IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received June 7, 2021, accepted June 19, 2021, date of publication June 24, 2021, date of current version July 1, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3092071

Observer-Based Fault Tolerant Control for a Class of Nonlinear Systems via Filter and Neural Network

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This work was supported in part by the Ministry of Education Humanities and Social Sciences Planning Fund Project under Grant SN18YJA630076, in part by the Research on Risk Warning and Control of Civil Aviation Employees' Unsafe Behavior based on Big Data, in part by the Program for Scientific Research Start-Up Funds of Guangdong Ocean University under Grant R20037, and in part by the ''Internet plus'' Double Seed Sugarcane Mechanized Precision Planting Technology Research and Demonstration under Grant 2020A03008.

ABSTRACT A filter and neural network (NN) based fault tolerant control (FTC) strategy is developed for a family of nonlinear systems expressed in strict feedback form in the event of unknown system dynamics and actuator failures. Specifically, adaptive neural network (ANN) is first utilized to facilitate the state observer design such that unmeasurable system states can be obtained. Note that ANN is only used when designing state observer instead of being used when designing controller. In our method, filter technique is introduced to construct virtual control inputs, which can not only reduce the adverse effects caused by ANN approximation errors and state estimation errors, but also deal with the expansion problem of the differential terms. Moreover, the fault tolerant tracking controller is designed by combining backstepping technique with the proposed NN with a novel weight updating law that is different from the above ANN. Theoretical analysis and simulation results demonstrate that the proposed FTC strategy can ensure that the tracking error converges to a small region of zero when there exist actuator faults and unknown system dynamics.

INDEX TERMS Fault tolerant control, filters, neural networks, backstepping technique.

I. INTRODUCTION

Control design for nonlinear systems has long been an active topic, and has gained considerable attention. Meanwhile, the existence of unknown system dynamics and actuator faults make the control design become more challenging. In the past decades, various control strategies [1]–[4] have been reported to deal with the control problem for nonlinear systems.

On the one hand, to accomplish control mission well, study effective and timely response control strategies is significant. Up to now, plenty of results have been developed. To mention a few, in [5], second-order multi-aircraft systems were investigated, and an anti-disturbance sliding mode controller was presented to ensure the convergence of the closed-loop system with unknown disturbances. In [6], a disturbance rejection tracking control scheme was proposed for an airplane in the presence of external time-dependent disturbances and internal uncertainties by utilizing a robust feedback control policy such that the controlled system is robust to the disturbances and uncertainties. In [7], the tracking control problem for vertical takeoff and landing aircraft with unknown disturbances was dealt with by designing a control method combined nonlinear velocity observer with fuzzy adaptive observer. The effectiveness of such method was verified via stability analysis and simulation results. In [8], a blended wing body aircraft with mismatched disturbances and output constraints was studied, in order to handle with the disturbances, a fixed time non-recursive observer was designed. Then, barrier Lyapunov function and nonsingular fast terminal sliding mode technique were used to design controller. Other similar results can refer to literatures [6], [9]–[12].

On the other hand, research on ensuring safety and reliability of equipments is extremely important. As time goes by, plants may inevitably be subjected to some failures caused

The associate editor coordinating the review of this manuscript and approving it for publication was Heng Wang.

by aging of components. Hence, fault tolerant control, which can accommodate the change of system models caused by failures, i.e., component faults, actuator faults and sensor faults, has been widely studied and developed. FTC methods are generally divided into passive FTC and active FTC. The main idea of passive FTC is to design a robust control strategy without changing the controller structure, such that the controlled system is insensitive to faults. While, the active FTC can adjust the parameters of the controller in the light of failures, even change the controller structure. Due to active FTC is more flexible to guarantee the control performance of closed-loop system when faults appear, it has attracted more attention. In [13], the FTC problem for overactuated aircraft with the unknown actuator failures and uncertainties was explored, a novel estimator-based sliding mode control scheme was investigated such that the controlled system is stable when there exist actuator faults and uncertainties. In [14], an active FTC strategy was proposed to deal with the actuator faults occurring to spacecraft attitude control system, such method can estimate and compensate faults timely and accurately. In [15], observers were designed to estimate the soft actuator faults occurring to VTOL aerial vehicles. Besides, experiments were conducted to verify the effectiveness of the proposed FTC method. Other fault tolerant control strategies can refer to [16]–[20].

It can be found that the design of an observer that can obtain fault information timely and accurately is important in ensuring a satisfactory fault-tolerant performance. Owing to the excellent capability in approximating the continuous functions, NN is regarded as an effective tool in handling with malfunctions. In [21], a control method combined robust technique with ANN was proposed for aircraft, such method can guarantee that the output tracking error converges to a small region of zero. In [22], an ANN-based control strategy was proposed for an airplane in the presence of parameter uncertainties and multi-disturbances. Such method utilized ANN to approximate the unknown functions appearing in systems, dynamic surface control was introduced to deal with the expansion of the differential terms in back-stepping technique. In [23], a rotary-wing unmanned aircraft was studied, neural network was introduced to cope with the external disturbances and measurement errors such that the designed ANN-based controller is robust to wind disturbances. It can be concluded that one similarity of these methods is that Lyapunov function is used to design ANN weight updating laws. However, such method only can ensure the convergence of NN weight. Hence, how to deal with the NN approximation error and further enhance the control performance are worth studying.

In this work, a filter and neural network based FTC strategy is proposed for a class of nonlinear systems with unknown dynamics, actuator faults and unmeasurable system states. First of all, ANNs are utilized to approximate the system unknown functions, and state observer is designed to estimate unmeasurable system states. Then, with the aid of back-stepping technique, filter technique and a NN with novel weight updating law, the fault tolerant controller is designed. The main contributions of this study are outlined as follows

1) Compared with literatures [24]–[26], ANN technique is only used to approximate unknown functions when designing state observer instead of being used to design controller in our method. Moreover, the virtual control inputs are designed by resorting to filter technique. The merits are that
   a) controller design process can be simplified;
   b) ANN approximation errors and state estimation errors can be compensated. Thus, the tracking errors can be further reduced;
   c) the problem of differential explosion caused by repeated differentiations is successfully overcome.
2) In contrast to ANN used in [27]–[30], a NN with novel weight updating law is developed. Such method can update the NN weight to its corresponding satisfactory value by minimizing a cost function, which can approximate and compensate the unknown functions and actuator faults more accurately. Thus, the control performance can be further enhanced.

This paper is organized as follows. The problem statement and some preliminaries are presented in Section II. Control design and stability analysis are given in Section III. Simulation results and conclusions are provided in Sections IV and V, respectively.

II. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following strict-feedback nonlinear systems described by

$$\begin{cases} \dot{x}_1(t) = g_1\left(x_1(t)\right)x_2(t) + f_1\left(x_1(t)\right) \\ \dot{x}_2(t) = g_2\left(\bar{x}_2(t)\right)x_3(t) + f_2\left(\bar{x}_2(t)\right) \\ \quad\vdots \\ \dot{x}_{n-1}(t) = g_{n-1}\left(\bar{x}_{n-1}(t)\right)x_n(t) + f_{n-1}\left(\bar{x}_{n-1}(t)\right) \\ \dot{x}_n(t) = g_n\left(\bar{x}_n(t)\right)u(t) + f_n\left(\bar{x}_n(t)\right) \\ y(t) = x_1(t), \end{cases} \tag{1}$$

with the system state vector $\bar{x}_i(t) = [x_1(t), x_2(t), \cdots, x_i(t)]^T$, $i = 1, 2, \cdots, n$, where n is a positive constant. $f_i\left(\bar{x}_i(t)\right) \in R$ and $g_i\left(\bar{x}_i(t)\right) \in R$ are nonlinear functions. Control input is $u(t) \in R$, while $y(t) \in R$ is the system output. System states, i.e., $x_2(t)$, $x_3(t)$, $\cdots$, $x_n(t)$, are unmeasurable except the system output y.

Assumption 1 [31]: Assume that the nonlinear function $g_i\left(\bar{x}_i(t)\right)$ is smooth and known. Moreover, if it is equal to zero, the studied system would be loss of control. Then, it is reasonable to assume that it is nonzero.

Assumption 2 [32]: It is assumed that the unknown drift function $f_i\left(\bar{x}_i(t)\right)$ satisfies Lipschitz condition.

Assumption 3 [31]: Assume that the system output reference $y_d(t)$ and its derivatives $\dot{y}_d(t)$ are known and bounded.

Actuator failure studied in this work is modeled as follows

$$u^o(t) = \beta(t)u(t) + \bar{u}, 0 < \beta(t) \le 1, \quad \forall t \ge 0, \tag{2}$$

where the actuator input is represented by $u(t)$, while $u^o(t)$ denotes its output. $\beta(t)$ denotes the partial loss of effectiveness fault, $\bar{u}$ represents the float fault. Let the moment when the fault occurs be denoted by t^f. When $t < t^f$, the actuator is healthy, which implies $\beta(t) = 1$ and $\bar{u} = 0$. When $t \geq t^f$, the fault occurs to the actuator, then the variable $0 < \beta(t) < 1$ or the variable $\bar{u} > 0$. In addition, since the studied system is a single input one, when $\beta(t) = 0$, the output of actuator is $u^o(t) = \bar{u}$, which denotes a stuck fault. In such case, the faults cannot be dealt with and thus, $\beta(t) = 0$ is not taken into consideration in this work.

