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Optimal Railway Disruption Bridging Using Heterogeneous Bus Fleets

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ABSTRACT Mass Rapid Transit (MRT) systems, i.e. subway systems, all over the world, are experiencing an increase in ridership. This also means that in case of an unplanned MRT service disruption, the number of affected passengers is larger, requiring a fast and comprehensive response. In this paper, we thus study the disruption management of MRT systems. We develop an optimization model to identify the optimal bridging plan in response to an MRT disruption, so that the negative effects of a disruption could be minimized. Our approach supports deployment of multiple types of bridging buses, reflecting the diversity of vehicle types in a typical public transportation provider's vehicle fleet. The optimization objective of our approach is to decrease the travel delay of passengers and increase the number of passengers who can be served. We demonstrate the effectiveness of our approach on a hypothetical case study in the central business district of Singapore. Moreover, we validate our analytical results with microscopic simulation, showing that our simplified analytical optimization approach can be used for disruption response planning. Some deviations indicate, however, that a combined simulation and optimization approach yields better results to obtain an effective bridging plan.

INDEX TERMS Data-driven optimization, mass rapid transit systems, microscopic simulation, reactive planning.

I. INTRODUCTION

As cities around the world are growing in population, public transportation becomes an even more important part of each city's transport backbone. For example, in 2015 in Beijing, approximately 45% of all trips were made by public transportation [24]. This comes as no surprise as public transportation offers a larger capacity and in highly congested areas even faster travel speeds compared to private vehicles. Often, public transportation infrastructure is developed complementary to road infrastructure, either underground (e.g. subway) or in the form of dedicated road space (e.g. Bus Rapid Transit (BRT) systems [20]) and is thus able to offer faster travel speeds. In Curitiba, Brazil, it has been reported that considering only peak hours, the average speed of the BRT system is 20 km/h compared to an average speed of 8 km/h for mixed traffic that includes private cars [20]. Moreover, it has been shown that the speed of taxis in New York City decreases starting from the city boundary to the city center (e.g. Manhattan) [28].

In addition to shorter travel times in some cities, public transportation is also often cheaper, making it the preference for many. However, frequent disruptions of public transportation can push commuters back to using private transportation modes. Unfortunately, disruptions in public transportation are inevitable. For example, in Melbourne almost 16,000 unplanned disruptions occurred only in the first half of 2011 [25]. During only one day of the same year, Singapore experienced disruption of 11 stations in the duration of 5 hours, with more than 100,000 passengers affected [12]. Between October 2018 and March 2021, 7 major disruptions happened in Singapore on different lines usually affecting multiple stations and lasting a few hours each [2]. The reason behind the majority of those disruptions

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was a power fault, although unfortunately in one event, one man was killed by a train after intruding a tunnel [1]. Even though this number does not seem large, those seven incidents represent only disruptions that were severe enough to hit the news headlines.

As disruptions cannot be completely avoided, there are two ways of easing their negative impact on the affected ridership – by *proactive* and *reactive* transportation planning. *Proactive* planning refers to designing a transportation system in such a way that it is more resilient, while *reactive* planning proposes what to do once a disruption occurs. Although designing transportation systems with the goal of minimizing the possibility of disruption is required (e.g. [12], [23], [24]), disruptions can never fully be ruled out. Therefore, in our work we focus on reactive planning by proposing how to develop an optimal bridging plan for the disruption period. Unlike during the normal operation hours when operators tend to organize their service with the goal of reducing their operation costs while maximizing benefits for passengers, during the disruption the main goal is to reduce travel delay for as many affected passengers as possible [7], [9]. In this paper, we present an optimization model that follows these two objectives – to *reduce travel delay* and *increase the number of served passengers* during a disruption.

Our model is based on our previous work [21], but has been significantly revised to enable the use of heterogeneous vehicle fleets as well as a passenger-based penalty parameter to avoid underserving of certain passengers or to prioritize passenger groups if needed. We evaluate our model using a hypothetical Mass Rapid Transit (MRT) disruption in the central business district of Singapore. Additionally, we extended the microscopic mobility simulator CityMoS [26] to validate our mathematical optimization model and to identify shortcomings stemming from the necessary simplifications for the model to be computationally feasible. We also show how a combined approach could be used to derive effective bridging plans in a cost and time-efficient way.

The remainder of the paper is structured as follows: In Section II we provide an overview of related work and how our method differs from the existing approaches. We present our optimization model in detail in Section III. Section IV describes our case study of a hypothetical MRT disruption. We discuss the results obtained with the optimization model in Section V. For validation purposes we introduce and compare a simulation-based approach in Section VI. Finally, Section VII concludes the paper.

II. LITERATURE REVIEW

Once a disruption happens, first it is important to understand how many passengers are affected in order to come up with a bridging plan with sufficient capacity to accommodate the affected passengers. However, even though the number of affected passengers might be estimated accurately in the first place, not all passengers are going to wait for a bridging service to continue with their travel plans, and an accurate demand modeling of the affected passengers is thus needed. Passenger behavior can be modeled using a logit model [6] or effects of balking and reneging can be considered [25], where the former refers to the passengers who leave the station immediately after a disruption happens and the latter one denotes the passengers who gradually decided to leave the station before the bridging vehicles were able to pick them up. Eventually, only the ones who stayed long enough can be served by the bridging service. In our model, we assume that the passengers will wait up to 30 minutes for a bridging vehicle to pick them up after which we assume that they would renege and the system does not need to serve them anymore. We take 30 minutes as a proxy of how long people are willing to wait. However, the maximum waiting time of passengers is an input parameter of our model and as such can be set up to some other value as well.

After a passenger demand is assumed, an optimal bridging plan must be developed. The challenge of organizing a bridging service can be divided into two sub-problems: finding a subset of potential bridging routes and assigning bridging vehicles to the chosen routes. However, even before finding the potential bridging routes and assigning vehicles to the routes, one can look at the transportation network properties in order to understand which stations are more important than others [16]. In our model, we are considering all stations and are not looking into network properties of the transportation system in order to distinguish among the stations.

Once a set of potential stations is derived, either in such a way that all stations are considered or only subset of them, the bridging routes can run in parallel with MRT lines [12] or a column generation algorithm can be used [13], [19]. Column generation is a well-known algorithm in which solving one complex problem begins with a small, manageable part of a problem, followed by solving that part, analyzing that partial solution to discover the next part of the problem, and then resolving the enlarged model. The described process is repeated until it achieves a satisfactory solution for the entire problem. Finally, bridging vehicles do not necessarily need to be assigned to only one route, but can change among different routes, which makes the problem of assigning bridging vehicles to the selected routes even more complex [11]. In our optimization model, we choose that the bridging vehicles run in parallel with MRT lines and assume that bridging vehicles stay on the same assigned route until the end of disruption.

The most common way for organizing a bridging service is to deploy bridging buses, which can be either mobilized from bus depots or retracted from their regular routes [7], [22]. However, bridging buses are not the only type of vehicles used in practice. In [27] the authors investigated how to partner with taxi companies to achieve a quicker response plan when a disruption occurs. The downside of using taxis as the only bridging vehicles is that their capacity is limited and consequently they cannot serve all affected passengers. However, in light of Mobility as a Service (MaaS) developments, different mobility packages are likely going to be available. In that sense, it is reasonable to assume that different passengers will be willing to pay different prices for the transportation service and the ones paying for a more reliable service could be prioritized when a disruption happens if desired by the provider. The advantage of our model is we use a penalty parameter for unserved passengers that could be set to different values if passenger prioritization is required.

