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A Facility Location and Allocation Model for Cooperative Fire Services

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ABSTRACT Designing efficient urban fire service systems is of crucial importance as prompt responses to emergencies and accidents can drastically reduce property loss and mortality. To achieve these goals, holistic location-allocation models for cooperative fire services must consider multiple factors, such as fire station size, vehicle quantity, vehicle type, response time, service reliability, and traffic condition. This paper proposes a mixed-integer linear program model that takes these factors into account by using two coverage metrics: one related to vehicle coverage, indicating service reliability, and the other related to time coverage, describing service accessibility. Integrating these two metrics, a full or partial time and vehicle coverage (FPTVC) standard is obtained that characterizes the efficiency of response of cooperative fire services. A case study and multiple sensitivity analyses based on historical data from Hefei (China) are performed to demonstrate the relationships between vehicle coverage proportion, time coverage rate, fire station number, fire engine quantity, FPTVC, and total budget. The solutions obtained are further analyzed using GIS-based maps, which validate that the proposed model can help develop enhanced fire service systems designs.

INDEX TERMS Cooperative fire services, fire station, time coverage, vehicle coverage.

I. INTRODUCTION

Strategic planning of urban fire service systems is a critical problem whose importance keeps on increasing due to the challenges posed by urban expansion and the need for efficiently using fire resources. In China, firefighters are major components of urban fire services. They attend to various types of fire accidents for which different numbers and types of fire vehicles must be dispatched. As part of their long term planning, local governments must therefore decide where to locate fire stations and what types of fire trucks to assign them. This problem is referred to as the *fire station location and vehicle assignment problem*. It is key in decreasing the risk of civilian casualties and property losses.

A stylized instance of the problem is depicted in Fig. 1. Potential fire station sites are chosen across the region of interest. They are represented using triangles. Fire events are aggregated into demand points, which are represented using green circles. Demand points are assigned a specific vehicle coverage standard, which specifies the minimum number and types of fire vehicles that the fire stations serving them should collectively have. Vehicle coverage standards are based on demand weights (which are themselves usually determined by fire risk.) Fire stations must be selected among potential sites so that each demand point is assigned to at least one nearby fire station. Fire departments will then be responsible for the rescue tasks of all demand points assigned to them. Depending on the demand points they served, fire stations will be allocated different numbers and types of fire vehicles. An example layout is presented in Fig. 1. Red triangles represent selected sites and arrows describe demand points they serve. Established fire stations have two types of fire vehicles (v1 and v2) to support the number of fire vehicles of each type required by the demand points. In describing this stylized instance, two important practical realities are ignored. First, it is not cost efficient to dedicate fire vehicles to be used only for a specific demand point. Pooling these resources will lead to better and less costly designs. Second, demand points should only be assigned to a fire station if it

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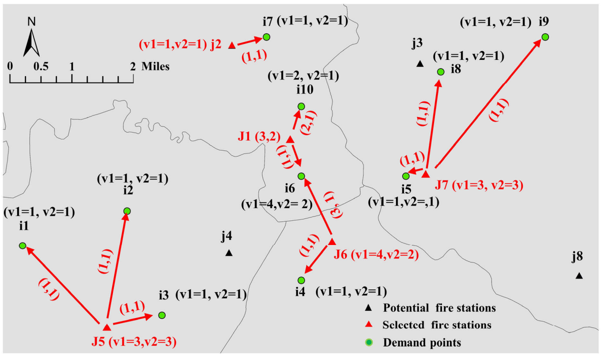

FIGURE 1. Feasible solution map.

can provide them with fast service as timely responses are crucial during emergencies.

The different vehicles assigned to a fire station will be used for all fire events occurring at its demand points. If a vehicle is busy attending a fire accident, it will be unable to tend to any new emergency for some period of time. It is unlikely however (although not impossible) that fire emergencies will occur simultaneously in all demand points. Local authorities therefore need to consider the probabilities of scenarios where multiple events coincide. In particular, they must determine the right number of vehicles to keep so as to guarantee that the probability that shared trucks are available to respond to emergencies is high. Hence, location-allocation models for fire services must consider vehicles' busy fractions so as to ensure reliability. This is often implemented through the notion of *vehicle coverage* which requires that each demand point is served by a minimum number of certain types of fire trucks, based on its demand weight. Details are provided in subsection III-A.

Another important aspect to consider is that response times to emergencies depend on station locations and traffic conditions, which fluctuate throughout the day. To capture this notion, previous studies usually compute *response time coverage*, which measures the fraction of demand sites that are reachable within a given response time. Travel times between stations and accident sites can be estimated in different ways. In some studies, they are obtained by averaging the speed of fire trucks and Euclidean distance between locations. In other studies, they are derived by considering multiple scenarios obtained by setting different average speeds or by using effective road lengths to simulate different traffic conditions. As cities grow and change, especially in China, traffic jam can be increasingly severe. It is therefore necessary to account for real traffic conditions when computing travel times. Details of how we propose to do so are described in subsection III-B

When designing a urban fire service system it is therefore important to consider the two critical measures described above to ensure reliable system performance. This paper introduces a model for the design of cooperative fire services that, to our knowledge, is the first that systematically considers these two coverage metrics. Specifically, this study introduces a model to determine an optimal location of fire stations and their corresponding vehicle assignment. The model seeks to maximize the demand area where the vehicle coverage (with a certain reliability) and the fire response time coverage under urban traffic conditions are within desired values. This integrated model is practical, as it can be used to solve a real-sized case study based on historical data collected from 2007 to 2011 for the city of Hefei, China.

The paper is organized as follows. Section II presents a literature review. Section III introduces the fire station location and vehicle assignment model for cooperative fire services. Section IV discusses how practical data is processed to create the inputs of the model. This includes, for instance, information about occurrence time of fire events and their locations. Section V discusses a practical case study for the central area in Hefei City and performs a range of numerical experiments to demonstrate the validity of the model. Section VI discusses the practical application and conceptual limitations of the method and presents ideas for further research.

