IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received June 9, 2021, accepted June 16, 2021, date of publication June 23, 2021, date of current version July 2, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3091713

Basin Eco-Compensation Strategy Considering a Cost-Sharing Contract

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This work was supported in part by the Fundamental Research Funds for the Central Universities under Grant B210207028 and Grant B210203060, and in part by the National Key Research and Development Program of China under Grant 2017YFC0405805-04.

ABSTRACT Based on differential game theory, this paper constructs a centralized game model, a Nash noncooperative game model and a game model with a cost-sharing contract. This paper discusses the interactive game strategy between the central government's participation in subsidies and the upstream and downstream sharing of the cost of pollution control, and we obtain and compare the optimal feedback strategies and trajectory of pollution control with time upstream and downstream of a basin. The introduction of the game model for cost-sharing contracts not only maximizes the benefits in the basin but also increases the amount of pollution control, improves the ecological environment of the basin, and enhances its capital attractiveness. In addition, it is found that the central government's subsidies have an impact on the decision-making behavior of local governments, especially in the case of introducing cost-sharing contracts. Central government subsidies can increase the enthusiasm of upstream and downstream local governments for long-term cooperation in pollution control and emission reduction.

INDEX TERMS Eco-compensation, differential game, cost sharing construct, basin social welfare.

I. INTRODUCTION

A basin is a relatively systematic, complete and independent hydrological unit centered on a river. The natural elements in basins are closely related, and the interaction between upstream and downstream aspects is obvious and acts as a highly integrated natural region. With the rapid increase in China's economic growth, the pressure from human activities placed on the ecological environment of basins is increasing daily, and the quality of basin ecosystems is declining [1]–[3]. The contradiction between economic society and the natural environment of basins is becoming increasingly prominent. As an important spatial carrier and pillar of economic development, basins have the characteristics of industrial concentration, the high intensity of economic activities and high-contact population density. As a quasi-public good, basins have the characteristics of nonexclusiveness, openness and competitiveness; if there is no overall planning from the overall level of the basin, then it is likely to produce negative externalities and lead to cross-border water pollution. There are many contradictions between upstream and downstream regions, especially in terms of economic development and ecological governance. Eventually, these contradictions lead to the phenomenon of "tragedy of the commons" and market failure in the development of watershed resources.

Basin ecological compensation is an effective way to coordinate the contradiction between basin environmental protection and economic development and is an institutional arrangement that uses a combination of market and economic means to regulate the relationship between the stakeholders of a basin [4]–[6]. Eco-compensation can also promote compensation activities between basins and increase the enthusiasm toward ecological protection; ultimately, the "win-win" of eco-environmental benefits and social and economic benefits will be realized. The ecological compensation of basins should abide by the principle of taking into account the interests of upstream and downstream regions and reflecting fairness and justice. The real costs of resource consumption and environmental protection should be reasonably compensated through a basin ecological compensation policy to promote

The associate editor coordinating the review of this manuscript and approving it for publication was Ahmed A. Zaki Diab.

the sustainable development of the basin in space. We should clarify the responsibilities and rights of the upstream and downstream areas and adhere to the principle of clear rewards and punishments. The upstream area needs to implement various watershed ecological environment management tasks to help the outbound water quality meet the prescribed standards, and the downstream area enjoys the watershed ecological service value provided by the upstream area, and thus, it needs to compensate the upstream area adequately [7]–[9]. Therefore, the watershed ecological compensation mechanism is an economic incentive method that internalizes the external effects of watershed ecological resources. The beneficiaries of a basin's environmental resources pay a certain fee to those who lose interest to encourage all relevant stakeholders in the basin to earnestly perform their ecological protection responsibilities, maintain the long-term operation of the ecological functions of the basin, and maximize the economic benefits of the ecological protection of the basin. One of the inevitable requirements for the sustainable development of basins is to reduce environmental pollution emissions [10], [11]. An important issue that must be solved urgently by the academic community, therefore, becomes how to incentivize upstream and downstream local governments to increase investment in pollution control and emission reduction and then improve the sustainable development level of basins. To reduce pollutant emissions and improve the basin environment, local governments in the upper and lower reaches of a basin need to cooperate closely to jointly formulate appropriate ecological compensation policies and benefit distribution mechanisms. The determination of ecological compensation standards is a complicated process, and the optimal allocation of pollution control costs is an important prerequisite for determining compensation standards [12]–[15]. Therefore, it is of important theoretical and practical significance to study the issue of cooperative pollution control and emission reduction between upstream and downstream local governments under the subsidy of the central government. The above authors focused on the study of economic transformation in forest areas, constructed an evolutionary game model including government ecological compensation and social capital, and obtained the necessary conditions for achieving an optimal and stable equilibrium strategy.

II. LITERATURE REVIEW

American environmental economists Seneca and Tosig pioneered the study of ecological compensation from the theory of compensation development. Subsequently, various countries around the world have adopted and carried out a variety of ecological compensation practices, for example, the comprehensive benefit compensation model for the water source area of Lake Biwa in Japan, the water rights trading model of the Delaware basin in the United States, and the national government fund model of the Sarapiqui basin in Costa Rica [16], [17]. China's ecological compensation mechanism has been constantly improved in recent years. After years of ecological compensation and cogovernance in Anhui and Zhejiang Provinces, the Xin'an Basin has become one of the regions with the best water quality in China, and it was included in the 2019 "Reform and Development Cases". This means that the improvement of the ecological environment of basins increasingly depends on upstream and downstream cooperative protection and ecological compensation. The nature of the conflict between the use and protection of water resources in the upper and lower reaches of a basin involves environmental protection and social and economic development. Due to the existence of this conflict of interest, ecological compensation within basins has a typical game feature. Therefore, the use of game theory has become an important tool for studying ecological compensation in basins [18]–[21].

Some scholars believe that to improve the basin environment and optimize the allocation of water resources, it is necessary to give full play to the constraint mechanism of the government [22]–[24]. With the deepening of the related research, an increasing number of stakeholders, such as social capital, enterprises and consumers in the basin, have been added to the design of the ecological compensation mechanism in basins [20], [25]–[27].

Due to the complexity of the economic environment and game problems and the imperfect rationality of participants, evolutionary games are an important method for scholars to study ecological compensation. Cui *et al.* [28] established an evolutionary game model composed of four participants: the government, financial institutions, enterprises, and consumers. Their research aimed to build a complete green financial system and enhance innovation capabilities and economic green transformation. Gao *et al.* [25] constructed an evolutionary game model including upstream and downstream local governments and the central government and analyzed the distribution of ecological benefits in the East Route of the South-to-North Water Transfer Project. Guo *et al.* [26] focused on the study of economic transformation in forest areas, constructed an evolutionary game model including government ecological compensation and social capital, and obtained the necessary conditions for achieving an optimal and stable equilibrium strategy. Another stream of the literature is based on differential games. Jiang *et al.* [29] established a differential game model for cross-basin pollution control in continuous time and then used optimal control theory to explore the optimal feedback equilibrium of the watershed environmental quality under three game situations. Wei *et al.* [30] constructed a differential game model between local governments and local enterprises in a basin, aiming to obtain a balance between sustainable economic development and environmental protection and identify how local enterprises can maximize benefits in terms of ecological compensation. By constructing a differential game model of ecological compensation, pollution control and emission reduction in the upstream and downstream regions, Chen *et al.* [31] found that an appropriate ecological compensation ratio can effectively improve the pollution control level of the two regions. The establishment of an investment ecological compensation mechanism is an effective measure for reducing pollution in basins in the long term. In addition, there are many researches on cost sharing contract in the traditional management research field such as supply chain [32], [33], which also enlightens this paper in the aspect of parameter setting.

In summary, many studies have addressed the optimization model of basin ecological compensation under different mathematical models and the calculation standard of basin dynamic water ecological compensation and explained the influence of subsidy behavior in basin ecological compensation. However, most of the literature has considered only the central government subsidy coefficient or the unidirectional compensation of the downstream to the upstream region. Few studies have considered the subsidy coefficient between upstream and downstream local governments as a decision variable. This paper considers that when both upstream and downstream local governments are involved in pollution control and emission reduction, a centralized and Nash noncooperative differential game model is constructed, and a cost-sharing contract is designed to coordinate the joint pollution control and emission reduction of both parties. This not only maximizes the benefits for basins but also increases the amount of pollution control, improves the ecological environment of basins, and enhances their capital attractiveness.