In a more broad sense, bus bridging services do not necessarily only have to be deployed in case of an MRT disruption. Namely, there is a body of research focusing on how to design temporary bus services and their routes early in the morning [14] or late in the night [15] to help affected passengers to transfer between different subway lines during that time. It has been shown that deploying bridging buses during those periods can improve connectivity in large-size subway networks, such as for example in Beijing. Although in both cases, bridging vehicles are used in order to substitute for inefficiencies in subway/MRT systems, in our work we focus on disruptions, which are sudden and unplanned. In that sense, an optimization model for disruptions has to be simplistic enough to be able to work in a real-time. On the other hand, first/last train timetabling is a problem that happens every day and thus is a part of proactive transportation planning, rather than reactive transportation planning strategies which are needed for disruptions.

The effectiveness of the proposed model is shown in comparison with alternative strategies on a real-world scenario in the central business district of Singapore. Using information collected by transportation smart cards (e.g. EZ-Link cards in Singapore) and public MRT/bus line information, we calculated the origin-destination passenger demand and the available complimentary capacity of the existing buses, information which is then used to develop an optimal bridging plan. Utilizing available complimentary capacity of the existing buses was previously proposed by Jin *et al.* [12], [13]. However, the aforementioned work did not consider that both the existing buses and the introduced ones can have different capacities, thus potentially miscalculating the available complimentary capacity.

In this paper, we consider more practical situations where multiple types of vehicles are available for bridging services. This assumption is more realistic because in reality, public transportation companies usually operate multiple types of vehicles. For example, Singapore's bus fleet consists of *single decker*, *double decker* and *articulated* buses. More specifically, both our approach and the approach of [21] identify the best bridging routes and their corresponding headways that minimize the travel delay and maximize the number of served passengers. However, the approach proposed in this paper additionally determines the best type of vehicles to be allocated on each of selected bridging routes. Compared with the approach proposed in [21], our model has more binary decision variables due to the consideration of bridging vehicle types. However, the increase in computational time is marginal. In particular, for the case we studied later, both optimization problems could be solved in a few minutes, which makes them suitable to use in a real-time situation.

Validation of an analytical model and evaluation of the performance of a bridging strategy can be carried out by means of simulation. Simulation of public transportation systems is often *macroscopic* in nature [17], meaning that individual vehicles such as buses and trains are not modeled as separate entities, but analysis is carried out using flows and capacities. There are several tools that support a *microscopic* analysis and that are able to capture effects such as bus bunching [3] or delays caused by traffic lights and heavy traffic. The most commonly used tools include VISSIM [10], SUMO [5], or MATSim [18]. We found that the majority of the available tools either does not support both relevant modes (i.e. bus and rail-based traffic), is not easily extendable or is limited in the other ways relevant for our study. We thus decided to base all simulation studies in this work on the CityMoS platform [26].

To conclude, contributions of our paper are threefold: our optimization model (i) supports utilization of different types of vehicles used for bridging purposes, (ii) allows prioritization among the affected passengers and (iii) is validated through a microscopic simulation. The first two points allow designing a personalized bridging service in case of an MRT disruption, which is aligned with the idea of MaaS. Namely, although it is impossible to offer mobility services without any disruptions, the passengers who will pay a higher premium can also expect to get a premium service both under normal circumstances, as well as during disruptions. Finally, there are plenty of studies on subway/MRT disruptions in which various mathematical optimization models were proposed, but only a few papers offered validation of the proposed theoretical model with simulation.

III. OPTIMIZATION METHODOLOGY

We developed a mathematical optimization model to derive the best bridging plan in the case of an MRT service disruption with two goals: to reduce the travel delay and increase the number of served passengers. Our model considers cases where multiple types of vehicles with different capacities are available for bridging services. Moreover, it also considers the existing bus service, that is, when an MRT disruption occurs, the disrupted passengers can either use the existing buses or the services provided by the bridging vehicles. The proposed model determines both the bridging service routes and the frequency of vehicles on each route.

A. IN/OUT PASSENGER FLOWS DATASET

The basis of our approach is a comprehensive passenger dataset of Singapore MRT and bus services for a duration of three months. Each record in this dataset consists of a timestamp of tap-in/out associated with an MRT station/bus stop identification, allowing us to reconstruct the traveled route. Additionally, we incorporated GPS coordinates of MRT stations/bus stops, as well as official records on MRT/bus service and MRT/bus line data including operational starting time and ending time, traveling time and frequency of MRTs/buses. Based on the dataset and MRT/bus line data, we elicit the following information:

- the number of passengers traveling between each pair of origin-destination stations;
- the time table and routes of MRT/bus services;
- the travel time of passengers;
- the available capacity of each MRT/bus line.

which is then used as an input for our model.

B. OPTIMIZATION MODEL

Before presenting our optimization model formulation, we have to define the following sets:

- $\mathcal{S}$ denotes the set of bridging buses;
- $\mathcal{R}$ denotes the set of routes r;
- $\mathcal{R}^0$ and $\mathcal{R}^+$ denote the set of the existing bus routes and the candidate bridging routes, respectively;
- k denotes a passenger group that includes all passengers who travel from origin o_k to destination s_k;
- $\mathcal{K}$ denotes the union of disjoint passenger groups k;
- $\mathcal{R}^k$ is used to denote the set of routes that connect origin o_k and destination s_k, that is, passengers in group k can and only take routes in $\mathcal{R}^k$;
- L_r denotes the set of edges/legs l for the route r. Our model aims at minimizing the total travel time of affected passengers in the disruption period, including both their riding time on either the existing or the introduced bridging vehicles and waiting time to board.

Furthermore, in order to discretize continuous time into time slots, we adopt the time-space network proposed by Jin *et al.* to model the time-dimension [13]. For each passenger group k, we thus discretize the whole time period into $\bar{u}$ periods associated with a demand $d_{(k,u)}$ such that $\sum_u d_{(k,u)} = D_k$, where D_k denotes the number of affected passengers in group k.

Our optimization model allows the usage of bridging vehicles with different capacities. Let $\mathcal{V} \subseteq S$ denote the set of vehicles of type v. We use Q^v to denote the capacity of a vehicle type v. Furthermore, let (r, h, n) represent the n^{th} round of a vehicle on the route r when the headway is h. The set of (r, h, n) is denoted by $\mathcal{B}_{(r,h)} = (r, h, n), \forall n = 1, 2, 3, \ldots, N_{(r,h)}$, where $N_{(r,h)}$ is the number of total rounds of bridging vehicles on the route r in the given period when the headway is set as h. One round is defined as a trip a bridging vehicle makes starting from its first assigned station to the last one. The union of $\mathcal{B}_{(r,h)}$ for all $h \in \mathcal{P}_r$ is denoted by notation $\mathcal{B}_r$.

