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The Cooperative Car-Following Model With Consideration of the Stimulatory Effect of Lane-Changing Types

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ABSTRACT Lane changing behavior is one of the most important tasks in driving. Cooperative lane change based on vehicle-vehicle (V2V) communication technology is getting extensive attention due to its potential to improve traffic safety and increase efficiency. In order to explore the internal mechanism of cooperative lane change behavior, the cooperative car-following model considering the stimulatory effect of lane-changing types is proposed in this paper. We investigate three lane changing types (free lane change, forced lane change, and cooperative lane change) to impact cooperative lane changing behavior. Based on this analysis, cooperative level function is presented. By using linear stability theory, the stability criterion of the proposed model is obtained. Simulation scenarios are set in three typical situations (starting process, stopping process and perturbation process) to analyze FVD model, cooperative driving for non-lane mode and cooperative driving for lane-changing mode. Simulation results indicate that cooperative driving of lane-changing mode has larger stable region compared to FVD model, cooperative driving for non-lane mode. Furthermore, the higher the level of cooperation, the greater the area of stability. Meanwhile, the simulation experiment also shows that response capability and smoothness of cooperative driving for lane-changing mode is be effectively promoted to the acceleration, velocity evolution.

INDEX TERMS Cooperative car-following model, lane-changing types, stimulatory effect, driving behavior.

I. INTRODUCTION

With the rapid increase in the number of vehicles around the world, the traffic accidents caused by lane changes can cause serious collisions and consequent traffic delays. According to research from European Union (EU), a large proportion of traffic accidents are caused by wrong lane changing maneuvers, and nearly 75% of lane change collisions are attributable to drivers [1]. Approximately 10% of traffic accidents are attributable to unstable traffic flow and complex traffic [2]. In addition, the research revealed that serious traffic accidents caused by improper lane changes were on the rise from 18% in 2005 year to 23.6% in 2014 year [3]. So, it is necessary to study lane changing strategies to improve driving safety and reduce traffic delays.

In recent years, connected and automated vehicles (CAVs) including vehicle to vehicle (V2V) communication and vehicle to infrastructure (V2I) communication have emerged to improve driving safety and traffic efficiency [4]–[8]. Lane changing behavior is extensive but complex work while driving. The drivers need adjust the current driving behavior to match the different stimulations of driving information such as vehicle velocity, relative motion between vehicles and lane conditions. Cooperative lane change has been widely concerned especially advancements of information and communication technologies (ICT) [9]–[12]. Li *et al.* proposed the safe driving patterns to represent collision free movements and the algorithm to determine cooperative driving and lane

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change [9], [10]. Silvlin *et al.* presented a low-complexity lane change maneuver which determines whether a lane change maneuver is desirable, and calculates the corresponding longitudinal and lateral control trajectory [11]. Hongil and Jung presented a design of a cooperative lane change protocol for a connected and automated vehicle based on an estimation of the communication delay [12].

In addition, various studies have been conducted to improve lane change maneuver and traffic safety via intelligence algorithm. Tang *et al.* proposed a lane changing predictor based on Adaptive Fuzzy Neural Network by fusing information from vehicle sensor to predict steering angles [13]. Tomar *et al.* [14] constructed a neural network with multilayer perceptron to predict the lane changing trajectory in future steps based on field data from the Next Generation Simulation (NGSIM). Ding *et al.* [15] developed a Back-Propagation (BP) Neural Network to predict lane-changing trajectory, and they also compared prediction results between BP Neural Network and Elman Network using the data collected from driving simulator data and NGSIM. Yu et al presented a game theory-based lane-changing model, which mimics human behavior by interacting with surrounding drivers using the turn signal and lateral moves [16]. The approach to modeling the interactions between vehicles during lane changing and lane merging were investigated based on Stackelberg game by proposed Yoo and Langari [17]. Wang *et al.* [18] presented a model for car-following and lane-changing control based on differential game.

Moreover, most existing lane-changing models are rule-based models [19]–[21]. While some models adopt more realistic utility-based approaches to capture drivers' decision-making processes [22]. Starting with the microscopic approach, the optimal velocity (OV) model proposed by Bando *et al.* is a significant one in the traffic flow theory [23]. Helbing and Tilch [24] proposed the generalized force model (GF model) to overcome unrealistic acceleration and decoration. To overcome this deficiency of the GF model, Jiang *et al.* [25] developed a full velocity difference (FVD) model. The OV extension models have been extensively investigated [26]–[28].

A lot of research focuses on the lane-discipline-based traffic flow models with single lane. However, lane boundaries can be vague and ambiguous in some countries and regions of the world such as China. Tang *et al.* [29] studied the stability of a car-following model on two lanes which incorporates the lateral effects in traffic. Jin *et al.* [30] proposed a non-lane-based full velocity difference car-following model to analyze the impact of lane width in traffic. Li *et al.* [31] proposed a new car-following model considering the effect of visual angle under the non-lane-discipline environment and the proposed CF model that considers the effects of both lateral gap and visual angle has larger stable region compared with FVD model. Li *et al.* [32]–[34] incorporated two-sided lateral gaps into consideration and presented a new non-lane-discipline-based car-following model. Zhao *et al.* [35] analyzed the different types of stimulus during the preceding vehicle's lane-merging process and the space gain effect produced by the preceding vehicle's lane-passing behavior. He *et al.* proposed cooperative driving and lane changing modeling for connected vehicles in the vicinity of traffic signals from cyber-physical perspective [36]. These works study about the non-lane-discipline traffic flow model from many aspects and many important research results are obtained.

The stimulatory effect of lane-changing types plays a very important role in safety traffic. In this paper, firstly, the cooperative car-following model with non-lane discipline is proposed. Secondly, we investigate three lane changing types (free lane change, forced lane change, and cooperative lane change) to impact cooperative lane changing behavior. Then, the stability of the proposed model is discussed. Finally, we verify the feasibility of the obtained analysis results through simulation experiment.

The remainder of this paper is arranged as follows. In section II, the cooperative car-following model with non-lane discipline is proposed and the stimulatory effect of lane-changing types is investigated. Section III discusses the stability of the proposed model and obtains the stability criterion. In section IV, numerical simulation is performed to demonstrate theoretical results. Finally, section V presents conclusions and summarizes the work.

II. MODEL

A. COOPERATIVE CAR FOLLOWING MODEL OF TWO LANES

The generic car-following model is an important method to study traffic phenomena and driving behavior, which focus on driving behavior of the interaction between vehicles moving on a single road without overtaking and lane changing shown in FIGURE 1. The generic car-following model can be expressed as nonlinear dynamic systems [23]:

$$\frac{dv_n(t)}{dt} = f(v_n(t), \Delta x_n(t), \Delta v_n(t)) \tag{1}$$

where x_n is the position of the nth vehicle at time t, v_n is the velocity of the nth vehicle at time t. Headway distance $\Delta x_n = x_{n+1} - x_n$, velocity difference $\Delta v_n = v_{n+1} - v_n$. The acceleration of vehicle will be decided by the nonlinear stimuli function which are usually be composed of the headway distance Δx_n, the velocity difference Δv_n and the vehicle's velocity v_n. In specific traffic, the accelerations of the vehicles change with the change of other variables.

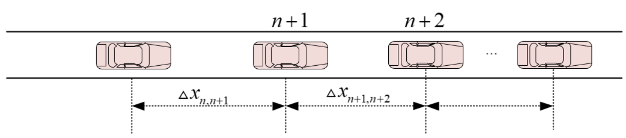


FIGURE 1. The single lane traffic model.