III. METHODOLOGY

A. MODEL HYPOTHESIS AND SYMBOL EXPLANATION

We take the ecological environmental protection of basins as the research object and study the influence of the ecological compensation mechanism of basins on the cooperative environmental governance of upstream and downstream areas. This article proposes that the upstream and downstream local governments and the central government are rational subjects. This paper constructs a centralized and Nash noncooperative differential game model and designs a cost-sharing contract to coordinate the overall ecological and environmental protection of the basin. Our goal is to maximize the benefits from improving the overall ecological environment of basins and improve their overall social welfare. The main symbols used in the model are described in TABLE 1.

TABLE 1. Notations and definitions.

Decision Variables	
$I_u(t)$，$I_d(t)$	Upstream and downstream local government investment in pollution control at time t
θ	The subsidy coefficient of the downstream local government to the upstream local government for pollution control
μ	The subsidy coefficient of the upstream local government to the downstream local government for pollution control
State Variables and Game Parameters	
w_u, w_d	Pollution control cost coefficient of upstream and downstream local governments
φ_u, φ_d	The central government's subsidy coefficient to upstream and downstream local governments
α, β	Sensitivity coefficient of pollution control amount to I_u and I_d
s	Influence coefficient of pollution control volume on the welfare effect of basins
R	Benefits of improving the basin environment
Π_u, Π_d	The influence coefficient of the basin welfare effect on the benefit of the upstream and downstream local governments

Assumption 1: Similar to the previous studies [34]–[37], the cost function is convex, so we use quadratic function to measure the cost function, which can meet the rising law of marginal cost:

$$C_i(I_i) = \frac{w_i}{2} I_i^2(t), \quad i \in \{u, d\} \tag{1}$$

where, w_i represents pollution control cost coefficient of i, I_i represents the upstream and downstream pollution control investment efforts.

Assumption 2: The amount of pollution control in the basin is affected by the input of pollution control and emission reduction of local governments in upstream and downstream.

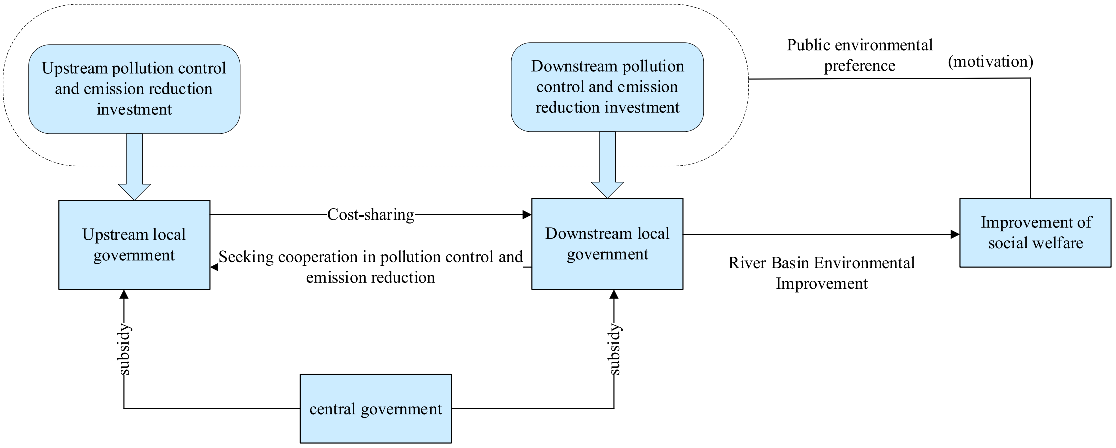


FIGURE 1. Decision-making relationship of upstream and downstream regions within a watershed.

TABLE 1. ***(Continued.)*** **Notations and definitions.**

Dependent Variables	
r	Pollutant reuse rate
γ	Natural decay rate of pollution control
ρ	Discount rate of upstream and downstream local governments

The increase of input can directly improve the amount of pollution control in the basin. With the passage of time and the aging of pollution control equipment, the amount of pollution control and emission reduction in the basin will decline to a certain extent [33], [37], [38]. Without loss of generality, the amount of pollution control can be described by the following differential equation :

$$\begin{cases} \dot{y}(t) = \alpha I_u(t) + \beta I_d(t) - \beta I_d(t) - (\gamma - r) y(t) \\ y(0) = y_0 \end{cases} \tag{2}$$

where $y(t)$ represents the amount of pollution control at time t, and y_0 is the initial pollution control amount. Parameters α and β are the Sensitivity coefficient of pollution control amount. Parameters γ and r represent natural decay rate of pollution control and pollutant reuse rate respectively.

Assumption 3: This article proposes that both upstream and downstream decisions are made based on complete information and does not consider the influence of other factors on pollution control and emission reduction [21], [39], [40].

Watershed pollution control and emission reduction are important ways to promote the coordinated development of the social economy and natural resource protection, and environmental improvement can increase the social welfare utility of basins. Advocating energy conservation and emission reduction, optimizing the industrial structure, and carrying out structural reforms of extensive economic growth methods are of great significance to the sustainable economic development of upstream and downstream regions.

The improvement of the basin environment can significantly improve the living environment of nearby residents, increase the vitality of the regional economy, and reduce the policy and capital costs of regional economic development. It is assumed that the social welfare effect function brought about by pollution control and emission reduction in the upper and lower reaches of a basin is as follows:

$$T(y(t), t) = sy(t) + T_0 \tag{3}$$

where T_0 represents the initial welfare utility of the basin.

Assume that in an infinite time interval, upstream and downstream areas have the same discount factor, ρ, at any time.

In sum, the objective functions of upstream and downstream local governments are as follows:

$$\begin{cases} R_u = \int_0^\infty -\rho t \left\{ \Pi_u T(y(t), t) + (\varphi_u - 1) C_u(I_u) \right\} dt \\ R_d = \int_0^\infty -\rho t \left\{ \Pi_d T(y(t), t) + (\varphi_d - 1) C_d(I_d) \right\} dt \end{cases} \tag{4}$$

For notational convenience, t is omitted below. Based on the above model assumptions, mutual decision-making relationship between variables is shown in Figure 1.

B. NASH NONCOOPERATIVE GAME SOLUTIONS (DECISION A)

When the upstream and downstream local governments are engaged in a Nash noncooperative game, the two parties will simultaneously and independently decide their respective investment in pollution control and emission reduction to maximize their own welfare utility. At this time, the optimal environmental management strategy combination of both parties is the static feedback Nash equilibrium. In this decision-making situation, $\theta(t)$ and $\mu(t)$ are equal to 0, and the upstream and downstream objective functions are, respectively, as follows:

$$\begin{cases} R_u^a = \max\limits_{I_u^a \geq 0} \displaystyle\int_0^{\infty} e^{-\rho t} \left\{ \Pi_u \left(sy(t) + T_0\right) \right. \\ \qquad \left. + \left(\varphi_u - 1\right) \dfrac{w_u}{2} \left(I_u^a\right)^2 (t) \right\} dt \\ R_d^a = \max\limits_{I_d^a \geq 0} \displaystyle\int_0^{\infty} e^{-\rho t} \left\{ \Pi_d \left(sy(t) + T_0\right) \right. \\ \qquad \left. + \left(\varphi_d - 1\right) \dfrac{w_d}{2} \left(I_d^a\right)^2 (t) \right\} dt \end{cases} \tag{5}$$

To obtain the Markov-refined Nash equilibrium of the Nash noncooperative game, a continuous bounded differential function is constructed that satisfies the Hamilton-Jacobi-Bellman equation:

$$\rho V_u(R) = \max_{I_u^a \geq 0} \left\{ \begin{array}{l} \Pi_u \left(sy(t) + T_0\right) + \left(\varphi_u - 1\right) \dfrac{w_u}{2} \left(I_u^a\right)^2 (t) \\ + V_u'(R) \left[\alpha I_u(t) + \beta I_d(t)\right. \\ \left. - \beta I_d(t) - (\gamma - r) y(t)\right] \end{array} \right\} \tag{6}$$

$$\rho V_d(R) = \max_{I_3^a \geq 0} \left\{ \begin{array}{l} \Pi_d \left(sy(t) + T_0\right) + \left(\varphi_d - 1\right) \dfrac{w_d}{2} \left(I_u^a\right)^2 (t) \\ + V_d'(R) \left[\alpha I_u(t) + \beta I_d(t)\right. \\ \left. - \beta I_d(t) - (\gamma - r) y(t)\right] \end{array} \right\} \tag{7}$$

By solving the above equation, we can obtain proposition 1.

