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A Novel Simple Format of Maxwell's Equations in SI Units

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ABSTRACT In the International System of Units (SI), distinct physical dimensions were assigned to the electric (E) and magnetic (H) fields as volt-per-meter and ampere-per-meter, respectively. To save the dimensional balance in the standard Maxwell's equations (MEs) in SI units, a pair of free-space constants, ϵ_0 and μ_0, with their dimensions of farad-per-meter and henry-per-meter were installed heuristically. Eventually, every quantity that participated in each ME in SI units has an individual physical dimension distinct from the other terms therein. This situation hampers the control of the dimensional balance in the processes of analytical manipulations with the MEs during the theoretical studies. Reformatting the free-space constants is performed in this article so that one new constant is obtained with its dimension of volt, and the other one has its dimension of ampere. These gave a handle to scale the electric and magnetic fields appropriately. Ultimately, the new fields are obtained with their common dimension of inverse-meter. Meanwhile, the standard differential procedures $\epsilon_0\frac{\partial}{\partial t}$ and $\mu_0\frac{\partial}{\partial t}$ from MEs are obtained in their simple common format of $\frac{\partial}{c\,\partial t}$, where c is the speed of light. The MEs in the novel format are exhibited for the fields in the free space, plasma, and dielectrics. The content of this article is destined for the researchers who deal with theoretical studies in electrodynamics, and the level of content is appropriate for and realizable by recent graduates, M.Sc. and Ph.D. students, and professionals.

INDEX TERMS Maxwell's equations, electrodynamics, time domain analysis.

I. INTRODUCTION

Since the 60s, the International System of Units (SI) has been assigned for general use in all scientific disciplines and engineering. The standard Maxwell's equations supplemented with appropriate constitutive relations, boundary and initial conditions are overloaded by the dimensions of the quantities. This fact hampers controlling the dimensional balance, which is especially virulent in theoretical studies. This article shows the way how to facilitate this problem. Maxwell's equations supplemented with the dynamic causal constitutive relations for the fields in plasma, Lorentz and Debye dielectrics are rearranged to the announced new format.

A. PREREQUISITES

The SI base units, which has been established in 1960, is a successor to Giovanni Giorgi's MKSA metric system proposed in 1901: see [1]- [3]. G. Giorgi noticed that mechanical and electric *energy* were expressible via fundamental units of length, mass, and time. He extended the classical definition of an *absolute* system of units on his MKSA [4]- [5]. The SI was founded on the same as Giorgi's set of four *basic* units: see **Table 1**.

TABLE 1. Basic SI units in electromagnetics.

Physical quantity	Name	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A

In **Table 2**, a set of frequently used units is presented. The dimension $\lfloor N \rfloor$ of the force unit newton, i.e., $\lfloor N \equiv kgm/s^2 \rfloor$, is exhibited especially to observe the connecting link between the mechanical units and ampere $\lfloor A \rfloor$.

In what follows, we specially extract the dimensions of physical quantities and select them by the floor brackets, like $\lfloor \# \rfloor$, to verify the dimensional balance in the equations.

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TABLE 2. Derived SI units with special names.

Physical quantity	Name	Symbol
force	newton	$\lfloor N = \frac{kgm}{s^2} \rfloor$
energy	joule	$\lfloor J = Nm \rfloor$
power	watt	$\lfloor W = \frac{Nm}{s} = \frac{J}{s} \rfloor$
electric charge	coulomb	$\lfloor C = As \rfloor$
electric tension	volt	$\lfloor V = \frac{Nm}{As} = \frac{W}{A} \rfloor$
electric resistance	ohm	$\lfloor \Omega = \frac{Nm}{A^2 s} = \frac{V}{A} \rfloor$
electric conductance	siemens	$\lfloor S = \frac{A^2 s}{Nm} = \frac{A}{V} \rfloor$
electric capacitance	farad	$\lfloor F = \frac{A^2 s^2}{Nm} = \frac{A}{V} s \rfloor$
inductance	henry	$\lfloor H = \frac{Nm}{A^2} = \frac{V}{A} s \rfloor$
magnetic flux	weber	$\lfloor Wb = \frac{Nm}{A} = V s \rfloor$
magnetic flux density	tesla	$\lfloor T = \frac{Wb}{m^2} = \frac{V}{m} \frac{s}{m} \rfloor$

B. NOVELTY

The shortest way to exhibit the novelty is doing that for the standard fields in the free space, *i.e.,*

$$\underbrace{\nabla\times}_{\lfloor 1/m \rfloor} \underbrace{\mathcal{H}(\mathbf{r},t)}_{\lfloor A/m \rfloor} = \underbrace{\epsilon_0}_{\lfloor F/m \rfloor} \underbrace{\partial/\partial t}_{\lfloor 1/s \rfloor} \underbrace{\mathcal{E}(\mathbf{r},t)}_{\lfloor V/m \rfloor} \tag{1a}$$

$$\underbrace{\nabla\times}_{\lfloor 1/m \rfloor} \underbrace{\mathcal{E}(\mathbf{r},t)}_{\lfloor V/m \rfloor} = \underbrace{-\mu_0}_{\lfloor H/m \rfloor} \underbrace{\partial/\partial t}_{\lfloor 1/s \rfloor} \underbrace{\mathcal{H}(\mathbf{r},t)}_{\lfloor A/m \rfloor} \tag{1b}$$

where $\mathbf{r}$ is the position vector of a point of observation, t is an observation time; ϵ_0 and μ_0 are defined in [2] as

$$\epsilon_0 = \frac{1}{c_0^2 4\pi\times 10^{-7}} \left\lfloor \frac{F}{m} = \frac{A^2 s^2}{Nm^2} \equiv \frac{A}{V}\frac{s}{m} \right\rfloor \tag{2a}$$

$$\mu_0 = 4\pi\times 10^{-7} \left\lfloor \frac{H}{m} = \frac{N}{A^2} \equiv \frac{V}{A}\frac{s}{m} \right\rfloor. \tag{2b}$$

Herein, c_0 is a dimension-free constant whose numerical value, $c_0 = 2.99792458\times 10^8$, specifies how many "meters per second," $\lfloor m/s \rfloor$, the speed of light has in SI units. The product of ϵ_0 and μ_0, taken as $1/\sqrt{\epsilon_0\mu_0} = c_0 \lfloor m/s \rfloor = c$, specifies the speed of light in vacuum, c, what was fixed in the set of *fundamental* SI constants in 1983: see [1], [4].

