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A Comparison Fault Diagnosis Algorithm for Star Networks

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ABSTRACT Fault diagnosis for a multiprocessor system is a process of identifying the faulty nodes in the system and is an important issue on the reliability of the system. As to the problem that there are few effective algorithms to diagnose faulty nodes in a given star network system in the literature, this paper proposes a precise fault diagnosis algorithm to identify faulty nodes in a star network system with a given syndrome under the comparison model. Such an algorithm contains three main parts. In the first part, we present an algorithm called Partition-Cycle for partitioning a cycle into sequences based on a given syndrome of the cycle. In the second part, we introduce an algorithm called Digout to diagnose these cycle sequences obtained the first part, which can diagnose each node in the cycle to be faulty or fault-free or unknown. In the third part, we design a diagnosis algorithm called Star-Digout to diagnose faulty nodes in an n-dimensional ($n \geqslant 6$) star networks, which is proved to contain a cycle that contains all nodes in the network and is not the same two nodes. Our theoretical analysis shows the time complexity of the diagnosis algorithm is $O(n!)$. Our simulation results show that our algorithm is a precise diagnosis algorithm for a star network system.

INDEX TERMS Fault diagnosis, star network, Hamiltonian cycle, the comparison model, multiprocessor system.

I. INTRODUCTION

With the rapid development of semiconductor technology, multiprocessor systems can contain hundreds and thousands of nodes. To ensure the reliability, the system should have ability to identify the faulty node and repair or replace it with a fault-free one. The process of identifying faulty nodes is called diagnosis of the system. The maximum number of faulty nodes in a system that the system can guarantee to identify is called the diagnosability of the system.

For a given multiprocessor system S, its interconnection network is usually abstracted as a graph $G = (V, E)$, where a vertex of G denotes a processor in S, for two vertex $u, v \in V$, $(u, v) \in E$ implies that their corresponding processors can communication each other. The choice of network topology is very important to the performance of a multiprocessor system. For example, a smaller diameter interconnection network is expected to cause less delay when a message is sent between two different processors. Due to its small diameter, high degree of fault-tolerance, low node degree and recursive, and structure symmetry, permutation star graphs have been proposed to model the interconnection network of multiprocessor systems and have been widely studied. [1] investigated several topological properties of the n-dimensional star graph S_n. In [2], the authors presented some novel structure properties and conditional diagnosability for star graphs under the PMC model. In 1996, Battayeb *et al.* studied the problem of embedding star networks into hypercube networks [3].

So far, there are two fault diagnosis approaches for the problem of locating faulty processors in a network system. One is system-level diagnosis, another is logical-circuit-level diagnosis. For interconnection networks in multiprocessor systems, since there exists large number of processors, to locate faulty processors in such interconnection networks, one trends to use system-level diagnosis rather than logical-circuit-level diagnosis (see [4]). To diagnose faulty processors, it is necessary to perform some test for diagnosed network systems and to obtain corresponding

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test results. For different system-level diagnosis models, their test assumption and the definition of their test results may be different. Over past years, two system-level diagnosis models, Preparata, Metze, and Chien's model (in brief, PMC model [5]) and the comparison model [6], were widely investigated and applied. The PMC model is the first system-level diagnosis model proposed by Preparata *et al.* [5]. In the PMC model, after node u sends a test task to node v, node v replies with a response message to node u. If the response is correct, then the result of node u testing node v is 0, denoted as $\sigma(u, v) = 0$; otherwise, the result of node u testing node v is 1, denoted as $\sigma(u, v) = 1$. The set of all test results for a test is called a syndrome. The comparison model is introduced by Sengupta and Dahbura [6]. In the comparison model, a comparator node k compares the outputs produced by two nodes i and j in response to the same input and task sent by k. If the output of node i is the same as that of node j, then the comparison result is 0, denoted by $\omega(k : i, j) = 0$; otherwise, the comparison result is 1, denoted by $\omega(k : i, j) = 1$. A syndrome ω in a test for the comparison model consists of all comparison results in the test. Sengupta and Dahbura [6] pointed out that the PMC model is a special case of the comparison model. In other words, the comparison model is more universal than the PMC model in terms of fault diagnosis approach.

To the best our knowledge, there are a few algorithms for the problem of fault diagnosis in interconnection networks under the PMC model in the existing literature. Chwa and Hakimi [7] proposed a fault diagnosis algorithm for asymmetric modular architectures under the PMC model. Dahbura and Masson [8] introduced a fault diagnosis algorithm with time complexity $O(N^{2.5})$ for diagnosable interconnection networks under the PMC model. These two algorithms cannot be used to diagnose the faulty nodes in interconnection networks under the comparison model. Ye *et al.* [9] proposed a pessimistic diagnosis algorithm for an n-dimensional hypercube under the comparison model, which costs $O(n2^n)$ time and cannot be used to determine the faulty nodes in a star network. They proposed a five-round fault diagnosis algorithm for identifying the faulty nodes in a Hamiltonian network under the PMC model [10], which can achieve almost complete diagnosis for a given Hamiltonian network with at least 4 node degree. To the best our knowledge, there are few papers to present a precise fault diagnosis algorithm for star networks under the comparison model.

In the paper, the problem of fault diagnosis for star networks under the comparison model is studied. We summarize our contributions in the paper as follows.

1. We present an algorithm to partition a given cycle into sequences based on a given syndrome under the comparison model. We prove that after the algorithm is finished, a given cycle with N nodes and t fault nodes ($N \geqslant 3t + 1$) can be divided sequences with the test result form as $0 \cdots 01 \cdots 10$, which can easily be used to determine faulty nodes in its corresponding sequence.

2. We introduce some important properties for sequences obtained by partitioning a cycle with N nodes and t fault nodes ($N \geqslant 3t + 1$). Using these properties, we present and prove that the upper bound of t is $\lfloor \sqrt{18 + 2N} - 5.5 \rfloor$, denoted by T such that after implementing Cycle-Partition for a N-node cycle with t faulty nodes ($N \geqslant 3t + 1$), a sequence, whose first node is fault-free, is always obtained provided that $t \leqslant T$.