Proposition 1: The optimal investment of upstream and downstream local governments for pollution control and emission reduction are as follows:

$$\begin{cases} \boldsymbol{I_u^{b*}} = \dfrac{\boldsymbol{(\Pi_u + \Pi_d) s\alpha}}{\boldsymbol{w_u m \left(1 - \varphi_u\right)}} \\ \boldsymbol{I_d^{b*}} = \dfrac{\boldsymbol{(\Pi_u + \Pi_d) s\beta}}{\boldsymbol{w_d m \left(1 - \varphi_d\right)}} \end{cases} \tag{8}$$

where $m = \rho + \gamma - r$. The optimal trajectory of the pollution control amount is as follows:

$$y^{a*} = \frac{A^a}{\gamma - r} - \left(\frac{A^a}{\gamma - r} - y_0\right) e^{-(\gamma - r)t} \tag{9}$$

where $A^a = \alpha \cdot I_u^{a*} + \beta \cdot I_d^{a*}$. The optimal benefits of the upstream and downstream local governments in the basin are as follows:

$$\begin{cases} R_u^{a*} = e^{-\rho t} \left[\Pi_u s y^{a*} + \dfrac{\Pi_u T_0}{\rho} + \dfrac{\alpha^2 \left(\Pi_u s\right)^2}{2\rho\left(1 - \varphi_u\right) w_u m^2} \right. \\ \qquad \left. + \dfrac{\beta^2 s^2 \Pi_u \Pi_d}{\rho\left(1 - \varphi_d\right) w_d m^2} \right] \\ R_d^{a*} = e^{-\rho t} \left[\Pi_d s y^{a*} + \dfrac{\Pi_d T_0}{\rho} + \dfrac{\beta^2 \left(\Pi_d s\right)^2}{\rho\left(1 - \varphi_d\right) w_d m^2} \right. \\ \qquad \left. + \dfrac{\alpha^2 s^2 \Pi_u \Pi_d}{2\rho\left(1 - \varphi_u\right) w_u m^2} \right] \end{cases} \tag{10}$$

Proof: See Appendix A.

C. COOPERATIVE GAME SOLUTIONS (DECISION B)

Suppose that under the constraints of the central government, local governments in the upper and lower reaches of a basin have formed a strong cooperation agreement and established a long-term cooperative relationship, that is, the upper and lower reaches acting as a coordinated control entity. In this case, the upstream and downstream local governments make decisions with the goal of maximizing the overall optimal benefits of the basin. Then, the objective function of the basin as a whole and the central government is as follows:

$$\begin{aligned} R_T^b = \max_{I_u^b \geq 0} \int_0^{\infty} e^{-\rho t} & \left\{ T_b^* \left(\Pi_u + \Pi_d\right) - \left(1 - \varphi_u\right) \frac{w_u}{2} \left(I_u^b\right)^2 \right. \\ & \left. - \left(1 - \varphi_d\right) \frac{w_d}{2} \left(I_d^b\right)^2 \right\} dt \end{aligned} \tag{11}$$

Similar to the first decision, we can obtain proposition 2.

Proposition 2: The optimal investment of upstream and downstream local governments in control pollution is as follows:

$$\begin{cases} I_u^{b*} = \dfrac{(\Pi_u + \Pi_d) s\alpha}{w_u m \left(1 - \varphi_u\right)} \\ I_d^{b*} = \dfrac{(\Pi_u + \Pi_d) s\beta}{w_d m \left(1 - \varphi_d\right)} \end{cases} \tag{12}$$

where $m = \rho + \gamma - r$. The optimal trajectory of the pollution control amount is as follows:

$$y^{b**} = \frac{A^b}{\gamma - r} - \left(\frac{A^b}{\gamma - r} - y_0\right) e^{-(\gamma - r)t} \tag{13}$$

where $A^b = \alpha \cdot I_u^{b*} + \beta \cdot I_D^{b*}$. The benefits of the upstream and downstream local governments in the basin are as follows:

$$\begin{aligned} R_T^{b**} = e^{-\rho t} & \left[\left(\Pi_u + \Pi_d\right) s T^{b**} \right. \\ & + \left(\frac{\alpha^2}{2 w_u (1 - \varphi_u)} + \frac{\beta^2}{2 w_d (1 - \varphi_d)} \right) \\ & \left. \times \frac{\left(\Pi_u + \Pi_d\right)^2 s^2}{\rho m^2} + \frac{\left(\Pi_u + \Pi_d\right) T_0}{\rho} \right] \end{aligned} \tag{14}$$

Proof: See Appendix B.

D. GAME SITUATION WITH A COST-SHARING CONTRACT (DECISION C)

This section realizes the coordination of upstream and downstream ecological compensation by considering cost-sharing contracts. To increase the enthusiasm of upstream and downstream local governments toward pollution control and emission reduction, in addition to central government subsidies, at the same time, a mutual incentive measure between upstream and downstream areas within the basin is adopted. That is, downstream local governments share the pollution control costs of upstream local governments, and the sharing ratio is θ, while upstream local governments share the cost of pollution control and emission reduction in the proportion of μ for downstream governments. In this case, the decision-making objective function of upstream and downstream local governments is as follows:

$$\begin{cases} R_u^c = \max\limits_{I_u^c \geq 0} \int_0^\infty e^{-\rho t} \{\Pi_u T - (1 - \varphi_u - \theta) \\ \qquad \times C_u\left(I_u^c\right) - \mu C_d\left(I_d^c\right)\} \, dt \\ R_d^c = \max\limits_{I_d^c \geq 0} \int_0^\infty e^{-\rho t} \{\Pi_d T - (1 - \varphi_d - \mu) \\ \qquad \times C_d\left(I_d^c\right) - \theta C_u\left(I_u^c\right)\} \, dt \end{cases} \tag{15}$$

Proposition 3: The optimal investment of upstream and downstream local governments in pollution control and the optimal subsidy coefficient between them are as follows:

$$\begin{aligned} I_u^{c^*} &= \frac{(\Pi_u + \Pi_d)\, s\alpha}{w_u m\, (1 - \varphi_u)} \\ I_d^{c^*} &= \frac{(\Pi_u + \Pi_d)\, s\beta}{w_D m\, (1 - \varphi_d)} \\ \mu^* &= \frac{\Pi_d\, (1 - \varphi_u)}{c + \Pi_d} \\ \theta^* &= \frac{\Pi_u\, (1 - \varphi_d)}{\Pi_u + \Pi_d} \end{aligned} \tag{16}$$

where $\mathrm{m} = \rho + \gamma - \mathrm{r}$. The optimal trajectory of the pollution control amount is as follows:

$$y^{c^*} = \frac{A^c}{\gamma - r} - \left(\frac{A^c}{\gamma - r} - y_0\right) e^{-(\gamma - r)t} \tag{17}$$

where $\mathrm{A^c} = \alpha \cdot \mathrm{I_u^{c^*}} + \beta \cdot \mathrm{I_d^{c^*}}$. In this case, the optimal welfare effect of the basin is as follows:

$$T^{c^*} = T_0 + s\left[\frac{A^c}{\gamma - r} - \left(\frac{A^c}{\gamma - r} - y_0\right) e^{-(\gamma - r)t}\right] \tag{18}$$

In this case, the optimal benefit of upstream and downstream local governments is as follows:

$$\begin{cases} R_u^{c^*} = e^{-\rho t}\left[\Pi_u s y^{c^*} + \dfrac{\Pi_u T_0}{\rho} + \dfrac{\alpha^2 s^2 (\Pi_u \Pi_d + \Pi_u^2)}{2\rho\, (1 - \varphi_u)\, w_u m^2} \right. \\ \qquad \left. + \dfrac{(\beta s)^2\, \Pi_u \Pi_d + \Pi_u^2)}{2\rho\, (1 - \varphi_d)\, w_d m^2}\right] \\ R_d^{c^*} = e^{-\rho t}\left[\Pi_d s y^{c^*} + \dfrac{\Pi_d T_0}{\rho} + \dfrac{\alpha^2 s^2 (\Pi_d{}^2 + \Pi_u \Pi_d)}{2\rho\, (1 - \varphi_u)\, w_u m^2} \right. \\ \qquad \left. + \dfrac{(\beta s)^2\, \Pi_d^2 + \Pi_u \Pi_d)}{2\rho w_d\, (1 - \varphi_d)\, m^2}\right] \end{cases} \tag{19}$$

Proof: See Appendix C.