The fields $\mathcal{E}$ and $\mathcal{H}$ in ($1a$) – ($1b$) have their *distinct* dimensions in SI units by definition. The field $\mathcal{E}$ has dimension of volt per meter, $\lfloor V/m \rfloor$, but the field $\mathcal{H}$ has dimension of ampere per meter, $\lfloor A/m \rfloor$. That is why the dimensional constants, ϵ_0 and μ_0, were *heuristically* installed in the MEs to provide the coincidence of the SI dimensions at the left-hand and right-hand sides in each equation, ($1a$) and ($1b$) .

The crucial idea resulting in the novel format of MEs was put forward at symposiums [6], [7] and implemented via solving the real time-domain problems [8], [9]. The central point is redefinition of the free-space constants as

$$\epsilon_0^{\mathrm{v}} \overset{\text{def.}}{=} \sqrt{\frac{1N}{\epsilon_0}} \left\lfloor V = \frac{Nm}{As} \right\rfloor \text{ and } \mu_0^{\mathrm{a}} \overset{\text{def.}}{=} \sqrt{\frac{1N}{\mu_0}} \lfloor A \rfloor \tag{3}$$

where N at the radicals is a force of one newton. The new constant ϵ_0^{v} has dimension of volt, $\lfloor V \rfloor$, and numerical value *close* [1] to 3.361×10^5. The constant μ_0^{a} has dimension of ampere, $\lfloor A \rfloor$, and its numerical value is close to 8.921×10^2.

One can use ϵ_0^{v} and μ_0^{a} as the *scaling factors* for the field vectors, $\mathcal{E}$ and $\mathcal{H}$, to factorize their dimensions as

$$\left.\begin{aligned} \underbrace{\mathcal{E}(\mathbf{r},t)}_{\lfloor V/m \rfloor} &= \underbrace{\epsilon_0^{\mathrm{v}}}_{\lfloor V \rfloor} \underbrace{\mathbb{E}(\mathbf{r},t)}_{\lfloor 1/m \rfloor} = \underbrace{3.361\times 10^5}_{\lfloor V \rfloor} \times \underbrace{\mathbb{E}(\mathbf{r},t)}_{\lfloor 1/m \rfloor} \\ \underbrace{\mathcal{H}(\mathbf{r},t)}_{\lfloor A/m \rfloor} &= \underbrace{\mu_0^{\mathrm{a}}}_{\lfloor A \rfloor} \underbrace{\mathbb{H}(\mathbf{r},t)}_{\lfloor 1/m \rfloor} = \underbrace{8.921\times 10^2}_{\lfloor A \rfloor} \times \underbrace{\mathbb{H}(\mathbf{r},t)}_{\lfloor 1/m \rfloor}. \end{aligned}\right\} \tag{4}$$

Notice that the "electric" and "magnetic" parts of the SI field dimensions (i.e., volt, $\lfloor V \rfloor$, and ampere, $\lfloor A \rfloor$, respectively) are relegated to the scaling factors, namely: ϵ_0^{v} and μ_0^{a}. Derivations of ϵ_0^{v} and μ_0^{a} are given in APPENDIX A.

Rearrangement of the standard MEs ($1a$) – ($1b$) to their announced simple format can be made via several algebraic manipulations. Let us start with equation ($1a$) . Replace formally $\mathcal{H}$ field by $\mu_0^{\mathrm{a}}\mathbb{H}$ at the left-hand side and $\mathcal{E}$ field by $\epsilon_0^{\mathrm{v}}\mathbb{E}$ at the right-hand side. Notice that $\epsilon_0\mathcal{E} = \epsilon_0\epsilon_0^{\mathrm{v}}\mathbb{E} = \sqrt{N\epsilon_0}\mathbb{E}$. After that, divide the left-hand and the right-hand sides of the equation by μ_0^{a} and take into account that $\sqrt{N\epsilon_0}/\mu_0^{\mathrm{a}} = 1/c$ where c is the speed of light. This yields the first equation in (5) . As long as $\sqrt{N\mu_0}/\epsilon_0^{\mathrm{v}} = 1/c$, as well, the second equation in (5) is equivalent to ($1b$) . Thus, the novel simple format of the MEs *in SI units* (!) is

$$\nabla\times\mathbb{H}(\mathbf{r},t) = \frac{\partial}{c\,\partial t}\mathbb{E}(\mathbf{r},t),\ \nabla\times\mathbb{E}(\mathbf{r},t) = -\frac{\partial}{c\,\partial t}\mathbb{H}(\mathbf{r},t) \tag{5}$$

where the fundamental *mechanical* units participate only. The dimensional balance in (5) is evident unlike the equations ($1a$) – ($1b$) . The operations ∇ and $\frac{\partial}{c\,\partial t}$ and the fields, $\mathbb{E}$ and $\mathbb{H}$, have the common dimension of $\lfloor 1/m \rfloor$ in (5). If one can consider $\mathbb{E}$ and $\mathbb{H}$ as the primary fields, then the fields $\mathcal{E}$ and $\mathcal{H}$ are the vectors *arithmetically* derived by formulas (4) .

Mathematical format of (5) one-to-one coincides with the Heaviside-Lorentz equations (HLEs) albeit the latter have been derived within the framework of the CGS metric system: see [3], [5]. There is one more coincidence between (5) and HLEs, namely: the electric and magnetic field vectors have their *common* dimensions. In HLEs, this dimension is in centimeter-gram-second as $\left\lfloor g^{\frac{1}{2}} cm^{-\frac{1}{2}} s^{-1} \right\rfloor$. But in (5) , $\mathbb{E}$ and $\mathbb{H}$ have the common SI dimension of $\lfloor 1/m \rfloor$.