When actuator fault is taken into consideration, the system dynamics can be rewritten as

$$\begin{cases} \dot{x}_1(t) = g_1\left(x_1(t)\right) x_2(t) + f_1\left(x_1(t)\right) \\ \dot{x}_2(t) = g_2\left(\bar{x}_2(t)\right) x_3(t) + f_2\left(\bar{x}_2(t)\right) \\ \quad \vdots \\ \dot{x}_{n-1}(t) = g_{n-1}\left(\bar{x}_{n-1}(t)\right) x_n(t) + f_{n-1}\left(\bar{x}_{n-1}(t)\right) \\ \dot{x}_n(t) = g_n\left(\bar{x}_n(t)\right)\left(\beta(t)u(t) + \bar{u}\right) + f_n\left(\bar{x}_n(t)\right) \\ y(t) = x_1(t), \end{cases} \tag{3}$$

To facilitate the control design, define $\varpi(\bar{x}_n, u) = g_n(\bar{x}_n)(\beta(t)u + \bar{u}) - g_n(\bar{x}_n)u$. Dynamics (3) can be rewritten as

$$\begin{cases} \dot{x}_1(t) = g_1\left(x_1(t)\right) x_2(t) + f_1\left(x_1(t)\right) \\ \dot{x}_2(t) = g_2\left(\bar{x}_2(t)\right) x_3(t) + f_2\left(\bar{x}_2(t)\right) \\ \quad \vdots \\ \dot{x}_{n-1}(t) = g_{n-1}\left(\bar{x}_{n-1}(t)\right) x_n(t) + f_{n-1}\left(\bar{x}_{n-1}(t)\right) \\ \dot{x}_n(t) = g_n\left(\bar{x}_n(t)\right) u(t) + f_n\left(\bar{x}_n(t)\right) + \varpi(\bar{x}_n, u) \\ y(t) = x_1(t), \end{cases} \tag{4}$$

Lemma 1 [33]: There exists a NN that can be capable of approximating any continuous nonlinear function $\mathcal{F}\left(\bar{\zeta}\right)$ within an arbitrary small error τ, which is bounded by a positive real constant $\bar{\tau}$ described as the following form

$$\left|\mathcal{F}(\bar{\zeta}) - W^{*T}\theta(\bar{\zeta})\right| \leq \bar{\tau}, \tag{5}$$

where $\bar{\zeta} = [\zeta_1, \zeta_2, \cdots, \zeta_p]^T$ is the NN input vector, $W^{*T}\theta(\bar{\zeta})$ is the ideal output of NN, $W^* = [w_1{}^*, \cdots, w_q{}^*]^T$ is the optimal weight vector, and described by

$$W^* = \arg\min_{w \in \Omega_w} \left\{ \sup_{\bar{\zeta} \in \Omega_{\bar{\zeta}}} \left|\mathcal{F}(\bar{\zeta}) - W^T\theta(\bar{\zeta})\right| \right\},$$

where $\Omega_{M_w} = \{W : \|W\| \leq M\}$ is a compact set, M is a positive constant. The basis function vector is denoted by $\theta(\bar{\zeta}) = [\vartheta_1(\bar{\zeta}), \vartheta_2(\bar{\zeta}), \cdots, \vartheta_q(\bar{\zeta})]^T$, each element is chosen as Gaussian function

$$\vartheta_j(\bar{\zeta}) = \exp\left(-\frac{\|\bar{\zeta} - c_j\|^2}{2b_j^2}\right), \quad j = 1, 2, \ldots, q,$$

where c_j and b_j are the center and width of NN. Then, we have

$$\mathcal{F}(\bar{\zeta}) = W^{*T}\theta(\bar{\zeta}) + \tau.$$

Lemma 2 [34]: For two positive constants a and b, there exist two constants p and q, when they satisfy $p > 1$, and $\frac{1}{p} + \frac{1}{q} = 1$, the following inequality holds

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}$$

Control objective: The control objective of this work is to design a fault-tolerant controller for the nonlinear system (1) with unknown system states, unknown drift functions and actuator faults such that output tracking error can converge to a small neighborhood of the origin.

III. NEURAL NETWORK-BASED FAULT-TOLERANT CONTROL DESIGN

In this section, we develop a FTC strategy for nonlinear system (1) with unknown drift functions, unmeasurable system states and actuator faults. Specifically, an ANN-based state observer is described in the first subsection, while the detailed design process of fault tolerant controller is given in the second subsection. The control block diagram to describe the proposed method is shown as Fig.1.

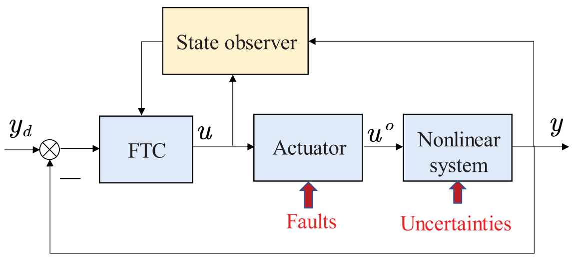

FIGURE 1. Block diagram of control system.

A. ADAPTIVE NEURAL NETWORK-BASED STATE OBSERVER

Since the states of system (1) are completely unknown, an ANN-based observer is designed to estimate the unknown states to facilitate the fault tolerant control design. According to Lemma 1, one can obtain

$$f_i\left(\hat{\bar{x}}_i\right) = \hat{W}_i^T\theta_i\left(\hat{\bar{x}}_i\right) + \epsilon_i \tag{6}$$

$$f_n\left(\hat{\bar{x}}_n(t)\right) + \varpi(\hat{\bar{x}}_n, u) = \hat{W}_n^T\theta_n\left(\hat{\bar{x}}_n\right) + \epsilon_n \tag{7}$$

where $\hat{\bar{x}}_j$, $j = 1, 2, \ldots, n$ is the state estimation, $\hat{W}_j$ is the weight approximation, ϵ_j is approximation error. Rewrite system (4) as the following form

$$\begin{cases} \dot{x} = \mathcal{A}x + \mathcal{K}y + \mathcal{B}_n g_n(\bar{x}_n)u + \sum_{i=1}^{n-1} \mathcal{B}_i\mathcal{G}_i\left(\bar{x}_{i+1}\right) \\ + \sum_{i=1}^{n} \mathcal{B}_i\hat{W}_i^T\theta_i\left(\hat{\bar{x}}_i\right) + \sum_{i=1}^{n} \mathcal{B}_i\Delta_i \\ y = \mathcal{C}^T x, \end{cases} \tag{8}$$

where

$$\mathcal{A} = \begin{bmatrix} -k_1 & & & \\ \vdots & & I_{n-1} & \\ -k_n & 0 & \cdots & 0 \end{bmatrix}$$
$$\mathcal{K} = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}$$
$$\mathcal{B}_i = [\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0, \ldots, 0}_{n-i}]^T$$
$$\mathcal{B}_n = [\underbrace{0, \ldots, 0}_{n-1}, 1]^T$$
$$\mathcal{C} = [1, \underbrace{0, \ldots, 0}_{n-1}]^T$$
$$x = [x_1, x_2, \ldots, x_n]^T$$
$$\Delta_i = f_i\left(\bar{x}_i\right) - \hat{W}_i^T \theta_i \left(\hat{\bar{x}}_i\right)$$
$$\Delta_n = f_n\left(\bar{x}_n(t)\right) + \varpi(\bar{x}_n, u) - \hat{W}_n^T \theta_n \left(\hat{\bar{x}}_n\right)$$
$$\mathcal{G}_i\left(\bar{x}_{i+1}\right) = g_i(\bar{x}_i)x_{i+1} - x_{i+1}$$

where the variable k_i, $i = 1, 2, \ldots, n$ is the parameter of the state observer, which should be designed to guarantee that the matrix $\mathcal{A}$ be a strict Hurwitz. Thus, for the matrix $\mathcal{Q} = \mathcal{Q}^T > 0$, there exists a positive matrix $\mathcal{P} = \mathcal{P}^T$ satisfying the following equation

$$\mathcal{A}^T \mathcal{P} + \mathcal{P}\mathcal{A} = -\mathcal{Q}. \tag{9}$$

The state observer is designed as follows

$$\begin{cases} \dot{\hat{x}} = \mathcal{A}\hat{x} + \mathcal{K}y + \mathcal{B}_n g_n(\hat{\bar{x}}_n)u(t) + \sum_{i=1}^{n-1} \mathcal{B}_i \mathcal{G}_i\left(\hat{\bar{x}}_{i+1}\right) \\ + \sum_{i=1}^{n} \mathcal{B}_i \hat{W}_i^T \theta_i \left(\hat{\bar{x}}_i\right) \\ y = \mathcal{C}^T x. \end{cases} \tag{10}$$

where $\hat{x}$ is the estimation of state vector x.

Remark 1: When designing state observer, the unknown functions and actuator faults are regarded as a lumped disturbance. Then, a NN is used to approximate it. Such approach is reasonable due to that the lumped disturbance is a function of system states.