II. LITERATURE REVIEW

Early models proposed for facility location include the *P-Median Location Problem (PMLP)* by Hakimi [8], the *Set Covering Location Problem (SCLP)* by Toregas*et al.* [9] and the *Maximal Covering Location Problem (MCLP)* by Church and ReVelle [10]. PMLP seeks to open p facilities so as to minimize the average weighted distance between demand points and their nearest open facility. SCLP minimizes the number of facilities to locate to ensure that all demands are served under a coverage threshold. Some early applications of SCLP to locate fire stations are developed by Plane and Hendrick [11] for the city of Denver, Colorado, and by Schreuder [12] for the city of Rotterdam, Netherlands. Finally, MCLP maximizes the total weighted number of demands covered under a covering threshold by building a set number of facilities. Applications of this model to fire station location can be found in [5], [12]–[19]. A thorough literature survey on the fire station problem can be found in Aleisa [20].

Various extensions of these three models have been developed by incorporating new aspects of emergency service operations. The articles reviewed next focus on vehicle coverage under a certain reliability and time coverage under traffic conditions, as these measures form the core of the model developed in this paper.

Research works studying vehicle coverage requirements are presented first. Schilling *et al.* [21] developed two extensions of MCLP that consider vehicle assignment: the Tandem Equipment Allocation Problem (TEAM) and the Facility Location & Equipment Emplacement Technique (FLEET). ReVelle and Hogan [22] presented a probabilistic version of MCLP and called it Maximum Available Location Problem (MALP). They sought to locate p facilities with the goal of maximizing the covered population that could find an available server within a certain probability. In ReVelle and Marianov [23], a similar extension was proposed for FLEET. Marianov and ReVelle [1] extended this model

TABLE 1. Comparison of the cooperative fire service modeling with other studies.

Author	Year	Fire station		Vehicle coverage		time coverage		Reliability	Case study
		Capacity	Type	Quantity	Type	Travel time	Travel condition		
Marianov et al. [1]	1992	✓	×	✓	✓	×	×	✓	×
Revelle et al. [2]	1995	✓	×	✓	✓	✓	×	×	×
Thomas et al. [3]	2007	✓	×	✓	×	✓	×	×	✓
Catay et al. [4]	2011	✓	×	✓	×	✓	×	×	✓
Wang et al. [5]	2016	✓	×	✓	✓	×	×	×	✓
Perez et al. [6]	2016	✓	×	✓	✓	×	✓	✓	✓
Rodriguez et al. [7]	2020	✓	×	✓	✓	✓	✓	✓	✓
The authors	–	✓	✓	✓	✓	✓	✓	✓	✓

to account for different types of fire vehicles. Later, Marianov and ReVelle [24] introduced the Queueing Maximal Availability Location Problem (Q-MALP), where the authors used queuing theory to relax the assumption of independence of servers' busyness in MALP. Alsalloum and Rand [25] extended emergency vehicle location models in two ways. The first was to replace the usual 0-1 coverage definition with the probability of covering demand within the target time. The second was to ensure that any demand arising within the service area of the station can be served by at least one vehicle. Beraldi and Bruni [26] introduced a new stochastic programming paradigm to consider probabilistic constraints for emergency service vehicle location problems. Sorensen and Church [27] combined the local busyness estimates of MALP with the maximum coverage objective of MCLP to integrate expected coverage and local reliability for emergency medical service location problems. Oran *et al.* [28] presented a new formulation of the facility location problem and vehicle routing problem with time windows with considerations of emergency priorities. Liu *et al.* [29] adopted a new double service coverage standards to design an emergency medical service vehicle allocation model that ensured acceptable service reliability with limited vehicle resources. Wang *et al.* [5] proposed a new partial coverage location model for cooperative fire services that includes multiple types of vehicles, different numbers of vehicles, constraints on the response distance, and a limited budget. However, the model failed to consider the traffic status and proved to be a nonlinear programming problem. Mohammadi *et al.* [30] developed a robust multi-objective model to integrate reliable location and vehicle routing decisions for disaster relief under fairness and aftershocks concerns.

Second, facility location-allocation problems that consider time coverage under traffic conditions are reviewed. There exists few studies involving this topic because acquiring the related data tends to be a difficult task. Most recently, Perez *et al.* [6] proposed a probabilistic fire station siting model to relocate the fire stations and reallocate fleets for the Santiago Fire Department (SFD). A deterministic approach simulating different traffic conditions was considered by computing the effects of various coverage distances. Rodriguez *et al.* [7] developed a facility location and equipment emplacement technique model with expected coverage for the location of fire stations in the Concepción province of Chile. The authors computed the average speed for each road type through collecting speed information of a sample of 40 roads during several time blocks of one day. Liu *et al.* [17] proposed a model to estimate the effective coverage rate of fire station services based on real-time travel times which were obtained by calling route-planning API of online maps together with scenario estimation methods.

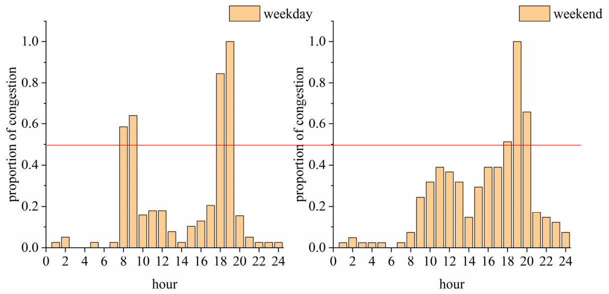

FIGURE 2. Proportion of congested roads during each hour of a day, broken down by weekday and weekend.

With respect to fire station location-allocation problems, studies similar to that conducted in this paper are summarized in Table 1. Five factors are presented. The first four consider whether or not cooperative fire service parameters (fire station, vehicle coverage, time coverage and reliability) are taken into account in the fire station location model. The last factor identifies whether the authors included a real case study to evaluate the performance of the model. The present work (last row on Table 1) integrates all those practical requirements. It describes both the vehicle coverage standard with a certain reliability and fire response time coverage under urban traffic conditions; see Section III for details of how this is achieved. The resulting integrated model captures the inherent complexity in a fire emergency rescue system with the goal of ensuring best possible system performance.