C. MOTIVATION AND GOAL OF THE STUDY

The simplicity of the way from ($1a$) – ($1b$) to (5) motivates to extend that approach on more complicated MEs for practical problems. The present study's goal is reformatting the equations for electromagnetic fields in plasma and in the

[1]Exact values of ϵ_0^{v} and μ_0^{a} can not be calculated because of that *irrational* number π which is present in the definitions ($2a$) – ($2b$) .

dielectrics of Lorentz and Debye types. Besides, the energetic and mechanical field characteristics should be rewritten in terms of the novel primary fields $\mathbb{E}$ and $\mathbb{H}$. In the analysis that follows, we imply that the space-time field vectors and the other field quantities are the *real-valued* functions of coordinates and time.

D. COMPOSITION OF THE PAPER

The MEs involving the electromagnetic field and induced plasma current density is considered in Section II simultaneously with the motion equation for the plasma current density. The MEs involving the electromagnetic field and the polarization vector for a dielectric are considered in Section III simultaneously with an appropriate motion equation for the polarization. The motion equation plays the role of a dynamic causal constitutive relation between the polarization and the applied electric field. The energetic field characteristics and the velocity of transporting power flow as the space-time functions are derived in Section IV. The mass and momentum as the mechanical equivalents of the field energetic characteristics are derived in Section V. In Section VI, discussion and conclusion are presented.

II. REFORMATTING MAXWELL's EQUATIONS FOR PLASMA

MEs jointly with the motion equation for the current density in non-magnetized, collisional plasma are

$$\underbrace{\nabla\times}_{\lfloor 1/m\rfloor}\underbrace{\mathcal{H}(\mathbf{r},t)}_{\lfloor A/m\rfloor} = \underbrace{\epsilon_0}_{\lfloor F/m\rfloor}\underbrace{\partial/\partial t}_{\lfloor 1/s\rfloor}\underbrace{\mathcal{E}(\mathbf{r},t)}_{\lfloor V/m\rfloor} + \underbrace{\mathcal{J}(\mathbf{r},t)}_{\lfloor A/m^2\rfloor} \tag{6a}$$

$$\underbrace{\nabla\times}_{\lfloor 1/m\rfloor}\underbrace{\mathcal{E}(\mathbf{r},t)}_{\lfloor V/m\rfloor} = \underbrace{-\mu_0}_{\lfloor H/m\rfloor}\underbrace{\partial/\partial t}_{\lfloor 1/s\rfloor}\underbrace{\mathcal{H}(\mathbf{r},t)}_{\lfloor A/m\rfloor} \tag{6b}$$

$$\underbrace{d/dt}_{\lfloor 1/s\rfloor}\underbrace{\mathcal{J}(\mathbf{r},t)}_{\lfloor A/m^2\rfloor} + \underbrace{\nu_p}_{\lfloor 1/s\rfloor}\underbrace{\mathcal{J}(\mathbf{r},t)}_{\lfloor A/m^2\rfloor} = \underbrace{\omega_p^2}_{\lfloor 1/s^2\rfloor}\underbrace{\epsilon_0}_{\lfloor F/m\rfloor}\underbrace{\mathcal{E}(\mathbf{r},t)}_{\lfloor V/m\rfloor} \tag{6c}$$

where $\mathcal{J}$ is the current density in plasma polarized by the field $\mathcal{E}$; ν_p is the parameter of electron collision frequency; $\omega_p = \sqrt{q_e^2\mathcal{N}_e/(m_e\epsilon_0)}$ is the plasma frequency; $\mathcal{N}_e$ is the parameter of volumetric density of electrons in plasma, q_e and m_e are the charge and mass of electron. Differential equation ($6c$) with time derivative establishes the *dynamic* constitutive relationship between the current density $\mathcal{J}$ and the applied field $\mathcal{E}$ inducing the plasma current [11], [12].

Execute scaling the current density $\mathcal{J}$ by μ_0^{a} from (3) as

$$\underbrace{\mathcal{J}(\mathbf{r},t)}_{\lfloor A/m^2\rfloor} = \underbrace{\mu_0^{\mathrm{a}}}_{\lfloor A\rfloor}\underbrace{\mathbb{J}(\mathbf{r},t)}_{\lfloor 1/m^2\rfloor} \cong \underbrace{8.921\times 10^2}_{\lfloor A\rfloor}\times\underbrace{\mathbb{J}(\mathbf{r},t)}_{\lfloor 1/m^2\rfloor}. \tag{7}$$

and substitute (7) with (4) to ($6a$) − ($6c$) . This yields the MEs ($8a$) − ($8c$) in the Heaviside-Lorentz format and simple motion equation ($8c$) for $\mathbb{J}$ with its dimension of $\lfloor 1/m^2\rfloor$ as

$$\nabla\times\mathbb{H}(\mathbf{r},t) = \frac{\partial}{c\,\partial t}\mathbb{E}(\mathbf{r},t) + \mathbb{J}(\mathbf{r},t) \tag{8a}$$

$$\nabla\times\mathbb{E}(\mathbf{r},t) = -\frac{\partial}{c\,\partial t}\mathbb{H}(\mathbf{r},t) \tag{8b}$$

$$\frac{d}{dt}\mathbb{J}(\mathbf{r},t) + \nu_p\mathbb{J}(\mathbf{r},t) = \omega_p k_p\mathbb{E}(\mathbf{r},t) \tag{8c}$$

where $k_p=\omega_p/c$ with its dimension of $\lfloor 1/m\rfloor$ is the wave number. The dimensional balance in the equations ($8a$) − ($8c$) is evident. In comparison with ($8a$) − ($8c$) , the original system of Maxwell's equations ($6a$) − ($6c$) looks as a "parade" of the derived SI units taken from **Table 1.**

III. REFORMATTING MAXWELL's EQUATIONS FOR DIELECTRICS OF LORENTZ AND DEBYE TYPES

The dielectrics of Lorentz and Debye types have different physical mechanisms of polarization: see [5], [13]. Therefore, the systems of Maxwell's equations jointly with the appropriate motion equation will be considered separately.

