initial states are assumed to be $x_0 = [0.2\ \ -0.5\ 0.4]^{\mathrm{T}}$ and $x_{f0} = \mathbf{0}$. The sampling instant and sampling interval distribution are shown in Fig. 2. Figs. 3–5 depict $z(t)$, $z_f(t)$ and estimation error $e(t)$, and also show the efficacy of the proposed filtering strategy.

V. CONCLUSION

In this paper, the problem of sampled-data H_∞ filtering for Lipschitz nonlinear systems is investigated by using the impulsive system approach. A sampled-data filter as the form of impulsive systems is introduced to estimate the states of the considered Lipschitz nonlinear systems. By constructing a time-varying Lyapunov functional, the criterion ensuring filtering error system asymptotic stability and guaranteeing an H_∞ performance is derived that is expressed in term of LMIs. Further, the desired filter parameter could be obtain by solving a convex optimization problem. The effectiveness of the proposed method is verified by two examples. Our future work will consider some network-induced complexities such as network-induced time-delays [42] and security-related issues [43].

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