III. MODEL FORMULATION

This section presents an integer linear programming model that extends the probabilistic location model proposed by Perez *et al.* [6]. The approach in [6] distinguishes different types of fire events and determines, for each, the number and type of vehicles needed to respond to them. In contrast, our model organizes demand point in three levels, depending on the occurrence of fire events. Also, the different periods considered in this paper to compute travel times are based on urban traffic conditions, which is different from the approach taken in [6] of evaluating these times for each month. Further, our model considers two new critical factors: traffic status and travel time. This paper combines them with gradual coverage to determine time coverage. The method of gradual coverage was first introduced in the Maximal Covering Location Problem in the Presence of Partial Coverage (MCLP-P) [31]. It assumes two radii R_1 and R_2 with $R_1 < R_2$ around each fire station. Every demand point located within a distance R_1 of the fire station is considered fully covered, while any demand point located at a distance between R_1 and R_2 is deemed to only be partially covered. Demand points at larger distance are considered to be not covered by that fire station. In the model presented, this notion of gradual coverage is applied to travel times, instead of distances.

In the following section, a location-allocation model is presented that incorporates vehicle coverage within a certain reliability and time coverage under urban traffic conditions. These two coverage measures are critical in providing adequate response to demand points. Specifically, the vehicle coverage (including types and numbers) is determined by the busy fraction of vehicles and allows for sufficient rescue ability to be present to serve emergency calls. The time coverage measures the ability to arrive at accident points within a limited time.

A. VEHICLE COVERAGE

A formula to compute the busy fraction of fire vehicles is presented in [6]. It serves as the basis for the extension presented next, that uses a different way of classifying demand points and a different set of time periods in which traffic is evaluated. In particular, for all $v \in V$, $t \in T$, and $i \in I$, the busy fraction q_{vi}^t of fire vehicles of type v within the pre-specified maximum response time of demand point i at period t is computed in the proposed model as

$$q_{vi}^t = \frac{F_{vi}^t}{\sum_{j \in W_i^t} \sum_{s \in S} x_{vjs}} \tag{1}$$

with

$$F_{vi}^t = \frac{\sum_{l \in L} b_{vl} \bar{t}_{vl} \sum_{j \in W_i^t} f_j^t}{24} \tag{2}$$

where

- V: Set of types of vehicles,
- T: Set of periods in the evaluation horizon,
- L: Set of demand levels,
- b_{vl}: Number of fire vehicles of type v dispatched to demand points of level l,
- $\bar{t}_{vl}$: Average duration (in hours) of demand points of level l which call for vehicle v,
- I: Set of demand points,
- J: Set of potential facility sites,
- W_i^t: Set of potential facilities that can provide service for point i,
- f_j^t: Frequency of calls toward facility j during period t,
- S: Set of types of fire stations,
- x_{vjs}: An integer variable representing the number of fire vehicles of type v that are placed at a facility j as a fire station of type s.

In (2), the numerator represents the total time occupied by fire vehicles of type $v \in V$ attending to calls of a point $i \in I$ in period $t \in T$ from nodes $j \in W_i^t$. The denominator denotes the available time of one vehicle (24 hours).

The probability of availability of at least a certain number of fire trucks is computed to assure effective response by the vehicles with strong reliability. Assuming a binomial distribution for the probability of the unavailability of vehicles, the busy probability for any r out of n vehicles is specified, as shown below:

$$\binom{n}{r}(1 - q_{vi}^t)^{n-r}(q_{vi}^t)^r. \tag{3}$$

There is an implicit assumption of independence between availabilities of the fire trucks. The parameter n represents the number of trucks of type v serving point i during period t, as shown below:

$$n = \sum_{j \in W_i^t} \sum_{s \in S} x_{vjs}. \tag{4}$$

Moreover, the probability of availability of two or more trucks for a point i is equal to the one that at most $(n-1)$ vehicles are busy.

$$\begin{aligned} &\mathbb{P}[\,\textit{two or more trucks free}\,] \\ &\quad = 1 - \mathbb{P}[\,\textit{all trucks busy}\,] \\ &\qquad - \mathbb{P}[\,(n-1)\,\textit{trucks busy}\,] = 1 - \binom{n}{n}(1 - q_{vi}^t)^{n-n}(q_{vi}^t)^n \\ &\qquad - \binom{n}{n-1}(1 - q_{vi}^t)^{n-(n-1)}(q_{vi}^t)^{n-1} \\ &\quad = 1 - (q_{vi}^t)^n - n(q_{vi}^t)^{n-1}(1 - q_{vi}^t). \end{aligned} \tag{5}$$

Since $q_{vi}^t = \frac{F_{vi}^t}{n}$, the above expression becomes

$$\begin{aligned} &\mathbb{P}[\,\textit{two or more trucks free}\,] \\ &\quad = 1 - \left(\frac{F_{vi}^t}{n}\right)^n - n\left(1 - \frac{F_{vi}^t}{n}\right)\left(\frac{F_{vi}^t}{n}\right)^{n-1}. \end{aligned} \tag{6}$$

Finally, this probability is required to be greater than or equal to α, leading to the requirement that

$$1 - \left(\frac{F_{vi}^t}{n}\right)^n - n\left(1 - \frac{F_{vi}^t}{n}\right)\left(\frac{F_{vi}^t}{n}\right)^{n-1} \geq \alpha, \qquad \forall t \in T, v \in V, i \in I. \tag{7}$$

Equation (7) shows the probability that at least two vehicles are free to serve point i during period t with a reliability of α. The function on the left-hand side is non-decreasing in n. Therefore if, for each $i \in I$, $v \in V$, and $t \in T$, we identify the minimum value of n, call it m_{vi}^t, for which (7) is tight, it will be true that (7) is satisfied for all $n \geq m_{vi}^t$. In words, m_{vi}^t represents the minimum number of trucks located within the coverage radius of point i served by at least two trucks with probability α within period t. This parameter is a function

TABLE 2. **Vehicle coverage standard.**

Set of demand levels	Vehicle numbers specified in UFRS	
	v1	v2
l_1	1	1
l_1	2	1
l_1	4	2