A. MAXWELL's EQUATIONS FOR A DIELECTRIC OF LORENTZ TYPE

The MEs in SI units for the fields in a lossy dielectric embedded in the background medium are

$$\underbrace{\nabla\times}_{\lfloor 1/m\rfloor}\underbrace{\mathcal{H}}_{\lfloor A/m\rfloor} = \underbrace{\partial/\partial t}_{\lfloor 1/s\rfloor}[\underbrace{\epsilon_0\mathcal{E}}_{\lfloor C/m^2\rfloor} + \underbrace{\mathcal{P}(\mathcal{E})}_{\lfloor C/m^2\rfloor}] \tag{9a}$$

$$\underbrace{\nabla\times}_{\lfloor 1/m\rfloor}\underbrace{\mathcal{E}}_{\lfloor V/m\rfloor} = \underbrace{-\mu_0}_{\lfloor H/m\rfloor}\underbrace{\partial/\partial t}_{\lfloor 1/s\rfloor}\underbrace{\mathcal{H}}_{\lfloor A/m\rfloor} \tag{9b}$$

where the macroscopic polarization vector $\mathcal{P} \equiv \mathcal{P}(\mathbf{r},t)$ is related to the bound charge carriers composing atom/molecules. Notation $\mathcal{P}(\mathcal{E})$ implies that the polarization vector $\mathcal{P}$ is induced by the applied field $\mathcal{E}$. Relationship between $\mathcal{P}$ and $\mathcal{E}$ is governed by the Newton motion equation as

$$\left(\partial^2/\partial t^2 + 2\Gamma\partial/\partial t + \omega_r^2\right)\underbrace{\mathcal{P}(\mathbf{r},t)}_{\lfloor C/m^2\rfloor} = \underbrace{\omega_p^2}_{\lfloor 1/s^2\rfloor}\underbrace{\epsilon_0\mathcal{E}'(\mathbf{r},t)}_{\lfloor C/m^2\rfloor} \tag{10a}$$

$$\underbrace{\epsilon_0\mathcal{E}'(\mathbf{r},t)}_{\lfloor C/m^2\rfloor} = \underbrace{\epsilon_0\mathcal{E}(\mathbf{r},t)}_{\lfloor C/m^2\rfloor} + \frac{1}{3}\underbrace{\mathcal{P}(\mathbf{r},t)}_{\lfloor C/m^2\rfloor} \tag{10b}$$

where Γ and ω_r are the intrinsic parameters of the dielectric with their dimensions of hertz, $\lfloor Hz{=}1/s\rfloor$; $\omega_p = \sqrt{\mathcal{N}_e q_e^2/(m_e\epsilon_0)}$ is a plasma frequency specified by a parameter of volumetric density $\mathcal{N}_e$ of the polarizable electrons in atoms/molecules of the dielectric; q_e and m_e are the charge and mass of electron. At the right-hand side of ($10a$) , the *local* field $\mathcal{E}'$ stands. The field $\mathcal{E}'$ is expressible via the *applied* field $\mathcal{E}$ and $\mathcal{P}$ by Mosotti formula ($10b$) : see Eq. (4.96) in [5].

Consider first the element $[\epsilon_0\mathcal{E}+\mathcal{P}(\mathcal{E})]$ from the right-hand side of ($9a$) . The summation of $\epsilon_0\mathcal{E}$ and $\mathcal{P}$ implies that $\epsilon_0\mathcal{E}$ and $\mathcal{P}$ have their *common* dimension. Scaling $\mathcal{E}$ as $\mathcal{E} = \epsilon_0^{\mathrm{v}}\mathbb{E}$ in (4) suggests that scaling $\mathcal{P}$ should be the same as for the product $\epsilon_0\mathcal{E} = \epsilon_0\epsilon_0^{\mathrm{v}}\mathbb{E}$. So, one more scaling factor appears as

$$\boxed{\epsilon_0^{\mathrm{c}} \overset{\text{def.}}{=} \epsilon_0\epsilon_0^{\mathrm{v}} = \sqrt{N\times\epsilon_0}\,\lfloor C/m\rfloor.} \tag{11}$$

One can verify that $\epsilon_0^{\rm c}$ has a numerical value close to 2.976×10^{-6} and the vector $\mathcal{P}$ can be scaled as

$$\underbrace{\mathcal{P}(\mathbf{r},t)}_{\lfloor C/m^2\rfloor}=\underbrace{\epsilon_0^{\rm c}}_{\lfloor C/m\rfloor}\underbrace{\mathbb{P}(\mathbf{r},t)}_{\lfloor 1/m\rfloor}\cong\underbrace{2.976\times10^{-6}}_{\lfloor C/m\rfloor}\times\underbrace{\mathbb{P}(\mathbf{r},t)}_{\lfloor 1/m\rfloor}. \quad (12)$$

So, the equation ($10b$) can be rewritten as

$$\epsilon_0\mathcal{E}'=\epsilon_0^{\rm c}(\mathbb{E}+\mathbb{P}/3). \quad (13)$$

Finally,[2] substituting (12) , (13) , and (4) to ($9a$)$-$($10b$) results in the Heaviside-Lorentz format for the system of Maxwell's equations in SI units for the fields $\mathbb{E}$, $\mathbb{H}$, and $\mathbb{P}$ in the dielectrics of Lorentz type as

$$\nabla\times\mathbb{H}(\mathbf{r},t)=\frac{\partial}{\partial\varsigma}[\mathbb{E}+\mathbb{P}] \quad (14a)$$

$$\nabla\times\mathbb{E}(\mathbf{r},t)=-\frac{\partial}{\partial\varsigma}\mathbb{H}(\mathbf{r},t) \quad (14b)$$

$$\left(\partial^2/\partial\varsigma^2+2\gamma\partial/\partial\varsigma+\kappa_r^2\right)\mathbb{P}(\mathbf{r},t)=k_p^2\,\mathbb{E}(\mathbf{r},\varsigma) \quad (14c)$$

where the vector $\mathbb{P}(\mathbf{r},t)$ is identical to the notation $\mathbb{P}(\mathbb{E})$ in ($9a$) and $\varsigma=ct$, $k_p=\omega_p/c$, $\gamma=\Gamma/c$, $\kappa_r=\Omega/c$, $\Omega=\sqrt{\omega_r^2-\omega_p^2/3}$.