3. We proposed a precise fault diagnosis algorithm for computing faulty nodes in a star network under the comparison. At first, we use an algorithm called Digout, which is introduced by us in the paper, to diagnose the nodes in the star network into fault-free nodes, faulty nodes and unknown nodes. Next, for each unknown node w, construct its $n-1$ different branch paths and check test results of each path. If there exists a branch Q_i with $T_{link}(S_n(w, Q_i)) = 1\overbrace{000 \cdots 000}^{n}$, then diagnose it to be faulty nodes. Otherwise, diagnose it to be fault-free.

The rest of this paper is organized as follow. In section II, related works are introduced. In section III, we shall introduce a cycle partition method to divide the Hamiltonian cycle into sequences and derive the fault bound T. Besides, a cycle diagnosis algorithm is also presented. In section IV, combining the theory of cycle diagnosis and the properties of n-dimensional star graphs, we propose a precise diagnosis algorithm, called Star-DigOut. Using Star-DigOut, we prove that the all faulty nodes can be detected in $O(N)$ time provided the number of faulty nodes does not exceed $n - 1$ for an n-dimensional ($n \geqslant 6$) star graph. The simulation results of the algorithm Star-DigOut are presented in the section V. Section VI draws a conclusion.

II. RELATED WORK

Over the past years, the problem of fault diagnosis for interconnection networks attracted a lot of attentions. System-level fault diagnosis model and logic-circuit-level fault diagnosis model are two fault diagnosis models for the problem of computing faulty nodes in network systems. In [11], Friedman and Simoncini pointed out that to solve the problem of fault diagnosis for interconnection networks, people tend to use a system-level fault diagnosis model rather than a logic-circuit-level fault diagnosis model and provided the explanations of related reasons. The PMC model, which are proposed by Preparata *et al.* in [5], is the first system-level fault diagnosis model. The authors introduced the concept of one-step t-fault diagnosable system and the concept of sequentially t-fault diagnosable system. Hakimi and Nakajima [12] studied the general theory on t-fault diagnosable analog systems. In [13], Barsi *et al.* modified the hypothesis of the PMC model and proposed the BGM model. Under the BGM model, when fault-free node u tests node v, if v is fault-free, then $\sigma(u, v) = 0$. Otherwise, $\sigma(u, v) = 0$. When faulty node u tests node v, if v is fault free then $\sigma(u, v) = 1$ or $\sigma(u, v) = 0$. Otherwise, $\sigma(u, v) = 1$, which is different from the corresponding test result of the PMC model that when a faulty node u tests a faulty node v, $\sigma(u, v) = 1$ or $\sigma(u, v) = 0$. In 1992, Sengupta and Dahbura [6] proposed another system-level diagnosis model called by the comparison model. In the comparison model, a test result $\omega(w : u, v)$ can be obtained by a comparator node w comparing the responses of two compared nodes u and v. If w is fault-free and the responses of u and v are the same, then $\omega(w : u, v) = 0$. If w is faulty, then $\omega(w : u, v) = 0$ or $\omega(w : u, v) = 1$ whether their response is the same or not. Overs the past years, in the literature, there are few papers for the problem of fault diagnosis in interconnection networks under the BGM model.

As a key measure of diagnostic capability, diagnosability is widely studied in the literature. Many results have been obtained for the diagnosabilities of interconnection networks. In [14], Hakimi and T introduced the characterization of connection assignment for diagnosable interconnection network under the PMC. Using this characterization, they proved that both the diagnosability of n-dimensional hypercube network Q_n and the diagnosability of n-dimensional star network S_n are $(n-1)$. To compute the t/k-diagnosability of Q_n under the PMC model, Somani and Peleg [15] introduced the concept of t/k-diagnosable system. Next, they proposed a sufficient condition and a necessary condition for testing whether a system is t/k-diagnosable or not. Using these two conditions, they proved that Q_n is T/k-diagnosable for $T = (k + 1)n - \frac{(k+1)(k+2)}{2} + 1$, where $k \leqslant n$ and $n \geqslant 4$. In [16], Lai *et al.* studied the problem of conditional diagnosability measures under the PMC model. They thought that the probability that all neighbor nodes are faulty in a real large-scale system is very small. To this end, they introduced the concept of conditionally t-diagnosable system and proved that the conditional diagnosability of Q_n is $4(n - 2) + 1$ for $n \geqslant 5$, 3 for $n = 3$ and 7 for $n = 4$. Furthermore, in [17], the authors investigated the relationship between two of classical diagnosability, strong diagnosability and conditional diagnosability for strong networks. They proved that for a strong network G is strongly t-diagnosable if and only if its conditional diagnosability is more than or equal to its classical diagnosability under the comparison model. They also proved that a regular strong network G is strongly t-diagnosable if and only if its conditional diagnosability is more than or equal to its classical diagnosability under the PMC model. Other results on diagnosabilities for interconnection networks can be found in [18]–[29].

Fault diagnosis algorithms are very important for the problem of fault diagnosis in interconnection networks. Over years, a few fault diagnosis algorithms have been obtained. In [30], the authors presented an algorithm for the system with $D_{\delta,t}$ testing interconnection assignments, which is a system $G = (V, E)$ with $V = \{v_1, v_2, \cdots, v_n\}$ and $E = \{(v_i, v_j)|j - i = \delta m(modulo\ \ n), 1 \leqslant m \leqslant t\}$, under the PMC. By modifying the hypothesis of the algorithm in [30], Dahbura and Masson [8] proposed a fault diagnosis algorithm for the system with $D\delta, t$ testing interconnection assignments. They claimed their algorithm can be applied under both of the PMC model and the BGM model. In 1984, Dahbura and Masson [8] introduced an algorithm with time complexity $O(N^{2.5})$ for the problem of fault diagnosis in interconnection networks under the PMC model, which is not suitable to be used diagnose the faulty nodes in interconnection networks under the comparison model. In 2012, Lai [31] proposed an system-level fault diagnosis algorithm for computing the faulty nodes in a hypercube network under the comparison model. Tsai [32] introduced a pessimistic diagnosis algorithm for hypercube-like networks under PMC model. Recent, Ye *et al.* [9] also proposed a pessimistic diagnosis algorithm for hypercube-like networks under the comparison model, which is different from the PMC model used in [32]. They proved that the time complexity is $O(n2^n)$. In [1], Ye and Hiesh presented a fault diagnosis algorithm for hypercube-like networks. They proved that their algorithm needs to cost $O(n^2 2^n)$ time, which is much bigger than $O(n2^n)$, the time complexity of the algorithm in [9].