The passengers in the group k arrive at the station o_k at time $\tilde{t}_{(k,u)}$, wait for a vehicle on the route $r \in \mathcal{R}^k$, board and travel on it for $c_{(k,r)}$ units of time. If the arriving vehicle is full, the passengers have to wait for the next one. Let $w((k, u), (r, h, n))$ denote the waiting time of passengers in the group (k, u) when taking the vehicle (r, h, n). We have: $w((k, u), (r, h, n)) = t_{((r,h,n),o_k)} - \tilde{t}_{(k,u)}$, where $t_{((r,h,n),o_k)}$ denotes the time when the vehicle (r, h, n) arrived at the station o_k, i.e. the origin station for the passenger group k.

Moreover, the passengers from group k cannot take vehicles that arrived at station o_k before their arrival time and we assume that they will not be willing to wait for a time longer than a limit $\bar{w}$. In addition, the passengers will only take vehicles that would transport them from the station o_k to s_k, that is, vehicles on the route $r \in \mathcal{R}^k$. Hence, we define the set Ω that excludes all impossible combinations: $((k, u), (r, h, n) : \Omega = \{((k, u), (r, h, n)) : t_{((r,h,n),o_k)} - \tilde{t}_{(k,u)} \geq 0, w((k, u), (r, h, n)) \leq \bar{w}, r \in \mathcal{R}^k\}$, where $\bar{w}$ is the limit of waiting time. We then generate the matrix of $w_{((k,u),(r,h,n))}$ for all (k, u) and (r, h, n) based on the travel time on each link and the bridging plan to use it as the input coefficients for our optimization model.

Our model will not only select the bridging routes, but at the same time it will also determine the frequency and headway of the vehicles on those selected routes and the allocation of available vehicle resources among each route. Here we make an assumption that on each route, only one type of vehicle can be allocated. We can then define a discrete set of bridging deployment plans as follows: $\mathcal{P}_r := \{(r, h) : h \in I, \forall h_r^{min} \leq h \leq h_r^{max}\}$, where each plan (r, h) is characterized by the route index r and the headway h; h_r^{min}, h_r^{max} denote the minimum and maximum allowed headways for route r, respectively. Let $\mathcal{P}^+$ be the union of $\mathcal{P}_r, \forall r \in \mathcal{R}^+$. The bridging deployment plans for the existing route $r \in \mathcal{R}$ are already determined and their headway is $h_r^0, \forall r \in \mathcal{R}^0$. The decision variables for our model are:

- $y_{((r,h),v)} \in \{0, 1\} : \forall r \in R^+, (r, h) \in \mathcal{P}_r, v \in \mathcal{V}$. $y((r, h), v)$ takes 1 if the bridging deployment plan (r, h) is employed with the vehicle type v assigned on this route, and 0 otherwise. The domain of $y_{((r,h),v)}$ can be thus defined as $y_{((r,h),v)} \in \{0, 1\}, \forall (r, h) \in \mathcal{P}^+$.
- $\chi_{((k,u),(r,h,n))} \geq 0$: the number of passengers in the group (k, u) who take the vehicle (r, h, n).
- $\eta(k, u)$: the number of passengers in the group (k, u) who are unable to board any vehicle by the end of the waiting time limit $\bar{w}$. This number is always equal or greater than zero, i.e. $\eta_{(k,u)} \geq 0, \forall (k, u)$.

Let c_k^0 denote the travel time of a single passenger in the group $k \in \mathcal{K}$ when there is no disruption (i.e. the usual MRT travel time). The bridging route selection process and the deployment problem can then be formulated as follows:

$$\min \sum_{((k,u),(r,h,n))\in\Omega} (c_{(k,r)} + w_{((k,u),(r,h,n))} - c_k^0)\chi_{((k,u),(r,h,n))} + \sum_{(k,u)} \theta_{(k,u)}\eta_{(k,u)} \tag{1}$$

$$s.t. \sum_{(r,h,n)} \chi_{((k,u),(r,h,n))} + \eta_{(k,u)} = d_{(k,u)}, \quad \forall (k, u) \tag{2}$$

$$\sum_{(k,u)} \gamma_{(k,(r,l))}\chi_{((k,u),(r,h,n))} \leq Q^0_{(r,h,n,l)}, \quad \forall r \in \mathcal{R}^0, \quad \forall (r, h, n) \in \mathcal{B}_r, \ \forall l \in L_r \tag{3}$$

$$\sum_{(k,u)} \gamma_{(k,(r,l))}\chi_{((k,u),(r,h,n))} \leq \sum_v Q^v y_{((r,h),v)}, \quad \forall r \in \mathcal{R}^+, \quad \forall (r, h, n) \in \mathcal{B}_r, \ \forall l \in L_r, \ \forall v \in \mathcal{V} \tag{4}$$

$$\sum_{h|(r,h)\in\mathcal{P}_r, v\in\mathcal{V}} y_{((r,h),v)} \leq 1, \quad \forall r \in \mathcal{R}^+ \tag{5}$$

$$\sum_{(r,h)\in\mathcal{P}^+} n_{(r,h)} y_{((r,h),v)} \leq A_v^{max}, \quad \forall v \in \mathcal{V} \tag{6}$$

$$\chi_{((k,u),(r,h,n))} = 0, \quad \forall r \notin \mathcal{R}^k \tag{7}$$

The objective function (1) minimizes: 1) the total increase in travel time for passengers taking either the existing buses or other bridging vehicles and 2) the number of passengers who cannot board, weighted by a penalty parameter $\theta_{(k,u)}$. By having two sub-objectives in our optimization function, we simultaneously want to *reduce the travel delay* and *increase the number of served passengers* during the disruption. As mentioned previously, the penalty parameter $\theta_{(k,u)}$ can be set up separately for each passenger instead of the whole passenger group k, which can then result in prioritizing passengers who are paying for a premium service. Furthermore, constraint (2) guarantees that the total number of passengers who boarded plus the number of passengers who did not board equals a passenger demand $d_{(k,u)}$.

Let $\gamma((k, (r, l))$ be a binary variable that takes 1 if the leg l is used by the passenger group k when they take the route r, and 0 otherwise. Constraints (3) and (4) ensure that on each leg $l \in L_r$ of each vehicle (r, h, n), the number of passengers on board (r, h, n) does not exceed the available vehicle capacity $Q^0_{(r,h,n)}$ of the existing bus routes and the total capacity Q^v of the introduced bridging vehicles, respectively.

Constraint (5) guarantees that the route r can either be selected with one specific headway h or not selected at all. Constraint (6) guarantees that the total number of each type of the bridging vehicles does not exceed the total number of vehicles available, where $n_{(r,h)}$ is the number of vehicles required for the route r with the headway set as h and A_v^{max} denotes the number of available vehicles for the type v. Constraint (7) ensures that passengers in the passenger group (k, u) cannot take any route r that is not in $\mathcal{R}^k$, meaning that does not connect the station o_k with the station s_k.