Integrating the considered information with the weighting coefficients into dynamical system, the acceleration of vehicle n considering cooperative driving system can be represented as [37]:

$$
\begin{aligned}
&\ddot{x}_{(l,n)}(t) \\
&\quad = f[v_{(l,n)}(t), \Gamma_1(\underset{j\in N(t)}{\Delta x_{(l,n+j)}(t)}), \Gamma_2(\underset{j\in N(t)}{\Delta v_{(l,n+j)}(t)})] \\
&\quad = f[v_{(l,n)}(t), \sum_{j=-K}^{K} \varphi_i(\Delta x_{(l,n+j)}(t)), \sum_{j=-K}^{K} \psi_j \Delta v_{(l,n+j)}(t)]
\end{aligned} \tag{2}
$$

where K is the number of the considered cooperative vehicles. $N(t)$ is the topological structure. Γ_1 and Γ_2 are the corresponding control policies. φ_j and ψ_j are supposed to define the importance of the interaction between vehicle n and its surrounding vehicles, corresponding to the weighed coefficients of cooperative relation.

In proposed model, φ_j and ψ_j satisfy the following three assumptions:

(1) For forward considered vehicles ($j = 1, 2, 3, \ldots, K$), $\varphi_K < \ldots < \varphi_3 < \varphi_2 < \varphi_1$ and $\psi_K < \ldots < \psi_3 < \psi_2 < \psi_1$.

(2) For the same system, the weighed coefficients of cooperative relation are same ($\varphi_j = \psi_j$).

(3) φ_j is defined as follows:

$$
\sum \varphi_j = 1, \quad \varphi_j = \begin{cases} \dfrac{2}{3^j} & \text{if } j \neq K \\ \dfrac{1}{3^{j-1}} & \text{if } j = K \end{cases} \tag{3}
$$

The existing car following model did not fully consider that lane change impacts the stability of traffic flow. However, Reference [29] studied that drivers are always afraid of the lane change in two-lane traffic flow to establish car following model of two-lane traffic. Considering the impact of lateral vehicles, Equation (2) is extended to a two-lane car following model:

$$
\begin{aligned}
\frac{dv_{(l,n)}(t)}{dt} = f(&v_{(l,n)}(t), \quad \Delta x_{(l,n),(l,n+1)}(t), \\
&\Delta_{(l,n),(l+1,n+1)}(t), \quad \Delta v_{(l,n),(l,n+1)}(t))
\end{aligned} \tag{4}
$$

where $\Delta x_{(l,n),(l,n+1)}(t) = x_{(l,n+1)}(t) - x_{(l,n)}(t)$ and $\Delta v_{(l,n),(l,n+1)}(t) = v_{(l,n+1)}(t) - v_{(l,n)}(t)$ are the headway and relative velocity between two vehicles n and $n + 1$ in the same lane l at time t, respectively. $\Delta_{(l,n),(l+1,n+1)}(t) = x_{(l+1,n+1)}(t) - x_{(l,n)}(t)$is the longitudinal distance between the n-th vehicle on the lane l and the preceding vehicle $n + 1$ in adjacent lane $l + 1$.

Based on full velocity difference model by proposed Helbing and Tilch [24], [25], the new dynamical equation is obtained:

$$
\frac{dv_{(l,n)}(t)}{dt} = \kappa_l \left(V_l(\Delta\bar{x}_{(l,n)}(t)) - v_{(l,n)}(t)\right) + \lambda_l \Delta v_{(l,n)}(t) \tag{5}
$$

where $V_l(\Delta\bar{x}_{(l,n)}(t))$ is the optimal velocity (OV) function which depends only on the headway distance $\Delta\bar{x}_{(l,n)}(t)$. $\Delta\bar{x}_{(l,n)}(t) = \beta_1 \Delta x_{(l,n),(l,n+1)}(t) + \beta_2 \Delta_{(l,n),(l+1,n+1)}(t)$, β_1 and β_2 are the weight of longitudinal distance ($\beta_1 + \beta_2 = 1 (0 \leq \beta_1 \leq 1, 0 \leq \beta_2 \leq 1)$). The longitudinal weight needs to consider the common effect of the lane and the adjacent lane. κ_l is the sensitivity coefficient and λ_l is the feedback gain. Eq. (5) means that the vehicle tend to brake when it is close to their predecessor, and to accelerate when the headway and speed difference are increasing.

For cooperative optimal velocity (OV) model, its mathematical dynamics formulation is described as:

$$
\begin{aligned}
\frac{dv_{(l,n)}(t)}{dt} &= \kappa_l [V(\sum_j \varphi_j \Delta\bar{x}_{(l,n+j)}(t)) - v_{(l,n)}(t)] \\
&\quad + \lambda_l \sum_j \varphi_j \Delta v_{(l,n_l+j)}(t)
\end{aligned} \tag{6}
$$

The optimal velocity function is calibrated with respect to the empirical data adopted by Helbing *et al.* [24], [25]:

$$
V(\Delta x) = V_1 + V_2 \tanh[C_1(\Delta x - l_c) - C_2] \tag{7}
$$

The function parameters are shown in TABLE 1.

B. THE STIMULATORY EFFECT OF LANE-CHANGING TYPES

In this paper, we take into account that the road is flat and have no slopes. All the vehicles are of the same type. For lane-changing vehicles, there are different motivations for free lane change, forced lane change, and cooperative lane change. The vehicle $S_{(l+1,n+1)}$ in present lane $l + 1$ suddenly accelerates to reach the target lane l and overtake the vehicle $T_{(l,n)}$. So, the vehicle $T_{(l,n)}$ needs to adjust quickly the current state to ensure safety. In addition, the vehicle $S_{(l+1,n+1)}$ observes the space headway and velocity variation of the vehicles in the target lane and then determines whether to change lane and the velocity of lane change.

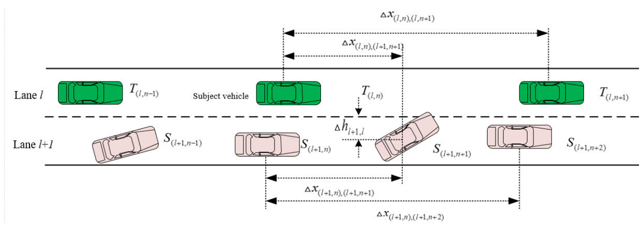


FIGURE 2. The basic notations adopted for the lane-changing model.

Shown in FIGURE 2, the subject vehicle $S_{(l+1,n+1)}$ changes lane from the present lane $l + 1$ to the target lane l. Before changing lane, its immediate preceding vehicle and immediate following vehicle are $S_{(l+1,n+2)}$and $S_{(l+1,n)}$ respectively. After changing lane, its immediate preceding vehicle and immediate following vehicle are $T_{(l,n+1)}$ and $T_{(l,n)}$ respectively. When the vehicle $T_{(l,n)}$ in target lane finds the behavior of lane change, the vehicle $T_{(l,n)}$ needs estimate the safety headway and lateral deviation between the vehicle $T_{(l,n)}$ and the subject vehicle $S_{(l+1,n+1)}$. The vehicle $T_{(l,n)}$ either decides to keep following the initial vehicle or adjust the following target vehicle.

There are three types of lane changing behavior.

Free lane change: there is no noticeable interaction between the subject and adjacent vehicle(s). So, the subject vehicle can drive in the target lane smoothly without acceleration change.