Remark 1: After the introduction of cost-sharing contracts, the overall benefits of basins were improved compared with the Nash noncooperative game situation and reached the level of the cooperative game situation. However, the respective economic growth of the upstream and downstream local governments may not be completely improved compared to the Nash noncooperative game situation, so to implement the cost-sharing contract game smoothly, we introduce transfer payment parameter E and amend the income of upstream and downstream local governments to the following:

$$\begin{aligned} R_u^c &= R_u^{c^*} + E \\ R_d^c &= R_D^{c^*} - E \end{aligned}$$

Compared with the Nash noncooperative game, the amount of change in the income of upstream and downstream local governments is as follows:

$$\begin{aligned} \Delta R_u &= R_u^c - R_u^{a^{**}} \\ \Delta R_d &= R_d^c - R_d^{a^{**}} \end{aligned}$$

$\mathrm{\Delta R_u} > 0$ and $\mathrm{\Delta R_d} > 0$ are necessary and sufficient conditions for the overall benefit of basins to achieve coordination. The value of transfer payment E is determined by negotiation between upstream and downstream local governments. The amended model is an improvement for upstream and downstream local governments and truly achieves a win-win situation. Before and after the model is revised, the overall benefits of basins remain unchanged.

IV. EQUILIBRIUM ANALYSIS

By comparing the pollution control volume, optimal pollution reduction trajectory, and increase in social welfare of the above three Nash noncooperative, centralized and cost-sharing decisions after the introduction of the cost sharing contract, the following propositions can be drawn.

Proposition 4: Compared with Nash noncooperative decision-making, after introducing an improved bilateral cost-sharing contract, the upstream and downstream local governments' pollution control and emission reduction input, pollution control amount, and social welfare effect have been correspondingly increased, and they have all been reached the level of centralized decision-making.

Proof: See Appendix D.

Proposition 5: In the three game situations, the upstream and downstream local governments' investment in pollution control and emission reduction are positively correlated with the central government's subsidy coefficient to them; upstream and downstream local governments' economic growth, pollution control, and watershed production are all positively correlated with the government's subsidy coefficient to them.

Proposition 5 shows that the willingness of upstream and downstream local governments to control pollution and reduce emissions depends not only on the sensitivity of their regional economic growth to the amount of pollution control but also on the central government's subsidy rate as well as other factors. When the natural attenuation rate (γ) of pollution control is large, the enthusiasm of upstream and downstream local governments toward pollution control and emission reduction will be reduced. By increasing the subsidy coefficient (φ_u and φ_d) of the central government to upstream and downstream local governments, the amount of pollution control can be increased.

The central government's pollution control and emission reduction subsidy policies for local governments can effectively encourage local governments to control pollution and reduce emissions, improve the living environment of regional residents and the regional investment environment, and increase the regional capital attractiveness of upstream and downstream regions.

Proof: See Appendix E.

Proposition 6: After the introduction of the bilateral cost-sharing contract, the downstream abatement cost sharing to the upstream is negatively related to the central government's subsidy rate to the upstream, and the upstream abatement cost sharing to the downstream is negatively related to the central government's subsidy rate to the downstream. This shows that after the introduction of the bilateral cost-sharing contract, the amount of abatement cost sharing among members of the basin system not only depends on their respective marginal profit as a percentage of the total marginal profit, but also on the central government's subsidy rate for upstream and downstream external abatement costs, that is, the external subsidy of the basin system affects the amount of cost sharing among its internal members.

Proof: See Appendix F.

V. NUMERICAL ILLUSTRATIONS

The establishment of the above differential game model provides a theoretical basis for upstream and downstream local governments to jointly adopt ecological compensation strategies. To illustrate the above proposition, further explain the relationship between decision parameters and make the conclusion more intuitive, we assign values to the relevant parameters with reference to the geographic location, economic development level and ecological environment of the upper and lower reaches of the Xin'an Basin. The upstream region usually serves as the "stabilizer" for the development of the economic belt. According to the statistical data from existing research, the economic aggregate and utility margin of the upper reaches of the study area are lower than those of the lower reaches.

A. XIN'ANJIANG BASIN

Xin'anJiang Basin is China's first cross-provincial ecological compensation demonstration area. The Xin'an River originates in Xiuning County, Huangshan City, Anhui Province, and flows eastward into Chun'an County, Zhejiang Province. It has a drainage area of 11,047 square kilometers and a mainstream length of 365 kilometers. The drainage area in Hangzhou is 5,718 square kilometers, and the river section is 128 kilometers long. The Xin'an River accounts for more than 60% of the annual average inflow water of Qiandao Lake, which is an important water supply in East China and an important ecological security barrier in the Yangtze River Delta.

Anhui and Zhejiang Provinces are located in the upper and lower reaches, respectively, of the Xin'an River, and the economic development level of Zhejiang Province is significantly better than that of Anhui Province. This makes the problems that the two provinces are most concerned with and most want to solve not very compatible. The upstream area focuses on making the best use of resources and quickly developing the economy, while the downstream area focuses on how to better sustain development in a good environment. If the upstream area does not pay attention to the environmental protection work of the Xin'an River in the process of its economic development, then as the pollutants are transmitted from top to bottom, the downstream area will face enormous ecological pressure. Therefore, downstream areas have a strong need to improve the water quality of the Xin'an River, which easily leads to conflicts of interest and contradictions between the two provinces. To study and resolve the contradiction between these two provinces, maximize the welfare of the basin and avoid falling into the "prisoner's dilemma", upstream and downstream local governments must jointly build an effective ecological compensation mechanism.

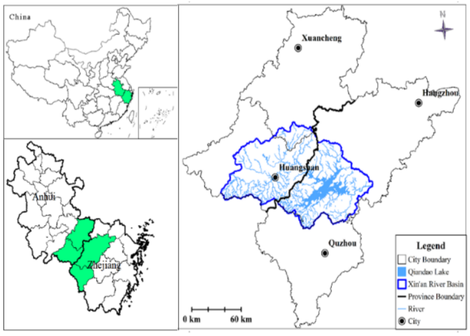


FIGURE 2. The location of Xin'anJiang basin.

Although there is a horizontal ecological compensation agreement for the Xin'an Basin, an increase in pollution control costs and industry losses in upstream areas and a gradual increase in water quality requirements in downstream areas have been found. Conflicts of interest still exist in the upper and lower reaches of Xin'an Basin. According to the per capita GDP, per capita financial income and other indicators of Huangshan city and Hangzhou city, we establish marginal quantities of $\Pi_{\mathrm{u}} = 5$ and $\Pi_{\mathrm{d}} = 9$, respectively. We set the sensitivity coefficient of the pollution control amount to local government input as $\alpha = 1$ and $\beta = 1.2$. Upstream areas need to maintain a better ecological environment and implement higher industry barriers to entry so that the upstream region will lose some development opportunities. The cost coefficients of pollution control efforts are $w_u = 2$ and $w_d = 2.5$. r and γ are limited by the current level of pollution treatment technology, and we set $r = 0.1$ and $\gamma = 0.2$, while a constellation of additional model parameters is held at $\rho = 0.2$, $s = 2$, $\varphi_u = 0.1$, and $\varphi_d = 0.2$. To more intuitively compare the equilibrium results of the differential game in the three situations of the Nash noncooperative game, centralized game situation and cost-sharing contract game, we set up TABLE 2.