TABLE 3. **Vehicle coverage standard based on $\alpha = 0.9$ reliability.**

Set of demand levels	Vehicle numbers specified by m_{vi}^t	
	v1	v2
l_1	4	4
l_1	5	4
l_1	8	4

TABLE 4. **Parameters.**

γ_t	Weight of period t
dem_i^t	Demand weight of point i during period t
C_{ij}^t	Coverage satisfaction of point i if provided by facility j during period t
m_{vi}^t	Number of vehicles of type v needed to satisfy the vehicle coverage standard of i with reliability α during period t
p_s	Maximum number of trucks in a fire station of type s
M_s^c	Construction cost of a fire station of type s
M_v^t	Purchase cost of a fire vehicle of type v
M	Total budget for fire stations and vehicles

TABLE 5. **Decision variables.**

$\theta_i^t \in \{0,1\}$	Binary decision describing if point i meets its vehicle coverage standard during period t or not
$x_{vjs} \in \mathbb{Z}_+$	Number of vehicles of type v placed at site j if that site is selected to be a fire station of type s
$u_{ijs}^t \in \mathbb{Z}_+$	Number of vehicles dispatched to point i in period t from site j, if selected to be a fire station of type s
$z_{js} \in \{0,1\}$	Binary decision describing if a fire station of type s is located at site j or not

of the location decisions (related to α-reliable coverage) and can be calculated before the solution of the location model. In fact, F_{vi}^t can be computed from historical fire rescue record data. It is set to 2 in this study. According to Urban Fire Rescue Specification (UFRS), vehicle types and quantities required for each demand point belonging to a particular demand level (called vehicle coverage standard, hereafter) are given in Table 2. Table 3 shows that demand points in each level possess a corresponding m_{vi}^t based on the vehicle coverage standard used in this paper. Specifically, when one demand point i in level 2 requires two fire trucks of type v1 and one fire truck of type v2, it means to assign five fire trucks of type v1 and four fire trucks of type v2 to customer i to ensure the service with a reliability of 0.9, meaning $m_{v1,i}^t = 5$ and $m_{v2,i}^t = 4$, respectively.

B. TIME COVERAGE

Gradual coverage based on traffic status at different time periods is adapted from MCLP-P [31], where all considerations are based on distances. Instead the response time from facilities to their responsible area is used. In particular, assuming that in period t, the travel time between fire station j and demand point i is T_{ij}^t, the gradual time coverage is computed using the formula

$$C_{ij}^t = \begin{cases} 1 & T_{ij}^t \le T_0 \\ \dfrac{1}{1 + exp(A(T_{ij}^t - \frac{T_0+T_1}{2}))} & T_0 < T_{ij}^t \le T_1 \\ 0 & T_{ij}^t > T_1, \end{cases} \tag{8}$$

where $A = 5$. The coverage decay function C_{ij}^t, which monotonically decreases in T_{ij}^t, captures the time coverage satisfaction that facility j provides to demand point i within period t. Parameters T_0 and T_1 are called the *maximum full coverage time* (set at two minutes) and the *maximum partial coverage time* (equal to six minutes). Coverage is full when $T_{ij}^t < T_0$ and null when $T_{ij}^t > T_1$. It is partial otherwise.

Because traffic changes throughout the day and throughout the week, the values of T_{ij} are not constant. To capture this feature, a method is presented next that groups different periods of the week based on their traffic status. This method is applied to the particular case of Hefei. The traffic information on the main roads in Hefei was crawled every ten minutes to analyze daily traffic conditions. This paper defines the periods where the proportion of congested roads is larger than 0.5 to be *vehicle peak periods* and the others to be *vehicle off-peak periods*, as shown in Figure 2. Using this approach, three peak periods are identified during the week: a weekday morning peak period from 7 to 9 o'clock, a weekday evening peak period from 17 to 19 o'clock, and a weekend evening peak period from 17 to 20 o'clock. The off-peak period occupies any other time during the week.

The shortest path between facility j and demand point i under period t was obtained through the route planning API in Gaode maps. For simplicity, the shortest path was crawled from one week of May in 2020 without considering long-term traffic conditions. For each time period, each path included a series of sub-routes, and each sub-route had its traffic status. This paper sets the maximum travel speed of fire trucks under various traffic conditions, such as 90 km/h for non-congested roads, 30 km/h for moderately congested roads, and 10 km/h for congested roads. The travel-speed rule was based on information from the Hefei Bureau. Finally, T_{ij}^t is obtained by summing the travel times of the subpaths under each period.

C. OPTIMIZATION MODEL

Having illustrated vehicle coverage (satisfying constraints on vehicle types and quantities) and time coverage (meeting travel time requirements under traffic conditions), the next step is to formulate the proposed location-allocation model. Parameters and decision variables are presented in Table 4 and Table 5, respectively. In these tables, indices t, i, j, v, and s are assumed to belong to T, I, J, V, and S, respectively.

The fire station location-allocation problem is formulated as follows:

$$P: \max \sum_{t\in T} \sum_{i\in I} \gamma_t dem_i^t \frac{\sum_{j\in J} \sum_{s\in S} C_{ij}^t u_{ijs}^t}{\sum_{v\in V} m_{vi}^t} \tag{9}$$

$$subject\ to: \ m_{vi}^t \cdot \theta_i^t \le \sum_{s\in S} \sum_{j\in W_i^t} x_{vjs}, \quad \forall t \in T, v \in V, \ i \in I \tag{10}$$

$$\sum_{v\in V} x_{vjs} \le p_s z_{js}, \ \forall j \in J, s \in S \tag{11}$$

$$\sum_{s\in S} z_{js} \le 1, \ \forall j \in J \tag{12}$$

$$u^t_{ijs} \le \sum_{v\in V} x_{vjs}, \ \forall t \in T, j \in J, s \in S \tag{13}$$

$$\sum_{j\in W^t_i} \sum_{s\in S} u^t_{ijs} = \sum_{v\in V} m^t_{vi} \cdot \theta^t_i, \ \forall t \in T, i \in I \tag{14}$$

$$\sum_{j\in J} \sum_{s\in S} M^c_s z_{js} + \sum_{v\in V} \sum_{j\in J} \sum_{s\in S} M^t_v x_{vjs} \le M \tag{15}$$

$$x_{vjs}, u^t_{ijs} \in \mathbb{Z}_+, \ \forall t \in T, v \in V, i \in I, j \in J, s \in S \tag{16}$$

$$\theta^t_i, z_{js} \in \{0, 1\}, \ \forall t \in T, v \in V, i \in I, j \in J, s \in S \tag{17}$$