III. THE HAMILTONIAN CYCLE DIAGNOSIS METHOD

It is known that a multiprocessor system can be modeled as a graph $G(V, E)$, where V denotes the set of all nodes in the system and E denotes the connection relationship between each pair of nodes, for $u, v \in V$, $(u, v) \in E$ if and only if node u can send messages to node v.

In the comparison model, it is necessary that some assumptions are made [6], which are described as follows.

1. All faulty nodes are permanent;
2. For each faulty node and each it's given task, it's output is incorrect;
3. The outcome of a comparison performed by a faulty node is unreliable;
4. Any two faulty nodes, when they are sent the same inputs and task, do not generate the same output.

In the comparison model, node $k \in V$ is a comparator for node $i \in V$ and $j \in V$ if and only if $(i, k) \in E$ and $(j, k) \in E$. We use $\omega(k : i, j)$ to represent the test result of comparator k testing nodes i and j. According to definition of the comparison model [6], when a comparator k compares the outputs generated by i and j, if these two output is not the same, then the comparison result $\omega(k : i, j) = 1$, whereas if these two outputs are the same, then the comparison result $\omega(k : i, j) = 0$. Figure 1 shows the possible comparison results for different conditions of three nodes in the comparison model. The collection of all comparison results is called a syndrome, denoted by ω.

A. CYCLE-PARTITION METHOD

A Hamiltonian path is a path in a system that visits each node exactly once. A Hamiltonian cycle is a Hamiltonian path that is a cycle. In the discussion on the Cycle-Partition method proposed by the section, some properties of Hamiltonian cycle under the comparison model are necessary, which are introduced as follows.

Comparator processors \ Of the two Compared Processors	TEST RESULT	
	None is faulty	At least one is faulty
Fault-free	0	1
Faulty	0 or 1	0 or 1

FIGURE 1. **Invalidation Rule of the Comparison Model.**

Lemma 1: Suppose that there are N nodes and t faulty nodes in a system with a Hamiltonian cycle. Let $u_1, u_2, \cdots, u_n$ denote these N nodes in a clockwise direction in the Hamiltonian cycle. If $N \geqslant 3t+1$, then there exist three nodes u_{i-1}, u_i and u_{i+1} such that $\omega(u_i : u_{i-1}, u_{i+1}) = 0$ in the Hamiltonian cycle.

Proof: Since there are at most t faulty nodes in the Hamiltonian cycle and this Hamiltonian cycle consists of at least $3t+1$ nodes, there must exist three consecutive fault-free nodes u_i,u_j,u_k such that $\omega(u_j : u_i, u_k) = 0$. ■

In the following, to partition the Hamiltonian cycle with N nodes and t faulty nodes into sequences under the comparison model, an algorithm called Cycle-Partition is introduced. A detail description for this algorithm can be found in Algorithm 1.

Algorithm 1 Algorithm of Cycle-Partition

1: Choose a test result 0 following a test result 1 in clockwise direction.
2: Proceed inspecting the following test result in clockwise direction. If the test result is 0, perform step 2 on the following test results. Otherwise, mark P for the node and go to step 3.
3: Proceed inspecting the following test result in clockwise direction. If the test result is 1, perform step 3 on the following test results. If the node was not previously marked and its test result is 0, mark X for the node and go to step 2. If the test result was previously marked, then the algorithm terminates.

Remark: in Step 1 of Algorithm 1, in the considered Hamiltonian cycle, the existence of a 0 is guaranteed by Lemma 1; the existence of a 1 is assured for the number of the faulty nodes in the Hamiltonian cycle is between 1 and t.

After finishing Algorithm 1, we obtain the sequences for the considered Hamiltonian cycle according to the following rules:

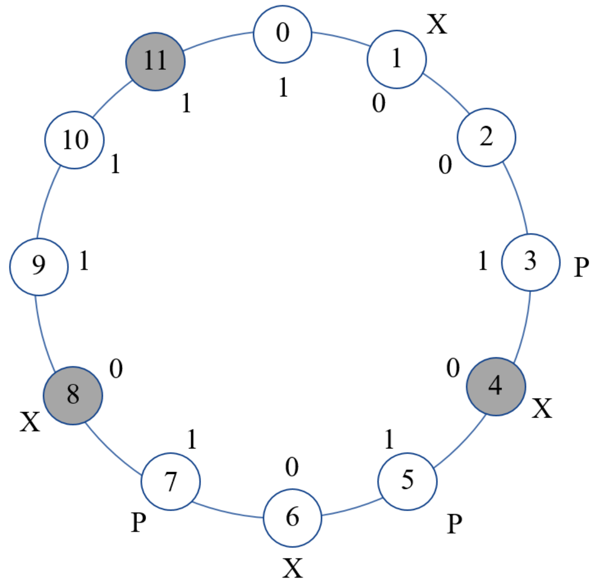

FIGURE 2. **A Hamiltonian cycle of 12 nodes.**

1) Each sequence of the Hamiltonian cycle consists of nodes comprised between two successive X-marked nodes and their connecting links in clockwise direction.

2)The first node marked with X in one sequence is the last node of the following sequence in clockwise direction.

For the sake of understanding the result after Algorithm 1, we consider an example shown in Figure 2, where the gray nodes are faulty nodes and the white nodes are fault-free nodes.