From TABLE 2, it can be seen that in the Nash noncooperative game, the upstream and downstream local government pollution control and emission reduction inputs decreased by 64.3% and 35.7%, respectively, compared to the centralized game situation; the amount of pollution treatment and the increase in output decreased by 48.2% and 44.2%, respectively; and the economic growth of the basin

TABLE 2. Equilibrium results of the differential game under different game situations.

Project	I_u	I_d	y	T	μ	θ	R_u	R_d	R_T
Nash noncooperative game	18.5	36	58.7	137.5	-	-	8693.2	12616.6	21309.8
Centralized game situation	51.8	56	113.3	246.6	-	-	-	-	29542.4
Cost-sharing contract game	51.8	56	113.3	246.6	0.58	0.29	10550.9	18991.5	29542.4

decreased by 27.9%, which indicated that there was a double marginal effect in the whole basin under the Nash noncooperative game. In the game to introduce cost-sharing contracts, upstream and downstream local governments have been able to increase the level of investment in pollution control and emission reduction, the amount of pollution control, and production. The overall economic increase in the basin is 38.6% higher than that in the Nash noncooperative game scenario and reaches the level of the centralized game situation. This means that the introduction of cost-sharing contracts has led to the coordination of ecological compensation policies in the basin as a whole.

B. SENSITIVITY ANALYSIS OF OPTIMAL EQUILIBRIUM FEEDBACK

To further illustrate the relationship between model parameters, this paper uses MATLAB R2019A to determine the effect of parameter s on the amount of pollution control (a), parameter γ on the amount of pollution control (b), parameter α on the amount of pollution control (c), and parameter w_d on the amount of pollution control (d).

1) CHANGES IN s

When the impact coefficient s of the pollution control amount on the welfare effect of the basin remains unchanged, the pollution control amount of the basin continues to increase over time. The function curve has a concave characteristic, indicating that the rate of increase in pollution control is gradually decreasing and tends to be stable; that is, the process of cooperation between upstream and downstream local governments in pollution control and emission reduction is stable and controllable. At any moment, the slope of the tangent line corresponding to the optimal treatment trajectory of the pollution treatment amount increases with an increase of s, which means that as the environment improves and promotes regional economic development, local governments in the upper and lower reaches of the basin will be increasingly more willing to control pollution, and the effect of cooperation between the two areas in pollution control and emission reduction will become stronger.

2) CHANGES IN γ

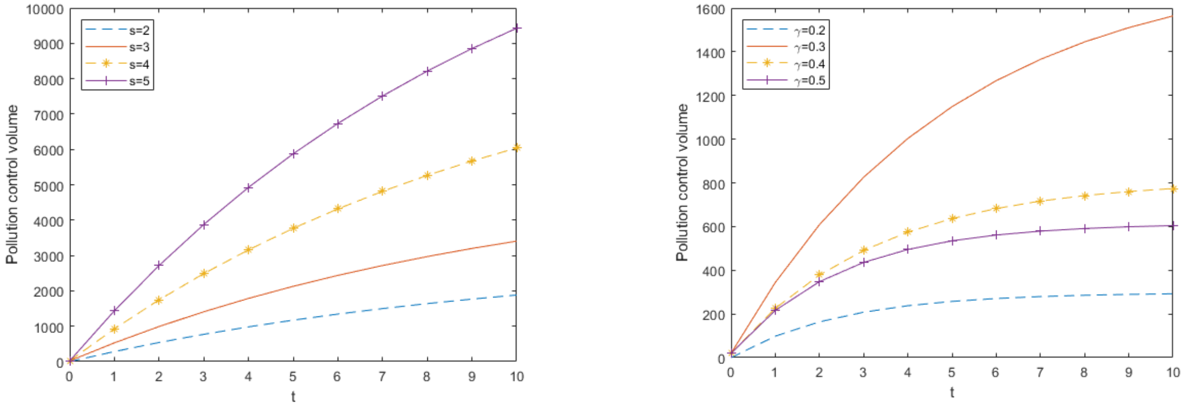
The increase in γ indicates that the aging degree of pollution treatment equipment invested in by the local government in the basin increases. At any same time, the larger γ is, the lower the value of the optimal trajectory is, and the effect of pollution control is thus less obvious. At this time, local governments need to promptly update pollution control equipment, introduce new pollution control technologies, and improve pollution control efficiency.

3) CHANGES IN α

At any moment, the amount of pollution control increases as the sensitivity coefficient α of the downstream local government's pollution control and emission reduction investment increases. This means that when the local government's pollution control and emission reduction efforts are transformed into a higher efficiency of pollution control, the effect of long-term cooperative ecological compensation in the upstream and downstream parts of the basin becomes more obvious. Comparing the two function curves in (c), the amount of pollution control in the decision to introduce a cost-sharing contract is higher than that of the Nash noncooperative game, and the amount of pollution treatment increases with the increase in the sensitivity coefficient α.

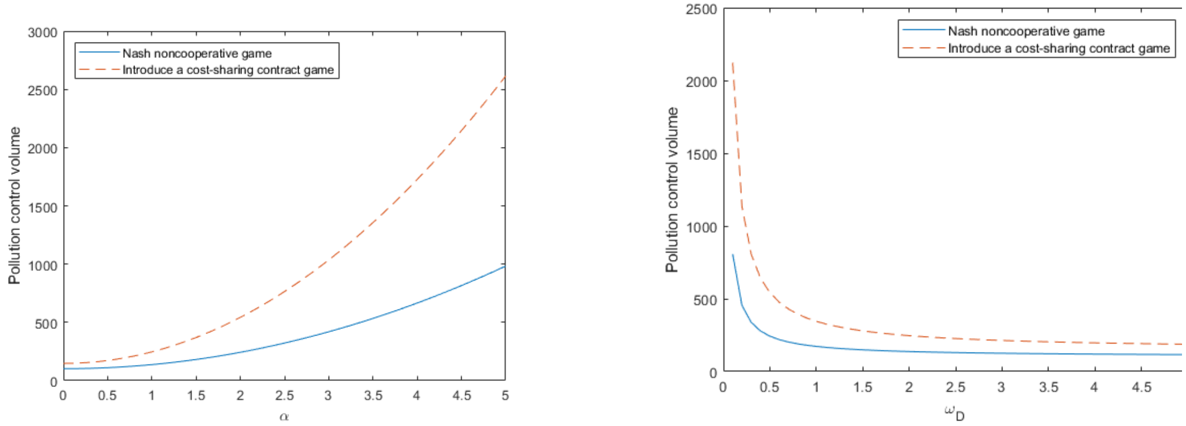
4) CHANGES IN w_D

By comparing the equilibrium results of introducing the cost-sharing contract game and Nash noncooperative game,



(a) Impacts of $\mathbf{s}$ on the amount of pollution control

(b) Impacts of $\boldsymbol{\gamma}$ on the amount of pollution control



(c) Impacts of $\boldsymbol{\alpha}$ on the amount of pollution control

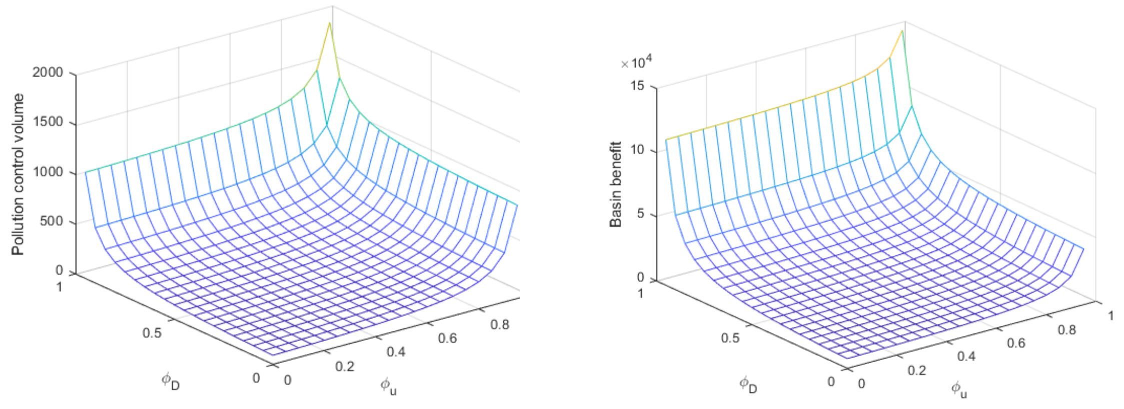
(d) Impacts of $\mathbf{w_D}$ on the amount of pollution control