Objective function (9) maximizes the weighted total service performance based on the time coverage standard for each demand point in each period, averaged over all vehicles required by the vehicle coverage standard. Constraint (10) imposes that if the demand point i at period t satisfies its vehicle coverage standard m^t_{vi}, then the number of vehicles assigned at station j is at least m^t_{vi}. Constraint (11) precludes allocating more vehicles to a station than its capacity. Constraint (12) indicates that no more than one fire station can be built at a site. Constraint (13) requires that the number of fire trucks assigned to point i at period t from site j, if this site is selected to be a station of type s, does not exceed the number of fire trucks placed at j. Constraint (14) means that when point i is covered at period t, the total number of fire vehicles dispatched from all available stations equals the sum of m^t_{vi} over v, which means every demand point served has received the required numbers and types of fire trucks. Constraint (15) limits the total budget for all fire stations and fire trucks. Constraints (16) and (17) impose integer and binary restrictions on the decision variables, respectively.

IV. DATA PREPARATION

A. DETERMINATION OF THE POTENTIAL FIRE STATIONS

The selection of candidate fire stations is achieved by considering the primary road network and by using the network analysis tool of GIS. The initial candidate sites for fire stations are selected to be all of the nodes in the road network dataset. Then, the analysis functions "buffer calculation" and "attribute selection" make it possible to remove the sites that are relatively close to existing fire stations and POIs (points of interests), where congestion is assumed to be large and therefore hamper rapid response times. This procedure produces 336 potential fire stations locations shown in Figure 3.

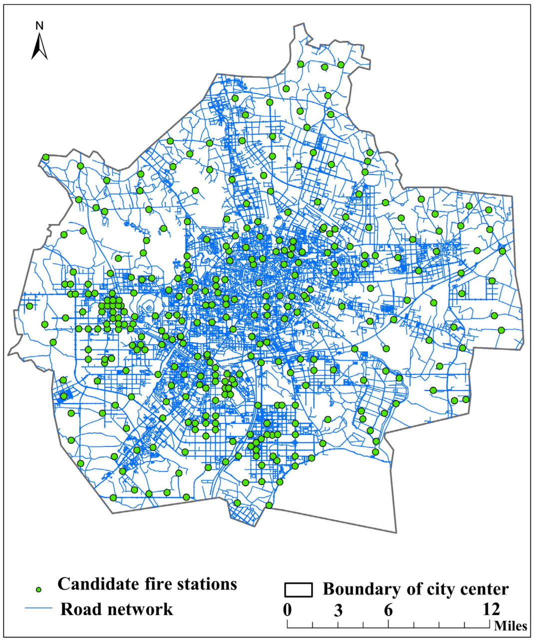

FIGURE 3. Candidate fire stations.

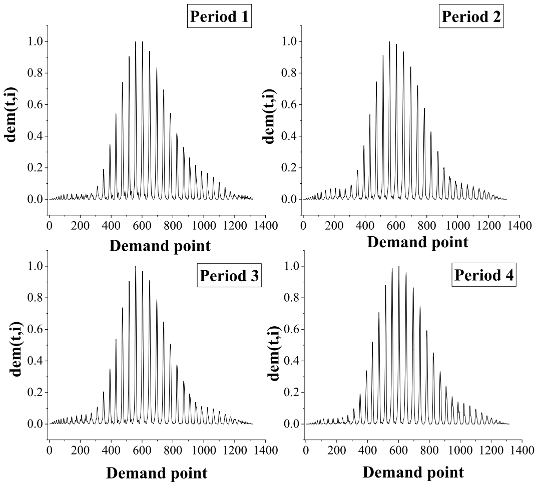

FIGURE 4. Demand weights of demand points at each period.

B. DETERMINATION FOR THE DISTRIBUTION OF DEMAND POINTS

This paper chooses Hefei (China) as the study area to examine the performance of the proposed model. The location of fire stations is guided by the spatial distribution of fire risk in Hefei. In particular, the discrete distribution of fire risk is used as demand weights for demand points. As mentioned above, this paper groups traffic patterns into four distinct periods: weekday morning peak, weekday evening peak, off-peak, and weekend evening peak.

Fire probability and fire consequences have a significant impact on fire risk. Fire probability can be expressed by annual average fire density (number of fires per square kilometer per year.) The direct fire property loss density (the amount of property loss per square kilometer) is used to represent the distribution of annual average fire consequence