According to Cycle-Partition, for the Hamiltonian cycle with 12 nodes, the results for the nodes marked with X or P can be shown in Figure 2 and the obtained sequences of the Hamiltonian cycle are as follows, which can visually be shown in Figure 3.

Sequence 1: 1, 2, 3, 4;
Test results of sequence 1: 0010.
Sequence 2: 4, 5, 6;
Test results of sequence 2: 010.
Sequence 3: 6, 7, 8;
Test results of sequence 3: 010.
Sequence 4: 8, 9, 10, 11, 0, 1;
Test results of sequence 4: 011110.

According to Cycle-Partition, we can easily obtain some useful properties about the sequences generated by Cycle-Partition for a Hamiltonian cycle with N nodes and t faulty nodes ($N \geqslant 3t+1$), which are summarized as follows:

Property 1: The test result of a sequence is always of the form: $0 \cdots 01 \cdots 10$, and if a sequence consists of three nodes and their connecting links, then the test result of the sequence must be 010.

Property 2: Every sequence contains at least one faulty node.

Property 3: If a Hamiltonian cycle with N nodes and t faulty nodes ($N \geqslant 3t+1$) is divided into s sequences by Cycle-Partition, then $2t \geqslant s$.

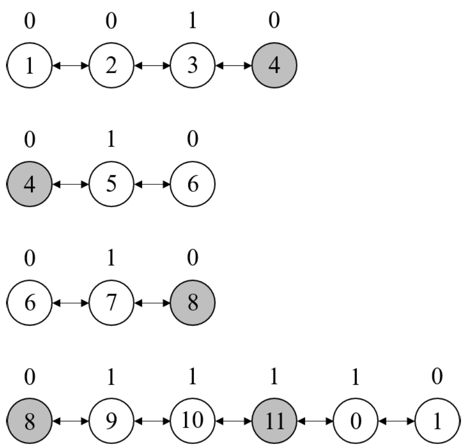

FIGURE 3. The sequences of a 12-nodes Hamiltonian cycle divided by Cycle-Partition.

Lemma 2: If a sequence consists of $x + 1$ test results of 0 and y test results of 1, then following two conditions hold:

i) In the sequence, if the first node is fault-free, then the first $x + 1$ consecutive nodes are fault-free and the $(x + 2)$th node is faulty.

ii)In the sequence, if the first node is faulty, then the first x consecutive nodes are faulty.

The proof of Lemma 2 is easily obtained by the definition of the comparison model and is omitted.

Lemma 3: Suppose a sequence consists of $x+1$ test results of 0 and y test results of 1, if the first node of the sequence is faulty, then there must exist at least $x + \lfloor \frac{y+1}{3} \rfloor$ faulty nodes in the sequence.

Proof: According to Lemma 2, we have that the first x consecutive nodes of the sequence are faulty. Consider the remaining $y + 1$ nodes in the sequence:

Case 1 $y = 1$, it is obvious that Lemma 3 holds;

Case 2 $y \geqslant 2$. In this case, according to the rule of the test result under the comparison model and Property 1, we obtain that there must exist at least one faulty in every continuous three nodes in the remaining $y + 1$ nodes of the sequence. Otherwise it would contradict to. Hence there are at least $\lfloor \frac{y+1}{3} \rfloor$ faulty nodes in the last $y+1$ consecutive nodes. Above all, the result of Lemma 3 is true. ■

Let t be the number of faulty nodes in an N-node ring with $N \geqslant 3t + 1$ and s be the number of sequences generated by Cycle-Partition for the N-node ring. Suppose that the ith sequence consists of $x_i + 1$ test results of 0 and y_i test results of 1, let $S_i = x_i + \lfloor \frac{y_i+1}{3} \rfloor$ and $S_{max} = max\{S_i, 1 \leqslant i \leqslant s\}$.

Lemma 4: For the ith sequence, if $S_i \geqslant t - \lfloor \frac{s-1}{2} \rfloor + 1$, then its first node is fault-free.

Proof: Let F_j be the set of faulty nodes in the jth sequence. To the contrary, assume the first node of the ith sequence is faulty. According to Lemma 3, we have that $|F_i| \geqslant S_i = x_i + \lfloor \frac{y_i+1}{3} \rfloor$. Consider the remaining $s - 1$ sequences. Note that $|F_j| \geqslant 1$ and there is at most one common faulty node shared by the jth sequence and the $(j+1)$th sequence, it can be obtained that $|F_j \cap F_{j+1}| \leqslant 1 (j \neq i$ and $j \neq i-1)$. Since the first node of the ith sequence is faulty, then the last node of the $(i - 1)$th sequence must be faulty. Thus, there exist at least $\lfloor \frac{s-1}{2} \rfloor$ faulty nodes in remaining $s-1$ sequences, which implies that $|F_1 \cup F_2 \cdots \cup F_{i-1} \cup F_{i+1} \cdots \cup F_s - F_i| \geqslant \lfloor \frac{s-1}{2} \rfloor$. Then $t = |F| = |F_1 \cup F_2 \cup \cdots \cup F_{i-1} \cup F_{i+1} \cup \cdots F_s - F_i| + |F_i| \geqslant x_i + \lfloor \frac{y_i+1}{3} \rfloor + \lfloor \frac{s-1}{2} \rfloor = S_i + \lfloor \frac{s-1}{2} \rfloor$, a contradiction to the assumption that $S_i \geqslant t - \lfloor \frac{s-1}{2} \rfloor + 1$. ■

For a given Hamiltonian cycle with the different distribution of faulty nodes, the results generated by our algorithm may be different. For a Hamiltonian cycle with N nodes and t faulty nodes ($N \geqslant 3t + 1$), assume that it has m different distributions of faulty nodes. Let $R(j, S_{max})$ denote the S_{max} of the jth distribution$(1 \leqslant j \leqslant m)$ and $R_{min} = min\{R(j, S_{max}), 1 \leqslant j \leqslant m\}$. According to Lemma 4, if the following inequality is true, then there must exist a sequence satisfying that its first node is fault-free.

$$t - \lfloor \frac{s-1}{2} \rfloor + 1 \leqslant R_{min} \tag{1}$$

Next, we discuss the upper bound of t, denoted by T, such that after implementing Cycle-Partition for a N-node cycle with t faulty nodes ($N \geqslant 3t + 1$), there always exists a sequence with fault-free first node provided $t \leqslant T$.