FIGURE 3. **Sensitivity analysis of some parameters.**

the amount of pollution control at any time is found to decrease with the increase in the pollution control and emission reduction cost coefficient of the downstream local government. This shows that the higher the pollution control cost per unit of pollution is, the worse the effect of long-term cooperation between upstream and downstream local governments on pollution control and emission reduction.

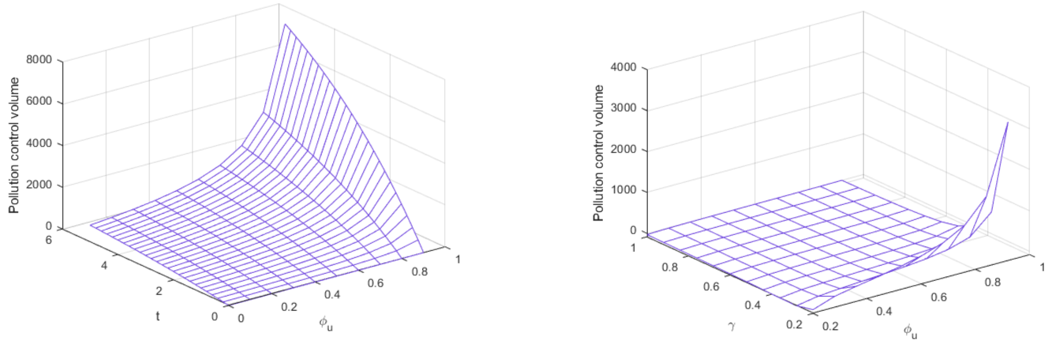
Figure 4 shows the impact of central government subsidies on pollution control and basin benefits after the introduction of a cost-sharing contract.

The amount of pollution control increases with the increase in the central government's subsidy coefficient to local governments, but the central government's subsidy rates φ_u and φ_d of the upstream and downstream areas, respectively, have different degrees of impact on the pollution control amount. According to formula (17), this is due to the difference in other relevant parameters of upstream and downstream local governments. After the introduction of a cost-sharing contract, the benefits of the upstream area increase with the increase in the central government's subsidy rate and increase with the increase in the government's downstream subsidy rate. This shows that under the influence of the internal cost-sharing mechanism in the basin, the central government can increase the benefits of the basin through subsidies upstream, and it can also increase its own benefits by increasing subsidies to its partners. Therefore, subsidies from the central government can help the basin achieve coordination. It can be seen from (g) that when the subsidy rate of the central government to the upstream and downstream local governments is the same, the amount of pollution control increases with the increase of time and gradually stabilizes; at a certain point in time, the pollution control volume increases with the increase of the central government's subsidy rate to downstream, indicating that the central government subsidy measures can be used to encourage local governments to invest in pollution control. It can be seen from (h) that the amount of pollution control decreases with the increase of the relative attenuation rate gamma, and increases with the increase in the subsidy rate of the central government to the downstream. This shows that when the relative attenuation rate of pollution control is relatively large, it will have a negative impact on the enthusiasm of local governments to invest in pollution control. At this time, measures such as updating pollution treatment equipment and upgrading



(e) Impacts of $\boldsymbol{\varphi}_{\mathbf{u}}$ and $\boldsymbol{\varphi}_{\mathbf{d}}$ on the amount of pollution control

(f) Impacts of $\boldsymbol{\varphi}_{\mathbf{u}}$ and $\boldsymbol{\varphi}_{\mathbf{d}}$ on basin benefits



(g) Impacts of $\boldsymbol{\varphi}_{\mathbf{u}}$ and $\mathbf{t}$ on the amount of pollution control

(h) Impacts of $\boldsymbol{\varphi}_{\mathbf{u}}$ and γ on the amount of pollution control

FIGURE 4. **The common influence of the two parameters on the equalization result.**

pollution treatment technology should be taken in a timely manner.

VI. CONCLUSION

This paper uses differential game theory to study the problem of horizontal and vertical ecological compensation cooperation among the upstream local government, the downstream local government and the central government. The time variable is introduced when constructing the model, and the influence of the natural decay rate of the pollution control amount and the pollutant recycling rate are considered. We construct three differential game strategies of the Nash noncooperative game, centralized game, and cost-sharing contract game and use inverse induction to obtain the game equilibrium value of these three strategies. Therefore, the model is dynamic and stable as a whole, and the results obtained by the model are reasonable and reliable. Finally, some conclusions are obtained through an analysis of numerical examples.

The cost sharing between upstream and downstream local governments makes the respective pollution control investment level, amount of pollution control, and economic increase in the upstream and downstream local governments reach the highest level of all three strategies and is equivalent to the centralized strategy. Effectively realizing the coordination of ecological compensation strategies in the basin not only helps realize the maximization of economic benefits but also improves the ecological environment of the basin. The equilibrium solutions of all three strategies can be established independently of the time parameter and have some practical implications for environmental management.

A basin is a special economic system, and the decisions made by upstream local governments often have the most direct impact on the overall basin environment. Therefore, downstream local governments and the central government should prioritize ensuring the long-term benefits of upstream local governments by establishing a sound vertical transfer payment mechanism and signing cost-sharing contracts. On the one hand, methods such as tax adjustment, policy preference, and special funds for ecological protection have been adopted to improve the central financial compensation mechanism to encourage upstream areas to control pollution and reduce emissions. On the other hand, the horizontal compensation mechanism should be improved as an effective supplement to central ecological compensation. The introduction of cost-sharing contracts not only reduces the financial pressure placed on the central government to bear ecological compensation but also effectively promotes the balance of revenue among regions, optimizes the distribution of resources among basins, and improves the efficiency of ecological compensation.

This article focuses on the boosting effect of cost-sharing contracts on pollution control and basin economic growth, as well as the effect of central government subsidies on upstream and downstream local governments' cooperative pollution control and emission reduction strategies. From a macro perspective, the conclusions of this article have confirmed the development trajectory of the watershed multiagent ecological compensation cooperation model, which can indicate an optimized path for the watershed environmental management model. From a micro perspective, the conclusions of this article can provide a theoretical basis for the central government and the upstream and downstream local governments in the basin to make scientific decisions regarding environmental management, pollution control and emission reduction incentive mechanism design as well as which game structure to choose.

Our research also has some limitations. The assumptions of the model building in this article are ideal, and the model involves fewer influencing factors. Future research can consider the assumption that the income function is a nonlinear function of pollution control, emission reduction input and economic increase. We can also take the subsidy coefficient of the central government as a decision variable, which one direction for future research.