B. THE FIELDS IN A DIELECTRIC OF DEBYE TYPE

Another type of dielectrics, whose molecules have *permanent* dipole moments, exists even in the absence of an external field. Also, they retain their moments under the action of the applied external field. This sort of dielectrics (solids and liquids) was attributed to Debye's type [5]. The MEs for the standard field vectors, $\mathcal{E}$ and $\mathcal{H}$ are the same as ($9a$) $-$ ($9b$) . But Debye's motion equation for the macroscopic polarization vector denoted here as $\mathring{\mathcal{P}}$ is

$$\left(2\Gamma\partial/\partial t+\omega_r^2\right)\underbrace{\mathring{\mathcal{P}}(\mathbf{r},t)}_{\lfloor C/m^2\rfloor}=\underbrace{\omega_p^2}_{\lfloor 1/s^2\rfloor}\underbrace{\epsilon_0\mathcal{E}(\mathbf{r},t)}_{\lfloor C/m^2\rfloor} \quad (15)$$

where $\mathcal{E}$ is the applied field, but not the local one as in ($10a$). The dipole moments of such molecules attempt to rotate in response to the action of a time-varying field, $\mathcal{E}$. The rotary inertia causes mutual collisions of all the molecules to the detriment of their temporal time-varying acceleration terms, $\frac{\partial^2}{\partial t^2}\mathcal{P}$, like in ($10a$) . J.W.P. Debye proposed to operate with the frictional term, $2\Gamma\frac{\partial}{\partial t}\,\mathring{\mathcal{P}}$, in the motion equation and ignored the acceleration term, $\frac{\partial^2}{\partial t^2}\,\mathring{\mathcal{P}}$. Substitution of the "scaling" formulas, (4) and (12) (available for the vector $\mathring{\mathcal{P}}$, as well) to the system of Maxwell's equations with (15) results in

$$\nabla\times\mathbb{H}(\mathbf{r},t)=\frac{\partial}{\partial\varsigma}[\mathbb{E}(\mathbf{r},t)+\mathring{\mathbb{P}}(\mathbf{r},t)] \quad (16a)$$

$$\nabla\times\mathbb{E}(\mathbf{r},t)=-\frac{\partial}{\partial\varsigma}\mathbb{H}(\mathbf{r},t) \quad (16b)$$

$$\left(2\gamma\partial/\partial\varsigma+k_r^2\right)\mathbb{P}(\mathbf{r},t)=k_p^2\,\mathbb{E}(\mathbf{r},t) \quad (16c)$$

[2]One can find additional details in APPENDIX B.

where $\varsigma=ct$, $\gamma=\Gamma/c$, $k_r=\omega_r/c$ and $k_p=\omega_p/c$; the parameter ω_p is the same as in ($14a$) $-$ ($14c$) .

Notice that the usual water is the polar molecule (H_2O). The human and animal tissues involve water in high doses. Hence, the tissues should be modeled by the polar dielectrics in studying their interaction with the electromagnetic radiation [14]. Apart from water, the other substances as N_2, O_2, O_3, C, CO, SO_2, HCl, CH_3CN are the polar molecules, as well.

C. LORENTZ FORCE LAW IN TERMS OF THE NEW FIELDS

The standard definition of the Lorentz force law in SI units is

$$\mathcal{F}(\mathbf{r},t)=q\,[\mathcal{E}(\mathbf{r},t)+\mathbf{v}\times\mu_0\mathcal{H}(\mathbf{r},t)] \quad (17)$$

where the fields $\mathcal{E}$ and $\mathcal{H}$ act on a point-like charge q (measurable in coulomb, $\lfloor C=As\rfloor$) moving with a given velocity $\mathbf{v}$. Substituting the fields $\mathcal{E}=\epsilon_0^{\rm v}\mathbb{E}$ and $\mathcal{H}=\mu_0^{\rm a}\mathbb{H}$ to (17) specifies this force in terms of the fields $\mathbb{E}$ and $\mathbb{H}$ as

$$\mathbb{F}(\mathbf{r},t)=q\,\epsilon_0^{\rm v}\left[\mathbb{E}(\mathbf{r},t)+\frac{\mathbf{v}}{c}\times\mathbb{H}(\mathbf{r},t)\right] \quad (18)$$

where the factor $q\,\epsilon_0^{\rm v}$ has the dimension of $\lfloor CV\rfloor$ can be rearranged to $q_0\,3.361\times10^5$ with q_0 as the *quantity symbol* of the charge q. In other words, the *number* q_0 specifies how many coulombs the charge q carries in (17) . Dimension $\lfloor CV\rfloor$ is equivalent to $\lfloor Nm\rfloor$: see **Table 2.** Because the fields $\mathbb{E}$ and $\mathbb{H}$ each has dimension of $\lfloor m^{-1}\rfloor$, one can see that $\mathbb{F}$ has the SI dimension of newton $\lfloor N=kgm/s^2\rfloor$.

IV. ENERGETIC CHARACTERISTICS VIA THE NEW FIELDS

The following identities will be used afterwards:

$$\epsilon_0\left(\epsilon_0^{\rm v}\right)^2=\mu_0\left(\mu_0^{\rm a}\right)^2=N \text{ and } \epsilon_0^{\rm v}\mu_0^{\rm a}=cN. \quad (19)$$

Setting $\mathcal{E}=\epsilon_0^{\rm v}\mathbb{E}$ and $\mathcal{H}=\mu_0^{\rm a}\mathbb{H}$ to the standard definition of the field energy results in

$$\mathbb{U}(\mathbf{r},t)=\frac{1}{2}\left[|\mathbb{E}(\mathbf{r},t)|^2+|\mathbb{H}(\mathbf{r},t)|^2\right]\left\lfloor\frac{N}{m^2}=\frac{J}{m^3}\right\rfloor \quad (20)$$

where the dimension of $|\mathbb{E}|^2$ and $|\mathbb{H}|^2$ as $\lfloor 1/m^2\rfloor$ was put in $\lfloor\#\rfloor$. The scaled version of the standard Pointing vector is

$$\mathbb{S}(\mathbf{r},t)=\frac{1}{c}[\mathcal{E}\times\mathcal{H}]\equiv[\mathbb{E}(\mathbf{r},t)\times\mathbb{H}(\mathbf{r},t)]\left\lfloor\frac{N}{m^2}\right\rfloor \quad (21)$$

where the dimension of $\mathbb{E}\times\mathbb{H}$ as $\lfloor 1/m^2\rfloor$ was placed in $\lfloor\#\rfloor$. The physical dimension of $\mathbb{U}$ and $\mathbb{S}$ coincides. Hence, $\mathbb{S}$ can be *a part* of the total field energy density, $\mathbb{U}$, which is accumulated in the Pointing vector for transferring outside.