C. OPTIMIZATION MODEL ASSUMPTIONS

The main assumption of our model is that the bridging vehicles on the same route belong to the same vehicle type. To relax this assumption, besides the routes to select and the headway to adopt, we would also need to determine which type of vehicles to use for each round of the bridging vehicle on each selected route. This would substantially increase the number of integer decision variables. In particular, let $|N|$ denote the average number of rounds of bridging vehicles over all candidate routes and let $|Y|$ denote the number of binary variables in our model. Then the optimization model with the relaxed assumption would introduce $|N| \times |Y|$ additional binary decision variables.

The introduction of additional decision variables and constraints results in significantly longer solving times. In particular, we tested the above-mentioned assumptions and constraints on one case study and found that it took up to 20 hours for the model to be solved with a fleet size of only 10 vehicles. For fleet sizes larger than 10, we could hardly solve the proposed model using off-the-shelf algorithms implemented in CPLEX. One potential way to solve this problem is to design a customized algorithm for this problem. However, this is beyond the scope of this paper and could serve as one possible direction for future research.

Moreover, to maintain a manageable parameter space and a reasonably short solving time, our optimization model is based on various assumptions and simplifications, such as:

- travel times of buses and MRT are estimated based on historical data and do not account for an increase of road traffic volume due to adding new bridging vehicles;
- inter-arrival times of the disrupted passengers are modeled as a constant within a given time interval and do not depend on time;
- capacities of bus stops are assumed to be unlimited meaning that each deployed bridging vehicle can make a stop at the bus stop no matter how many other bridging vehicles are already at the same bus stop, and finally
- bus dwell times are assumed to be static and do not take into consideration how many passengers are boarding or alighting buses at each stop.

all of which are addressed later in simulation.

The main reason why travel times are static is because they are used as input parameters of the model. With that being said, if they would depend on the real traffic conditions due to the effect of bridging services, then they would be a part of the model, which would significantly affect the solving time as we would have a circular dependence. The reason why we can model different traffic conditions in simulation is because it takes a fixed bridging plan from the optimization model, which then does not change during the whole simulation period. The inter-arrival times of the disrupted passengers can be modeled to be constant within the discrete time interval in the optimization model as the time intervals are small and the demand does not significantly change during one discrete time interval. Finally, although in theory the capacities of bus stops are unlimited, defining the minimum headway h_r^{min} for each route r prevents the unlimited number of buses to be simultaneously on the same bus stop.

IV. CASE STUDY

In this section, we consider a hypothetical MRT disruption in a business district of Singapore during morning peak hours (i.e. 7-9 AM). The study area including MRT and bus lines is shown in Figure 1. We model a disruption affecting the links from station A to station D of the purple line for the whole morning peak hour period. We apply our optimization approach to find the best bridging plan for the assumed disruption. First we start with generating the candidate bridging routes by replicating the MRT services, considering all possible routes parallel to the disrupted purple MRT line (i.e. for 4 disrupted MRT stations we generate 11 candidate bridging routes). Then, once all routes are generated, our optimization model finds the optimal bridging plan by simultaneously choosing the optimal bridging routes and assigning the available bridging vehicles to the chosen routes.

The passenger demand and the available capacity of existing buses are derived from historical smart card data which indicates that almost 6,000 passengers would be affected. For each passenger, his/her origin and destination MRT station are known from the data, while the route he/she is taking is modeled with the shortest path in the network. When a disruption happens, the affected passengers have to turn to other transportation services to reach their destination, such as to existing bus lines (represented by dashed lines in Figure 1), temporary bridging routes specifically allocated for the disruption, taxis, or bicycles.

Using the data, we calculated that the existing buses on relevant lines would make around 80 rounds passing through the disrupted area during the affected hours, with an average available capacity of approx. 74 passengers per bus per link. Although these buses seem to provide sufficient capacity to serve all affected passengers, in reality only one existing bus line serves station D. Station D, with its connection to the red and yellow MRT lines would be the ideal location for passengers to continue their journey should their original destination not lie within the disrupted area itself.

In our case study, we use multiple types of bridging buses, where both the total number of buses (i.e. $|\mathcal{S}|$) and the number of buses of each type (i.e. $|\mathcal{V}|$) are given as input parameters of our model. The buses currently operating in Singapore can be generally divided into three types: *single decker* bus with an average capacity of 88, *double decker* bus with an average capacity of 131, and *articulated* bus with an average capacity of 148. We assume that the distribution of the three types of bridging buses (i.e. single decker, double decker and articulated) in our optimization model follows the one of the currently operating buses in Singapore (i.e. 70.12%, 24.93% and 4.95%, respectively). Distributions of different types of buses given different fleet sizes are shown in Figure 2.

The parameters of the optimization model are set as follows: the minimum and maximum headways of the bridging vehicles are set to 1 and 31 minutes respectively, with the minimum incremental value set to be 1 minute. The range for headways is set considering factors such as the current headways of the existing buses, the high morning peak demand and the capacity of bus stops. Furthermore, the limit of the waiting time $\bar{w}$ is set to 30 minutes, after which we assume that the passengers leave the system. The given time is arbitrarily chosen and can be changed as it is a parameter.

The penalty parameter $\theta(k, u)$ changes with u, which denotes the arrival time of the passenger group k and is calculated as $120 - u$. 120 minutes was taken as a constant as it denotes the whole duration of the assumed disruption. This penalty parameter ensured that the passengers who came first were served first as well. Please note here that although our mathematical model allows using this parameter to prioritize passengers on various criteria (e.g. a premium they are paying for the service), in this case study we are assuming it depends only on the arrival time of the passenger group k. The reason why we chose to define it as such is because our simulation does not support passenger prioritization and is based on a first-come-first-serve approach. And since we want to use simulation to validate our mathematical model, we decided to define the penalty parameter as described.

Bridging vehicles are allocated to serve throughout the disruption period. In particular, no bridging vehicle will depart after the MRT service is restored (i.e. after 9 AM in this case), but those vehicles that have already departed before 9 AM will continue to serve until they reach their designated destination. Finally, our model was programmed in Python and solved using CPLEX v12.9 running on a computer with Intel(R) Xeon(R) Silver 4116 CPU @ 2.10GHz and 62.5GiB. The solver found a solution within several minutes. The reason why we chose the study area of only 4 MRT stations in the central business district of Singapore is because our main focus in this paper is to validate our proposed mathematical model. Namely, those 4 MRT stations see a large passenger flow during the morning rush hours. Nevertheless, in future work, a more efficient solving algorithm could be developed for deriving the optimized bridging plans for disruptions affecting a larger number of MRT stations.

V. OPTIMIZATION RESULTS

First, we performed a sensitivity analysis to investigate the impact of the bridging vehicle fleet size by steadily increasing the number of available bridging vehicles from 0 to 15 (refer to Figures 3 and 4). We chose this range as 14 bridging vehicles were sufficient to serve almost all affected passengers. As mentioned before, the distribution of bus types follows the pattern of different types of buses observed on Singapore's streets. Nevertheless, as this is an input to our optimization model, a service operator can define the numbers of each type of available bridging vehicles based on their respective fleet. We also studied cases where the bridging vehicle fleet only consists of single decker buses and articulated buses, as those two types present buses with the smallest and largest capacities of buses observed in Singapore.