Forced lane change: Vehicles must change lanes due to a turn or traffic accident ahead or lane use restrictions. This type of lane change is followed by deceleration of the adjacent vehicle. Generally, the subject vehicle does not use turn signals or uses them very briefly before changing lanes. The adjacent vehicle does not slow down until part of the subject vehicle has entered the target lane.

Cooperative lane change: The merging vehicle sends a lane-changing request to the adjacent vehicle by turning on the turn signal. The adjacent vehicle evaluates the request and may either cooperate by slowing down or not cooperate. The subject vehicle reevaluates the response based on the new gap and the speed of the lag vehicle. If the lane-changing criteria are satisfied, a cooperative lane change is executed.

Reference [35] defined three kinds of lane changing processes of lane changing vehicles for different spacing. However, for target lane, these three kinds of lane changing processes could be regarded as the following vehicles are stimulated to varying degrees under different conditions when the behaviors of lane change occur:

(1) When the lane changing vehicles are in free lane change, the following vehicles in target lane have enough space and response time, even if they maintain their current driving state. $\Delta x_{(l,n),(l,n+1)} = x_{(l,n+1)} - x_{(l,n)}$ and $\Delta x_{(l,n),(l+1,n+1)} = x_{(l+1,n+1)} - x_{(l,n)}$ are longitudinal spacings between $T_{(l,n)}$ with $T_{(l,n+1)}$ and between $T_{(l,n)}$ with $S_{(l+1,n+1)}$ respectively shown in FIGURE 2. The vehicle $T_{(l,n)}$ could maintain current driving state if the following conditions are satisfied as:

$$\begin{aligned}&\mathit{Type}\ 1\\ &\mathit{if}: \Delta x_{(l,n),(l+1,n+1)}(t_0) > L_{safe}\\ &\mathit{and}\ \Delta \tilde{x}_{(l,n),(l+1,n+1)}(t_0 + t_{LC})\big|_{t=t_0} > L_{safe}\end{aligned} \tag{8}$$

Shown in FIGURE 2, for the subject vehicle, Type 1 is the stimulus of free lane change. t_0 is the initial time before lane change. t_{LC} is the duration of lane change. $\Delta x_{(l,n),(l+1,n+1)}(t_0)$ and $\Delta \tilde{x}_{(l,n),(l+1,n+1)}(t_0 + t_{LC})\big|_{t=t_0}$ are the space headway between vehicle $S_{(l+1,n+1)}$ and vehicle $T_{(l,n)}$ at time t_0 and estimated headway at time $t_0 + t_{LC}$(lane change process is complete). L_{safe} is the acceptable safe spacing.

$$\begin{aligned}\Delta \tilde{x}_{(l,n),(l+1,n+1)}(t_0 + t_{lc}) = \Delta x_{(l,n),(l+1,n+1)}(t_0)\\ + S_{LC(l+1,n+1)} - T_{LC(l,n)}\end{aligned} \tag{9}$$

$T_{LC(l,n)}$, $S_{LC(l+1,n+1)}$ are the distance traveled longitudinally of $T_{(l,n)}$ and $S_{(l+1,n+1)}$ during lane change which satisfy $T_{LC(l,n)} = v_{(l,n)}(t_0) \cdot t_{LC} + 0.5 \cdot a_{\max} \cdot t_{LC}^2$ and $S_{LC(l+1,n+1)} = v_{(l+1,n+1)}(t_0) \cdot t_{LC}$. $v(t_0)$ is the velocity at time t_0, $a_{\max}$ is reasonable maximum lateral acceleration.

(2) When the lane changing vehicles are cooperative lane change, the following vehicles are affected by the lane change action and decide to accelerate and decelerate. This paper only considers the success of lane change, which is manifested as different deceleration trend of the following vehicles under the stimulatory effect of lane change. Therefore, the following conditions are satisfied:

$$\begin{aligned}&\mathit{Type}\ 2\\ &\mathit{if}: \quad \Delta x_{(l,n),(l+1,n+1)}(t_0) > L_{safe}\\ &\mathit{and} \quad \Delta \tilde{x}_{(l,n),(l+1,n+1)}(t_0 + t_{LC})\big|_{t=t_0} < L_{safe}\end{aligned} \tag{10}$$

(3) When the lane changing vehicles are forced lane change, the following vehicles are obviously stimulated and decelerate sharply to avoid collision. When the subject vehicle $S_{(l+1,n+1)}$ occurs lane change occurs, the headway of vehicle $T_{(l,n)}$ is satisfied:

$$\mathit{Type}\ 3 \quad \mathit{if}: \Delta x_{(l,n),(l+1,n+1)}(t_0) < L_{safe} \tag{11}$$

When lane change occurs, on the one hand, the vehicle $T_{(l,n)}$ need to keep tracking the preceding vehicle $T_{(l,n+1)}$. On the other hand, the vehicle $T_{(l,n)}$ estimates the interaction of lane change and makes corresponding driving decisions according to the state information of the subject vehicle. Therefore, combining equations (9-11) and expanding equations (4), the dynamic model of different stimulus types can be obtained as:

$$\frac{dv_{(l,n)}(t)}{dt} = \begin{cases} f(\Delta x_{(l,n),(l,n+1)}, \Delta v_{(l,n),(l,n+1)}, v_{(l,n)}), & \text{Type 1}\\ f(\Delta x_{(l,n),(l,n+1)}, \Delta x_{(l,n),(l+1,n+1)}, \Delta v_{(l,n),(l+1,n+1)}, & \\ \Delta v_{(l,n),(l+1,n+1)}, v_{(l,n)}, \Delta h_{l+1,l}), & \text{Type 2}\\ f(\Delta x_{(l,n),(l+1,n+1)}, \Delta v_{(l,n),(l+1,n+1)}, v_{(l,n)}) & \text{Type 3}\end{cases} \tag{12}$$

where $\Delta h_{l+1,l}$ is vertical distance of the subject vehicle $S_{(l+1,n+1)}$ in the lane $l+1$ to the target lane l. $\Delta h_{l+1,l} < 0$ indicates that the lane edge line has not been crossed. Otherwise, the lane edge line has been crossed.

Based on above different stimuli, equation (6) following model is used as the basis:

$$\frac{dv_{(l,n)}(t)}{dt} = \begin{cases} \kappa_l\left(V(\Delta x_{(l,n),(l,n+1)}(t)) - v_{(l,n)}(t)\right) + \lambda_l \Delta v_{(l,n),(l,n+1)}(t),\\ \quad \text{Type 1}\\ \kappa_l\left(V_{LC}(\Delta x_{(l,n),(l,n+1)}(t), \Delta x_{(l,n),(l+1,n+1)}(t)) - v_{(l,n)}(t)\right)\\ \quad + \lambda_l G_{LC}(\Delta v_{(l,n),(l,n+1)}(t), \Delta v_{(l,n),(l+1,n+1)}(t)),\\ \quad \text{Type 2}\\ \kappa_l\left(V(\Delta x_{(l,n),(l+1,n+1)}(t)) - v_{(l,n)}(t)\right)\\ \quad + \lambda_l \Delta v_{(l,n),(l+1,n+1)}(t),\\ \quad \text{Type 3}\end{cases} \tag{13}$$

where

$$\begin{aligned}
&V_{LC}(\Delta x_{(l,n),(l,n+1)}(t), \Delta x_{(l,n),(l+1,n+1)}(t)) \\
&\quad = V(C(\Delta h_{l+1,l})\Delta x_{(l,n),(l,n+1)}(t) \\
&\qquad + (1 - C(\Delta h_{l+1,l}))\Delta x_{(l,n),(l+1,n+1)}(t)), \\
&\qquad G_{LC}(\Delta v_{(l,n),(l,n+1)}(t), \Delta v_{(l,n),(l+1,n+1)}(t)) \\
&\quad = C(\Delta h_{l+1,l})\Delta v_{(l,n),(l,n+1)}(t) \\
&\qquad + (1 - C(\Delta h_{l+1,l}))\Delta v_{(l,n),(l+1,n+1)}(t) \qquad (14)
\end{aligned}$$

In equation (14), $C(\cdot)$is cooperative level function of following vehicle T_{n_l} for lane change [35].

$$C(\Delta h_{l+1,l}) = c \cdot H(-\Delta h_{l+1,l}) \cdot \frac{|\Delta h_{l+1,l}|}{\Delta h_{\max}} \qquad (15)$$

where c is the cooperative coefficient of the vehicle $T_{(l,n)}$with target vehicle $S_{(l+1,n+1)}(c \in [0, 1])$. $H(\cdot)$is Heaviside function. $\Delta h_{\max}$ is maximum distance from lane line.