The following Theorem is given to prove that the designed ANN-based state observer is stable and can estimate system states within a small bounded error.

Theorem 1: For the strick-feedback nonlinear system (1) in the presence of actuator fault modeled as equation (2). When an ANN-based state observer is designed as (10), the state estimation $\hat{x}$ can be guaranteed to converge to actual state x within a small bounded error.

Proof: Let the error between system state x and its estimation $\hat{x}$ be denoted by $\tilde{x} = x - \hat{x}$. Accordingly, the estimation error dynamic can be described as

$$\dot{\tilde{x}} = \mathcal{A}\tilde{x} + \sum_{i=1}^{n} \mathcal{B}_i \Delta_i + \mathcal{B}_n(g_n(\bar{x}_n)u(t) - g_n(\hat{\bar{x}}_n)u(t)) + \sum_{i=1}^{n-1} \mathcal{B}_i(\mathcal{G}_i(\bar{x}_{i+1}) - \mathcal{G}_i(\hat{\bar{x}}_{i+1})) \tag{11}$$

Choose $\mathcal{V}_0 = \tilde{x}^T \mathcal{P} \tilde{x}$ as a Lyapunov function candidate. The time derivative of $\mathcal{V}_0$ is calculated as

$$\dot{\mathcal{V}}_0 \leqslant \tilde{x}^T \mathcal{P} \dot{\tilde{x}} + \dot{\tilde{x}}^T \mathcal{P} \tilde{x}. \tag{12}$$

Substitute equation (11) into (12) yields

$$\begin{aligned} \dot{\mathcal{V}}_0 \leqslant\ & \tilde{x}^T \mathcal{P}(\mathcal{A}\tilde{x} + \sum_{i=1}^{n} \mathcal{B}_i \Delta_i + \mathcal{B}_n(g_n(\bar{x}_n)u(t) - g_n(\hat{\bar{x}}_n)u(t)) \\ & + \sum_{i=1}^{n-1} \mathcal{B}_i(\mathcal{G}_i(\bar{x}_{i+1}) - \mathcal{G}_i(\hat{\bar{x}}_{i+1}))) + (\mathcal{A}\tilde{x} + \sum_{i=1}^{n} \mathcal{B}_i \Delta_i \\ & + \mathcal{B}_n(g_n(\bar{x}_n)u(t) - g_n(\hat{\bar{x}}_n)u(t)) \\ & + \sum_{i=1}^{n-1} \mathcal{B}_i(\mathcal{G}_i(\bar{x}_{i+1}) - \mathcal{G}_i(\hat{\bar{x}}_{i+1})))^T \mathcal{P}\tilde{x} \\ \leqslant\ & \tilde{x}^T(\mathcal{P}\mathcal{A} + \mathcal{A}^T\mathcal{P})\tilde{x} + 2\tilde{x}^T \mathcal{P} \sum_{i=1}^{n} \mathcal{B}_i \Delta_i \\ & + 2\tilde{x}^T \mathcal{P}\mathcal{B}_n(g_n(\bar{x}_n)u(t) - g_n(\hat{\bar{x}}_n)u(t)) \\ & + 2\tilde{x}^T \mathcal{P} \sum_{i=1}^{n-1} \mathcal{B}_i(\mathcal{G}_i(\bar{x}_{i+1}) - \mathcal{G}_i(\hat{\bar{x}}_{i+1})) \end{aligned} \tag{13}$$

For the purpose of simplicity, let $\psi_i(\bar{x}_i) = f_i(\bar{x}_i)$, $i = 1, 2, \ldots, n-1$, $\psi_n(\bar{x}_n) = f_n(\bar{x}_n(t)) + \varpi(\bar{x}_n, u)$ and $\Psi(\bar{x}_n, u(t)) = g_n(\bar{x}_n)u(t)$. Then, equation (13) can be rewritten as

$$\begin{aligned} \dot{\mathcal{V}}_0 \leqslant\ & \tilde{x}^T\left(\mathcal{P}\mathcal{A} + \mathcal{A}^T\mathcal{P}\right)\tilde{x} + 2\tilde{x}^T \mathcal{P} \sum_{i=1}^{n} \mathcal{B}_i\left(\psi_i(\bar{x}_i) - \psi_i(\hat{\bar{x}}_i)\right) \\ & + 2\tilde{x}^T \mathcal{P} \sum_{i=1}^{n} \mathcal{B}_i(\psi_i(\hat{\bar{x}}_i) - \hat{W}_i^T \theta_i(\hat{\bar{x}}_i)) \\ & + 2\tilde{x}^T \mathcal{P}\mathcal{B}_n(\Psi(\bar{x}_n, u(t)) - \Psi(\hat{\bar{x}}_n, u(t))) \\ & + 2\tilde{x}^T \mathcal{P} \sum_{i=1}^{n-1} \mathcal{B}_i(\mathcal{G}_i(\bar{x}_{i+1}) - \mathcal{G}_i(\hat{\bar{x}}_{i+1})). \end{aligned} \tag{14}$$

Assume that $\psi_i(\bar{x}_i)$ and $\Psi(\bar{x}_n, u(t))$ are Lipschitz with Lipschitz constants ℓ_{f_i} and ℓ_{g_n} respectively [32]. We have

$$|\psi_i\left(\bar{x}_i\right) - \psi_i\left(\hat{\bar{x}}_i\right)| \leq \ell_{f_i}\|\bar{x}_i - \hat{\bar{x}}_i\| \leq \ell_{f_i}\|\tilde{\bar{x}}\|$$
$$|\Psi(\bar{x}_n, u(t)) - \Psi(\hat{\bar{x}}_n, u(t))| \leq \ell_{g_n}\|\bar{x}_n - \hat{\bar{x}}_n\| \leq \ell_{g_n}\|\tilde{\bar{x}}\|$$
$$|\mathcal{G}_i(\bar{x}_{i+1}) - \mathcal{G}_i(\hat{\bar{x}}_{i+1})| \leq \ell_{\mathcal{G}_i}\|\bar{x}_{i+1} - \hat{\bar{x}}_{i+1}\| \leq \ell_{\mathcal{G}_i}\|\tilde{\bar{x}}\|$$

Then,

$$\begin{aligned} \dot{\mathcal{V}}_0 \leqslant\ & -\tilde{x}^T \mathcal{Q}\tilde{x} + 2\tilde{x}^T \mathcal{P} \sum_{i=1}^{n} \mathcal{B}_i \ell_{f_i}\left\|\tilde{\bar{x}}\right\| + 2\tilde{x}^T \mathcal{P} \sum_{i=1}^{n} \mathcal{B}_i \epsilon_i \\ & + 2\tilde{x}^T \mathcal{P}\mathcal{B}_n \ell_{g_n}\left\|\tilde{\bar{x}}\right\| + 2\tilde{x}^T \mathcal{P} \sum_{i=1}^{n-1} \mathcal{B}_i \ell_{\mathcal{G}_i}\left\|\tilde{\bar{x}}\right\| \end{aligned} \tag{15}$$

By resorting to Lemma 2, one can obtain

$$\tilde{x}^T\mathcal{P}\sum_{i=1}^{n}\mathcal{B}_i\ell_{f_i}\|\tilde{x}\| \leqslant \frac{1}{2}\|\tilde{x}\|^2+\frac{1}{2}\|\mathcal{P}\|^2\sum_{i=1}^{n}\ell_{f_i}^2\|\tilde{x}\|^2 \tag{16}$$

$$\tilde{x}^T\mathcal{P}\sum_{i=1}^{n}\mathcal{B}_i\epsilon_i \leqslant \frac{1}{2}\|\tilde{x}\|^2+\frac{1}{2}\|\mathcal{P}\|^2\sum_{i=1}^{n}\|\epsilon_i\|^2 \tag{17}$$

$$\tilde{x}^T\mathcal{P}\sum_{i=1}^{n-1}\mathcal{B}_i\ell_{\mathcal{G}_i}\|\tilde{x}\| \leqslant \frac{1}{2}\|\tilde{x}\|^2+\frac{1}{2}\|\mathcal{P}\|^2\sum_{i=1}^{n-1}\ell_{\mathcal{G}_i}^2\|\tilde{x}\|^2 \tag{18}$$

It follows immediately from (17)-(19) that

$$\begin{aligned}\dot{\mathcal{V}}_0 \leqslant & -\tilde{x}^T\mathcal{Q}\tilde{x}+\|\tilde{x}\|^2+\|\mathcal{P}\|^2\sum_{i=1}^{n}\ell_{f_i}^2\|\tilde{x}\|^2\\ &+\|\tilde{x}\|^2+\|\mathcal{P}\|^2\sum_{i=1}^{n}\|\epsilon_i\|^2\\ &+\|\tilde{x}\|^2+\|\mathcal{P}\|^2\ell_{g_n}^2\|\tilde{x}\|^2\\ &+\|\tilde{x}\|^2+\|\mathcal{P}\|^2\sum_{i=1}^{n-1}\ell_{\mathcal{G}_i}^2\|\tilde{x}\|^2\end{aligned} \tag{19}$$

Let $c_0 = \lambda_{\min}(\mathcal{Q}) - 4 - \|\mathcal{P}\|^2\sum_{i=1}^{n}\ell_{f_i}^2 - \|\mathcal{P}\|^2\ell_{g_n}^2 - \|\mathcal{P}\|^2\sum_{i=1}^{n-1}\ell_{\mathcal{G}_i}^2$ and $d_0 = \|\mathcal{P}\|^2\sum_{i=1}^{n}\|\epsilon_i\|^2$. According to Lyapunov stability theory, if $c_0 \geq 0$, the state estimation error $\tilde{x}(t)$ would converge to a compact, which can be arbitrary small by tuning the parameter k_i.