In Figures 3 and 4 we compare performances of three approaches: 1) *Conventional approach*, 2) approach with one type of vehicles (*Single decker bus* and *Articulated bus*), and 3) approach with *Multiple types of vehicles*. The *Conventional approach* of designing bridging services does not consider the complementary capacities of the existing buses when allocating the bridging vehicles and refers to the current approach when responding to a disruption. The approach with one type of vehicles we presented in our previous work [21]. The aforementioned model does take into account the complementary capacity of the existing buses, but does not allow multiple types of vehicles to be deployed, as the model proposed in this paper allows. All three scenarios can be run using the proposed optimization model. Namely, our model accepts a boolean variable as an input parameter indicating whether complementary capacity is used and as discussed previously the total number and the types of bridging vehicles are also

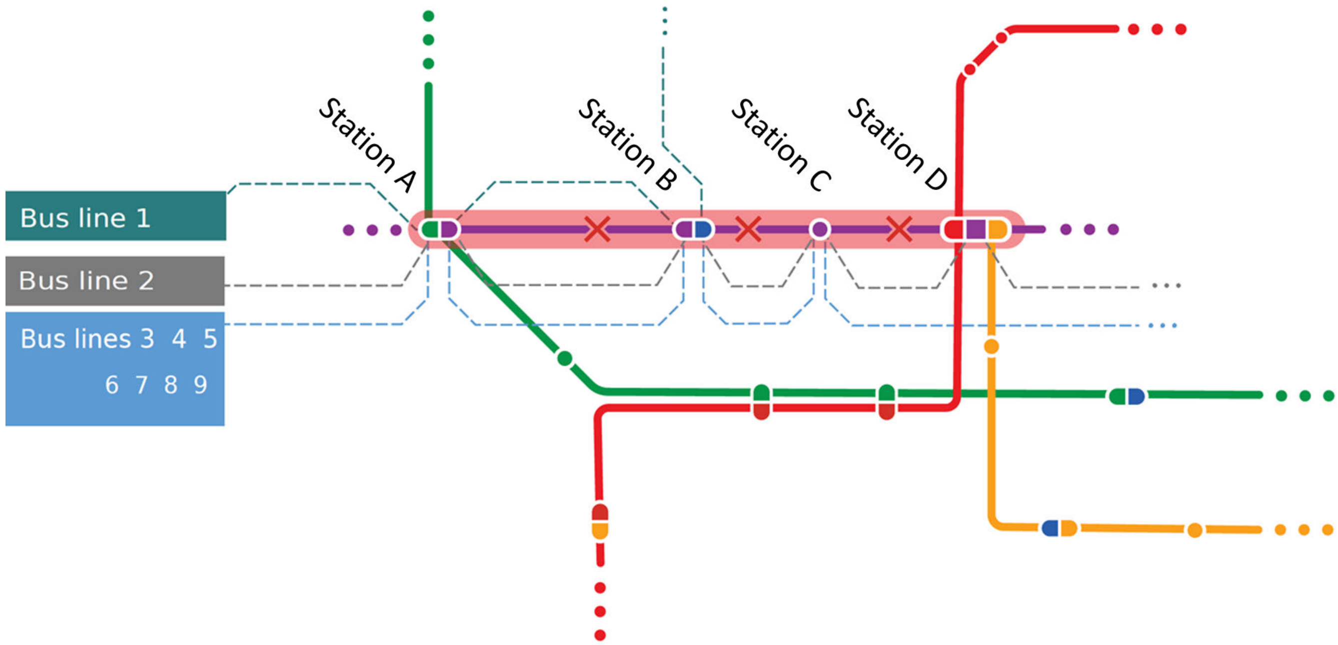

FIGURE 1. Region of interest for the case study. Affected MRT stations are station A, B, C, and D.

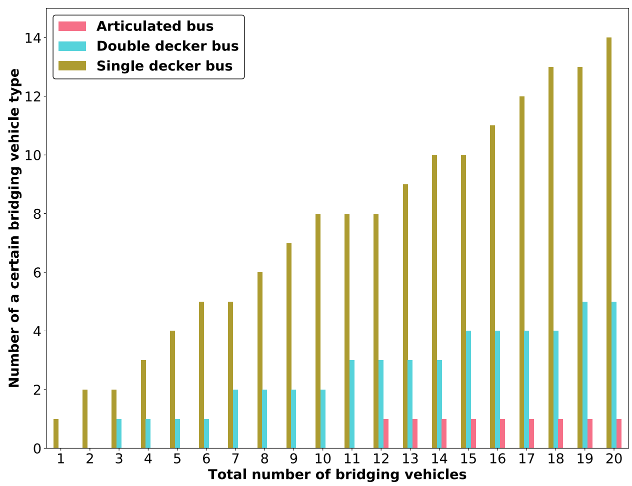

FIGURE 2. Distribution of different bus types.

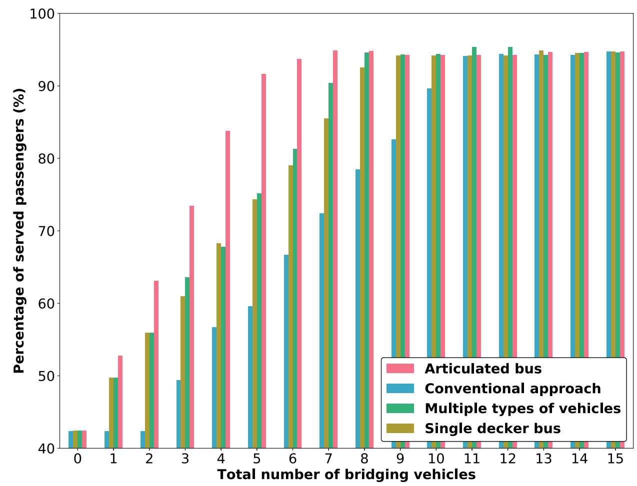

FIGURE 3. The percentage of served passengers with different approaches.

input parameters of our model and can be set accordingly to match the described scenarios.

Figure 3 compares the percentage of served passengers across three different approaches. The x-axis depicts the number of available bridging vehicles, while the y-axis starts at 40% (instead of 0%) as already the existing buses could serve slightly more than 40% of the disrupted passengers. Each group of bars represents one configuration of bridging vehicles where the total number of available vehicles is fixed. For readability reasons, we limit the x-axis to 15 bridging vehicles as after that, all approaches performed almost identically. Generally speaking, the percentage of served passengers should increase as bridging vehicles are added, however, as our optimization function has two objectives - to *reduce the travel delay* and to *increase the number of served passengers*, this is not necessarily the case. This leads to the fact that between two consequential bridging configurations the percentage of served passengers can increase if it results in a shorter travel delay and vice versa.