Finally, we propose the cooperative car-following model considering the stimulatory effect of lane-changing types.

$$\frac{dv_{(l,n)}(t)}{dt} = \begin{cases}
\kappa_l[V(\sum_j \varphi_j \Delta x_{(l,n+j)}(t)) - v_{(l,n)}(t)] \\
\quad + \lambda_l \sum_j \varphi_j \Delta v_{(l,n+j)}(t), \\
\quad \text{Type 1} \\
\kappa_l[V_{LC}(\sum_j \varphi_j \Delta x_{(l,n+j)}(t), \sum_j \varphi_j \Delta x_{(l+1,n+j)}(t)) \\
\quad - v_{(l,n)}(t)] + \lambda_l G_{LC}(\sum_j \varphi_j \Delta v_{(l,n+j)}(t), \\
\quad \sum_j \varphi_j \Delta v_{(l+1,n+j)}(t)), \\
\quad \text{Type 2} \\
\kappa_l[V(\sum_j \varphi_j \Delta x_{(l+1,n+j)}(t)) - v_{(l,n)}(t)] \\
\quad + \lambda_l \sum_j \varphi_j \Delta v_{(l+1,n+j)}(t), \\
\quad \text{Type 3}
\end{cases} \qquad (16)$$

where

$$\begin{aligned}
&= V_{LC}(\sum_j \varphi_j \Delta x_{(l,n+j)}(t), \sum_j \varphi_j \Delta x_{(l+1,n+j)}(t)) \\
&= V(C(\Delta h_{l+1,l}) \sum_j \varphi_j \Delta x_{(l,n+j)}(t) \\
&\quad + (1 - C(\Delta h_{l+1,l})) \sum_j \varphi_j \Delta x_{(l+1,n+j)}(t)), \\
&\quad G_{LC}(\sum_j \varphi_j \Delta v_{(l,n+j)}(t), \sum_j \varphi_j \Delta v_{(l+1,n+j)}(t)) \\
&= C(\Delta h_{l+1,l}) \sum_j \varphi_j \Delta v_{(l,n+j)}(t)) \\
&\quad + (1 - C(\Delta h_{l+1,l})) \sum_j \varphi_j \Delta v_{(l+1,n+j)}(t)). \qquad (17)
\end{aligned}$$

Next, we analyze the linear stability of the proposed model (16).

Because the patterns for Type 1 and Type 3 are similar, we need to make them simple. By discretizing Eq. (16) of Type 1 and Type 3, we get the asymmetric difference equation in terms of headway as follows:

$$\begin{aligned}
&x_{(l,n)}(t + 2\tau) \\
&\quad = x_{(l,n)}(t + \tau) + \tau[V(\sum_j \varphi_j \Delta x_{(l,n+j)}(t))] \\
&\qquad + \tau\lambda_l \sum_j \varphi_j[\Delta x_{(l,n+j)}(t + \tau) - \Delta x_{(l,n+j)}(t)] \qquad (18)
\end{aligned}$$

We discretize Eq. (16) of Type 2 and get the asymmetric difference equation as follows:

$$\begin{aligned}
&x_{(l,n)}(t + 2\tau) \\
&= x_{(l,n)}(t + \tau) + \tau\{V_{LC}(C(\Delta h_{l+1,l}) \sum_j \varphi_j \Delta x_{(l,n+j)}(t) \\
&\quad + (1 - C(\Delta h_{l+1,l})) \sum_j \varphi_j \Delta x_{(l+1,n+j)}(t))\} \\
&\quad + \tau\lambda_l\{C(\Delta h_{l+1,l}) \sum_j \varphi_j \Delta x_{(l,n+j)}(t + \tau) - \Delta x_{(l,n+j)}(t)) \\
&\quad + (1 - C(\Delta h_{l+1,l})) \sum_j \varphi_j \Delta x_{(l+1,n+j)}(t + \tau) \\
&\quad - \Delta x_{(l+1,n+j)}(t))\} \qquad (19)
\end{aligned}$$

III. LINEAR STABILITY ANALYSIS

Stability analysis is an important feature of the proposed model which was extensively studied by the researchers. In this paper, we mainly study linear stability of the proposed model to reveal the impact of the stimulatory effect of lane-changing types. Firstly, for the sake of analysis, we need some assumptions. Secondly, the linear stability is analyzed in detail and the analytical solution is obtained. Finally, we analyze the influence of the stimulatory effect of lane-changing types on stability in combination with traffic behavior.

Assumption 1: All vehicles are the same and run on the same road conditions.

Assumption 2: The motion state of all vehicles at the beginning is steady and uniformly distributed, which move with the identical headway distance and velocity.

Based on two assumptions, we conduct the stability analysis of the proposed model.

Theorem 1: The uniform traffic flow in Eq. (16) for Type 1 and Type 3 is unstable if

$$\tau > \frac{\sum_j \varphi_j(2j - 1)}{3V' - 2\lambda} \qquad (20)$$

Proof: Assume the vehicles are evenly distributed and the relative velocity is 0 on two lanes (l and $l + 1$). So, the initial state is described as:

$$x_n^0(t) = nh + Vt \qquad (21)$$

Set $y_n(t)$ be a small fluctuation in the steady state $x_n^0(t)$ of the vehicle $C_{(l,n)}$ applied to lane l.

$$x_n(t) = x_n^0(t) + y_n(t) \tag{22}$$

Substituting formula (22) into formula (18) and using linearization method, we get

$$\begin{aligned} y_n(t+2\tau) = {} & y_n(t+\tau) + \tau V' \sum_j \varphi_j (y_{n+j}(t) - y_{n+j-1}(t)) \\ & + \tau\lambda \sum_j \varphi_j [y_{n+j}(t+\tau) - y_{n+j-1}(t+\tau) \\ & - y_{n+j}(t) + y_{n+j-1}(t)] \end{aligned} \tag{23}$$

We set Fourier series expansion $y_n(t)$ expressed by:

$$y_n(t) = A\exp(ink + zt) \tag{24}$$

Substituting formula (24) into formula (23) and proceeding with Taylor expansion, we obtain

$$\begin{aligned} & 1 + (2\tau z) + \frac{1}{2}(2\tau z)^2 \\ & = 1 + (\tau z) + \frac{1}{2}(\tau z)^2 + \tau V' \sum_j \varphi_j e^{ijk}(1 - e^{-ik}) \\ & + \tau\lambda \sum_j \varphi_j [e^{ijk+z\tau} - e^{i(j-1)k+z\tau} - e^{ijk} + e^{i(j-1)k}] \end{aligned} \tag{25}$$