Umov had proved that the velocity of transportation of energy in *any* wave process is a simple fraction with the power flow at the numerator and the stored energy at the denominator [15]. According to Poynting's theorem [16], the standard SI power flow is $\mathcal{E}\times\mathcal{H}=c\,\mathbb{S}$ as it follows from (21) . The definition of the *total* stored SI field energy in (20) is also standard. The velocity of transferring the total field energy is

$$\mathbb{V}(\mathbf{r},t)=\frac{c\,\mathbb{S}(\mathbf{r},t)}{\mathbb{U}(\mathbf{r},t)}=c\,\frac{2\,[\mathbb{E}(\mathbf{r},t)\times\mathbb{H}(\mathbf{r},t)]}{|\mathbb{E}(\mathbf{r},t)|^2+|\mathbb{H}(\mathbf{r},t)|^2}. \quad (22)$$

Evidently, the power flow transfers the field energy with *local* velocity varying in space-time as $0 \leq \mathbb{V}(\mathbf{r},t) \leq c$, albeit the field itself propagates with its constant *phase* velocity c.

V. MECHANICAL EQUIVALENTS OF THE ENERGETIC FIELD CHARACTERISTICS

Recently, Gerald Kaiser defined the mechanical equivalents of the local electromagnetic fields [17]. He has done that under the strict restriction that the electric and magnetic fields have their *common* dimension. To this aim, he used Heaviside-Lorentz equations, which have been originally derived in the past in a version of the CGS units. We rearranged Maxwell's equations in SI units to the Heaviside-Lorentz format where the fields $\mathbb{E}$ and $\mathbb{H}$ have the common SI dimension. So, we can repeat Kaiser's technique in the SI units. It has not been done before, as far as we know.

The *scalar* $\mathbb{U}$ and the *vector* $\mathbb{S}$ in (20) and (21), respectively, have common their dimension. To make them *summable* compose two *scalars* as $\mathfrak{U} = \sqrt{\mathbb{U}^2}$ and $\mathfrak{I} = \sqrt{\mathbb{S}\cdot\mathbb{S}}$ what yields

$$\mathfrak{U}(\mathbf{r},t) = \sqrt{\frac{1}{4}\left(\mathbb{E}^2+\mathbb{H}^2\right)^2} = \sqrt{\frac{1}{4}\left(\mathbb{E}^4+2\mathbb{E}^2\mathbb{H}^2+\mathbb{H}^4\right)} \quad (23)$$

$$\mathfrak{I}(\mathbf{r},t) = \sqrt{(\mathbb{E}\times\mathbb{H})\cdot(\mathbb{E}\times\mathbb{H})} = \sqrt{\mathbb{E}^2\mathbb{H}^2-(\mathbb{E}\cdot\mathbb{H})^2} \quad (24)$$

The product $[\mathbb{E}\times\mathbb{H}]\cdot[\mathbb{E}\times\mathbb{H}]$ can be found out by applying identity (B.8) from [5]. Combination of $\mathfrak{U}$ and $\mathfrak{I}$ as

$$\mathrm{R}(\mathbf{r},t) = \sqrt{\mathfrak{U}^2-\mathfrak{I}^2} = \frac{1}{2}\sqrt{\left(\mathbb{E}^2-\mathbb{H}^2\right)^2+4(\mathbb{E}\cdot\mathbb{H})^2} \quad (25)$$

was named by Kaiser as the *reactive (rest) energy density*[3] and studied in [17]. The field vectors of the plane waves (homogeneous and inhomogeneous) in the free space are *locally* orthogonal. The modal fields in cavities and waveguides are locally orthogonal, as well. For all these cases, $\mathbb{E}\cdot\mathbb{H} = 0$ in the analysis that follows.

Einstein's famous formula available for any material body at the state of rest establishes a general relationship between the given energy of the body and its equivalent mass as

$$\mathrm{E} = \mathrm{m}c^2 \text{and "mass at rest" is} \mathrm{m} = \mathrm{E}/c^2, \quad (26)$$

see [18], for example. One can substitute serially $\mathfrak{U}$, $\mathfrak{I}$, R as E to (26) what yields the set of equivalent mass as

$$\mathrm{m}_{\mathfrak{U}}(\mathbf{r},t) = \frac{\mathfrak{U}}{c^2} = \frac{|\mathbb{E}|^2+|\mathbb{H}|^2}{2c_0^2}\left\lfloor\frac{N}{m^2}\frac{s^2}{m^2} \equiv \frac{kg}{m^3}\right\rfloor \quad (27)$$

$$\mathrm{m}_{\mathfrak{I}}(\mathbf{r},t) = \frac{\mathfrak{I}}{c^2} = \frac{1}{c_0^2}|\mathbb{E}|\,|\mathbb{H}|\left\lfloor\frac{kg}{m^3}\right\rfloor \quad (28)$$

$$\mathrm{m}_{\mathrm{R}}(\mathbf{r},t) = \frac{\mathrm{R}}{c^2} = \frac{1}{2c_0^2}\left(|\mathbb{E}|^2-|\mathbb{H}|^2\right)\left\lfloor\frac{kg}{m^3}\right\rfloor \quad (29)$$

[3] In the past, it was established that the analogous to $E^2 - H^2$ and $E\cdot H$ constructions are the local scalar Lorentz invariants of the space-time fields in arbitrary inertial reference frame [18].

where *number* $c_0 = 2.99792458\times10^8$ is the *quantity symbol* of the speed of light, c.