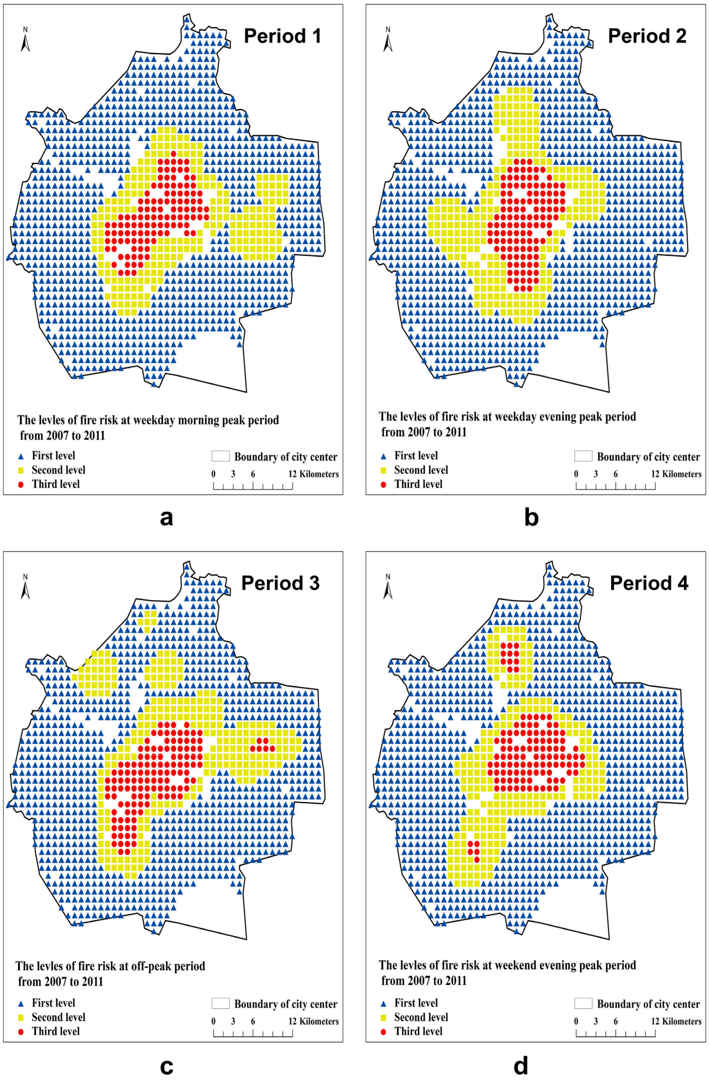

FIGURE 5. Distribution of annual average fire risk at each period from 2007 to 2011.

in this research due to the lack of historical data for other types of fire losses.

The kernel density analysis in GIS is a tool to express the fire probability distribution and fire consequence for each period. It requires a "smoothing" parameter that this paper sets to be a 5km fixed window width. Then, the study area is subdivided into a total of 1317 grid cells, each with dimension 1km by 1km. Every cell has two values for each period: the fire probability and the fire consequence, which are obtained from the kernel density surface using the "values extract to points" tool in GIS.

The rank distribution of fire risk at each period is obtained as the product of fire density and kernel density of fire property loss for the corresponding period. In other words, the fire risk at the center of each cell is the product of its fire probability and fire consequence. The values of fire risk at each period were denoted by dem_i^t in Table 3. They are standardized between zero to one and graphically represented in Figure 4. For simplicity, this paper divides fire risk in the study area into three levels: first level (about 70 percent of demand points with the lowest fire risk), second level (about 20 percent with the medium fire risk), and third level (the remaining 10 percent with the highest fire risk). The results are shown in Figure 5.

C. OTHER DATA

The remaining data is set according to local fire department records. The costs of a vehicle of type $v1$ and a vehicle of type $v2$ are ¥200000 and ¥300000, respectively. The construction costs for a station of type $s1$ and a station of type $s2$ are ¥6000000 and ¥4000000, respectively. Finally, the capacities of the a station of type $s1$ and a station of type $s2$ are set at 6 and 3, respectively.

V. RESULTS AND DISCUSSION

This paper uses Gurobi to solve the proposed model as it is an integer programming problem. Gurobi is run on a notebook with a 2.6 GHz CPU and 8.00 GB memory. It is set to terminate either when a relative MIP optimality gap smaller

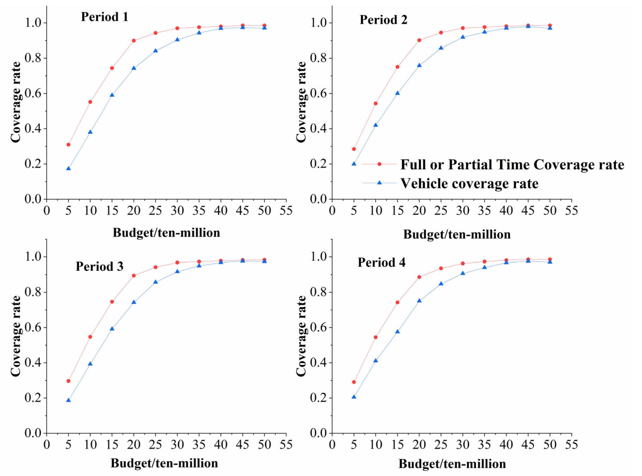

FIGURE 6. Relationships between FPTC-rate, vehicle coverage rate, and budget at each period.

than 10% has been achieved, or when it has exceeded a maximum running time set to 3 hours. Gurobi provides a feasible near-optimal solution for all experimental results. These results are analyzed in the following subsections.

A. SENSITIVITY ANALYSIS FOR RESCUE TIME SERVICE AND FIRE ENGINES SERVICE

The *full or partial time coverage rate* (FPTC-rate) is defined as the percentage of demand points covered within six minutes without considering fire engine requirements. The vehicle coverage rate indicates the percentage of demand points served under the vehicle coverage standard based on 90 percent reliability.

Figure 6 shows the trade-off among the total budget, FPTC-rate, and vehicle coverage rate for each period. Higher total budgets for the Hefei urban area yield higher rates of FPTC and vehicle coverage, although the rates of increase gradually flatten as higher budgets are considered. This flattening occurs in each period when the total budget is approximately ¥200 million for FPTC-rate and ¥250 million for vehicle coverage rate. In words, these results suggest that ¥200 million is sufficient in providing adequate response time coverage service but not in offering vital fire engine coverage service. It thus appears that the vehicle coverage standard is more stringent than the time coverage standard in our case. It also appears that locating fire service areas solely based on traditional fire response time might yield unreliable service.

B. SENSITIVITY ANALYSIS FOR FIRE STATIONS AND FIRE TRUCKS

Figure 7 depicts, as a function of the total budget, the optimal number of fire stations of each type to locate and the optimal number of fire trucks of each type to acquire. The number of located fire stations, the numbers of acquired vehicles of type $v1$ and $v2$ increase approximately linearly with the budget. However, the number of stations of type $s2$ sharply decreases as the budget grows beyond ¥400 million. Therefore, when a sufficient budget is available, it seems beneficial to establish stations of type $s1$ that have a larger capacity.