Let t' be the number of test results of 1, then $N - t'$ is the number of test results of 0. Consider the following cases:

Case1: $\frac{N-t'}{s}$ *is not an integer*:

$$R(j, S_{max}) \geqslant \lceil \frac{N-t'}{s} \rceil + \lfloor \frac{\lfloor \frac{t'}{s} + 1 \rfloor}{3} \rfloor. \tag{2}$$

Case2: $\frac{N-t'}{s}$ *is an integer*:

$$R(j, S_{max}) \geqslant \lceil \frac{N-t'}{s} \rceil + \lfloor \frac{\lceil \frac{t'}{s} + 1 \rceil}{3} \rfloor. \tag{3}$$

Note that the right side of the above inequalities (2) and (3) decreases as t' increases. Since a Hamiltonian cycle with T faulty nodes have at most $3T$ test results of 1, when $t' = 3T$, the right sides of the above inequalities get the minimum. In order to obtain the upper bound T, we let

$$R_{min} = \lceil \frac{N-3T}{s} \rceil + \lfloor \frac{\lfloor \frac{3T}{s} + 1 \rfloor}{3} \rfloor \tag{4}$$

According to (1) and (4), we can get the following inequality.

$$T - \lfloor \frac{s-1}{2} \rfloor + 1 \leqslant \lceil \frac{N-3T}{s} \rceil + \lfloor \frac{\lfloor \frac{3T}{s} + 1 \rfloor}{3} \rfloor. \tag{5}$$

After transposition, (5) is as follows:

$$T - \lceil \frac{N-3T}{s} \rceil - \lfloor \frac{\lfloor \frac{3T}{s} + 1 \rfloor}{3} \rfloor \leqslant \lfloor \frac{s-1}{2} \rfloor - 1. \tag{6}$$

It is very difficult to obtain the solution of (6), but we can consider the solution of the following inequality:

$$T - \frac{N-3T}{s} - \frac{T}{s} + 1 \leqslant \frac{s-1}{2} - 2. \tag{7}$$

It is obvious that for independent variable T, the solution of (7) is also that of (6). After simplifying (7), we have:

$$T \leqslant \frac{s^2 - 7s + 2N}{2(s+2)} \tag{8}$$

On the other hand, according to Cycle-Partition, we have the following result.

$$2T \geqslant s \tag{9}$$

For these two inequalities (8) and (9), using the method of solving extreme value for a given function by function derivative, we can obtain a upper bound of T as follows:

$$T \leqslant \sqrt{18 + 2N} - 5.5.$$

Since T is an integer, the upper bound can be denoted as $\lfloor\sqrt{18+2N} - 5.5\rfloor$. According to Lemma 4, for a Hamiltonian cycle with N nodes and at most T faulty nodes ($N \geqslant 3T + 1$), after partitioning it by Partition-Cycle, we can obtain a sequence, whose first node is fault-free and in which at least one faulty node can be identified.

B. ALGORITHM OF DigOut

In the previous section, it was shown that for a Hamiltonian cycle with N nodes and a fault bound $T = \lfloor\sqrt{18+2N} - 5.5\rfloor$, there always exists a sequence obtained by Cycle-Partition such that the first node of it is fault-free provided the number of faulty nodes in the Hamiltonian cycle does not exceed fault bound T. In this section, we shall use the identified nodes in this sequence to identify more nodes in other sequences. In the section, to identify the faulty nodes in this given Hamiltonian cycle, we present a cycle diagnosis algorithm DigOut. See Algorithm 2 for details. After the executing of DigOut, the nodes in the cycle will be divided into three parts: faulty nodes, fault-free nodes and unknown nodes.

Algorithm 2 Algorithm of DigOut

Require:

An N-node cycle with their test results and fault bound T ($N \geqslant 3T + 1$).

Ensure:

The sequences and the states of all nodes: {faulty, fault-free, unknown}.

1: Partition the cycle into sequences in clockwise direction by Cycle-Partition.

2: For each sequence i (with $x_i + 1$ test results 0 and y_i test results 1), if $|S_i| \geqslant T - \lfloor\frac{s-1}{2}\rfloor + 1$, then mark the first $x + 1$ consecutive nodes as fault-free and the $(x + 2)$th node as faulty.

3: Output the faulty nodes set, the fault-free nodes set and the unknown nodes set.

Remark: In [10], the authors proposed a five-round diagnosis algorithm for Hamiltonian networks under the PMC model, which does not work for the problem of fault diagnosis in Hamiltonian networks under the comparison model. As mentioned in the part of introduction, the PMC model is only a special case of the comparison model. As a result, this algorithm, DigOut, is better than the algorithm proposed by [10] in some sense.

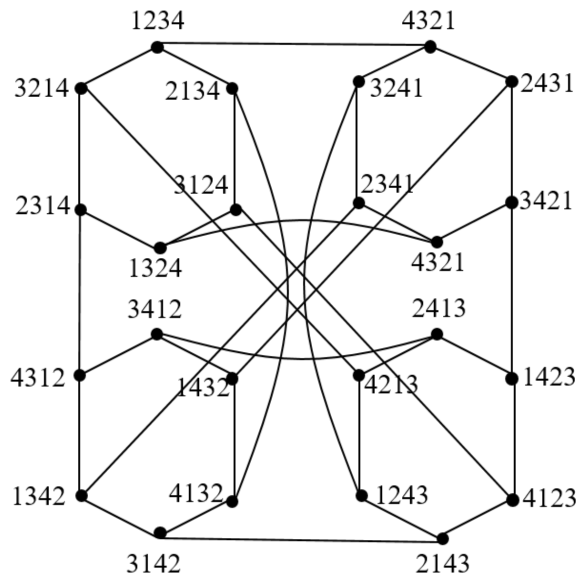

FIGURE 4. 4-dimensional star graph with 24 nodes.