Results from Figure 3 show that the *Conventional approach* always performs worse or equal compared to the other approaches. Moreover, we observe that the number of served passengers does not reach 100% for any approach due to the maximum waiting time limit $\bar{w}$ that was set to 30 minutes and the minimum headway time set to 1 minute. When a disruption occurs, a large number of passengers are affected at once, leading to a peak in demand. Due to limited

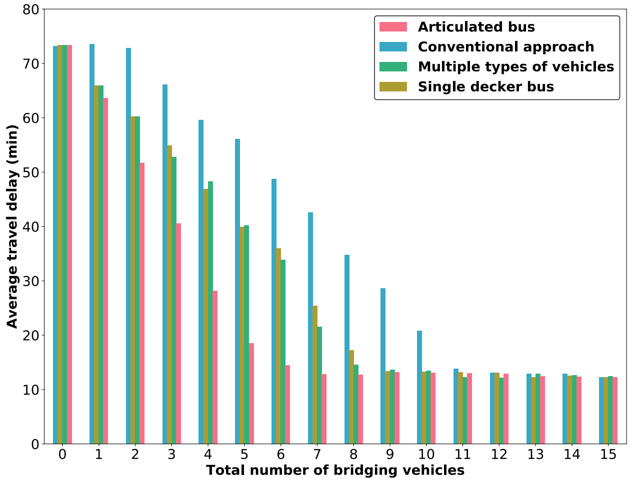

FIGURE 4. The average travel delay with different approaches.

road and bus stop capacities, we can only deploy a finite number of bridging vehicles on the most affected routes. Consequently, a limited capacity and waiting time result in some of the affected passengers not being served within 30 minutes. Those passengers are then marked as unserved.

Figure 4 shows the average travel delay of all affected passengers across three different approaches where for the unserved passengers we assume that their travel delay is equal to the total duration of disruption, i.e. 120 minutes. For the served passengers, their travel delay is equal to summation of their waiting time for either the existing or the bridging vehicle and possibly a longer travel time as buses tend to be slower than the MRT. Similarly to Figure 3, x-axis depicts the number of available bridging vehicles, while the y-axis represents the average travel delay measured in minutes. As expected, the average travel delay of all affected passengers and the percentage of unserved passengers decrease almost linearly as the bridging vehicle fleet size increases.

Figures 3 and 4 show that ignoring the complementary capacity of the existing buses leads to allocating the bridging buses to serve passengers who could have already been served with the existing buses. This results in an oversupply on some routes and undersupply on the others, leading to the *Conventional approach* to perform worse or equal compared to other approaches both in terms of the served passengers and the average travel delay. Moreover, the performance of the *Multiple types of vehicles* approach lies between the performance of the bridging plans with only the smallest *Single decker buses* and only the largest *Articulated buses*.

The results of our sensitivity analysis gave one optimal bridging plan for every configuration of bridging vehicles. Please note that the bridging plan for each configuration is calculated separately and it is completely independent from the bridging plans for other configurations of bridging vehicles. In the interest of space, we will show only one example of an optimal bridging plan for 12 bridging vehicles. A total of 12 available bridging vehicles would include 8 single decker buses, 3 double decker buses, and 1 articulated bus (following the distribution shown in Figure 2). Table 1 shows the optimal bridging plan for this case.

TABLE 1. An optimal bridging plan where N stands for the number of allocated bridging vehicles on each route.

Bridging route	Headway	Vehicle type	N
A - D	4 min	single decker	7
B - C - D	24 min	single decker bus	1
A - B - D	10 min	double decker bus	3
A - C - D	29 min	articulated bus	1

The results from Figure 3 showed that with this bridging plan, more than 95% of the affected passengers could be served, with about 40% passengers served by existing buses and about 55% served by bridging vehicles. We can thus conclude that the existing bus capacities are of a critical importance for the design of bridging deployment plans. Table 1 shows that all bridging vehicles are allocated to serve station D, but start from different affected MRT stations. Two facts could explain this observation: first, station D connects three MRT lines and second, all passengers whose destination is beyond the disrupted area are diverted to station D to take the other operating lines. Moreover, when the purple line is disrupted, station D is only served by one existing bus line (i.e. bus line 2 shown in Figure 1.)

VI. VALIDATION

Our optimization model is based on various assumptions and simplifications, as discussed in Section III-C. The main reason for those assumptions was to maintain a manageable parameter space and a reasonably short solving time of the model. Nevertheless, we wanted to check how those simplifications affected the results our model produced. That is why we decided to use simulation for the validation purposes. Before developing a simulation framework, we had to choose a type of simulation to use. The use of a *macroscopic* simulation model would largely be based on similar simplifications and might even incorporate the same equations as our optimization model, making it an invalid choice for the purpose of validation. We thus decided to use *microscopic* simulation that does not make use of these simplifications and can be considered as an entirely different and independent approach.

In the simulation, travel times of buses depend on the traffic conditions on the road, dwell times at each bus stop are calculated based on passenger demand on that stop and bus stops are set to have a constant capacity of a maximum of two buses simultaneously allowed to dwell at the same bus stop. Those assumptions are different from the optimization model, as discussed in Section III-C. Please note that the microscopic simulation itself does not yield a bridging deployment plan, but is only used to evaluate the bridging plans that were generated by our optimization model. Using simulation-based optimization would be infeasible as each combination of input parameters requires running of the simulation. While there are approaches to explore the parameter space more efficiently [4], the run time of simulation-based optimization would be in stark contrast with the requirements of reactive bridging service planning.

A. SIMULATION MODEL

As a simulator, we chose the City Mobility Simulator (CityMoS) [26], which is an agent-based, discrete-event and sub-microscopic traffic simulator that supports private vehicles, buses, rail-based transportation, and individual passengers. The simulation engine makes use of parallel computing techniques which allows us to maintain fast turn-around times when evaluating our optimization bridging deployment plans. We started with the implementation of our simulation by modeling the relevant part of the Singapore MRT network (see Figure 1), which we parameterized using real, publicly available information (i.e. train sizes and inter-arrival times).

We extended CityMoS to allow for disruptions, stopping passengers from boarding trains at affected MRT stations and forcing passengers to alight the train when it reaches such an MRT station. We extended the passenger routing model to allow for mode changes so that passengers would try to adapt to the changing conditions by finding a new route using the bridging services. Additionally, we implemented a bridging fleet manager which reads bridging plans output by the optimization model and assigns buses accordingly.

Since we did not model the entire MRT network, the first MRT stations inside the study area were modeled as so-called *entry points*. We assumed that trains do not depart empty from these stations, but that the number of boarding passengers at the entry points represents the number of passengers who would be on the train if the entire network was modeled. The numbers were derived from the same historical smart card data as we used for the optimization model. However, slight differences are possible as discussed in Section VI-B.

We then modeled the relevant parts of the bus system for the evaluation of the bridging plans. This includes the pre-existing lines that passengers could take instead of the MRT, as well as the new bridging routes. We reduced the capacity of the existing lines according to the real demand to account for the passengers who were on the bus when it entered the study area. The capacity and frequency of the bridging vehicles were set according to the bridging plans. The dwelling time of the bus was based on the number of boarding and alighting passengers, assuming parallel boarding and alighting [8]. Furthermore, we assigned one bus stop to each MRT station as its *associated* stop.

Passengers affected by the disruption walk to the associated bus stop. While generally, passengers in CityMoS experience delays from walking to platforms or bus stops, we did not model walking times from the MRT station to the bus stop as for this to exhibit a high level of realism it would require a detailed 3D model of the area to capture delays inside the station, traffic light timings if the bus stop is on the other side of the road, and most importantly, real-world data to validate it. If this was available, it would be straightforward to extend both the optimisation approach and the simulation with a distribution of walking speeds to estimate travel time

FIGURE 5. Screenshot of the CityMoS environment showing bus and MRT operation. Passenger information for each station, stop, train, and bus can be displayed.

delays caused by walking from the disrupted MRT station to the bus stop. To capture delays from walking to the bus stop, the optimization model itself would not need to be extended, as this travel delay would only need to be added in the model's evaluation function.