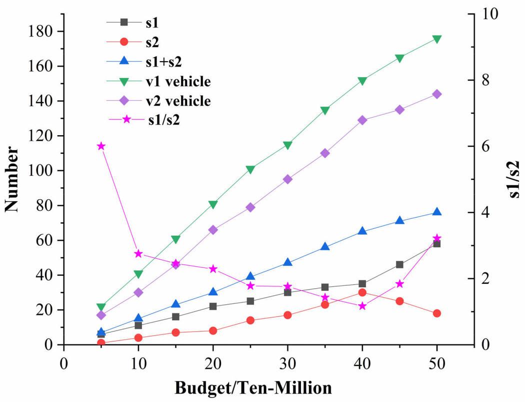

FIGURE 7. Relationships among two kinds of fire stations, two types of fire engines, and total budget.

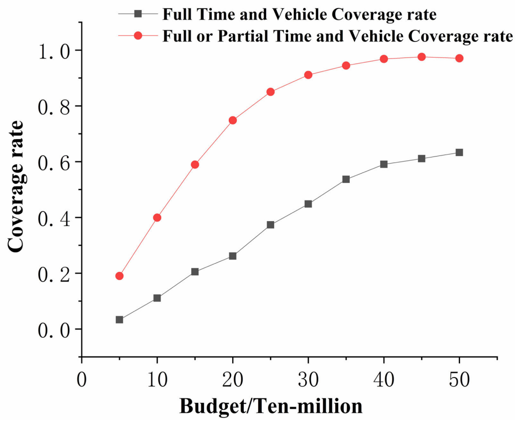

FIGURE 8. Relationships among the FPTVC-rate, the FTVC-rate, and total budget.

Additionally, the curve depicting the ratio between the number of stations of type $s1$ and of type $s2$ (s1/s2) as a function of the total budget first decreases but then increases when the budget reaches ¥400 million. This indicates that, when the budget is insufficient, the establishment of stations of type $s2$ dominates. one could then extrapolate that the popular current practice of building small fire stations is in fact an ''optimal'' reaction to limited budgets of fire departments.

C. SENSITIVITY ANALYSIS FOR COOPERATIVE FIRE SERVICES

The importance of the interplay between response time and availability of vehicles was highlighted in previous sensitivity analyses. To investigate it further, two new indicators are computed. The *full or partial time and vehicle coverage rate* (FPTVC-rate) is defined as the percentage of demand points served within six minutes based on the 90% reliability vehicle coverage standard in all periods. The *full time and vehicle coverage rate* (FTVC-rate) is defined as the percentage of demand points covered within two minutes based on the 90% reliability vehicle coverage standard in all periods.

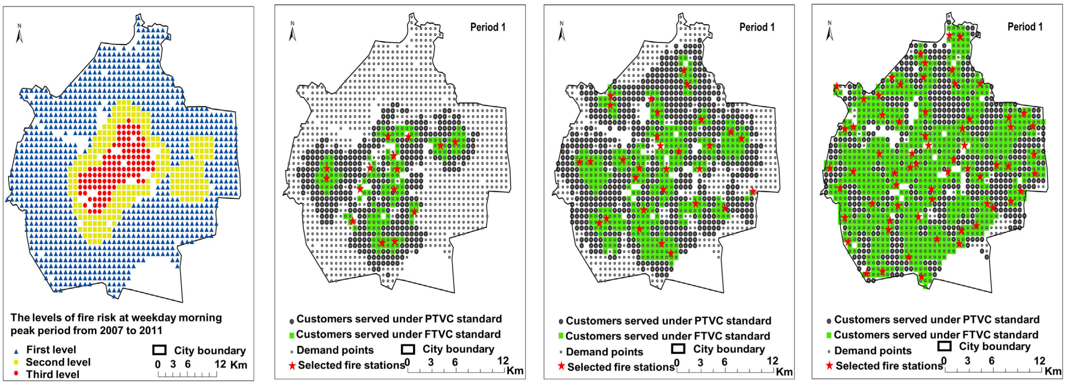

FIGURE 9. Distribution of demand points served under FTVC and PTVC standard during weekday morning peak period for budget ¥100 million, ¥200 million, and ¥400 million, respectively.

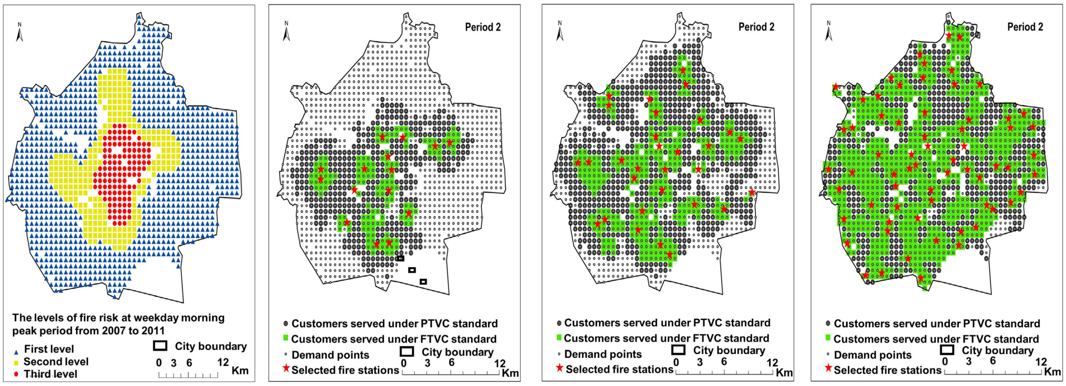

FIGURE 10. Distribution of demand points served under FTVC and PTVC standard during weekday peak evening period for budget ¥100 million, ¥200 million, and ¥400 million, respectively.

TABLE 6. Coverage rates under FPTVC standard during all periods for three different budgets.

Periods	Coverage rate under FPTVC standard		
	¥100 million	¥200 million	¥400 million
T1	40.0%	74.3%	96.9%
T2	41.7%	75.9%	97.0%
T3	39.2%	74.2%	96.6%
T4	40.9%	75.0%	96.7%

T1: weekday morning peak
T2: weekday evening peak
T3: off-peak
T4: weekend evening peak

Figure 8 shows that the FTVC-rate increases approximately linearly with the total budget. It also suggests that achieving a fire response time of two minutes for each area in the city would require a large budget. It is unlikely that such a budget would be available to city authorities. In contrast, Figure 8 shows that FPTVC-rates close to one can be achieved with much smaller budgets. Further, the increase in FPTVC-rates (with six minutes of firefighting time) as a function of the budget seems to flatten out drastically when the budget reaches ¥250 million. This result supports the discussion presented in Section V-A.