APPENDIX A
PROOF OF PROPOSITION 1

From the foregoing, we can see that the optimal economic increase function of the upstream local government at time t is:

$$R_u^a = \max_{I_u^b \geq 0, I_d^b \geq 0} \int_0^\infty e^{-\rho t} \left\{ J_u \cdot T(t) - \left(1-\varphi_u^a\right) C_u\left(I_u^a\right) \right\} dt \tag{20}$$

Let $R_u^a(y^a) = e^{-\rho t} F_u(y^a)$, according to the optimal control theory, $F_u(y^a)$ satisfies Hamilton-Jacobi-Bellman equation for $\forall y^b \geq 0$.

$$\begin{aligned} \rho F_u\left(y^a\right) = \max_{I_u^a \geq 0} & \left[J_u \cdot T(t) - \left(1-\varphi_u^a\right) C_u\left(I_u^a\right) \right. \\ & \left. + F_u'\left(y^a\right)\left(\alpha \cdot I_u^a + \beta \cdot I_D^a - (\gamma - r) y^a\right) \right] \end{aligned} \tag{21}$$

The Hessian of the I_u^a and I_D^a is:

$$H = \begin{bmatrix} -\left(1-\varphi_u^a\right) w_u & 0 \\ 0 & -\left(1-\varphi_d^a\right) w_D \end{bmatrix} \tag{22}$$

By $|H| > 0$ and $-\left(1-\varphi_u^a\right) w_u < 0$, we can see that Hessian Matrix negative semi-definite. Therefore, $\rho F_T\left(T_a^*\right)$ is a concave function, we can find the first-order partial derivative of I_u^a for $\rho F_T\left(T_a^*\right)$, and the maximum value can be obtained:

$$I_u^{a^*} = \frac{\alpha F_u'\left(y^a\right)}{\left(1-\varphi_u^a\right) w_u} \tag{23}$$

Similarly, we can set:

$$\begin{aligned} \rho F_d\left(y^a\right) = \max_{I_u^a \geq 0} & \left[\Pi_d \cdot T - (1-\varphi_d) C_d\left(I_d^a\right) \right. \\ & \left. + F_d'\left(y^a\right)\left(\alpha \cdot I_u^a + \beta \cdot I_d^a - (\gamma - r) y^a\right) \right] \end{aligned} \tag{24}$$

$\rho F_d(y^a)$ is a concave function about I_d^a, we can find the first-order partial derivative of I_D^a for $\rho F_d(y^a)$, and the maximum value can be obtained:

$$I_d^{a^*} = \frac{\beta F_d'\left(y^a\right)}{\left(1-\varphi_d^a\right) w_d} \tag{25}$$

Substituting (23) and (25) into function (21) and (24), we can obtain:

$$\begin{cases} \rho F_u\left(y^a\right) = \left(\Pi_u s - (\gamma - r) F_u'\left(y^a\right)\right) y^a + \Pi_u T_0 \\ \qquad + \dfrac{\alpha^2 \left(F_u'\left(y^a\right)\right)^2}{2\left(1-\varphi_u^b\right) w_u} + \dfrac{\beta^2 F_d'\left(y^a\right) F_u'\left(y^a\right)}{\left(1-\varphi_d^b\right) w_d} \\ \rho F_d\left(y^a\right) = \left(\Pi_d s - (\gamma - r) F_d'\left(y^a\right)\right) y^a + \Pi_d T_0 \\ \qquad + \dfrac{\beta^2 \left(F_d'\left(y^a\right)\right)^2}{2\left(1-\varphi_d^a\right) w_d} + \dfrac{\alpha^2 F_d'\left(y^a\right) F_u'\left(y^a\right)}{\left(1-\varphi_u^a\right) w_u} \end{cases} \tag{26}$$

Suppose the linear structures of $\rho F_u(y^a)$ and $\rho F_d(y^a)$ are $F_u(y^a) = k_2 y^a + b_2$ and $F_d(y^a) = k_3 y^a + b_3$, it's easy to know $F_u'(y^a) = k_2$ and $F_d'(y^a) = k_3$. Substituting $F_u(y^a)$, $F_d(y^a)$, $F_u'(y^a)$ and $F_d'(y^a)$ into (44) and (45), we can obtain:

$$\begin{cases} k_2 = \dfrac{\Pi_u s}{m} \\ k_3 = \dfrac{\Pi_d s}{m} \\ b_2 = \dfrac{\Pi_u T_0}{\rho} + \dfrac{\alpha^2 (\Pi_u s)^2}{2\rho\left(1-\varphi_u^a\right) w_u m^2} + \dfrac{\beta^2 s^2 \Pi_u \Pi_d}{\rho\left(1-\varphi_D^a\right) w_D m^2} \\ b_3 = \dfrac{\Pi_d T_0}{\rho} + \dfrac{\beta^2 (\Pi_d s)^2}{2\rho\left(1-\varphi_d^a\right) w_d m^2} + \dfrac{\alpha^2 s^2 \Pi_u \Pi_d}{\rho\left(1-\varphi_u^a\right) w_u m^2} \end{cases} \tag{27}$$

Substituting k_2 and k_3 into (26), we can get $I_u^{a^*}$ and $I_d^{a^*}$, then substituting $I_u^{a^*}$ and $I_d^{a^*}$ in (3), y^{a^*} and T_a^* can be obtained. Further, substituting (27) in $F_u\left(y^{a^*}\right)$ and $F_d\left(y^{a^*}\right)$, the equilibrium solution $R_u^{a^*}$ and $R_d^{a^*}$ can be obtained.

APPENDIX B
PROOF OF PROPOSITION 2

According to the optimal control theory, from equation (15), it can be seen that the optimal revenue function of the upstream local government at time t is:

$$R_u^c = \max_{I_u^c \ge 0} \int_t^{\infty} e^{-\rho t} \left\{ \Pi_u \cdot T - (1-\varphi_u-\theta)\frac{w_u}{2} I_u^2 - \mu \frac{w_d}{2} I_d^2 \right\} dt \tag{28}$$

Let $R_u^c = e^{-\rho t} F_u(y^c)$, $F_u(y^c)$ satisfies the HJB equation for any $y^c \ge 0$,

$$\rho F_u\left(y^c\right) = \max_{I_u^c \ge 0} \Big[\Pi_u \cdot T(t) - (1-\varphi_u-\theta)\frac{w_u}{2} I_u^2 - \mu \frac{w_d}{2} I_d^2 + F_u'\left(y^c\right)\left(\alpha \cdot I_u^c + \beta \cdot I_d^c - (\gamma - r) y^c\right) \Big] \tag{29}$$

Find the first-order partial derivative of I_u^c for equation (29) and set it to zero, we can get

$$I_u^c = \frac{\alpha F_u'(y^c)}{w_u(1-\varphi_u-\theta)} \tag{30}$$

Similarly, the optimal income function of the downstream local government at time t is:

$$R_d^c = e^{-\rho t} F_d\left(y^c\right) \tag{31}$$

According to the optimal control theory, $F_d(y^c)$ satisfies the following equation for all $y^c \ge 0$:

$$\rho F_d\left(y^c\right) = \max_{I_u^c \ge 0} \Big[\Pi_d \cdot T - (1-\varphi_d-\mu)\frac{w_d}{2} I_d^2 - \theta \frac{w_u}{2} I_u^2 + F_d'\left(y^c\right)\left(\alpha \cdot I_u^c + \beta \cdot I_d^c - (\gamma - r) y^c\right) \Big] \tag{32}$$

Find the first derivative with respect to I_d^c for the formula (32), and set it to zero, we can get:

$$I_d^c = \frac{\beta F_d'(y^c)}{w_d(1-\varphi_d-\mu)} \tag{33}$$

When the bilateral cost-sharing contract is introduced, the overall economic growth of the basin is higher than that of decentralized decision-making, and reaches the level of centralized decision-making. Reflected in the model, that is, the need to make

$$I_d^c = I_d^{b*}$$
$$I_u^c = I_u^{b*}$$

Therefore

$$\begin{cases} \mu = 1 - \varphi_u - \dfrac{(1-\varphi_u)(\rho+\gamma-r) F_u'(y^c)}{s(\Pi_u + \Pi_d)} \\ \theta = 1 - \varphi_d - \dfrac{(1-\varphi_d)(\rho+\gamma-r) F_d'(y^c)}{s(\Pi_u + \Pi_d)} \end{cases} \tag{34}$$