All passengers were modeled as individual agents with their own origin and destination according to historical data. They would board and alight trains and buses according to their fastest route. The simulation allows us to track trains through the network, where information about each station includes waiting and transiting passengers as shown in Figure 5. When passengers are waiting at a bus stop, they board arriving buses in a greedy fashion, that is, they would try to board the first bus that brings them to either their destination or to the next MRT station on their route that is not affected by the disruption. In the simulation, the passengers queue in a first-in first-out fashion. When the disruption occurs at 7:00 AM, all the passengers on the affected trains alight at the first disrupted MRT station and are transferred to the associated bus stop. Other passengers who start their journey inside the disrupted zone walk directly to the associated bus stop instead of the affected MRT station.

To validate the simulation model itself, we compared the real bus and MRT travel times in Singapore with the ones produced by our simulation. We calibrated the travel speeds and traffic light timings so that the simulated travel times do not deviate from the real world travel times by more than 10%. Since the passenger demand, as well as the bus and MRT schedules were based on real data, there was no need for calibration. For car-following, we used the intelligent driver model and while for lane-change models we employed MOBIL, we hardly witnessed any lane changes by buses except for the ones necessary to follow their routes.

B. COMPARING OPTIMIZATION AND SIMULATION MODELS

We collected detailed information about all passengers, i.e. the buses they boarded and their travel times, as well as statistics for all buses and MRT trains that traversed the study area. For validation purposes, we compared our *Multiple types of vehicles* optimization approach with the results obtained from the simulation model, using two main metrics: the average

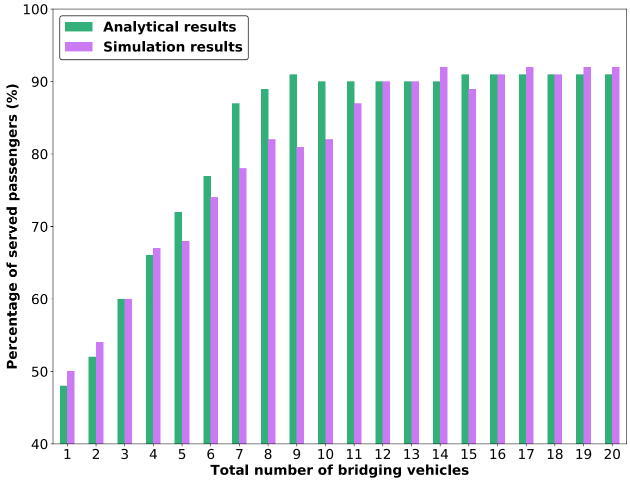

FIGURE 6. **The percentage of served passengers for optimization and simulation.**

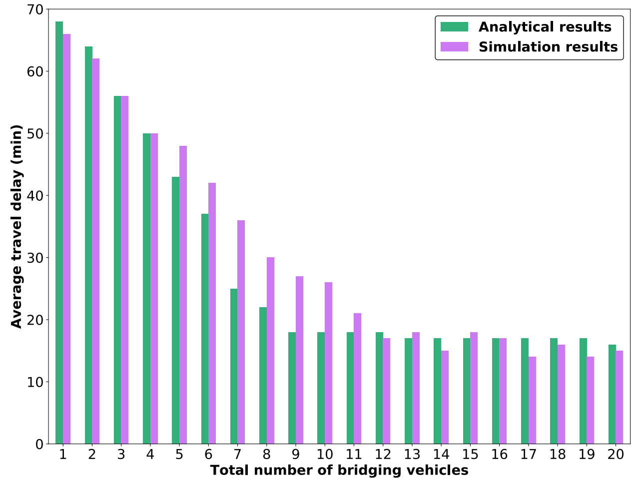

FIGURE 7. **The average travel delay for optimization and simulation.**

travel delay (expressed in minutes) and the percentage of served passengers for each configuration of bridging vehicles. Similarly to comparing the three different approaches presented in Figures 3 and 4, here we also performed a sensitivity analysis to investigate the impact of the bridging vehicle fleet size by steadily increasing the number of available bridging vehicles from 0 to 20.

Figure 6 compares the percentage of served passengers, i.e. the number of disrupted passengers who can be served within 30 minutes, for our optimization (i.e. denoted as *Analytical results*) and simulation models. The x-axis presents the number of available bridging vehicles, while the y-axis starts at 40% for the same reasons as previously described. The difference is that this time the x-axis starts at 1 (instead at 0 as in previous figures) as we are only interested in comparing configurations of bridging vehicles where the number of bridging vehicles is larger than 0, i.e. the bridging vehicles that were actually deployed.

Results show that the analytical and the simulation results are in agreement up until around 4 bridging vehicles and then later on from 12 bridging vehicles onwards, where both models seem to have reached their saturation points. However, for configurations of 5 to 11 bridging vehicles, the percentage of served passengers predicted by the optimization model is higher than for the simulation model. In terms of an average delay (see Figure 7), we observe that the results of simulation and optimization models follow the same patterns as for the percentage of served passengers.

The main source of differences between the optimization and simulation models comes from the different number of non-empty rounds that the bridging vehicles would make through the disrupted area in both cases. The bridging plan (which one example is shown in Table 1) sets the total number of buses and their headways for each of the bridging routes. Once a bridging vehicle reaches the end of its route, it is deployed to go back to the beginning of the route, making one round. When comparing the total number of rounds the

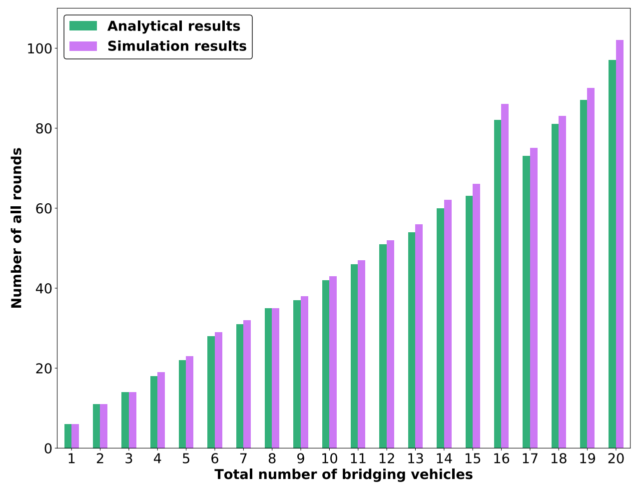

FIGURE 8. **The total number of bridging vehicle rounds.**

bridging vehicles are making both in the simulation and in the optimization, the numbers are virtually the same as shown in Figure 8. As mentioned before, the reason behind this is because the number and frequency of the bridging vehicles in simulation are set according to the bridging plan proposed by the optimization model.