The equivalent mass stored in Pointing vector is proportional to c^{-2} (see (Vb) and $\mathcal{E}\times\mathcal{H} = c\,\mathbb{S}$ (see (21)). So, the definition of the vector of *mechanical momentum* for the equivalent mass transferred by the power flow density is

$$\mathrm{P}(\mathbf{r},t) = \frac{\mathcal{E}\times\mathcal{H}}{c^2} = \frac{1}{c_0}\frac{2\,[\mathbb{E}\times\mathbb{H}]}{|\mathbb{E}|^2+|\mathbb{H}|^2}\left\lfloor\frac{kg}{m^2s} \equiv \frac{kg}{m^3}\frac{m}{s}\right\rfloor \quad (30)$$

where the dimension $\lfloor Nm^{-2}\rfloor$ of the cross product is taken into account.

VI. DISCUSSION AND CONCLUSION

Main novelty is the definition of the new formats for the free-space constants proposed recently in [6], [7] as

$$\left.\begin{aligned} \epsilon_0^{\mathrm{v}} &\overset{def.}{=} \sqrt{\frac{N}{\epsilon_0}} = 3.361\times10^5\left\lfloor V = \frac{Nm}{As}\right\rfloor \\ \mu_0^{\mathrm{a}} &\overset{def.}{=} \sqrt{\frac{N}{\mu_0}} = 8.921\times10^2\lfloor A\rfloor \end{aligned}\right\} \quad (31)$$

where ϵ_0 and μ_0 are the free-space constants in SI units and N implies a force of 1 (one) newton. The standard electric and magnetic fields $\mathcal{E}$ and $\mathcal{H}$ have *distinct* dimensions of $\lfloor V/m\rfloor$ and $\lfloor A/m\rfloor$, respectively. Scaling them as

$$\mathcal{E}(\mathbf{r},t) = \epsilon_0^{\mathrm{v}}\mathbb{E}(\mathbf{r},t), \quad \mathcal{H}(\mathbf{r},t) = \mu_0^{\mathrm{a}}\mathbb{H}(\mathbf{r},t) \quad (32)$$

refers the dimensions of $\lfloor V\rfloor$ and $\lfloor A\rfloor$ to the scaling factors ϵ_0^{v} and μ_0^{a}. Meanwhile, the new fields, $\mathbb{E}$ and $\mathbb{H}$, remain in the SI units with the *common* dimension of $\lfloor 1/m\rfloor$.

Substitution of (32) to the standard Maxwell's equations simplifies them the novel (in SI units!) *symmetric* form as (5). The time derivative appears in the novel format as $\frac{\partial}{c\,\partial t}$ (c is the speed of light) instead of $\epsilon_0\frac{\partial}{\partial t}$ and $\mu_0\frac{\partial}{\partial t}$ originally. Numerical algorithms developed for that symmetrical format can provide *stability* of time-intensive computations.

The equivalent mass and mechanical momentum of the electromagnetic fields are calculated as the functions of time. It gives hints for an in-depth analysis of the energetic wave processes accompanying the propagation of the electromagnetic fields. The control of the physical dimensions in manipulations with Maxwell's equations in the novel format is much simpler than dealing with the standard ones.

One can find in publications [19]- [21] information about the implementation of MEs to analytically solve the boundary-value problems and the causal dynamic constitutive relations for the time-domain fields.

Modern communication widely applies digital signals (like Walsh functions) each of which is a chain of the unit-step functions of time [22]. Method of matrix exponential (instead of Fourier or Laplace transformations) is a powerful tool for solving the initial-boundary-value problems of electrodynamics and analysis of the digital signals [23]. The SI metric system has become obligatory for studies in all scientific disciplines since its inception in 1960. Nowadays, multiscale and multiphysics disciplines are appealing for the development of modern techniques and technologies, *i.e.,* Quantum

Electromagnetics [24]. Possibly, applying a factorization of the physical dimensions like (4) may be useful for further development of these modern topics.

APPENDIX A
DERIVATIONS OF ϵ_0^{v} AND μ_0^{a}

Essential simplifying *format* of the MEs in SI units requires introducing two new *conventional* constants as ϵ_0^{v} and μ_0^{a}. We derive them by using the field energy density specified as

$$\mathcal{W}(\mathbf{r},t) = \mathcal{W}_E(\mathbf{r},t) + \mathcal{W}_H(\mathbf{r},t) \qquad \text{(A.1)}$$

where $\mathcal{W}_E$ and $\mathcal{W}_H$ are the volumetric energy densities stored in the electric and magnetic fields individually. They are

$$\mathcal{W}_E = \frac{1}{2}\underbrace{\epsilon_0}_{\lfloor \frac{F}{m} \rfloor}\underbrace{\mathcal{E}\cdot\mathcal{E}}_{\lfloor \frac{V^2}{m^2} \rfloor} \text{ and } \mathcal{W}_H = \frac{1}{2}\underbrace{\mu_0}_{\lfloor \frac{H}{m} \rfloor}\underbrace{\mathcal{H}\cdot\mathcal{H}}_{\lfloor \frac{A^2}{m^2} \rfloor}. \qquad \text{(A.2)}$$

Simplifying the dimensions of $\mathcal{W}_E$ and $\mathcal{W}_H$ results in

$$\left\lfloor \frac{F}{m} \right\rfloor \left\lfloor \frac{V^2}{m^2} \right\rfloor \equiv \left\lfloor \frac{N}{m^2} \right\rfloor \text{ and } \left\lfloor \frac{H}{m} \right\rfloor \left\lfloor \frac{A^2}{m^2} \right\rfloor \equiv \left\lfloor \frac{N}{m^2} \right\rfloor. \qquad \text{(A.3)}$$