IV. A PRECISE DIAGNOSIS ALGORITHM FOR STAR GRAPHS

A. THE PROPERTIES OF STAR GRAPH AND THE COMPARISON MODEL

An n-dimensional star graph, denoted by S_n, is a graph with the node set $V(S_n) = \{a_1a_2a_3\cdots a_n | a_1a_2a_3\cdots a_n$ is a permutation of $1, 2, 3, \cdots n\}$ and the edge set $E(S_n) = \{(a_1a_2a_3\cdots a_n, a_ia_2a_3\cdots a_{i-1}a_1a_{i+1}\cdots a_n) | 2 \leqslant i \leqslant n\}$. There is an edge between two nodes in S_n if and only if they can be obtained from each other by swapping the leftmost number with one of the other $n-1$ numbers. For a node $v \in V(S_n)$, we use $add(v)$ to denote the address of the node v.

Lemma 5 [23]: In an n-dimensional star graph, there are no odd cycles and there are even cycles with length l where $l \geqslant 6$ and $l \leqslant n!$.

Definition 1: Let $G = (V, E)$ be a network system. For a node $v_0 \in V$, a branch path of node v_0 is a path denoted by $P = v_0v_1v_2\cdots v_k, (v_i, v_{i+1}) \in E, 0 \leqslant i \leqslant k-1, v_i \neq v_j (i \neq j, 0 \leqslant i, j \leqslant k)$.

For the convenience of discussion, we use $G(v_0, P)$ to denote the branch path P of node v_0.

Lemma 6: For each node v of an n-dimensional ($n \geqslant 5$) star graph S_n, there exist $n-1$ different branch paths of node v, say $Q_1, Q_2, \cdots, Q_{n-1}$(see Figure 5), such that the following conditions hold.

i) Each branch path $Q_i (1 \leqslant i \leqslant n-1)$ contains $n+3$ nodes.

ii) $V(Q_i) \cap V(Q_j) = \{v\}$, where $1 \leqslant i, j \leqslant n-1$ and $i \neq j$.

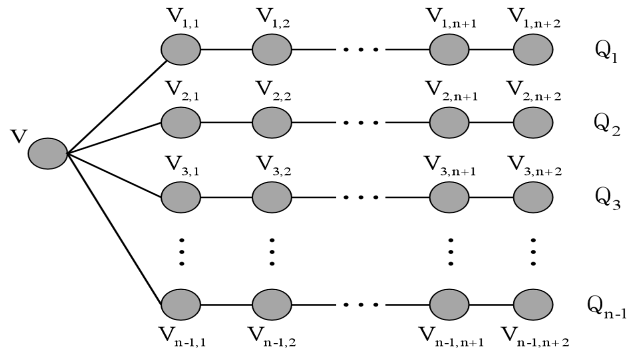

FIGURE 5. An illustration of $n-1$ different branch paths of node v in S_n.

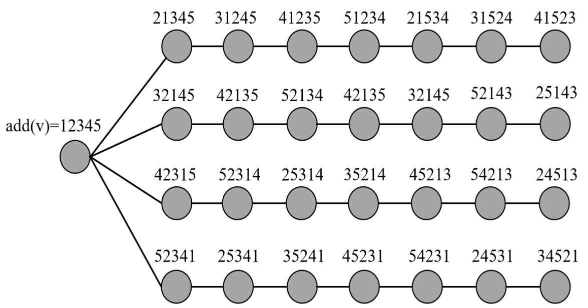

FIGURE 6. An example of $S_5(v)$ with $add(v) = 12345$.

Proof: For each node v of an n-dimensional ($n \geqslant 5$) star graph with address $a_1a_2a_3\cdots a_n$, we introduce a method to construct $Q_1, Q_2, \cdots, Q_{n-1}$: Construction:

$V(Q_i) = \{v, v_{i,1}, v_{i,2}, \cdots, v_{i,n+2} | where\ 1 \leqslant i \leqslant n-1\}$ and the address of node $v_{i,1}$ is obtained by swapping the first position with the $(i+1)$th position from the left of the address of node v: $add(v_{i,1}) = a_{i+1}a_2a_3\cdots a_ia_1a_{i+2}\cdots a_n$. And the addresses of the other $n+1$ nodes can be obtained by executing following rules:

begin:

$k = 2$;

for($j = 2 : j \leqslant n+2; j++;$)

{

if($k \geqslant n+1$) then $k = 2$;

if($k == i+1$) then $k = k+1$;

$add(v_{i,j})$ can be obtained by swapping the first position with the kth position from the left of the address of node $v_{i,j-1}$.

$k = k+1$;

}

end

According to above rules, for any two different nodes $u_1, u_2 \in V(S_n(v, Q_i))$ ($1 \leqslant i \leqslant n-1$), we claim that $add(u_1) \neq add(u_2)$. In fact, for any two nodes $v_{i,j} \in V(Q_i), v_{k,l} \in V(Q_k)$ where $1 \leqslant i, k \leqslant n-1, 1 \leqslant j, l \leqslant n+2$ with $i \neq k$, since the $(i+1)$th position of $add(v_{i,j})$ is a_1 and number $(k+1)$th position of $add(v_{k,l})$ is a_1, $add(v_{i,j}) \neq add(v_{k,l})$. ■

An example: 4 different branch paths of node v with $add(v) = 12345$ in S_4 can be shown in Figure 6.