B. FAULT TOLERANT CONTROL DESIGN

In this subsection, the proposed fault tolerant control strategy for nonlinear system (1) is demonstrated. It should be stressed that the virtual control inputs are designed by resorting to filter technique, while, the real control input is designed by using a filter and a NN with novel weight updating law. The detailed design process is given as follows.

Step 1: In this step, the output of ANN used to approximate the unknown drift function f_1 is first designed. Next, with the help of a filter, the virtual control input v_1 is constructed. It must be pointed out that ANN is only used to design state observer instead of being used to design virtual control input v_1 proposed in our method.

Let the tracking errors be defined as

$$\xi_1 = y - y_d \tag{20}$$

$$\xi_2 = \hat{x}_2 - v_1, \tag{21}$$

where the desired signal y_d is pre-defined. Select a Lyapunov function candidate as

$$V_1 = \frac{1}{2}\xi_1^2 + \frac{1}{2\eta_1}\tilde{W}_1^T\tilde{W}_1, \tag{22}$$

where η_1 is a positive constant, which should be designed. $\tilde{W}_1$ denotes the weight approximation error between the optimal weight vector and its approximation, i.e., $\tilde{W}_1 = W_1^* - \hat{W}_1$. Take the derivative of V_1 yields

$$\dot{V}_1 = \xi_1\dot{\xi}_1 - \frac{1}{\eta_1}\tilde{W}_1^T\dot{\hat{W}}_1. \tag{23}$$

By recalling equations (4), (20) and (21), the time derivative of ξ_1 is immediately obtained as

$$\begin{aligned}\dot{\xi}_1 &= \dot{y} - \dot{y}_d\\ &= g_1x_2 + f_1 - \dot{y}_d\\ &= g_1\tilde{x}_2 + g_1\hat{x}_2 + f_1 - \dot{y}_d\\ &= g_1\tilde{x}_2 + g_1\xi_2 + g_1v_1 + f_1 - \dot{y}_d.\end{aligned} \tag{24}$$

Hence,

$$\dot{V}_1 = \xi_1(g_1\tilde{x}_2 + g_1\xi_2 + g_1v_1 + f_1 - \dot{y}_d) - \frac{1}{\eta_1}\tilde{W}_1^T\dot{\hat{W}}_1. \tag{25}$$

Let f_1 be approximated by utilizing ANN, based on Lemma 1, we have $f_1 = W_1^{*T}\theta_1(\hat{x}_1) + \tau_1$. Hence, $\dot{V}_1$ can be rewritten as

$$\begin{aligned}\dot{V}_1 = \xi_1(&g_1\tilde{x}_2 + g_1\xi_2 + g_1v_1 + W_1^{*T}\theta_1(\hat{x}_1)\\ &+\tau_1 - \dot{y}_d) - \frac{1}{\eta_1}\tilde{W}_1^T\dot{\hat{W}}_1.\end{aligned} \tag{26}$$

Design virtual control input v_1 as follows

$$v_1 = \frac{1}{g_1}\left(-\lambda_1\xi_1 - \hat{W}_1^T\theta_1(\hat{x}_1) + \dot{y}_d\right). \tag{27}$$

where λ_1 is a positive constant, which should be designed. Substitute v_1 into equation (26) yields

$$\begin{aligned}\dot{V}_1 &= \xi_1(g_1\tilde{x}_2 + g_1\xi_2 - \lambda_1\xi_1 - \hat{W}_1^T\theta_1(\hat{x}_1) + W_1^{*T}\theta_1(\hat{x}_1)\\ &\quad+\tau_1) - \frac{1}{\eta_1}\tilde{W}_1^T\dot{\hat{W}}_1\\ &= \xi_1(g_1\tilde{x}_2 + g_1\xi_2 - \lambda_1\xi_1 + \tilde{W}_1^T\theta_1(\hat{x}_1) + \tau_1)\\ &\quad-\frac{1}{\eta_1}\tilde{W}_1^T\dot{\hat{W}}_1\\ &= -\lambda_1\xi_1^2 + g_1\xi_1\tilde{x}_2 + g_1\xi_1\xi_2 + \xi_1\tilde{W}_1^T\theta_1(\hat{x}_1)\\ &\quad+\xi_1\tau_1 - \frac{1}{\eta_1}\tilde{W}_1^T\dot{\hat{W}}_1.\end{aligned} \tag{28}$$

Design the weight updating law $\dot{\hat{W}}_1$ as

$$\dot{\hat{W}}_1 = \eta_1\xi_1\theta_1(\hat{x}_1) - \lambda_{w_1}\hat{W}_1, \tag{29}$$

where $\lambda_{w_1} > 0$ should be designed. Consequently, we have

$$\begin{aligned}\dot{V}_1 &= -\lambda_1\xi_1^2 + g_1\xi_1\tilde{x}_2 + g_1\xi_1\xi_2 + \xi_1\tau_1 + \frac{\lambda_{w_1}}{\eta_1}\tilde{W}_1^T\hat{W}_1\\ &= -\lambda_1\xi_1^2 + g_1\xi_1\tilde{x}_2 + g_1\xi_1\xi_2 + \xi_1\tau_1\\ &\quad+\frac{\lambda_{w_1}}{\eta_1}\tilde{W}_1^T\left(W_1^* - \tilde{W}_1\right)\\ &= -\lambda_1\xi_1^2 - \frac{\lambda_{w_1}}{\eta_1}\tilde{W}_1^T\tilde{W}_1 + g_1\xi_1\tilde{x}_2 + g_1\xi_1\xi_2\\ &\quad+\xi_1\tau_1 + \frac{\lambda_{w_1}}{\eta_1}\tilde{W}_1^TW_1^*.\end{aligned} \tag{30}$$

By resorting to Lemma 2, one can obtain

$$\begin{aligned}\dot{V}_1 \leqslant & -\lambda_1\xi_1^2 - \frac{\lambda_{w_1}}{\eta_1}\tilde{W}_1^T\tilde{W}_1 + \frac{1}{2}\xi_1^2 + \frac{1}{2}|g_1\tilde{x}_2|^2\\ &+\frac{1}{2}\xi_1^2 + \frac{1}{2}|g_1\xi_2|^2 + \frac{1}{2}\xi_1^2 + \frac{1}{2}\tau_1^2\end{aligned}$$

$$
\begin{aligned}
&+\frac{\lambda_{w1}}{2\eta_1}\left\|\tilde{W}_1\right\|^2+\frac{\lambda_{w1}}{2\eta_1}\left\|W_1^*\right\|^2\\
&\leqslant-\left(\lambda_1-\frac{3}{2}\right)\xi_1^2-\frac{\lambda_{w1}}{2\eta_1}\left\|\tilde{W}_1\right\|^2+\frac{1}{2}\left|g_1\tilde{x}_2\right|^2\\
&+\frac{1}{2}\left|g_1\xi_2\right|^2+\frac{1}{2}\tau_1^2+\frac{\lambda_{w1}}{2\eta_1}\left\|W_1^*\right\|^2. \qquad (31)
\end{aligned}
$$

According to Lyapunov stability theory, when $\lambda_1 > \frac{3}{2}$, and the tracking error ξ_2 is ensured to be convergent, the tracking error ξ_1 and weight approximation error $\tilde{W}_1$ can converge to a small region of zero. Accordingly, the NN utilized to approximate f_1 is designed, and its output is $\hat{W}_1^T\theta_1(\hat{x}_1)$.

Remark 2: The NN output $\hat{W}_1^T\theta_1(\hat{x}_1)$ is only used to design state observer. The virtual control input v_1 designed as equation (27) is to help to prove the convergence of NN weight. Also, it is used as comparison to show that the virtual control input designed by using the proposed method, which will be specified in afterward content, can simplify the control design process and can result in a smaller tracking error.

Next, the virtual control input v_1 designed by using the proposed method is given as follows.

Design the following filter and let ξ_1 be its input, we have

$$\dot{\hat{\xi}}_1=p_1\left(\xi_1-\hat{\xi}_1\right). \qquad (32)$$

where $p_1 > 0$ is the filter parameter, which should be designed.