Set $z = z_1(ik) + z_2(ik)^2 + \cdots$ and ignore the higher-order terms of (ik). We obtain the solution of First order (ik) and second order $(ik)^2$ terms, respectively

$$z_1 = V' \tag{26}$$

and

$$z_2 = \frac{1}{2}V'[\sum_j \varphi_j(-1+2j) + 2\lambda\tau - 3V'\tau] \tag{27}$$

For small disturbance with long wave mode, if z_2 is positive, the initial stable traffic flow will become unstable. If z_2 is negative, the initial stable traffic flow will remain stable. Thus, the uniform traffic flow can be unstable when the following conditions are met

$$\tau > \frac{\sum_j \varphi_j(2j-1)}{3V' - 2\lambda} \tag{28}$$

Theorem 2: The uniform traffic flow in Eq. (16) for Type 2 is unstable if

$$\tau > \frac{\sum_j \varphi_j(2j-2) + C(\Delta h_{l+1,l})}{3V'_{LC} - 2\lambda_l} \tag{29}$$

Proof: The assumptions include formulas (21) and (22). In addition, we assume that the following condition is met

$$y_{(l+1,n+1)}(t) = \frac{1}{2}(y_{(l,n)}(t) + y_{(l,n+1)}(t)) \tag{30}$$

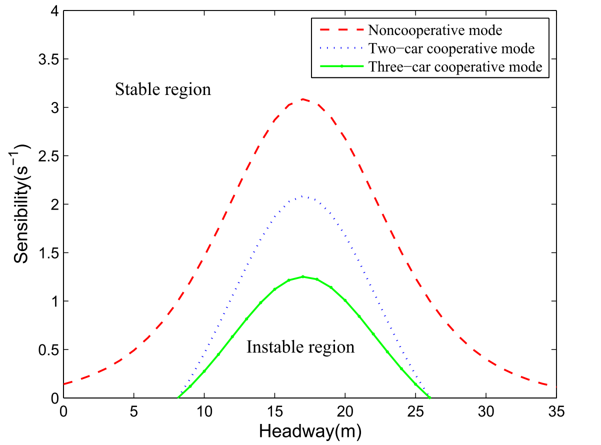


FIGURE 3. Neutral stability curves with different cooperative vehicles n for non-lane mode.

Substituting the above assumptions into formula (19), we get

$$\begin{aligned} e^{2\tau z} = {} & e^{\tau z} + \tau V'_{LC}\{C(\Delta h_{l+1,l}) \sum_j \varphi_j [e^{ijk} - e^{i(j-1)k}] \\ & + (1 - C(\Delta h_{l+1,l})) \sum_j \varphi_j [\frac{1}{2}e^{ijk} - \frac{1}{2}e^{i(j-2)k}]\} \\ & + \tau\lambda_l\{C(\Delta h_{l+1,l}) \sum_j \varphi_j [e^{ijk+\tau z} - e^{i(j-1)k+\tau z} \\ & - e^{ijk} + e^{i(j-1)k}] \\ & + (1 - C(\Delta h_{l+1,l})) \sum_j \varphi_j [\frac{1}{2}e^{ijk+\tau z} - \frac{1}{2}e^{i(j-2)k+\tau z} \\ & - \frac{1}{2}e^{ijk} + \frac{1}{2}e^{i(j-2)k}]\} \end{aligned} \tag{31}$$

Formula (31) is processed by the Taylor series and ignores the higher-order terms. Then, we extract the first and second order terms.

$$z_1 = V'_{LC} \tag{32}$$

and

$$z_2 = \frac{1}{2}V'_{LC}(\sum_j \varphi_j(2j-2) + C(\Delta h_{l+1,l}) + 2\lambda_l\tau - 3V'\tau) \tag{33}$$

If uniform traffic flow can be unstable for long wave small disturbances, the following conditions are met

$$\tau > \frac{\sum_j \varphi_j(2j-2) + C(\Delta h_{l+1,l})}{3V'_{LC} - 2\lambda_l} \tag{34}$$

This completes the proof. □

Shown in FIGURE 3, the neutral stability curves with different cooperative vehicles n for non-lane mode is displayed. In FIGURE 3, the stable region is above the curve and the instable region lies below the curve. It is found that the stability region becomes larger and larger as the number of

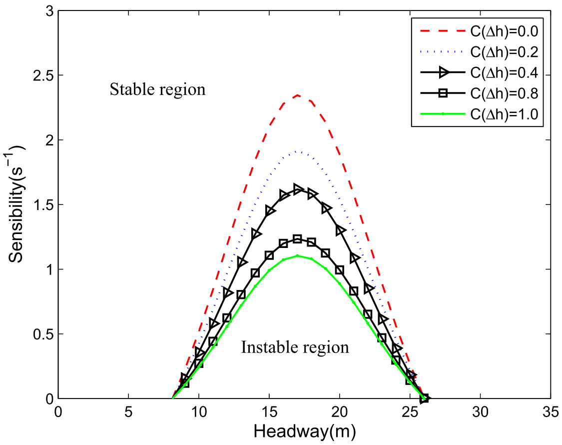


FIGURE 4. Neutral stability curves with different cooperative level $C(\Delta h)$ for lane-changing mode.

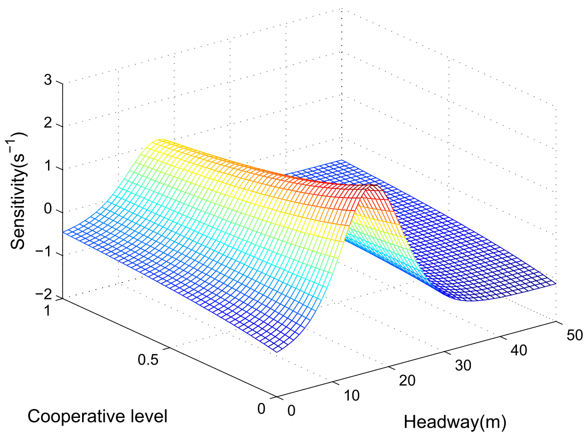


FIGURE 5. The critical diagram with different cooperative level $C(\Delta h)$ for lane-changing mode.

cooperative cars increases. Conversely, the instability region gets smaller and smaller.

The neutral stability curves with different cooperative level $C(\Delta h)$ for lane-changing mode is shown in FIGURE 4. FIGURE 5 is the corresponding three-dimensional view of FIGURE 4, which shows that continuous change process for cooperation level, driving sensibility and stability area. When $C(\Delta h) = 0$, the instable region is the largest which implies that the drivers is completely uncooperative and ungracious. When $C(\Delta h) = 1$, the stable region is the largest which means the result of complete cooperation and humility on the part of the driver. From FIGURE 4 and 5, it is effectively revealed that the higher driver's cooperative level is, the larger the stable region is.

IV. NUMERICAL SIMULATION

In this section, we carry out numerical experiments to demonstrate traffic behavior of the cooperative car-following model considering the stimulatory effect of lane-changing types.

A. PARAMETER SETTING

We set the parameters required for numerical simulation, as shown in TABLE 1.

TABLE 1. The values of parameters for the proposed models.

Parameter	Value	Unit
V_1	6.75	m/s
V_2	7.91	m/s
C_1	0.13	m^{-1}
C_2	1.57	-
l_c	5	m
k_l	0.35	s^{-1}
λ_l	0.3	s^{-1}
v_{max}	15	m/s
N	11	-
L_s	500	m
h_c	7.5	m
t_{LC}	3	s
a_{max}	5	m/s^2

Assume that the vehicles move with the initial velocity $v_0 = 14.6m/s$ in the two-lane traffic flow system. In order to analyze the proposed model, β_1 and β_2 are set 0.8 and 0.2, respectively. c is 0, 0.8. The space between the preceding vehicle and following vehicle maintain at an acceptable safe distance.