Figure 9 shows the differences in the number of rounds that are non-empty (i.e. that are carrying the affected passengers). It can be seen that the number of non-empty rounds for both models is roughly the same until 4 introduced bridging vehicles. That is the reason why the percentage of served passengers and the average travel delay shown in Figures 6 and 7 are the same both for simulation and optimization models. With 12 or more bridging vehicles, the performances of optimization and simulation models are again in agreement as the number of non-empty rounds in the simulation is large enough to serve all disrupted passengers who can be theoretically served.

The reason for the difference in results is that the optimization model is unable to capture effects such as bus bunching

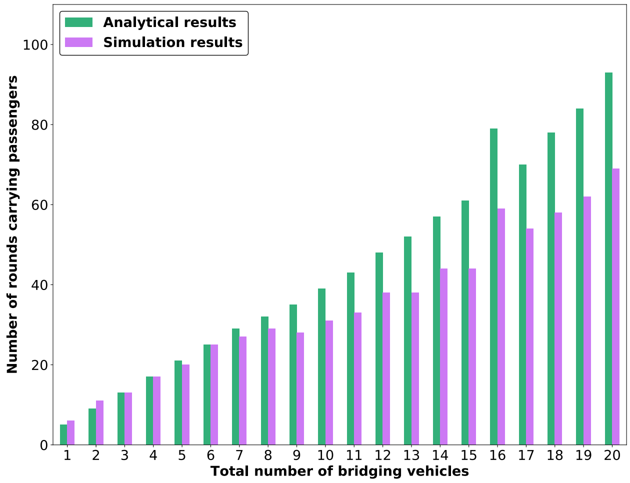

FIGURE 9. The number of bridging vehicle rounds carrying passengers.

where a bus is catching up with another bus of the same line but cannot overtake it. This will result in two buses of the same line arriving at the bus stop in short succession. In the simulation this would result in all passengers boarding the first bus while the second bus would remain empty. Additionally, in the optimization model the passenger demand curve is assumed to be constant over a time slot whereas in the simulation, a large number of passengers arrive with each MRT train, causing an uneven distribution of arrivals. As discussed before, due to the maximum waiting time which is set to 30 minutes and a high demand being created immediately after a disruption happens, not all of affected passengers could be served even in theory.

What can be also seen from Figure 8 is that the number of total rounds for the configuration with 16 bridging vehicles is larger compared to the configurations with 15 and 17 bridging vehicles. The reason behind this is that the bridging deployment plan for each configuration is calculated separately with the purpose of reducing the average travel delay and the percentage of unserved passengers and does not depend on the bridging deployment plan of the previous or the next configuration. More concretely, the increase in the number of rounds for the configuration with 16 bridging vehicles comes from the fact that instead of having 4 double decker buses deployed on the same route, as it is the case with 15 and 17 bridging vehicles, the bridging deployment plan for configuration with 16 bridging vehicles deploys one double decker bus on a new line. As the total time needed to make that round is shorter, that bridging vehicle can make more rounds (please refer to Table 2).

The remaining differences between the results for the optimization and simulation models can be explained by the differences in passenger demand between the two models. As shown in Figure 10, for the simulation model passenger demand slightly changes across the different configurations of bridging vehicles. Unlike for the optimization model where the passenger demand is an input, for the simulation

TABLE 2. Deployment plans with 15, 16 and 17 bridging vehicles (N stands for the number of allocated bridging vehicles on each route).

Bridging route	Headway	Vehicle type	N
Deployment plan with 15 bridging vehicles			
A - D	3 min	single decker	9
B - C - D	24 min	single decker bus	1
A - B - C - D	8 min	double decker bus	4
A - C - D	29 min	articulated bus	1
Deployment plan with 16 bridging vehicles			
A - D	3 min	single decker	9
A - B - C - D	16 min	single decker bus	2
A - B	7 min	double decker bus	1
B - C - D	8 min	double decker bus	3
A - C - D	29 min	articulated bus	1
Deployment plan with 17 bridging vehicles			
A - D	3 min	single decker	9
B - C - D	8 min	single decker bus	3
A - B - C - D	8 min	double decker bus	4
A - C - D	29 min	articulated bus	1

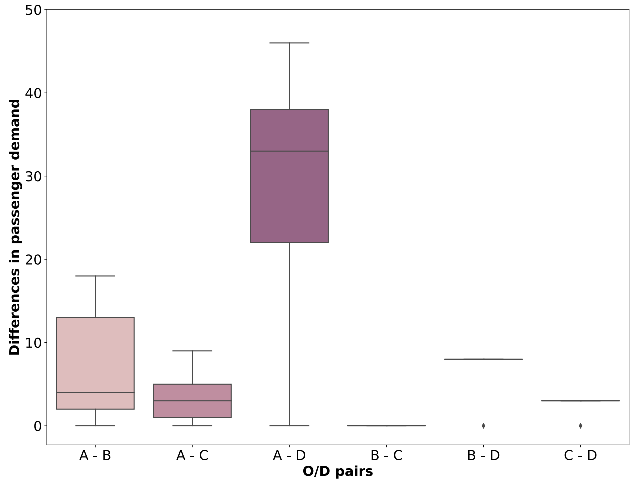

FIGURE 10. The oscillations in a passenger demand for the simulation model.

model, the demand is calculated within the simulation itself and can vary among the different configurations. Box plots in Figure 10 show that for different configurations of bridging vehicles these variations can be up to 40 passengers for example for the passengers traveling between stations A and D. In the optimization model we are using the same demand across all configurations, set to the mean value of all simulation values.

The main take-away from the validation is that our optimization model yields a valid bridging plan and can be used as guidance to policy makers. A simulation-based approach can be then used to offer more fine-grained insights into the performance of a bridging plan and capture effects a mathematical model could not. We thus advocate the use of this combined approach for an effective disruption response.

VII. CONCLUSION AND DISCUSSION

In this paper, we proposed an optimization model to allocate bridging services in response to MRT disruptions. The model considers the case where a heterogeneous bus fleet is available to be used as bridging services. Moreover, the model allows to prioritize which groups of passengers / individual passengers should be served first, allowing for a premium service during disruption to be designed. The output of this model includes the routes to select, their corresponding headways and the type of vehicles to be used on each route.

Once when a disruption happens, the affected passengers who are served can either be transferred by a bridging vehicle to their final destination if that station was one the same line and was disrupted or to the first station on the line that was not affected by a disruption. In that sense, our optimization model can be perceived as designed for disruptions on a line. The reason behind that is that among seven severe MRT disruptions that have happened in Singapore in the last three years, six of them happen along the same MRT line, indicating that this type is a more frequent one [2].

We demonstrated our model using a case study in the central business district of Singapore. The results showed that our approach could generate bridging deployment plans that would effectively reduce the average travel delay of affected passengers and the number of passengers who could not board the bridging vehicles. We validated our approach using a detailed microscopic simulation model and found that in some cases the analytical results were too optimistic, meaning that they underestimated the number of bridging vehicles needed to serve all affected passengers. Rather than diminishing the importance of the analytical approach to solve a disruption problem, our results emphasize the need of using a combined approach when designing bridging plans.

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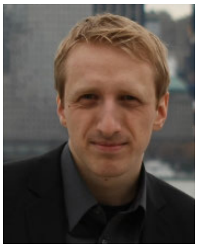

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