Let the errors $\xi_1-\hat{\xi}_1$ and $\dot{\xi}_1-\dot{\hat{\xi}}_1$ be denoted by $\tilde{\xi}_1$ and $\dot{\tilde{\xi}}_1$. It is easy to prove that these two errors are bounded. Here, we assume that the bounds of $\tilde{\xi}_1$ and $\dot{\tilde{\xi}}_1$ are $\bar{\xi}_1$ and $\bar{\dot{\xi}}_1$, respectively. We have the following equation

$$
\begin{aligned}
\dot{\hat{\xi}}_1&=\dot{\xi}_1-\dot{\tilde{\xi}}_1\\
&=g_1\tilde{x}_2+g_1\hat{x}_2+f_1-\dot{y}_d-\dot{\tilde{\xi}}_1. \qquad (33)
\end{aligned}
$$

Further, we have

$$\dot{\hat{\xi}}_1-g_1\hat{x}_2+\dot{\tilde{\xi}}_1=g_1\tilde{x}_2+f_1-\dot{y}_d. \qquad (34)$$

Since the convergence of NN weights has been proved in the above content, the Lyapunov function (23) is redesigned as

$$V_1=\frac{1}{2}\xi_1^2$$

Take derivative of V_1, and substitute (34) into it, we have

$$
\begin{aligned}
\dot{V}_1&=\xi_1\left(g_1\tilde{x}_2+f_1-\dot{y}_d+g_1\xi_2+g_1v_1\right)\\
&=\xi_1\left(\dot{\hat{\xi}}_1-g_1\hat{x}_2+\dot{\tilde{\xi}}_1+g_1\xi_2+g_1v_1\right) \qquad (35)
\end{aligned}
$$

Then, the virtual control input v_1 can be designed as

$$v_1=\frac{1}{g_1}\left(-\lambda_1\xi_1+g_1\hat{x}_2-\dot{\hat{\xi}}_1\right). \qquad (36)$$

Consequently,

$$
\begin{aligned}
\dot{V}_1&=\xi_1\left(\dot{\tilde{\xi}}_1+g_1\xi_2-\lambda_1\xi_1\right)\\
&=-\lambda_1\xi_1^2+g_1\xi_1\xi_2+\xi_1\dot{\tilde{\xi}}_1. \qquad (37)
\end{aligned}
$$

Since $\dot{\tilde{\xi}}_1$ is bounded by a positive constant $\bar{\dot{\xi}}_1$, by using Lemma 2, we have

$$\dot{V}_1\leqslant-\left(\lambda_1-1\right)\xi_1^2+\frac{1}{2}\left|g_1\xi_2\right|^2+\frac{1}{2}\bar{\dot{\xi}}_1^2. \qquad (38)$$

According to Lyapunov stability theory, when the control parameter $\lambda_1 > 1$ satisfies, and the boundedness of tracking error ξ_2 is ensured, the tracking error ξ_1 can converge to a small compact set.

Remark 3: In existing literatures, ANN output $\hat{W}_1^T\theta_1(\hat{x}_1)$ is used to design virtual control input shown as equation (27). It can be seen from equation (31) that such method cannot compensate the NN ideal approximation error τ_1 and state estimation error $\tilde{x}_2$, which would lead to larger control efforts. While, the virtual control input designed by using the proposed method is shown as equation (36). Compared to the equation (31), it can be found from equation (38) that some positive terms are eliminated and thus, the tracking errors can be further reduced.

Step i: Similar to step 1, this step details the design of ANN weight updating law and the virtual control input v_i.

Let the tracking errors be defined as

$$
\begin{aligned}
\xi_i&=\hat{x}_i-v_{i-1} \qquad (39)\\
\xi_{i+1}&=\hat{x}_{i+1}-v_i. \qquad (40)
\end{aligned}
$$

Select a Lyapunov function candidate as

$$V_i=\frac{1}{2}\xi_i^2+\frac{1}{2\eta_i}\tilde{W}_i^T\tilde{W}_i, \qquad (41)$$

where $\eta_i > 0$ is a positive constant, which should be designed. $\tilde{W}_i$ denotes the NN weight approximation error between the optimal weight vector and its approximation, i.e., $\tilde{W}_i=W_i^*-\hat{W}_i$. Take the derivative of V_i yields

$$\dot{V}_i=\xi_i\dot{\xi}_i-\frac{1}{\eta_i}\tilde{W}_i^T\dot{\hat{W}}_i. \qquad (42)$$

By recalling equalities (4), (39) and (40), the time derivative of ξ_i is immediately obtained

$$
\begin{aligned}
\dot{\xi}_i&=\dot{\hat{x}}_i-\dot{v}_{i-1}\\
&=\dot{x}_i-\dot{\tilde{x}}_i-\dot{v}_{i-1}\\
&=g_ix_{i+1}+f_i-\dot{\tilde{x}}_i-\dot{v}_{i-1}\\
&=g_i\hat{x}_{i+1}+g_i\tilde{x}_{i+1}+f_i-\dot{\tilde{x}}_i-\dot{v}_{i-1}\\
&=g_i\xi_{i+1}+g_iv_i+g_i\tilde{x}_{i+1}+f_i-\dot{\tilde{x}}_i-\dot{v}_{i-1}. \qquad (43)
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\dot{V}_i=\xi_i(&g_i\xi_{i+1}+g_iv_i+g_i\tilde{x}_{i+1}+f_i-\dot{\tilde{x}}_i\\
&-\dot{v}_{i-1})-\frac{1}{\eta_i}\tilde{W}_i^T\dot{\hat{W}}_i. \qquad (44)
\end{aligned}
$$

Let f_i be approximated by utilizing ANN, based on Lemma 1, we have $f_i = W_i^{*T}\theta_i(\hat{\bar{x}}_i)+\tau_i$. Hence, $\dot{V}_i$ can be rewritten as

$$
\begin{aligned}
\dot{V}_i=\xi_i(&g_i\xi_{i+1}+g_iv_i+g_i\tilde{x}_{i+1}-\dot{\tilde{x}}_i+W_i^{*T}\theta_i(\hat{\bar{x}}_i)\\
&+\tau_i-\dot{v}_{i-1})-\frac{1}{\eta_i}\tilde{W}_i^T\dot{\hat{W}}_i. \qquad (45)
\end{aligned}
$$

Design v_i as follows

$$v_i = \frac{1}{g_i}\left(-\lambda_i\xi_i - \hat{W}_i^T\theta_i(\hat{\bar{x}}_i) + \dot{v}_{i-1}\right). \tag{46}$$

Substitute v_i into equation (45) yields

$$\begin{aligned}\dot{V}_i &= \xi_i(g_i\tilde{x}_{i+1} + g_i\xi_{i+1} - \lambda_i\xi_i - \hat{W}_i^T\theta_i(\hat{\bar{x}}_i) + W_i^{*T}\theta_i(\hat{\bar{x}}_i) \\ &\quad +\tau_i - \dot{\hat{x}}_i) - \frac{1}{\eta_i}\tilde{W}_i^T\dot{\hat{W}}_i \\ &= \xi_i(g_i\tilde{x}_{i+1} + g_i\xi_{i+1} - \lambda_i\xi_i + \tilde{W}_i^T\theta_i(\hat{\bar{x}}_i) + \tau_i - \dot{\hat{x}}_i) \\ &\quad -\frac{1}{\eta_i}\tilde{W}_i^T\dot{\hat{W}}_i \\ &= -\lambda_i\xi_i^2 + g_i\xi_i\tilde{x}_{i+1} + g_i\xi_i\xi_{i+1} + \xi_i\tilde{W}_i^T\theta_i(\hat{\bar{x}}_i) \\ &\quad +\xi_i\tau_i - \xi_i\dot{\hat{x}}_i - \frac{1}{\eta_i}\tilde{W}_i^T\dot{\hat{W}}_i. \end{aligned} \tag{47}$$

Design the NN weight updating law as

$$\dot{\hat{W}}_i = \eta_i\xi_i\theta_i(\hat{\bar{x}}_i) - \lambda_{w_i}\hat{W}_i, \tag{48}$$

where $\lambda_{w_i} > 0$ should be designed. Consequently, we have

$$\begin{aligned}\dot{V}_i &= -\lambda_i\xi_i^2 + g_i\xi_i\tilde{x}_{i+1} + g_i\xi_i\xi_{i+1} + \xi_i\tau_i \\ &\quad -\xi_i\dot{\hat{x}}_i + \frac{\lambda_{w_i}}{\eta_i}\tilde{W}_i^T\hat{W}_i \\ &= -\lambda_i\xi_i^2 + g_i\xi_i\tilde{x}_{i+1} + g_i\xi_i\xi_{i+1} + \xi_i\tau_i - \xi_i\dot{\hat{x}}_i \\ &\quad +\frac{\lambda_{w_i}}{\eta_i}\tilde{W}_i^T\left(W_i^* - \tilde{W}_i\right) \\ &= -\lambda_i\xi_i^2 - \frac{\lambda_{w_i}}{\eta_i}\tilde{W}_i^T\tilde{W}_i + g_i\xi_i\tilde{x}_{i+1} + g_i\xi_i\xi_{i+1} \\ &\quad +\xi_i\tau_i - \xi_i\dot{\hat{x}}_i + \frac{\lambda_{w_i}}{\eta_i}\tilde{W}_i^T W_i^*. \end{aligned} \tag{49}$$