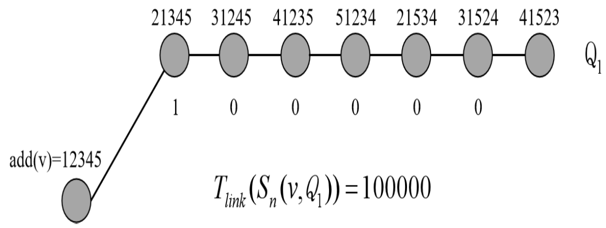

FIGURE 7. An example of $T_{link}(S_n(v, Q_1))$.

For each branch $S_n(v, Q_i)$, let $T_{link}(S_n(v, Q_i))$ denote to a test result link which consists of the test results of the following nodes: $v_{i,1}, v_{i,2}, \cdots, v_{i,n+2}$. For example, $T_{link}(S_4(v, Q_1))$ can be shown by Figure 7.

Lemma 7: Let $_n$ be n-dimensional ($n \geqslant 5$) star graph with at most $(n-1)$ faulty nodes and σ be a syndrome under the comparison model. For any node v in an n-dimensional ($n \geqslant 5$) star graph S_n and its $n-1$ different branch paths $Q_1, Q_2, \cdots, Q_{n-1}$ obtained by the construction rule of Lemma 6, if the number of faulty nodes in S_n does not exceed $n-1$, then the following conditions hold:

i) If there exists a branch $S_n(v, Q_i)$ such that $T_{link}(S_n(v, Q_i)) = 1\overbrace{000\cdots000}^{n} \subset \sigma$, where $1 \leqslant i \leqslant n-1$, then v is faulty.

ii) If there does not exist a branch $S_n(v, Q_i)$ such that $T_{link}(S_n(v, Q_i)) = 1\overbrace{000\cdots000}^{n} \subset \sigma$, where $1 \leqslant i \leqslant n-1$, then v is fault-free.

Proof:

i) Without loss of generality, let $T_{link}(S_n(v, Q_1)) = 1\overbrace{000\cdots000}^{n}$. If $v_{1,2}$ is faulty, then $v_{1,3}, v_{1,4}, \cdots, v_{1,n+1}$ are all faulty, which implies that there are at least n faulty nodes in the star graph, which is an contradiction to the assumption. Hence, $v_{1,2}$ is fault-free, which implies that $v_{1,1}$ is fault-free and v is faulty.

ii) Assume that there is no branch $S_n(v, Q_i)$ such that $T_{link}(S_n(v, Q_i)) = 1\overbrace{000\cdots000}^{n}$, where $1 \leqslant i \leqslant n-1$. Consider following cases:

Case1: If there exists a branch $S_n(v, Q_i)$ such that $T_{link}(S_n(v, Q_i)) = \overbrace{000\cdots000}^{n+1}$,

Similar argument of condition i) can be used to prove that each node in $V(Q_i)$ is fault-free.

Case2: The other possible cases.

There is at least one test result 1 in the $T_{link}(S_n(v, Q_i))$, which implies there is at least one faulty node in $V(Q_i) - \{v\}$. Overall, there are at least $n-1$ faulty nodes in $V(Q_1) \cup V(Q_2) \cup \cdots \cup V(Q_{n-1}) - \{v\}$, which implies that v is fault-free. ■

Lemma 8: For a system modelled by an n-dimensional ($n \geqslant 5$) star graph $G = (V, E)$ and a syndrome ω under the comparison model, after Digout is used to diagnose this system modelled by the star graphs with the syndrome ω, there exist at most $\lceil \frac{4n^2+12n-15}{8} \rceil$ unknown nodes in the system provided the number of faulty nodes does not exceed $(n-1)$ in the system.

Proof: According to Lemma 1, there exists a Hamiltonian cycle with length $n!$. The cycle can be divided into sequences by Cycle-Partition. Since $(n-1) - \lfloor\sqrt{18+2*(n!)} - 5.5\rfloor \leqslant 0$ for $n \geqslant 5$, then in these sequences obtained by Cycle-Partition, there exists at least a sequence such that the first node of this sequence is fault-free. Assume that the cycle is divided into s sequences by Cycle-Partition, then $1 \leqslant s \leqslant 2(n-1)$(Property 3). Let L_i with $x_i + 1$ test results 0 and y_i test results 1 be the ith $(1 \leqslant i \leqslant s)$ sequence obtained by Cycle-Partition.

For the sake of convenience, we call the first x_i nodes of S_i as the previous part of L_i.

For L_i, according to Lemma 7 and the assumption that the number of nodes in the system does not exceed (n-1), we can obtain that if $x_i+1+\lfloor\frac{s-1}{2}\rfloor > n-1$, then the first x_i+1 nodes are fault-free. And then, if $x_i+1+\lfloor\frac{s-1}{2}\rfloor \leqslant n-1$, then the first x_i nodes of L_i may be unknown. In other words, the nodes in the previous part of L_i may be diagnosed to be unknown. On the other hand, according to above discussion, it is easily known that there are at most $s-1$ sequences such that the nodes in their previous part may be diagnosed to be unknown. At the same time, since there is at most $(n-1)$ faulty nodes, there are at most $3(n-1)$ nodes with test results 1 such that they may be diagnosed to be unknown. Therefore, there are at most $3(n-1)+(s-1)(n-2-\lfloor\frac{s-1}{2}\rfloor)$ nodes which can be diagnosed to be unknown. Let $f(s) = (s-1)(n-2-\lfloor\frac{s-1}{2}\rfloor)$. Now, we discuss the maximum value of this function. Consider the following cases:

Case 1: $s = 2m$, where $1 \leqslant m \leqslant n-1$.

$f(s) = (2m-1)(n-2-\lfloor\frac{2m-1}{2}\rfloor) = (2m-1)(n-2-(m-1)) = (2m-1)(n-m-1) = -2m^2+(2n-1)m-(n-1)$. It is easily seen that when $m = \frac{2n-1}{4}$, the function gets the maximum value $\frac{(2n-3)^2}{8}$.

Case 2: $s = 2m+1$, where $0 \leqslant m \leqslant n-2$.