B. DYNAMIC PERFORMANCE

1) STARTING PROCESS

In this part, we simulate the vehicle from stationary to starting process. 11 vehicles (including a guide vehicle and 10 other regular vehicles) is platooned on the road with uniform safe spacing $h_c = 7.5m$. These vehicles may be affected by adjacent lanes. When $t = 0s$, all vehicles will launch. The head vehicle begin to accelerate to max speed and other vehicles follow the head vehicle and accelerate until all vehicles are traveling with the optimal speed.

Through simulation comparison, the acceleration evolution profile and velocity evolution profile of starting process for FVD model, cooperative driving of non-lane mode and cooperative driving of lane-changing mode are shown in FIGURE 6 and 7. As shown FIGURE 6 and 7 (a), (b) and (c), the acceleration amplitude of all vehicles of FVD model is about 3m/s^2 and the velocity response of the following vehicles is the slowest, followed by cooperative driving of non-lane mode. FIGURE 6 and 7 demonstrates that cooperative driving for lane-changing mode is able to best respond to changes of the preceding vehicles and owns minimum acceleration amplitude compared to the FVD model and cooperative driving for non-lane mode. In a sense, the proposed model contributes to enhancing the traffic efficiency.

2) STOPPING PROCESS

The stopping process is studied and set up as follows. All vehicles decelerate from maximum speed (optimal speed) to stop. At the time $t = 0s$, the 11 vehicles travel with

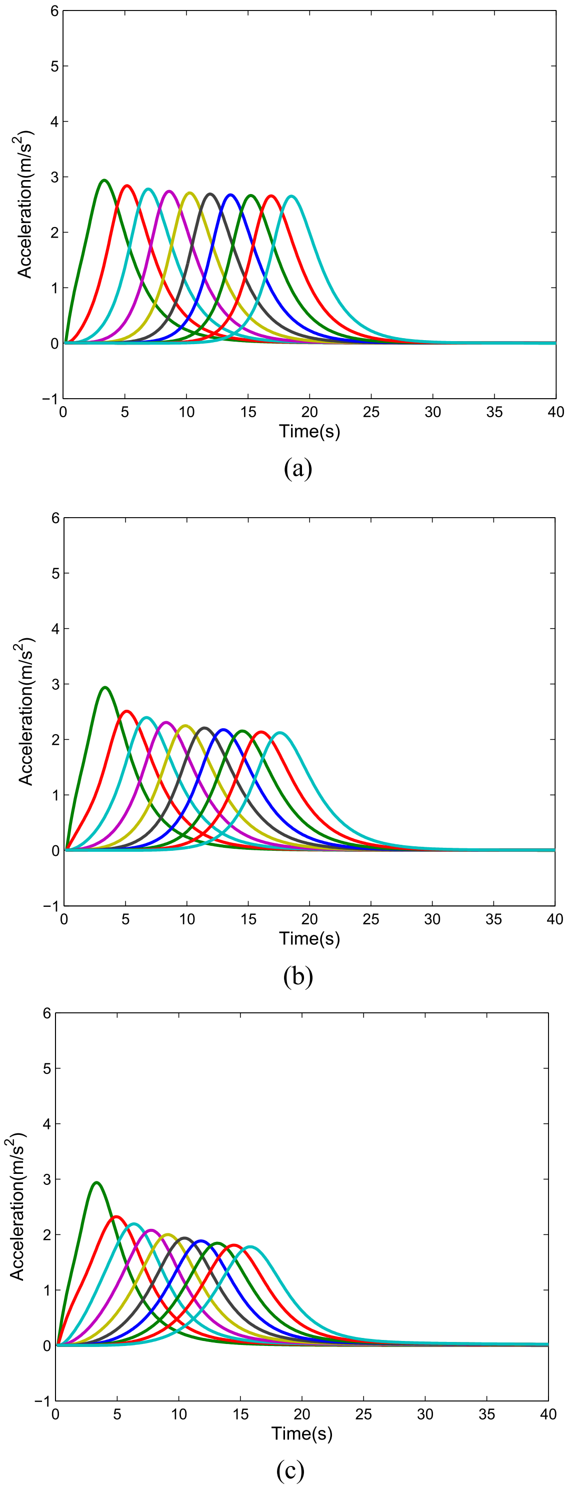


FIGURE 6. **The acceleration evolution profile of starting process (a) FVD model, (b) cooperative driving for non-lane mode(n=3,C(h)=0), (c) cooperative driving for lane-changing mode(n=3,C(h)=0.8).**

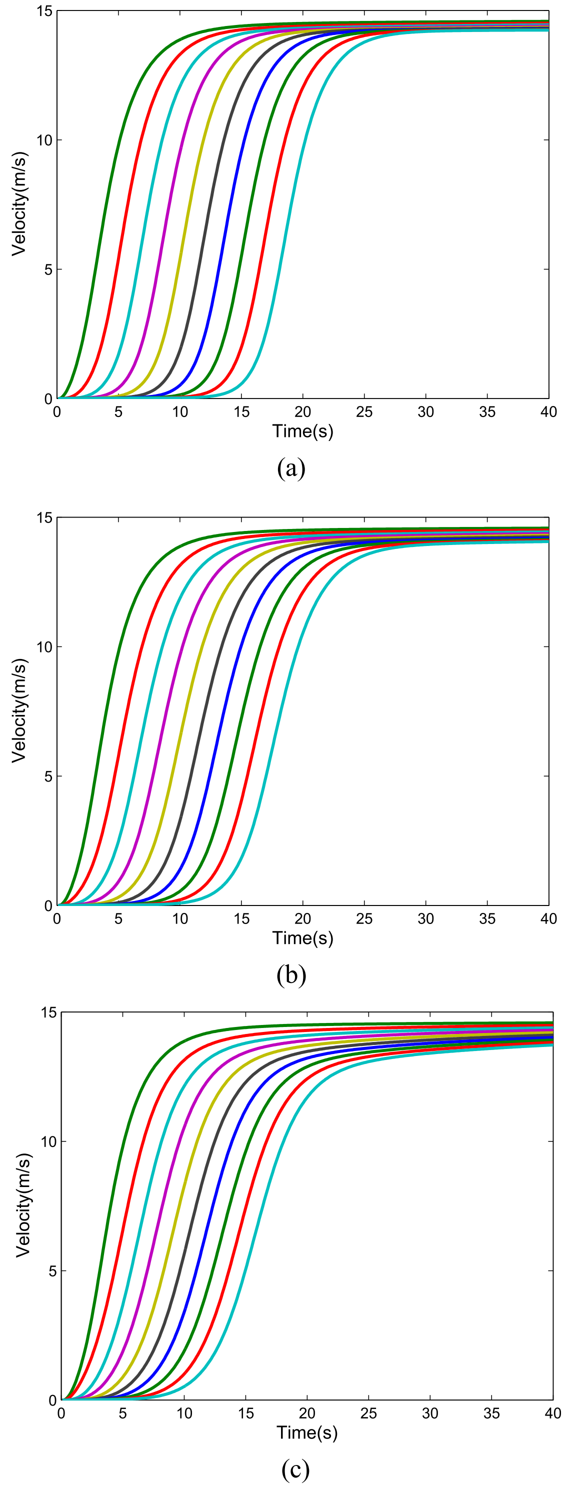
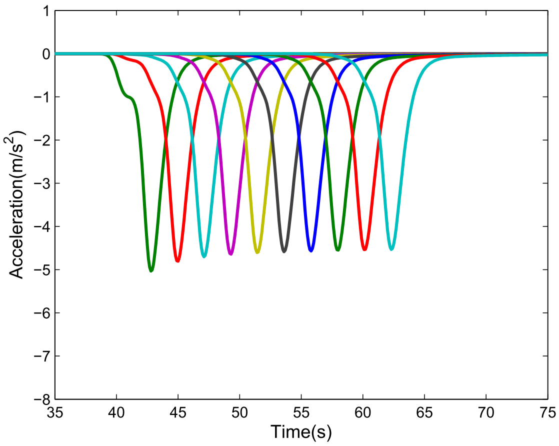