By resorting to Lemma 2, one can obtain

$$\begin{aligned}\dot{V}_i &\leqslant -\lambda_i\xi_i^2 - \frac{\lambda_{w_i}}{\eta_i}\tilde{W}_i^T\tilde{W}_i + \frac{1}{2}\xi_i^2 + \frac{1}{2}\left|g_i\tilde{x}_{i+1}\right|^2 \\ &\quad +\frac{1}{2}\xi_i^2 + \frac{1}{2}\left|g_i\xi_{i+1}\right|^2 + \frac{1}{2}\xi_i^2 + \frac{1}{2}\tau_i^2 + \frac{1}{2}\xi_i^2 \\ &\quad +\frac{1}{2}\dot{\hat{x}}_i^2 + \frac{\lambda_{w_i}}{2\eta_i}\left\|\tilde{W}_i\right\|^2 + \frac{\lambda_{w_i}}{2\eta_i}\left\|W_i^*\right\|^2 \\ &\leqslant -\left(\lambda_i - 2\right)\xi_i^2 - \frac{\lambda_{w_i}}{2\eta_i}\left\|\tilde{W}_i\right\|^2 + \frac{1}{2}\left|g_i\tilde{x}_{i+1}\right|^2 \\ &\quad +\frac{1}{2}\left|g_i\xi_{i+1}\right|^2 + \frac{1}{2}\tau_i^2 + \frac{\lambda_{w_i}}{2\eta_i}\left\|W_i^*\right\|^2 + \frac{1}{2}\dot{\hat{x}}_i^2. \end{aligned} \tag{50}$$

According to Lyapunov stability theory, when $\lambda_i > 2$, and the tracking error ξ_{i+1} is guaranteed to be bounded, the tracking error ξ_i and weight approximation error $\tilde{W}_i$ can converge to a small region of zero. Accordingly, the NN utilized to approximate f_i is designed, and its output is $\hat{W}_i^T\theta_i(\hat{\bar{x}}_i)$.

It should be stressed that the virtual control input v_i designed as equation (46) is used to help to prove the convergence of NN weight instead of being used to design the virtual control input in this work. Meanwhile, $\hat{W}_i^T\theta_i(\hat{\bar{x}}_i)$ is only used to design state observer.

Next, the design process of virtual control input v_i proposed in this work is given as follows.

Design the following filter and let ξ_i be its input, we have

$$\dot{\hat{\xi}}_i = p_i\left(\xi_i - \hat{\xi}_i\right). \tag{51}$$

where $p_i > 0$ is the filter parameter, which should be designed.

Let the errors $\xi_i - \hat{\xi}_i$ and $\dot{\xi}_i - \dot{\hat{\xi}}_i$ be denoted by $\tilde{\xi}_i$ and $\dot{\tilde{\xi}}_i$. It is easy to prove that these two errors are bounded. Here, we assume that the bound of $\tilde{\xi}_i$ and $\dot{\tilde{\xi}}_i$ are $\bar{\xi}_i$ and $\bar{\dot{\xi}}_i$, respectively. We have the following equation

$$\begin{aligned}\dot{\tilde{\xi}}_i &= \dot{\xi}_i - \dot{\hat{\xi}}_i \\ &= g_i\tilde{x}_{i+1} + g_i\hat{x}_{i+1} + f_i - \dot{\hat{x}}_i - \dot{v}_{i-1} - \dot{\hat{\xi}}_i. \end{aligned} \tag{52}$$

That is

$$\dot{\tilde{\xi}}_i - g_i\hat{x}_{i+1} + \dot{\hat{\xi}}_i = g_i\tilde{x}_{i+1} + f_i - \dot{\hat{x}}_i - \dot{v}_{i-1}. \tag{53}$$

Since the convergence of NN weights has been proved in the above content, the Lyapunov function is redesigned as

$$V_i = \frac{1}{2}\xi_i^2$$

Take derivative of V_i, and substitute (53) into it, we have

$$\begin{aligned}\dot{V}_i &= \xi_i\left(g_i\xi_{i+1} + g_iv_i + g_i\tilde{x}_{i+1} + f_i - \dot{\hat{x}}_i - \dot{v}_{i-1}\right) \\ &= \xi_i\left(\dot{\tilde{\xi}}_i - g_i\hat{x}_{i+1} + \dot{\hat{\xi}}_i + g_i\xi_{i+1} + g_iv_i\right). \end{aligned} \tag{54}$$

Then, the virtual control input v_i designed by the proposed method can be given by

$$v_i = \frac{1}{g_i}\left(-\lambda_i\xi_i + g_i\hat{x}_{i+1} - \dot{\hat{\xi}}_i\right). \tag{55}$$

Substitute equation (55) into (54), one can obtain

$$\begin{aligned}\dot{V}_i &= \xi_i\left(\dot{\tilde{\xi}}_i + g_i\xi_{i+1} - \lambda_i\xi_i\right) \\ &= -\lambda_i\xi_i^2 + g_i\xi_i\xi_{i+1} + \xi_i\dot{\tilde{\xi}}_i. \end{aligned} \tag{56}$$

Since $\dot{\tilde{\xi}}_i$ is bounded by a positive constant $\bar{\dot{\xi}}_i$, by using Lemma 2, we have

$$\dot{V}_i \leqslant -\left(\lambda_i - 1\right)\xi_i^2 + \frac{1}{2}\left|g_i\xi_{i+1}\right|^2 + \frac{1}{2}\bar{\dot{\xi}}_i^2. \tag{57}$$

Remark 4: Compared with equation (50), it can be seen from equation (57) that the proposed method can compensate the state estimation error $\dot{\hat{x}}_i^2$, the ideal NN approximation error τ_i, and the positive term $\frac{1}{2}\left|g_i\tilde{x}_{i+1}\right|^2$, which can further reduce the tracking error. In addition, the control parameter λ_i should be $\lambda_i > 2$ when the virtual control input v_i designed as equation (46). While, when the virtual control input v_i designed via our method, it only requires the parameter $\lambda_i > 1$. Moreover, it can be seen from equation (52) that the derivative of virtual control input $\dot{v}_{i-1}$ is included in the variable $\dot{\tilde{\xi}}_i$. Thus, the proposed method avoids using other methods, such as dynamic surface control or command filter, to deal with the expansion problem caused by differential terms, which simplifies the control design.

Step n: This step designs an ANN and the proposed NN with novel weight updating law. Note that the ANN that approximates the unknown term $f_n + \varpi$ is utilized when designing state observer. Another NN is designed to approximate another unknown term that will be specified in the following content. Besides, the fault tolerant tracking control input $u(t)$ by using the proposed method is constructed in this step.

Let the tracking errors be defined as

$$\xi_{n-1} = \hat{x}_{n-1} - v_{n-2} \tag{58}$$
$$\xi_n = \hat{x}_n - v_{n-1}. \tag{59}$$

Select a Lyapunov function candidate as

$$V_n = \frac{1}{2}\xi_n^2 + \frac{1}{2\eta_n}\tilde{W}_n^T\tilde{W}_n, \tag{60}$$

where $\eta_n > 0$ is a positive constant, which should be designed. $\tilde{W}_n$ denotes the NN weight approximation error between the optimal weight vector and its approximation, i.e., $\tilde{W}_n = W_n^* - \hat{W}_n$. Take the derivative of V_n yields

$$\dot{V}_n = \xi_n\dot{\xi}_n - \frac{1}{\eta_n}\tilde{W}_n^T\dot{\hat{W}}_n. \tag{61}$$

By recalling equalities (4), (58) and (59), the time derivative of ξ_n is immediately obtained

$$\begin{aligned}\dot{\xi}_n &= \dot{\hat{x}}_n - \dot{v}_{n-1}\\ &= \dot{x}_n - \dot{\tilde{x}}_n - \dot{v}_{n-1}\\ &= g_n u + f_n + \varpi - \dot{\tilde{x}}_n - \dot{v}_{n-1}.\end{aligned} \tag{62}$$

Then, we have

$$\dot{V}_n = \xi_n(g_n u + f_n + \varpi - \dot{\tilde{x}}_n - \dot{v}_{n-1}) - \frac{1}{\eta_n}\tilde{W}_n^T\dot{\hat{W}}_n. \tag{63}$$

Let $f_n + \varpi$ be approximated by utilizing NN, based on Lemma 1, we have $f_n + \varpi = W_n^{*T}\theta_n(\hat{\bar{x}}_n) + \tau_n$. Hence, $\dot{V}_n$ can be repressed as

$$\dot{V}_n = \xi_n(g_n u - \dot{\tilde{x}}_n - \dot{v}_{n-1} + W_n^{*T}\theta_n(\hat{\bar{x}}_n) + \tau_n) - \frac{1}{\eta_n}\tilde{W}_n^T\dot{\hat{W}}_n. \tag{64}$$

Design u as follows

$$u = \frac{1}{g_n}\left(-\lambda_n\xi_n - \hat{W}_n^T\theta_n(\hat{\bar{x}}_n) + \dot{v}_{n-1}\right). \tag{65}$$