$f(s) = 2m(n-2-\lfloor\frac{2m}{2}\rfloor) = 2m(n-2-m) = -2m^2 + (2n-4)m$. It is easily seen that when $m = \frac{n-2}{2}$, the function gets the maximum value $\frac{(n-2)^2}{2}$.

Therefore, after Digout is finished, in the considered system, there are at most $\lceil\frac{4n^2+12n-15}{8}\rceil$ nodes that can be diagnosed to be unknown. ■

Next, we shall introduce a fast precise diagnosis algorithm, whose time complexity is $O(N)$ for n-dimensional star graphs $(N = n!)$.

B. A DIAGNOSIS ALGORITHM FOR STAR GRAPHS

In this section, we present a precise diagnosis algorithm to diagnose a n-dimensional $(n \geqslant 5)$ star graph network system, called by Star-DigOut, which can identified all faulty nodes in the system provided the number of faulty nodes in the system does not exceed $n-1$. The detail description for Star-DigOut can be found in Algorithm 3.

Remark: According to Lemma 8, we have that after the execution of step 1), there are at most $O(n^2)$ unknown nodes in the system. Furthermore, according to Lemma 6 and Lemma 7, step 2) and step 3) can identify all unknown nodes as faulty or fault-free.

Algorithm 3 Algorithm of Star-DigOut

Require: An n-dimensional $(n \geqslant 5)$ star graph given by $G = (V, E)$ with their test results and the fault bound $n-1$. Let $T = F = U = \varnothing$.

Ensure: The statue of all nodes: faulty or fault-free. **1:** Use Digout to diagnose the Hamiltonian cycle which belongs to the given n-dimensional star graph. For each node $v \in V$, if v is marked with faulty, then $F = F \cup \{v\}$. If v is marked with fault-free, then $T = T \cup \{v\}$. Otherwise, $U = U \cup \{v\}$.

2: For each node $w \in U$, use the Construction of Lemma 2 to produce $n-1$ different branch paths of node w. Meanwhile, check each test results link:
If there exists a branch, say Q_i, such that
$T_{link}(S_n(w, Q_i)) = 1\overbrace{000\cdots000}^{n}$, then $F = F \cup \{w\}$.
Otherwise, $T = T \cup \{w\}$.
If $|F| = n-1$, then $T = T \cup U$ and goto step 3).

3: Output the nodes set T, F.

Theorem 1: For an n-dimensional $(n \geqslant 6)$ star graph with $N = n!$ nodes, the time complexity of algorithm Star-DigOut is $O(N)$.

Proof: Step 1) needs $O(N)$ time. In step 2), constructing subgraph for each unknown node and checking its test results links needs $O(n)$ time. And there are at most $O(n^2)$ unknown nodes in the system, which implies that step 2) needs $O(n^2) * O(n)$ time. Step 3) needs $O(1)$ time. Hence the time complexity of algorithm Star-DigOut is $O(N) + O(n^3) + O(1) = O(N)$. ■

In the next section, some simulations are presented to show the efficiency of the algorithm Star-DigOut.

V. SIMULATION

In this section, the performance of the algorithm Star-DigOut is evaluated by the computer simulation. We randomly deployed t faulty nodes in an n-dimensional star graph $G = (V, E)$, where $n \geqslant 5$ and $1 \leqslant t \leqslant n-1$, and assume that the faulty nodes present test results 1 and 0 with probability 0.5 and 0.5, respectively.

To obtain simulation results, we first obtain a syndrome. According to the definition of the comparison model, this syndrome ω is a set of comparator w testing its two neighbors u and v by comparing their responses, namely $\omega = \{\omega(w : u, v) | w, u, v \in V, (w, u) \in E, ((w, v) \in E\}$. A syndrome ω is a compatible syndrome with the fault set F if and only if for any $\omega(w : u, v) \in \omega$, if $w, u, v \in V - F$, then $\omega(w : u, v) = 0$, and if $w \in V - F$ and $u, v\cap \neq \phi$, then $\omega(w : u, v) = 1$. After a syndrome had been generated, we checked if it was compatible with fault set F consisting of t nodes deploying previously in n-dimensional star graph $G = (V, E)$.

TABLE 1. The number of faulty nodes detected by the algorithm Star-DigOut under the comparison model.

dimension n	6	7	8	9	10	11
faulty nodes t	5	6	7	8	9	10
detected faulty nodes	5	6	7	8	9	10

If not, we renounced such a syndrome. Otherwise, it was regarded as a candidate syndrome for testing. By repeating this procedure, we obtained all syndrome candidates for each performance measure. For each candidate syndrome, we run our algorithm and obtained the corresponding fault set F, repeating 100000 times, we took the intersection of all Fs as the fault set F' determined by the candidate syndrome. At last, we took the intersection of these $F's$, which are determined by these corresponding candidate syndrome, respectively, as the simulation result.

Table 1 presents the results of our simulations. And it is clear that the algorithm successfully identifies the all faulty nodes in the system. The hardware and software used to perform the simulation are Intel Core i5-5200U CPU 2.20 GHz, 8 GB DRAM, 64-bit Windows 7 OS, and Java is used to program the algorithm.

VI. CONCLUSION

We present a novel method to partition a Hamiltonian cycle with N nodes and t fault nodes ($N \geqslant 3t+1$) into sequences, such that the first node of one of them is fault-free. At the same time, we have theoretically derived a upper bound for the number of faulty nodes t, $T = \lfloor \sqrt{18+2N} - 5.5 \rfloor$. Based on th partition method and the result that any star graph has a Hamiltonian cycle, we introduce a fast precise diagnosis algorithm for an n-dimensional ($n \geqslant 6$) star graph under the comparison model, which consists of Algorithm 1, Algorithm 2 and Algorithm 3. We prove that the time complexity of this algorithm for an n-dimensional ($n \geqslant 6$) star graph is $O(N)$, where $N = n!$. Similar fault diagnosis algorithms may be able to diagnose other network structures such as hypercubes, exchanged hypercubes and augmented hypercubes, and will be investigated by us in the future.

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