Consider first the energy density $\mathcal{W}_E$ from (A.2) supposing that the field $\mathcal{E}$ can be scaled as $\mathcal{E} = \epsilon_0^{\mathrm{v}}\mathbb{E}$, where the scaling factor ϵ_0^{v} is unknown, as yet, but the new field, $\mathbb{E}$, has the dimension of the inverse meter, $\lfloor 1/m \rfloor$. Substituting these suppositions to the formula for $\mathcal{W}_E$ from (A.2) yields

$$\begin{aligned}\mathcal{W}_E\left\lfloor \frac{N}{m^2} \right\rfloor &= \underleftrightarrow{\epsilon_0(\epsilon_0^{\mathrm{v}})^2}\frac{1}{2}\mathbb{E}\cdot\mathbb{E}\left\lfloor \frac{1}{m^2} \right\rfloor \\ &= \underleftrightarrow{1N}\frac{1}{2}\mathbb{E}\cdot\mathbb{E}\left\lfloor \frac{1}{m^2} \right\rfloor,\end{aligned} \qquad \text{(A.4)}$$

and definition for the scaling factor, ϵ_0^{v}, follows as

$$\epsilon_0(\epsilon_0^{\mathrm{v}})^2 = 1N \quad \text{and hence, } \boxed{\epsilon_0^{\mathrm{v}} = \sqrt{1N/\epsilon_0}}. \qquad \text{(A.5)}$$

Consideration of the energy density $\mathcal{W}_H$ with the field $\mathcal{H}$ scaled as $\mathcal{H} = \mu_0^{\mathrm{a}}\mathbb{H}$ can be done analogously and results in

$$\begin{aligned}&\mathcal{W}_H\left\lfloor \frac{N}{m^2} \right\rfloor = \underleftrightarrow{\mu_0(\mu_0^{\mathrm{a}})^2}\frac{1}{2}\mathbb{H}\cdot\mathbb{H}\left\lfloor \frac{1}{m^2} \right\rfloor = \underleftrightarrow{1N}\frac{1}{2}\mathbb{H}\cdot\mathbb{H}\left\lfloor \frac{1}{m^2} \right\rfloor \\ &\mu_0(\mu_0^{\mathrm{a}})^2 = 1N \quad \text{and hence, } \boxed{\mu_0^{\mathrm{a}} = \sqrt{1N/\mu_0}}.\end{aligned}$$

APPENDIX B
CONSTITUTIVE RELATION $\mathbb{D}(\mathbb{E})$

The electric displacement vector, $\mathcal{D}$, is defined as

$$\mathcal{D} = \epsilon_0\mathcal{E} + \mathcal{P}(\mathcal{E}) \qquad \text{(B.1)}$$

where the right-hand side is alluded in formula (9a). Substituting $\mathcal{E} = \epsilon_0^{\mathrm{v}}\mathbb{E}$ to $\epsilon_0\mathcal{E}$ yields $\epsilon_0\mathcal{E} = \epsilon_0\epsilon_0^{\mathrm{v}}\mathbb{E} = \sqrt{N\epsilon_0}\mathbb{E}$. As long as the therms $\epsilon_0\mathcal{E}$ and $\mathcal{P}(\mathcal{E})$ are *summarizable* in (B.1), the vector $\mathcal{P}(\mathcal{E})$ should be scaled in the same way as $\mathcal{P}(\mathcal{E}) = \sqrt{N\epsilon_0}\mathbb{P}(\mathbb{E})$. This yields the novel format of the displacement vector, $\mathbb{D}$, as

$$\mathbb{D} = \mathbb{E} + \mathbb{P}(\mathbb{E}) \equiv \mathbb{D}(\mathbb{E}) \qquad \text{(B.2)}$$

where the vectors $\mathbb{D}$, $\mathbb{E}$, and $\mathbb{P}$ have common their dimension of $\lfloor 1/m \rfloor$. The notation $\mathbb{P}(\mathbb{E})$ implies the inducing the polarization vector, $\mathbb{P}$, by the field in a dielectric, $\mathbb{E}$, in accordance with appropriate motion equation (e.g., (14c)). The notation $\mathbb{D}(\mathbb{E})$ symbolizes the constitutive relation between $\mathbb{D}$ and $\mathbb{E}$.

Combination of equations (10b) and (10a) yields

$$\left(\partial^2/\partial t^2 + 2\Gamma\partial/\partial t + \Omega^2\right)\mathcal{P}(\mathbf{r},t) = \epsilon_0\omega_p^2\mathcal{E}(\mathbf{r},t) \qquad \text{(B.3)}$$

where $\Omega = \sqrt{\omega_r^2 - \omega_p^2/3}$. For an isotropic homogeneous dielectric, the vectors $\mathcal{E}$ and $\mathcal{P}$ are presentable as

$$\mathcal{E} = E(t)\,\vec{\mathcal{E}}(\mathbf{r}) \text{ and } \mathcal{P} = \epsilon_0 p(t)\,\vec{\mathcal{E}}(\mathbf{r}) \qquad \text{(B.4)}$$

where the amplitude $E(t)$ should be given and $p(t)$ is sought for. Substituting (B.4) to (B.3) yields

$$\left(\partial^2/\partial t^2 + 2\Gamma\partial/\partial t + \Omega^2\right)p(t) = \omega_p^2 E(t) \qquad \text{(B.5)}$$

In the *frequency domain,* the amplitudes are presentable as

$$E(t) = E(\omega)e^{i\omega t} \quad \text{and } p(\omega)e^{i\omega t} \qquad \text{(B.6)}$$

where E and p are constants, $i = \sqrt{-1}$, ω is the frequency parameter, $-\infty < \omega < \infty$, and time t varies as $-\infty < t < \infty$. Substituting (B.6) to (B.5) results in standard constitutive relation $\vec{\mathcal{D}}(\mathbf{r},\omega) = \epsilon_0\varepsilon(\omega)\,\vec{\mathcal{E}}(\mathbf{r},\omega)$ where $\varepsilon(\omega)$ is the dimensionless relative permittivity specified as

$$\varepsilon(\omega) = 1 + \omega_p^2/\left[\Omega^2 - \omega^2 + i\,2\Gamma\omega\right]. \qquad \text{(B.7)}$$

Constitutive relation (B.1) in the new format is $\vec{\mathbb{D}}(\mathbf{r},\omega) = \varepsilon(\omega)\,\vec{\mathbb{E}}(\mathbf{r},\omega)$ where $\varepsilon(\omega)$ is the same as in (B.7). This relation in the frequency domain is *noncausal* because of supposition (B.6) where $-\infty < t < \infty$.

Studying this constitutive relation in the *time domain* directly needs in supplementing the motion equation with appropriate *initial condition.* That yields a *causal solution.* One can find several examples of exact analytical causal solutions in [10].

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