Substitute u into equation (64) yields

$$\begin{aligned}\dot{V}_n &= \xi_n(-\lambda_n\xi_n - \hat{W}_n^T\theta_n(\hat{\bar{x}}_n) + W_n^{*T}\theta_n(\hat{\bar{x}}_n)\\ &\quad + \tau_n - \dot{\tilde{x}}_n) - \frac{1}{\eta_n}\tilde{W}_n^T\dot{\hat{W}}_n\\ &= \xi_n(-\lambda_n\xi_n + \tilde{W}_n^T\theta_n(\hat{\bar{x}}_n) + \tau_n - \dot{\tilde{x}}_n)\\ &\quad - \frac{1}{\eta_n}\tilde{W}_n^T\dot{\hat{W}}_n\\ &= -\lambda_n\xi_n^2 + \xi_n\tilde{W}_n^T\theta_n(\hat{\bar{x}}_n) + \xi_n\tau_n\\ &\quad - \xi_n\dot{\tilde{x}}_n - \frac{1}{\eta_n}\tilde{W}_n^T\dot{\hat{W}}_n.\end{aligned} \tag{66}$$

Design the weight updating law as

$$\dot{\hat{W}}_n = \eta_n\xi_n\theta_n(\hat{\bar{x}}_n) - \lambda_{w_n}\hat{W}_n, \tag{67}$$

where $\lambda_{w_n} > 0$ should be designed. Consequently, we have

$$\begin{aligned}\dot{V}_n &= -\lambda_n\xi_n^2 + \xi_n\tau_n - \xi_n\dot{\tilde{x}}_n + \frac{\lambda_{w_n}}{\eta_n}\tilde{W}_n^T\hat{W}_n\\ &= -\lambda_n\xi_n^2 + \xi_n\tau_n - \xi_n\dot{\tilde{x}}_n + \frac{\lambda_{w_n}}{\eta_n}\tilde{W}_n^T\left(W_n^* - \tilde{W}_n\right)\\ &= -\lambda_n\xi_n^2 - \frac{\lambda_{w_n}}{\eta_n}\tilde{W}_n^T\tilde{W}_n + \xi_n\tau_n - \xi_n\dot{\tilde{x}}_n\\ &\quad + \frac{\lambda_{w_n}}{\eta_n}\tilde{W}_n^T W_n^*.\end{aligned} \tag{68}$$

By resorting to Lemma 2, we have

$$\begin{aligned}\dot{V}_n &\leqslant -\lambda_n\xi_n^2 - \frac{\lambda_{w_n}}{\eta_n}\tilde{W}_n^T\tilde{W}_n + \frac{1}{2}\xi_n^2 + \frac{1}{2}\tau_n^2 + \frac{1}{2}\xi_n^2\\ &\quad + \frac{1}{2}\dot{\tilde{x}}_n^2 + \frac{\lambda_{w_n}}{2\eta_n}\left\|\tilde{W}_n\right\|^2 + \frac{\lambda_{w_n}}{2\eta_i}\left\|W_n^*\right\|^2\\ &\leqslant -\left(\lambda_n - 1\right)\xi_n^2 - \frac{\lambda_{w_n}}{2\eta_n}\left\|\tilde{W}_n\right\|^2\\ &\quad + \frac{1}{2}\tau_n^2 + \frac{\lambda_{w_n}}{2\eta_n}\left\|W_n^*\right\|^2 + \frac{1}{2}\dot{\tilde{x}}_n^2.\end{aligned} \tag{69}$$

According to Lyapunov stability theory, when $\lambda_n > 1$, the tracking error ξ_n and weight approximation error $\tilde{W}_n$ can converge to a small region of zero. Accordingly, the ANN utilized to approximate $f_n + \varpi$ is designed, and its output is $\hat{W}_n^T\theta_n(\hat{\bar{x}}_n)$.

Note that the control input u designed as equation (65) is given to help to prove the convergence of NN weight, and $\hat{W}_n^T\theta_n(\hat{\bar{x}}_n)$ designed above is used when designing state observer instead of being used when designing the proposed control input u.

Next, the design process of control input u by using the proposed method is given as follows.

Design the following filter and let ξ_n be its input, we have

$$\dot{\hat{\xi}}_n = p_n\left(\xi_n - \hat{\xi}_n\right). \tag{70}$$

where $p_n > 0$ is the filter parameter, which should be designed.

Let the errors $\xi_n - \hat{\xi}_n$ and $\dot{\xi}_n - \dot{\hat{\xi}}_n$ be denoted by $\tilde{\xi}_n$ and $\dot{\tilde{\xi}}_n$. It is easy to prove that these two errors are bounded. Here, we assume that the bounds of $\tilde{\xi}_n$ and $\dot{\tilde{\xi}}_n$ are $\bar{\xi}_n$ and $\dot{\bar{\xi}}_n$, respectively. According to equation (62), we have the following equation

$$\begin{aligned}\dot{\tilde{\xi}}_n &= \dot{\xi}_n - \dot{\hat{\xi}}_n\\ &= g_n u + f_n + \varpi - \dot{\tilde{x}}_n - \dot{v}_{n-1} - \dot{\hat{\xi}}_n.\end{aligned} \tag{71}$$

Let the unknown term $f_n + \varpi - \dot{\tilde{x}}_n - \dot{v}_{n-1} - \dot{\hat{\xi}}_n$ be approximated by using another NN, whose output is $\hat{W}_{n+1}^T\theta_{n+1}(\hat{\bar{x}}_n)$. According to Lemma 1, we have

$$\dot{\tilde{\xi}}_n = g_n u + {W_{n+1}^*}^T\theta_{n+1}(\hat{\bar{x}}_n) + \tau_{n+1}. \tag{72}$$

Furthermore, we reconstruct another variable as follows

$$\dot{\chi}_n = g_n u + \hat{W}_{n+1}^T \theta_{n+1}(\hat{\bar{x}}_n). \tag{73}$$

Subtract (73) from (72), we have

$$\dot{\hat{\xi}}_n - \dot{\chi}_n = \tilde{W}_{n+1}^T \theta_{n+1}(\hat{\bar{x}}_n) + \tau_{n+1}. \tag{74}$$

Design a cost function as follows

$$E_\xi = \frac{1}{2}(\dot{\hat{\xi}}_n - \dot{\chi}_n)^2. \tag{75}$$

Then, according to gradient decent algorithm, the weight updating law $\dot{\hat{W}}_{n+1}$ is designed as

$$\begin{aligned}\dot{\hat{W}}_{n+1} &= -\eta_{n+1}\frac{\partial E_\xi}{\partial \hat{W}_{n+1}} \\ &= \eta_{n+1}\left(\dot{\hat{\xi}}_n - \dot{\chi}_n\right)\theta_{n+1}(\hat{\bar{x}}_n). \end{aligned} \tag{76}$$

where $\eta_{n+1} > 0$ denotes the learning rate, which should be designed.

Remark 5: The NN weight updating algorithm designed in equation (76) is different from the ANN weight updating law designed in (67). The former can lead to a better approximation performance for the reason that the proposed NN weight can be updated to its corresponding satisfactory value by minimizing a cost function designed as equation (75), while the latter can only guarantee the convergence of weight. In addition, the proposed NN also can approximate and compensate the state estimation error, and can deal with the problem of differential explosion caused by repeated differentiations.

Next, we design the control input u by using the proposed NN with output $\hat{W}_{n+1}^T \theta_{n+1}(\hat{\bar{x}}_n)$.

Choose a Lyapunov function as the following form

$$V_n = \frac{1}{2}\xi_n^2. \tag{77}$$

Take derivative of V_n yields

$$\dot{V}_n = \xi_n\left(g_n u + f_n + \varpi - \dot{v}_{n-1} - \dot{\hat{x}}_n - \dot{\hat{\xi}}_n\right). \tag{78}$$

Redesign the control input as

$$u = \frac{1}{g_n}\left(-\lambda_n \xi_n - \hat{W}_{n+1}^T \theta_{n+1}(\hat{\bar{x}}_n)\right). \tag{79}$$

Then,

$$\begin{aligned}\dot{V}_n &= \xi_n\left(-\lambda_n \xi_n + \tilde{W}_{n+1}^T \theta_{n+1}(\hat{\bar{x}}_n) + \tau_{n+1}\right) \\ &= -\lambda_n \xi_n^2 + \xi_n \tau_{n+1}. \end{aligned} \tag{80}$$

By utilizing Lemma 2, we have

$$\dot{V}_n \leq -(\lambda_n - \frac{1}{2})\xi_n^2 + \frac{1}{2}\tau_{n+1}^2. \tag{81}$$

According to Lyapunov stability theory, when $\lambda_n > \frac{1}{2}$, the tracking error ξ_n can converge to a small region of zero. Similarly, it can be inferred that the convergence of tracking error ξ_i, $i = 1, 2, \ldots, n-1$ is ensured.