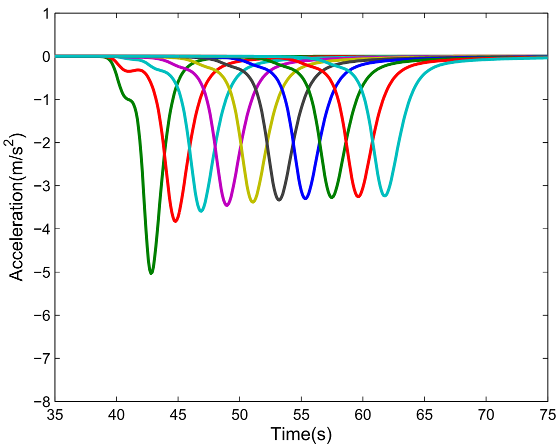
FIGURE 7. **The velocity evolution profile of starting process (a) FVD model, (b) cooperative driving for non-lane mode(n=3,C(h)=0), (c) cooperative driving for lane-changing mode(n=3,C(h)=0.8).**

the same speed and the same spacing. For purposes of comparison, we use the same parameters for FVD model, cooperative driving of non-lane mode and cooperative driving of lane-changing mode. The simulation of the stopping process is shown in FIGURES 8 and 9.

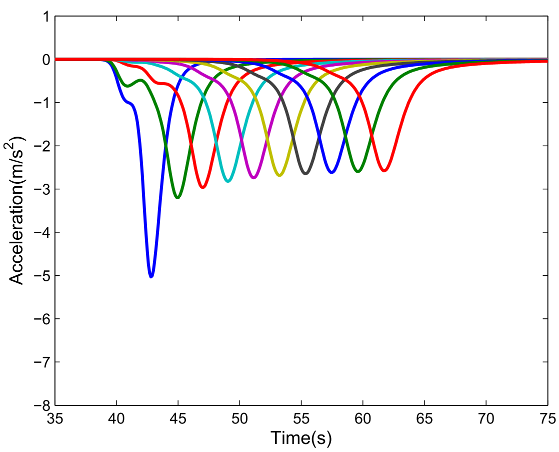
We find that the deceleration amplitude of FVD model is greater than -4m/s^2 and the velocity response of the following vehicles is the slowest. The deceleration amplitude of cooperative driving of non-lane mode is greater than -3m/s^2. However, Cooperative driving for lane-changing



(a)

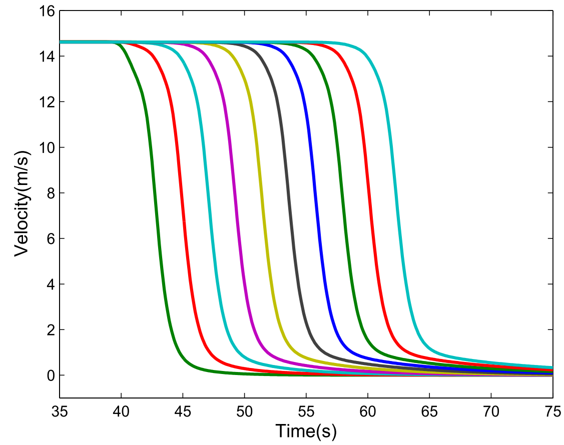


(b)

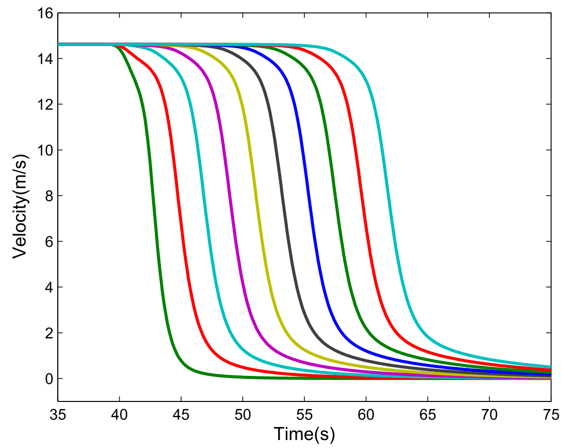


(c)

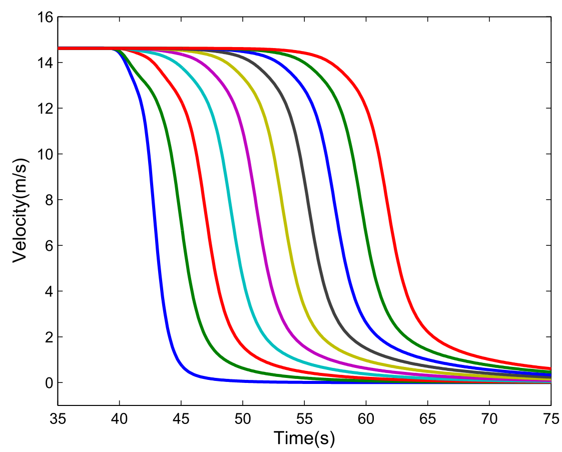
FIGURE 8. The deceleration evolution profile of stopping process (a) FVD model, (b) cooperative driving for non-lane mode(n=3,C(h)=0), (c) cooperative driving for lane-changing mode(n=3,C(h)=0.8).



(a)



(b)



(c)

FIGURE 9. The velocity evolution profile of stopping process (a) FVD model, (b) cooperative driving for non-lane mode(n=3,C(h)=0), (c) cooperative driving for lane-changing mode(n=3,C(h)=0.8).

mode is able to best respond the change and owns minimum deceleration amplitude less than -3m/s^2. The velocity evolution profile of cooperative driving for lane-changing mode is the smoothest compared with the other two modes. So, the proposed model contributes to improving traffic safety and comfort.