D. GIS-LAYER TREND ANALYSIS OF DEMAND POINTS SERVED UNDER THREE KINDS OF STANDARDS

The *FPTVC standard* is said to be met for a demand point when it is covered within 6 minutes under the 90% vehicle coverage standard. Similarly, the *FTVC standard* is said to be met for a demand point when it is covered within two minutes while the *PTVC standard* is said to be met for a demand point when it is covered from two minutes to six minutes. Figures 9–12 illustrate the layout trends for optimized fire station locations and demand points served under the FTVC standard and the PTVC standard during the four periods when considering three different budgets. The located fire stations are represented as red pentagrams. The demand points served under the FTVC standard are colored green, and those covered under the PTVC standard are colored black. The distribution trends of demand points served in each period are similar because the demand weights in each period are similarly distributed (leftmost picture in each figure.) Additionally, fire service areas are gradually opened starting from the central regions to the remote areas as the budget grows. Intuitively, the model focuses on level 3 areas occupying a relatively large proportion of demand weights to maximize the objective. This provides some validation to the model as such strategy indeed produces strong fire response

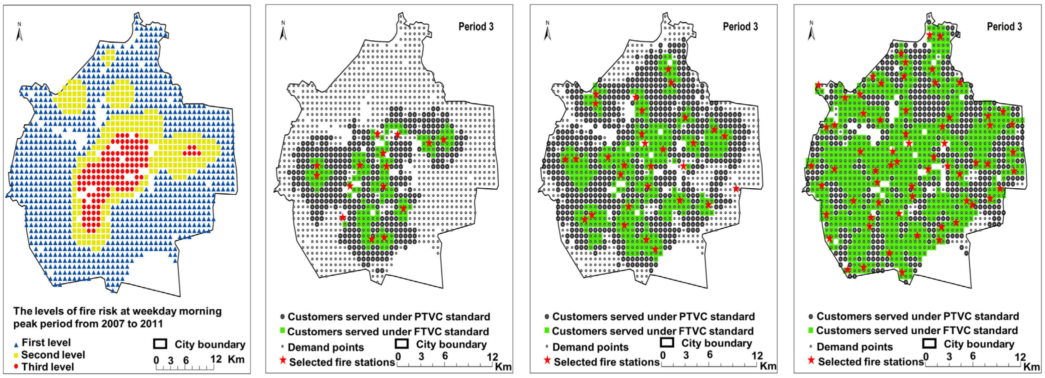

FIGURE 11. **Distribution of demand points served under FTVC and PTVC standard during off-peak period for budget ¥100 million, ¥200 million, and ¥400 million, respectively.**

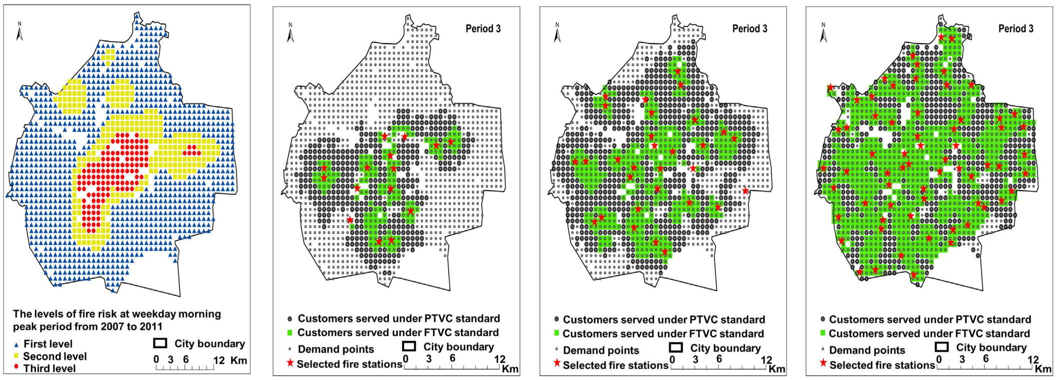

FIGURE 12. **Distribution of demand points served under FTVC and PTVC standard during weekday evening peak period for budget ¥100 million, ¥200 million, and ¥400 million, respectively.**

reliability among all periods. Table 6 presents the specific coverage rate values under the FPTVC standard.

VI. CONCLUSION

Fire and rescue services are fundamental to the safety of humans, property, and the natural environment. Efficient fire prevention and protection can significantly mitigate the loss of lives and can reduce economic damage. This paper proposes a comprehensive location model to select the best sites for fire stations together with determining the types and numbers of vehicles they should be assigned. This model is an integer program that can be solved efficiently using commercial solvers, even for practical instances such as the large metropolitan area of Hefei, China. It integrates multiple considerations, including the change of traffic status over multiple periods, the use of different types and numbers of vehicles, response time, and service reliability. It does so through the use of two coverage measures: vehicle coverage accounting for fire service reliability and time coverage describing fire service accessibility. A series of sensitivity analyses are conducted to reveal how indicators, such as FPTC-rate, vehicle coverage rate, FPTVC-rate, and fire station number change as a function of the available budget. Additionally, based on the FPTVC standard, the spatial distribution of fire and rescue services at each period under three different budgets is presented. such results can provide guidelines for local policy-makers.

There exist several other directions for future research. This paper computes fire risk as the product of fire probability and fire consequence. A more detailed approach, perhaps using random forest and geographically weighted regression, could be used to capture a series of other influential factors such as socioeconomic conditions (population, road, railway, etc.), climate and topographic conditions (temperature, humidity, elevation, etc.) to predict the fire risk. Moreover, collaboration and cooperation with local or regional fire response operators should be undertaken, so as to identify any practical impediment in the implementation of the prescriptions of the model. Given the uncertainties in fire station siting problems, an emerging modeling paradigm called distributionally robust optimization (DRO) could also be helpful to capture random factors not currently considered in the model; see [32]. Therefore, another promising avenue of research is to explore ways to develop practical DRO approaches to the location of fire stations under fire uncertainty for urban emergency fire services.

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