IV. SIMULATIONS

In this section, a simulation study is conducted on an invert system [35] to verify the effectiveness of the proposed fault-tolerant control strategy. It should be stressed that the proposed method is an improved version compared with the existing ANN-based FTC since it provides a novel FTC strategy by resorting to filter and novel NN weight updating algorithm. Thus, an ANN-based FTC method presented in literature [36] is used as a comparison. The dynamics of the invert system is given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_2(\bar{x}_2) + g_2(\bar{x}_2)u \\ y = x_1 \end{cases} \tag{82}$$

with

$$f_2(\bar{x}_2) = \frac{g sin(x_1) - \frac{mlx_2^2 sin(x_1)cos(x_1)}{m+m_c}}{l(\frac{4}{3} - \frac{mcos^2(x_1)}{m+m_c})}$$

$$g_2(\bar{x}_2) = \frac{\frac{cos(x_1)}{m+m_c}}{l(\frac{4}{3} - \frac{mcos^2(x_1)}{m+m_c})}$$

where x_1 and x_2 are the system states, the state x_2 can not be measured, while the output $y = x_1$ is measurable. The nonlinear function $f_2(\bar{x}_2)$ is unknown, while the nonlinear $g_2(\bar{x}_2)$ is known and can be used when designing controller. The system parameters are $g = 9.8m/s^2$, $m = 0.1kg$, $m_c = 1kg$ and $l = 0.5m$. The desired output reference is chosen as $y_d = sin(\frac{2}{\pi}t)$. The actuator fault is modeled as

$$u^o = \begin{cases} u, & t < 20s \\ (0.8 + e^{-4t})u + 0.5, & t \geq 20s \end{cases} \tag{83}$$

The control objective is to design a fault tolerant controller for the system (82) with actuator fault (83) such that the system output tracks the desired signal.

TABLE 1. Control parameters of proposed method.

$Parameter$	$Value$	$Parameter$	$Value$
λ_1	6.98	λ_2	6.38
η	1.2	b	10.06
p_1	40	p_2	40
k_1	40	k_2	40

The proposed filter and NN-based fault tolerant tracking controller is given as equation (79) with NN updating law shown as (76). The control parameters of proposed method are given in Table 1. The initial states are chosen as $x_0 = [0, 0]^T$. The simulation results are illustrated in Figs. 2-3. Specifically, Fig. 2 draws the output tracking performance and tracking error. It can be seen from Fig. 2 that the system output obtained by using the proposed control method has a quite small fluctuation when actuator occurs after $t = 20s$, and the tracking error is also small. Fig. 3 depicts the output of proposed NNs, it can be found that the designed NN can approximate the unknown function with small error even after actuator failure.

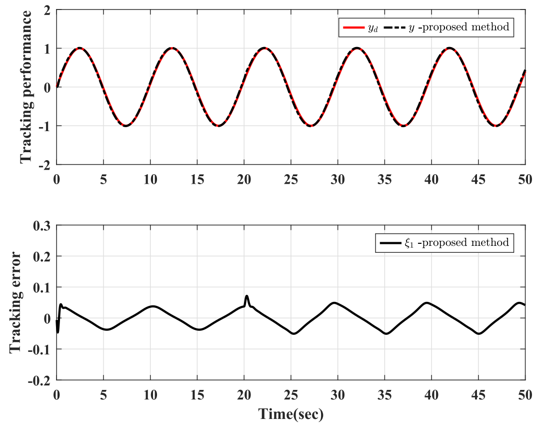

FIGURE 2. Tracking performance and tracking errors by using the proposed method.

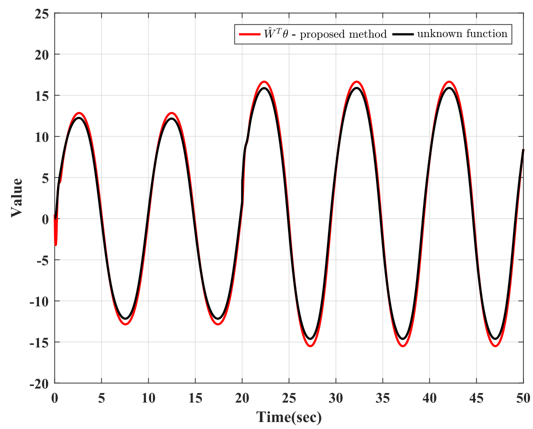

FIGURE 3. Outputs of proposed NN.

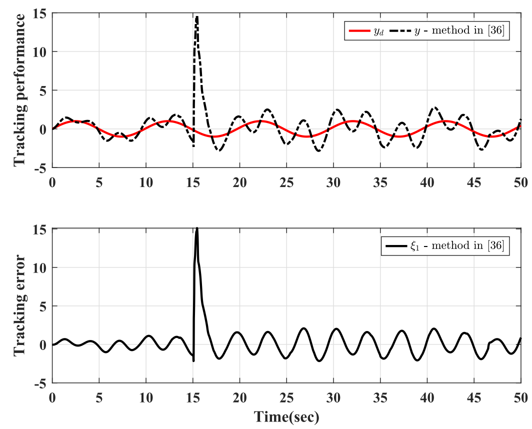

FIGURE 4. Tracking performance and tracking errors by using the method in [36] when the controller parameters are chosen as shown in Table 1 besides designing the ANN parameters as $\eta = 9.74$ and $\lambda_w = 0.64$.

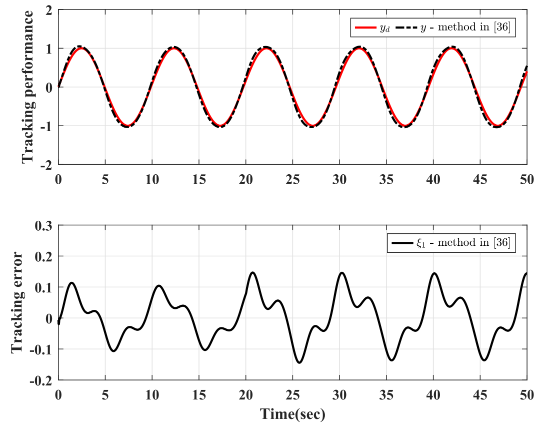

FIGURE 5. Tracking performance and tracking errors by using the method in [36] when the controller parameters are chosen as shown in Table 1 besides design $\eta = 9.74$, $\lambda_w = 0.64$, $\lambda_1 = 35$ and $\lambda_2 = 10$.

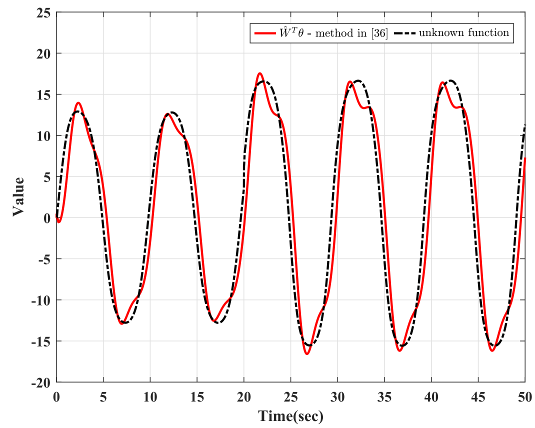

FIGURE 6. Outputs of ANN proposed in literature [36].

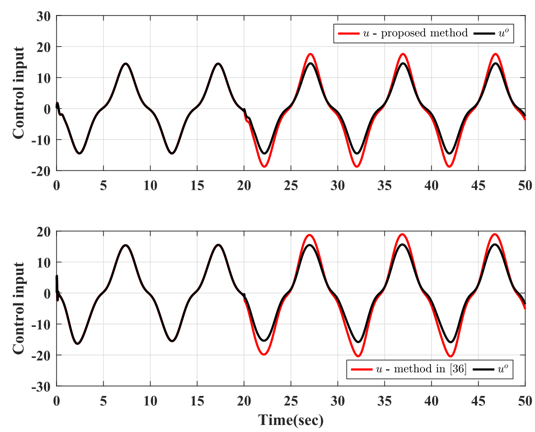

FIGURE 7. The first subplot shows the actuator input and output of the proposed method, while the second subplot depicts those of the method in paper [36].

To make a fair and reliable comparison, the controller parameters of the ANN-based fault tolerant controller designed according to literature [36] are same as those of the proposed method besides designing the ANN parameters as $\eta = 9.74$ and $\lambda_w = 0.64$. The simulation results are shown in Fig. 4. It can be found that the tracking performance is extremely worse since the ANN approximation performance is not as well as the proposed NN approximation performance. To achieve a good tracking performance, the controller parameters λ_1 and λ_2 should be carefully readjusted. Then, chose them as $\lambda_1 = 35$ and $\lambda_2 = 10$, the simulation results are shown in Figs. 5-6, it can be seen that the tracking performance becomes better. However, the tracking performance is still worse than that of the proposed control method. Finally, Fig. 7 compares the evolutions of the control input by using the proposed method and the method in [36]. It can be seen that the proposed method requires less control efforts, but can achieve a better control performance.

Note that the control parameters are tuned by the same method based on uniform design table to guarantee that the comparative results are convincing. In such method, control parameter sets are generated by resorting to uniform design table. Each control parameter set is applied, and the one that can result in the best control performance is what we want.

V. CONCLUSION

In this paper, by resorting to filter technique and online learning algorithm, a novel NN-based fault tolerant control strategy for nonlinear systems with unknown drift function and actuator faults was proposed. The designed filter and NN can successfully compensate the state observer errors, ANN approximation errors and actuator faults. Furthermore, the expansion problem caused by derivative terms of backstepping technique also can be dealt with. Theoretical analysis and simulation results demonstrate that the proposed method can be efficient in coping with time-varying actuator failures and unknown system dynamics appearing in the controlled system with immeasurable system states. Although the proposed control method is proved to be effective, it can not be used to solve the control problem for the systems with unknown control direction. Hence, in the future, we would take efforts to investigate such control problem.

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