3) PERTURBATION PROCESS

In actual traffic, emergency braking often occurs because of the perturbation of uncertain road factors, which could lead to traffic accidents and uncomfortable driving. Let's consider a traffic situation like this. A queue of 6 vehicles is moving at a desired speed and constant spacing on the road. The guiding

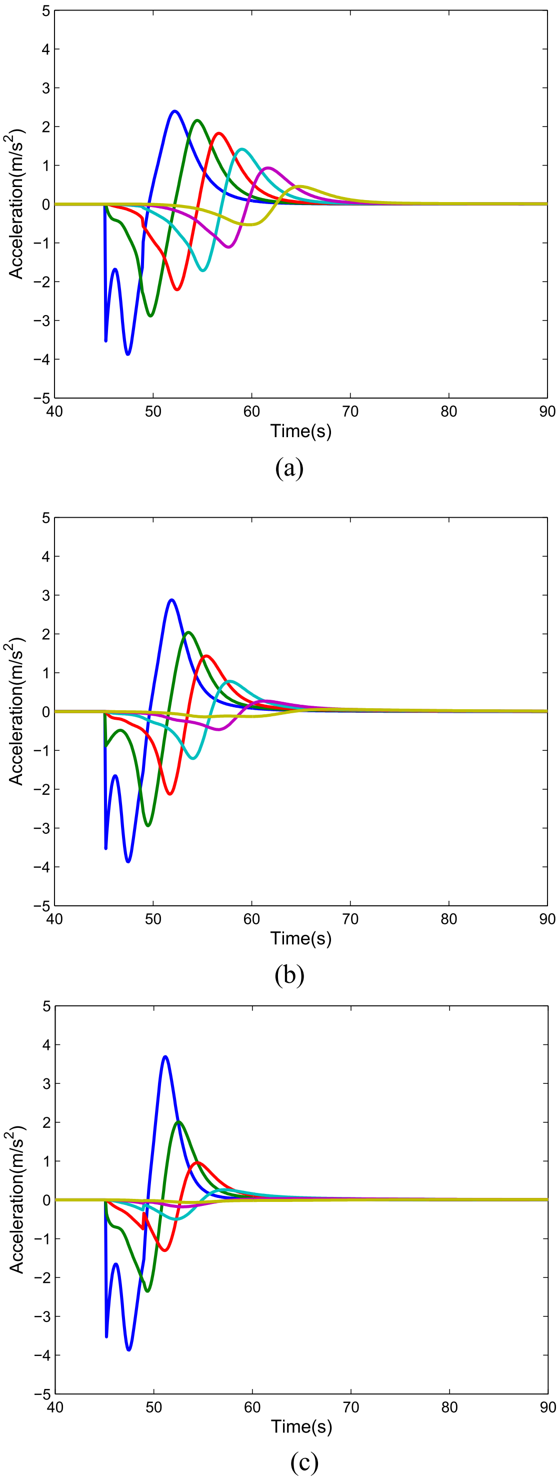


FIGURE 10. The acceleration evolution profile of perturbation process (a) FVD model, (b) cooperative driving for non-lane mode (n=3, C(h)=0), (c) cooperative driving for lane-changing mode (n=3,C(h)=0.8).

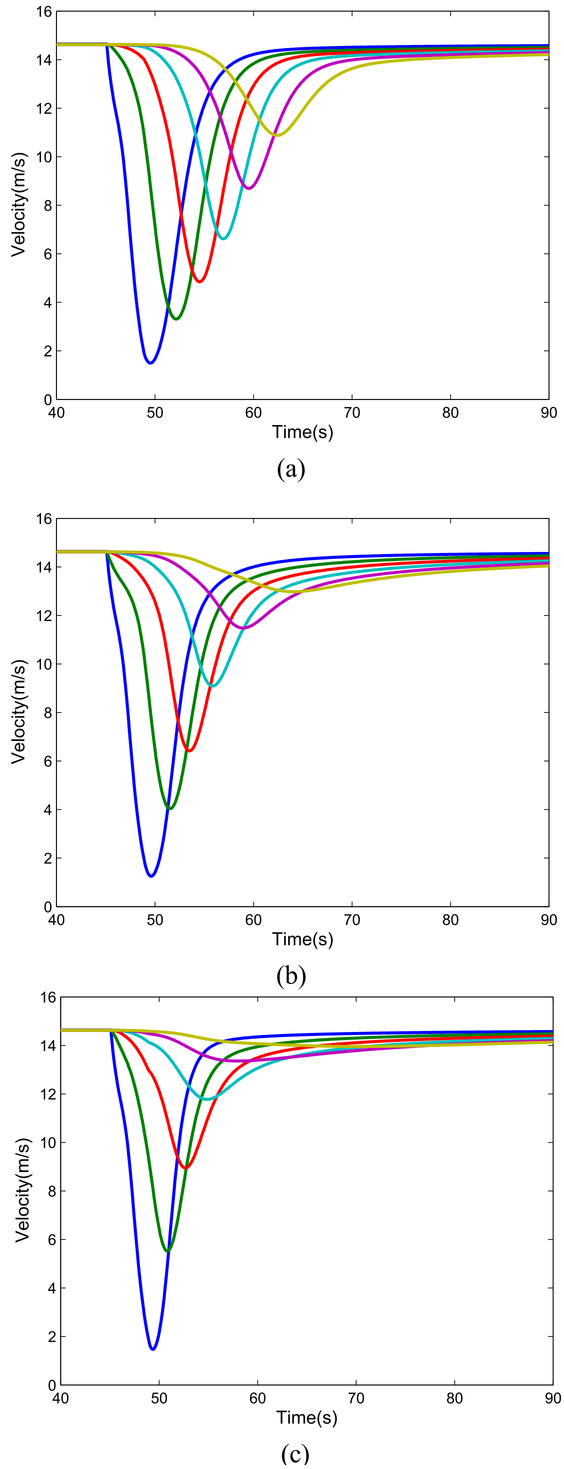


FIGURE 11. The velocity evolution profile of perturbation process (a) FVD model, (b) cooperative driving for non-lane mode(n=3,C(h)=0), (c) cooperative driving for lane-changing mode(n=3,C(h)=0.8).

vehicle is suddenly interfered with due to the lane change of other vehicles.

For safety, the guiding vehicle brakes for 2 seconds at the time of 45 seconds and then accelerates to desired velocity. The acceleration evolution and velocity evolution of perturbation process is shown in FIGURE 10 and 11. We can observe that all vehicles accelerate first and then accelerate. The guiding vehicle has a minimum deceleration around 48 seconds and a minimum speed around 50 seconds. Shown in FIGURE 10 and 11 (a, b, c), we observe that the vehicles of

FVD mode react more slowly and are affected by the deceleration of the preceding vehicles. Cooperative driving for non-lane mode is effectively improved, but the acceleration/ deceleration amplitudes of following vehicle is still relatively large. The effect of cooperative driving for lane-changing mode is the most obvious. The amplitude of acceleration and deceleration is obviously alleviated, and the 5th and 6th vehicles are not affected by the disturbance of preceding vehicle. According to the analysis results, the performance of cooperative driving for lane-changing mode is better than the FVD model, AD-CF model and cooperative driving for non-lane mode when vehicle platoon occurs sudden brake because of perturbation.

V. CONCLUSION

We investigate the cooperative car-following model considering the stimulatory effect of lane-changing types in this paper. Free lane change, forced lane change, and cooperative lane change three driving behavior are discussed. In order to reveal the internal mechanism of cooperative lane change behavior. The stability criterion of the proposed model is obtained via linear stability theory. According to the stability criterion, we obtain the stability range. Cooperative level contributes to expanding the range of stability. Simulation results indicate that cooperative driving of lane-changing mode has larger stable region compared to FVD model, cooperative driving for non-lane mode. Furthermore, the higher the level of cooperation, the greater the area of stability. Meanwhile, we analyze starting process, stopping process and perturbation process for FVD model, cooperative driving for non-lane mode and cooperative driving for lane-changing mode. The simulation experiments indicate that cooperative driving for lane-changing mode is be effectively promoted to traffic safety and comfort. In future works, we will research more detailed and practical lane changing rules for cooperative lane-changing mode.

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(Dong Chen and Xiaoming Xiong contributed equally to this work.)

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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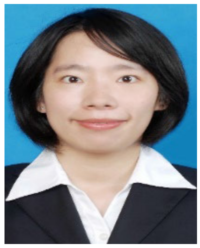

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