Substituting formulas (30) and (33) into formulas (29) and (32) respectively, we can get:

$$\rho F_u\left(y^c\right) = \left(\Pi_u s - (\gamma-r) F_u'\left(y^c\right)\right) + \Pi_u T_0 + \frac{\alpha^2 F_u'(y^c) F_d'(y^c)}{w_d(1-\varphi_d-\mu)} - \frac{\alpha^2 \mu F_u'(y^c) F_d'(y^c)}{2w_d(1-\varphi_d-\mu)^2} + \frac{(\beta F_u'(y^c))^2}{2w_u(1-\varphi_u-\theta)^2} \tag{35}$$

$$\rho F_d\left(y^c\right) = \left(\Pi_d s - (\gamma-r) F_d'\left(y^c\right)\right) + \Pi_d T_0 + \frac{(\alpha F_d'(y^c))^2}{2w_d(1-\varphi_d-\mu)} - \frac{\theta(\beta F_u'(y^c))^2}{2w_u(1-\varphi_u-\theta)} + \frac{\beta^2 F_u'(y^c) F_d'(y^c)}{w_u(1-\varphi_u-\theta)} \tag{36}$$

According to the characteristics of equations (35) and (36), let $F_u(y^c)$ and $F_d(y^c)$ have the analytical expressions of y^c as $F_u(y^c) = k_4 y^c + b_4$ and $F_d(y^c) = k_5 y^c + b_5$. Where k_4, b_4, k_5 and b_5 are all constants, It is easy to know that $F_u'(y^c) = k_4$, $F_d'(y^c) = k_5$.

Substituting $F_u(y^c)$, $F_d(y^c)$, $F_u'(y^c)$ and $F_d'(y^c)$ into equations (35) and (36), we can get e_3^*, e_5^*. Substituting e_3^* and e_5^* into $F_u'(y^c)$ and$F_d'(y^c)$ respectively, we can get $F_u'(y^c)^*$, $F_d'(y^c)^*$. Then substituting $F_u'(y^c)^*$, $F_d'(y^c)^*$ into equations (30), (33), (34), we can get I_d^{c*}, I_u^{c*}, μ^* and θ^*:

$$\begin{cases} \mu^* = \dfrac{\Pi_d(1-\varphi_u)}{\Pi_u + \Pi_d} \\ \theta^* = \dfrac{\Pi_u(1-\varphi_d)}{\Pi_u + \Pi_d} \\ I_u^{c*} = \dfrac{(\Pi_u + \Pi_d) s\alpha}{w_u m(1-\varphi_u)} \\ I_d^{c*} = \dfrac{(\Pi_u + \Pi_d) s\beta}{w_d m(1-\varphi_d)} \end{cases}$$

APPENDIX C
PROOF OF PROPOSITION 3

This process of proof is akin to Proposition 1, and it is easy proved.

APPENDIX D
PROOF OF PROPOSITION 4

Differencing between the upstream local government's optimal pollution control and emission reduction investment before and after the introduction of the cost sharing contract can be obtained:

$$\begin{cases} I_u^{c*} - I_u^{a*} = \dfrac{\Pi_d s\alpha}{w_u m(1-\varphi_u)} > 0 \\ I_d^{c*} - I_d^{a*} = \dfrac{\Pi_u s\alpha}{w_d m(1-\varphi_d)} > 0 \end{cases} \tag{37}$$

Therefore $I_u^{c*} > I_u^{a*}$ and $I_d^{c*} > I_d^{a*}$, in the same way, we can get $I_u^{c*} = I_u^{b*}$, $I_d^{c*} = I_d^{b*}$, $I_u^{b*} > I_u^{a*}$ and $I_d^{b*} > I_d^{a*}$.

We make a difference between the amount of pollution control and the increase in output before and after the introduction of the cost sharing contract. We can get that, (38), as shown at the top of the next page.

Therefore $y^{c*} > y^{a*}$ and $T^{c*} > T^{a*}$, in the same way, we can get $y^{c*} = y^{b*}$, $y^{b*} > y^{a*}$, $T^{c*} = T^{b*}$ and $T^{b*} > T^{a*}$.

APPENDIX E
PROOF OF PROPOSITION 5

Seeking the first-order partial derivative of I_u^{c*} with respect to φ_u in (21), we can get:

$$\frac{\partial I_u^{c*}}{\partial \varphi_u} = \frac{(\Pi_u + \Pi_d) s\alpha}{w_u m(1-\varphi_u)^2} > 0 \tag{39}$$

In the same way, $\frac{\partial I_d^{c*}}{\partial \varphi_d} > 0$.

$$\begin{cases} y^{c^*} - y^{a^*} = \left[\dfrac{\alpha^2 s^2 \Pi_d}{(\gamma - r)(1 - \varphi_u) w_u m} + \dfrac{\beta^2 s^2 \Pi_u}{(\gamma - r)(1 - \varphi_d) w_d m}\right]\left(1 - e^{-(\gamma - r)t}\right) + y_0 e^{-(\gamma - r)t} > 0 \\ T^{c^*} - T^{a^*} = s\left[\dfrac{\alpha^2 s^2 \Pi_d}{(\gamma - r)(1 - \varphi_u) w_u m} + \dfrac{\beta^2 s^2 \Pi_u}{(\gamma - r)(1 - \varphi_d) w_d m}\right]\left(1 - e^{-(\gamma - r)t}\right) + y_0 e^{-(\gamma - r)t} > 0 \end{cases} \tag{38}$$

Seeking the first-order partial derivative of T^{c^*} with respect to φ_u in (21), we can get:

$$\frac{\partial T^{c^*}}{\partial \varphi_u} = \frac{(\Pi_u + \Pi_d)\alpha^2 s^2}{(\gamma - r) w_u m (1 - \varphi_u)^2}\left(1 - e^{-(\gamma - r)t}\right) + y_0 e^{-(\gamma - r)t} > 0 \tag{40}$$

In the same way, $\frac{\partial T^{c^*}}{\partial \varphi_d} > 0$.

To sum up, it is easily prove that $\frac{\partial R^{c^*}}{\partial \varphi_u} > 0$ and $\frac{\partial R^{c^*}}{\partial \varphi_d} > 0$, the conclusion of proposition 5 can be attained, thus completing the proof.

APPENDIX F
PROOF OF PROPOSITION 6

Seeking the first-order partial derivatives of φ_u and φ_d for μ^* and θ^* in Eq. (16), we can get:

$$\begin{cases} \dfrac{\partial \mu^*}{\partial \varphi_u} = -\dfrac{\Pi_d}{\Pi_u + \Pi_d} < 0 \\ \dfrac{\partial \theta^*}{\partial \varphi_d} = -\dfrac{\Pi_u}{\Pi_u + \Pi_d} < 0 \end{cases} \tag{41}$$

Therefore, it can be seen that the external subsidies affect the cost sharing among its internal members.

Make a difference in the economic increase of the upstream local government before and after the introduction of the cost sharing contract. we can get:

$$R_u^{c^*} - R_u^{a^*} = e^{-\rho t}[\Pi_u s\left(y^{c^*} - y^{a^*}\right) + \frac{\beta^2 s^2\left(\Pi_u^2 - \Pi_u \Pi_d\right)}{2\rho(1 - \varphi_d)^2 w_d m^2} + \frac{\alpha^2 s^2 \Pi_u \Pi_d}{2\rho(1 - \varphi_u) w_u m} \tag{42}$$

Seeking the first-order partial derivatives of $R_u^{c^*} - R_u^{a^*}$ for φ_u, we can get:

$$\frac{\partial\left(R_u^{c^*} - R_u^{a^*}\right)}{\partial \varphi_u} = \left[\frac{\beta^2 s^2\left(\Pi_u^2 - \Pi_u \Pi_d\right)}{2\rho(1 - \varphi_d)^2 w_d m^2} + \frac{\left(1 - e^{-(\gamma - r)t}\right)\alpha^2 s^2 \Pi_d}{(\gamma - r)(1 - \varphi_u) w_u m}\right] e^{-\rho t} \tag{43}$$

Clearly, $\frac{\partial\left(R_u^{c^*} - R_u^{a^*}\right)}{\partial \varphi_u} > 0$. Proposition 6 is proved.

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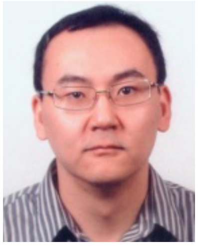

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